Systematic Uncertainties from Gribov Copies in CG-Fixed Correlation Functions

Based on arXiv:2408.05910

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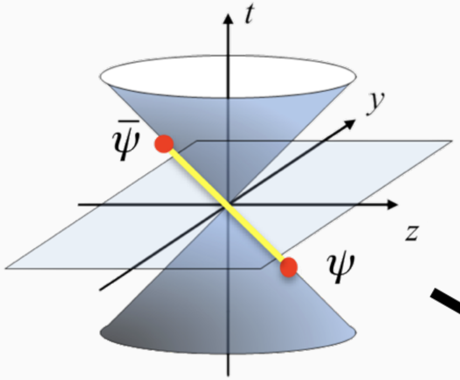
Contents

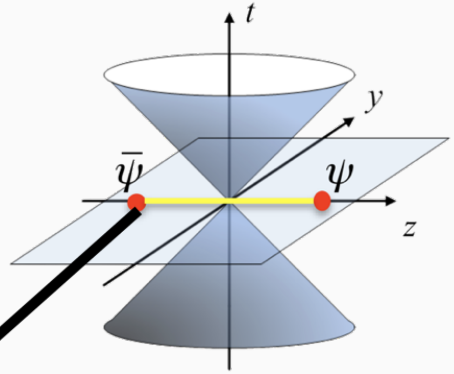
Background

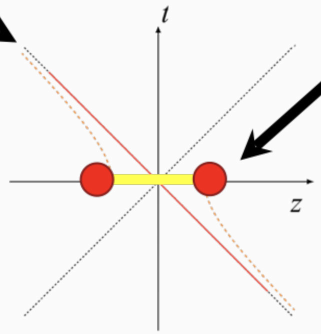
Large-Momentum Effective Theory(LaMET)

First-principle calculations of the x-dependence of parton distribution functions



X. Ji, Phys.Rev.Lett. 110, 262002 (2013)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)





Light-cone distribution:

Separated on the time axis;

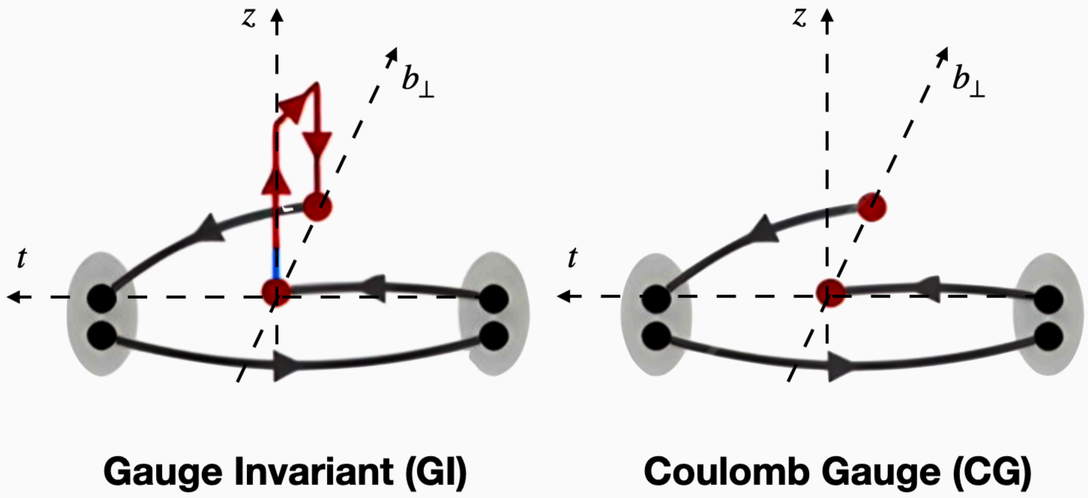
Cannot be calculated on the lattice

Quasi distribution:

Equal time in large-P hadron state;

Directly calculable on the lattice

Quasi-distributions in CG without Wilson Line



Yong's talk yesterday

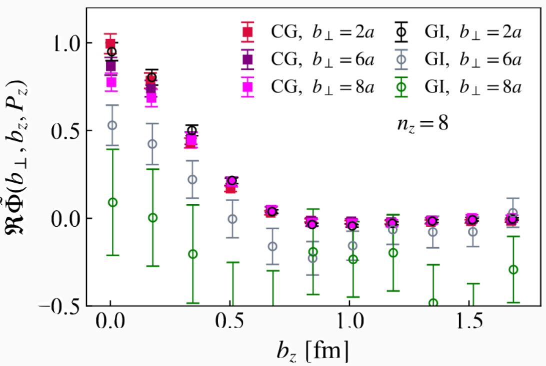
The quasi-TMD matrix elements of the pion under CG are defined as

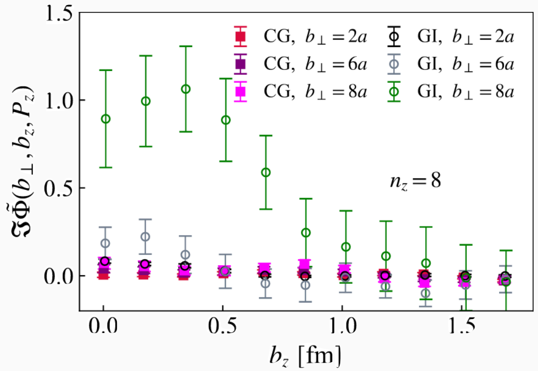
$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t}\langle P \,|\, \bar{\psi}(z)\gamma^t\psi(0)\,|_{\vec{\nabla}\cdot\vec{A}=0}\,|\, P\rangle$$

Y. Zhao, 2311.01391

Quasi-TMD in Coulomb Gauge without Wilson Line

Compared with the GI method, CG method has much better signal, especially for TMDs.

