Progress toward Moments of Light-Cone Distribution Amplitudes from a Heavy-Quark Operator Product Expansion

Fermilab

THE HOPE COLLABORATION

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Outline

Light-Cone Distribution Amplitude

$$\langle 0|\bar{\psi}(z)\gamma^\mu\gamma^5 W[z,-z]\psi(-z)|M(\mathbf{p})\rangle = i f_M p^\mu \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\varphi_M(\xi)$$

- Represents overlap between meson and $q\bar{q}$ pair with momenta $(1\pm\xi)/2$
- Relates quark-level process (e.g. electroweak interactions) to hadron-level interactions (experimental observables)
- Universal (relates to multiple different processes)

$B \to \pi\pi$ Decay

- Rate depends on CKM matrix elements and terms such as

$$f^{+}(0) \int_0^1 dx \, T_i^{I}(x) \varphi_\pi(x) + \int_0^1 d\xi \, dx \, dy \, T_i^{II}(\xi, x, y) \varphi_B(\xi) \varphi_\pi(x) \varphi_\pi(y)$$

  where $T_i^{I,II}$ are known functions and φ_B, φ_π are distribution amplitudes
- Complicated decay form (various ways to subdivide quark energies) – require convolution of functions to capture structure
- Sensitive to symmetry violation and thus complex phase of CKM matrix

Pion Electromagnetic Form Factor

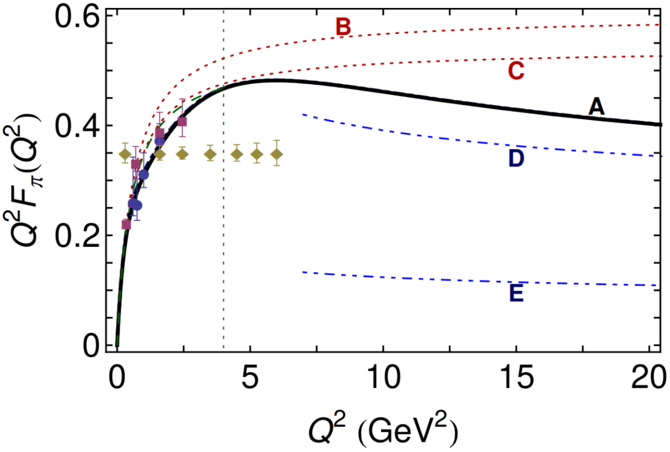
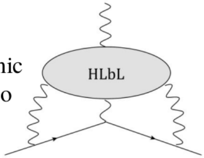


Figure credit: L. Chang et al., nucl-th/1307.0026, PRL **111**, 141802

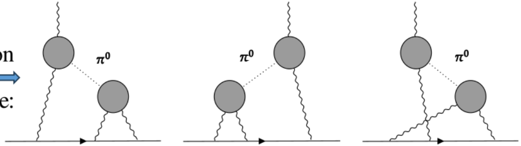
Hadronic Light-By-Light

$O(\alpha^3)$ hadronic contribution to muon g-2



main contribution

pseudoscalar pole: π^0, η, η'-pole



- Pion pole depends on $\pi \to \gamma^*\gamma^*$ transition form factor $F_{\pi\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)$
- $F_{\pi\gamma^*\gamma^*}$ can be factorized into other (calculable) contributions times $\varphi(\xi)$

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \int_0^1 du \int d^4x \; e^{-ik\cdot x} \frac{(x\cdot K)P^2 - (x\cdot P)(K\cdot P)}{K^2P^2 - (K\cdot P)^2} \phi_\pi(x^2,u) H(x^2,0)$$

SO(4) average

$$= -2i \int_0^1 du \int d^4x \frac{J_2(kx)}{k^2} \phi_\pi(x^2,u) H(x^2,0) \longrightarrow \text{Lattice input}$$

$\downarrow$ Physical pion structure function

Adapted from Tian Lin @ Lattice 2024

Lattice Determination of LCDA

$$\langle 0|\bar{d}(-z)\gamma_\mu\gamma_5\mathcal{W}[-z,z]u(z)|\pi^+(p)\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\varphi_\pi(\xi)$$

- z is light-like separation ($z^2 = 0$)
- Euclidean space $\Rightarrow$ light-cone is single point
- Need indirect methods
  - Quasi-PDF [Ji, 1305.1539, PRL **110**, 262002]
  - Pseudo-PDF [Radyushkin, 1705.01488, PRD **96** 034025; Radyushkin, 1909.08474, PRD **100** 116011]
  - Expansion into moments

Lattice Determination of LCDA

- Expansion of LCDA into Mellin moments

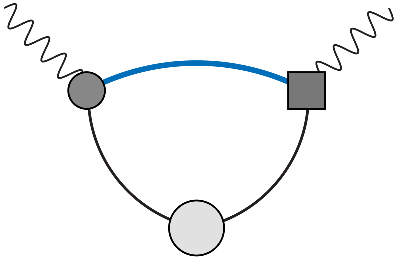
$$\langle \xi^n \rangle = \int_{-1}^{1} d\xi \, \xi^n \varphi_\pi(\xi)$$

- Moments can be written in terms of local derivative operators
- Second moment computed with this approach [Bali et al., 1903.08038, JHEP **2020**, 37]
- For $n > 2$, mix with lower-dimensional lattice operators – diverge instead of converging as $a \to 0$
- Alternative approach to compute moments needed

Hadronic Tensor

$$V^{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} \langle 0|\, T\left[A^{\mu}(x/2)A^{\nu}(-x/2)\right] |\pi^+(p)\rangle$$

- Transition between pion and two axial current insertions
- Amenable to lattice calculations

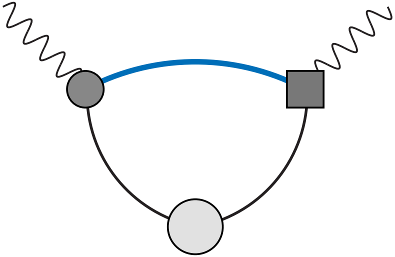


Source: arXiv:hep-lat/0507007, PRD **73**, 014501

Hadronic Tensor

$$V^{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} \langle 0|\, T\left[A^\mu(x/2)A^\nu(-x/2)\right] |\pi^+(p)\rangle$$

