Proton Transversity GPDs from lattice QCD

Joshua Miller

Temple University

Preliminary work by:

J. Miller, S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, F. Steffens

LaMET 2024
College Park, Maryland
08/12/2024

Outline

❖ Work extends the approach for the unpolarized and axial cases

❖ Theoretic Formulation

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Outline

❖ Work extends the approach for the unpolarized and axial cases

❖ Theoretic Formulation

❖ Lattice Formulation (focus of this talk)

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Outline

❖ Work extends the approach for the unpolarized and axial cases

❖ Theoretic Formulation

❖ Lattice Formulation (focus of this talk)

- ❖ Background
- ❖ Lattice Methodology
- ❖ Results
  - Matrix Elements
  - Lorentz invariant amplitudes
  - Quasi-GPDs
  - Light-Cone GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Generalized Parton Distributions

❖ **GPDs are rich in information:**

- Reflect spatial distribution of partons in transverse plane
- Hadron mechanical properties are stored in GPDs
- Information on spin

❖ **… but not well studied:**

- extracted from off-forward kinematic (unlike PDFs)
- Multi-variable quantities; dependence upon $x,\ t$ and ξ (unlike PDFs)
- Inferred from Compton form factors from experimental data (e.g., DVCS)
- Other processes proposed (**SDHEP [J. Qiu et al, arXiv:2205.07846]**) still require theoretical developments

❖ **Transversity proton GPDs:**

- Four GPDs: $H_T,\ E_T,\ \widetilde{H}_T,\ \widetilde{E}_T$

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

Methodology on the Lattice

❖ Choice of frame:

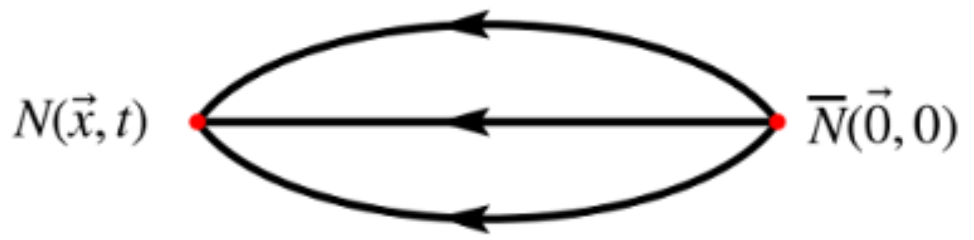
- Symmetric: $\vec{p_i} = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p_f} = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \vec{p}_f = P_3\hat{z}$

Methodology on the Lattice

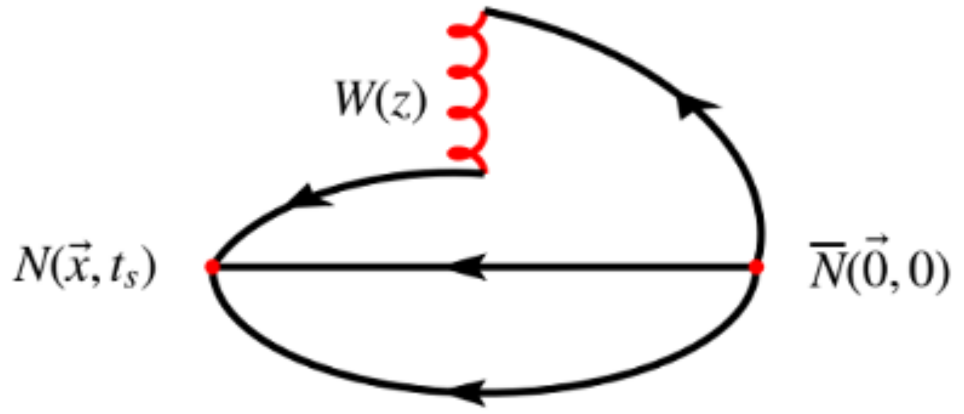
❖ Choice of frame:

- Symmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P) | N(P) \rangle$

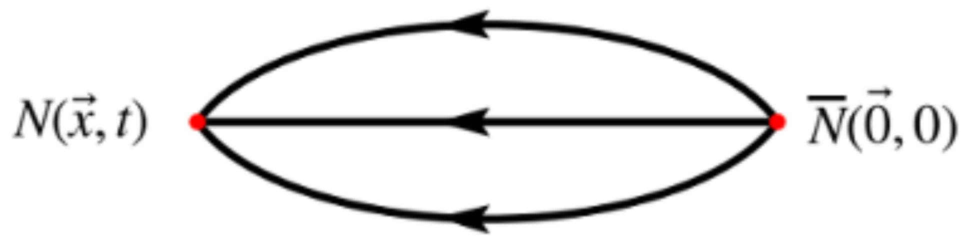


$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

Methodology on the Lattice

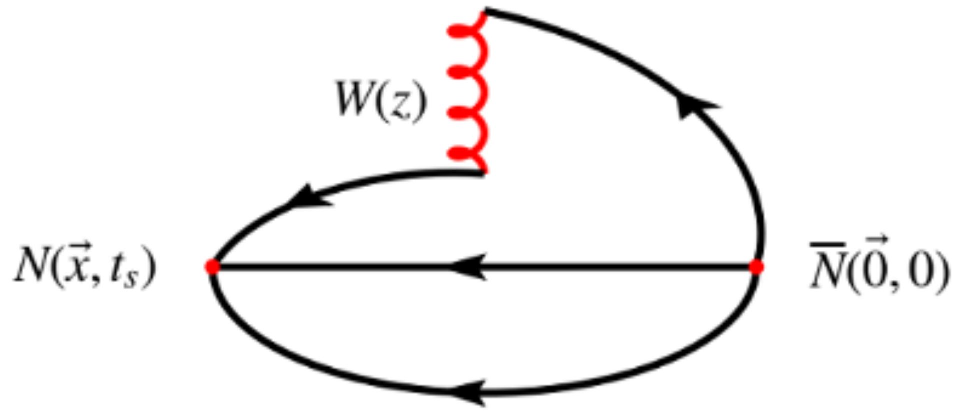
❖ Choice of frame:

- Symmetric: $\vec{p_i} = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p_f} = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$



$\langle N(P) | N(P) \rangle$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

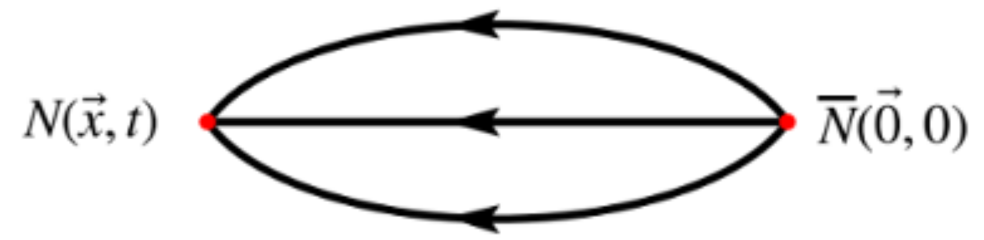
❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C^{3\text{pt}}_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

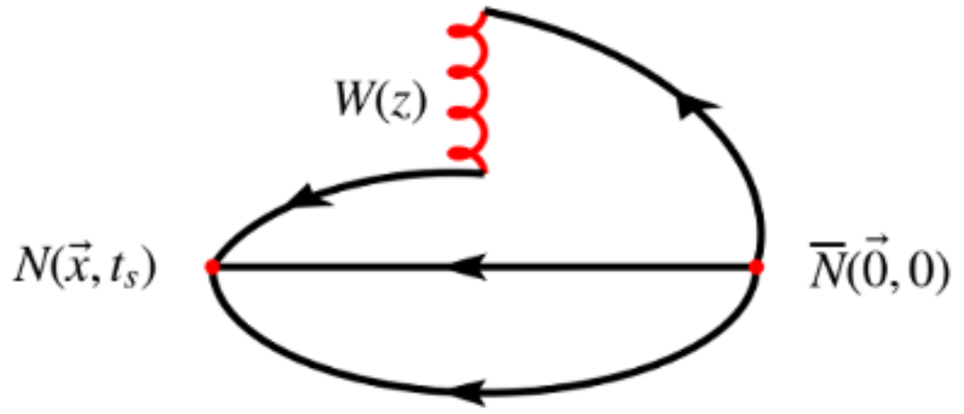
Methodology on the Lattice

❖ Choice of frame:

- Symmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$



$\langle N(P) | N(P) \rangle$



$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen

❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C^{3\text{pt}}_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

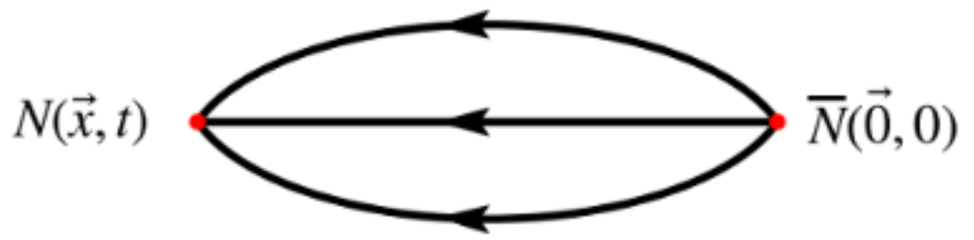
❖ Parameterize the matrix elements in terms of (Lorentz invariant) amplitudes