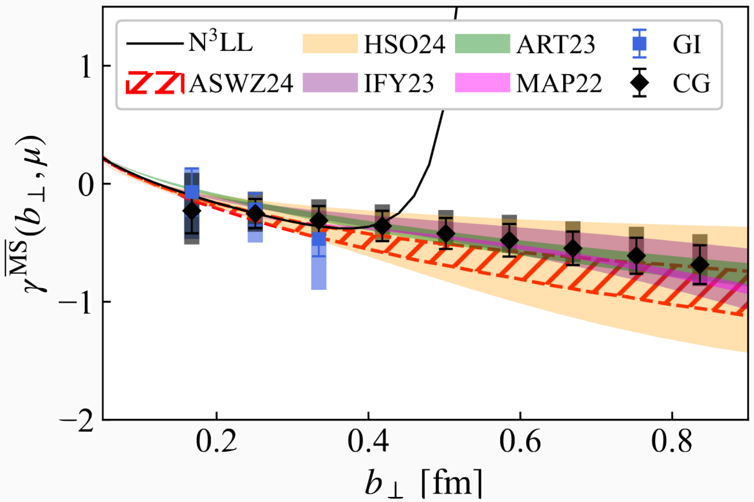




Pion Quasi-TMDWF

Artur's talk yesterday

Xiang's talk yesterday

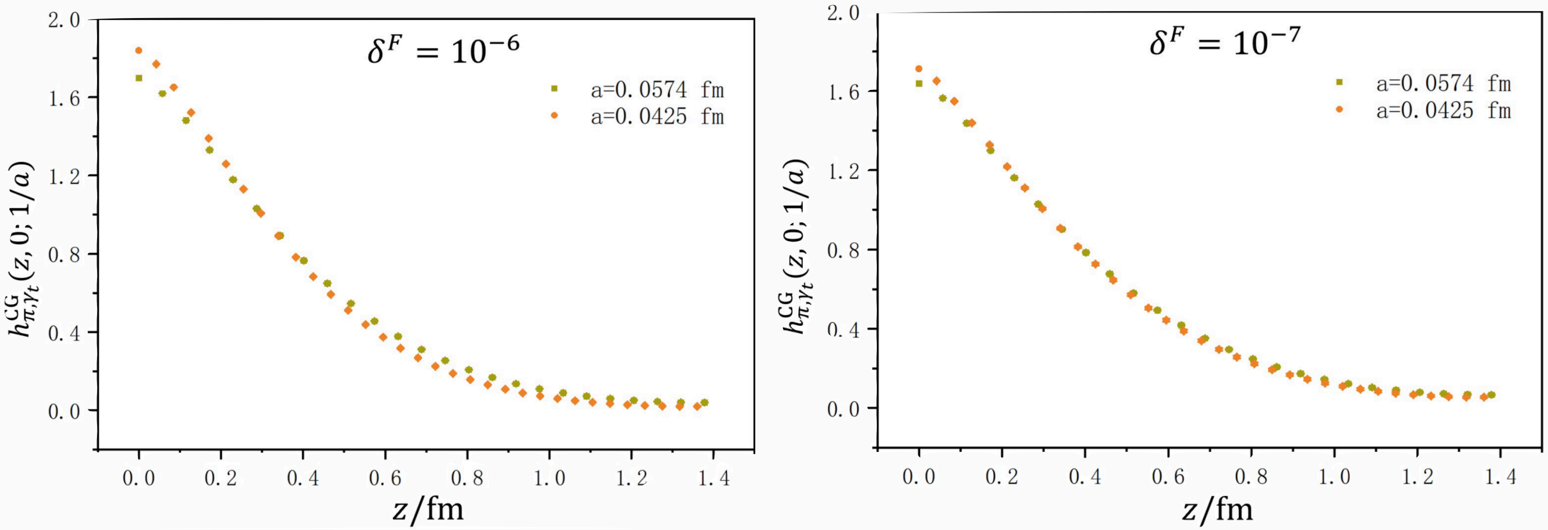


Collins-Soper kernel

D. Bollweg, et al., Phys.Lett.B 852 (2024)

Dependence on the Gauge Fixing Precision

Pion PDF using Coulomb gauge method depends on the gauge fixing precision.