- Transition between pion and two axial current insertions
- Amenable to lattice calculations
- Free to make currents flavor changing (and intermediate quark heavy)



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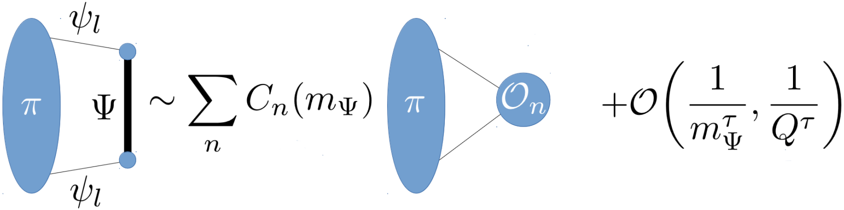
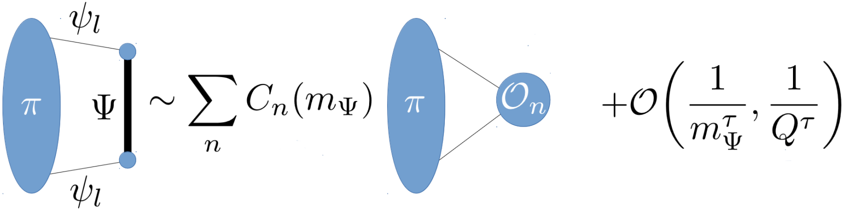
Operator Product Expansion (OPE)

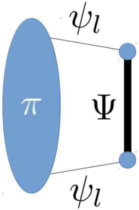


Operator Product Expansion (OPE)

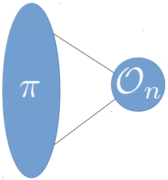


$$\left\langle 0 \left| \gamma^\mu \frac{-i(i\not{D} + \not{q}) + m_\Psi}{(iD+q)^2 + m_\Psi^2} \gamma^\nu \right| \pi^+(\mathbf{p}) \right\rangle = \left\langle 0 \left| \gamma^\mu \frac{i(i\not{D} + \not{q}) + m_\Psi}{q^2 + (iD)^2 + m_\Psi^2} \sum_{n=0}^{\infty} \left(\frac{2iq \cdot D}{q^2 + (iD)^2 + m_\Psi^2} \right)^n \gamma^\nu \right| \pi^+(\mathbf{p}) \right\rangle$$

Operator Product Expansion (OPE)



$$\sim \sum_n C_n(m_\Psi)$$



$$+\mathcal{O}\left(\frac{1}{m_\Psi^\tau}, \frac{1}{Q^\tau}\right)$$

$$\left\langle 0 \left| \gamma^\mu \frac{-i(i\not{D}+\not{q})+m_\Psi}{(iD+q)^2+m_\Psi^2} \gamma^\nu \right| \pi^+(\mathbf{p}) \right\rangle = \left\langle 0 \left| \gamma^\mu \frac{i(i\not{D}+\not{q})+m_\Psi}{q^2+(iD)^2+m_\Psi^2} \sum_{n=0}^{\infty} \left(\frac{2iq\cdot D}{q^2+(iD)^2+m_\Psi^2} \right)^n \gamma^\nu \right| \pi^+(\mathbf{p}) \right\rangle$$

$$= \sum_{n=0}^{\infty} \left\langle 0 \left| \gamma^\mu \frac{i(i\not{D}+\not{q})+m_\Psi}{q^2+m_\Psi^2} \left(\frac{2p\cdot q}{q^2+m_\Psi^2} \right)^n \gamma^\nu \right| \pi^+(\mathbf{p}) \right\rangle \langle \xi^n \rangle + O\left(\frac{\Lambda_{\mathrm{QCD}}}{(q^2+m_\Psi^2)^{1/2}} \right)$$

using $D|\pi^+(\mathbf{p})\rangle \to \xi p|\pi^+(\mathbf{p})\rangle$, $D^2 \sim O(\Lambda_{\mathrm{QCD}}^2)$

Heavy Quark Operator Product Expansion (HOPE)

$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0 \\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu) \langle \xi^n \rangle(\mu) + O\left(\frac{\Lambda_{\text{QCD}}}{\tilde{Q}}\right)$$

$$\tilde{Q}^2 = -q^2 - m_\Psi^2 \,, \qquad \tilde{\omega} = \frac{p \cdot q}{\tilde{Q}^2}$$

- Odd moments vanish by isospin symmetry

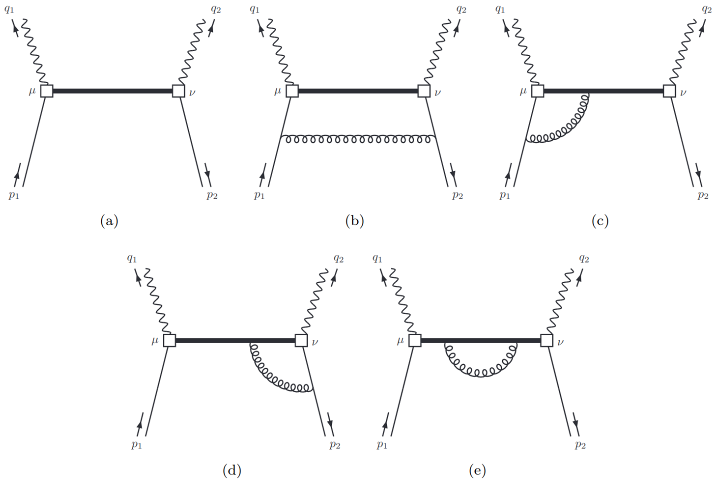
Heavy Quark Operator Product Expansion (HOPE)

