Transverse Momentum Distributions from Lattice QCD without Wilson Lines

2024 Meeting on Lattice Parton Physics from Large-Momentum Effective Theory (LaMET 2024)
Aug 13, 2024

YONG ZHAO

Xiang Gao, Wei-Yang Liu and Yong Zhao, Phys.Rev.D 109 (2024),
Yong Zhao, arXiv: 2311.01391.

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NATIONAL LABORATORY

Outline

- **Introduction**
  - Large-Momentum Effective Theory approach to TMDs
- **Coulomb gauge method**
  - Universality class in LaMET
  - Coulomb-gauge quasi-TMD
  - Large-momentum factorization formula
  - Soft function
  - Transverse gauge link
- **Discussions**

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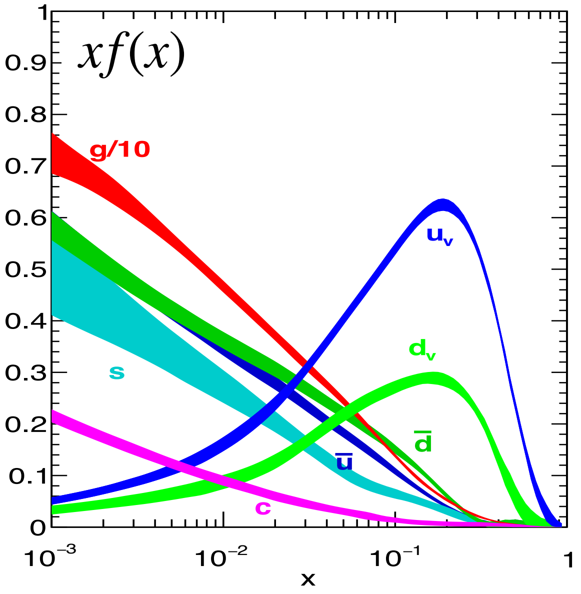
3D imaging of the proton

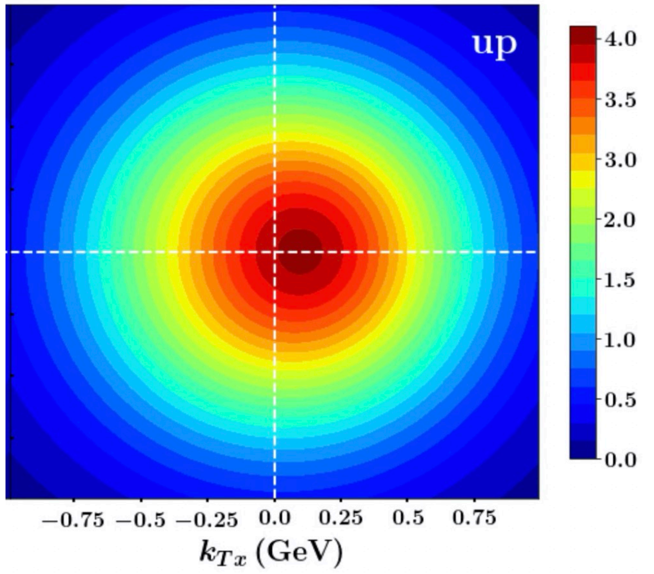
PDFs



NNPDF, EPJ C77 (2017)

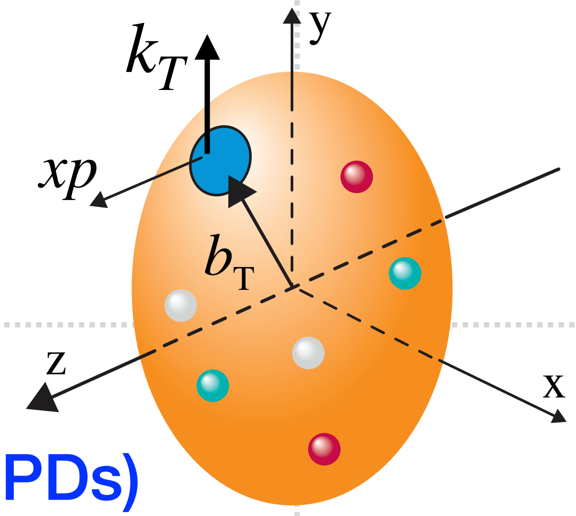
$\int d^2\vec{k}_T$

Transvers momentum distributions (TMDs)



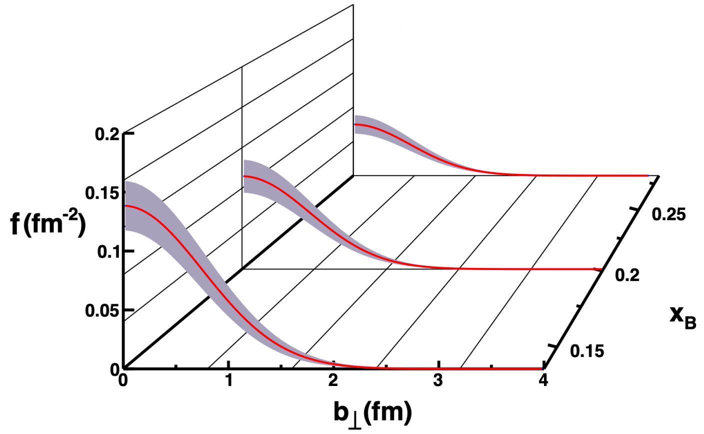
Cammarota, et al. (JAM), PRD **102** (2020).

$\int d^2\vec{b}_T$



$\int d^2\vec{b}_T$

Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

$\int d^2\vec{k}_T$

Wigner distributions/Generalized TMDs

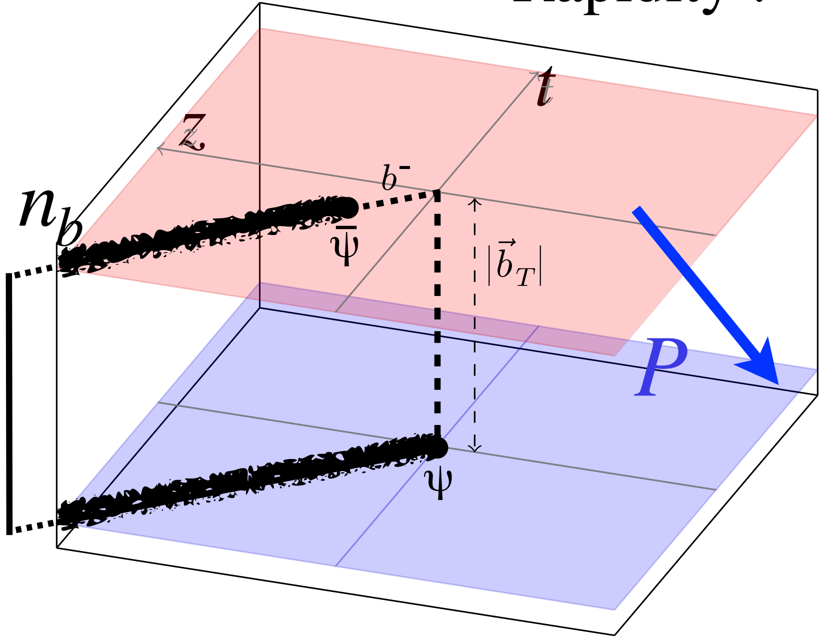
$$W(x, \vec{k}_T, \vec{b}_T)$$

Transverse Momentum Distributions (TMDs)

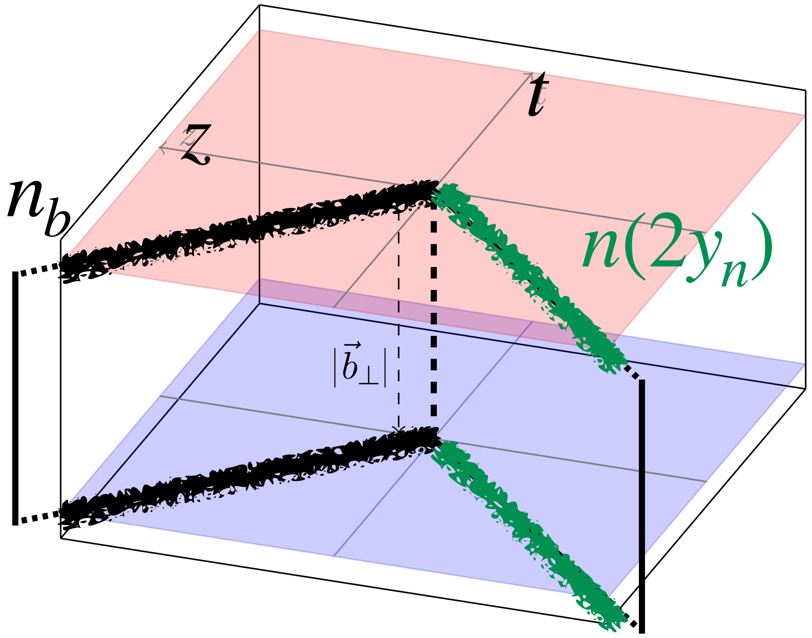
- Beam function:
- Soft function :

$$n_b^2 = 0$$

$$\text{Rapidity}: \quad y_B = \frac{1}{2}\ln\left|\frac{n_b^+}{n_b^-}\right| = -\infty$$



