Progress Towards a Hybrid Renormalized Gluon PDF

William Good[1]*
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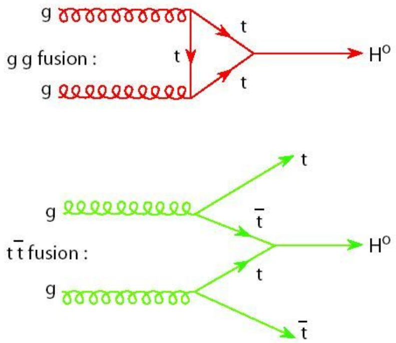
Introduction

- The gluon parton distribution function (PDF) provides important input to high energy experiments, such as Higgs production and J/ψ photo-production



See section 3.1.1 of H.W. Lin, FBS 64:58, 2023.

Image credit: Energy Blog, Denis Silverman

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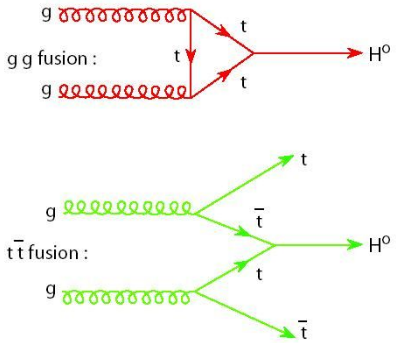

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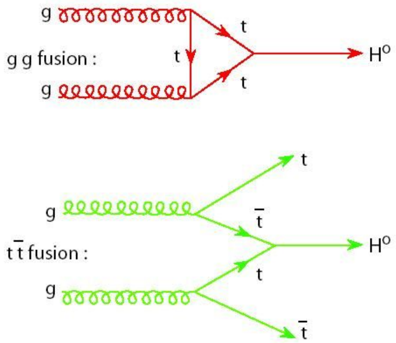

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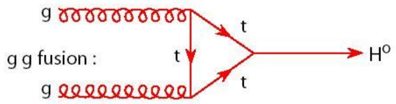
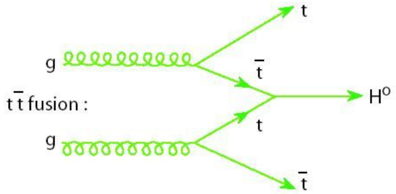
- All previous lattice studies of the gluon PDF have used the pseudo-PDF methodology, which relies on fitting to a model of the PDF

Radyushkin PRD 96:034025 (2017)





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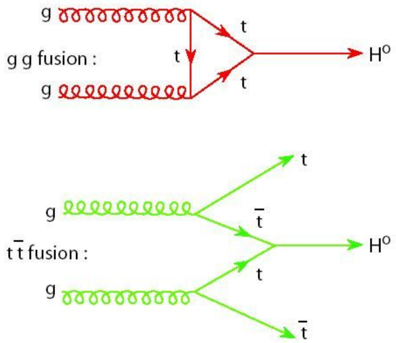
- We present an explorative study of unpolarized gluon PDF operators from the LaMET framework



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Wang, *et al*. PRD 100:074509 (2019)
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Operator and Matrix Elements

- The relevant operators for the gluon PDF are typically of this form or combinations of this form

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- $W(z,0)$ is a straight Wilson line in the z-direction with length z:

$$W(z,0) = \mathcal{P} \exp\left[-ig \int_0^z dz' A^z(z')\right]$$

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- The relevant matrix elements for a hadron H are

$$h^{\mathrm{B}}(z,P_z) = \langle H(P_z)|O(z)|H(P_z)\rangle$$

Some Historical Overview

- 2018: our group made some guesses on what operators could work and compared to the Fourier transform of a pheno. PDF **Fan, *et al.*, PRL 121:242001 (2018)**
- 2019: several operators were shown to be MR and qPDF matching kernels were derived. One of the operators with the cleanest signal from the 2018 study was shown to be not MR
  Zhang, *et al.*, PRL 121:142001 (2019)
  Wang, et al. PRD 100:074509 (2019)
- 2020: A new operator was identified which was MR, but only the pseudo-PDF matching kernels were given explicitly **Balitsky, *et al.*, PLB 808:135621 (2020)**
- Onwards: the new operator became very popular and was used to get PDFs through the pseudo-PDF method in several studies

Fan, *et al.* Int. J. Mod. Phys. A 36:12:2150080 (2021)
Khan, *et al.* (HadStruc) PRD 104:094516 (2021)
Fan, *et al.* PLB 823:136778 (2021)
Salas-Chavira, *et al.* PRD 106:094510 (2022)
Fan, WG, Lin. PRD 108:014508 (2023)
Delmar, *et al.*PRD 108:094515 (2023)

Operators of Focus

Zhang, et al., PRL 121:142001 (2019)
Wang, et al. PRD 100:074509 (2019)
Yao, *et al*. JHEP 11(2023)021

- A paper came out last year detailing hybrid renormalization matching for two of the previously identified operators:
  - Summation for i,j over transverse indices only. Summation for μ over all Lorentz indices

$$O^{(1)}(z) = F^{zi}(z)W(z,0)F^{z}_{\ i}(0) \qquad O^{(2)}(z) = F^{z\mu}(z)W(z,0)F^{z}_{\ \mu}(0)$$

Thanks to Jian-Hui Zhang for additional information about $O^{(2)}(z)$ matching and Wilson coefficients

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- We want to see if we can achieve reasonable signal for these operators and compare them to what has been identified as a clean, seemingly less contaminated operator used in pseudo-PDF studies:

Balitsky, *et al.*, PLB 808:135621 (2020)

$$O^{(3)}(z) = F^{ti}(z)W(z,0)F^{t}_{\ i}(0) - F^{ij}(z)W(z,0)F_{ij}(0)$$

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Lattice Information

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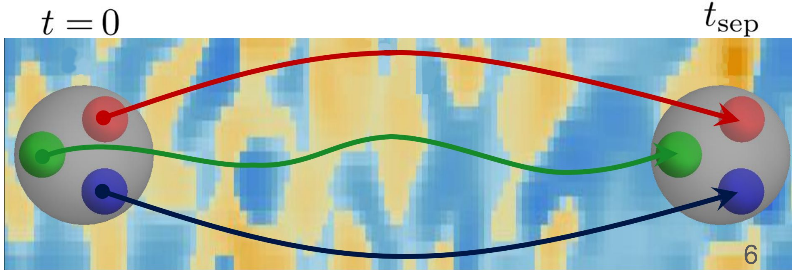

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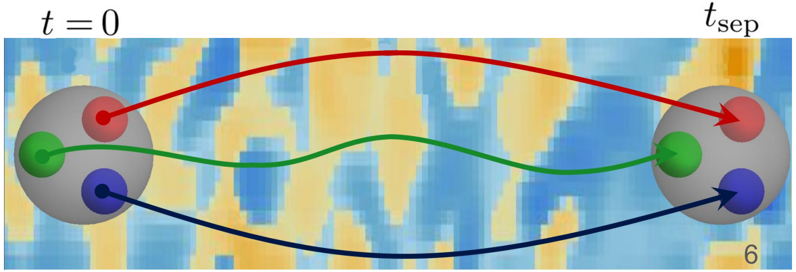
- We use lattice spacing $a \approx 0.12$ fm, and tune valence pion masses to $M_\pi \approx$ 310 and 690 MeV with volume $24^3 \times 64$



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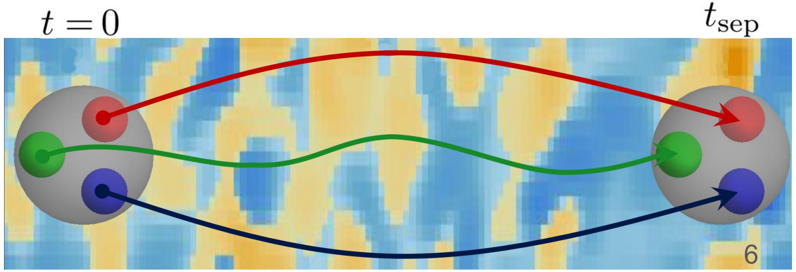

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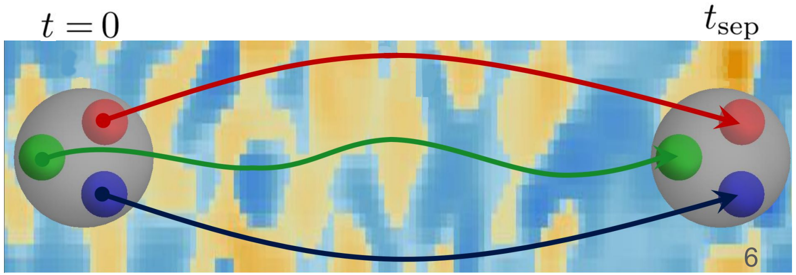

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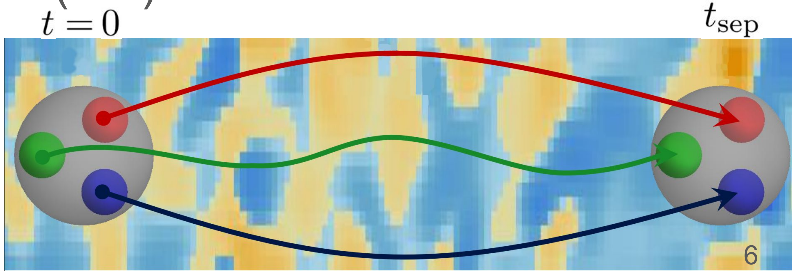

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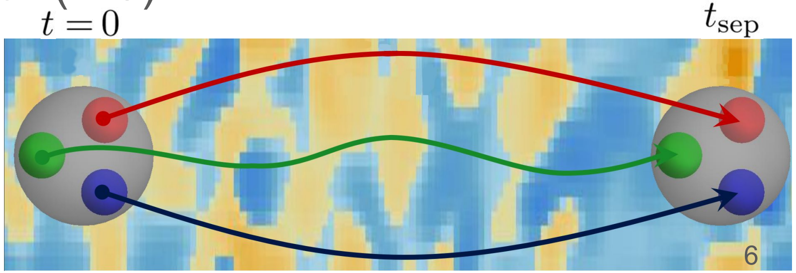
- We look at the “strange” nucleon (N_s), light nucleon (N_l), strange pion (η_s), and the light pion (π)



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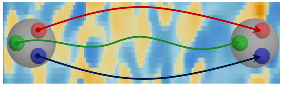

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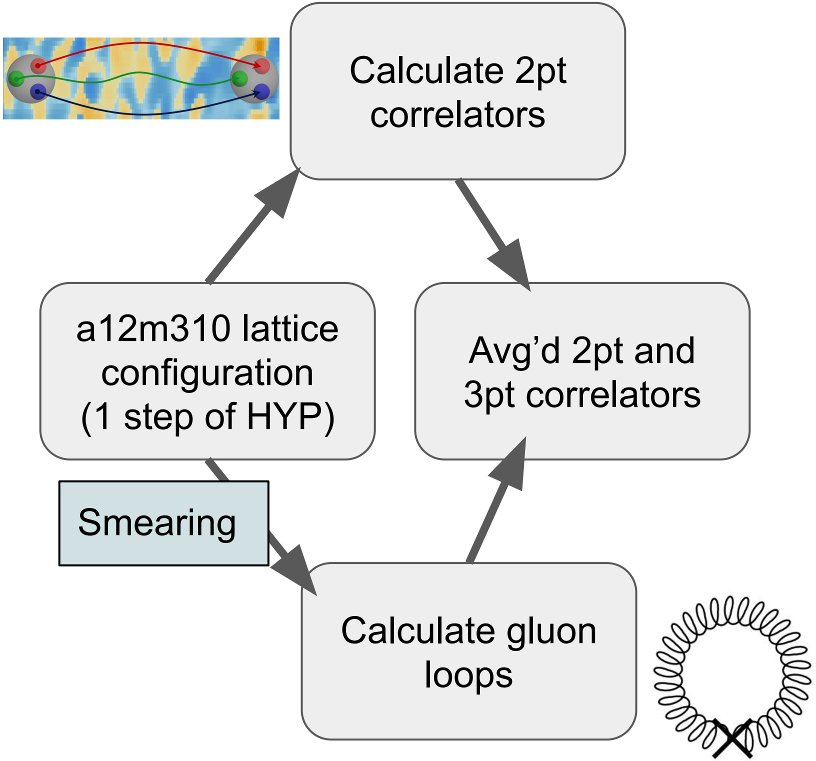

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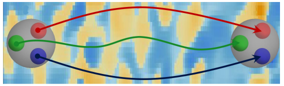

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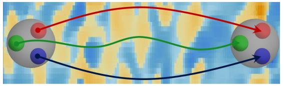

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$h^{\mathrm{R}}(z, P_z)$

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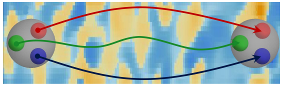

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Fourier Transform to quasi-PDF

$$x\tilde{g}(x, P_z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{2\pi P_z} e^{ixP_z z} h^{\mathrm{R}}(z, P_z)$$

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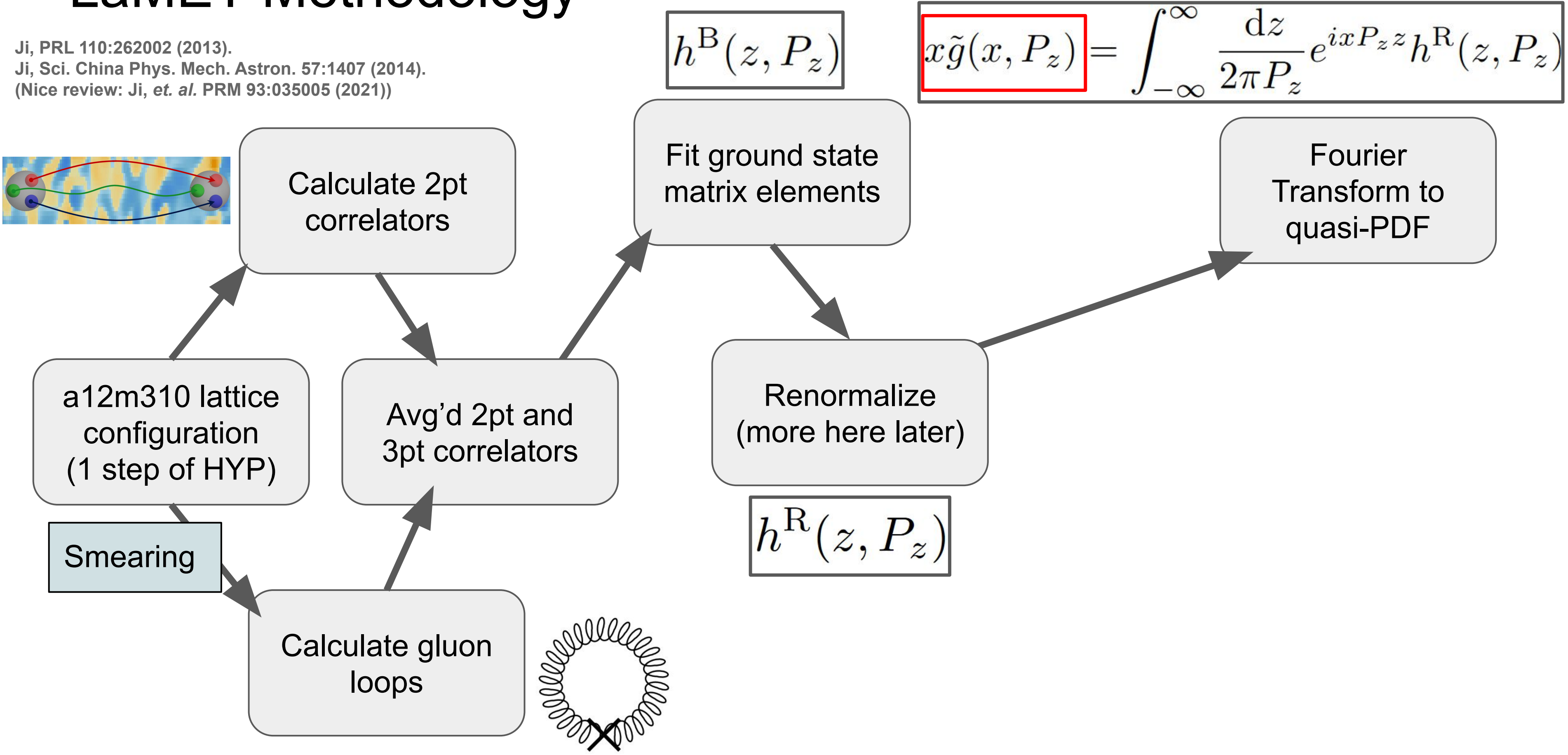

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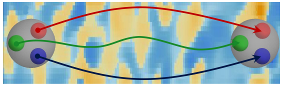

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Match to the Light-Cone PDF

$$x\tilde{g}(x, P_z) = \int_{-1}^{1} \mathrm{d}y\, K(x, y, \mu, P_z) y g(y, \mu)$$

2pt and 3pt Analysis

- Can fit the correlators to a two-state simultaneous fit:

$$C_H^{\mathrm{2pt}}(P_z,t) = |A_{H,0}|^2 e^{-E_{H,0}t} + |A_{H,1}|^2 e^{-E_{H,1}t} + \dots$$

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$$C_H^{3\mathrm{pt}}(z,P_z,t,t_{\mathrm{sep}}) = |A_{H,0}|^2\langle 0|O^{(i)}|0\rangle e^{-E_{H,0}t_{\mathrm{sep}}} + |A_{H,0}||A_{H,1}|\langle 0|O^{(i)}|1\rangle e^{-E_{H,1}(t_{\mathrm{sep}}-t)}e^{-E_{H,0}t}$$
$$+|A_{H,0}||A_{H,1}|\langle 1|O^{(i)}|0\rangle e^{-E_{H,0}(t_{\mathrm{sep}}-t)}e^{-E_{H,1}t} + |A_{H,1}|^2\langle 1|O^{(i)}|1\rangle e^{-E_{H,1}t_{\mathrm{sep}}} + \ldots$$

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- We can plot the ratio of the 3pt to 2pt which goes to $\langle 0|O^{(i)}|0\rangle$ at large separation times

$$\boxed{R_H(P_z;t_{\text{sep}},t) = \frac{C_H{}^{3\text{pt}}(P_z;t_{\text{sep}},t)}{C_H{}^{2\text{pt}}(P_z;t_{\text{sep}})}}$$

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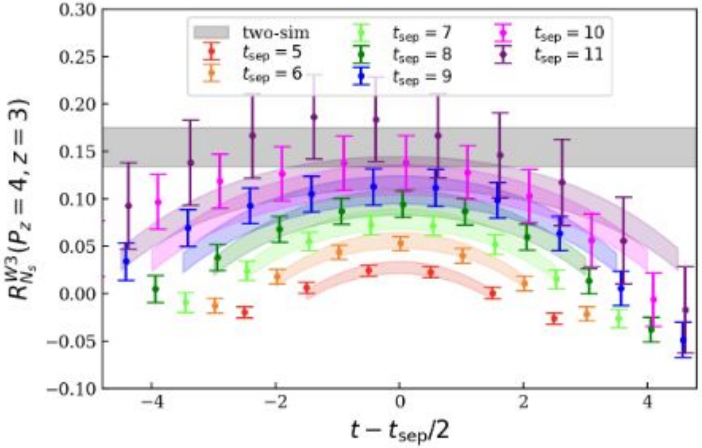
$\boxed{N_s \text{ W3}}$

$$\boxed{R_H(P_z;t_{\text{sep}},t) = \frac{{C_H}^{3\text{pt}}(P_z;t_{\text{sep}},t)}{{C_H}^{2\text{pt}}(P_z;t_{\text{sep}})}}$$

$O^{(2)}(z)$



Thanks to Kinza Hasan for these plots

2pt and 3pt Analysis

- Can fit the correlators to a two-state simultaneous fit:

$$C_H^{2\text{pt}}(P_z,t) = |A_{H,0}|^2 e^{-E_{H,0}t} + |A_{H,1}|^2 e^{-E_{H,1}t} + \ldots$$