$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0 \\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu) \langle \xi^n \rangle(\mu) + O\left(\frac{\Lambda_{\text{QCD}}}{\tilde{Q}}\right)$$

$$\tilde{Q}^2 = -q^2 - m_\Psi^2\,, \qquad \tilde{\omega} = \frac{p\cdot q}{\tilde{Q}^2}$$

- Odd moments vanish by isospin symmetry
- Need to suppress $\Lambda_{\text{QCD}}/\tilde{Q}$ corrections $\Rightarrow$ need either q or m_Ψ large
- Large-q (short-distance) studied in [Braun and Müller, 0709.1348, EPJC **55**, 349; Bali et al., 1807.06671, PRD **98** 094507]
- This talk will discuss large-m_Ψ approach

Wilson Coefficients



Detmold, AVG, Kanamori, Lin, Perry, Zhao, 2103.09529, PRD **104**, 074511

Hadronic Tensor

$$V^{\mu\nu}(q,p) = \int d^4x\, e^{iq\cdot x} \left\langle 0 \left| \mathcal{T}\left[A^\mu\left(\frac{x}{2}\right) A^\nu\left(-\frac{x}{2}\right)\right] \right| \pi^+(p) \right\rangle$$

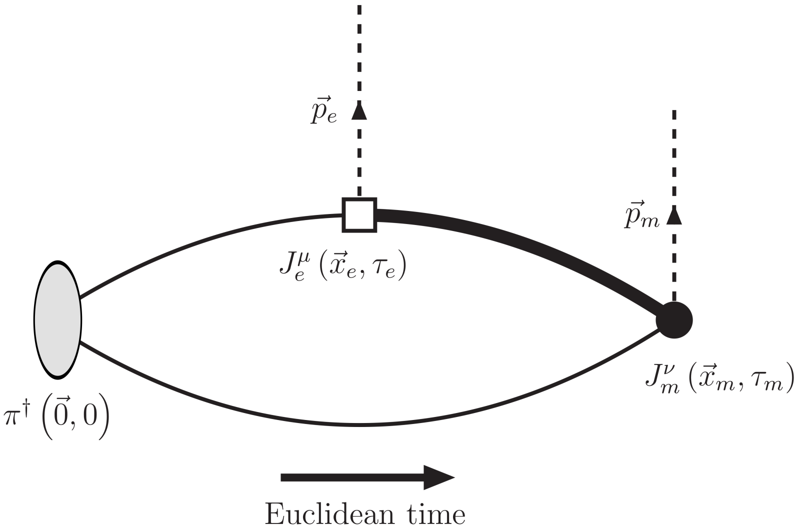
$$\int dq_4 e^{-iq_4\tau} V^{\mu\nu}(q,p) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \left\langle 0 \left| \mathcal{T}\left[A^\mu\left(\frac{\mathbf{x}}{2},\frac{\tau}{2}\right) A^\nu\left(-\frac{\mathbf{x}}{2},-\frac{\tau}{2}\right)\right] \right| \pi^+(\mathbf{p}) \right\rangle$$

- Inverse FT of $V^{\mu\nu}$ calculable on lattice in terms of 2-point and 3-point functions

$$C_2(\tau) = \langle \mathcal{O}_\pi(\tau)\mathcal{O}_\pi^\dagger(0)\rangle$$
$$C_3(\tau_e,\tau_m) = \langle A^\mu(\tau_e)A^\nu(\tau_m)\mathcal{O}_\pi^\dagger(0)\rangle$$

- Isolation of ground state relies on sufficiently large separation between 0 and $\min\{\tau_e,\tau_m\}$

Hadronic Tensor



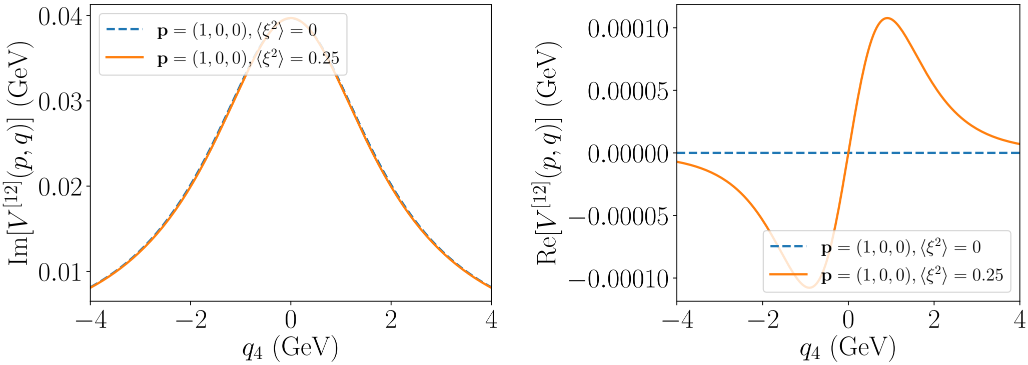
Choice of Kinematics

$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0 \\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu) \langle \xi^n \rangle(\mu) + O\left(\frac{\Lambda_{\text{QCD}}}{\tilde{Q}}\right)$$

- Wilson coefficients $C_W^{(n)}(\mu = 2\text{ GeV})$ calculated to 1-loop
- Fit parameters: f_π, m_Ψ, $\langle \xi^2 \rangle$
- Contribution of second moment $\langle \xi^2 \rangle$ suppressed by