Hadronic matrix element



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\text{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

Transverse Momentum Distributions (TMDs)

- Beam function:
- Soft function :

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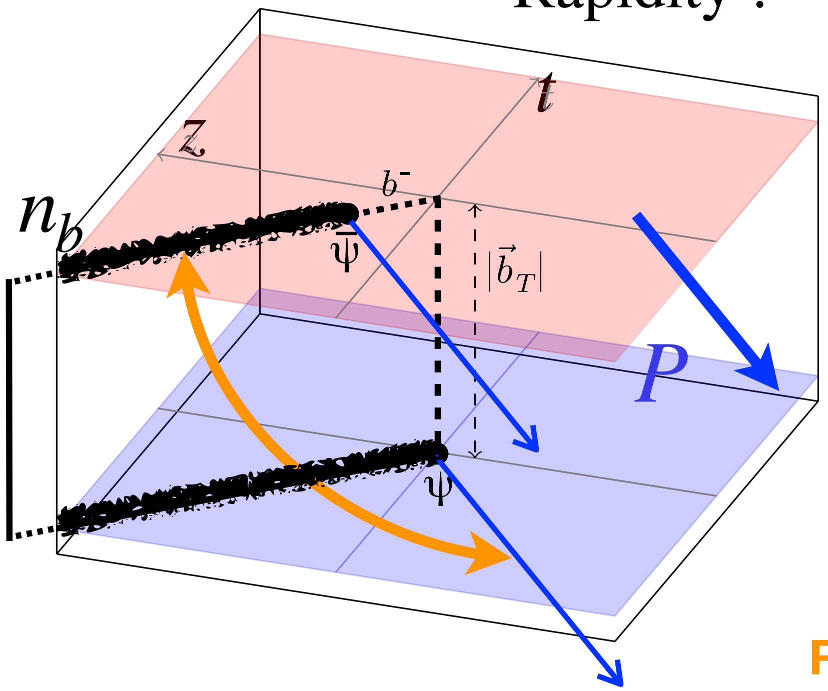
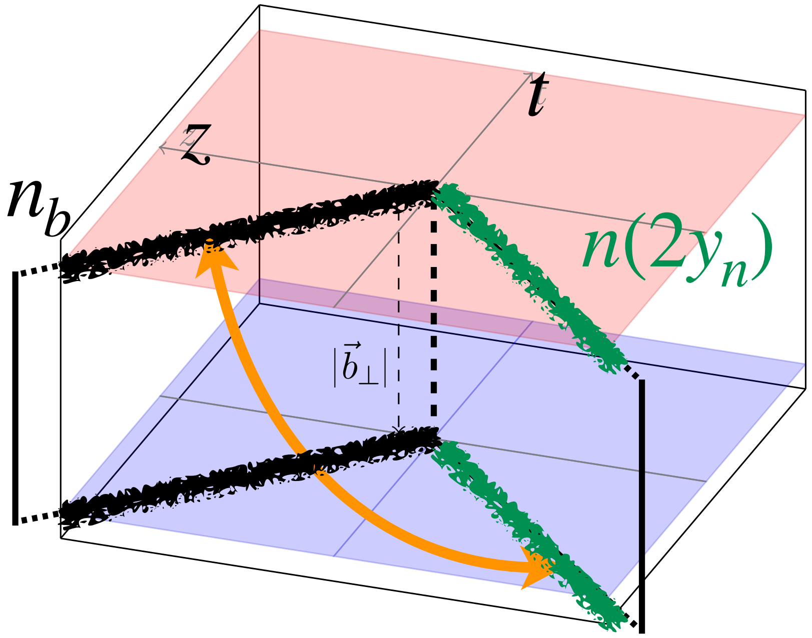


Hadronic matrix element

Rapidity divergences



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

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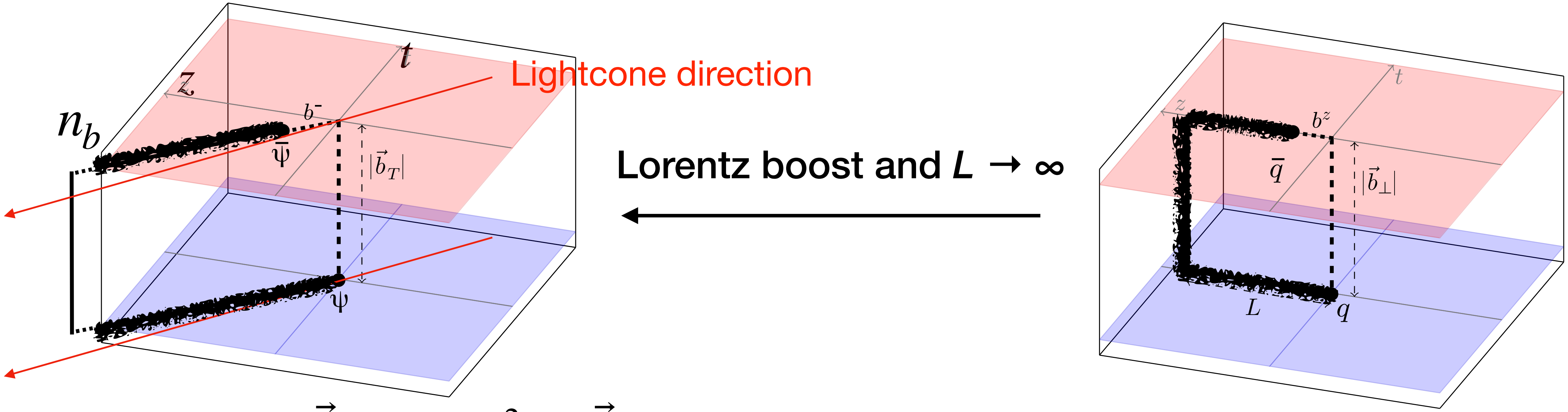
Rapidity divergence regulator

TMDs from LaMET

- Beam function (in Collins scheme):
- Quasi beam function :



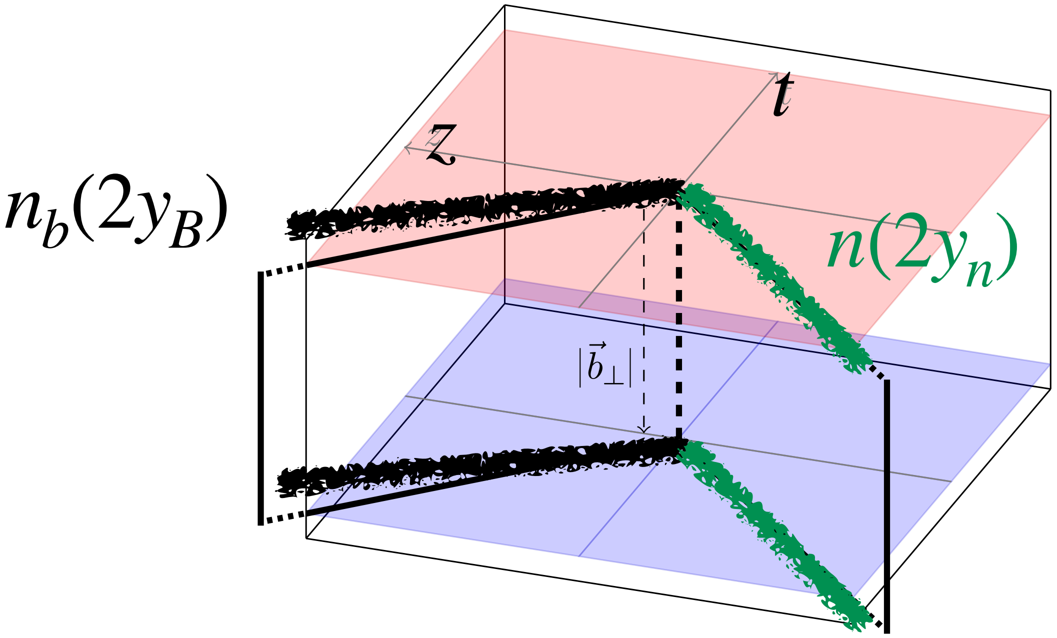
$$n_b^\mu(y_B) = (n_b^+, n_b^-, \vec{0}_\perp) = (-e^{2y_B}, 1, \vec{0}_\perp)$$

Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹

Equal-time Wilson lines, directly calculable on the lattice🙂

Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

Soft factor



Collins-Soper kernel

$$\xrightarrow{y_n - y_B \to \infty} S_r(b_T, \mu)\ e^{-2(y_n - y_B)\gamma_\zeta(b_T,\mu)}$$