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- We can plot the ratio of the 3pt to 2pt which goes to $\langle 0|O^{(i)}|0\rangle$ at large separation times

N_s W3

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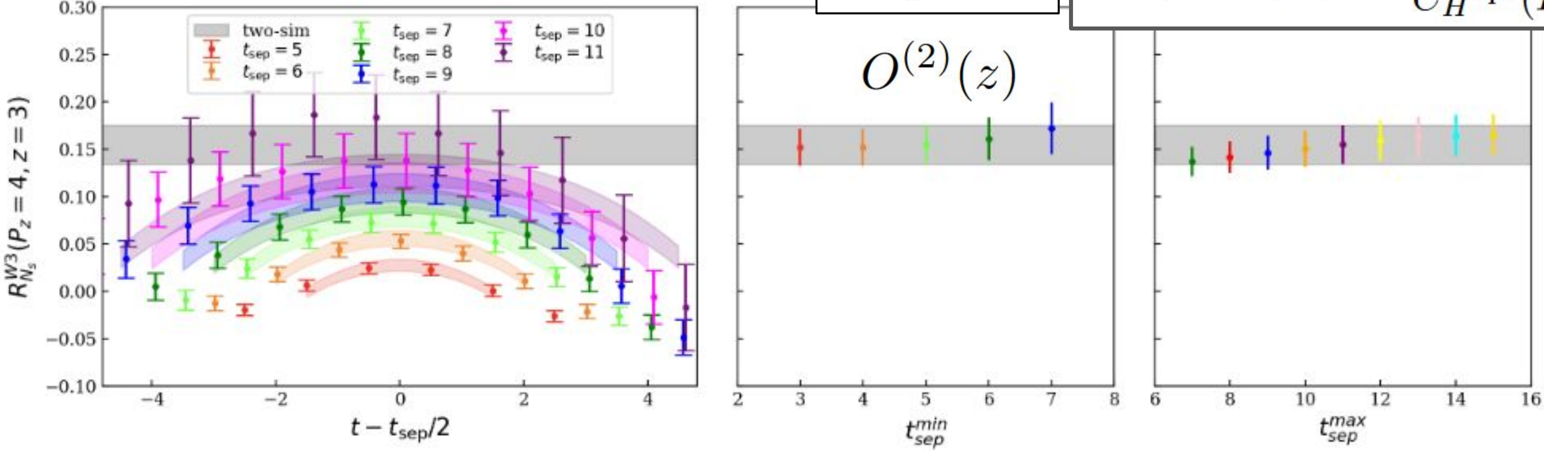

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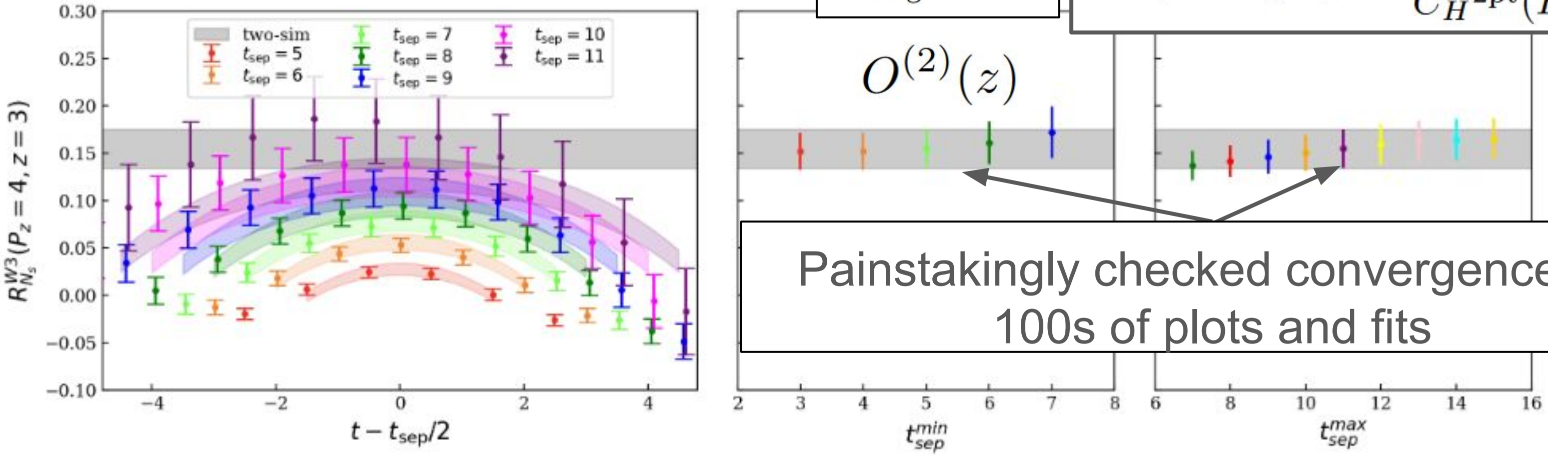
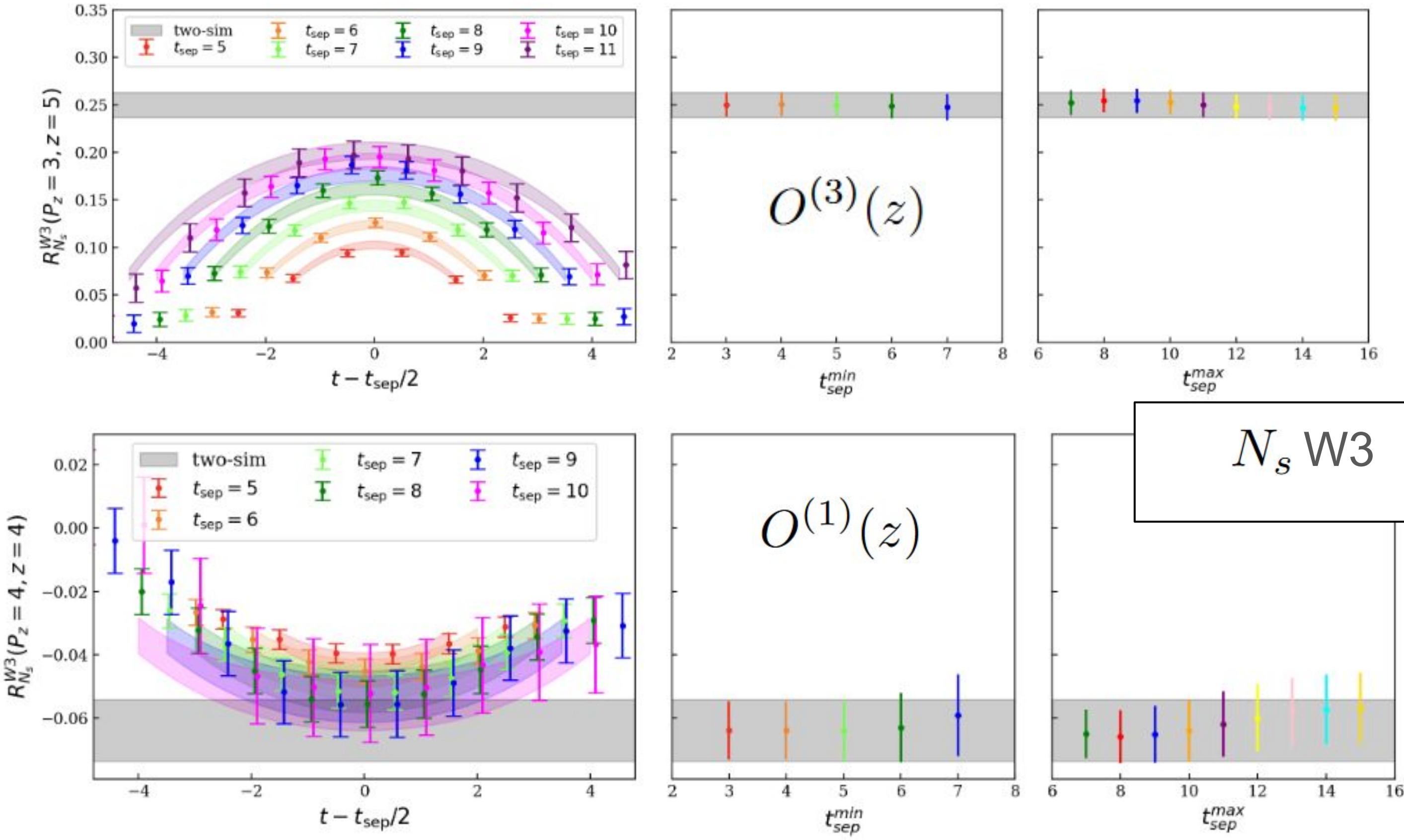


Painstakingly checked convergence on 100s of plots and fits

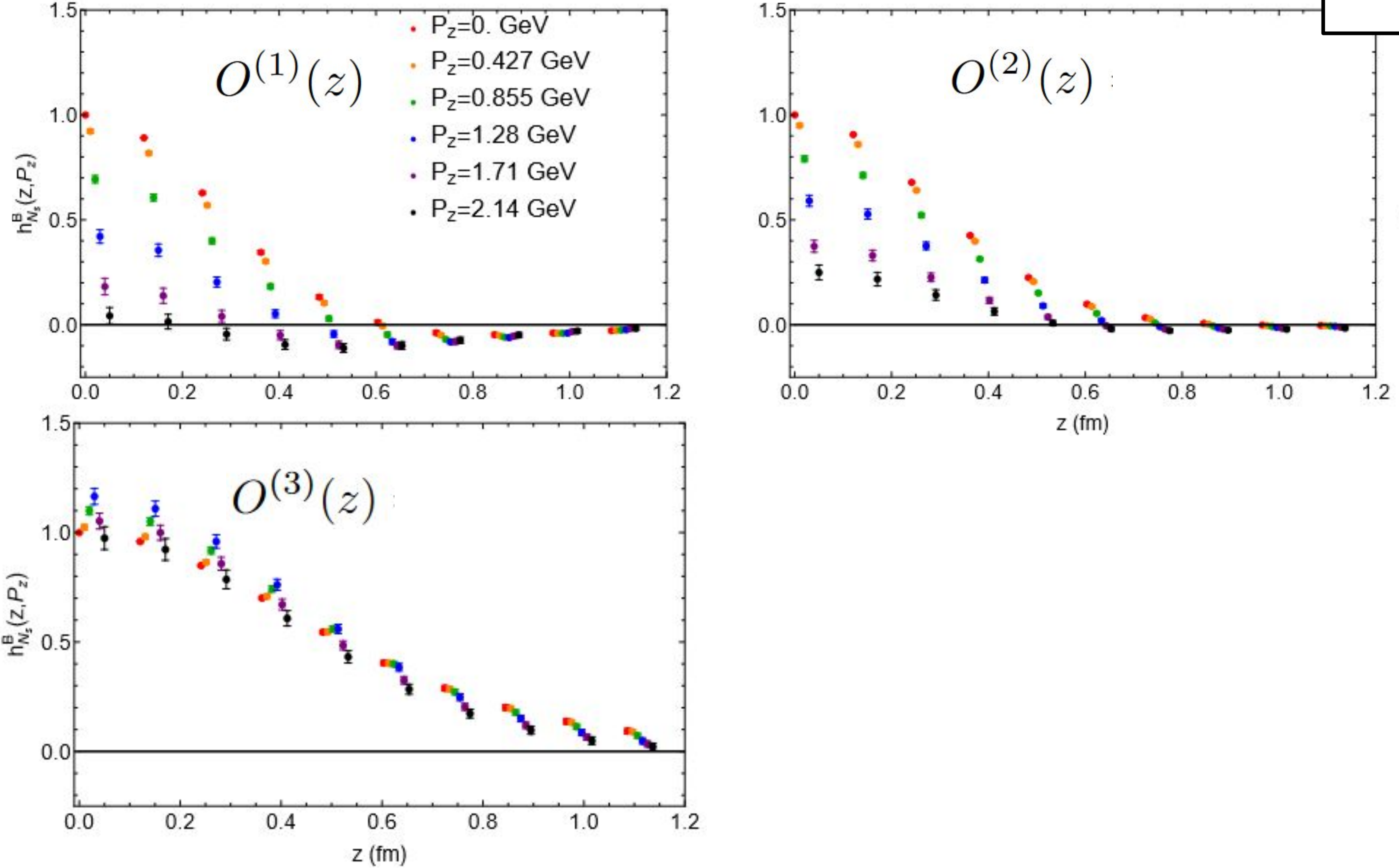
More Ratio Plots



N_s W3

Bare Matrix Element Plots

N_s W3



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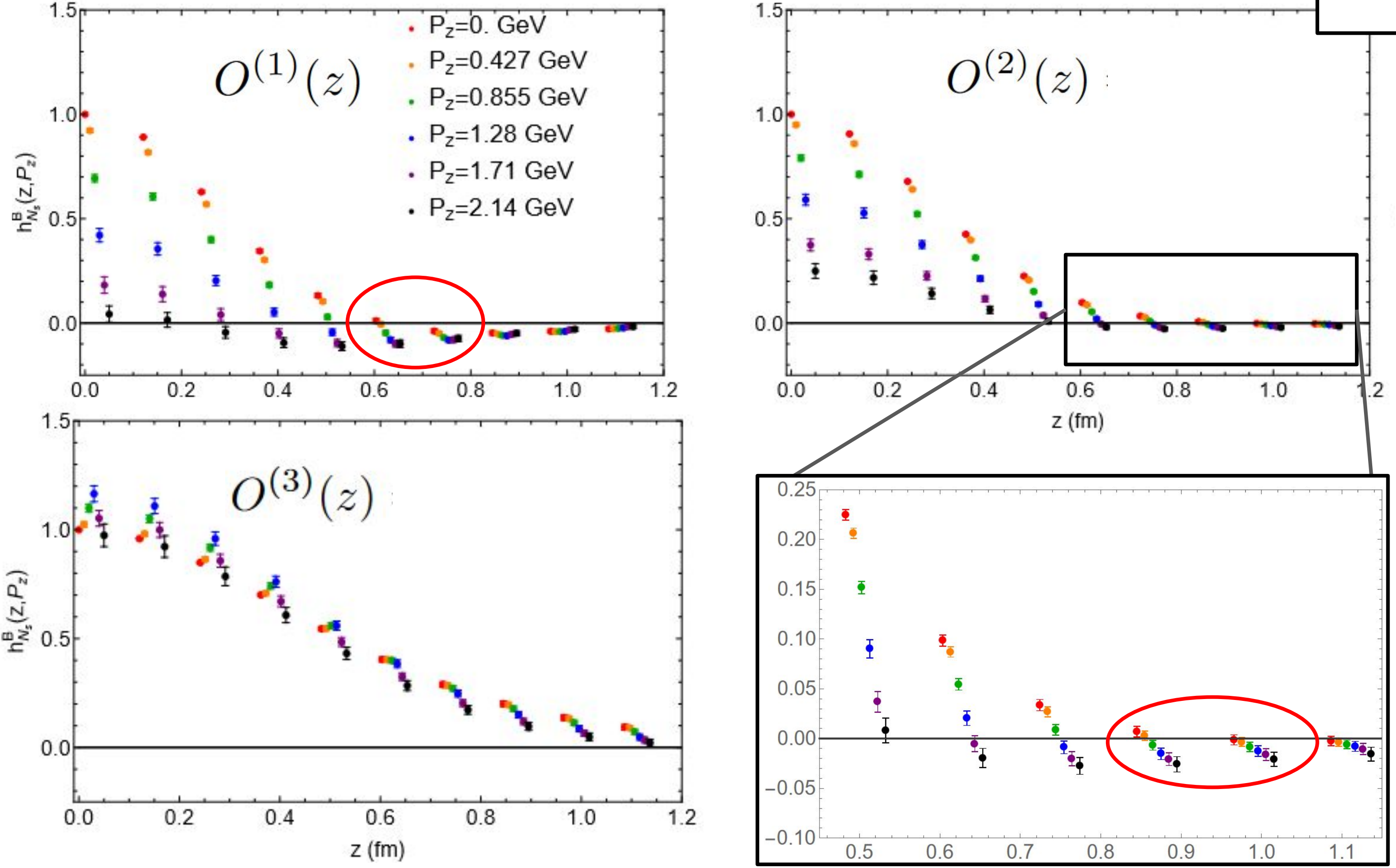

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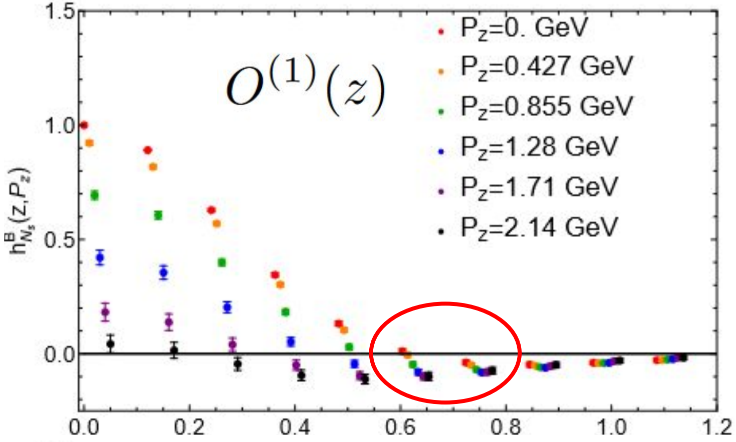
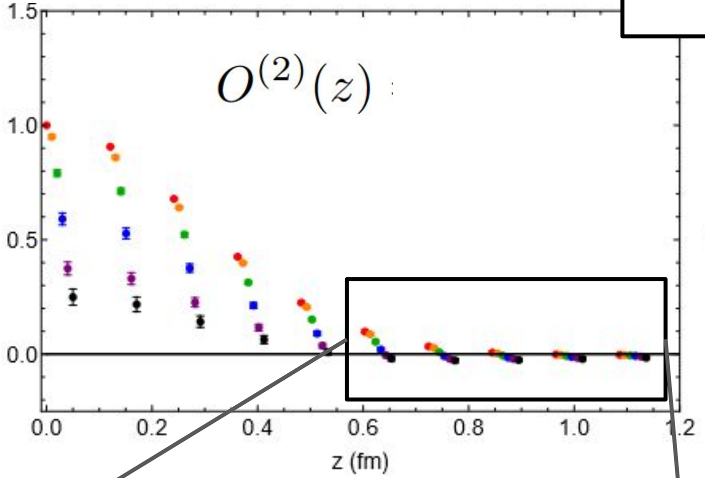
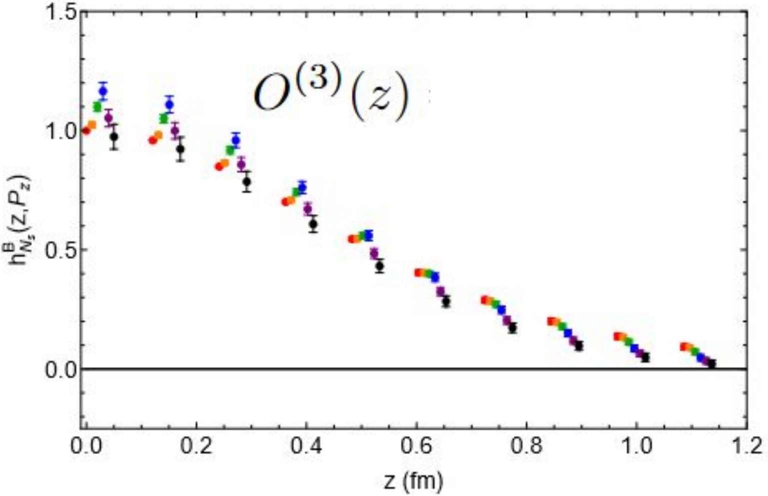
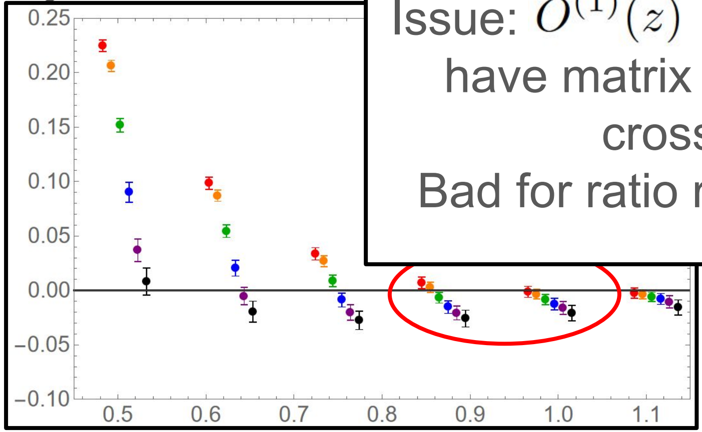