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

Methodology on the Lattice

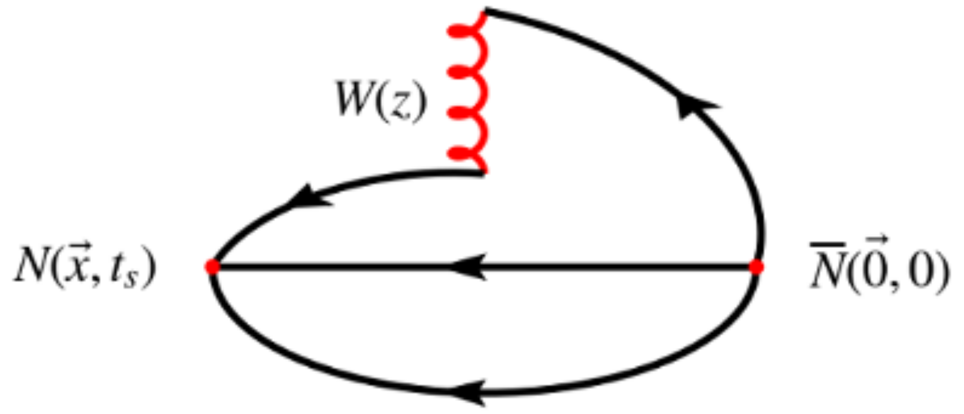
❖ Choice of frame:

- Symmetric: $\vec{p_i} = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p_f} = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$



$\langle N(P)\,|\,N(P)\rangle$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P_f)\,|\,\bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle$

❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C^{3\text{pt}}_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s-\tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s-\tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

❖ Parameterize the matrix elements in terms of (Lorentz invariant) amplitudes

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

→ 12 linearly independent Lorentz invariant amplitudes

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

- Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

GPDs defined in SF with A_i in SF

⟶ $H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$

⟶ $E_T = 2A_2 - A_4$

⟶ $\widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$

⟶ $\widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$$

GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i+E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4 + \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_8$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{(E_i+E_f)^2}{2P}zA_8$$

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$$

GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4 + \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{(E_i + E_f)^2}{2P}zA_8$$

❖ Renormalization functions: RI-MOM

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$$

GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i+E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4 + \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_8$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{(E_i+E_f)^2}{2P}zA_8$$

❖ Renormalization functions: RI-MOM

❖ Transform from position to momentum space (Backus-Gilbert) [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$$

GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4 + \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{(E_i + E_f)^2}{2P}zA_8$$

❖ Renormalization functions: RI-MOM

❖ Transform from position to momentum space (Backus-Gilbert) [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

❖ Extract light cone-GPDs using one-loop matching formalism [Liu, et al., Phys. Rev. D 100, 034006 (2019)]

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2 + \Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m) - P_3^2)}{2m^2}zA_{T11}\right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

$$\Pi^s_i \neq \Pi^a_i$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Coefficients become more complicated

Amplitudes frame independent

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

$$\Pi^s_i \neq \Pi^a_i \qquad A^s_{Ti} = A^a_{Ti}$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Coefficients become more complicated

Amplitudes frame independent

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

$$\Pi^s_i \neq \Pi^a_i \qquad A^s_{Ti} = A^a_{Ti}$$

Frame dependence of matrix elements due to kinematic coefficients of A_{Ti}

Lattice Setup

❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

Lattice Setup

❖ $N_f = 2+1+1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u,d,s,c	260	4	3.0

❖ Calculation of symmetric and asymmetric frame

- Symmetric frame:
  Each $\vec{\Delta}$ requires new calculation
- Asymmetric frame:
  Several $\vec{\Delta}$ values grouped in the same production run (e.g. $\{\vec{\Delta} = (100), (200), (300), \ldots\}$)

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.38	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.77	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	269	8	17216
asymm	±1.25	(±1, ±1, 0)	0.34	0	16	195	8	24960
asymm	±1.25	(±2,0,0), (0,±2,0)	0.65	0	8	269	8	17216
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.81	0	16	195	8	24960
asymm	±1.25	(±2,±2,0)	1.24	0	16	195	8	24960
asymm	±1.25	(±3,0,0), (0,±3,0)	1.38	0	8	269	8	17216
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.52	0	16	195	8	24960
asymm	±1.25	(±4,0,0), (0,±4,0)	2.29	0	8	269	8	17216

Lattice Setup

- $N_f = 2+1+1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

- Calculation of symmetric and asymmetric frame
  - Symmetric frame:
    Each $\vec{\Delta}$ requires new calculation
  - Asymmetric frame:
    Several $\vec{\Delta}$ values grouped in the same production run
    (e.g. $\{\vec{\Delta} = (100), (200), (300), \ldots\}$)

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.38	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.77	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	269	8	17216
asymm	±1.25	(±1, ±1, 0)	0.34	0	16	195	8	24960
asymm	±1.25	(±2,0,0), (0,±2,0)	0.65	0	8	269	8	17216
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.81	0	16	195	8	24960
asymm	±1.25	(±2,±2,0)	1.24	0	16	195	8	24960
asymm	±1.25	(±3,0,0), (0,±3,0)	1.38	0	8	269	8	17216
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.52	0	16	195	8	24960
asymm	±1.25	(±4,0,0), (0,±4,0)	2.29	0	8	269	8	17216

Computationally efficient setup

Lattice Setup

❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.38	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.77	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	269	8	17216
asymm	±1.25	(±1, ±1, 0)	0.34	0	16	195	8	24960
asymm	±1.25	(±2,0,0), (0,±2,0)	0.65	0	8	269	8	17216
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.81	0	16	195	8	24960
asymm	±1.25	(±2,±2,0)	1.24	0	16	195	8	24960
asymm	±1.25	(±3,0,0), (0,±3,0)	1.38	0	8	269	8	17216
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.52	0	16	195	8	24960
asymm	±1.25	(±4,0,0), (0,±4,0)	2.29	0	8	269	8	17216

❖ Calculation of symmetric and asymmetric frame

- Symmetric frame:
  Each $\vec{\Delta}$ requires new calculation
- Asymmetric frame:
  Several $\vec{\Delta}$ values grouped in the same production run
  (e.g. $\{\vec{\Delta} = (100), (200), (300), \ldots\}$)

Computationally efficient setup

❖ Strategy: decomposition of amplitudes for each kinematic setup $(\pm P_3, \pm \vec{\Delta}, \pm z)$

Lattice Setup

❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

❖ Calculation of symmetric and asymmetric frame

- Symmetric frame:
  Each $\vec{\Delta}$ requires new calculation
- Asymmetric frame:
  Several $\vec{\Delta}$ values grouped in the same production run
  (e.g. $\{\vec{\Delta} = (100), (200), (300), \ldots\}$)

Computationally efficient setup

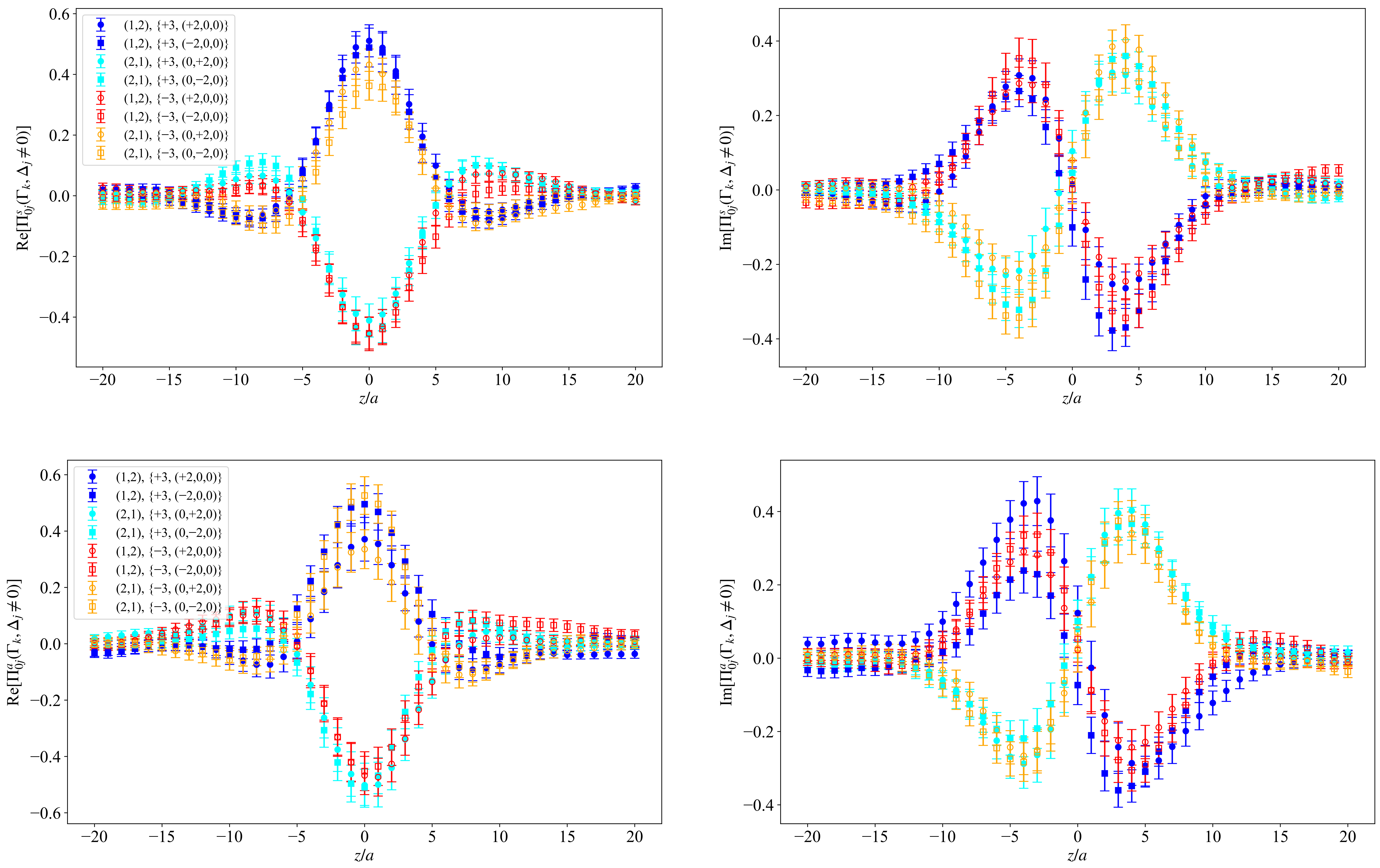
frame	P_3 [GeV]	$\mathbf{\Delta}$ [$\frac{2\pi}{L}$]	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.38	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.77	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	269	8	17216
asymm	±1.25	(±1, ±1, 0)	0.34	0	16	195	8	24960
asymm	±1.25	(±2,0,0), (0,±2,0)	0.65	0	8	269	8	17216
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.81	0	16	195	8	24960
asymm	±1.25	(±2,±2,0)	1.24	0	16	195	8	24960
asymm	±1.25	(±3,0,0), (0,±3,0)	1.38	0	8	269	8	17216
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.52	0	16	195	8	24960
asymm	±1.25	(±4,0,0), (0,±4,0)	2.29	0	8	269	8	17216

