LCDA moments of meson

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Outline

- Motivation and Formula
- Results for first moments
- Results for second moments
- Summary

Motivation

Light-cone distribution amplitudes (LCDAs) plays an important role in the hardon hard exclusive reactions.

There are two ways to extract information of LCDA on lattice:

1. Calculate the quasi-DA on the lattice and match it to the LCDA through LaMET.
2. Calculate moments using local operator on lattice and reconstruct the LCDA.

$$\xi_2^\pi = 0.300(41),\ \xi_2^K = 0.258(32)$$

Gegenbauer moments	a_1	a_2	a_3	a_4
π	—	0.258(70)(52)	—	0.122(46)(31)
K	-0.108(14)(51)	0.170(14)(44)	-0.043(06)(22)	0.073(08)(21)

Results from J. Hua et al. (Lattice Parton Collaboration), Phys. Rev. Lett. 129, 132001 (2022).

M	RI′	order	$\langle\xi^2\rangle_M$	a_2^M
π	SMOM	N^{3}LO	$0.240^{+6}_{-6}(2)_r(3)_a(2)_m$	$0.116^{+16}_{-17}(4)_r(9)_a(5)_m$
π	SMOM	NNLO	$0.234^{+6}_{-6}(4)_r(4)_a(2)_m$	$0.101^{+17}_{-17}(12)_r(10)_a(5)_m$
π	SMOM	NLO	$0.227^{+6}_{-6}(5)_r(5)_a(2)_m$	$0.078^{+18}_{-19}(16)_r(13)_a(5)_m$

Results from Bali, G.S., Braun, V.M., Bürger, S. *et al*. *J. High Energ. Phys.* **2019**, 65 (2019).

Formula

LCDA definition: $\langle\Omega|\bar{q}_2(z_2 n_\mu) e^{i\int_{z_2}^{z_1} A_\mu(x n_\mu)dx} \not{n}\gamma_5 q_1(z_1 n_\mu)|M(\vec{p})\rangle = i f_M p\cdot n \int_{-1}^{1} dx\, e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_M(x,\mu^2)$

Moments definition

$$\xi = x - (1-x) = 2x - 1$$

$$\langle\xi^n\rangle_M(\mu^2) = \int_{-1}^{1} dx(2x-1)^n \phi_M(x,\mu^2)$$

The moments could be extracted from hadron matrix elements of twist-2 operators:

$$M^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{q}_2(x)\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}}\gamma_{\rho)}\gamma_5 q_1(x)$$

For first moments:

$$\mathcal{O}^-_{\rho\mu}(x) = \bar{q}_2(x)\left[\overleftarrow{D}_{(\mu} - \overrightarrow{D}_{(\mu}\right]\gamma_{\rho)}\gamma_5 q_1(x)\,, \qquad \langle\Omega|\mathcal{O}^-_{\rho\mu}|M(\vec{p})\rangle = i f_M p_{(\rho}p_{\mu}\langle\xi\rangle$$

For second moments:

$$\mathcal{O}^\pm_{\rho\mu\nu}(x) = \bar{q}_2(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} \pm 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 q_1(x)\,, \qquad \langle\Omega|\mathcal{O}^-_{\rho\mu\nu}|M(\vec{p})\rangle = i f_M p_{(\rho}p_{\mu}p_{\nu)}\langle\xi^2\rangle$$

$$\langle\Omega|\mathcal{O}^+_{\rho\mu\nu}|M(\vec{p})\rangle = i f_M p_{(\rho}p_{\mu}p_{\nu)}\langle 1^2\rangle$$

The $\mathcal{O}^-_{\rho\mu\nu}$ and $\mathcal{O}^+_{\rho\mu\nu}$ would mixing under the renormalization.

$$Z_q^{-1}\sum_{m''}^{M} Z_{mm''}\sum_{i}^{d}\mathrm{Tr}\left[\Lambda^{(i)}_{m''}(p_1,p_2)\Lambda^{(i)}_{m',\mathrm{tree}}(p_1,p_2)\right] = \sum_{i}^{d}\mathrm{Tr}\left[\Lambda^{(i)}_{m,\mathrm{tree}}(p_1,p_2)\Lambda^{(i)}_{m',\mathrm{tree}}(p_1,p_2)\right]\Big|_{p_1^2=p_2^2=(p_1-p_2)^2}$$

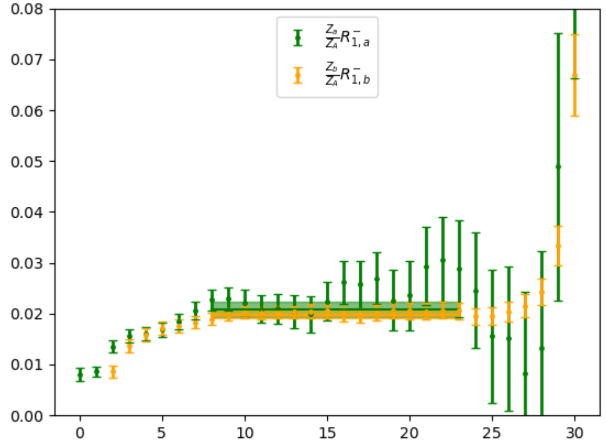
Pseudo scalar meson

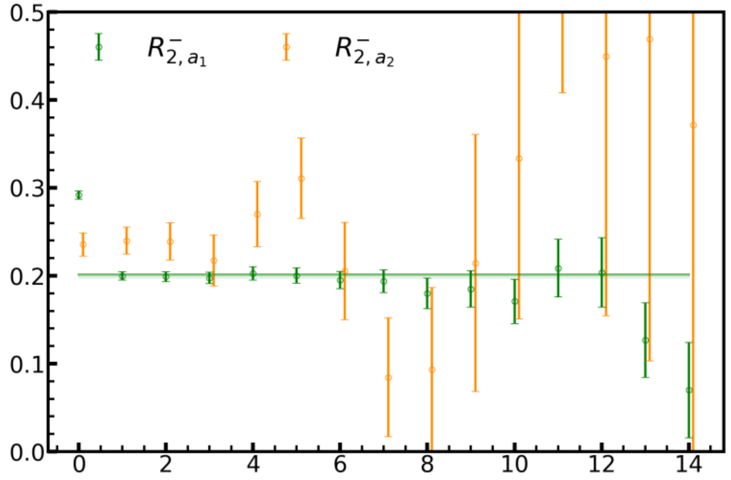
For first moments:

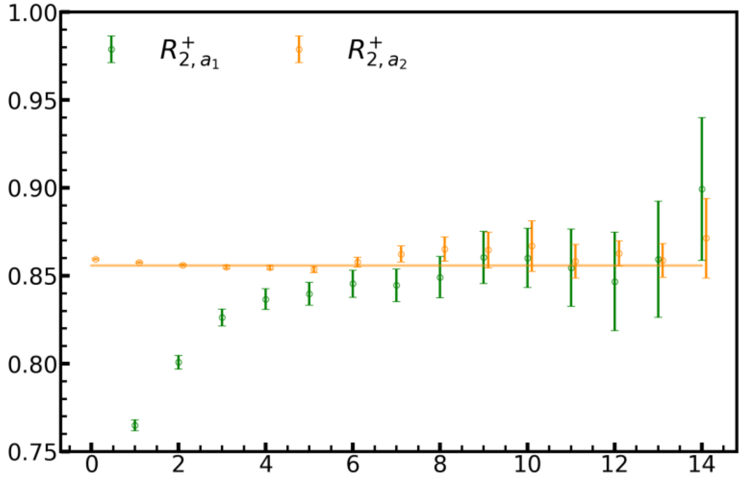
$$R^{-}_{1,a} = -\frac{i}{3}\sum_{i=1}^{3}\frac{1}{p_i}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{-}_{4i}(\vec{x},t)P(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle A_4(\vec{x},t)P(\vec{0},0)\rangle} \qquad R^{-}_{1,b} = -\frac{4E}{3E^2+p^2}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{-}_{44}(\vec{x},t)P(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle A_4(\vec{x},t)P(\vec{0},0)\rangle}$$

For second moments:

$$R^{\pm}_{2,a_1} = -\frac{1}{3}\sum_{i\neq j}^{3}\frac{1}{p_i p_j}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{\pm}_{4ij}(\vec{x},t)P(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle A_4(\vec{x},t)P(\vec{0},0)\rangle} \qquad R^{\pm}_{2,a_2} = -\frac{1}{3}\sum_{i=1}^{3}\frac{p_i}{p_1p_2p_3}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{\pm}_{123}(\vec{x},t)P(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle A_i(\vec{x},t)P(\vec{0},0)\rangle}$$







a=0.12fm using gaussian point source

Vector meson

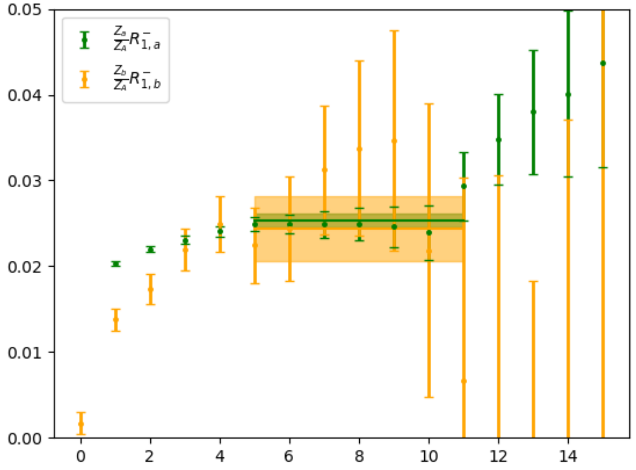
For first moments:

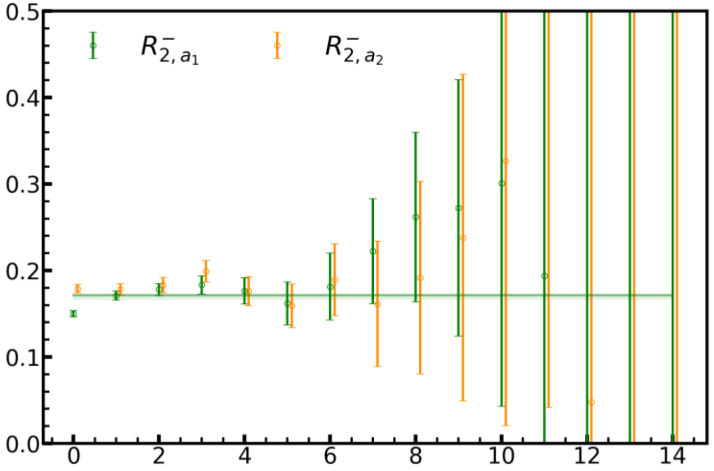
$$R^{-}_{1,a} = \frac{2}{3}\sum_{i=1}^{3}\frac{1}{E}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{-}_{4i}(\vec{x},t)V_i(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle V_i(\vec{x},t)V_i(\vec{0},0)\rangle} \quad R^{-}_{1,b} = -\frac{4E}{3E^2+p^2}\sum_{i=1}^{3}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{-}_{44}(\vec{x},t)V_i(\vec{0},0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle V_4(\vec{p},t)V_i(\vec{0},0)\rangle}$$