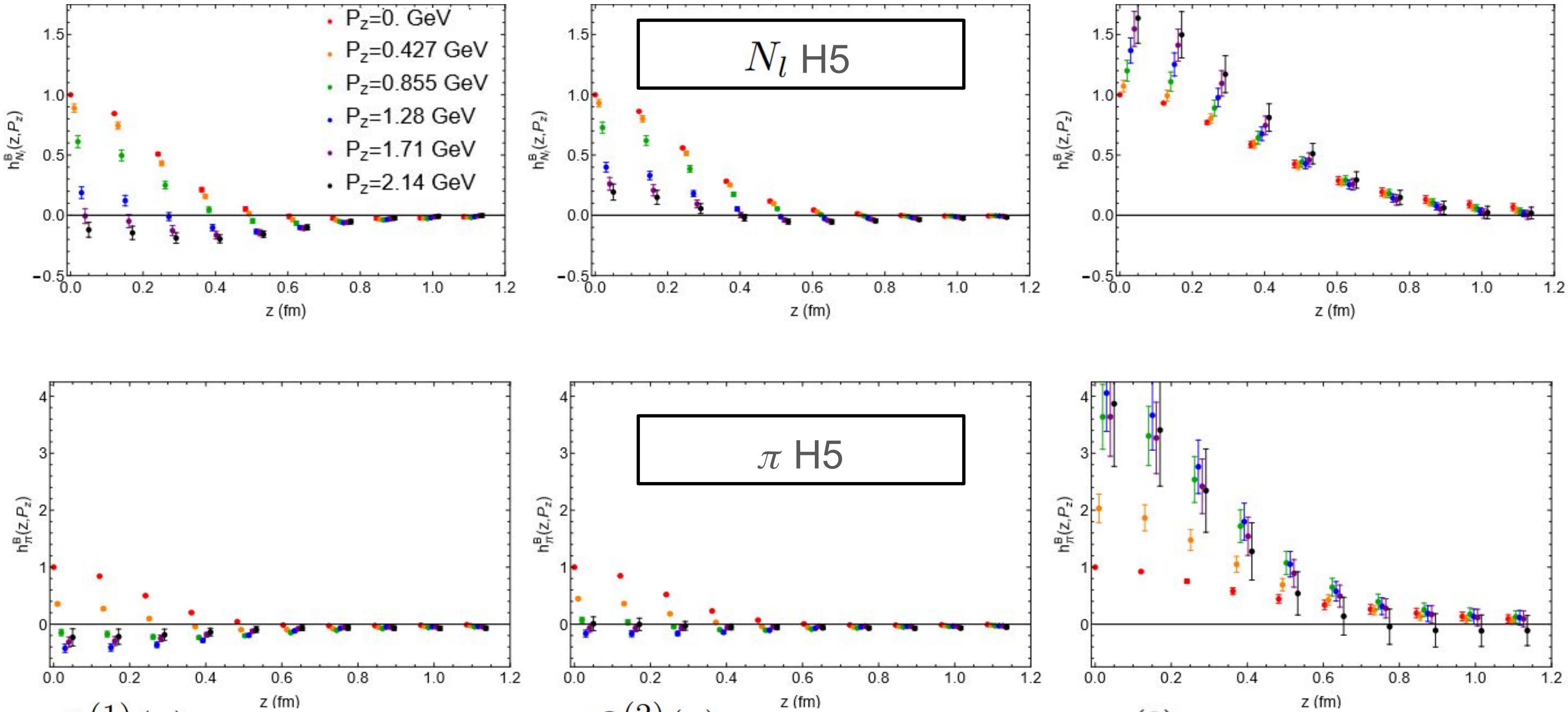






Issue: $O^{(1)}(z)$ and $O^{(2)}(z)$ both have matrix elements that cross zero.
Bad for ratio renormalization

Similar Issues in Other Hadrons and Smearing Choice



$O^{(1)}(z)$ $O^{(2)}(z)$ $O^{(3)}(z)$

Hybrid-Ratio Scheme

Ji, *et al*. Nucl. Phys. B. 964:115311 (2021)

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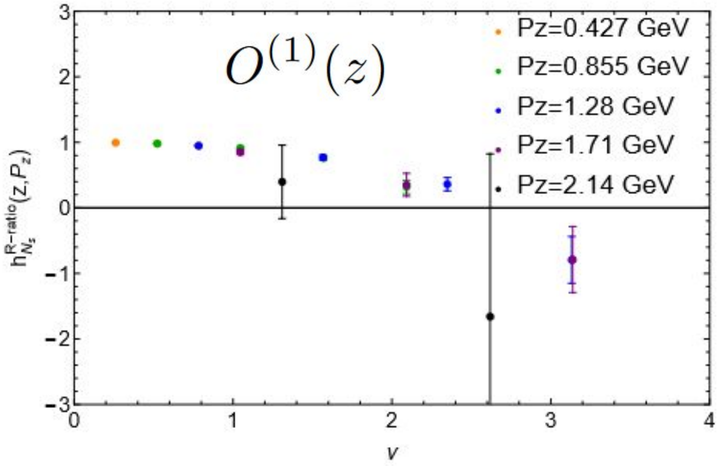
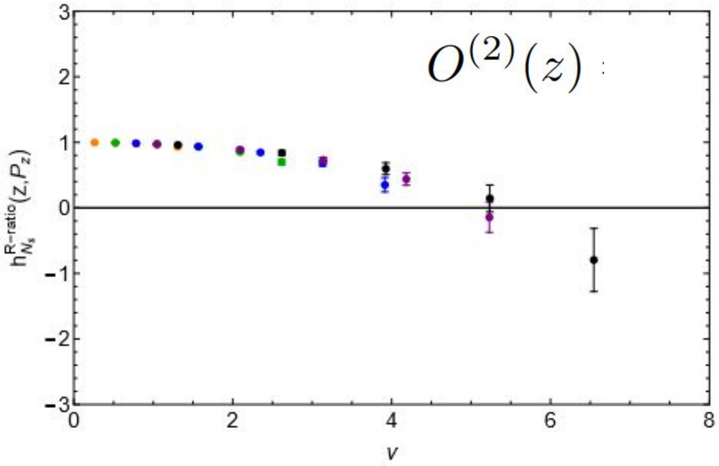
- $\delta m + m_0$ can be fit by matching to the Wilson coefficients for the given operator
- The hybrid-ratio scheme agrees with the standard ratio scheme for $z_s \to \infty$

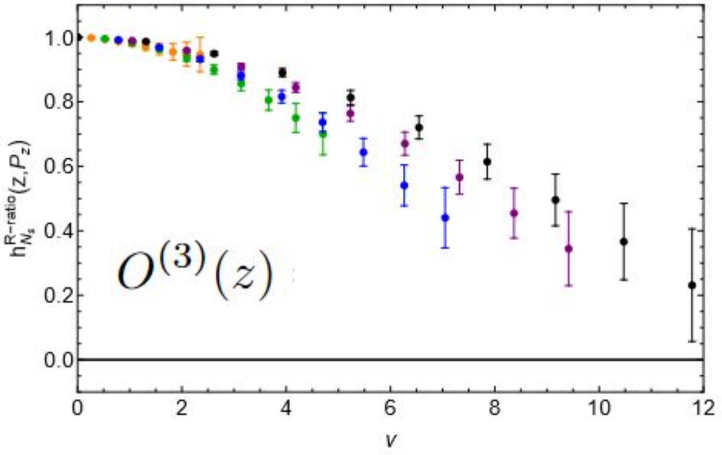
Ratio Renormalized MEs $z_s \to \infty$

N_s W3





$$\nu = zP_z$$



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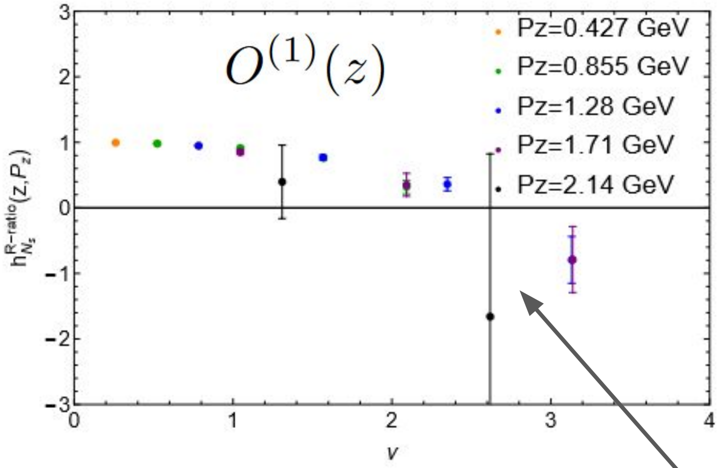

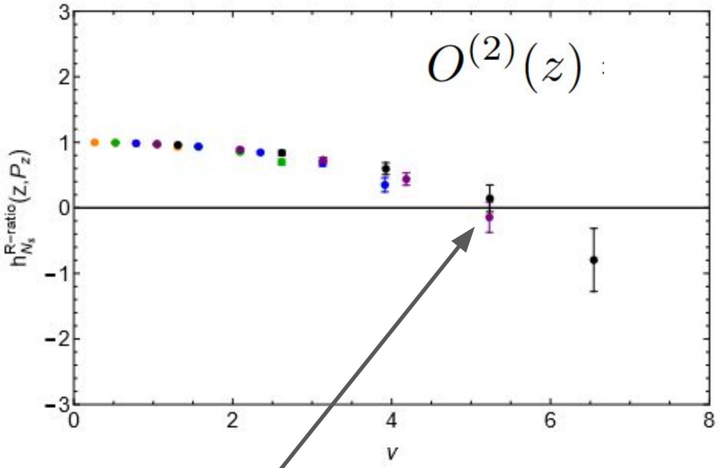

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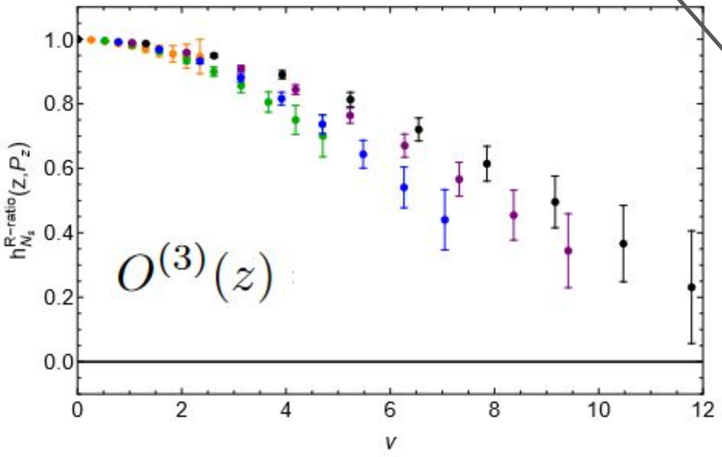
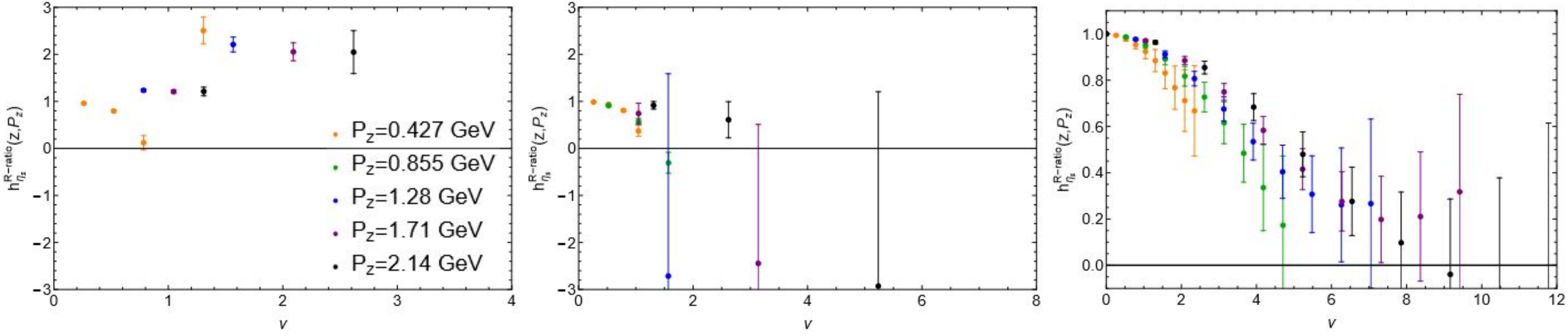


$O^{(1)}(z)$ and $O^{(2)}(z)$ both have many noisy points removed for clarity and cross zero, which is unexpected. $O^{(3)}(z)$ is much cleaner and better behaved.

Things are Even Worse for Other Hadrons and Smearing

η_s, H5

$O^{(1)}(z)$ $O^{(2)}(z)$ $O^{(3)}(z)$



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η_s, H5

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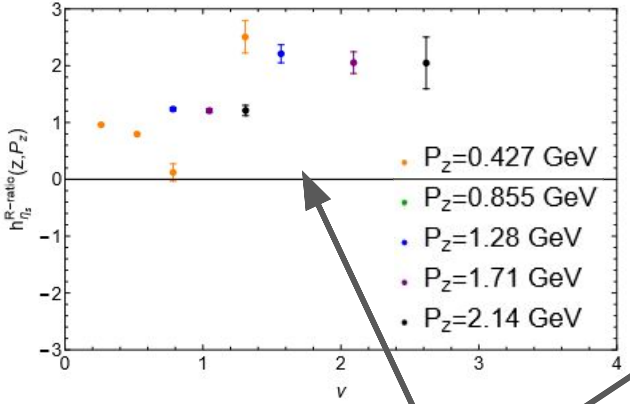
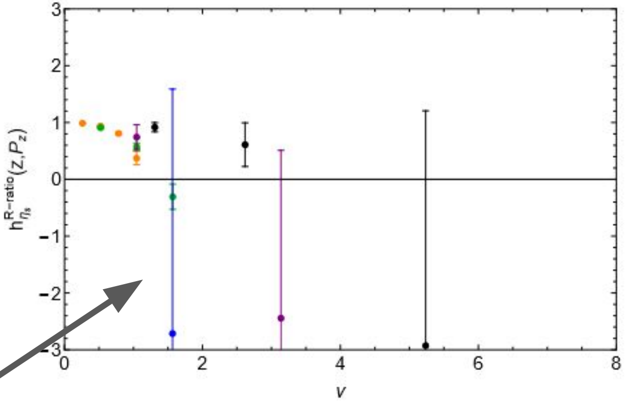


$O^{(2)}(z)$



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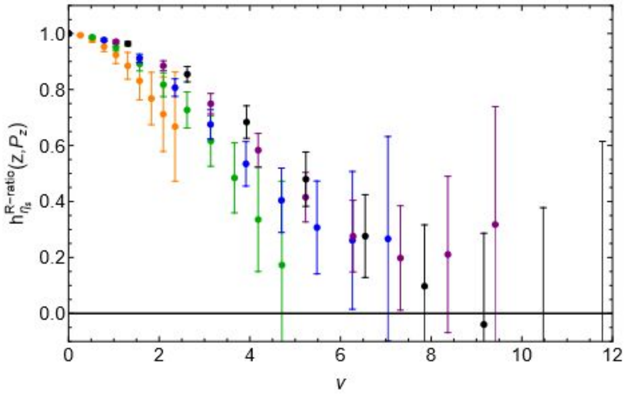

Very crazy behavior in the first two operators (likely due to zero crossings and other noise)

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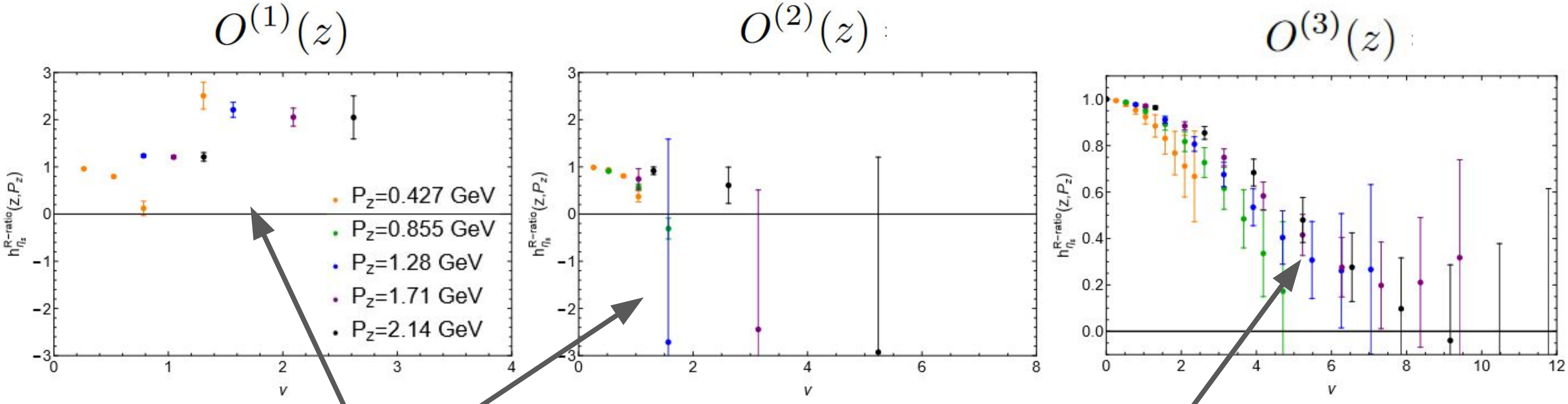

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Noisier, but still reasonable behavior in $O^{(3)}(z)$

Compare to Pheno. Results

Hou *et al*. (CTEQ) PRD 103(1):014013 (2021)
Yao, *et al*. JHEP 11(2023)021

- Took the CT18 nucleon gluon PDF at $\overline{\mathrm{MS}}$ scale μ = 2 GeV to the quasi-PDF using the ratio scheme kernels and Fourier transformed to position space

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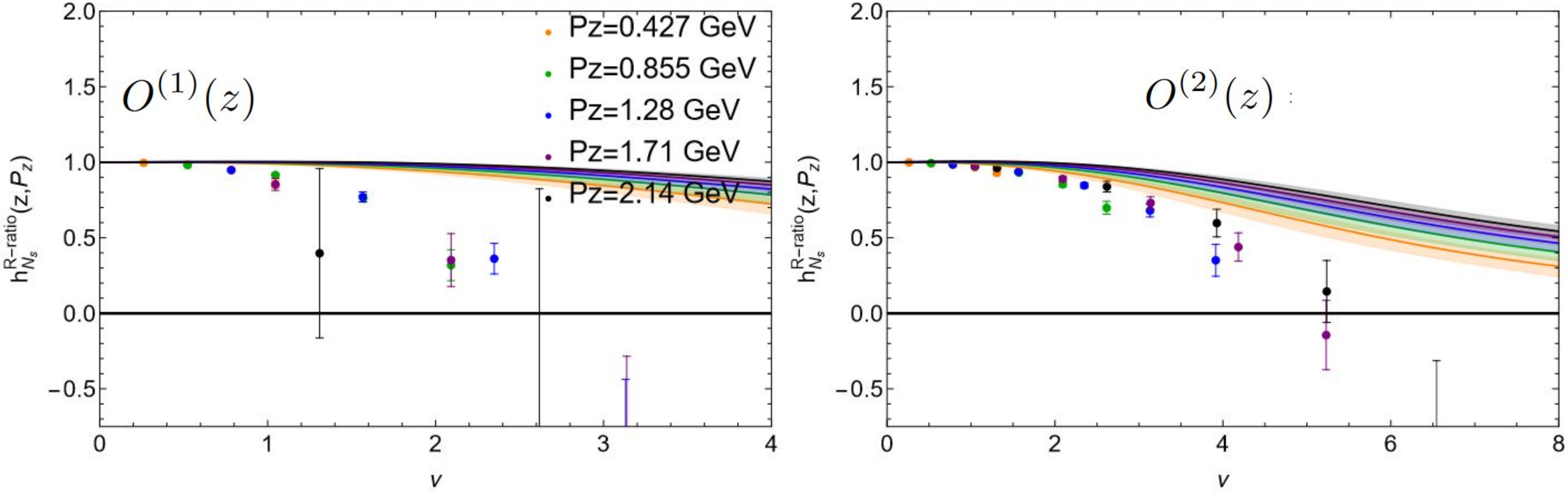