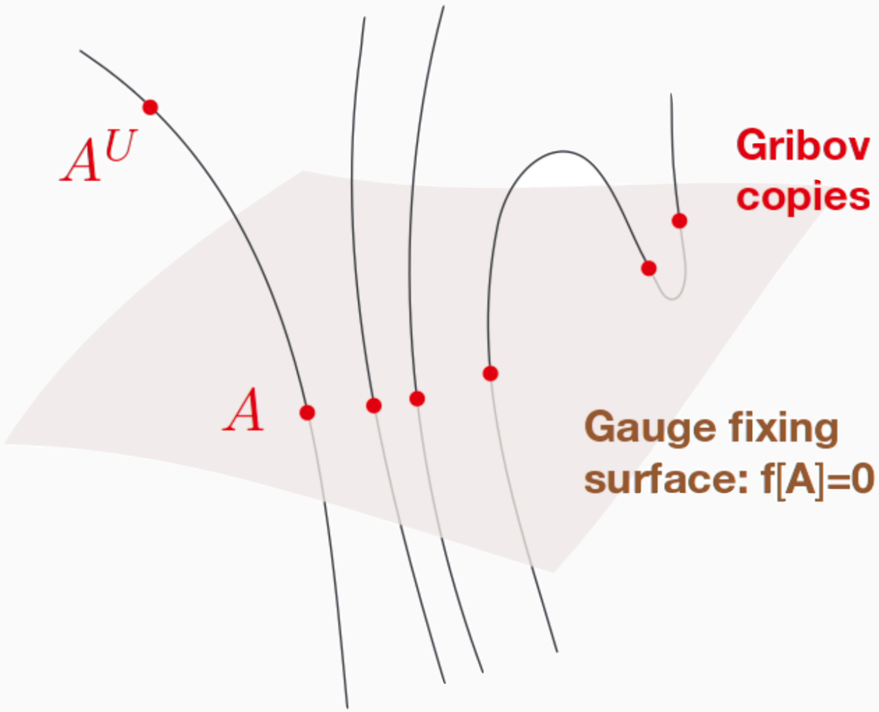


Yi-Bo's talk

K. Zhang, et al.(LPC), 2405.14097

Gribov Copies

- The gauge fixing condition may have many solutions in Lattice QCD.



Ph. D. Thesis of Diego Fiorentini

Faddeev-Popov operator

- The existence of Gribov copies is related to the zero mode of the Faddeev-Popov operator

Gauge condition: $\mathscr{F} = \vec{\nabla} \cdot \vec{A} = 0$ Infinitesimal transformation: $\delta \vec{A} = -\vec{D}\omega$

Variation: $\delta \mathscr{F} = \vec{\nabla} \cdot \delta \vec{A} = -\vec{\nabla} \cdot (\vec{D}\omega)$

Faddeev-Popov: $\mathscr{M} \equiv \dfrac{\delta \mathscr{F}}{\delta \omega} = -\vec{\nabla} \cdot \vec{D}$ For QED, there is no Gribov copy in Landau gauge.

if $\mathscr{M}\theta = 0$, then $\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot (\vec{A} - \vec{D}\theta) \equiv \vec{\nabla} \cdot \vec{A}' = 0$

$* \, D_{\mu ab}\omega_b = \partial_\mu \omega_a - g f^{cab} A^c_\mu \omega_b$

Methodology

Gauge Fixing in Lattice QCD

Continuous Theory

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\begin{aligned}\delta F_{\mathrm{CG}}[A,\Omega] &= -\sum_{\mu=1}^{3}\int d^4x (D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega} \\ &= -\sum_{\mu=1}^{3}\int d^4x(\partial_\mu\theta_a - g f^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega} \\ &= \sum_{\mu=1}^{3}\int d^4x\theta_a(\partial_\mu A^{\mu a}_{\Omega})\end{aligned}$$

$* A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$

Lattice Theory

$$F_{\mathrm{CG}}[U,\Omega] \equiv -\Re\left[\mathrm{Tr}\sum_x\sum_{\mu=1}^{3}\Omega^\dagger(x+\hat{\mu})U_\mu(x)\Omega(x)\right]$$

Find stationary points of the functional value.

Criteria of Gauge Fixing

- Variation of the functional

$$\delta F / F < 10^{-8}$$

- Residual gradient of the functional

$$\theta^G \equiv \frac{1}{V} \sum_x \theta^G(x) \equiv \frac{1}{V} \sum_x \mathrm{Tr}\left[\Delta^G(x) \left(\Delta^G\right)^{\dagger}(x)\right]$$