❖ Strategy: decomposition of amplitudes for each kinematic setup $(\pm P_3, \pm \vec{\Delta}, \pm z)$

❖ Exploitation of $\tilde{A}_i$ symmetry properties with respect to $(\pm P_3, \pm \vec{\Delta}, \pm z)$

Matrix Elements: $\Pi_j^{s/a}(\Gamma_k, \Delta_j \neq 0)$

$|P_3| = 1.25\ \text{GeV} \quad -t = 0.69\ \text{GeV}$



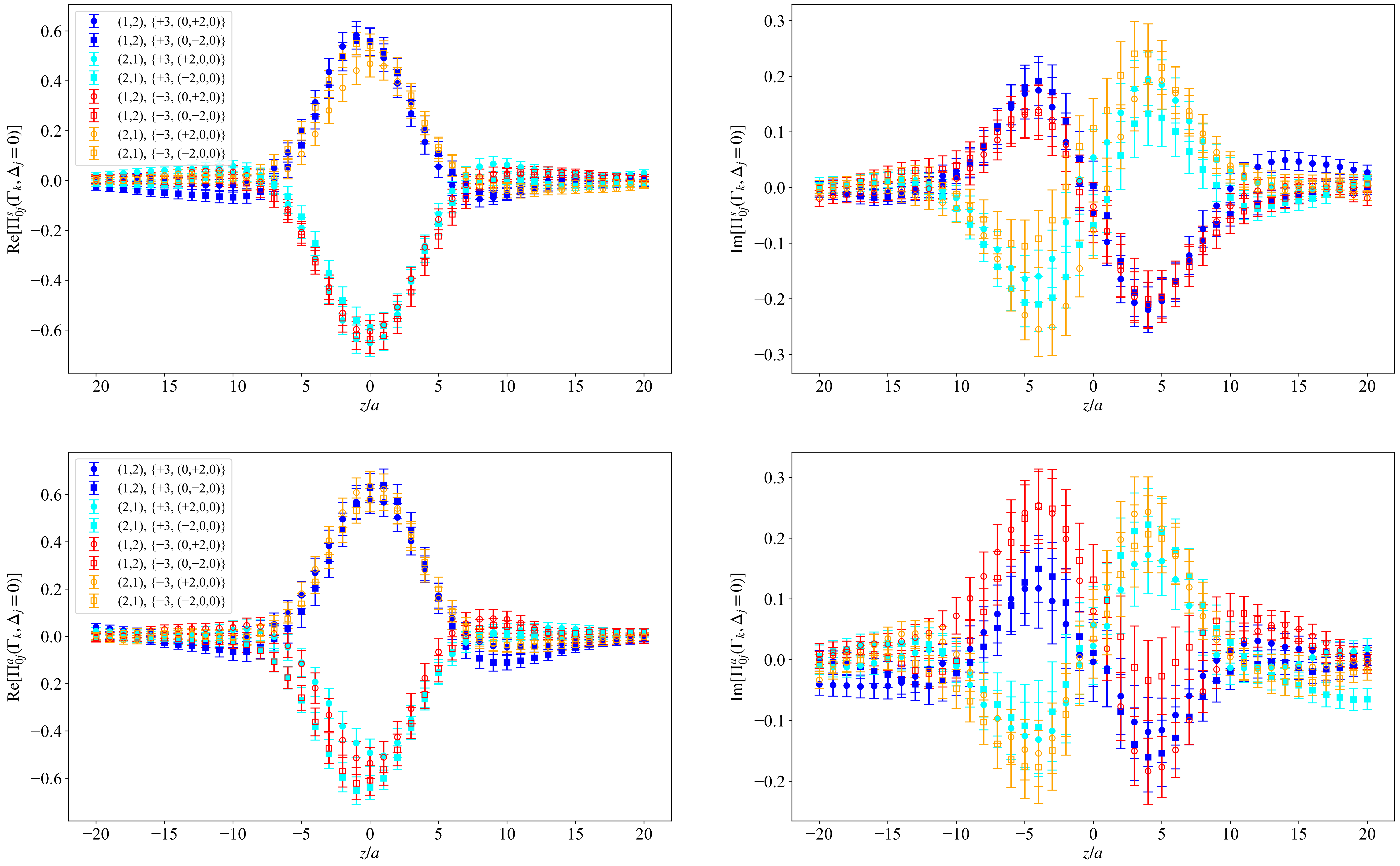
$|P_3| = 1.25\ \text{GeV} \quad -t = 0.65\ \text{GeV}$

$$\Pi^s_{01}(\Gamma_2) = K\Bigg(\frac{(E+m)P_3\Delta_2^2}{4m^4}A_{T2} - \frac{(E+m)\Delta_2^2}{4m^2}zA_{T3} + \frac{P_3(\Delta_1^2+4m(E+m))}{8m^3}A_{T4} - \frac{(\Delta_1^2+4m(E+m))}{8m}zA_{T5} + \frac{(E+m)P_3}{2m^2}A_{T10}\Bigg)$$

$$\Pi^s_{02}(\Gamma_1) = K\Bigg(-\frac{(E+m)P_3\Delta_1^2}{4m^4}A_{T2} + \frac{(E+m)\Delta_1^2}{4m^2}zA_{T3} - \frac{P_3(\Delta_2^2+4m(E+m))}{8m^3}A_{T4} + \frac{(\Delta_2^2+4m(E+m))}{8m}zA_{T5} - \frac{(E+m)P_3}{2m^2}A_{T10}\Bigg)$$

Matrix Elements: $\Pi_j^{s/a}(\Gamma_k,\, \Delta_j = 0)$

$|P_3| = 1.25\,\text{GeV} \quad -t = 0.69\ \text{GeV}$



$|P_3| = 1.25\,\text{GeV} \quad -t = 0.65\ \text{GeV}$

$$\Pi^s_{01}(\Gamma_2) = K\bigg(\frac{(E+m)P_3\Delta_2^2}{4m^4}A_{T2} - \frac{(E+m)\Delta_2^2}{4m^2}zA_{T3} + \frac{P_3(\Delta_1^2 + 4m(E+m))}{8m^3}A_{T4} - \frac{(\Delta_1^2 + 4m(E+m))}{8m}zA_{T5} + \frac{(E+m)P_3}{2m^2}A_{T10}\bigg)$$

$$\Pi^s_{02}(\Gamma_1) = K\bigg(-\frac{(E+m)P_3\Delta_1^2}{4m^4}A_{T2} + \frac{(E+m)\Delta_1^2}{4m^2}zA_{T3} - \frac{P_3(\Delta_2^2 + 4m(E+m))}{8m^3}A_{T4} + \frac{(\Delta_2^2 + 4m(E+m))}{8m}zA_{T5} - \frac{(E+m)P_3}{2m^2}A_{T10}\bigg)$$

Matrix Elements to Amplitudes

❖ 24 separate matrix elements for a given kinematic case

Matrix Elements to Amplitudes

- 24 separate matrix elements for a given kinematic case
  - Some do not contribute at zero skewness
  - Some are linearly dependent!
  - Ex: If $\Delta_1 = \Delta_2$, then $\Pi_{01}(\Gamma_2) = -\Pi_{02}(\Gamma_1)$

Matrix Elements to Amplitudes

- 24 separate matrix elements for a given kinematic case
  - Some do not contribute at zero skewness
  - Some are linearly dependent!
  - Ex: If $\Delta_1 = \Delta_2$, then $\Pi_{01}(\Gamma_2) = -\Pi_{02}(\Gamma_1)$

We reduce the data to 12 independent Matrix Elements to disentangle the A_i

Matrix Elements to Amplitudes

- 24 separate matrix elements for a given kinematic case
  - Some do not contribute at zero skewness
  - Some are linearly dependent!
  - Ex: If $\Delta_1 = \Delta_2$, then $\Pi_{01}(\Gamma_2) = -\Pi_{02}(\Gamma_1)$

We reduce the data to 12 independent Matrix Elements to disentangle the A_i

Example: $\overrightarrow{\Delta} = (\Delta, 0, 0)$

Symmetric Frame

$$zA_{T1} \propto \Pi^s_{12}(\Gamma_1)$$

Asymmetric Frame

$$zA_{T1} \propto \Pi^a_{02}(\Gamma_1),\ \Pi^a_{02}(\Gamma_3),\ \Pi^a_{12}(\Gamma_1),\ \Pi^a_{12}(\Gamma_3)$$