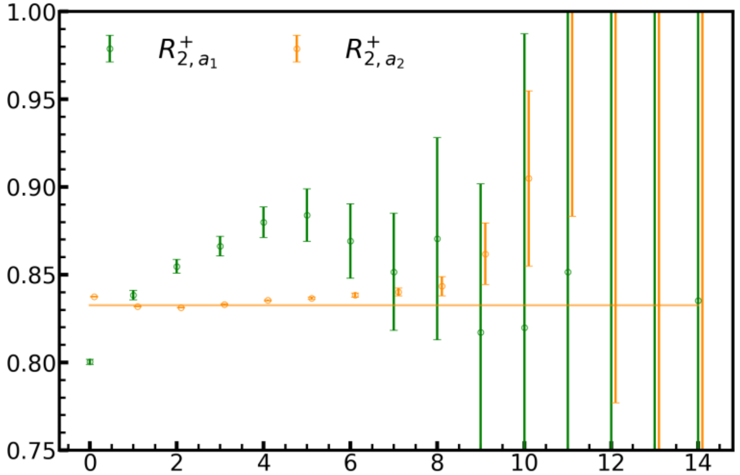
$$\langle V_\rho V_\sigma\rangle \propto g_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_V^2}$$
$$\langle O^{-}_{\mu\rho} V_\sigma\rangle \propto p_{(\mu}\left(g_{\rho)\sigma} - \frac{p_{\rho)}p_\sigma}{M_V^2}\right)$$
$$\langle O^{-}_{\mu\nu\rho} V_\sigma\rangle \propto p_{(\mu}p_\nu\left(g_{\rho)\sigma} - \frac{p_{\rho)}p_\sigma}{M_V^2}\right)$$

For second moments:

$$R^{\pm}_{2,a_1} = -\frac{1}{3}\sum_{i\neq j}^{3}\frac{1}{p_i p_j}\frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{\pm}_{4ij}(\vec{x},t)V_k(0,0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle V_4(\vec{x},t)V_k(0,0)\rangle} \qquad R^{\pm}_{2,a_2} = -\frac{1}{3}\sum_{i=1}^{3}\frac{p_i}{p_1p_2p_3}\frac{\sum_{j=1}^{3} p_j\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \mathcal{O}^{\pm}_{123}(\vec{x},t)V_j(\vec{0},0)\rangle}{\sum_{j=1}^{3} p_j\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle V_i(\vec{x},t)V_j(\vec{0},0)\rangle}$$







a=0.12fm using gaussian point source

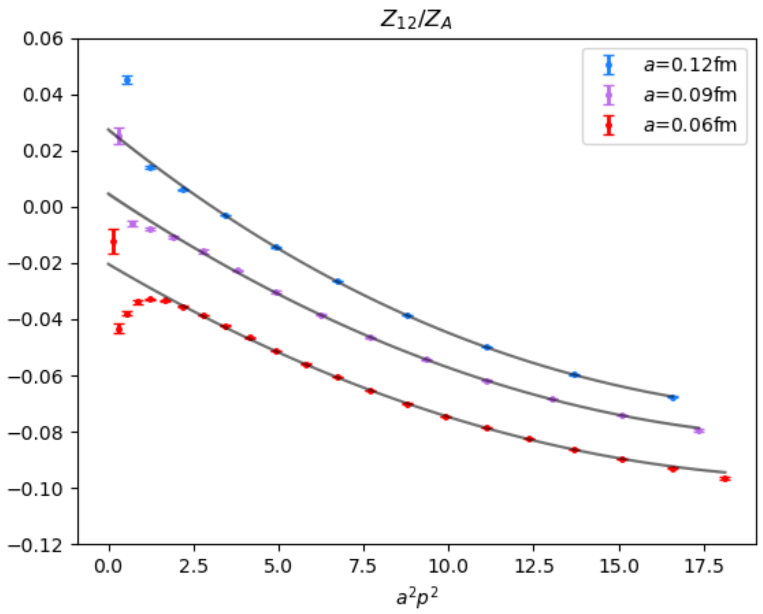
Renormalization

The $\mathcal{O}^{-}_{\rho\mu\nu}$ and $\mathcal{O}^{+}_{\rho\mu\nu}$ would mixing under the renormalization.

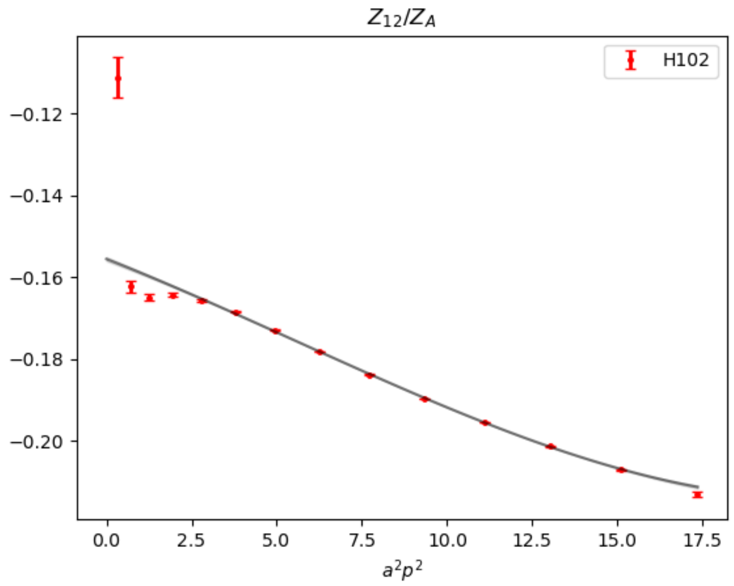
$$Z_q^{-1}\sum_{m''}^{M} Z_{mm''}\sum_{i}^{d}\mathrm{Tr}\left[\Lambda^{(i)}_{m''}(p_1,p_2)\Lambda^{(i)}_{m',\mathrm{tree}}(p_1,p_2)\right] = \sum_{i}^{d}\mathrm{Tr}\left[\Lambda^{(i)}_{m,\mathrm{tree}}(p_1,p_2)\Lambda^{(i)}_{m',\mathrm{tree}}(p_1,p_2)\right]\Big|_{p_1^2=p_2^2=(p_1-p_2)^2}$$

$$\begin{cases} {\mathcal{O}^{-}_{\rho\mu\nu}}^{(R)} = Z_{11}{\mathcal{O}^{-}_{\rho\mu\nu}}^{(B)} + Z_{12}{\mathcal{O}^{+}_{\rho\mu\nu}}^{(B)} \\ {\mathcal{O}^{+}_{\rho\mu\nu}}^{(R)} = Z_{22}{\mathcal{O}^{+}_{\rho\mu\nu}}^{(B)} \end{cases}$$

using 1step-HYP smearing clover action



using normal clover action



smearing can help reduce the mixing effect.