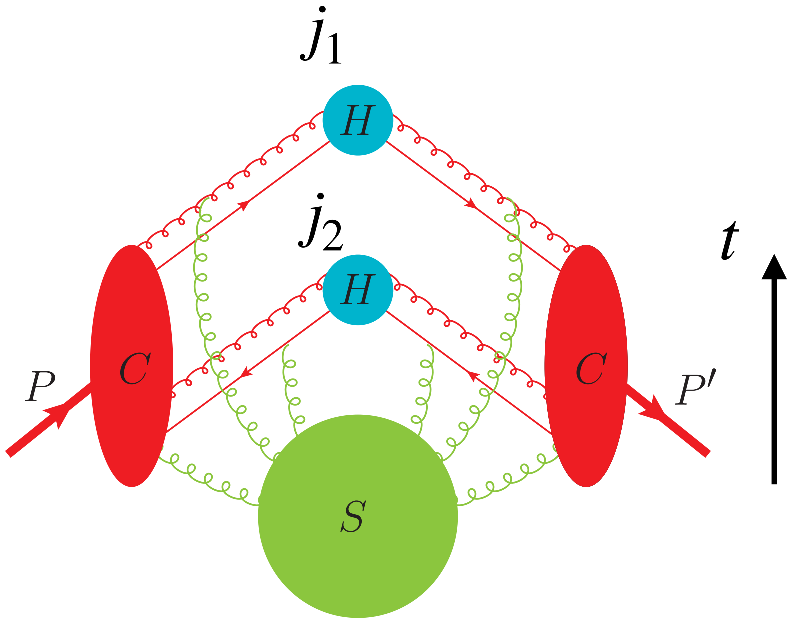
Reduced soft factor

Light-meson form factor:

$$F(b_T, P^z) = \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle$$

$$\overset{P^z \gg m_N}{=} S_r(b_T, \mu) \int dx dx'\ H(x, x', \mu)$$

$$\times\, \Phi^\dagger(x, b_T, P^z, \mu) \Phi(x', b_T, P^z, \mu)$$



$\Phi(x, b_T, P^z, \mu)$: **quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\big(\mu, x\tilde{P}^z\big) \ \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

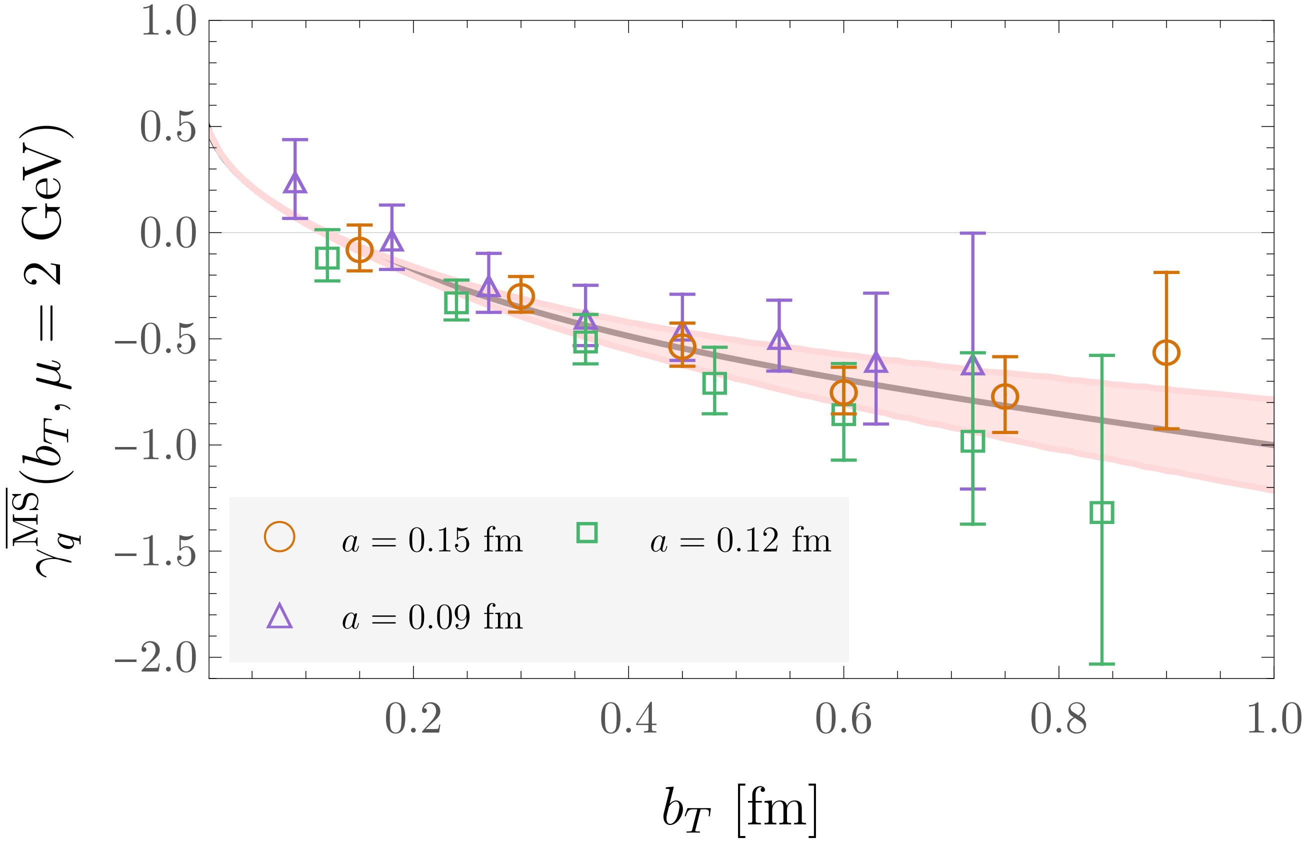
$$\gamma_\zeta(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\big(\mu, x\tilde{P}^z\big)}$$

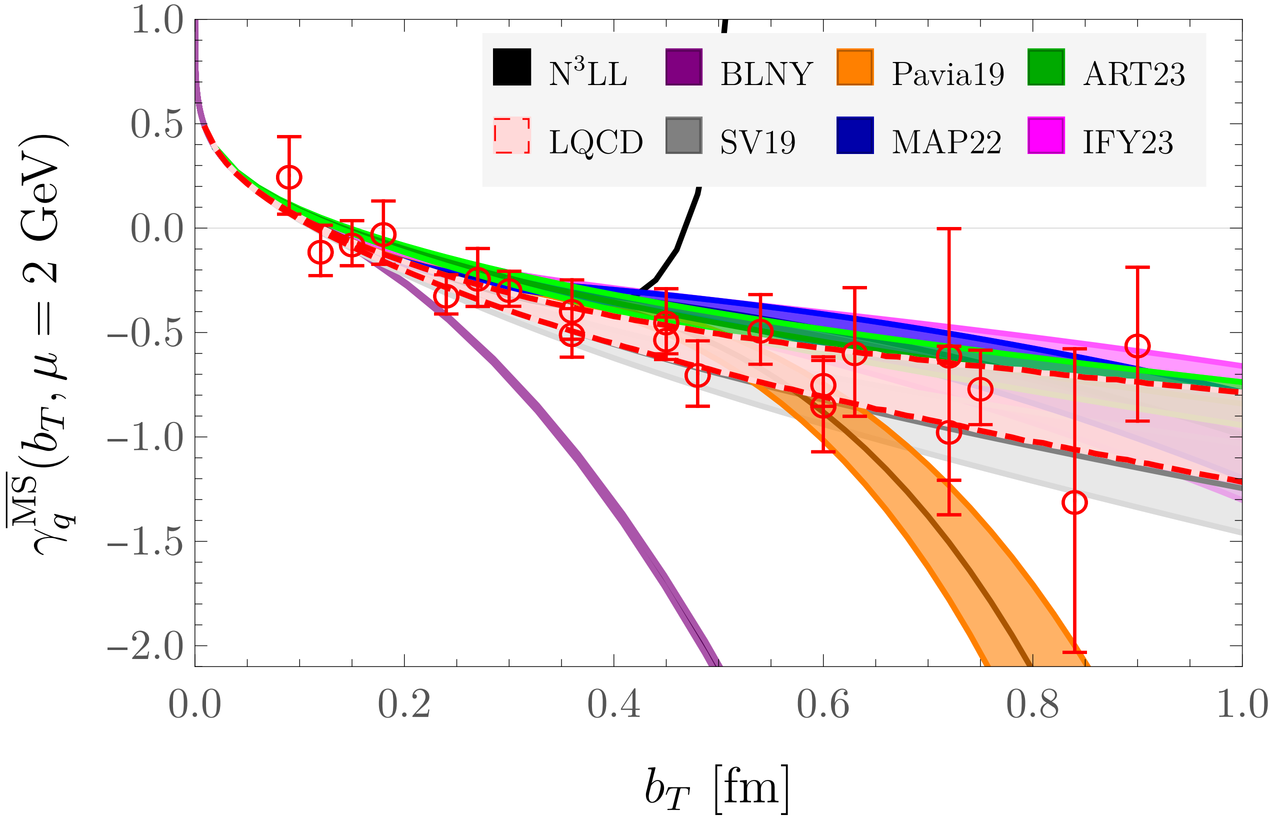
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

State-of-the-art determination of the Collins-Soper kernel

- Physical quark masses
- Continuum limit with a = 0.15, 0.12, 0.09 fm
- Controlled renormalization and Fourier transform
- Next-to-next-to-leading logarithmic (NNLL) order

See Artur Avkhadiev's talk.