Matrix Elements to Amplitudes

- 24 separate matrix elements for a given kinematic case
  - Some do not contribute at zero skewness
  - Some are linearly dependent!
  - Ex: If $\Delta_1 = \Delta_2$, then $\Pi_{01}(\Gamma_2) = -\Pi_{02}(\Gamma_1)$

We reduce the data to 12 independent Matrix Elements to disentangle the A_i

Example: $\vec{\Delta} = (\Delta,0,0)$

Symmetric Frame

$$zA_{T1} \propto \Pi^s_{12}(\Gamma_1)$$

Asymmetric Frame

$$zA_{T1} \propto \Pi^a_{02}(\Gamma_1),\ \Pi^a_{02}(\Gamma_3),\ \Pi^a_{12}(\Gamma_1),\ \Pi^a_{12}(\Gamma_3)$$

- Disentangling amplitudes gets more difficult in (any) non-symmetric frame compared to the symmetric one
- Amplitudes are frame independent by construction
- Coefficients are frame dependent

Transversity Amplitudes

Ratio

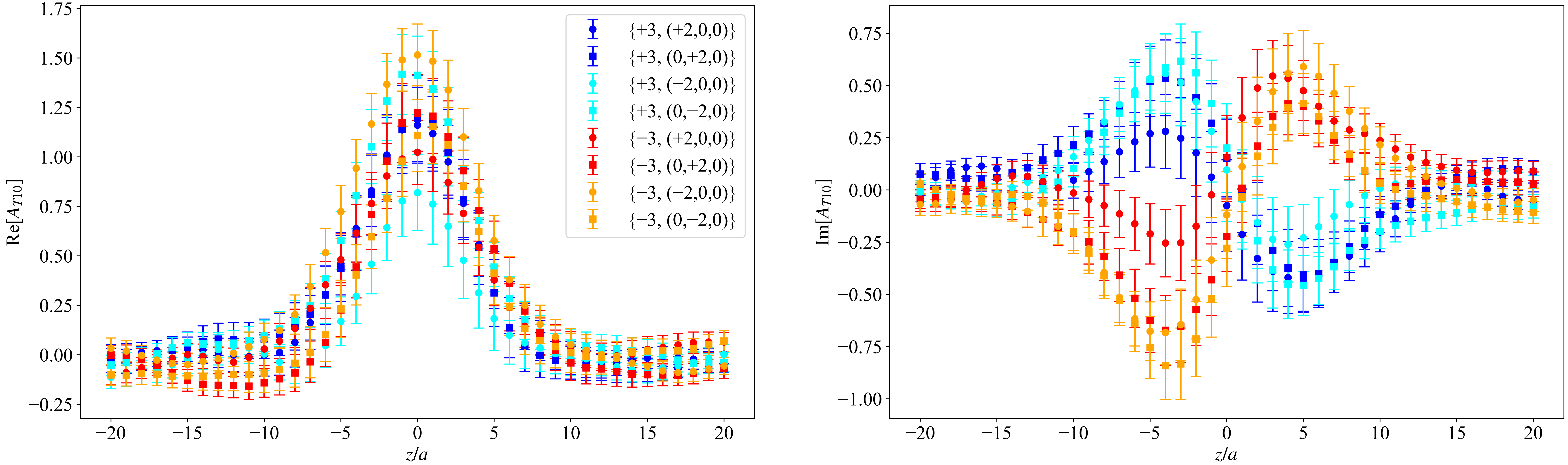
Transversity Amplitudes

Ratio → *Plateau*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}

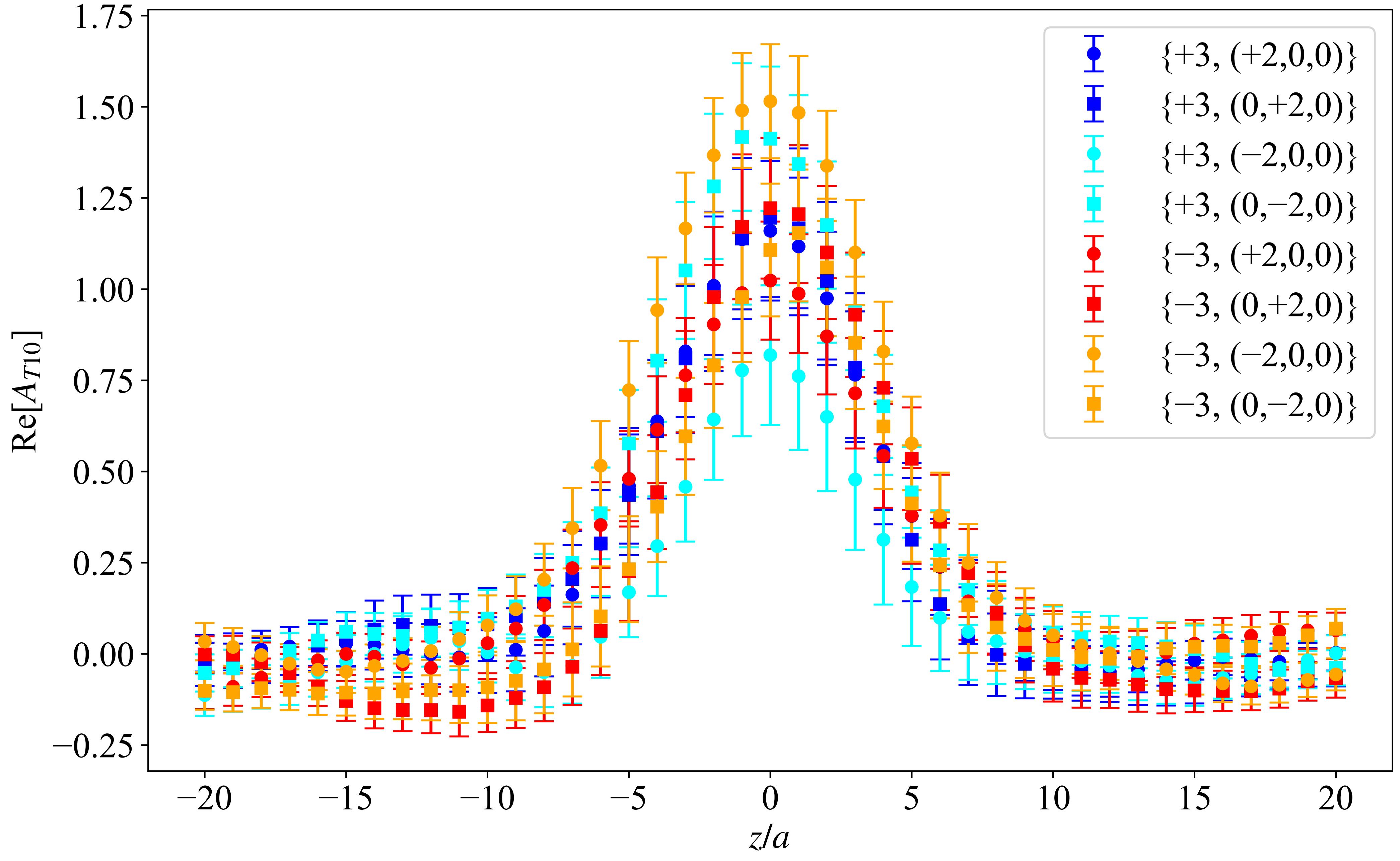


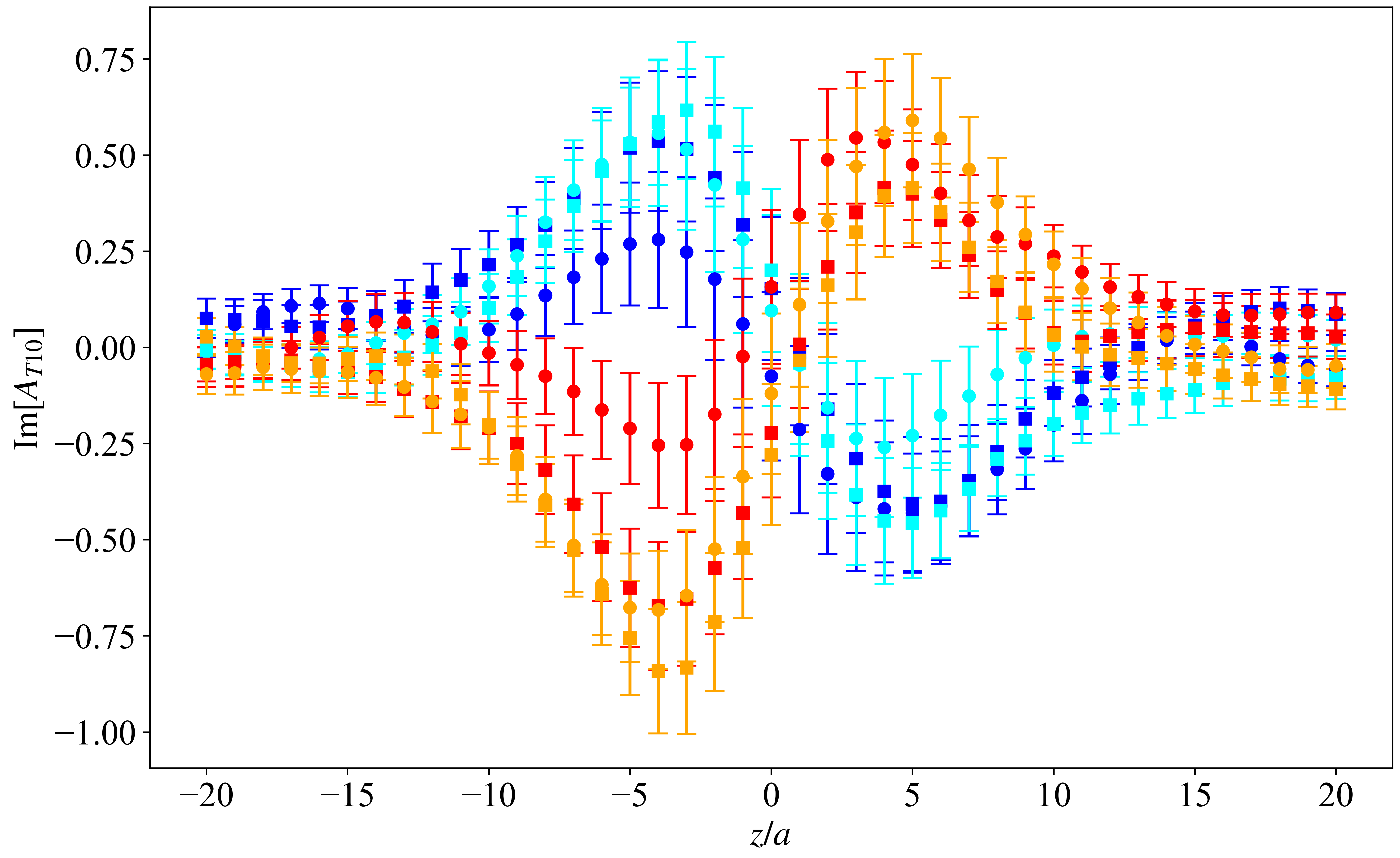
Raw data shows a clear signal

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}





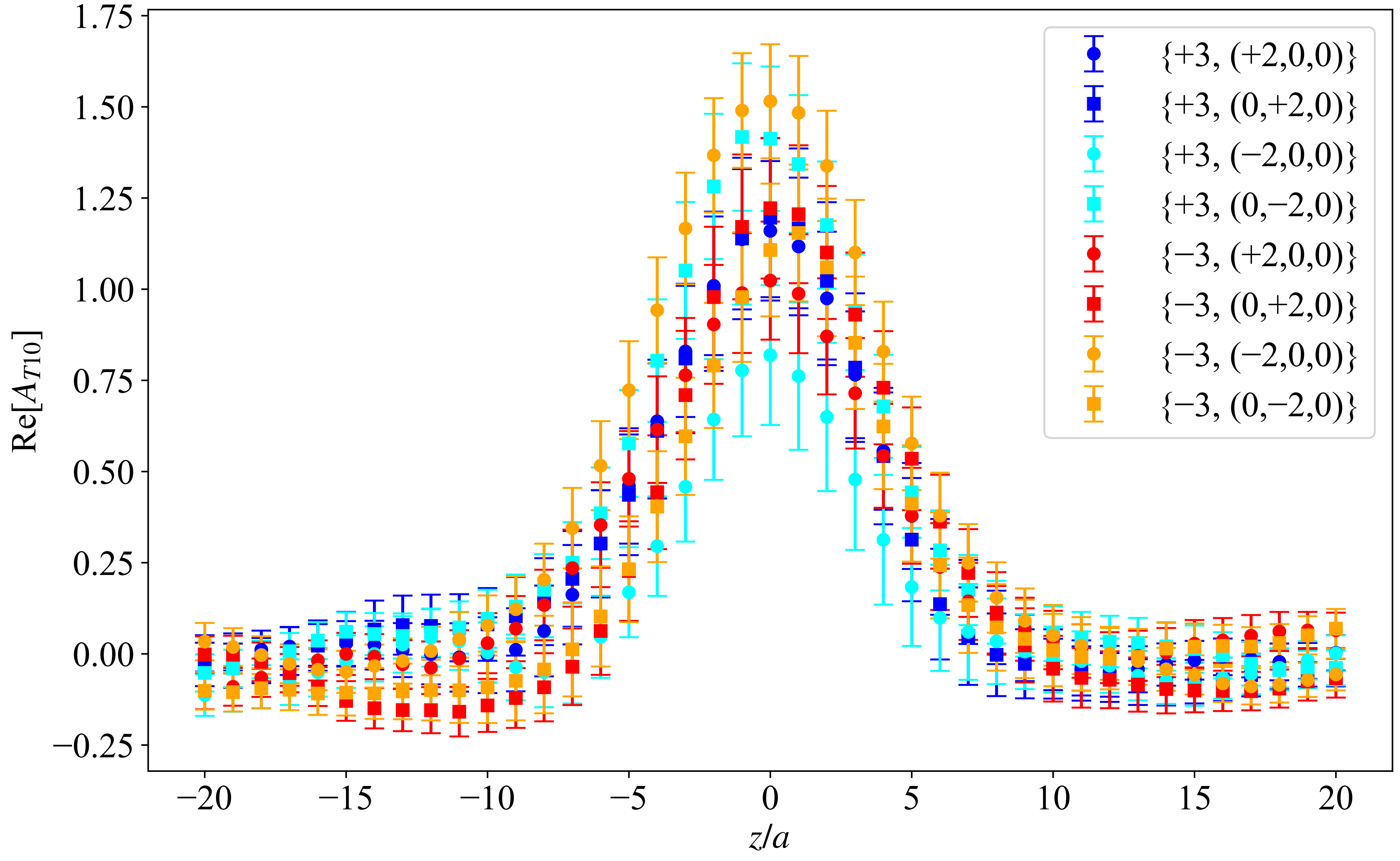
Raw data shows a clear signal

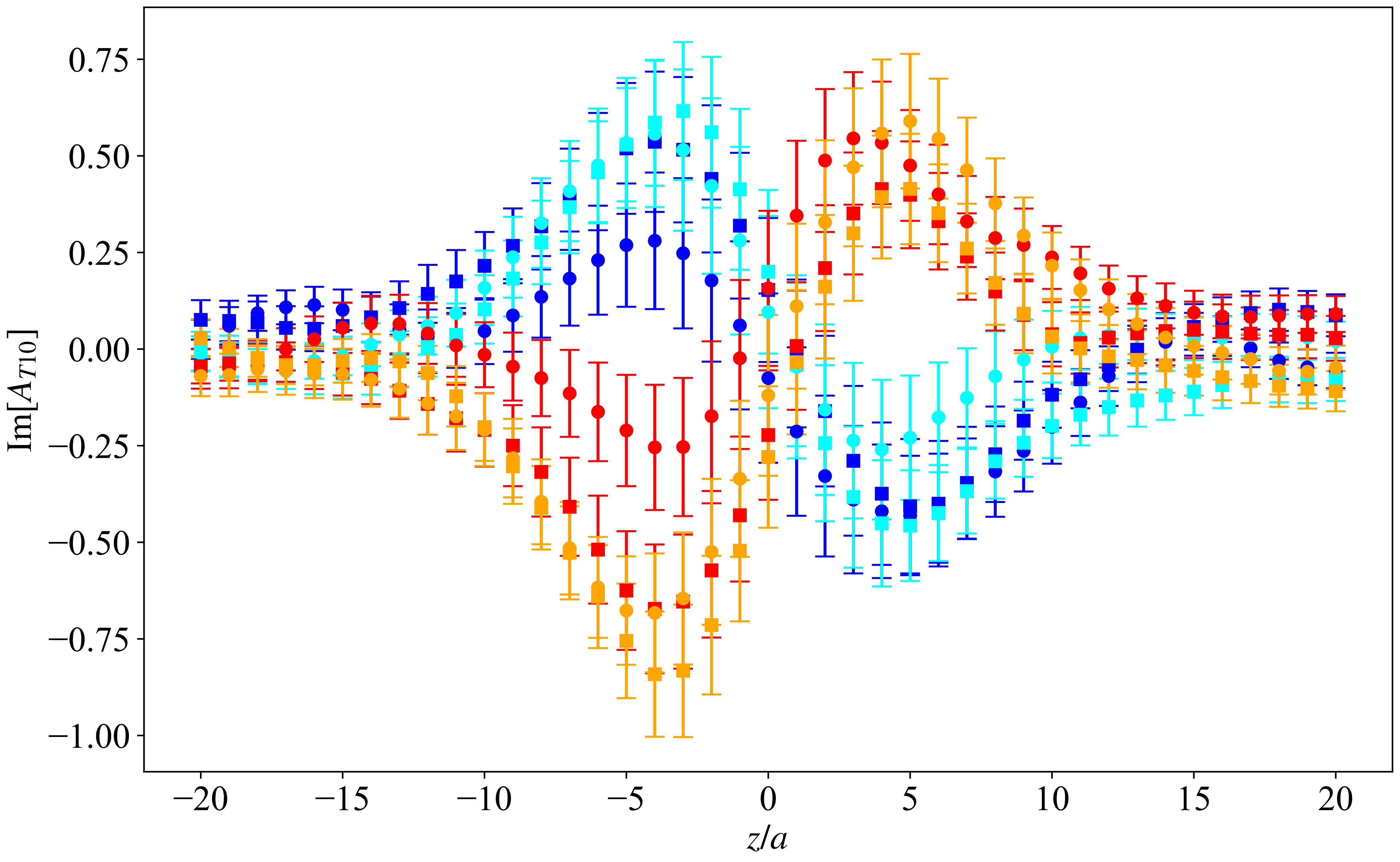
→ *Average*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}