Lattice set up

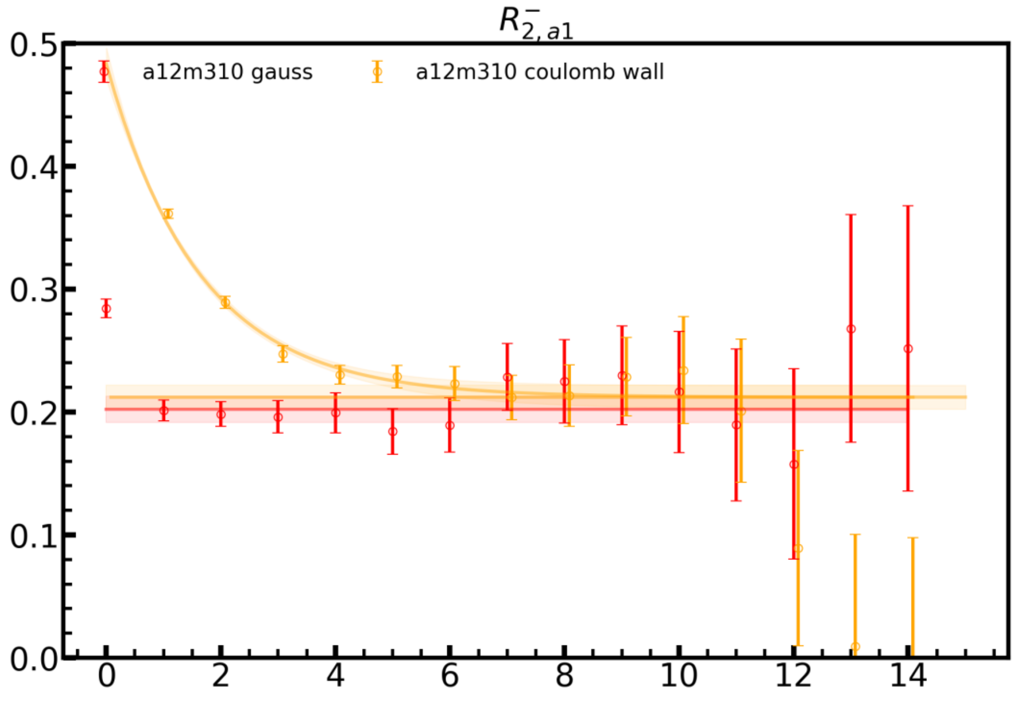
Fermion action use the 1step-HYP smearing clover action on MILC ensembles

	a12m310		a12m130	a09m310		a06m310	
L^3xT	24^3x64		48^3x64	32^3x96		48^3x144	
Lattice spacing	~ 0.12fm		~ 0.12fm	~ 0.09fm		~ 0.06fm	
m_π	~310MeV		~130MeV	~310MeV		~310MeV	
c_{sw}	1.0508		1.0508	1.0424		1.03493	
am_l	-0.0695	-0.0785	-0.0785	-0.0514	-0.0580	-0.0398	-0.0439
am_s	-0.0312	-0.0191	-0.0191	-0.0227	-0.0174	-0.0242	-0.0191

Hardon matrix elements: Coulomb wall source and gaussian point source.

Renormalization: Volume source with $p = \frac{2\pi}{n_s}(n, n, 0, 0)$ and $p = \frac{2\pi}{n_s}(n, 0, n, 0)$ (n = 2, 3 … $\frac{n_s}{2} - 1$)