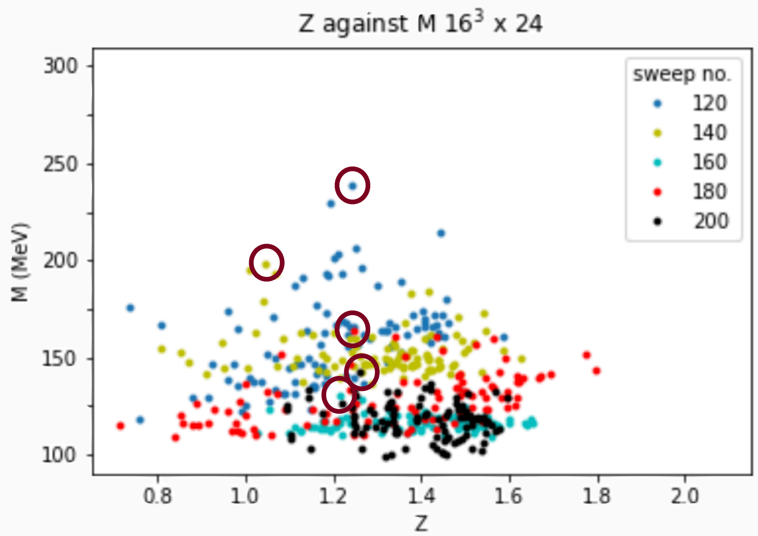
$$* \; \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x - \hat{\mu})\right)$$

Different Gribov copies can be distinguished by the difference of functional values ΔF.

Two kinds of impact from Gribov Copies

- Lattice Gribov noise: not separable from the statistical uncertainty; related to the distribution across Gribov copies.

- Measurement distortion: systematic uncertainty; related to the bias of strategies to choose representative from copies.



G. Kalusche, et al., 2405.17301

Strategies to Choose Representative in Gribov Copies

- **Mother-daughter method**: do a random gauge transformation before gauge fixing to get different copies.

- "First it": choose the first instance of gauge fixing;

- "Smallest f": choose the instance with the smallest functional value among all instances; (Fundamental Modular Region)

N. Vandersickel, et al., Phys.Rept. 520 (2012)

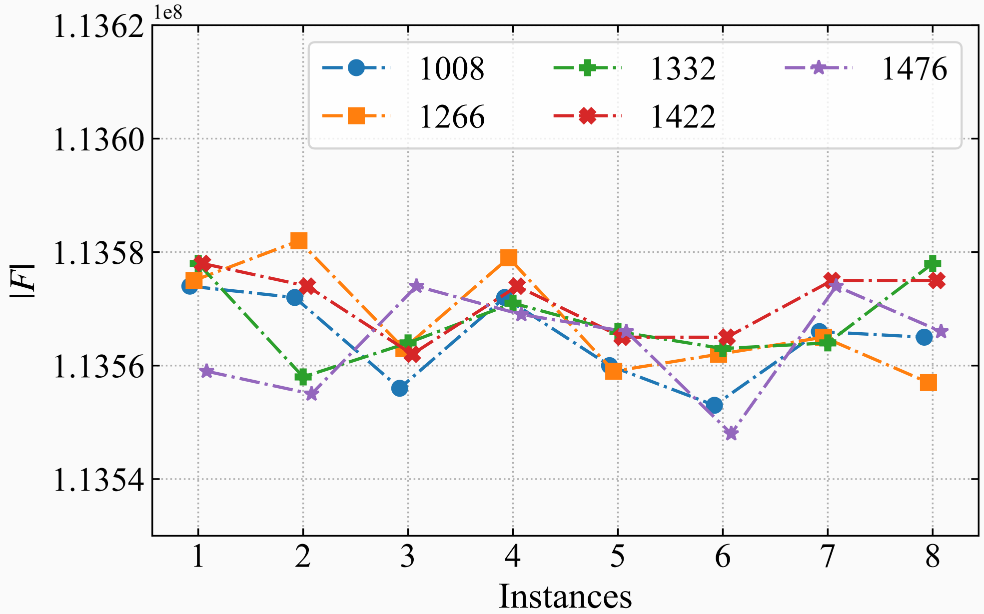
Numerical Results

Functional Values of Gribov Copies

Lattice setup: 2+1 flavor HISQ ensemble by HotQCD

a	$L_s^3 \times L_t$	$m_q a$	$m_\pi L_t$	#Cfgs	(#ex, #sl)	Inv precision
0.06 fm	$48^3 \times 64$	-0.0388	5.85	100	(1, 8)	ex: 10^{-10}; sl: 10^{-4}

We have 100 configurations, do the Coulomb gauge fixing to get 8 instances on each configuration.



Quark Propagator under the Coulomb Gauge

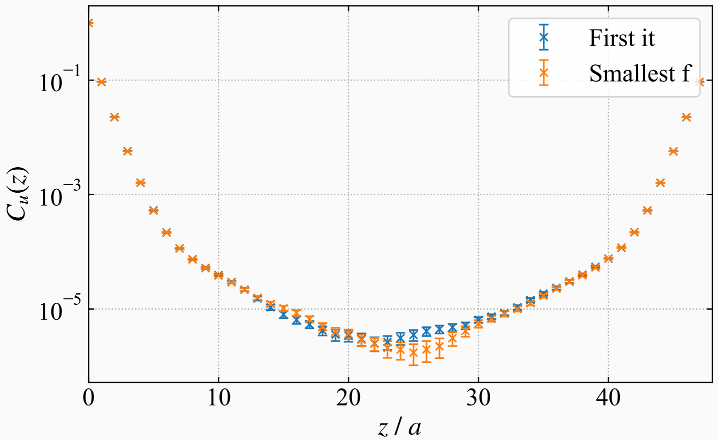
- Quark Propagator:

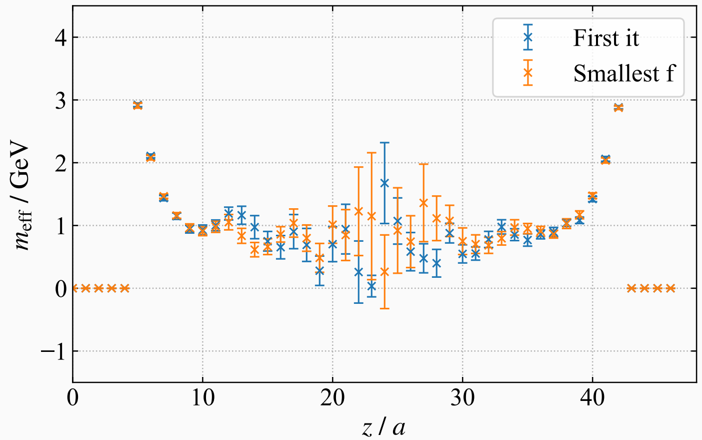
$$C_u(z) = \langle \mathrm{Tr}[u(z)\bar{u}(0)] \rangle$$

- Effective Mass:

$$\frac{C_u(z)}{C_u(z+1)} = \frac{\cosh(m_{\mathrm{eff}} \cdot (z - L_s/2))}{\cosh(m_{\mathrm{eff}} \cdot (z + 1 - L_s/2))}$$

Both 2pt and meff show a good consistency between two strategies.





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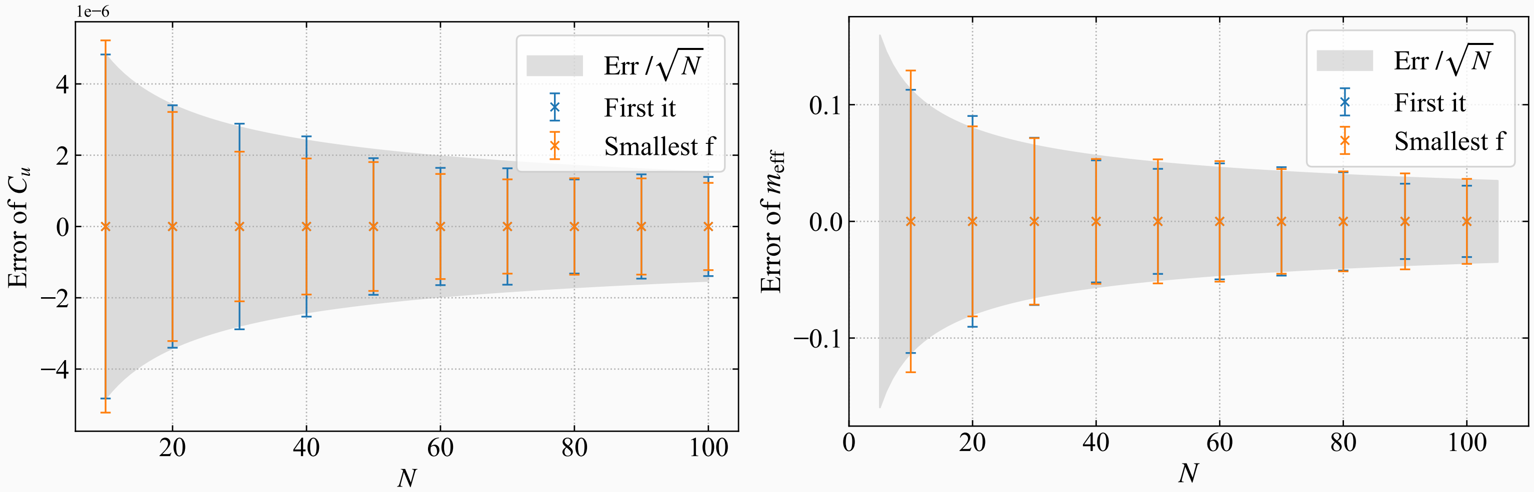
The behavior of error is consistent with $1/\sqrt{N}$ when varying the number of configurations N.