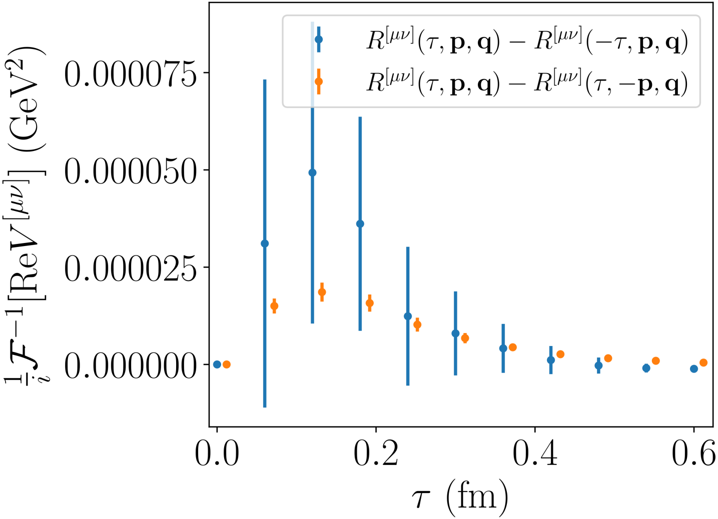
$$\frac{|\tilde{\omega}|^2}{2^2 \times 3} = \frac{1}{3}\left|\frac{p \cdot q}{\tilde{Q}^2}\right|^2 \lesssim 10^{-2}$$

Choice of Kinematics



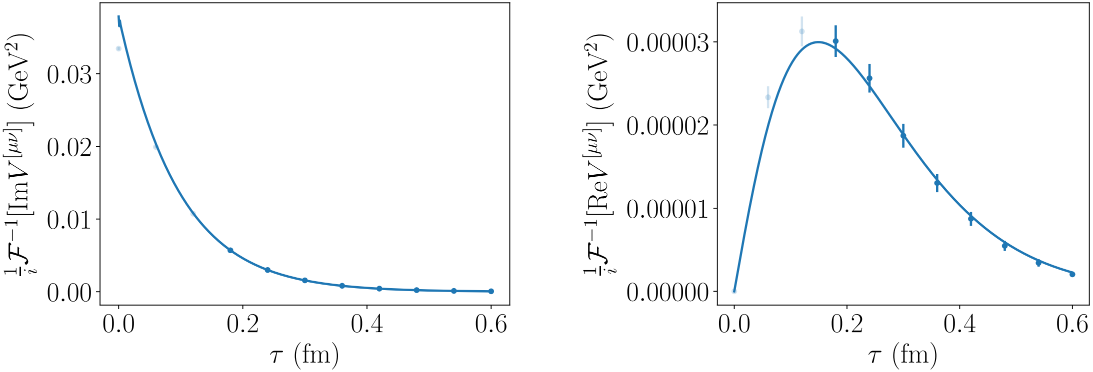
$$\mathbf{p} = (1,0,0) = (0.64\ \text{GeV}, 0, 0)$$
$$2\mathbf{q} = (1,0,2) = (0.64\ \text{GeV}, 0, 1.28\ \text{GeV})$$

Noise Reduction



Fitting Hadronic Tensor

- Fit ratio of 2- and 3-point correlators to inverse FT of OPE

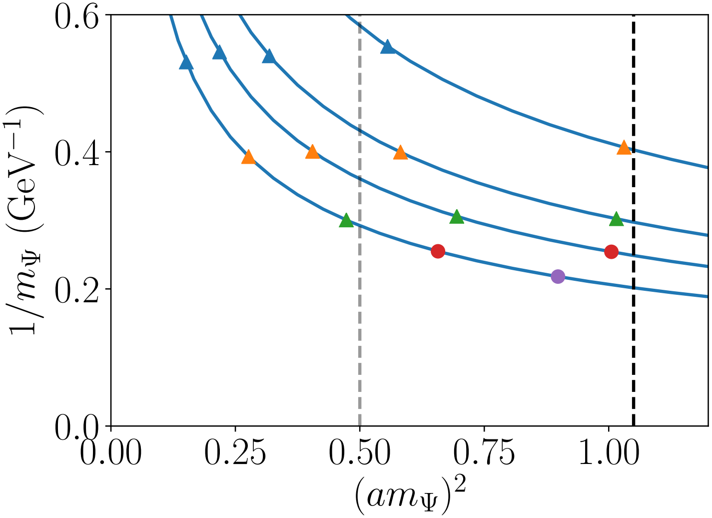


$f_\pi = 149 \pm 1$ MeV

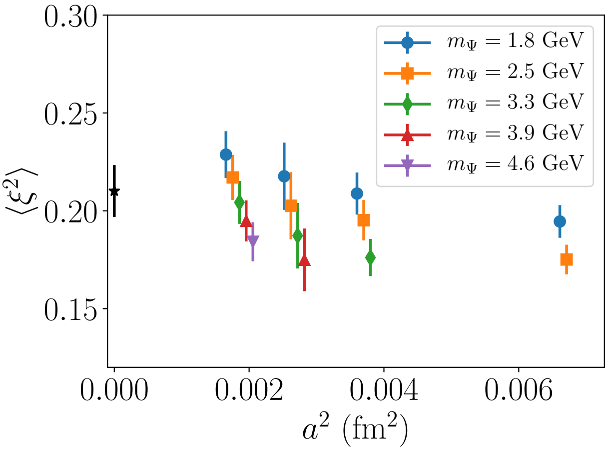
$m_\Psi = 1.85 \pm 0.01$ GeV

$\langle \xi^2 \rangle = 0.209 \pm 0.011$

Ensembles Used

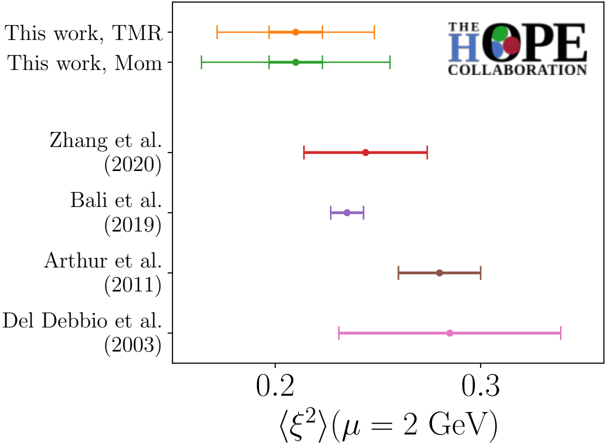


Continuum Extrapolation



$$\text{data} = \langle \xi^2 \rangle + \frac{A}{m_\Psi} + Ba^2 + Ca^2 m_\Psi + Da^2 m_\Psi^2 \Rightarrow \langle \xi^2 \rangle = 0.210 \pm 0.013 \text{ (stat.)}$$