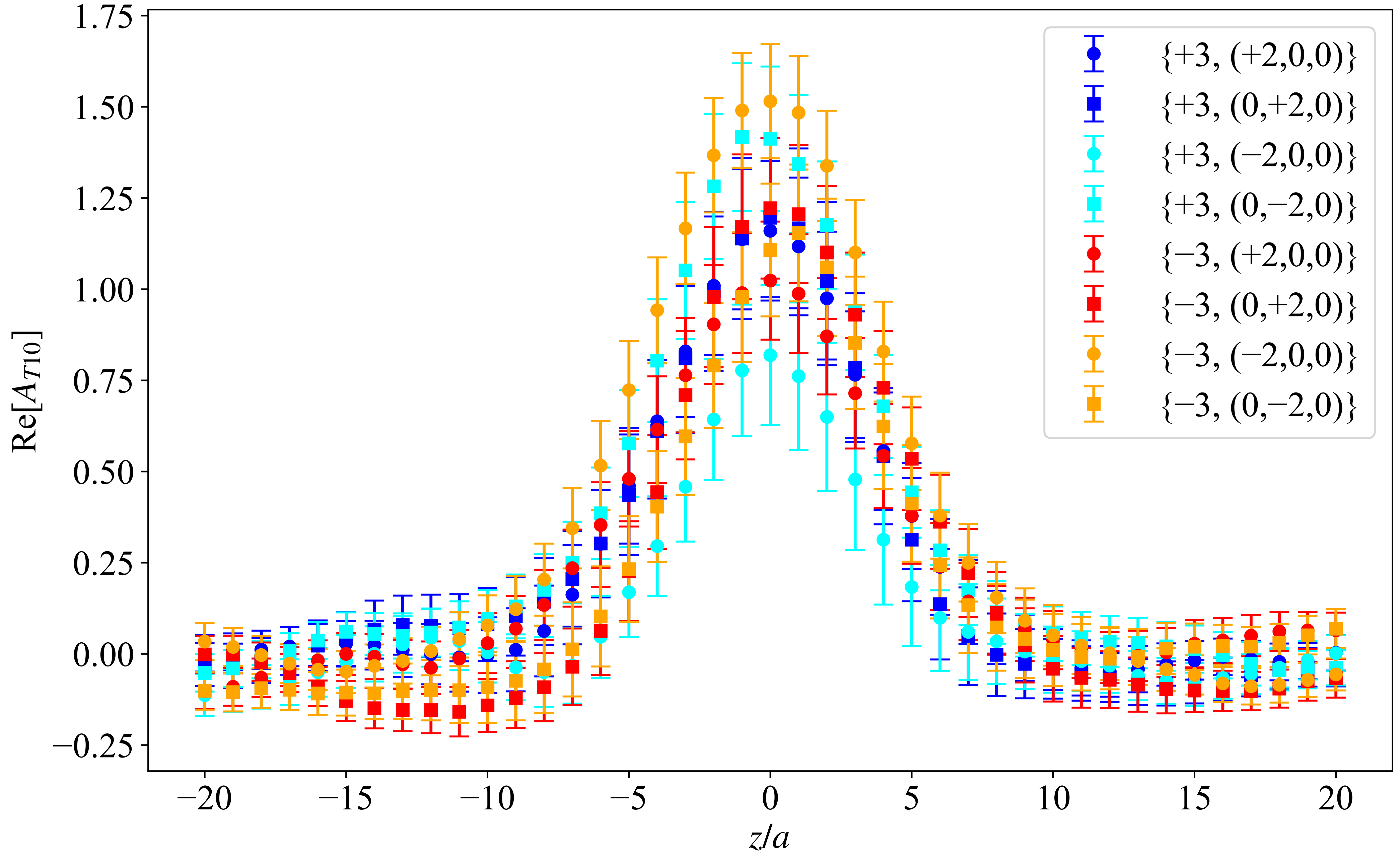
Raw data shows a clear signal

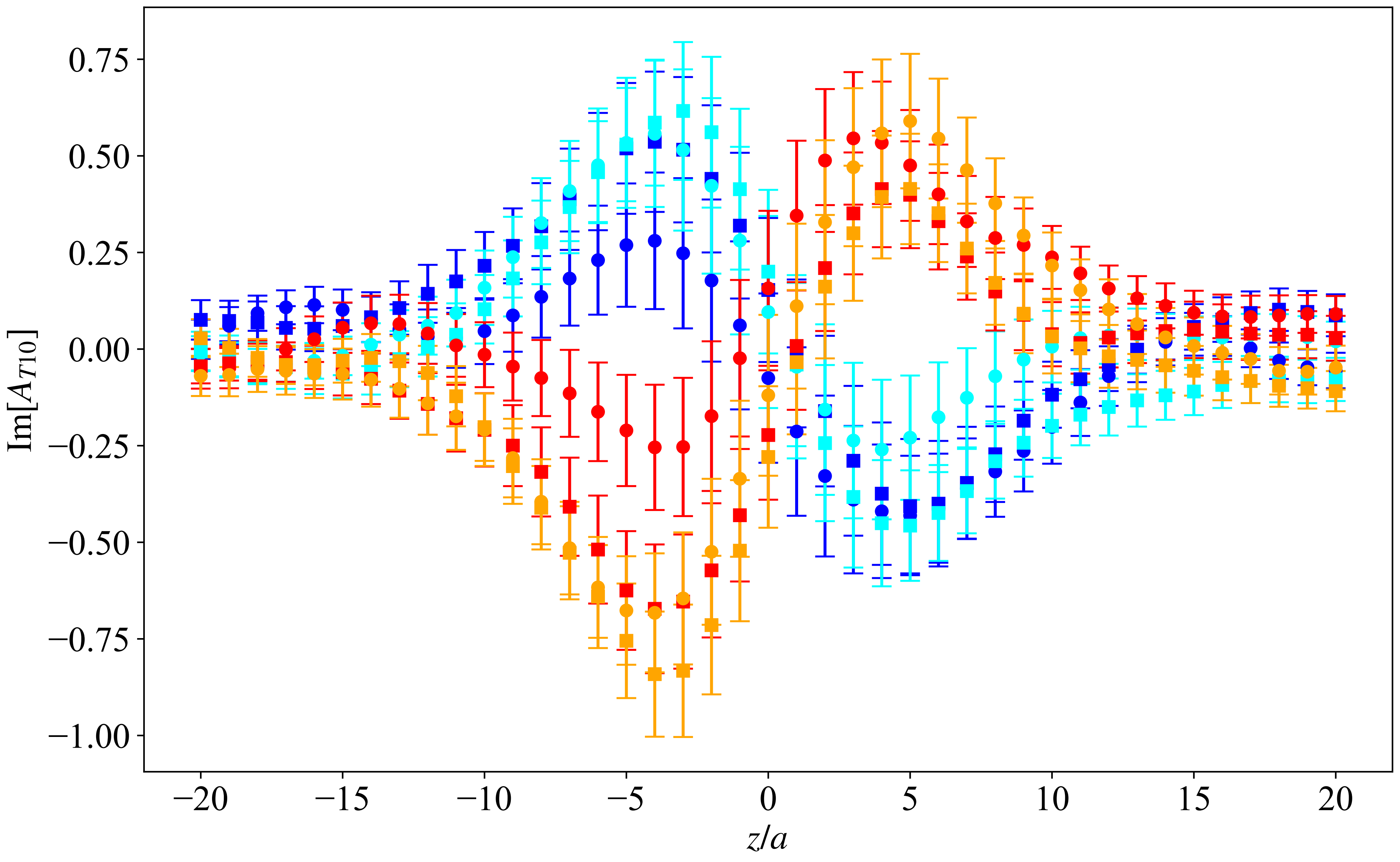
→ *Average* → *Quasi-GPDs*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