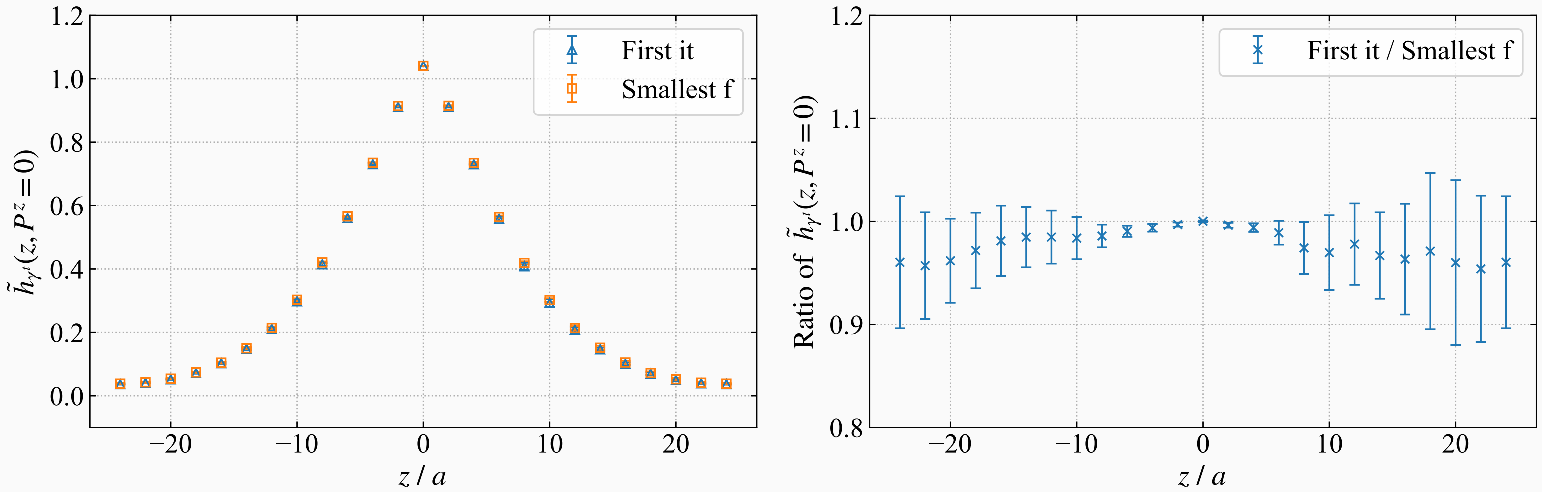
$z = 10\ a$

Quasi-distribution under the Coulomb Gauge

- Quasi-distribution:

$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t}\langle \overrightarrow{P} = \vec{0} \,|\, \bar{\psi}(z)\gamma^t\psi(0) \,|_{\vec{\nabla}\cdot\vec{A}=0} \,|\, \overrightarrow{P} = \vec{0}\rangle$$

The conclusion holds for both collinear and TMD case because of 3D rotational symmetry.



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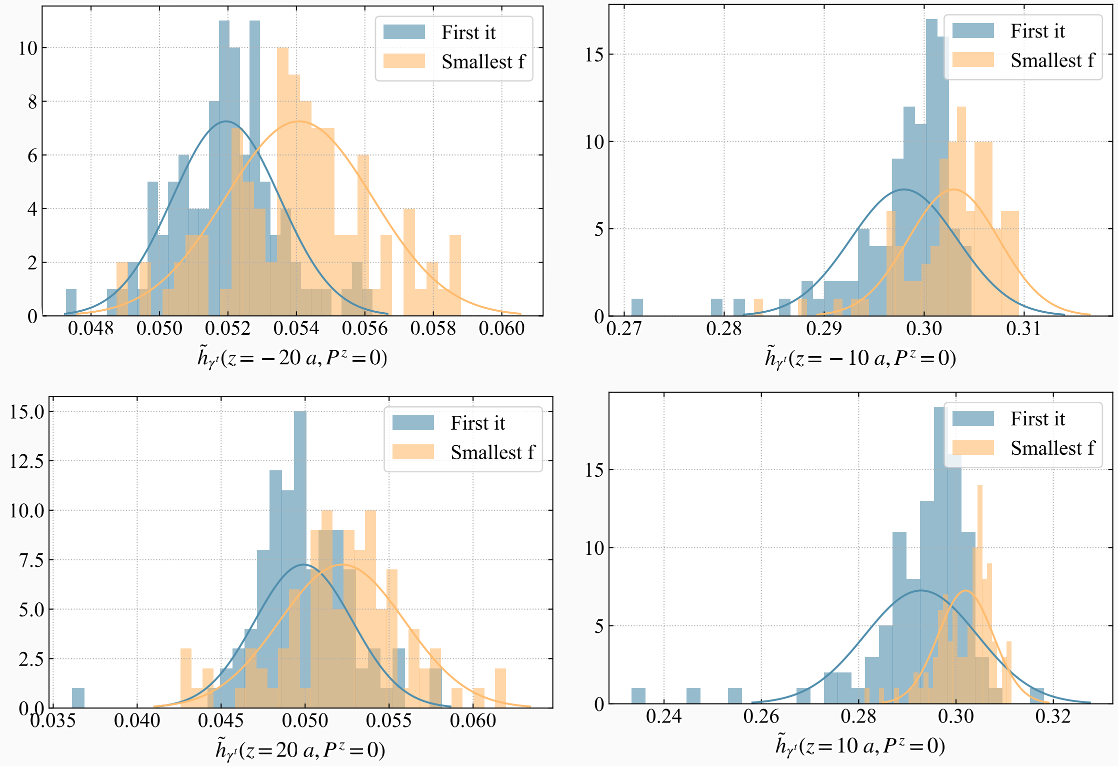