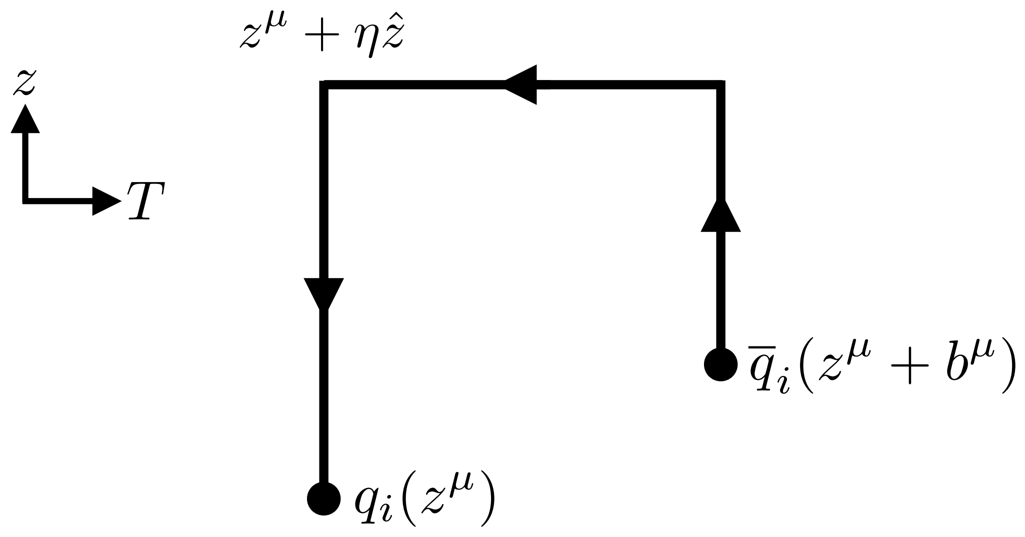




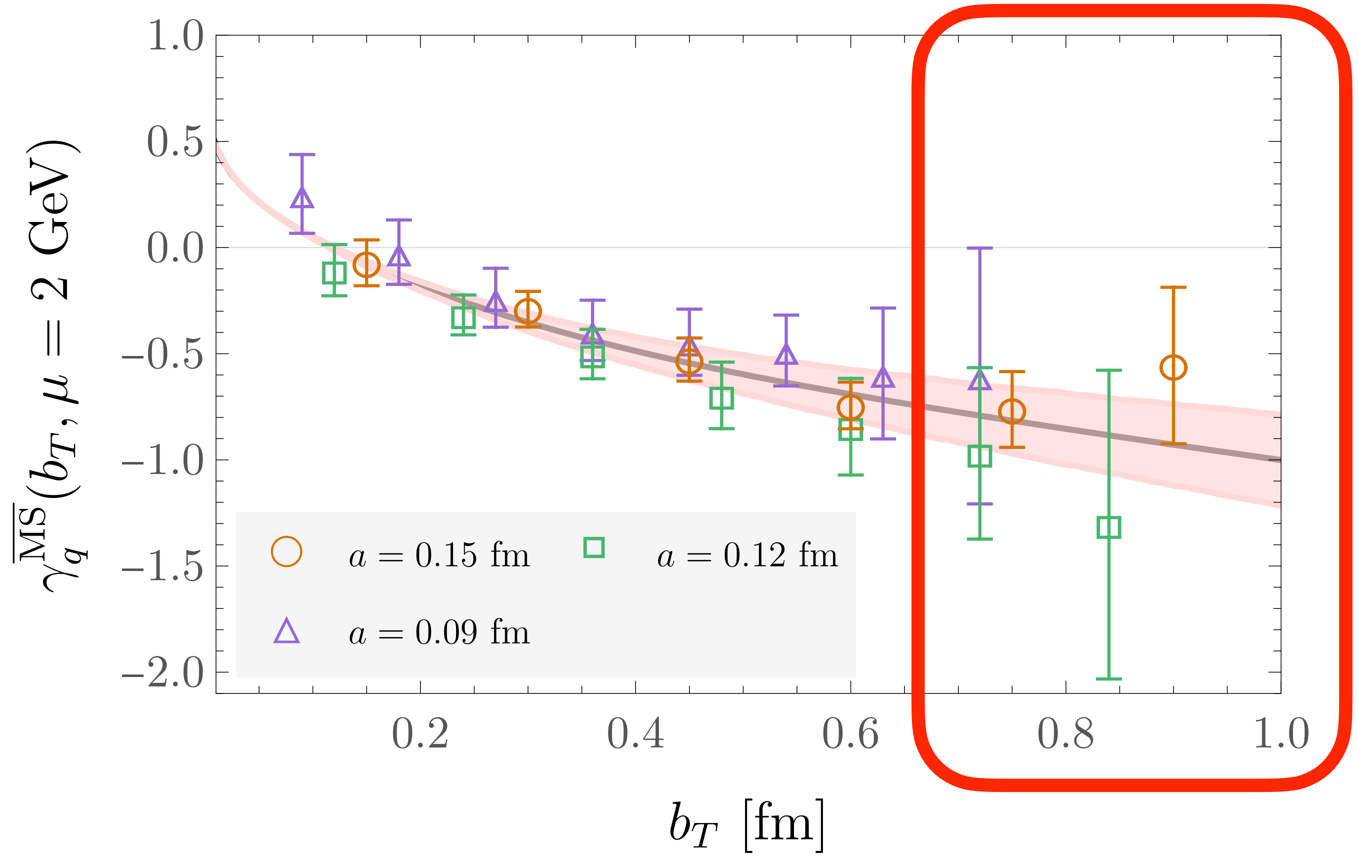
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, Phys.Rev.D **108** (2023);
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, 2402.06725, PRL **132** (2024).

Systematics in lattice calculation

Staple-shaped Wilson line



$$\eta \gg \{b^z, b_T\},\ xP^z \gg 1/b_T$$



- Signal-to-noise ratio suppressed suppressed by the linear divergences, which becomes worse at larger b_T. Smearing or gradient flow is required to reach reasonable precision;

- Operator mixings induced by the staple geometry, which are currently subtracted with RI-type renormalization schemes.

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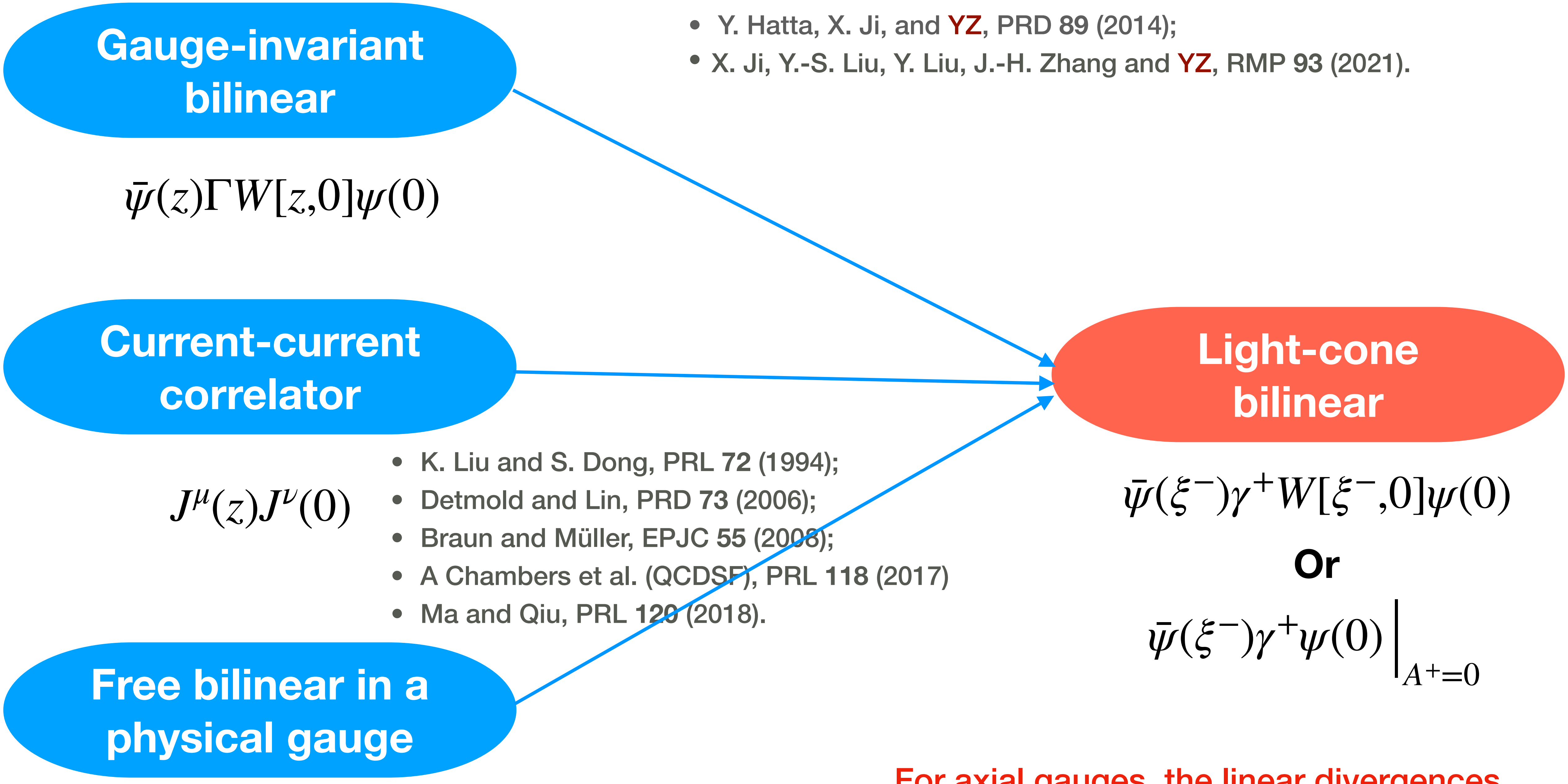
Universality in LaMET



- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Gauge-invariant bilinear

$$\bar{\psi}(z)\Gamma W[z,0]\psi(0)$$

Current-current correlator

$$J^\mu(z)J^\nu(0)$$

- K. Liu and S. Dong, PRL **72** (1994);
- Detmold and Lin, PRD **73** (2006);
- Braun and Müller, EPJC **55** (2008);
- A Chambers et al. (QCDSF), PRL **118** (2017)
- Ma and Qiu, PRL **120** (2018).

Free bilinear in a physical gauge

$$\bar{\psi}(z)\Gamma\psi(0)\Big|_{G(A)=0} \qquad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}$$

Light-cone bilinear

$$\bar{\psi}(\xi^-)\gamma^+W[\xi^-,0]\psi(0)$$

Or

$$\bar{\psi}(\xi^-)\gamma^+\psi(0)\Big|_{A^+=0}$$

For axial gauges, the linear divergences still appear in the quark self-energy 🙁

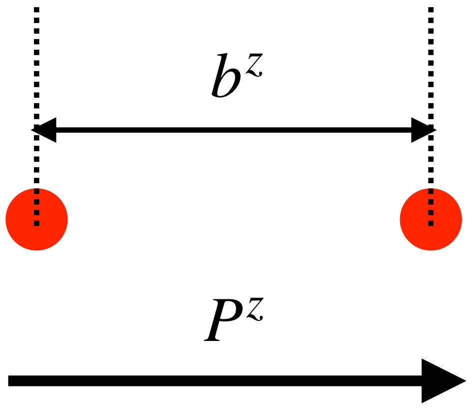
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Quasi-TMD in the Coulomb gauge

$$\tilde{h}(\vec{b}, \vec{P}, \mu) = \frac{1}{2P^t}\langle P | \bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0} | P\rangle$$

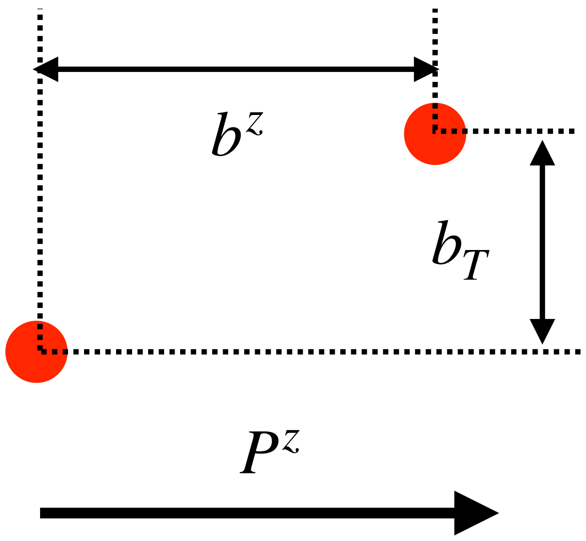
$$\tilde{f}(x, b_T, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{db^z}{2\pi}\, e^{ixP^z b^z}\, \tilde{h}(b^z, b_T, P^z, \mu)$$

Quasi-PDF



X. Gao, W.-Y. Liu and YZ, Phys.Rev.D 109 (2024)

Quasi-TMD



YZ, arXiv: 2311.01391.

Quasi-TMD under the infinite boost

$$\tilde{h}(\vec{b}, \vec{P}, \mu) = \frac{1}{2P^t}\langle P|\,\bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|P\rangle$$

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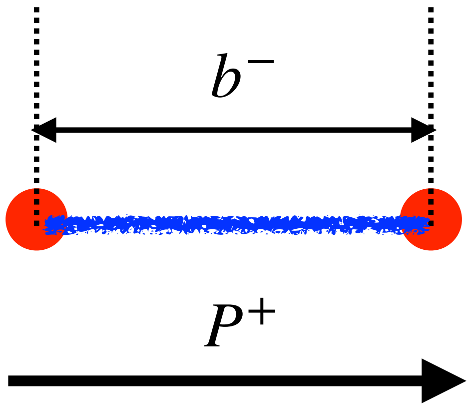
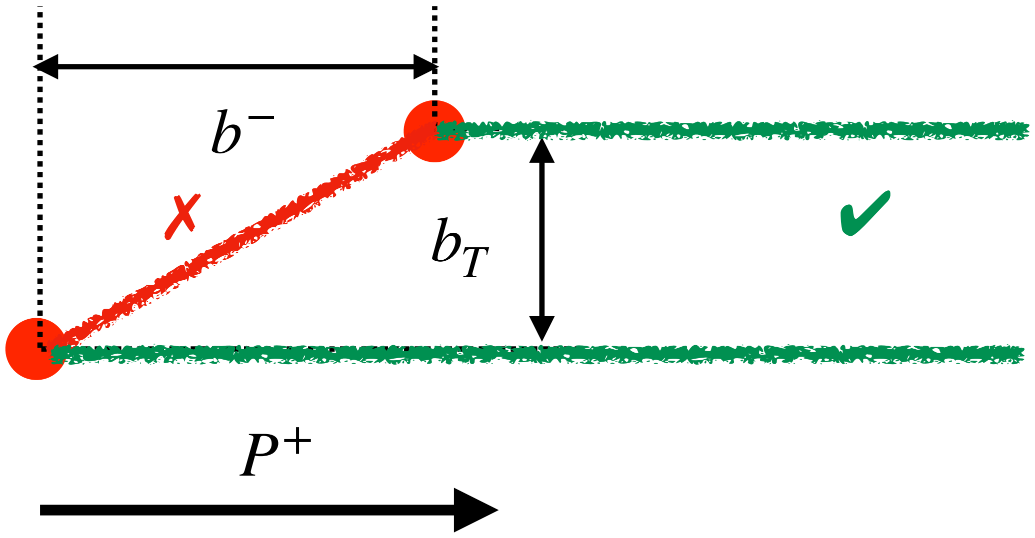
PDF



TMD



Parton distributions probe the correlation of energetic quarks and gluons, which can be formulated by fixing a physical gauge condition.

$$G(A) = 0, \quad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}, A^+$$

Quasi-TMD under the infinite boost

- **Gauge-invariant extension:**

P. A. M. Dirac, Can. J. Phys. **33** (1955);
M. Lavelle and D. McMullan, Phys. Rept. **297** (1997).

$$\Psi_C(x) = U_C(x)\psi(x)$$

$$\vec{\nabla}\cdot\left[U_C\vec{A}U_C^{-1} + \frac{i}{g}U_C\vec{\nabla}U_C^{-1}\right] = 0$$

Under arbitrary compact gauge transformation $U(x)$

$$\psi_C(x) \to U\psi_C(x), \quad U_C \to U_C U^{-1}, \quad \Psi_C(x) \to \Psi_C(x)$$

Compact perturbative solution:

$$U_C = \sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\omega_n$$

$$\omega_1 = -\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\frac{\omega_2}{2!} = \frac{1}{\nabla^2}\left(\vec{\nabla}\cdot\left(\omega_1^\dagger\vec{\nabla}\omega_1\right) - [\vec{\nabla}\omega_1, \cdot\vec{A}]\right),$$

$$\cdots$$

$$U_C(A)\Big|_{\vec{\nabla}\cdot\vec{A}=0} = 1$$

- **Infinite boost limit along the *z* direction:**

$$-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A}(x) = i\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot x}\frac{1}{k_z^2+k_\perp^2}\left[k^z\tilde{A}^z(k) + k_\perp\cdot\tilde{A}_\perp(k)\right]$$

$$\approx i\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot x}\frac{k^+}{(k^+)^2+\epsilon^2}\tilde{A}^+(k)$$

$$= \frac{1}{2}\left[\int_{-\infty^-}^{x^-} + \int_{+\infty^-}^{x^-}\right]d\eta^-\;A^+(x^+,\eta^-,x_\perp) \equiv \frac{1}{\partial^+_{\rm pv}}A^+(x)$$