<u>*Example*</u>: A_{T10}





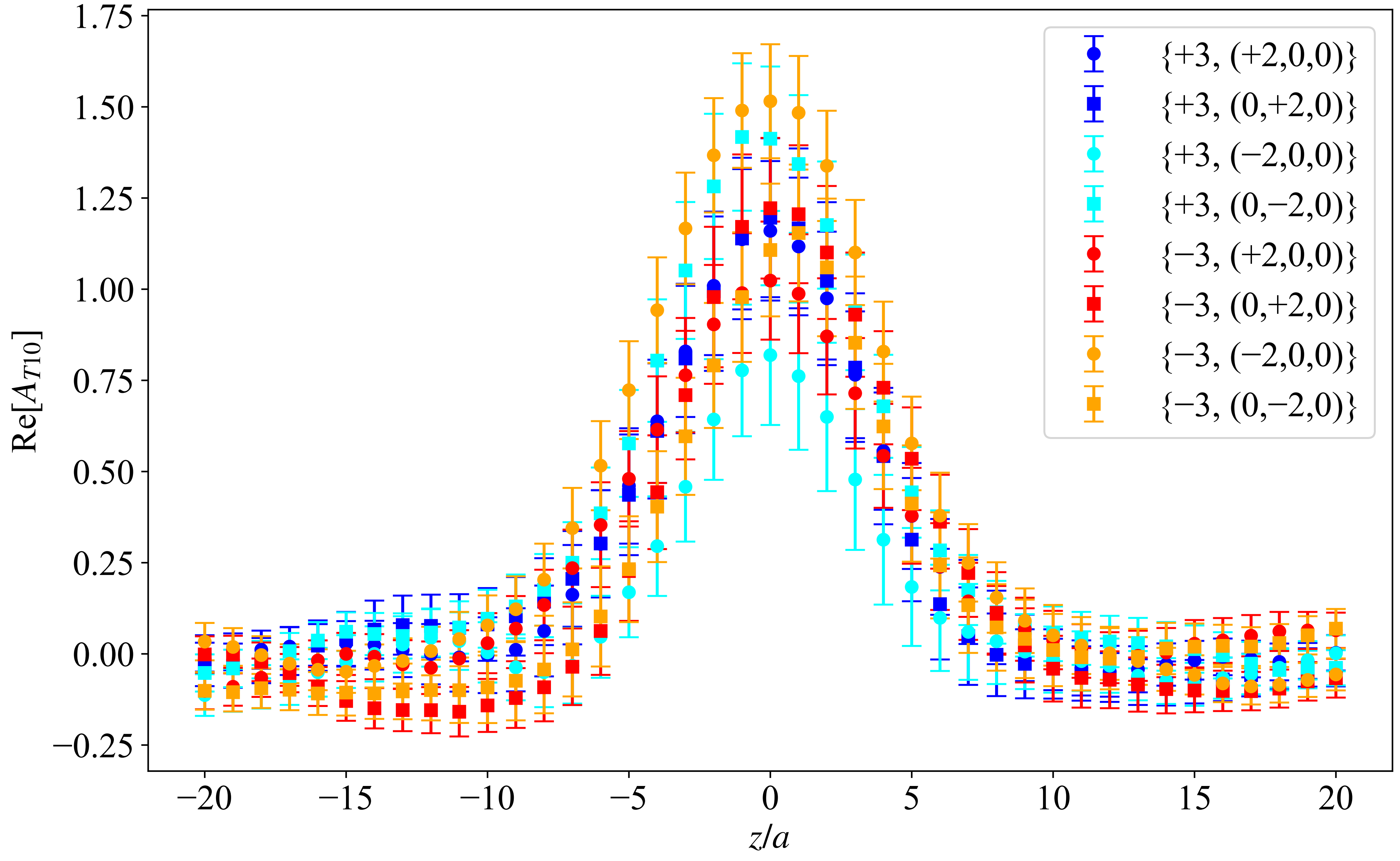
Raw data shows a clear signal

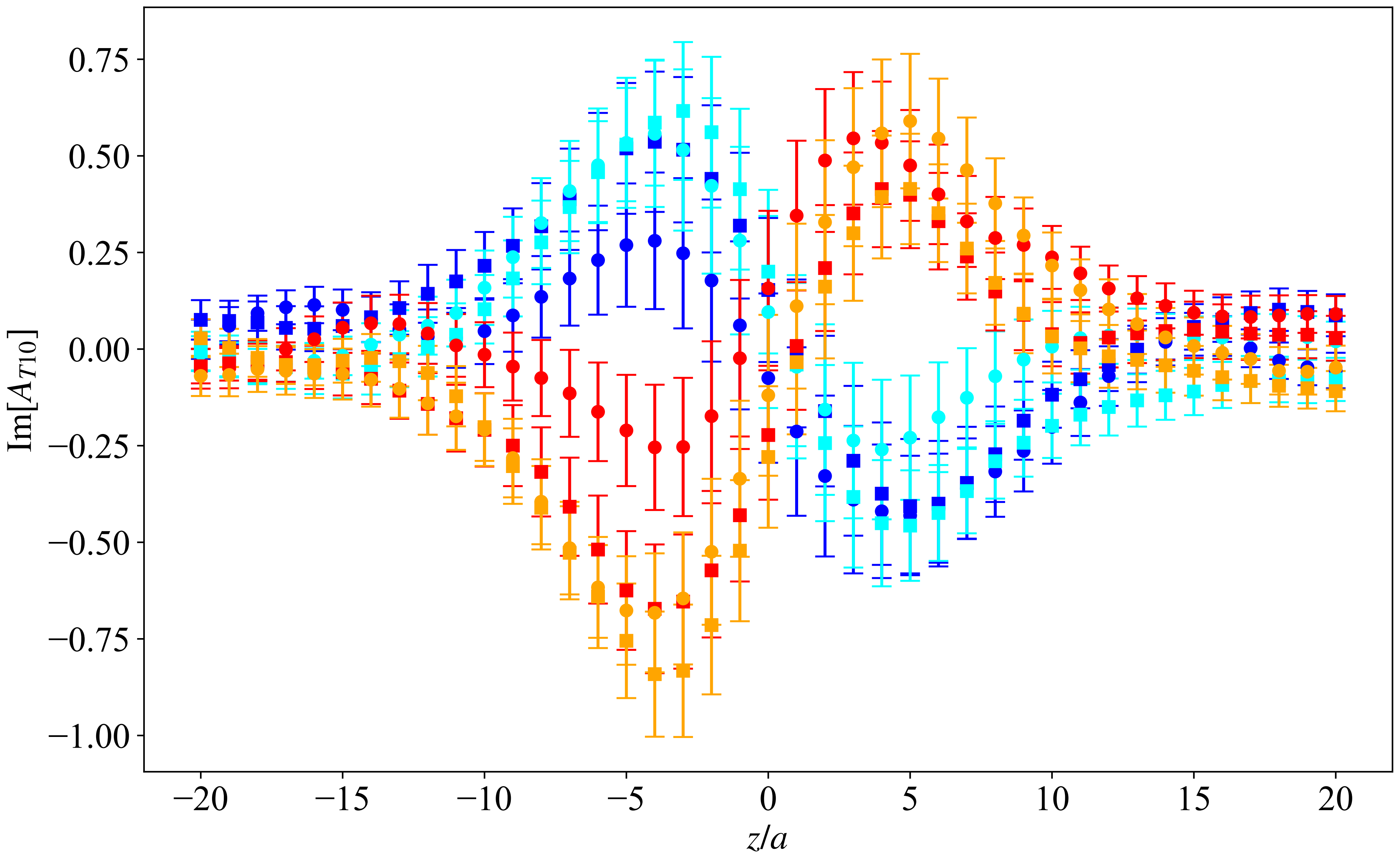
→ *Average* → *Quasi-GPDs* → *Backus-Gilbert*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}





Raw data shows a clear signal

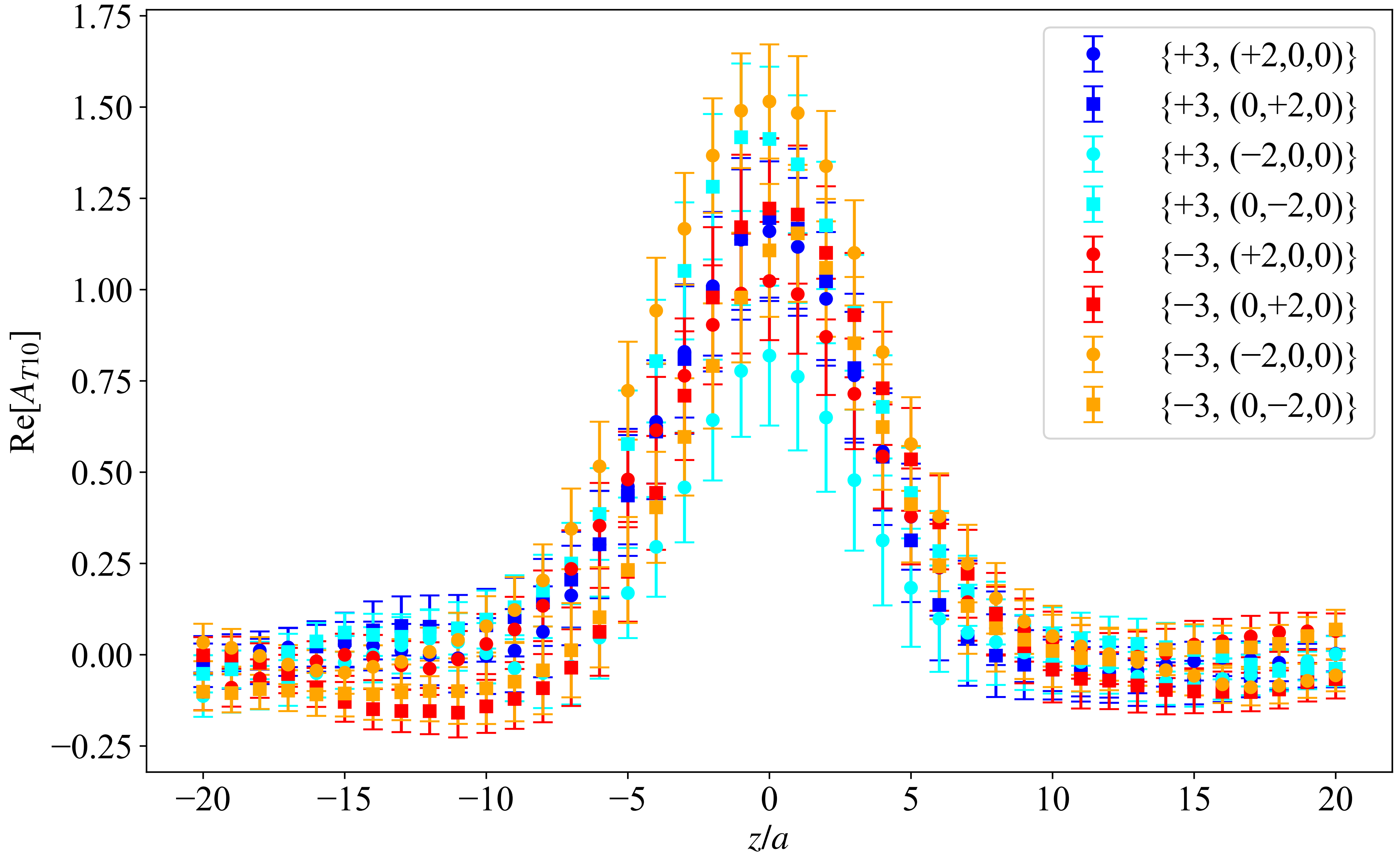
→ *Average* → *Quasi-GPDs* → *Backus-Gilbert* → *Match*