N_s W3

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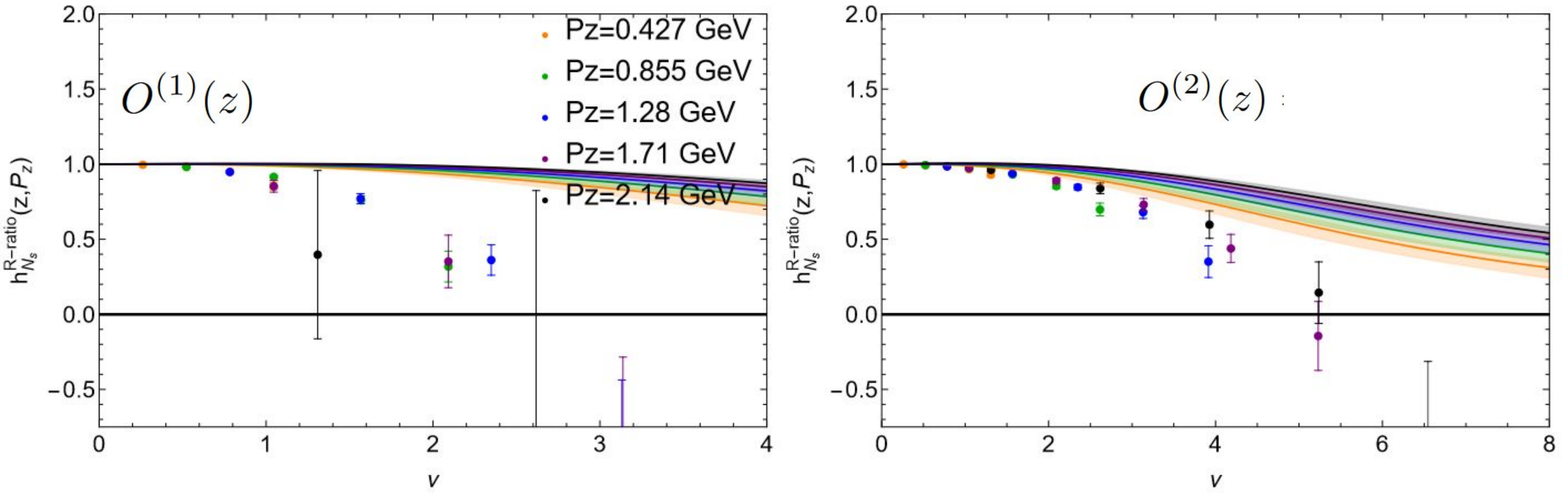
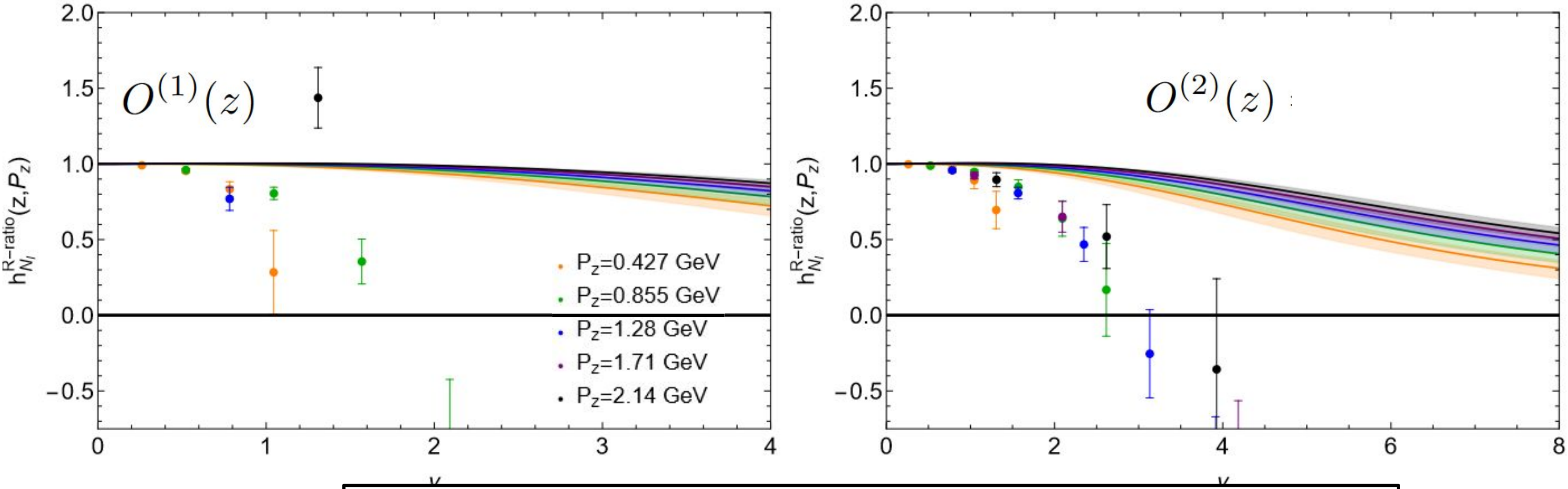


N_s W3

The decay is too fast and the crossing over zero is not expected from the pheno. results

Similar Results with in H5 N_l Case

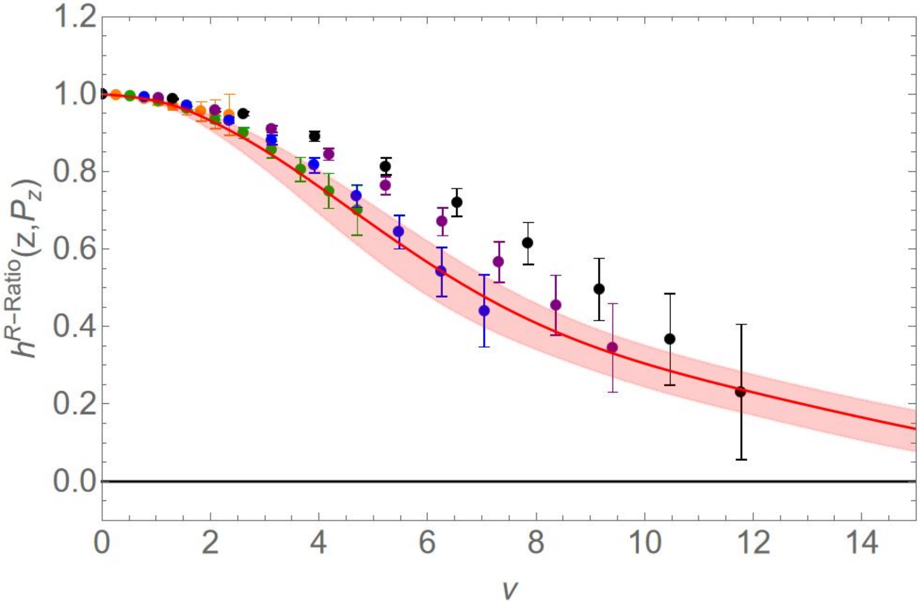


The decay and zero crossing is even worse in this case

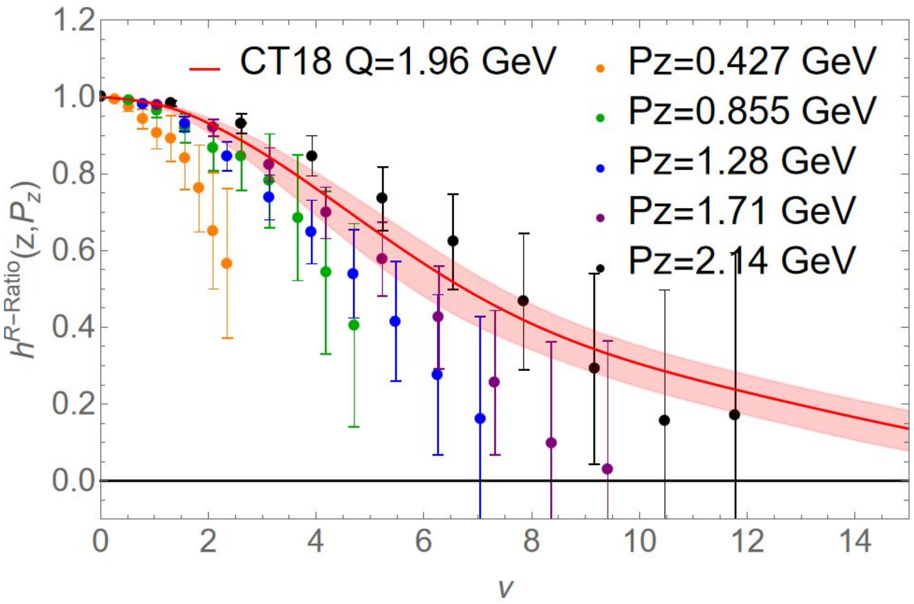
$O^{(3)}(z)$ Compared to Pheno. Matrix Elements

Hou *et al.* (CTEQ) PRD 103(1):014013 (2021)
Balitsky, et al., PLB 808:135621 (2020)

- Use pseudo-PDF matching kernel at z = a on the same CT18 PDF



N_s W3



N_l H5

Fitting $\delta m + m_0$

Ji, *et al*. Nucl. Phys. B. 964:115311 (2021)
Gao, *et al*. PRL 128:142003 (2022)
Zhang, *et al*. PLB 844:128081 (2023
Hou, *et al*. (LPC) Nucl. Phys. B. 969:115443 (2021)

- We essentially fit the discrepancy between the matrix elements with the Wilson coefficient at short distances to a linear model to obtain $\delta m + m_0$

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$$L_z = \ln\left(\frac{4e^{-2\gamma_E}}{\mu^2 z^2}\right) \qquad A^{(1)} = \frac{11}{6} \quad B^{(1)} = 4 \qquad A^{(2)} = \frac{11}{6} \quad B^{(2)} = \frac{14}{3}$$

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- μ is the renormalization scale and γ_{E} is the Euler-Mascheroni constant
- We only have the Wilson coefficients for $O^{(1)}(z)$ and $O^{(2)}(z)$

Fit Results

- Due to the coarse lattice spacing, we interpolate the data in the small-z range. We do this, then fit to three points {z-0.02 fm, z, z +0.02 fm}, varying z

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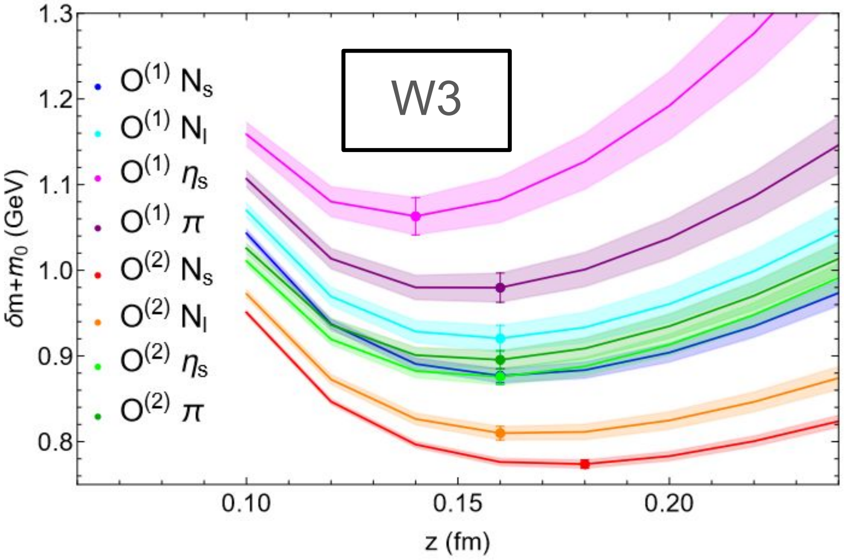

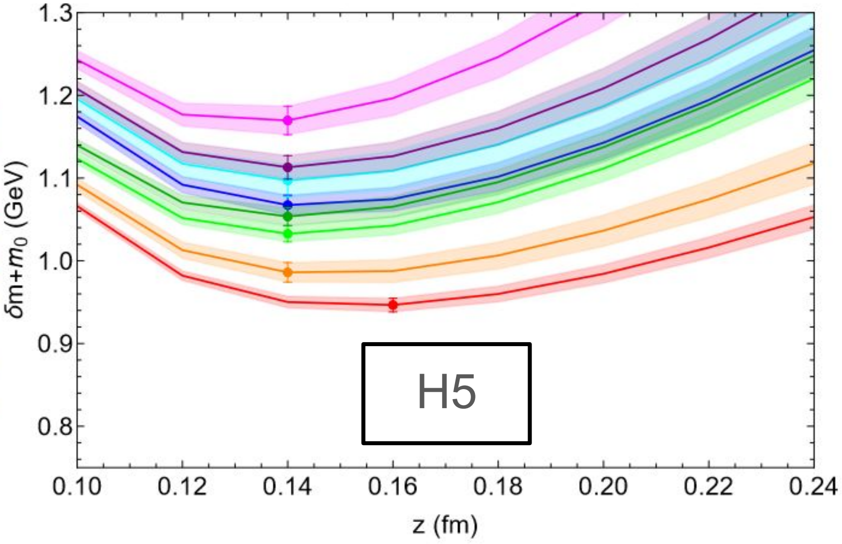

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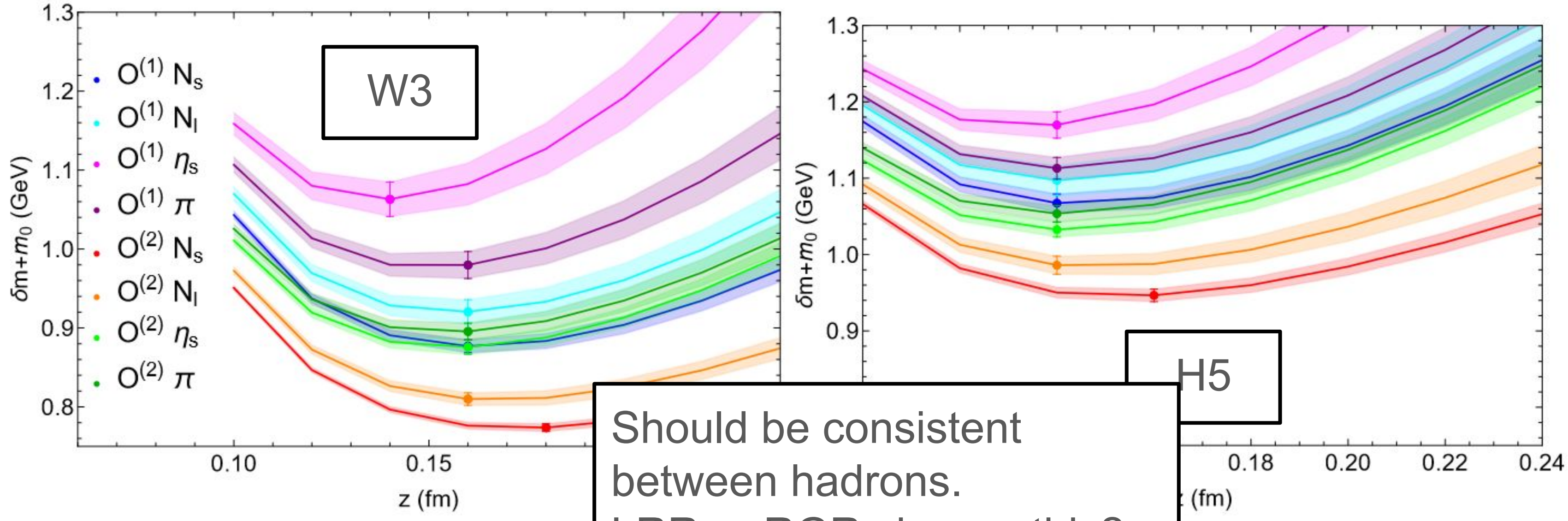

Should be consistent between hadrons.
LRR or RGR change this?

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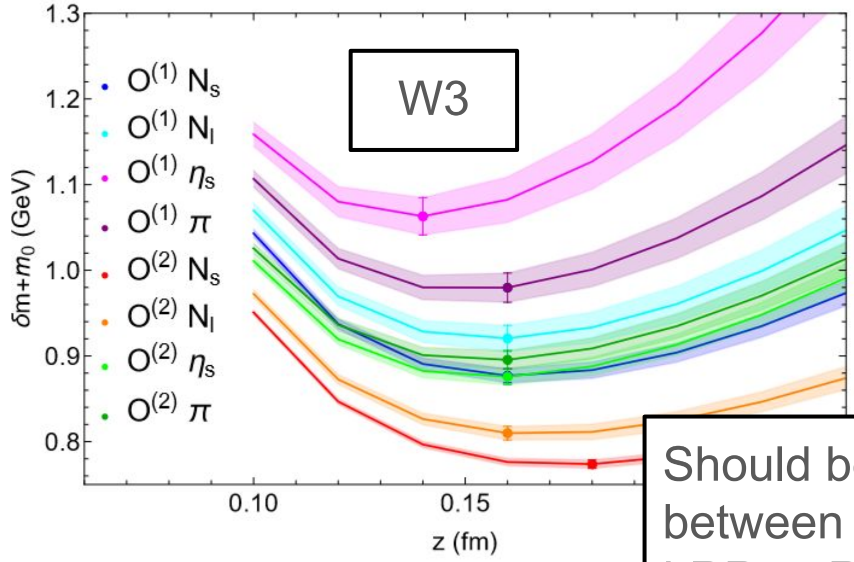