Principle-value (P.V.) prescription:

$$\frac{k^+}{(k^+)^2+\epsilon^2} = \frac{1}{2}\left[\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\right]$$

Past pointing

Future pointing

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$$\cdots$$

$$U_C(A)\Big|_{\vec{\nabla}\cdot\vec{A}=0} = 1$$

- **Infinite boost limit along the *z* direction:**

$$\frac{\omega_n}{n!} \to \frac{1}{\partial^+_{\rm pv}}\left(\ldots\left(\frac{1}{\partial^+_{\rm pv}}\left(\left(\frac{1}{\partial^+_{\rm pv}}A^+\right)A^+\right)A^+\right)\ldots A^+\right)$$

Path-ordered integral for future/past pointing $1/\partial^+$

$$U_C \to \mathscr{P}\exp\left[-ig\int_{x^-}^{\mp\infty^-}dy^- A^+(y^-)\right] \equiv W_n^\dagger(x, \mp\infty^-)$$

Infinite "P.V. prescribed" light-like Wilson line

Factorization formula

- Power counting: $\lambda \sim \Lambda_{\rm QCD}/P^+ \ll 1$

Collinear $(k^+, k^-, k_\perp) = (1, \lambda^2, \lambda)P^+,$

$(b^+, b^-, b_\perp) = (\lambda^{-2}, 1, \lambda^{-1})(P^+)^{-1}$

Soft $(k^+, k^-, k_\perp) = (\lambda, \lambda, \lambda)P^+,$

$(b^+, b^-, b_\perp) = (\lambda^{-1}, \lambda^{-1}, \lambda^{-1})(P^+)^{-1}$

SCET$_{\rm II}$

- Bauer, Pirjol and Stewart, PRD **65** (2002), **66** (2002), **68** (2003)
- Beneke, Chapovsky, Diehl and Feldmann, NPB 643 (2002)
- Bauer et al., PRD **66** (2002)

$$\psi = \psi_n + \psi_s + \psi_{us} + \ldots$$

$$\{\psi_n, \psi_s, \psi_{us}\} \sim \{\lambda, \lambda^{3/2}, \lambda^3\}P^+$$

$$A^\mu = A^\mu_n + A^\mu_s + A^\mu_{us} + \ldots$$

$$A^\mu_n = (A^+_n, A^-_n, A^\perp_n) \sim (1, \lambda^2, \lambda)P^+$$

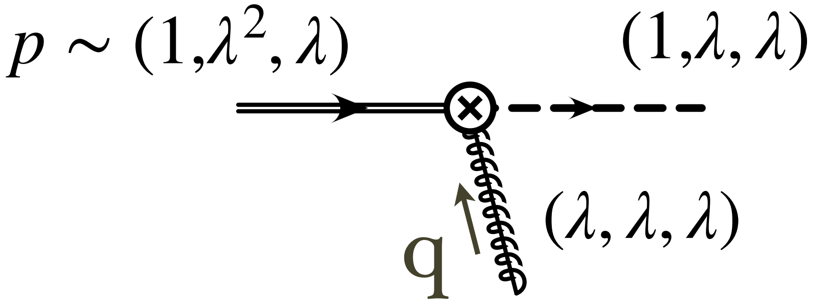
$$\{A^\mu_s, A^\mu_{us}\} \sim \{\lambda, \lambda^2\}P^+$$

- **Collinear, soft and ultra-soft modes decouple in the SCET Lagrangian, allowing for factorization of matrix elements**

- **Since $(1, \lambda^2, \lambda) + (\lambda^2, \lambda^2, \lambda^2) \sim (1, \lambda^2, \lambda)$, interaction with the ultrasoft modes do not change the collinear modes. They are separated through a field redefinition, which can be withheld if unnecessary.**

Factorization formula

- Power counting: $\lambda \sim \Lambda_{\rm QCD}/P^+ \ll 1$



$$(p+q)^2 \sim (\sqrt{\lambda}P^+)^2 \gg \Lambda_{\rm QCD}$$

Decoupling of soft and collinear modes $\Rightarrow U_C(A) = U_C(A_s)U_C(A_n) \equiv U_C^s U_C^n \approx U_C^s W_n^\dagger$

$U_C^s(A_s)$ cannot be expanded because

$$-\frac{1}{\nabla^2}\overrightarrow{\nabla}\cdot\overrightarrow{A}_s(x) = i\int\frac{d^4k_s}{(2\pi)^4}\ e^{-ik_s\cdot x}\frac{\vec{k}_s\cdot\overrightarrow{\tilde{A}}_s(k_s)}{k_x^2+k_y^2+k_z^2}$$

$$\vec{k}_s \sim (\lambda,\lambda,\lambda)P^+$$

Factorization formula

- **QCD**
- **SCET$_{\text{II}}$**

$\hat{\mathscr{P}}$: (collinear) label momentum

$$\begin{tikzcd} \Psi_C(x) = U_C(x)\psi(x) \arrow[r, "\text{Collinear expansion}"] & e^{i\hat{\mathscr{P}}\cdot x}U_C^s(x)W_n^\dagger(x)\xi_n(x) + O(\lambda^2) \arrow[d] \\ & e^{i\hat{\mathscr{P}}\cdot x}U_C^s(x)S_n(x)W_n^\dagger(x)\xi_n(x) \end{tikzcd}$$

Soft gauge transformation

$U_C^s \to U_C^s V_s^{-1}$,

$S_n \to V_s S_n$

SCET gauge invariance

Bauer, Pirjol and Stewart, PRD **65** (2002)

Matching QCD dressed quark field to SCET

$$U_C\psi(b) = e^{i\hat{\mathscr{P}}\cdot b}C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](b) + O(\lambda^2)$$

$C(\hat{\mathscr{P}}^+/\mu)$

Matching coefficient

Matching quasi-TMD correlator to SCET

$$\langle P|\Psi_C^\dagger(b)\gamma^t\Psi_C(0)|P\rangle = \frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\left\langle P\middle|\left[\bar{\xi}_n W_n S_n^\dagger (U_C^s)^\dagger\right](b)\; C(\hat{\mathscr{P}}^+/\mu)^\dagger\; \gamma^+ C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](0)\middle|P\right\rangle$$

Factorization formula

$$\langle P|\Psi_C^\dagger(b)\gamma^t\Psi_C(0)|P\rangle = \frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\Big\langle P\Big|\left[\bar{\xi}_n W_n S_n^\dagger (U_C^s)^\dagger\right](b)\ C(\hat{\mathscr{P}}^+/\mu)^\dagger\ \gamma^+ C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](0)\Big|P\Big\rangle$$

Equal time separation: $b^\mu = (0, b_\perp, b^z)$

- Mode separation: soft modes decouple from the collinear modes and can be treated as the background field;
- Multipole expansion: $b^z \sim 1/(xP^z) \sim O(1)/P^z, \quad b_\perp \sim O(\lambda^{-1})/P^z$

$$U_C^s(b) = U_C^s(b_\perp) + b^z\partial_z U_C^s(b_\perp) + \ldots \approx U_C^s(b_\perp) + O(\lambda)$$

$$\frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\Big\langle P\Big|\left[\bar{\xi}_n W_n\right](b) C\big(\hat{\mathscr{P}}^+/\mu\big)^\dagger \gamma^+ C\big(\hat{\mathscr{P}}^+/\mu\big)\left[W_n^\dagger \xi_n\right](0)\Big|P\Big\rangle$$

Beam with zero-bin subtraction

$$\times\frac{1}{N_c}\langle 0|T\left[S_n^\dagger(b_\perp)(U_C^s)^\dagger(b_\perp)U_C^s(0)S_n(0)\right]|0\rangle$$

Soft S_C^0

- Operator definition obtained for the first time!
- Can be regarded as the "zero-bin" of quasi-beam function

$$\frac{P^z}{2P^t}\int\frac{db^z}{2\pi}e^{ixP^zb^z}\ \Big\downarrow\ \frac{1}{\sqrt{2}}\delta(\mathscr{P}^z - xP^z) = \delta(\mathscr{P}^+ - xP^+)$$

$$\tilde{B}(x, b_\perp, \mu, P^z) = |C(xP^+/\mu)|^2 B(x, b_\perp, \ldots, xP^+)S_C^0(b_\perp, \ldots) + O(\lambda^2)$$

Manohar and Stewart, PRD **76** (2007)