Gaussian vs Coulomb wall

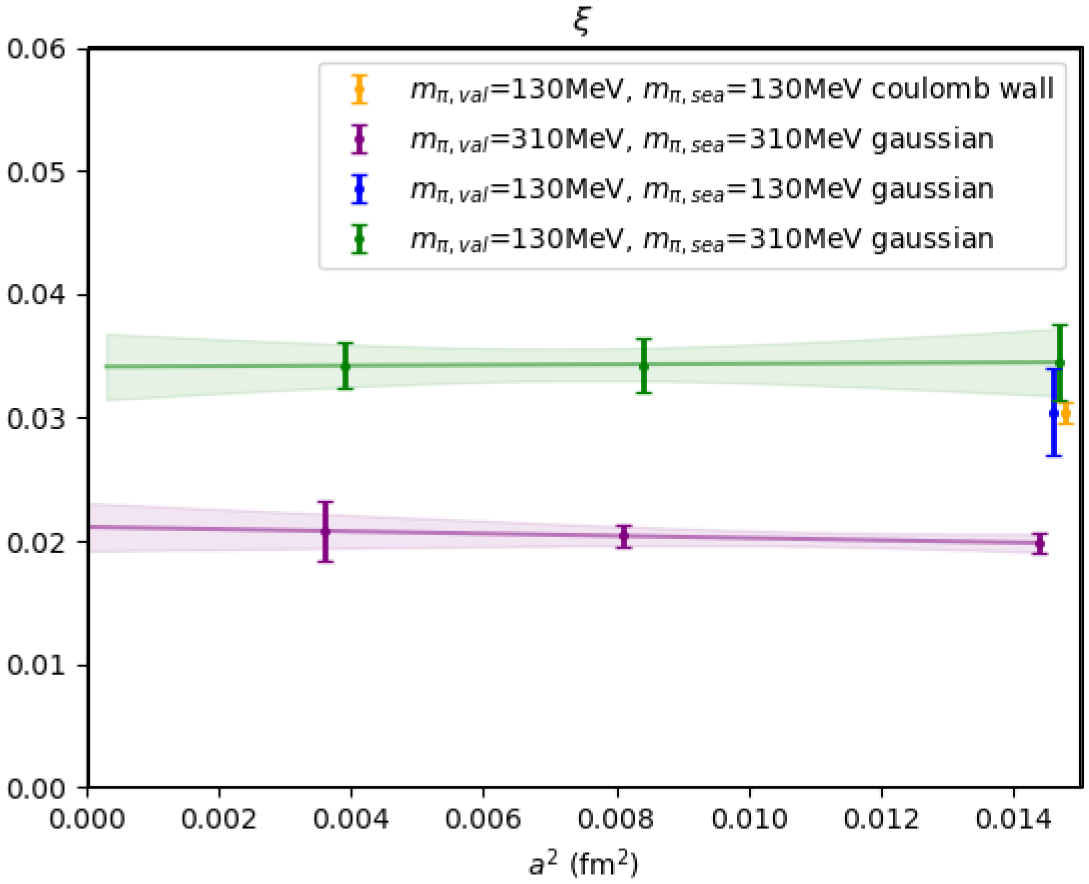


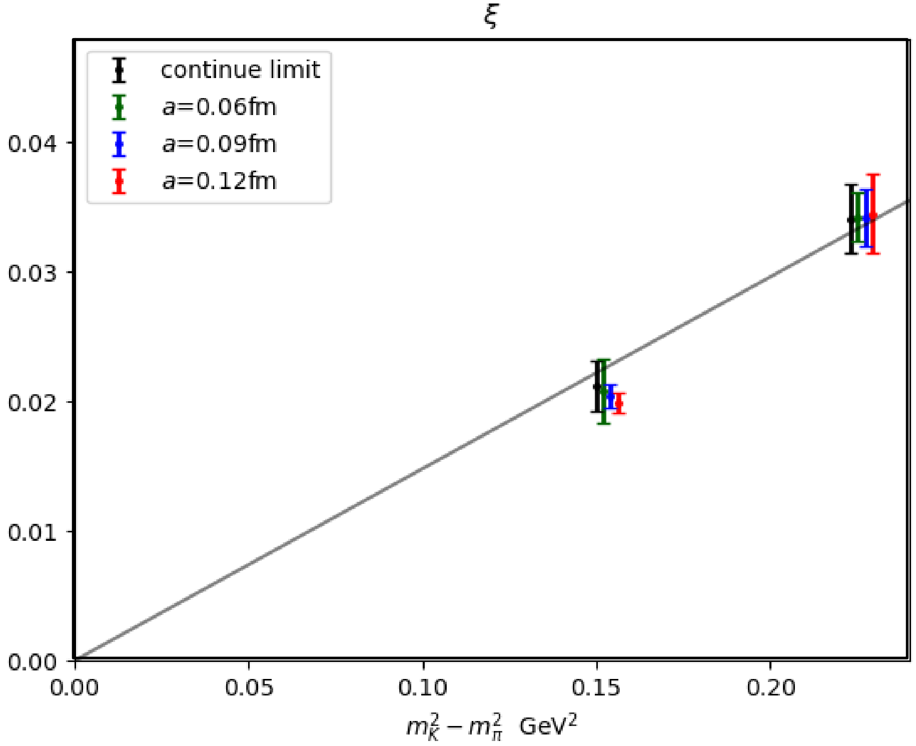
Gaussian use constant fit
Coulomb wall use two-state fit

The Coulomb wall has a smaller error bar

The Gaussian point source has a smaller excited state effect.