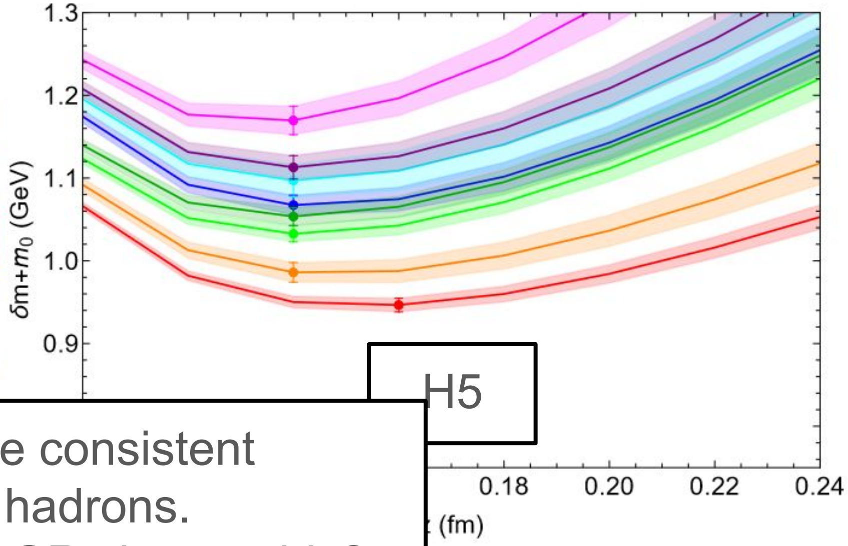

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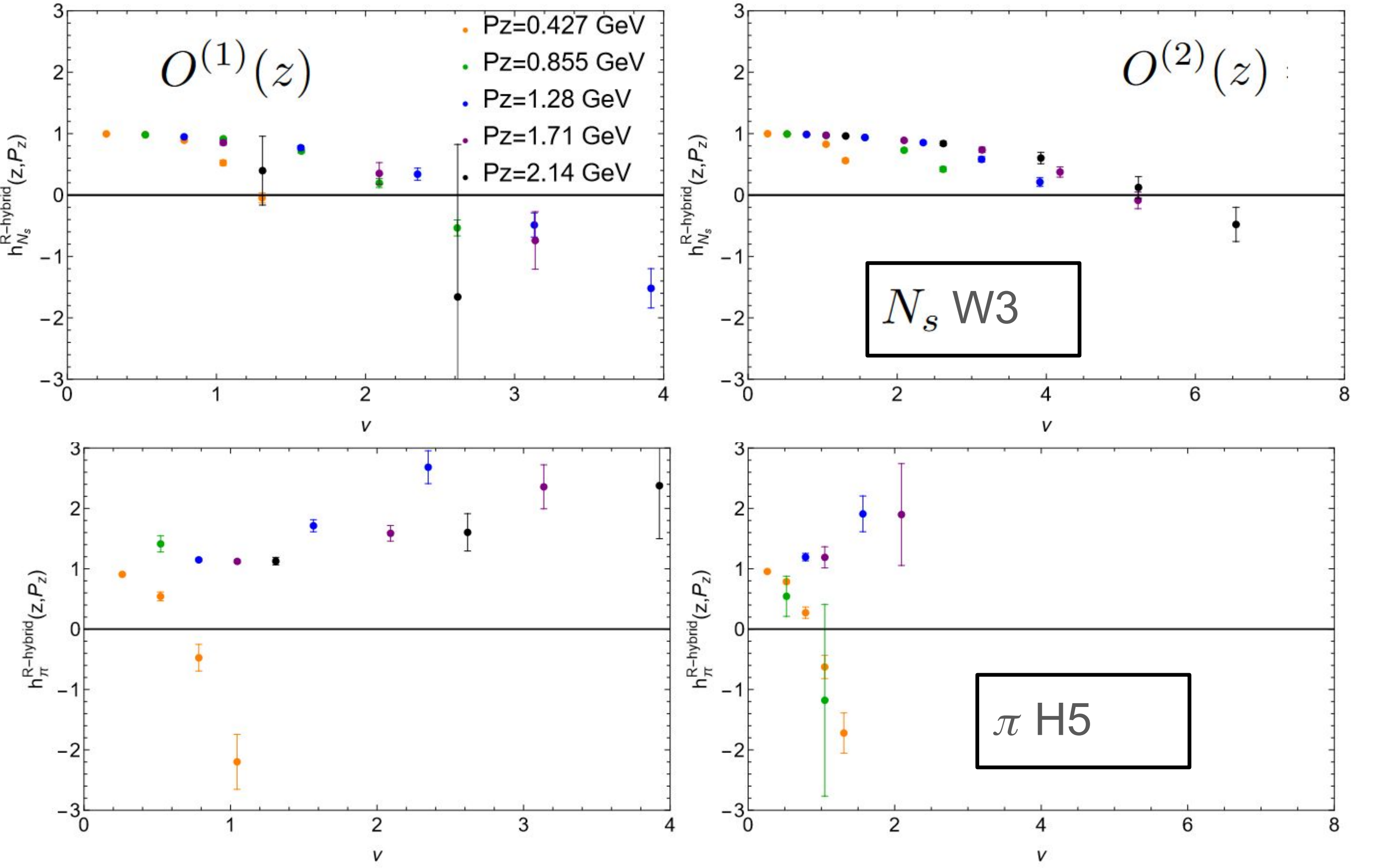
Does Hybrid Renormalization Change Anything?



$\boxed{z_s = 0.24 \text{ fm}}$

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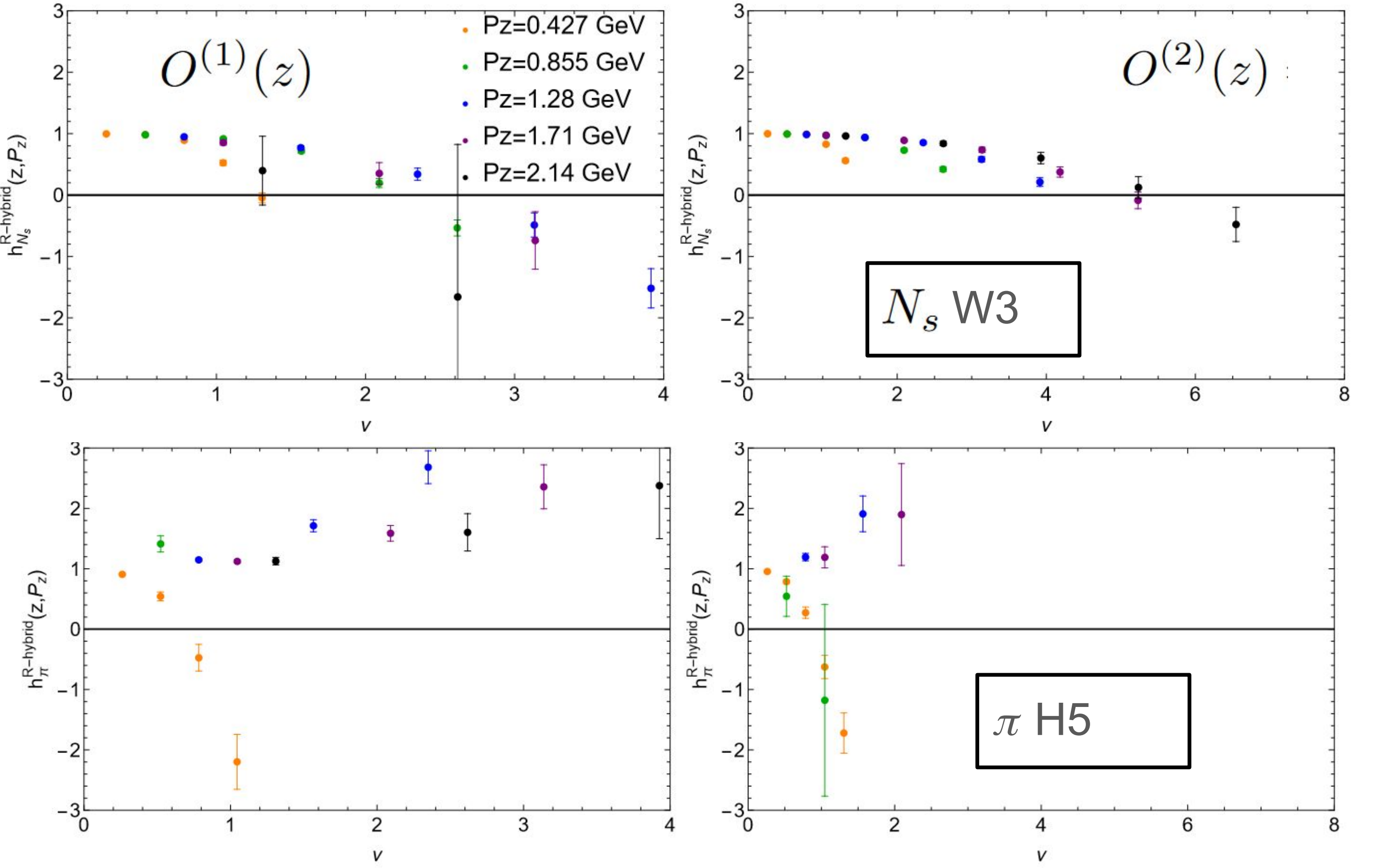

$z_s = 0.24$ fm

With hybrid renormalization we still see zero crossings, lots of noise and divergent behavior

Compare to Pheno. Results

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Yao, *et al*. JHEP 11(2023)021

- Took the same pheno. PDF as before and transformed it similarly using the hybrid-ratio kernels and a Fourier transform to position space

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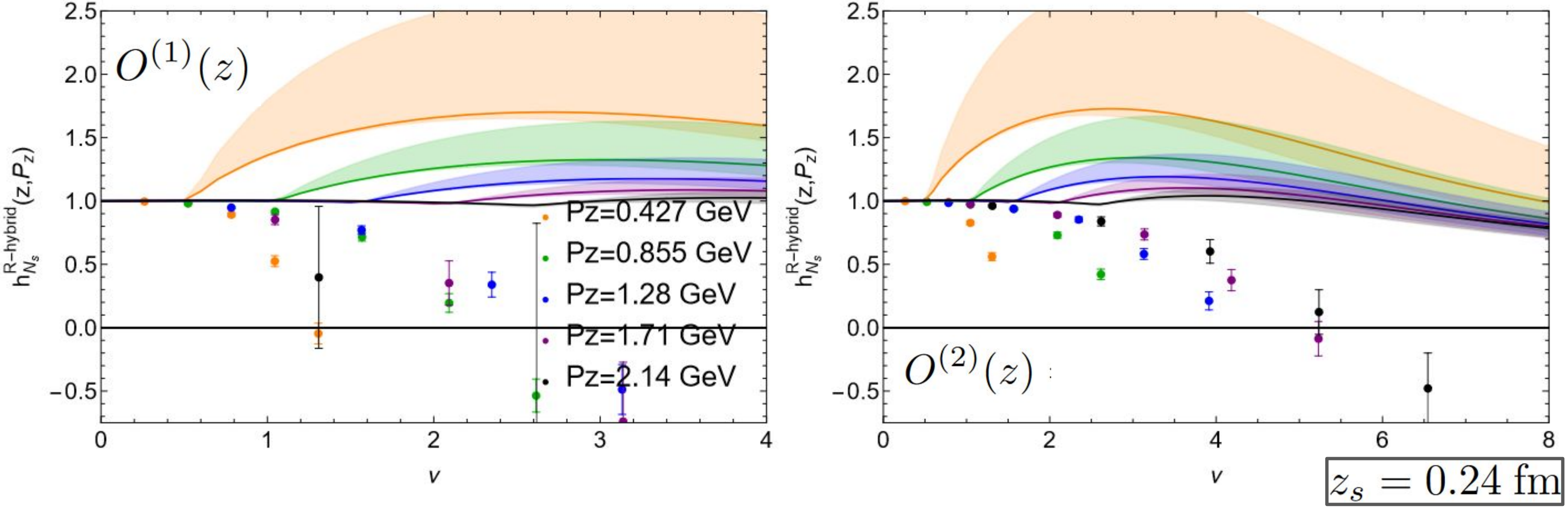

N_s W3

Compare to Pheno. Results

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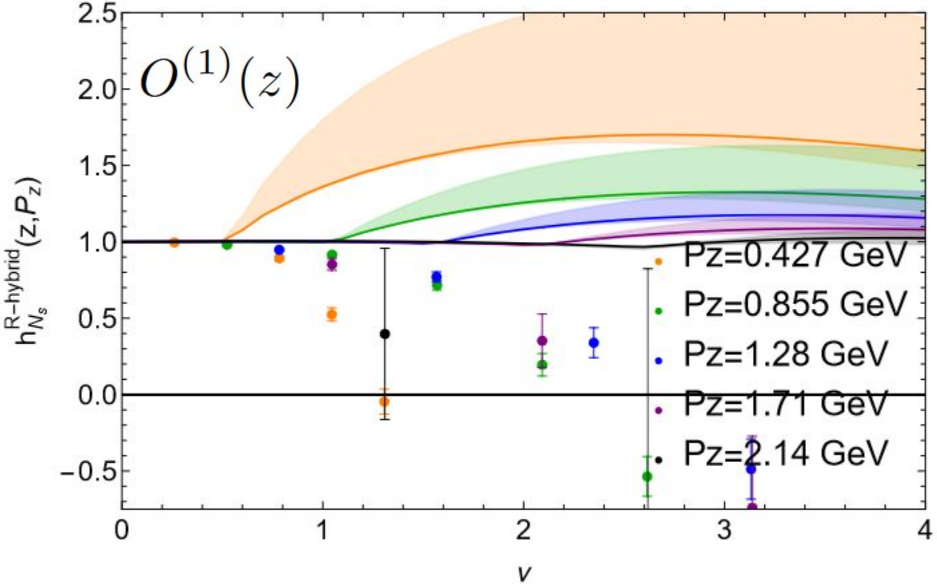
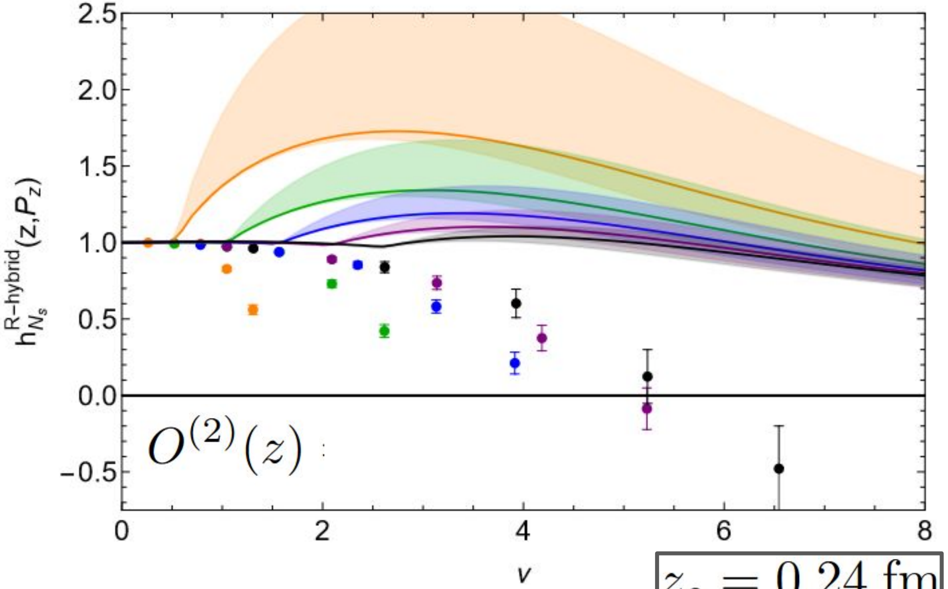


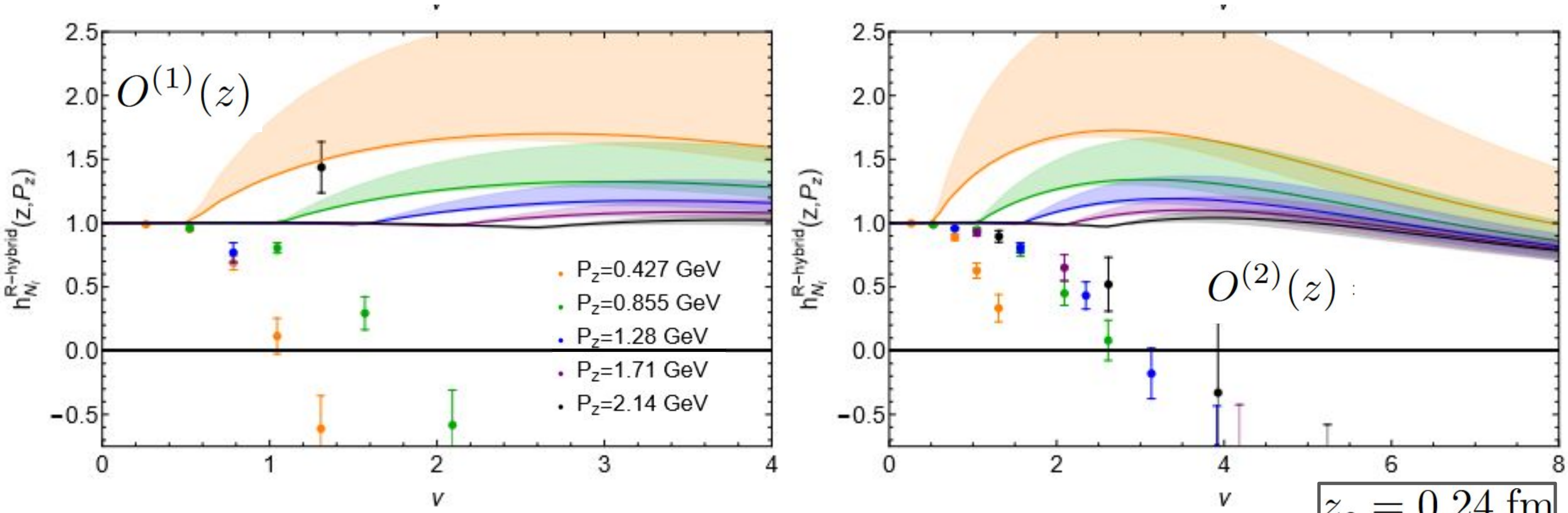


$z_s = 0.24$ fm

N_s W3

Hybrid renormalized pheno. matrix elements have a large bump and still don't capture the zero crossing

Light Nucleon Results for Hybrid-Renormalization



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N_l W3

Even worse decay and divergences

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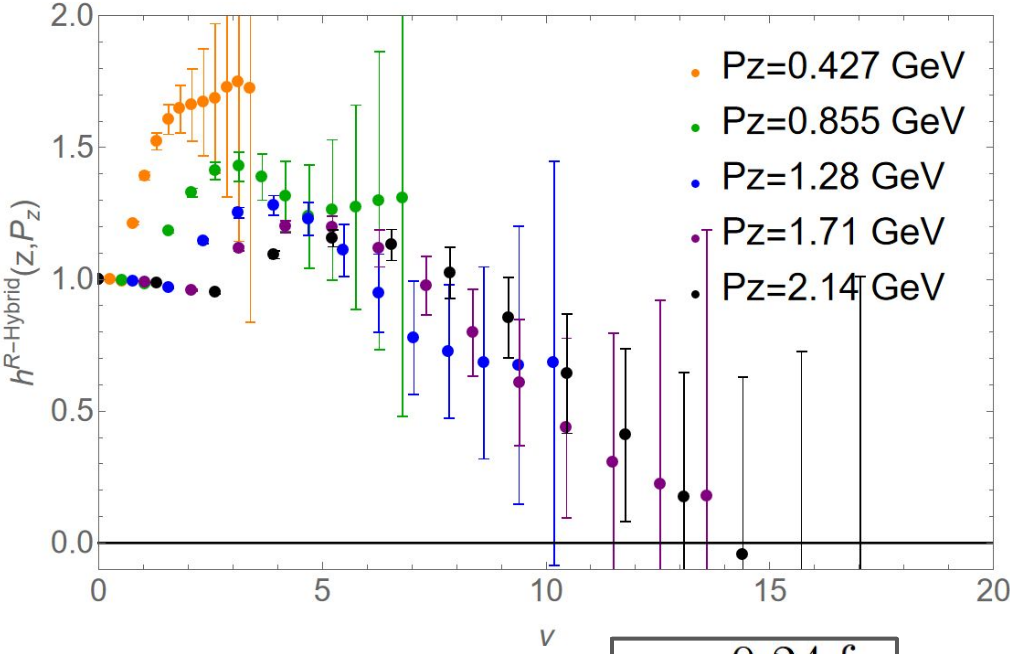

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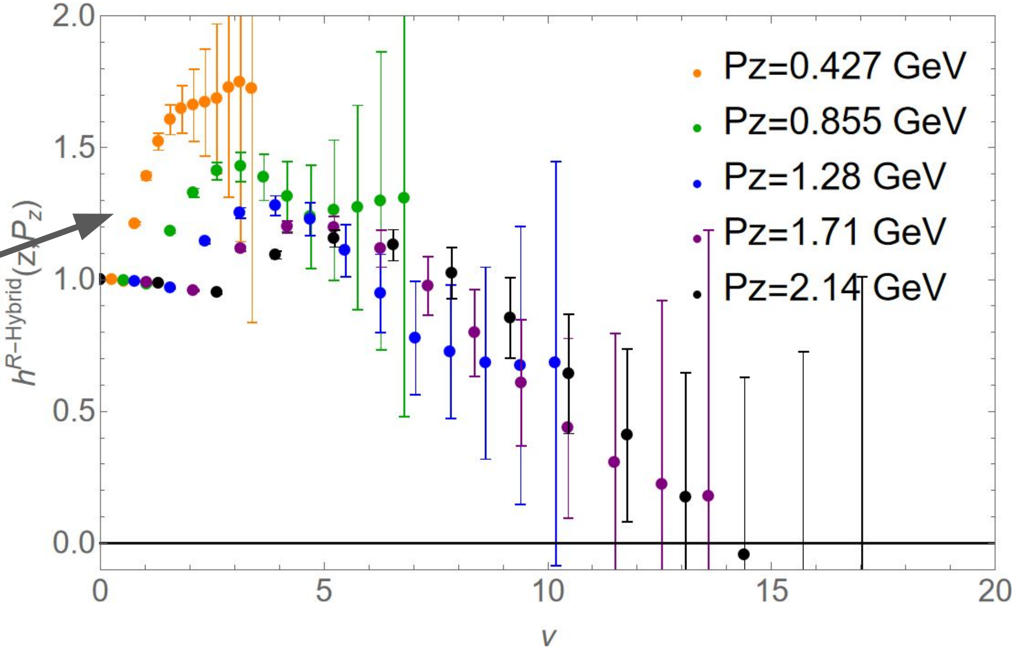