Factorization formula

$$\tilde{B}(x, b_\perp, \mu, P^z) = |C(xP^+/\mu)|^2 B(x, b_\perp, \ldots, xP^+) S_C^0(b_\perp, \ldots) + O(\lambda^2)$$

"…": UV and rapidity regulators

Physical (or subtracted) TMD PDF:

$$f(x, b_\perp, \mu, \zeta) = B(x, b_\perp, \ldots, xP^+) S(b_\perp, \ldots, y_n)$$

Collins-Soper scale

$$\zeta = 2(xP^+)^2 e^{-2y_n}$$

$$\downarrow$$

$$\frac{\tilde{B}(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, y_n)} = |C(xP^+/\mu)|^2 f(x, b_\perp, \mu, \zeta) + O(\lambda^2)$$

$$\tilde{S}_C(b_\perp, \mu, y_n) \equiv \frac{S_C^0(b_\perp, \ldots)}{S(b_\perp, \ldots, y_n)}$$

Quasi soft factor

$$\frac{d}{dy_n} \ln \tilde{S}_C(b_\perp, \mu, y_n) = \gamma_\zeta(b_\perp, \mu)$$

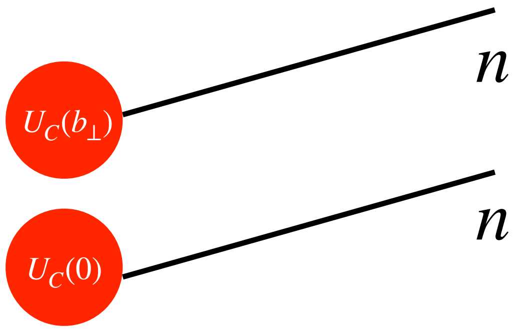
Or

$$\frac{\tilde{B}(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} = |C(xP^+/\mu)|^2 \exp\left[\frac{1}{2}\gamma_\zeta(b_\perp, \mu) \ln \frac{2(xP^+)^2}{\zeta}\right] \times f(x, b_\perp, \mu, \zeta) + O(\lambda^2)$$

Same form as the gauge-invariant quasi-TMD

Quasi soft factor

$$\tilde{S}_C(b_\perp, \mu, y_n) \equiv \frac{S_C^0(b_\perp, \ldots)}{S(b_\perp, \ldots, y_n)}$$



Not directly calculable on the lattice.

$$S_C^0 = \frac{1}{N_c}\langle 0 | T\left[S_n^\dagger(b_\perp)(U_C^s)^\dagger(b_\perp)U_C^s(0)S_n(0)\right] | 0 \rangle$$

Light-meson form factor:

$$F(b_\perp, P^z) = \langle \pi(-P) | j_1(b_\perp) j_2(0) | \pi(P) \rangle$$

$$= \int dx_1 dx_2 \; H_F(x_1, x_2, P^z, \mu)$$

Hard kernel: known at 1-loop

$$\times \frac{\phi(x_1, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} \frac{\phi(x_2, b_\perp, \mu, P^z)]}{\tilde{S}_C(b_\perp, \mu, 0)}$$

$\phi(x, b_T, \mu, P^z)$: **Coulomb gauge quasi-TMD wave function** ✔

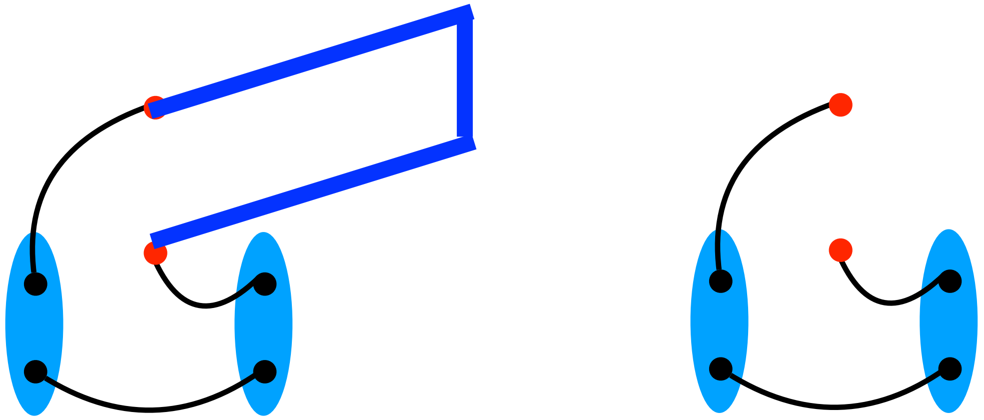
$\phi^* = \phi$ due to the P.V. prescription

Outline

- **Introduction**
  - Large-Momentum Effective Theory approach to TMDs
- **Coulomb gauge method**
  - Universality class in LaMET
  - Coulomb-gauge quasi-TMD
  - Large-momentum factorization formula
  - Soft function
  - Transverse gauge link
- **Discussions**

Advantages

- Significantly improved statistical precision (at large b_T);

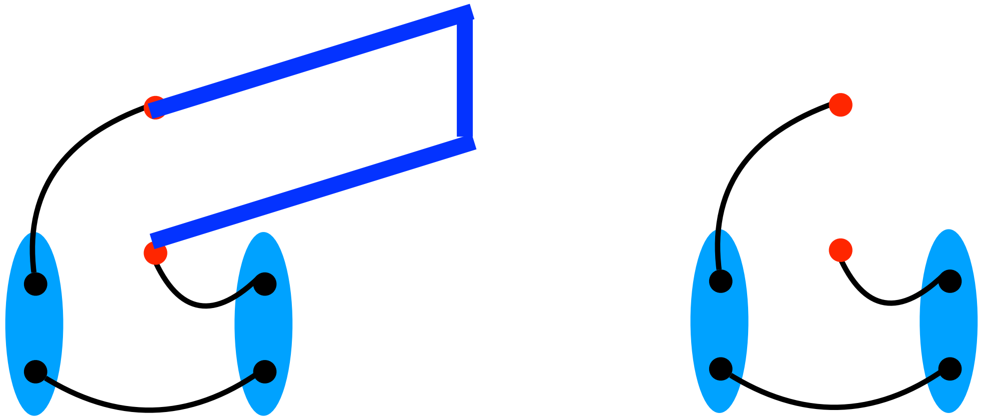


- Absence of linear power divergence and multiplicative renormalization;

- Access to larger off-axis momenta.

Advantages

- Significantly improved statistical precision (at large b_T);



- Absence of linear power divergence and multiplicative renormalization;

- Access to larger off-axis momenta.

See Xiang Gao's talk.

Gauge fixing and Gribov copies

- Find the gauge transformation U that minimizes

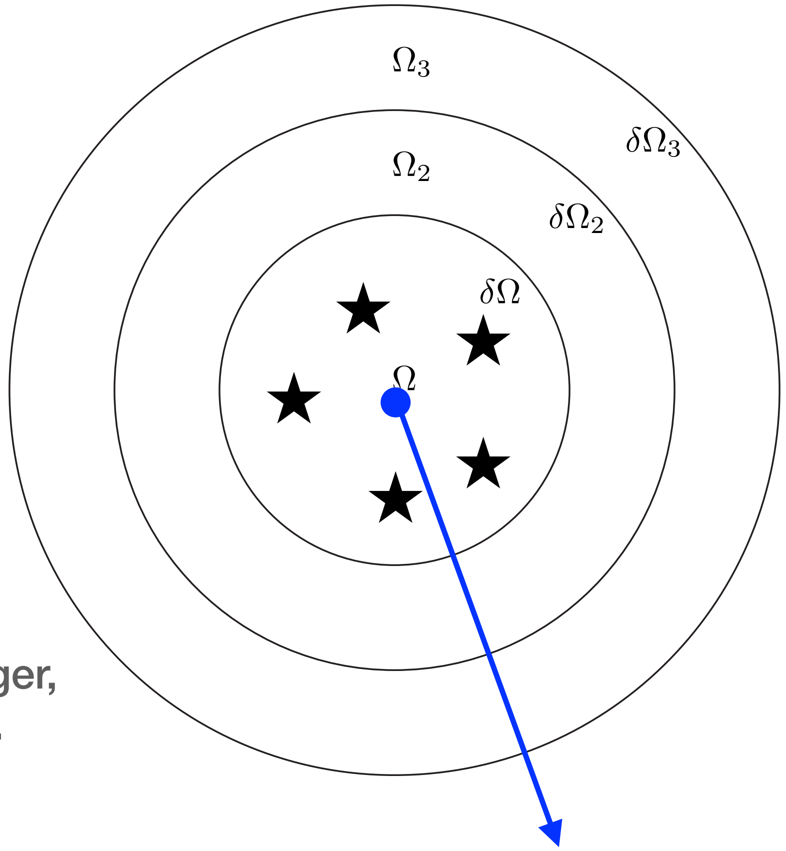
$$F[A^U] = \mathrm{Tr}\int dx A_i^U(x) A_i^U(x) \rightarrow \delta F = \int dx\ \theta^a(x)\partial_i A_i^a(x)$$

$$\delta F = \int dx\ \theta^a(x)\partial_i A_i^a(x) = 0$$

$$\delta^2 F = \int dx\ \theta^a(x)(-\partial_i D_i^{ab})\theta^b(x) > 0$$

First Gribov region Ω

Vandersickel and Zwanziger, Phys. Rept. **520** (2012).