K First Moments



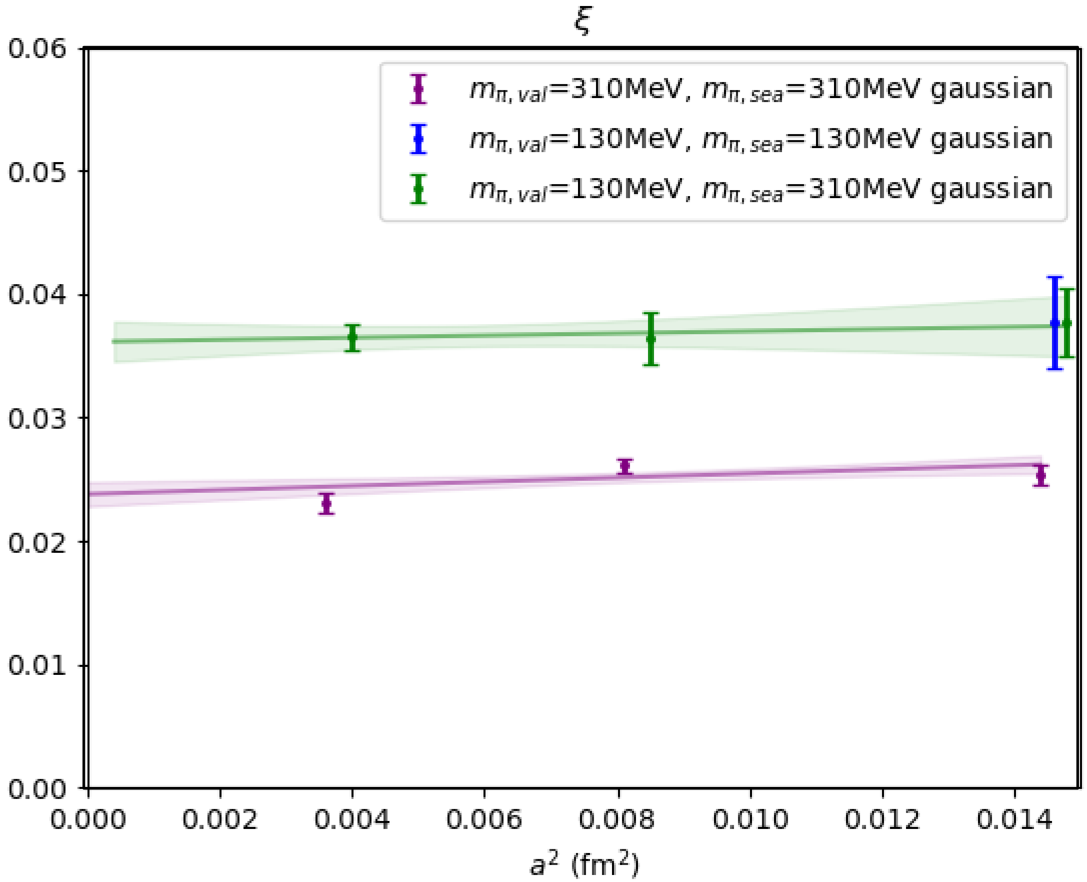


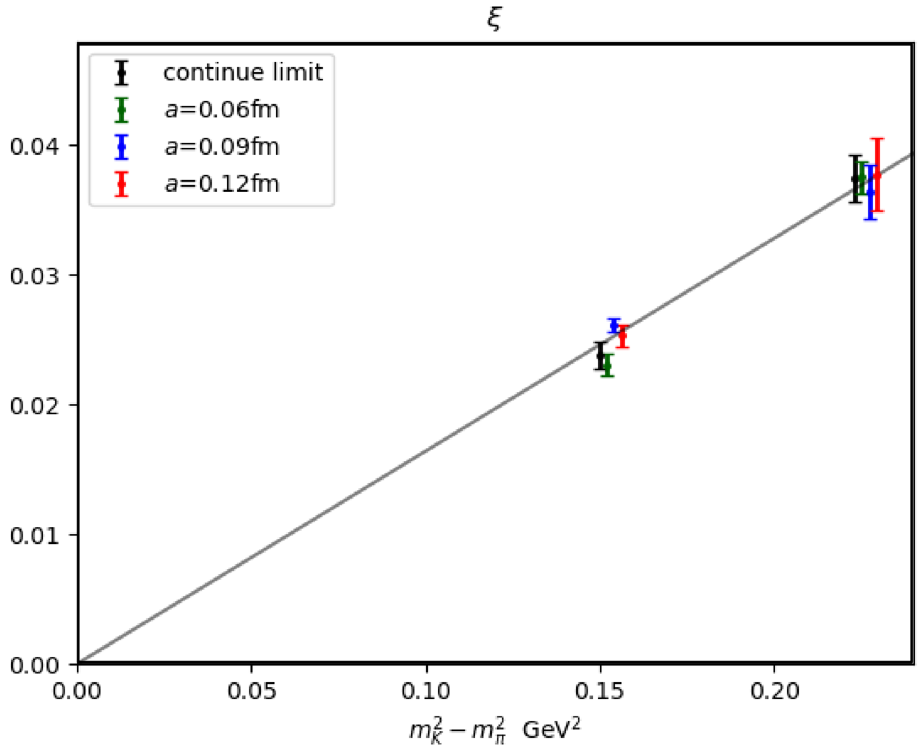
our preliminary results: $\langle \xi \rangle = 0.0341(27)$

M	RI′	order	$\langle \xi^1 \rangle_M$	a_1^M
K	SMOM	NNLO	$0.0320^{+11}_{-12}(3)_r(13)_a(11)_m$	$0.0533^{+18}_{-19}(6)_r(22)_a(18)_m$
K	SMOM	NLO	$0.0327^{+11}_{-12}(6)_r(14)_a(11)_m$	$0.0545^{+18}_{-20}(9)_r(23)_a(18)_m$

Result from Bali, G.S., Braun, V.M., Bürger, S. *et al*. *J. High Energ. Phys.* **2019**, 65 (2019).