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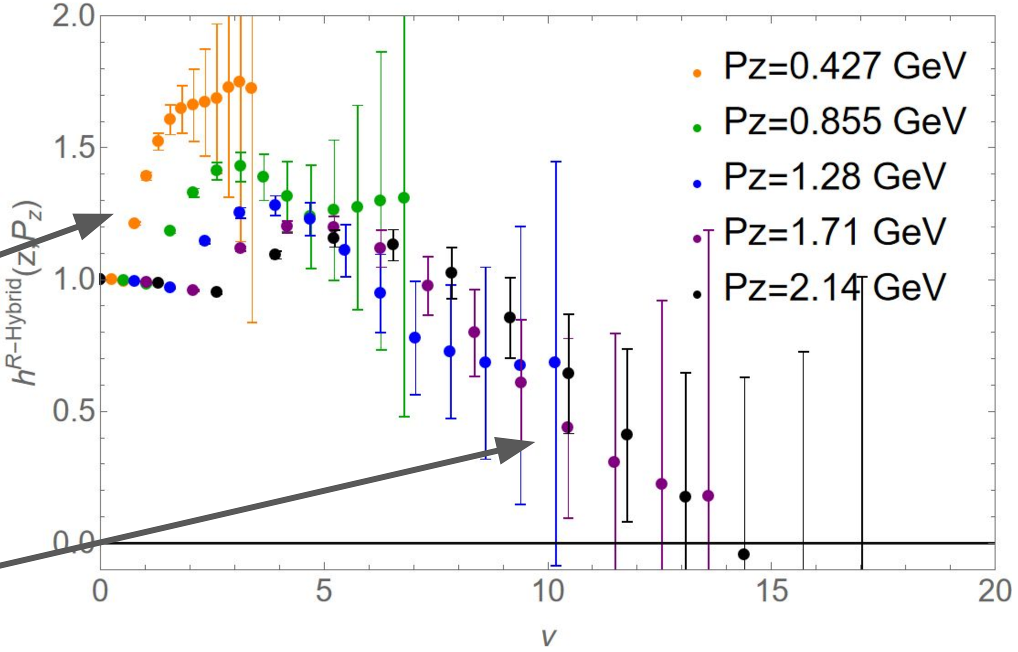

Bump after ν_s as seen in pheno. results

Reasonable signal and seemingly nice decay for P_z = 1.71 GeV. Let's follow this and see what happens

Extrapolating to Large ν

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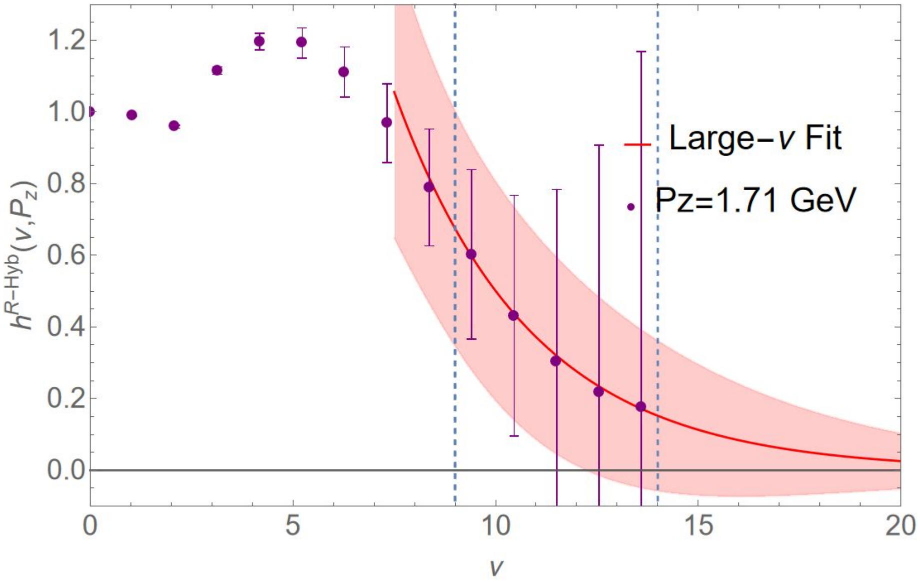

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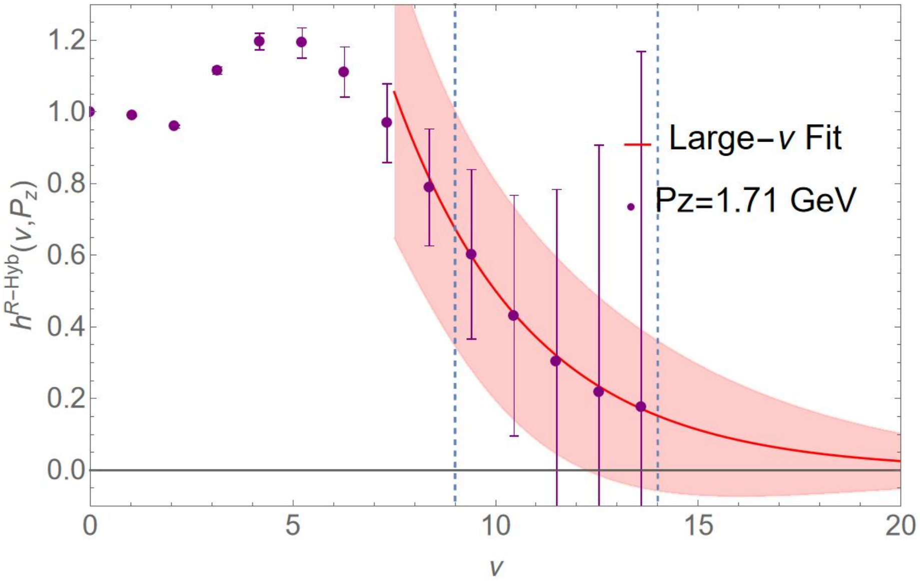

The Imagined Gluon Quasi-PDF

- We interpolate the small-ν data and use the extrapolation after around ν=10 in order to get a Fourier transform of the matrix elements, giving us the first* nucleon gluon quasi-PDF from lattice data

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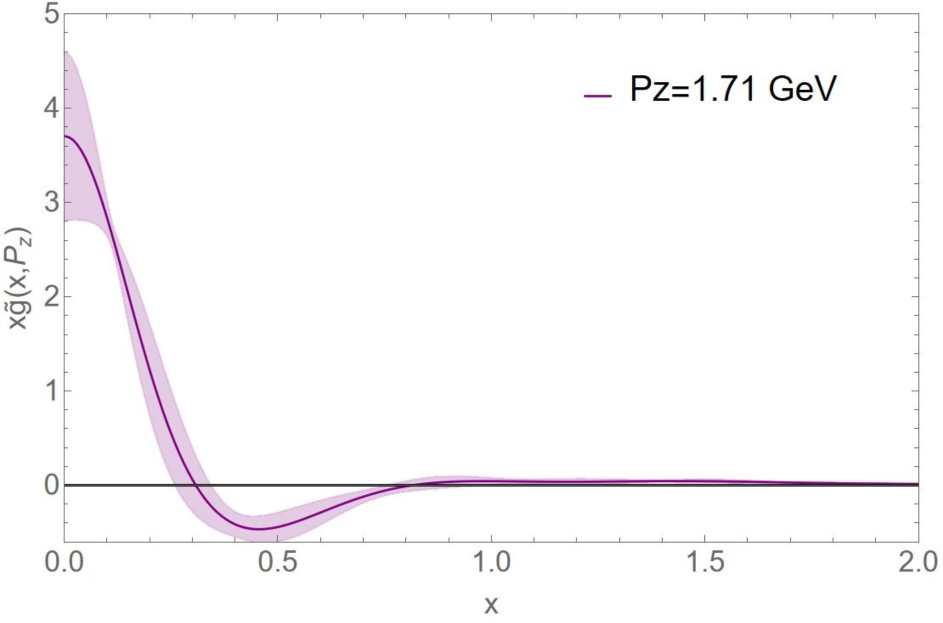

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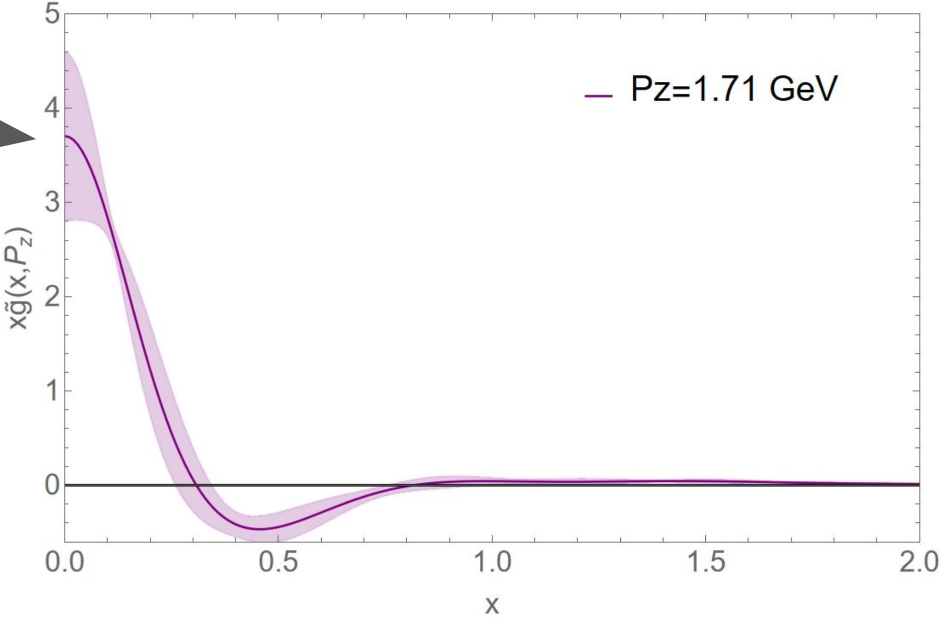

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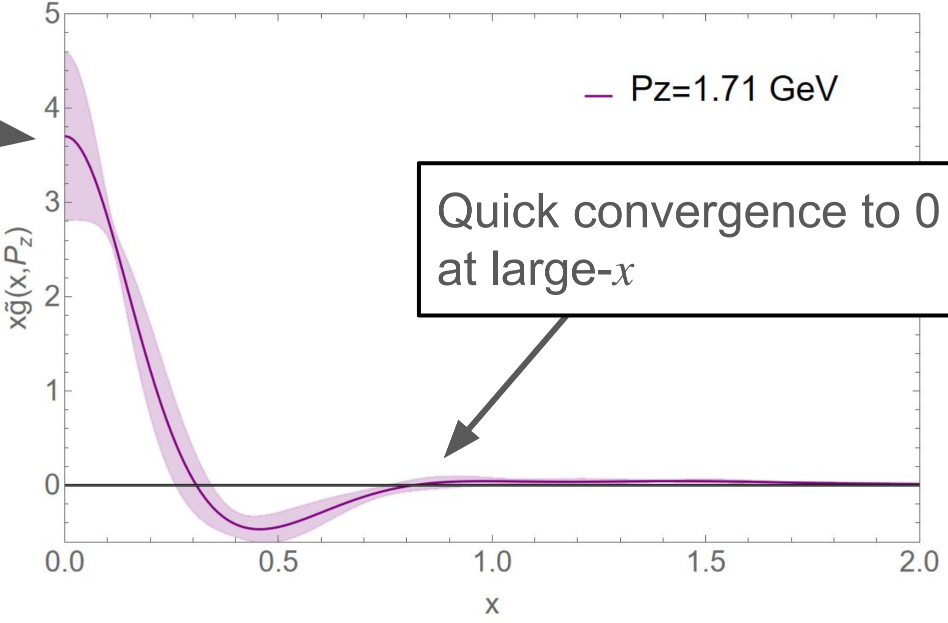

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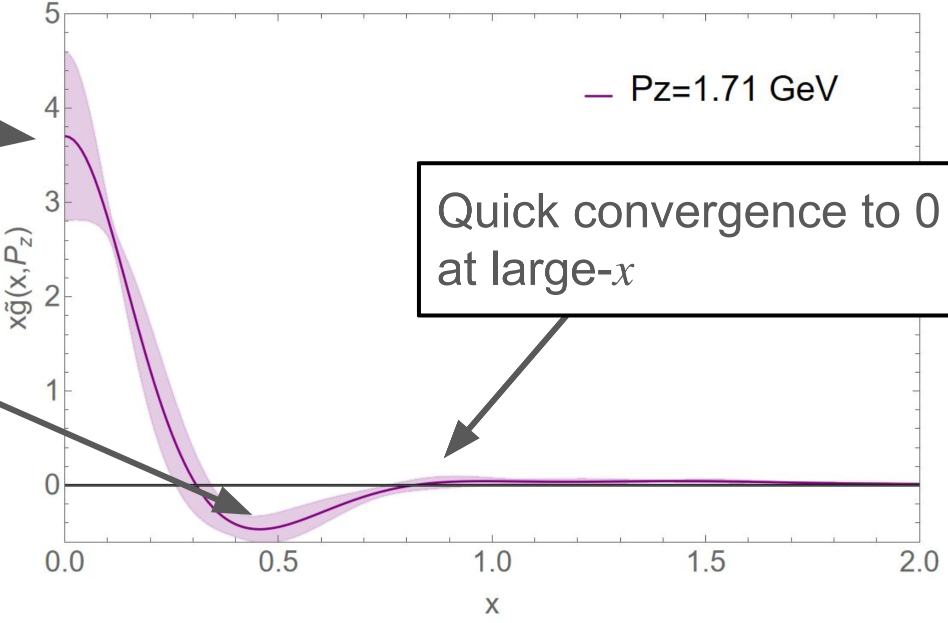
Dip below zero (hopefully this would go away with real hybrid-renorm. or when matching to the light-cone)



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Coulomb Gauge Fixing

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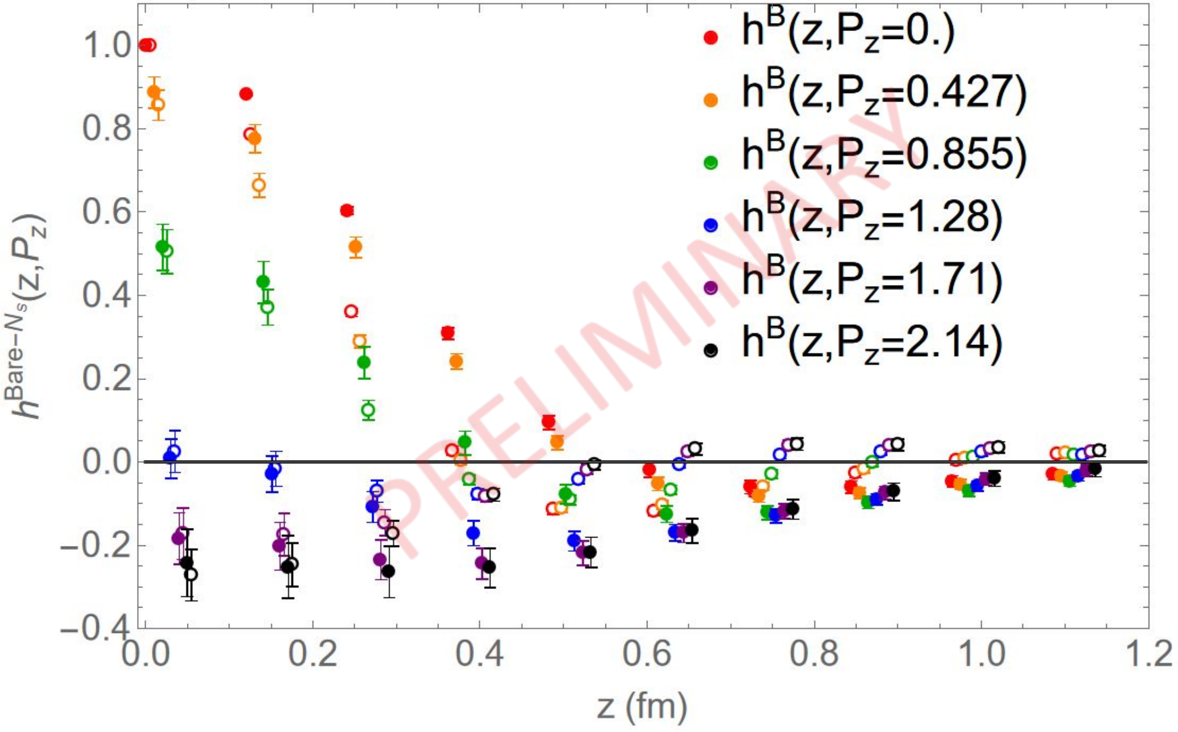
- After smearing, we implement gauge fixing with a precision of 10^{-7} for the calculation of the gluon loops
- As a first look, we only consider the strange nucleon with Wilson-3 smearing for $O^{(1)}(z)$

Bare Matrix Elements

- Bare CG (open circles) and gauge invariant (GI) (closed circles) matrix elements:



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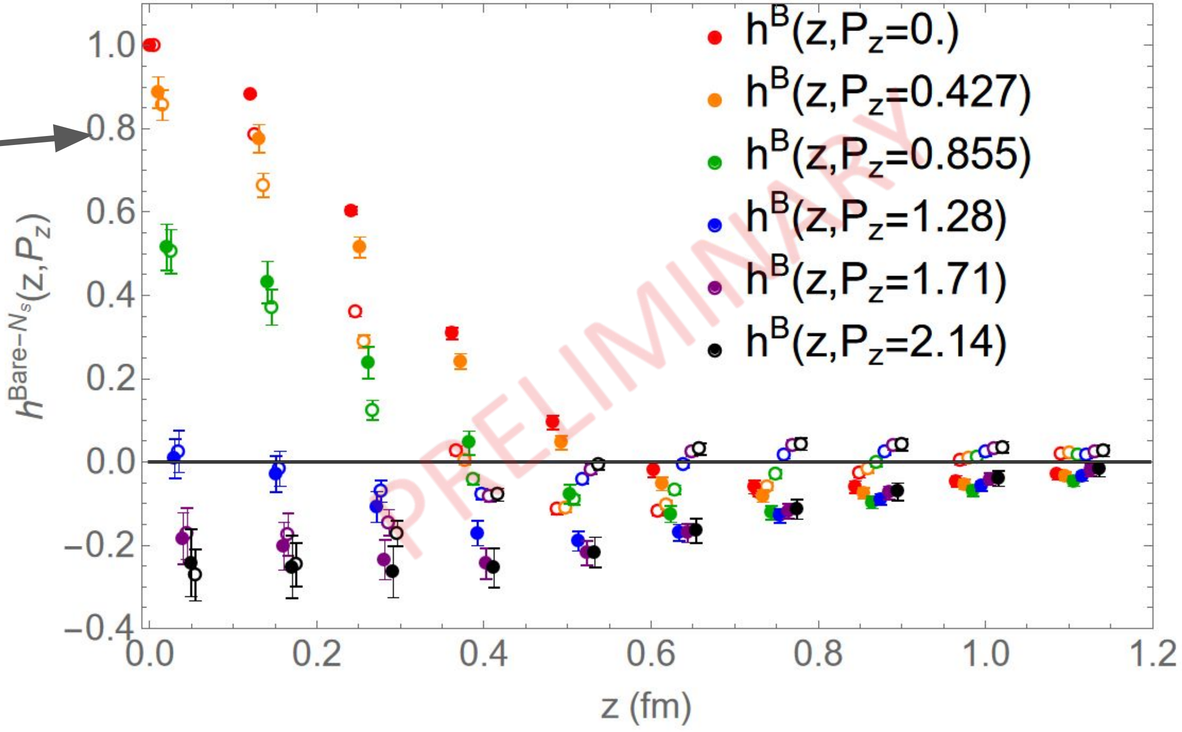
Agreement with local operator