Comparison to Literature



Detmold, AVG, Kanamori, Lin, Mondal, Perry, Zhao, 2109.15241, PRD **105**, 034506

Dynamical Ensembles

- Redo calculation CLS $2+1$-flavor isoclover ensembles
- Good control over lattice spacing dependence
- Also allow control over m_π
  - Previously uncontrolled systematic
- Also examine excited states more carefully

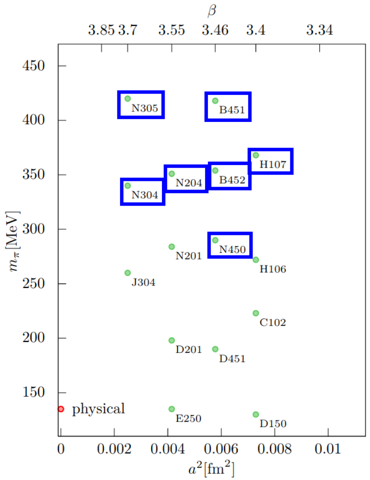
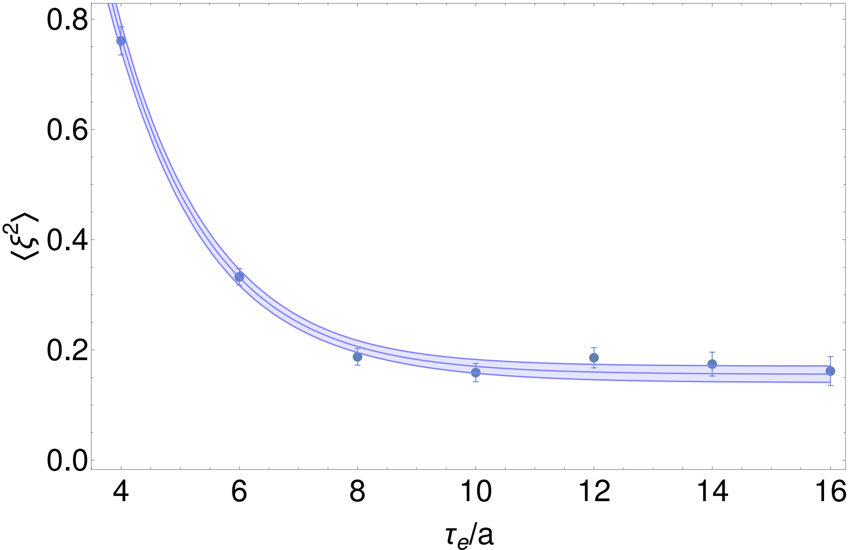
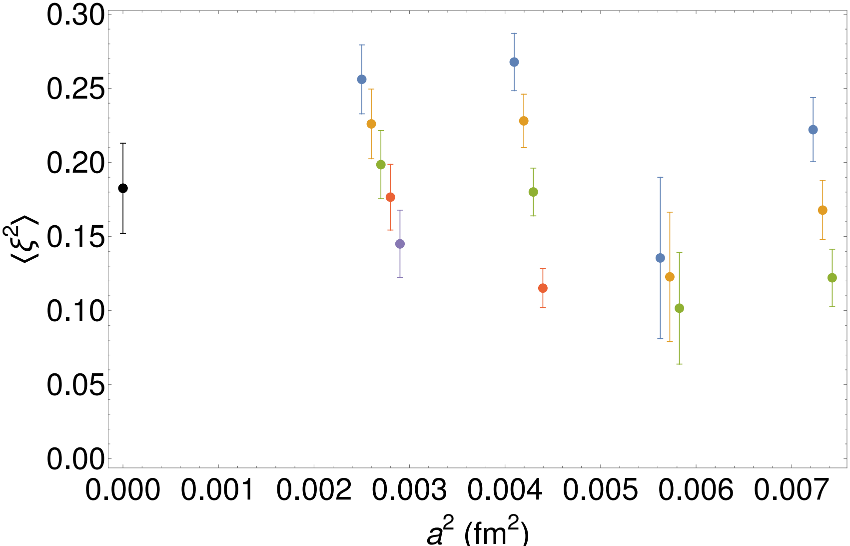


Figure credit: S. Collins @ CERN (2019)

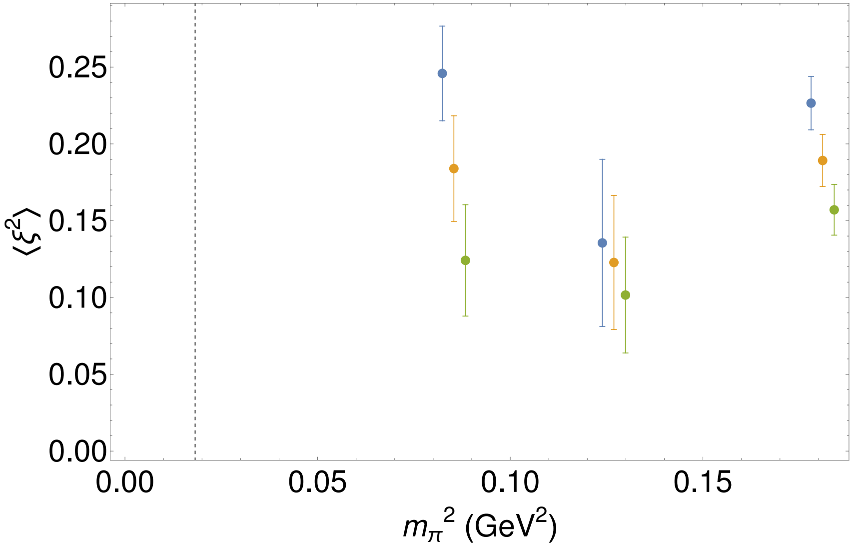
Excited State Contamination



Lattice Spacing Dependence (Preliminary)