Quasi-distribution under the Coulomb Gauge

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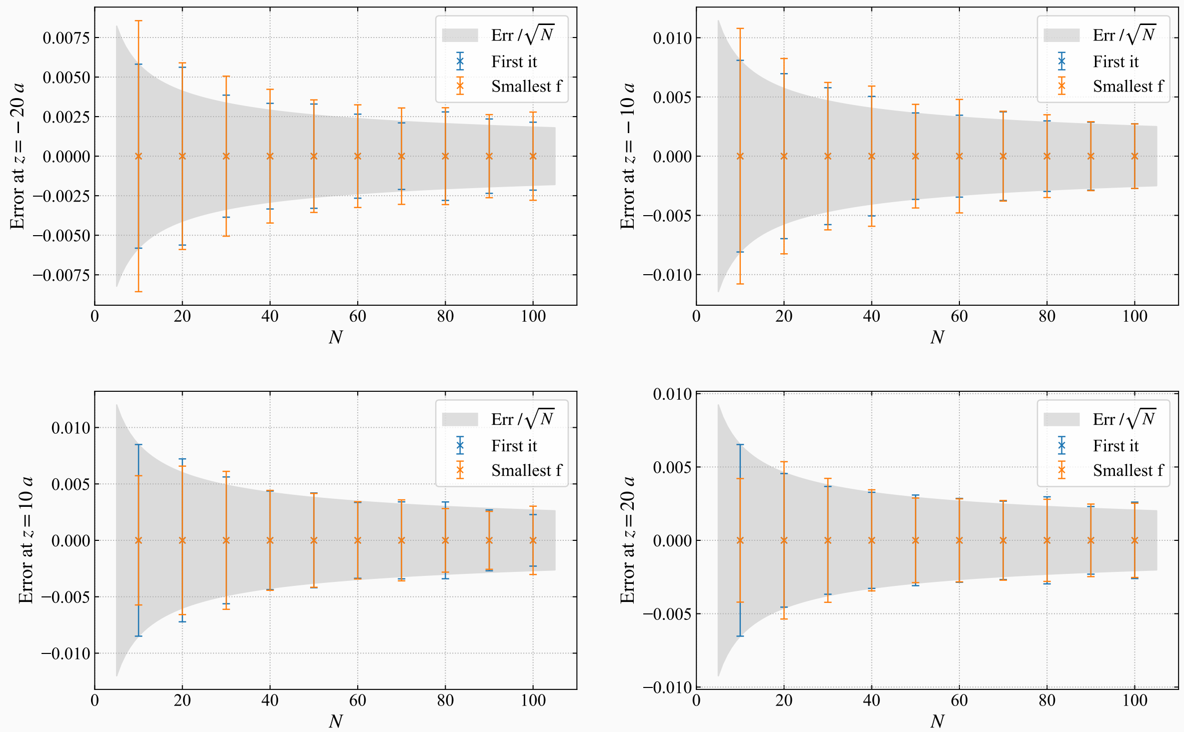
$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t} \langle \vec{P} = \vec{0} \,|\, \bar{\psi}(z)\gamma^t \psi(0) \,|_{\vec{\nabla}\cdot\vec{A}=0} \,|\, \vec{P} = \vec{0} \rangle$$



$t_{\text{sep}} = 8\ a,\ \tau = 4\ a$

Summary

Summary

- Gribov copies stem from multiple solutions of gauge condition;
- Two impacts of Gribov copies: noise and distortion;
- Gribov noise is undistinguishable from the statistical noise;
- No significant distortion on quark propagator & quasi-distribution;

Prospect

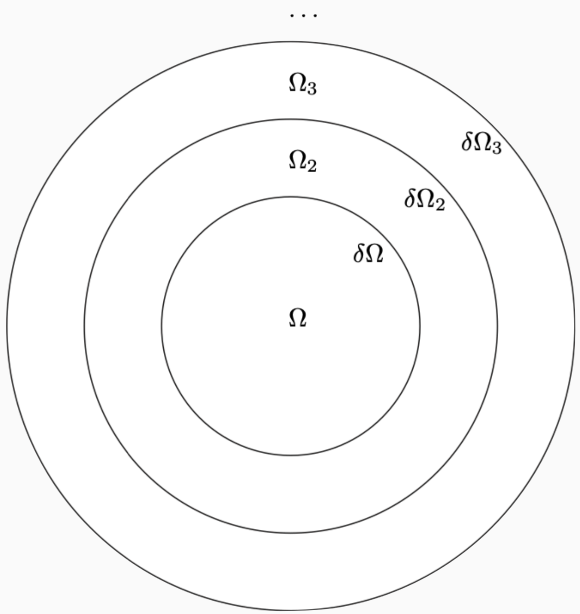
- Including more strategies;
- Including more instances;
- Study the gluon propagator for the gluon parton distribution;

Back Up

First Gribov Region

- First Gribov Region: Faddeev-Popov operator is positive definite;

- Gribov Horizon: Faddeev-Popov determinant is zero.