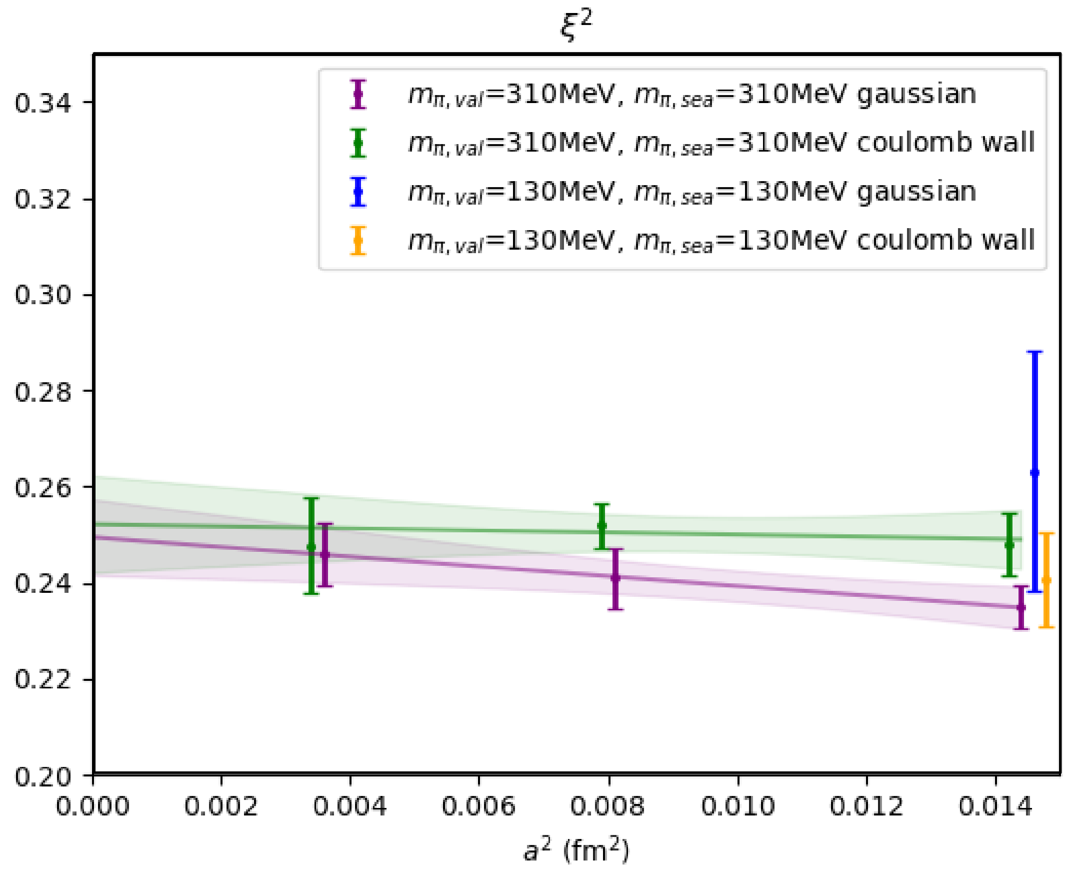
K^* First Moments

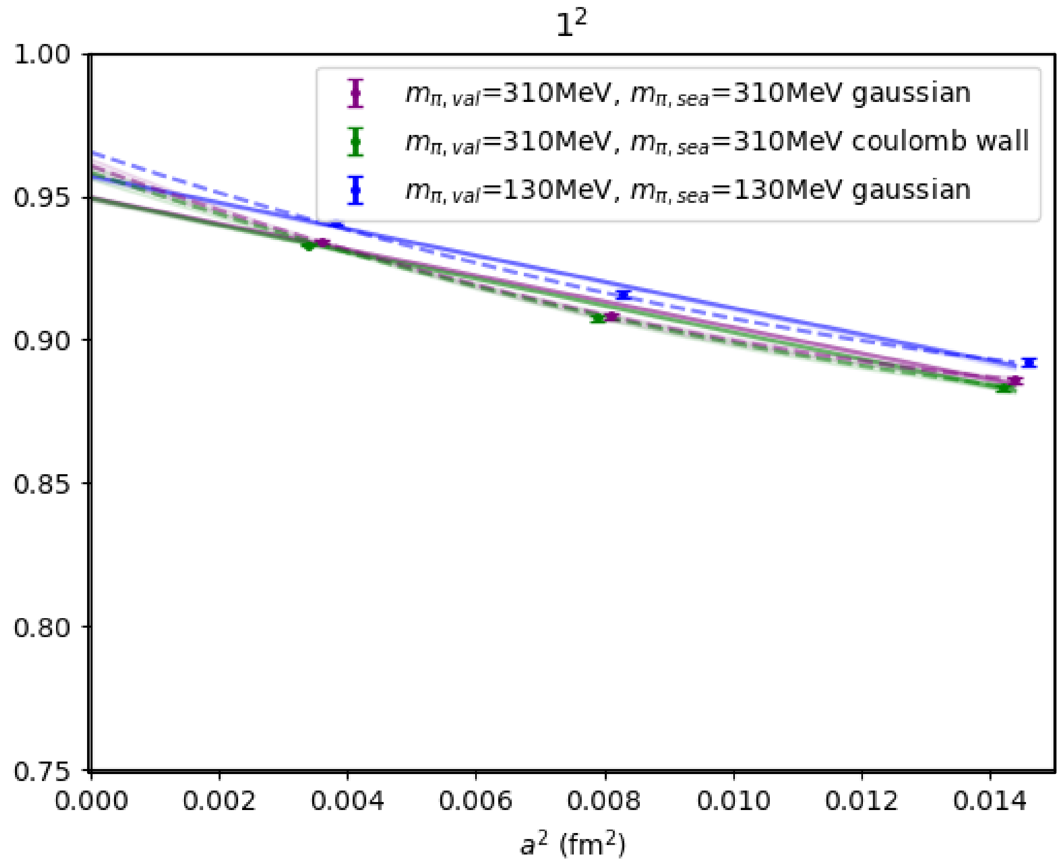




our preliminary results: $\langle \xi \rangle = 0.0374(19)$

π Second Moments



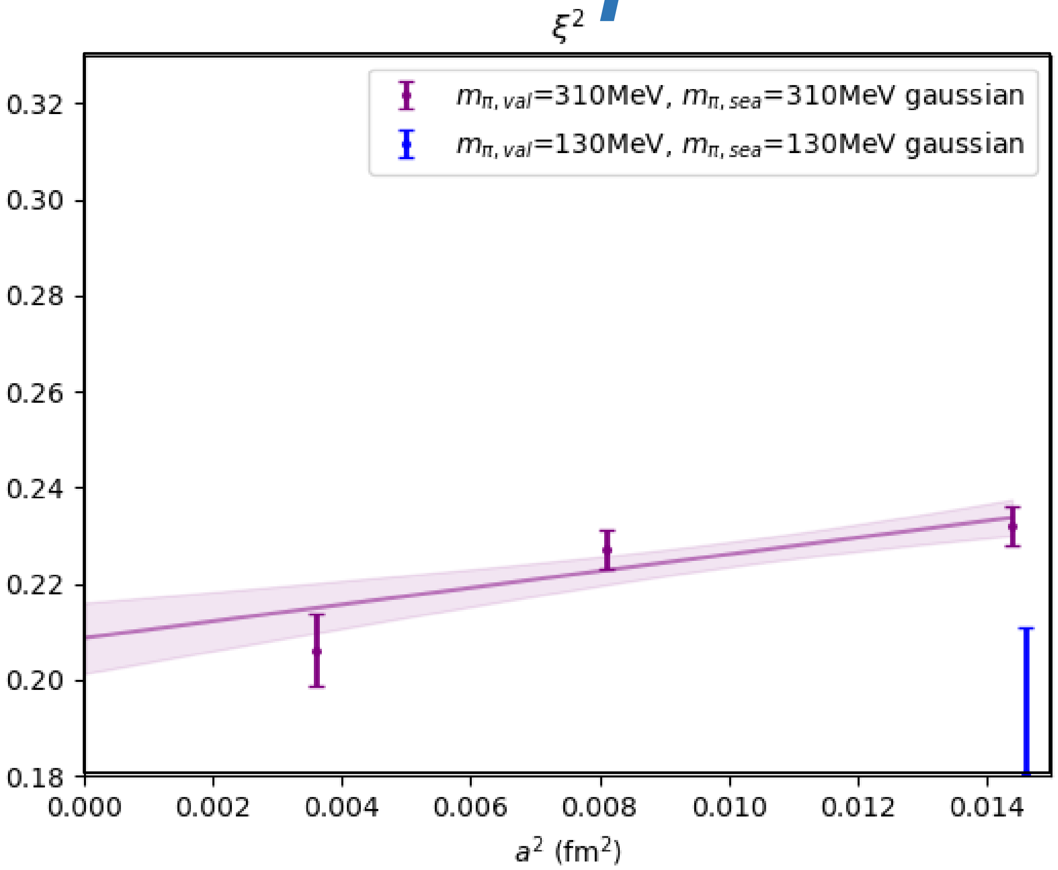


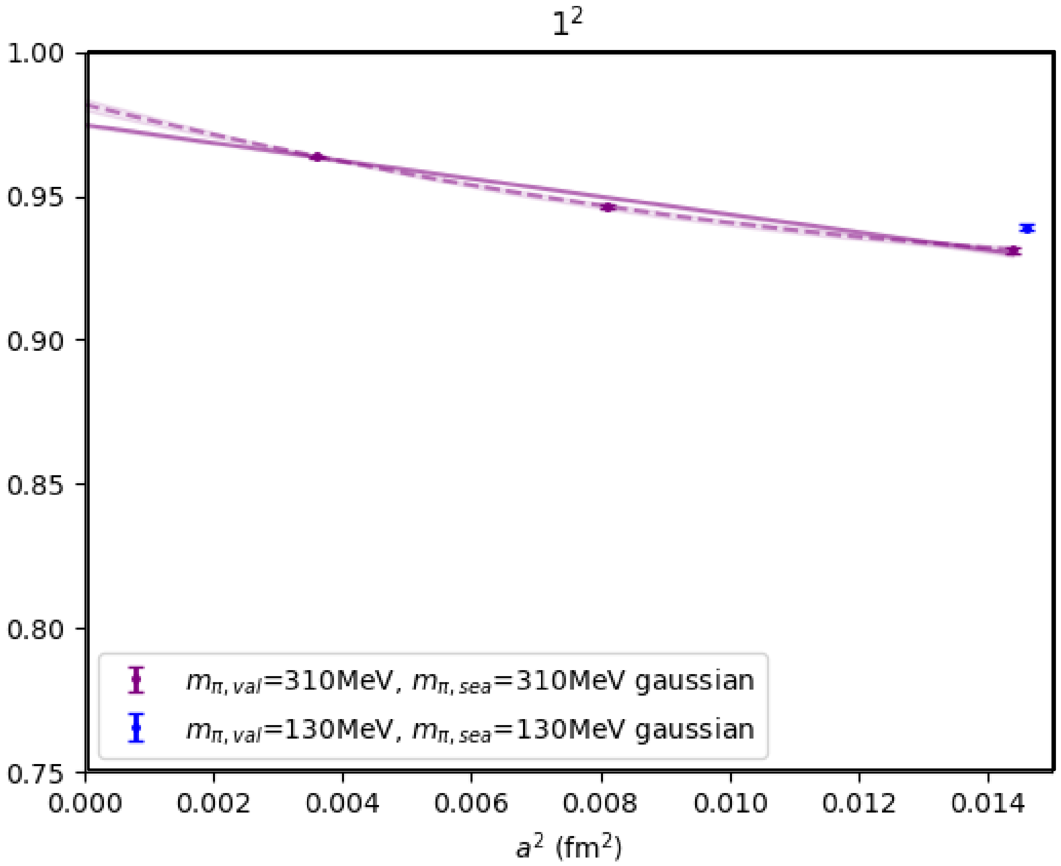
our preliminary results: $\langle \xi^2 \rangle = 0.2494(79)$

M	RI$'$	order	$\langle \xi^2 \rangle_M$
π	SMOM	N^{3}LO	$0.240^{+6}_{-6}(2)_r(3)_a(2)_m$
π	SMOM	NNLO	$0.234^{+6}_{-6}(4)_r(4)_a(2)_m$
π	SMOM	NLO	$0.227^{+6}_{-6}(5)_r(5)_a(2)_m$

Result from Bali, G.S., Braun, V.M., Bürger, S. *et al*. *J. High Energ. Phys.* **2019**, 65 (2019).

ϕ Second Moments





our preliminary results: $\langle \xi^2 \rangle = 0.2087(73)$

Preliminary Results

	a12m310	a09m310	a06m310	continue
π	0.2349(45)	0.2409(61)	0.2460(66)	0.2494(79)
K	0.2289(31)	0.2342(42)	0.2283(61)	0.2330(66)
	a12m130			
π	0.263(25)			
K	0.249(14)			

$$\xi_2^{\pi} = 0.300(41),\ \xi_2^{K} = 0.258(32)$$

Results from J. Hua et al. (Lattice Parton Collaboration), Phys. Rev. Lett. 129, 132001 (2022).

	a12m310	a09m310	a06m310	continue
ρ	0.2373(55)	0.2295(53)	0.2078(98)	0.208(10)
K^*	0.2322(46)	0.2292(46)	0.2050(90)	0.2106(88)
ϕ	0.2320(39)	0.2271(39)	0.2062(73)	0.2087(73)
	a12m130			
ρ	0.193(70)			
K^*	0.192(51)			
ϕ	0.181(30)			

Except the physical point, the mass dependence of second moments are very small

Summary

- Compute the first moments for the K and the second moments for the K and π. it is consistent with the previous calculation of RQCD collaboration.

- Compute the first moments for the K^* and the second moments for the ρ, K^* and ϕ.