Pion Mass Dependence (Preliminary)

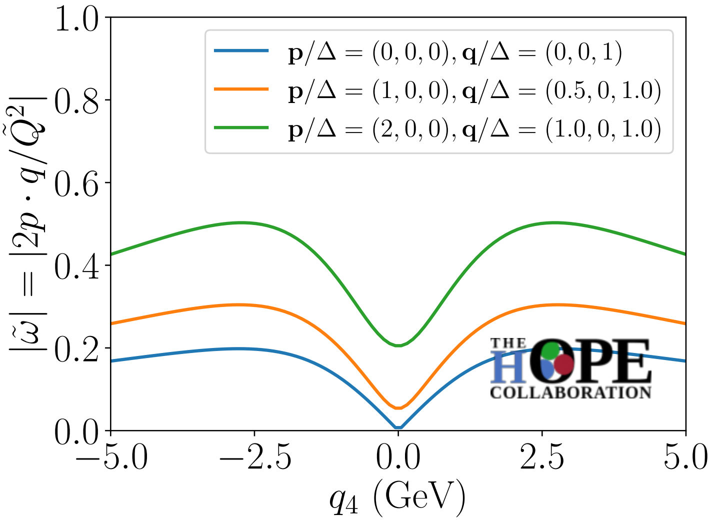


Suppression of Signal

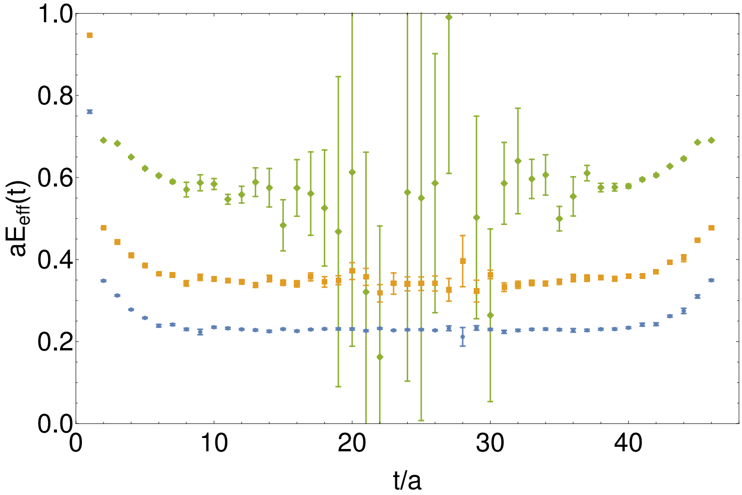
$$V^{\mu\nu}(p,q) = \frac{2if_\pi \varepsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} \sum_{\substack{n=0 \\ \text{even}}}^{\infty} \frac{\tilde{\omega}^n}{2^n(n+1)} C_W^{(n)}(\tilde{Q}, m_\Psi, \mu)\langle \xi^n \rangle(\mu) + O\left(\frac{\Lambda_{\text{QCD}}}{\tilde{Q}}\right)$$

- $\langle \xi^n \rangle$ suppressed by $(\tilde{\omega}/2)^n \sim 0.1^n$
- Splitting signal into real/imaginary parts facilitates extraction of $\langle \xi^2 \rangle$
- Trick works best for $n = 2$ (only 2 channels available)

Higher Momentum



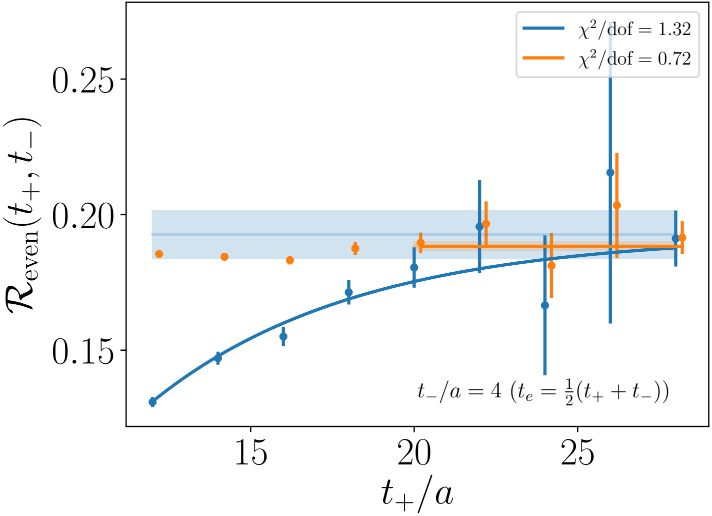
Signal-to-Noise Problem



Variational Method

- Excited state contamination more severe at larger **p**
- Also steeper penalty for extending Euclidean time
- Solution – better interpolating operator to create pion
- Pion can be created with $\bar{\psi}\gamma_5\psi$ or $\bar{\psi}\gamma_4\gamma_5\psi$
- Use both and take optimal linear combination to reduce excited state contamination

Variational Method



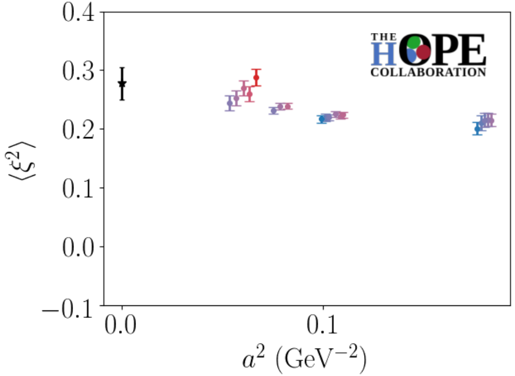
Truncated Solver Method

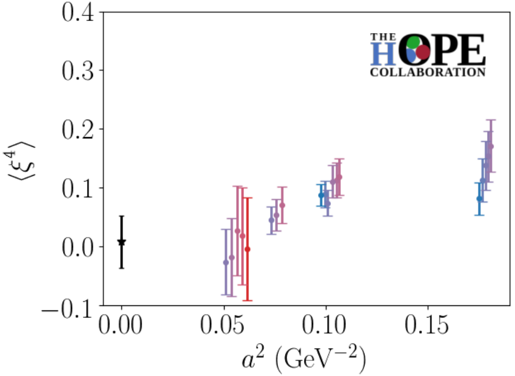
- Lattice calculations typically use solver precision of $\varepsilon_{\text{fine}} \sim 10^{-10}$
- Sloppier precision of $\varepsilon_{\text{sloppy}} \sim 10^{-5}$ cheaper but biases results
- Correct for bias with 1 fine solve/config [Bali et al., CPC 181, 1570 (0910.3970)]:

$$\langle \mathcal{O} \rangle_{\text{true}} = \left(\langle \mathcal{O} \rangle_{\text{fine}} - \langle \mathcal{O} \rangle_{\text{sloppy}} \right) + \sum_{i=1}^{N} \langle \mathcal{O}_i \rangle_{\text{sloppy}}$$

- In practice, bias correction $\left(\langle \mathcal{O} \rangle_{\text{fine}} - \langle \mathcal{O} \rangle_{\text{sloppy}} \right) \ll$ stat error with $\varepsilon_{\text{sloppy}} = 10^{-5}$
- Reduces compute cost by factor of $\gtrsim 2$ with little additional noise
  - Can use single precision for sloppy solves and smearing

Preliminary Results





$\langle\xi^2\rangle = 0.28 \pm 0.03$ (stat.)

$\langle\xi^4\rangle = 0.01 \pm 0.05$ (stat.)

Kaon LCDA

$$\langle 0|\bar{d}(-z)\gamma_\mu\gamma_5\mathcal{W}[-z,z]u(z)|\pi^+(p)\rangle = ip_\mu f_\pi \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\varphi_\pi(\xi)$$

- Isospin symmetry in π $\rightarrow$ even in z $\rightarrow$ odd moments vanish

Kaon LCDA

$$\langle 0|\bar{s}(-z)\gamma_\mu\gamma_5\mathcal{W}[-z,z]u(z)|K^+(p)\rangle = ip_\mu f_K \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\varphi_K(\xi)$$

- Isospin symmetry in π $\rightarrow$ even in z $\rightarrow$ odd moments vanish
- For kaon, $m_s \neq m_l \rightarrow \langle\xi^1\rangle, \langle\xi^3\rangle, \cdots \neq 0$
- Odd moments suppressed by powers of $m_s - m_l$: numerically small
- Additional fit parameters in OPE fits to hadronic tensor

Extracting Moments

- Different linear combinations of $V(p,q) \to$ different moments

$$V^{12}_{\text{even}}(p,q) \equiv \frac{1}{2}\left[V^{12}_K(p,q) - V^{12}_K(p,-q)\right] = N^{12}(p,q) \sum_{n \text{ even}} \left(\frac{\tilde{\omega}}{2}\right)^n \langle \xi^n \rangle_K$$

$$V^{12}_{\text{odd}}(p,q) \equiv \frac{1}{2}\left[V^{12}_K(p,q) + V^{12}_K(p,-q)\right] = N^{12}(p,q) \sum_{n \text{ odd}} \left(\frac{\tilde{\omega}}{2}\right)^n \langle \xi^n \rangle_K$$

Extracting Moments

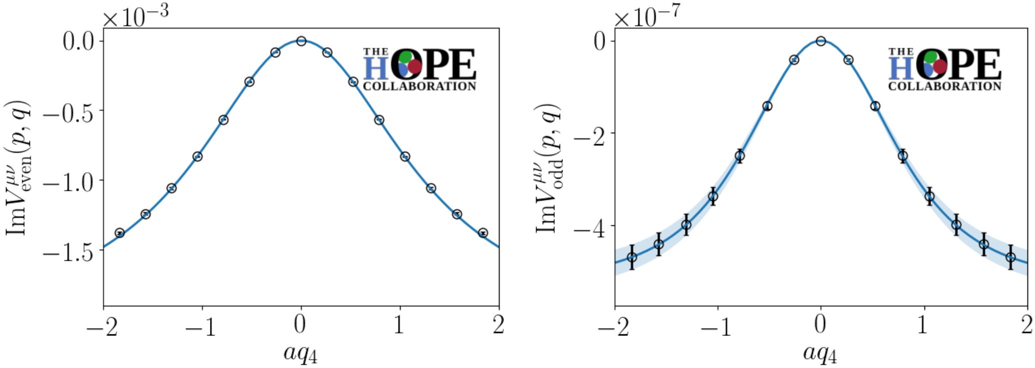
- Different linear combinations of $V(p, q) \to$ different moments

$$V^{12}_{\text{even}}(p, q) \equiv \frac{1}{2}\left[V^{12}_{K}(p, q) - V^{12}_{K}(p, -q)\right] = N^{12}(p, q) \sum_{n \text{ even}} \left(\frac{\tilde{\omega}}{2}\right)^{n} \langle \xi^n \rangle_K$$

$$V^{12}_{\text{odd}}(p, q) \equiv \frac{1}{2}\left[V^{12}_{K}(p, q) + V^{12}_{K}(p, -q)\right] = N^{12}(p, q) \sum_{n \text{ odd}} \left(\frac{\tilde{\omega}}{2}\right)^{n} \langle \xi^n \rangle_K$$