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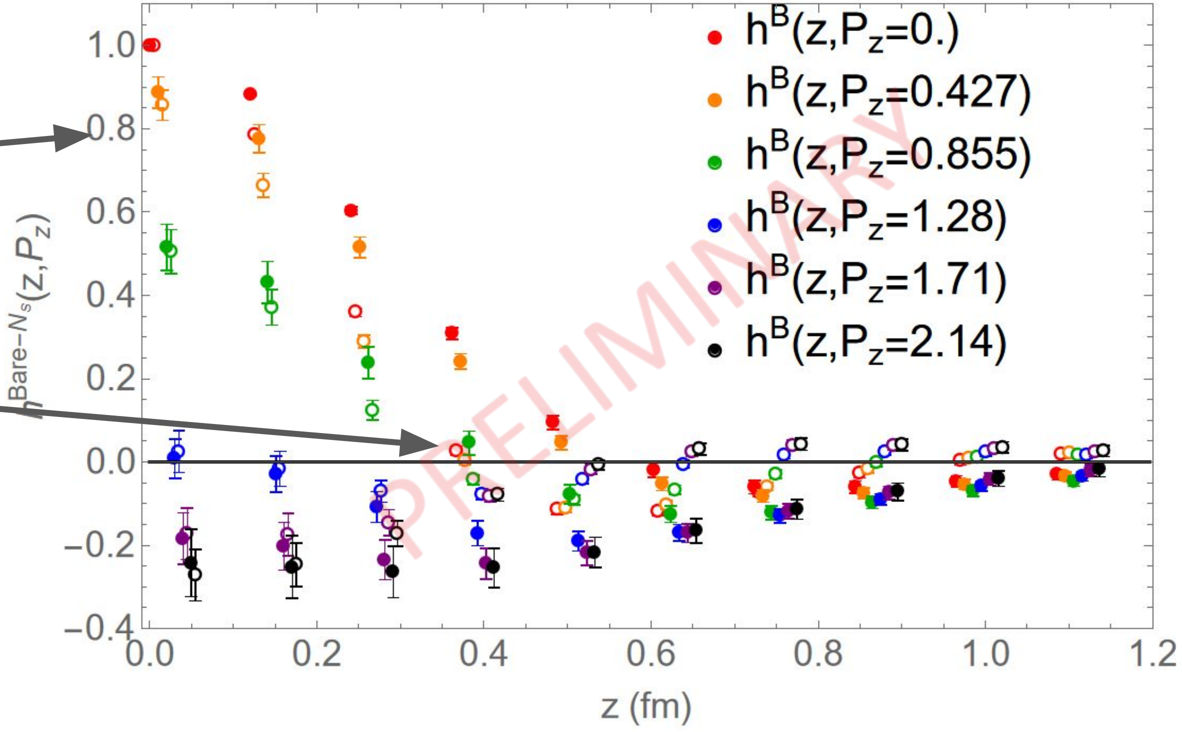

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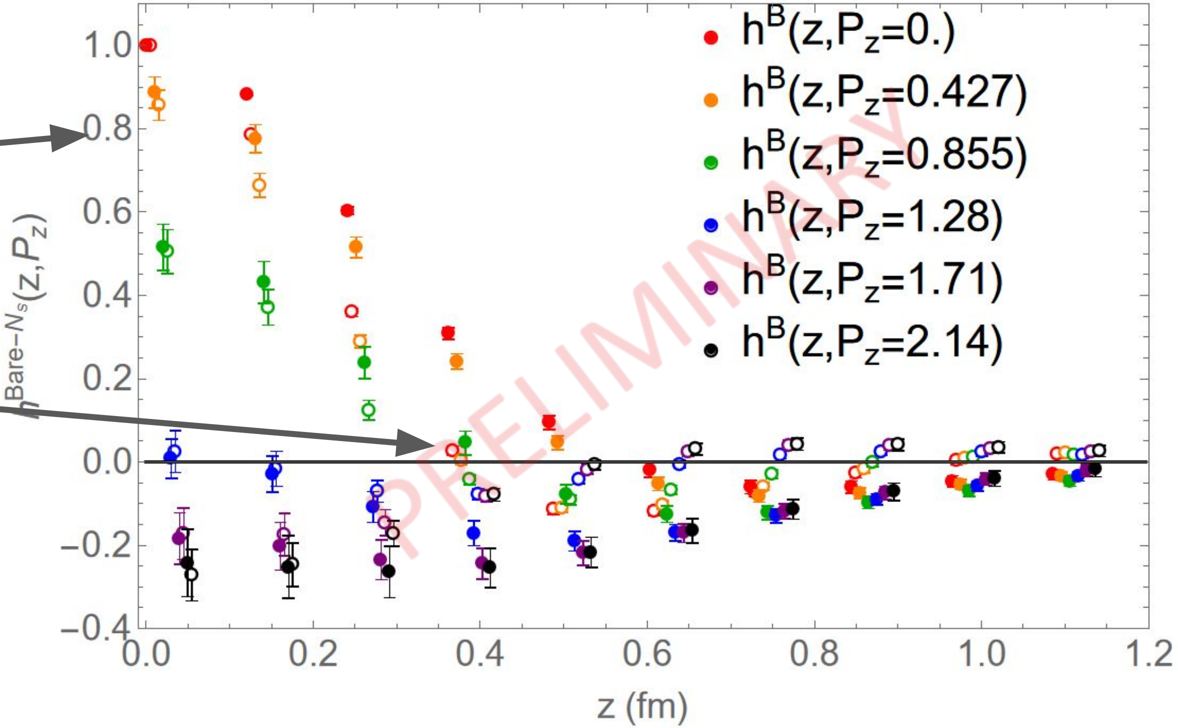
Hard to tell if there's signal improvement here, buth the important part is hybrid renormalization



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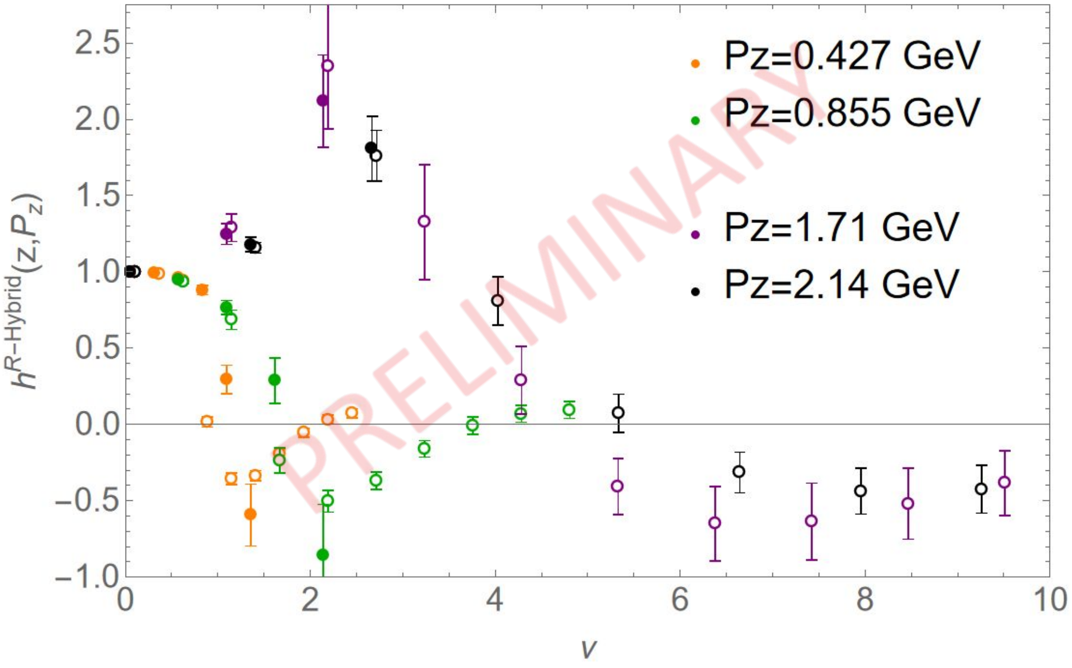

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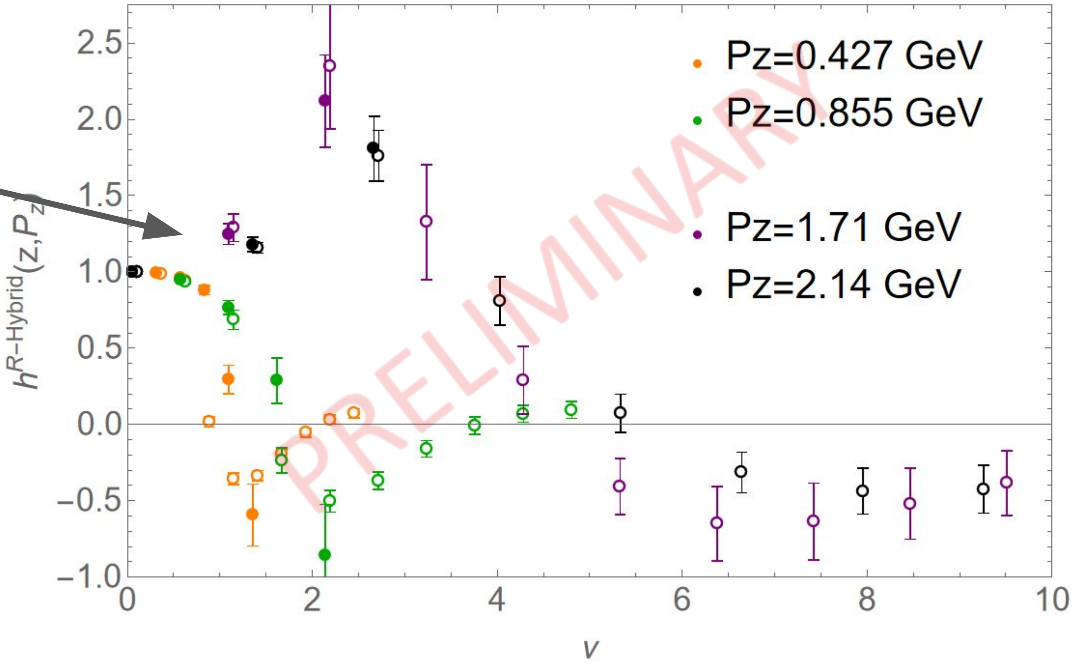

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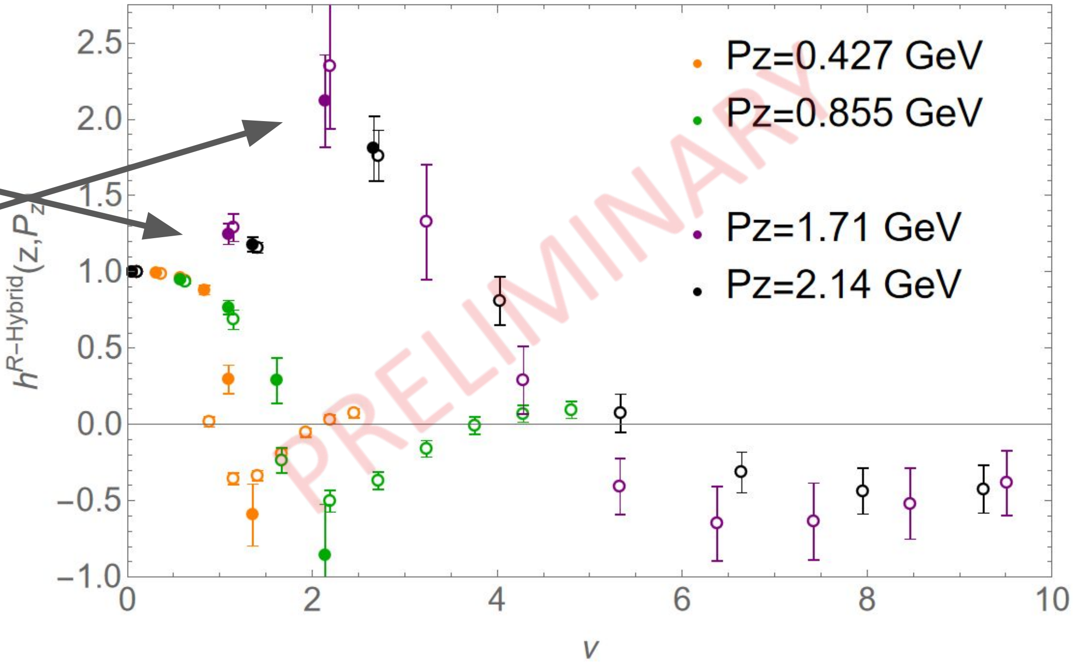

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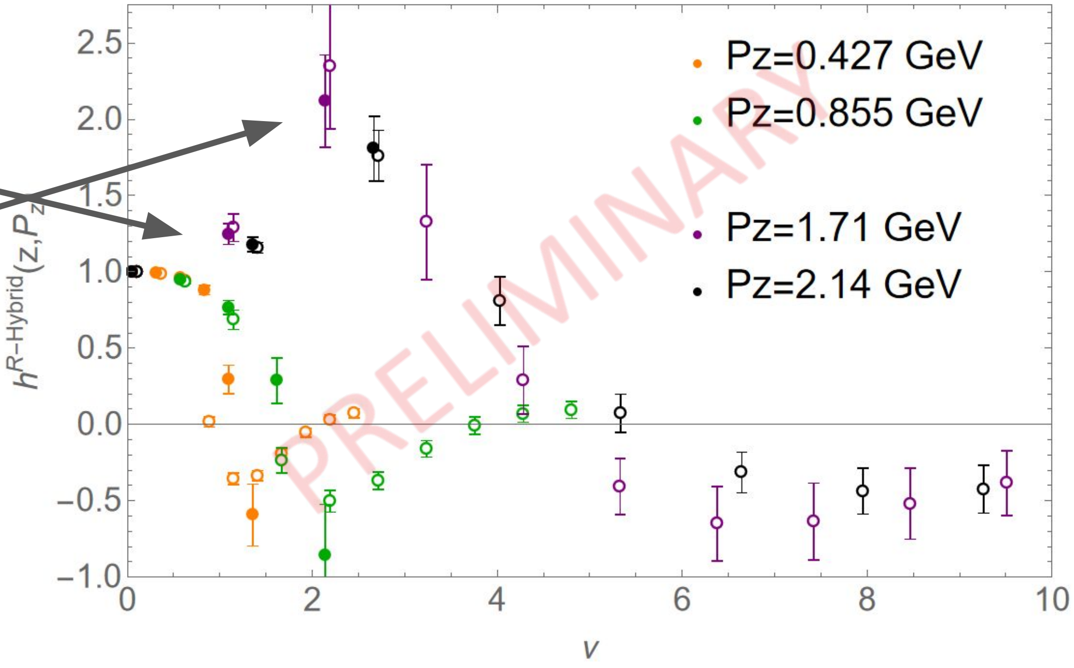
Still a sign change and non-monotonic Pz behavior: Maybe Pz dependent contamination?



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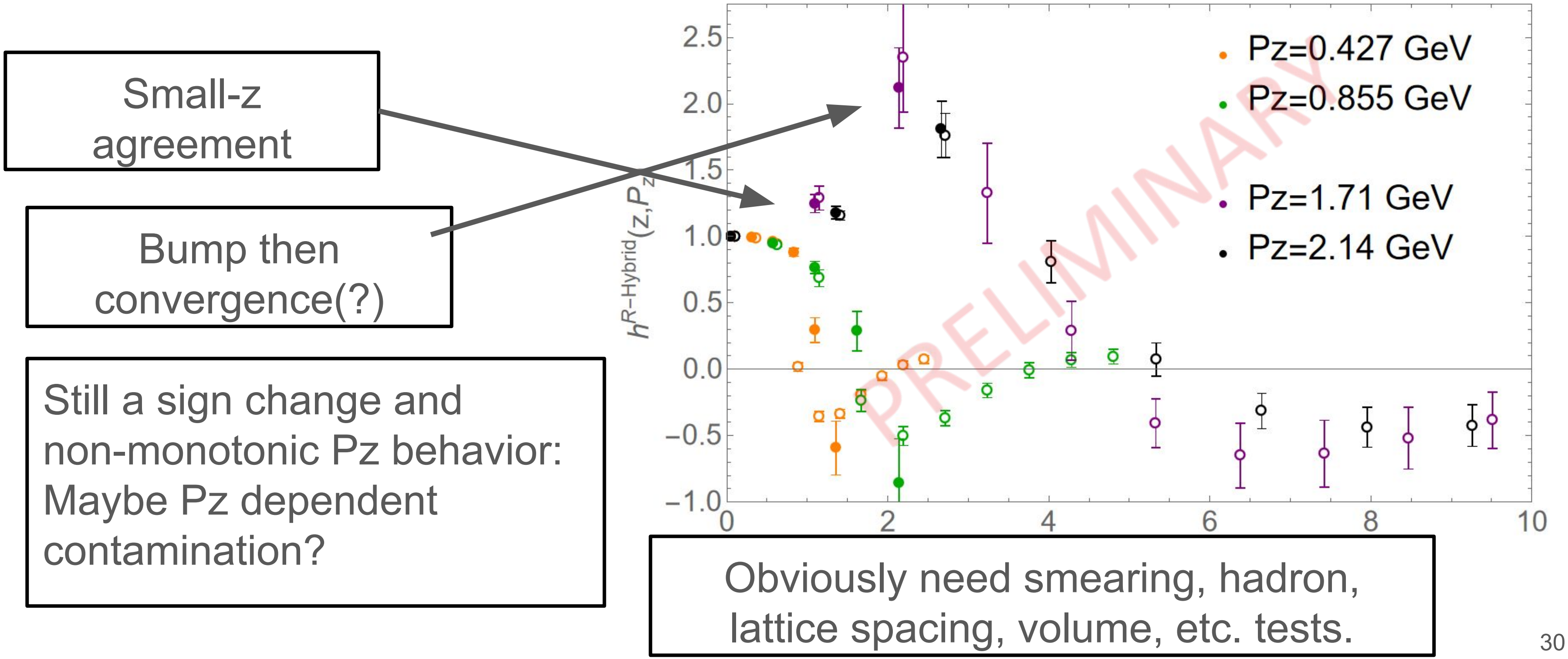

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- We need to (and are) considering further ways to improve signal

Bill's Birthday Wishes

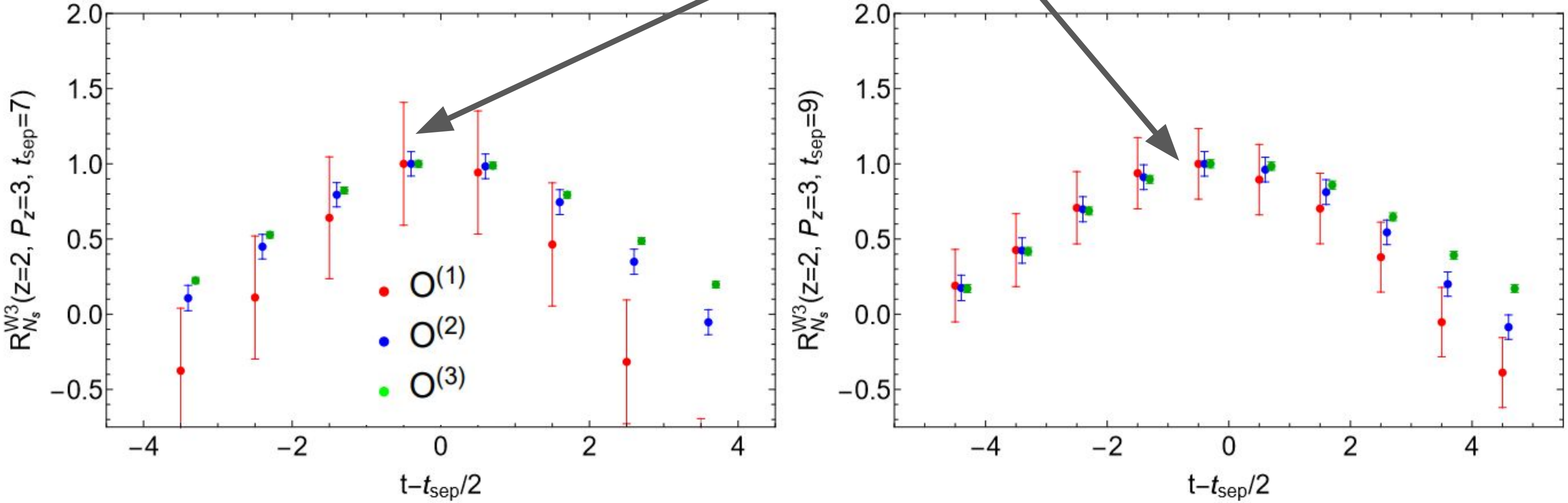
- Hybrid-ratio matching kernels and Wilson coefficients for $O^{(3)}(z)$ (and the other operators not mentioned here)
- Proof (or disprove) that Coulomb gauge fixing is valid for the gluon operators
  - Matching kernels for each operator in this study, if valid
- Any other algorithmic improvements
- A million dollars