N. Vandersickel, et al., Phys.Rept. 520 (2012)

First Gribov Region

Take minimum point of the functional

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\delta F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x (D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}$$

$$= -\sum_{\mu=1}^{3}\int d^4x (\partial_\mu\theta_a - g f^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}$$

$$= \sum_{\mu=1}^{3}\int d^4x \theta_a(\partial_\mu A^{\mu a}_{\Omega})$$

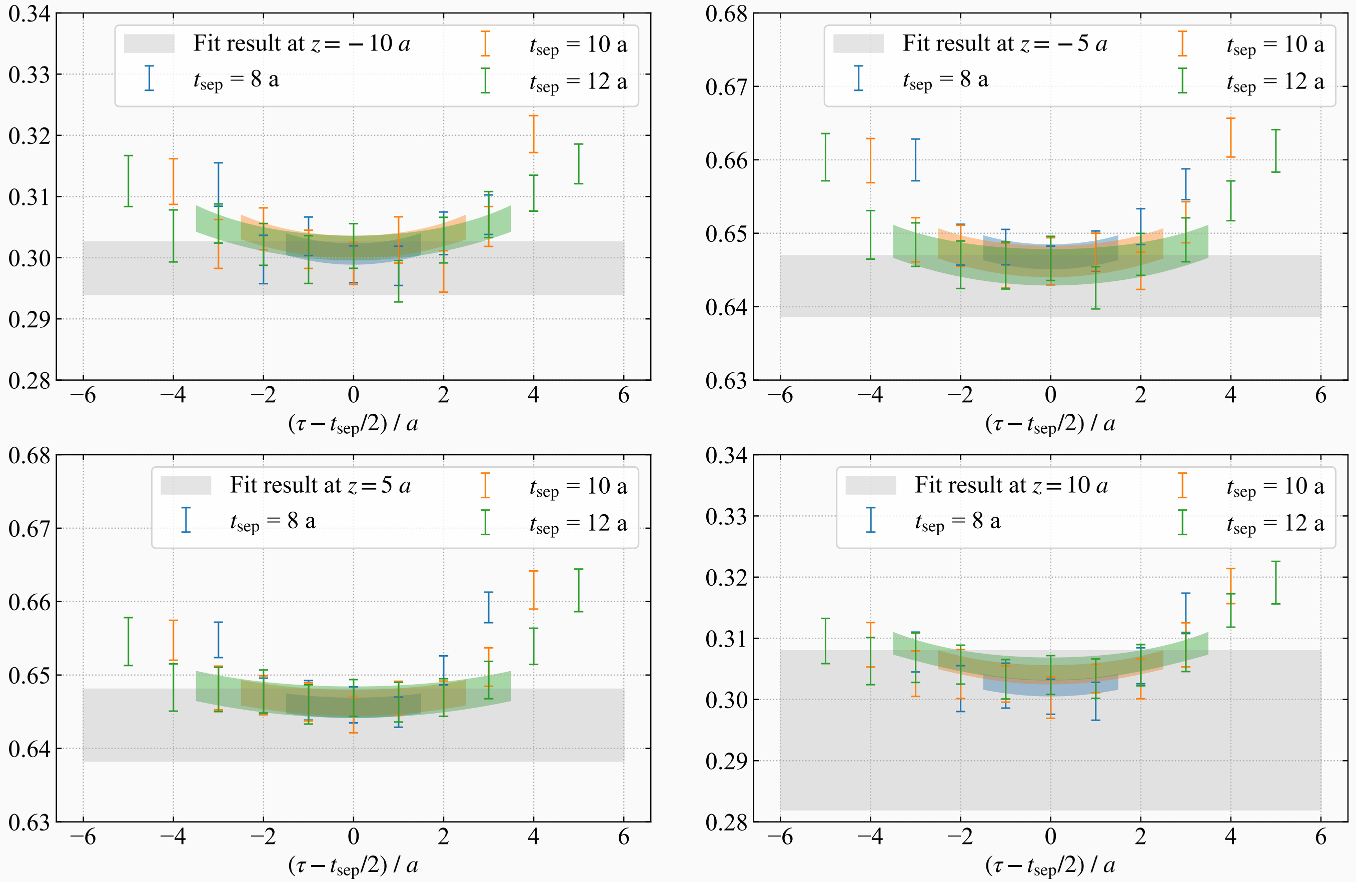
$$\delta^2 F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x \partial_\mu\theta_a\delta A^{\mu a}_{\Omega} = -\sum_{\mu=1}^{3}\int d^4x \theta_a(\partial_\mu D^{ab}_{\mu})\theta^b$$

$\delta^2 F_{\mathrm{CG}}[A,\Omega] \geq 0$ for $\forall\theta \Longrightarrow \mathscr{M} = -\partial_\mu D^{ab}_{\mu}$ is positive definite

In the first Gribov region

* $A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$

Ground State Fit of First it



Ground State Fit of Smallest f