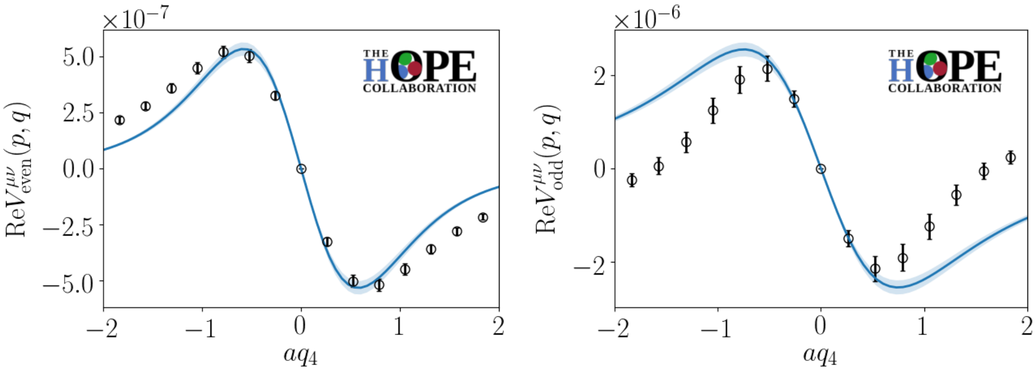
- Further separation into different channels by considering real/imaginary parts (even/odd in τ)
  - 4 channels total

Zeroth and First Moments



- Good fits to tree-level OPE $\rightarrow$ good prospects for extracting $\langle \xi^1 \rangle_K$

Higher Moments



- Tree-level OPE poor fit for higher moments
- Need perturbative corrections to extract $\langle \xi^2 \rangle_K$
  - Computation in progress

Conclusion

- Pion LCDA important but challenging – good to have complementary methods
- $\langle \xi^2 \rangle(\mu = 2 \text{ GeV}) = 0.210 \pm 0.036$ (quenched) – compatible with other methods
- Moving to dynamical ensembles with lighter masses
- Preliminary extraction of $\langle \xi^4 \rangle$ – working to increase statistics
- Explorations of kaon LCDA and nonvanishing odd moments

Thank you to R. Edwards, B. Joó, S. Ueda and others for software development (Chroma, QPhiX, Bridge++) and to ASRock, DiCOS, Barcelona Supercomputing Center, and RIKEN for computational resources!

Multi-Mass Solver

- Need to invert Dirac operator $D = \not{D} + m$ for quark propagators
- Can write $D = 1 - \kappa H$ with $\kappa = \frac{1}{2(m+4)}$ and

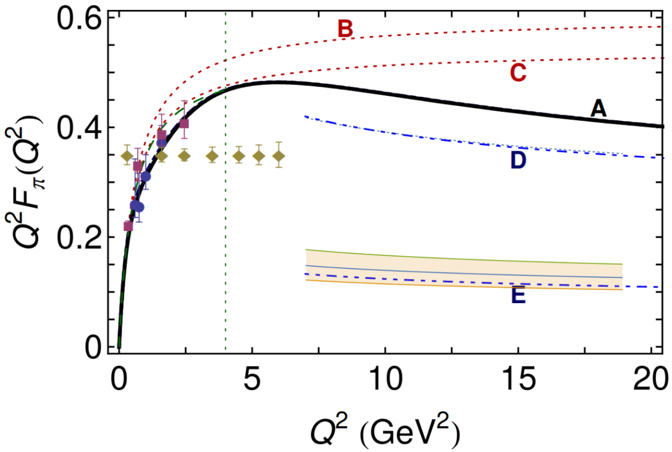
$$H(n|m) = \sum_{\mu} (1 - \gamma_\mu) U_\mu \delta_{n+\hat{\mu},m}$$

- Can expand D^{-1} as *hopping expansion*

$$D^{-1}\psi = \sum_{j=0}^{\infty} \kappa^j H^j \psi$$

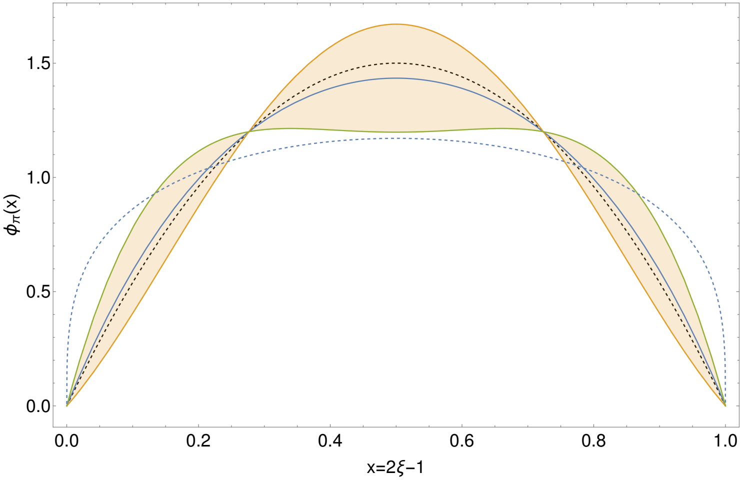
- H is independent of m – can reuse $H^j\psi$ for all masses
- More expensive for single mass but can be competitive with enough masses

Pion Electromagnetic Form Factor



Adapted from L. Chang et al., nucl-th/1307.0026

Reconstruction of LCDA



Comparison of LCDA Models

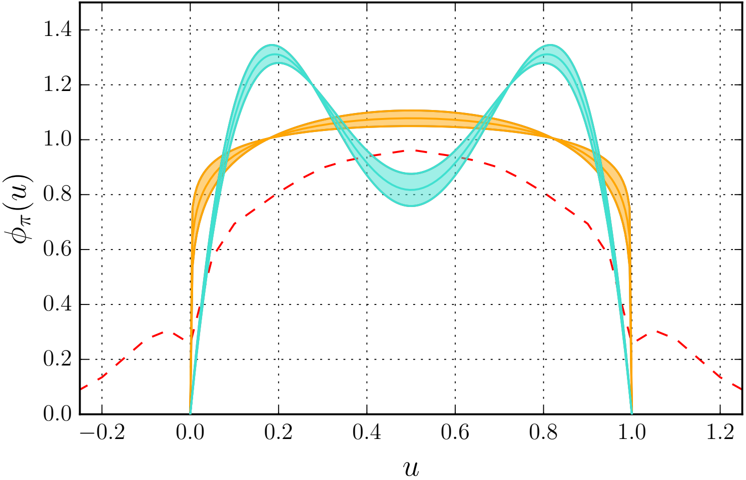


Figure credit: Bali et al., 1807.06671, PRD **98**, 094507