$A \approx 0$, perturbative region, matchable to the light-cone, which is free from Gribov copies.

- There are still multiple local extrema (Gribov copies) in the first Gribov region.
- Lattice study shows that measurement distortion by Gribov copies are negligible compared to gauge noise.
- However, in the continuum, a nonzero—however small—distortion would still make one wonder which copy has the right IR physics for light-cone.

See Jinchen He's talk.

X. Gao, J. He, R. Zhang and YZ, arXiv: 2408.05910

Is the distortion bounded and power suppressed in LaMET expansion?

T-odd TMDs

- *T*-odd light-cone TMD:

P.V. prescription = anti-symmetric boundary condition $A_\perp^\mu(\infty^-) = -A_\perp^\mu(-\infty^-)$

Need a transverse link to define the *T*-odd TMDs

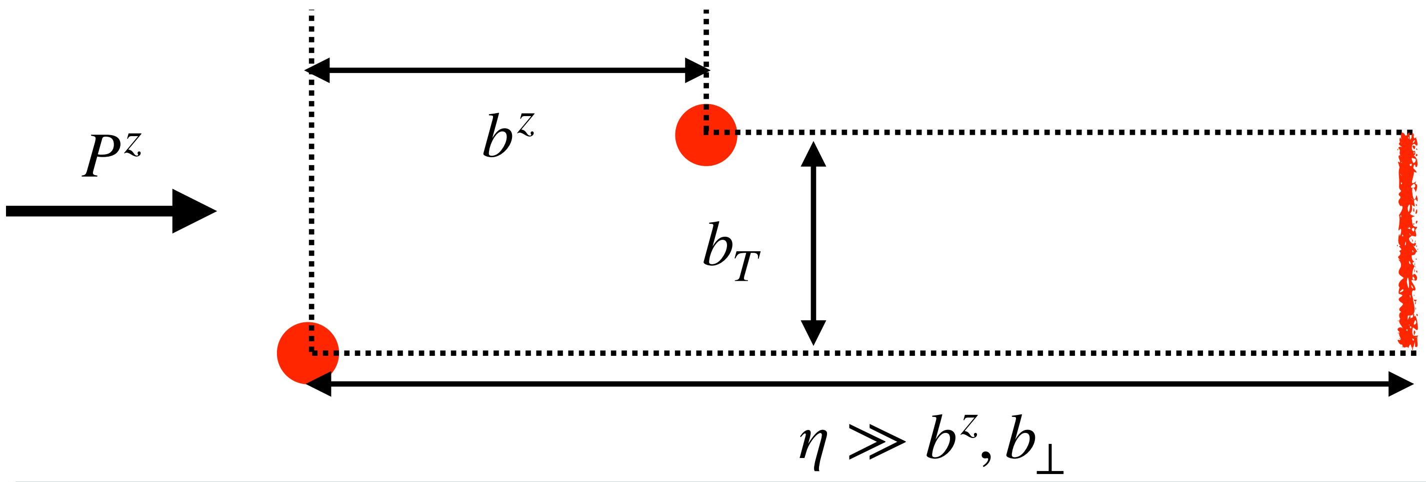
Contribution in a Feynman diagram: $\dfrac{e^{-i\infty^- k^+}}{[k^+]_{\rm pv}} = -i\pi\delta(k^+)$

- Ji and Yuan, PLB **543** (2002)
- Belitsky, Ji and Yuan, NPB **656** (2003)

- Coulomb-gauge (CG) quasi-TMD?

$$\tilde{h}(\vec{b}, \vec{P}, \mu, \pm) = \frac{1}{2P^t}\langle P | \bar{\psi}(\vec{b})\mathscr{W}_\perp(\pm\infty\hat{z}; b_\perp, 0_\perp)\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0} | P\rangle$$

Quasi-TMD with a transverse link



- However, this idea does not work because boundaries in the CG and light-cone gauge are not related.

- Since the *T-odd* TMDs are zero-mode effects, can they be defined with topological properties of the configurations in the CG?

Gluon TMDs

- Factorization formula can be derived using SCET as well.
- No linear power divergence.
- Mixing with gauge-variant operators. However, the number of mixings is finite under the constraint of SO(3) symmetry.
- Could be more susceptible to the Gribov noise, but it may still easily beat the statistical precision achieved with Wilson line operators.

One-loop check

$$\tilde{B}^{(1)}(x, b_\perp, \mu, p^z, \epsilon) \qquad (26)$$

$$= \frac{\alpha_s C_F}{2\pi}\left[-\frac{1+x^2}{1-x}\left(\frac{1}{\epsilon}+L_b\right)+(1-x)\right]_+^{(0,1)}$$

$$+\,\delta(1-x)\frac{\alpha_s C_F}{2\pi}\left[\frac{5}{2}L_b-3L_p-\frac{(L_b-L_p)^2}{2}-\frac{23}{2}+\frac{\pi^2}{2}\right],$$

$$\tilde{S}_C^{(1)}(b_\perp, \mu, y_n) = \frac{\alpha_s C_F}{2\pi}(1-2y_n)L_b\,, \qquad (27)$$

$$f^{(1)}(x, b_\perp, \mu, \zeta, \epsilon) \qquad (28)$$

$$= \frac{\alpha_s C_F}{2\pi}\left[-\frac{1+x^2}{1-x}\left(\frac{1}{\epsilon}+L_b\right)+(1-x)\right]_+^{(0,1)}$$

$$+\,\delta(1-x)\frac{\alpha_s C_F}{2\pi}\left[-\frac{L_b^2}{2}+L_b\left(\frac{3}{2}+\ln\frac{\mu^2}{\zeta}\right)+\frac{1}{2}-\frac{\pi^2}{12}\right],$$

$$\tilde{B}^{(1)} - \delta(1-x)\tilde{S}_C^{(1)} - f^{(1)}$$

$$= \delta(1-x)\frac{\alpha_s C_F}{2\pi}\left[-\frac{L_p^2}{2}-3L_p-12+\frac{7\pi^2}{12}\right] \qquad (29)$$

Quasi soft factor checked with both off-the-light-cone (Collins) rapidity regulator and the η regulator, which give the same answer.

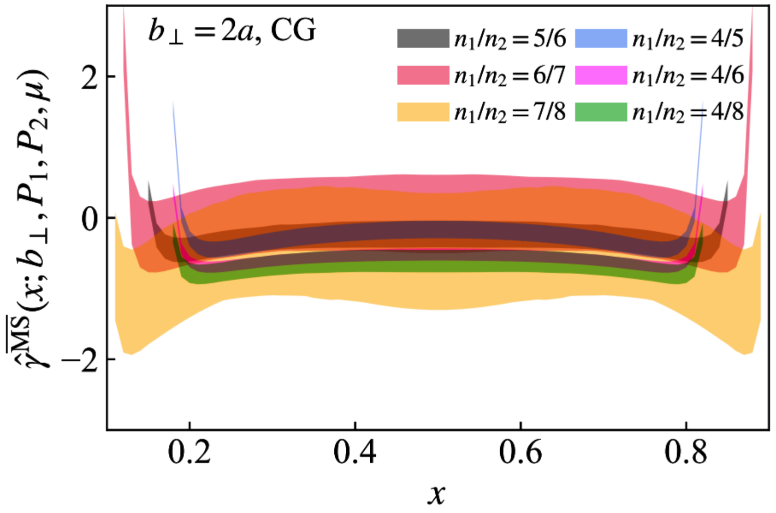
High-order corrections

- Known to be difficult due to non-covariant nature, but it is just one scale in massless integrals.

- No complete 2-loop result so far, even for the quark wave function renormalization.

- Linear renormalon in the matching coefficient.
  - Corresponds to a linear power correction of order $\Lambda_{\mathrm{QCD}}/P^z$;
  - However, strength of the renormalon is not easy to estimate. An NNLO calculation can offer a lot of insight.
  - Lattice calculation of the Collins-Soper kernel suggests that the result converges in P^z quite well.

Y. Liu and Y. Su, JHEP **2024** (2024)



D. Bollweg, X. Gao, S. Mukherjee and YZ, Phys.Lett.B **852** (2024)

Summary

- The Coulomb-gauge quasi-TMD can be factorized into the physical TMDs at large momentum;

- The factorization formula can be derived using SCET, which also leads to the operator definition of the quasi soft factor.

- It corresponds to the principle value prescription for the light-like Wilson lines, which can be used to calculate T-even TMDs;

- It can significantly reduce the statistical error, simplify the renormalization and access higher off-axis momenta, thus providing a more efficient way to calculate the TMDs.