Backup

Operator Signal Comparison

- As a first test of which operators are performing better, we can plot the ratio of the 3pt to the 2pt correlators for each operator
- We show the strange nucleon with W3 smearing
- Normalized to such that mean of the center (left) point is 1 so that these can be directly compared

$$R_H(P_z; t_{\text{sep}}, t) = \frac{C_H{}^{3\text{pt}}(P_z; t_{\text{sep}}, t)}{C_H{}^{2\text{pt}}(P_z; t_{\text{sep}})}$$



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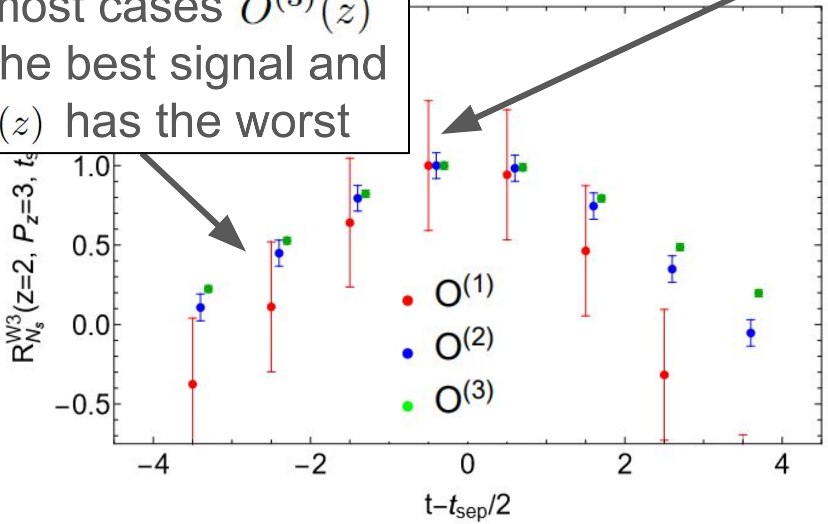
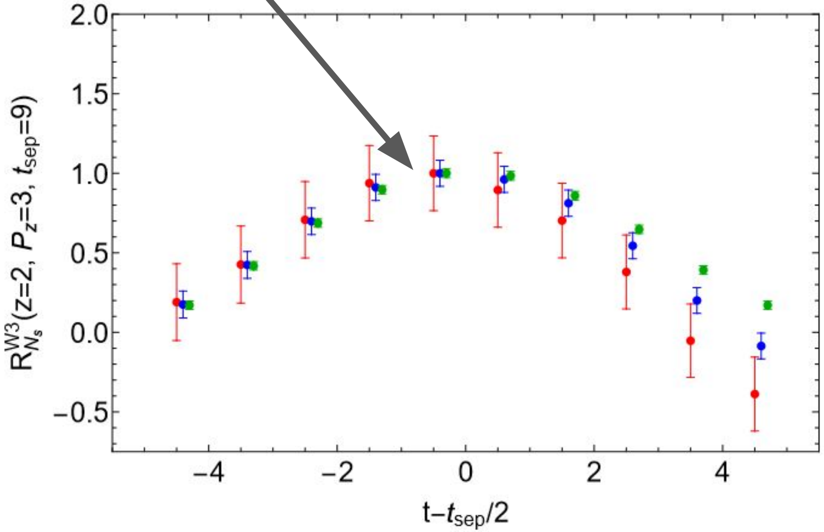
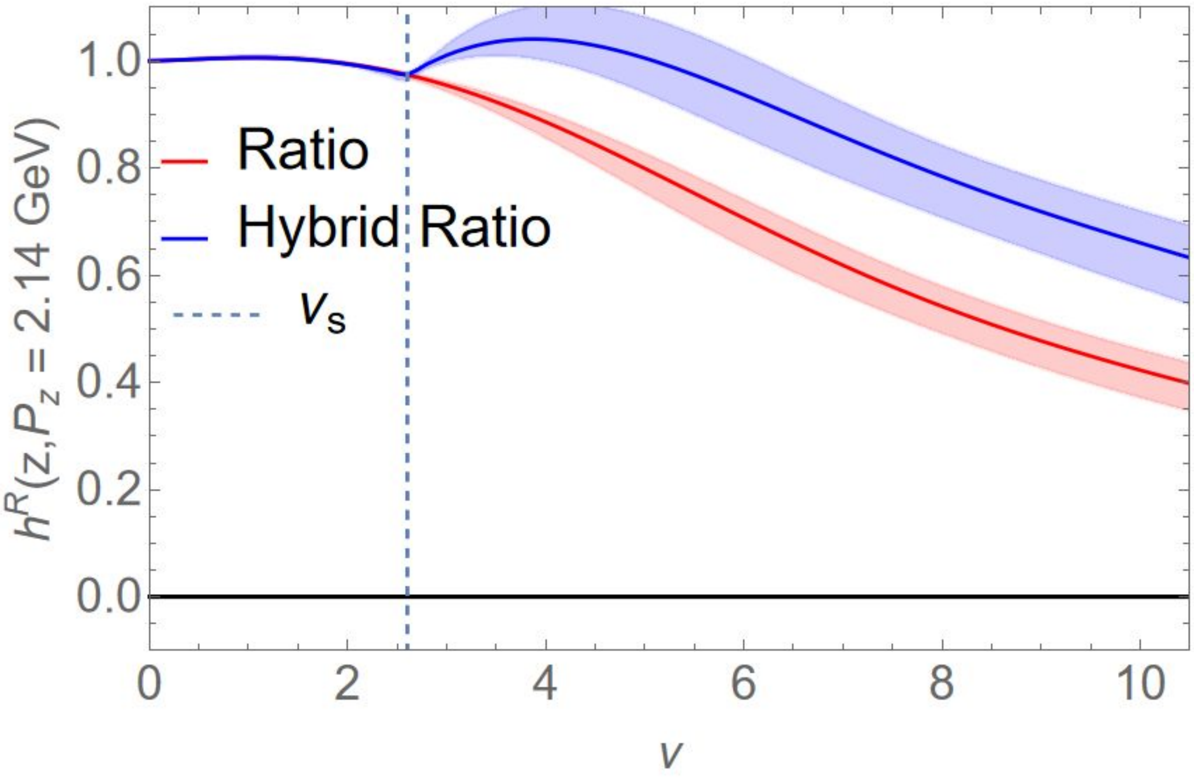
In most cases $O^{(3)}(z)$ has the best signal and $O^{(1)}(z)$ has the worst





Comparing Pheno. Ratio and Hybrid

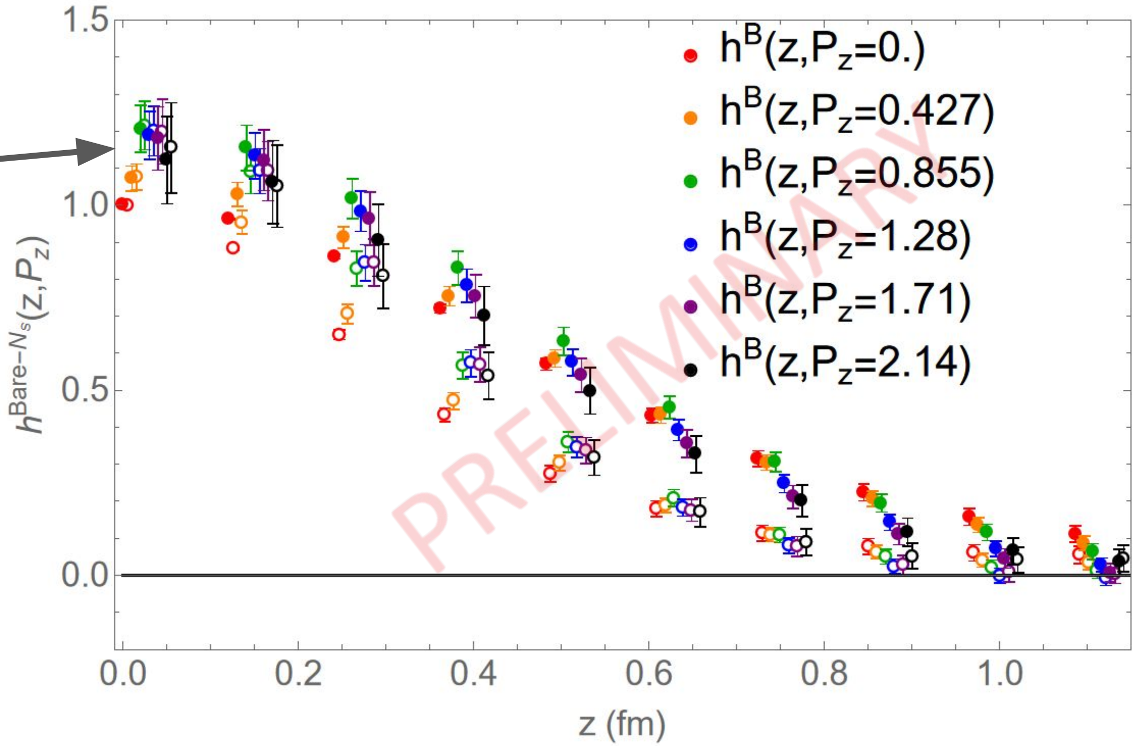


(Also, good agreement with very large z_s)

$O^{(3)}(z)$ Bare Matrix Elements

- Bare CG (open circles) and gauge invariant (GI) (closed circles) matrix elements:

Agreement with local operator



$O^{(3)}(z)$ Hybrid-Ratio Renormalized Matrix Elements

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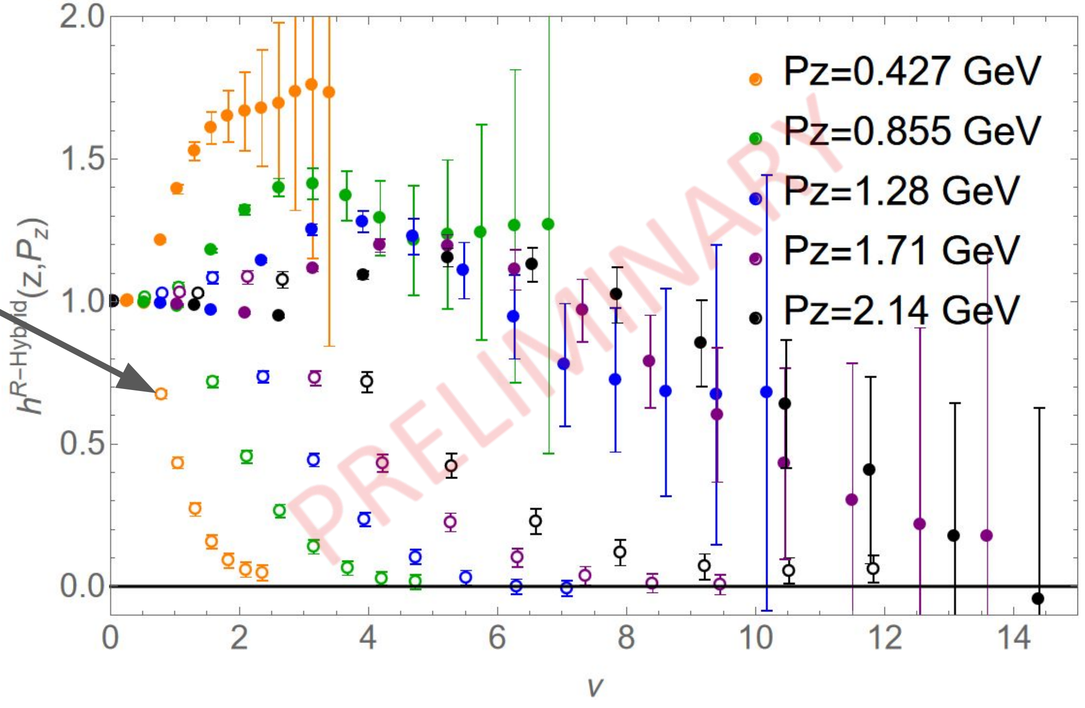
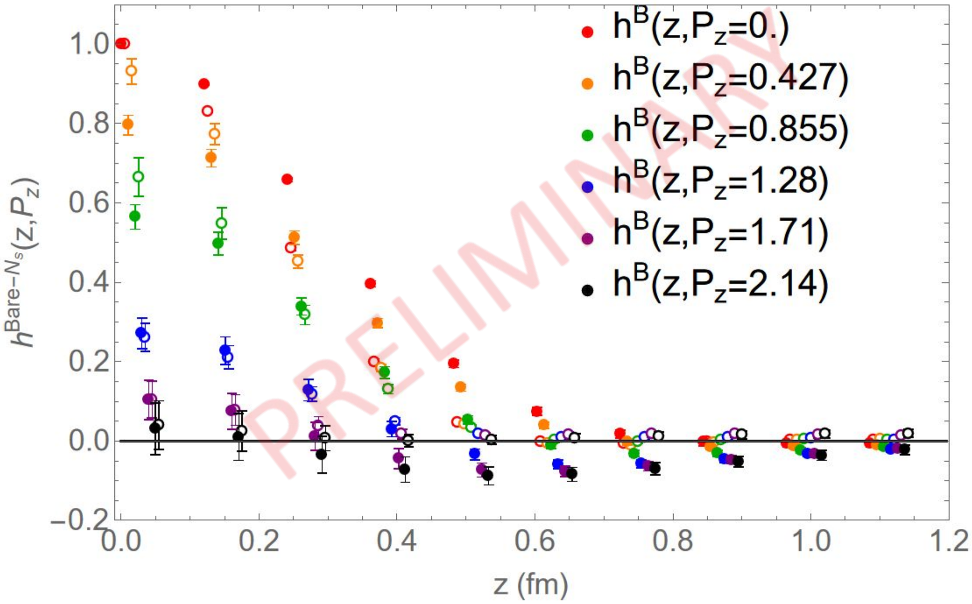
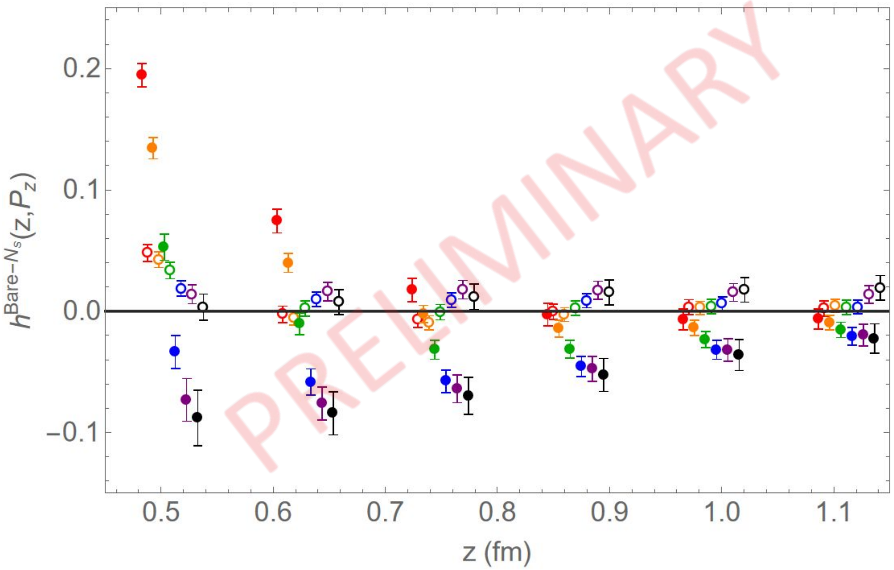
VERY rapid decay?



O^(2) CG Fixing Results

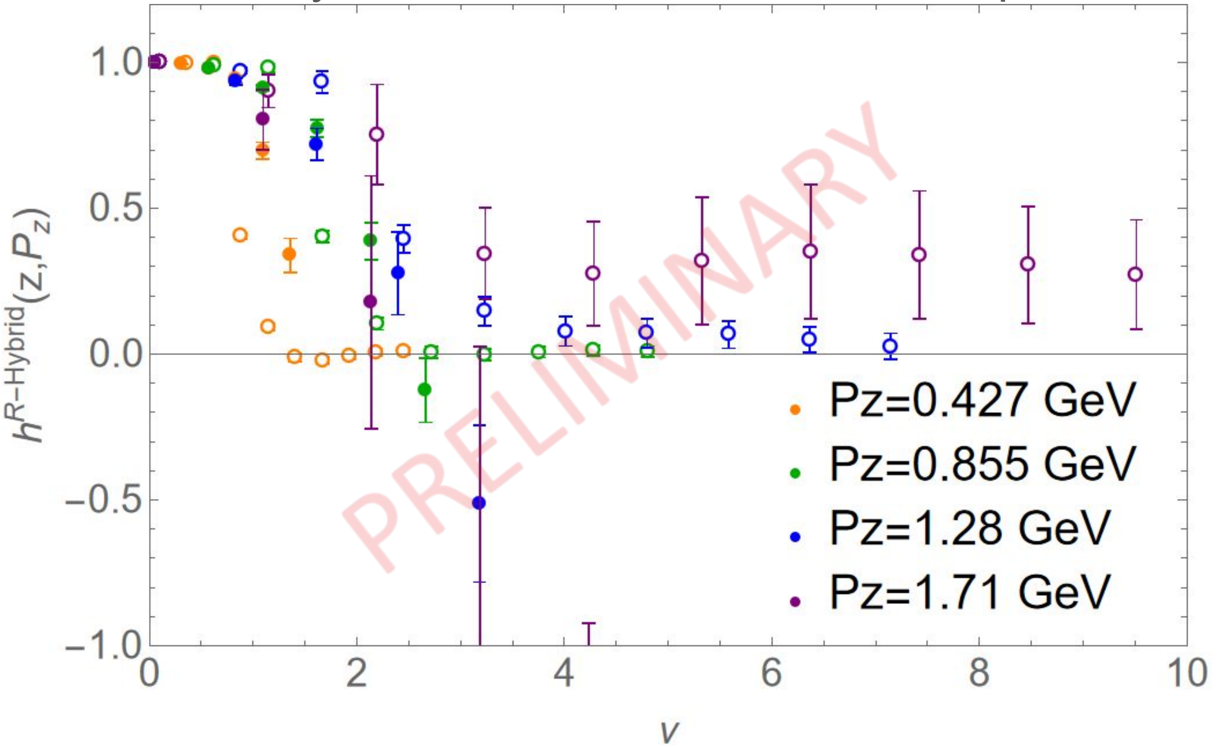
- Less zero crossings





O^(2) CG Fixing Results

Agreement at small-z. Noisy Pz behavior No Pheno. to compare with



Other Concern About O^(1)

Lorentz decomposition of the operator:

$$M_{3i;i3} = 2p_3^2 \mathcal{M}_{pp} + 2z_3^2 \mathcal{M}_{zz} + 2z_3 p_3 \left(\mathcal{M}_{zp} + \mathcal{M}_{pz} \right) - 2\mathcal{M}_{gg} \,, \tag{2.8}$$

No information about the relevant matrix element at P_z = 0

Balitsky, *et al.*, PLB 808:135621 (2020)