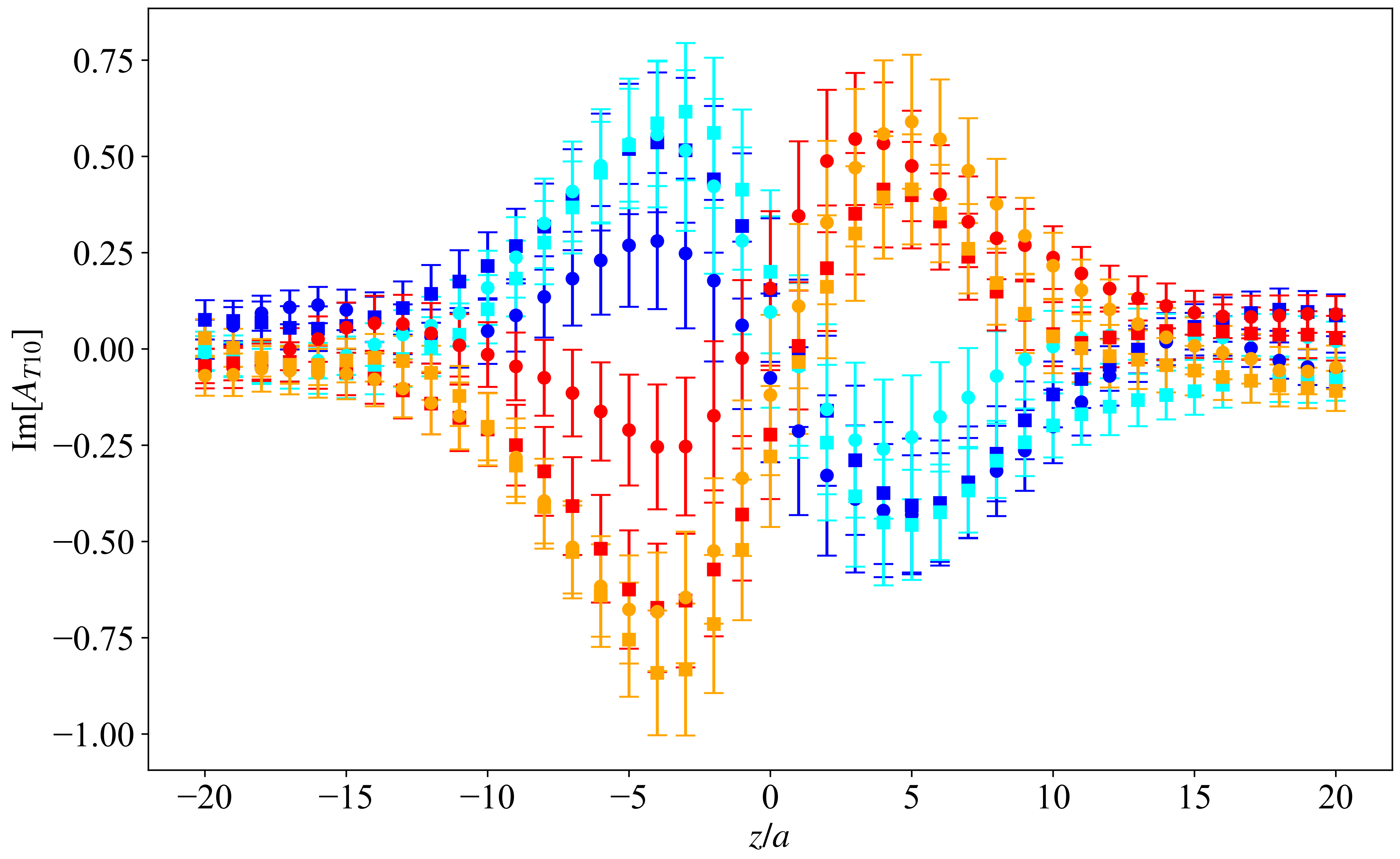
Amplitude Symmetry

❖ Symmetry properties of amplitudes

$$A^*_{Ti}(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_{Ti}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) \qquad i = 1,\ 2,\ 4,\ 7,\ 8,\ 10,\ 11$$

$$-A^*_{Ti}(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_{Ti}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) \qquad i = 3,\ 5,\ 6,\ 9,\ 12$$



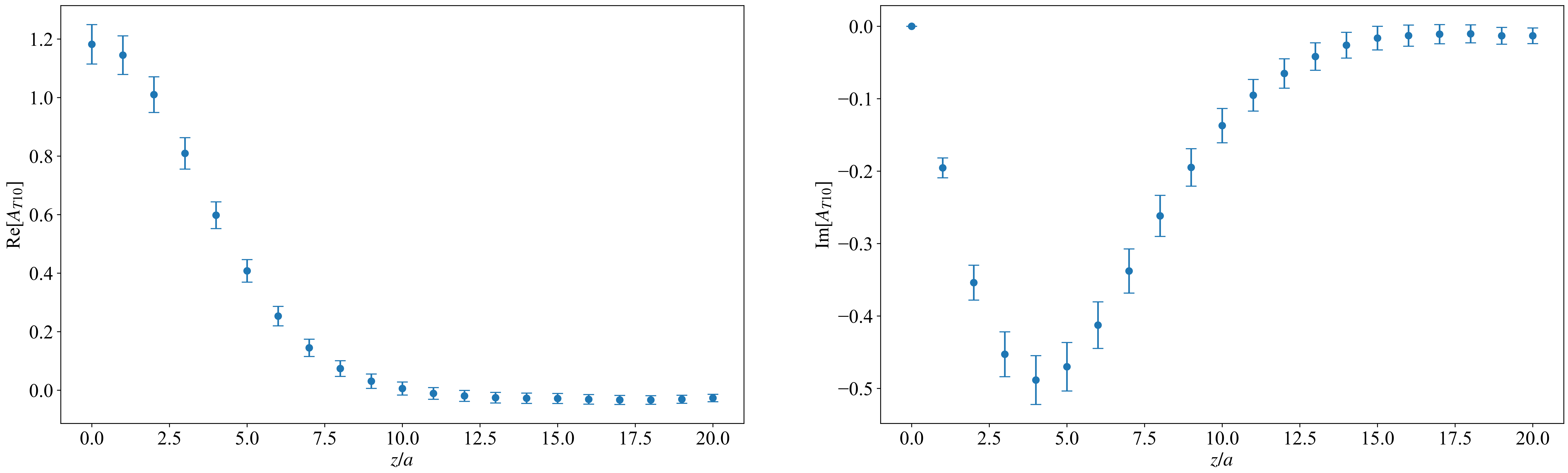


Amplitude Symmetry

- Symmetry properties of amplitudes

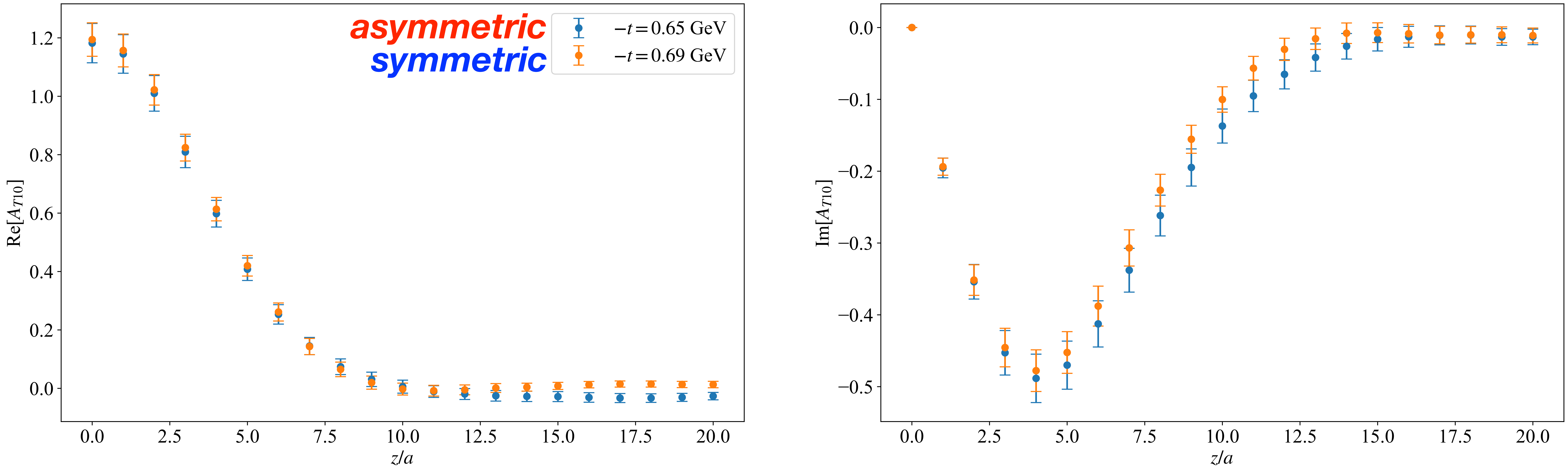
$$A^*_{Ti}(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_{Ti}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 1,\ 2,\ 4,\ 7,\ 8,\ 10,\ 11$$

$$-A^*_{Ti}(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_{Ti}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 3,\ 5,\ 6,\ 9,\ 12$$



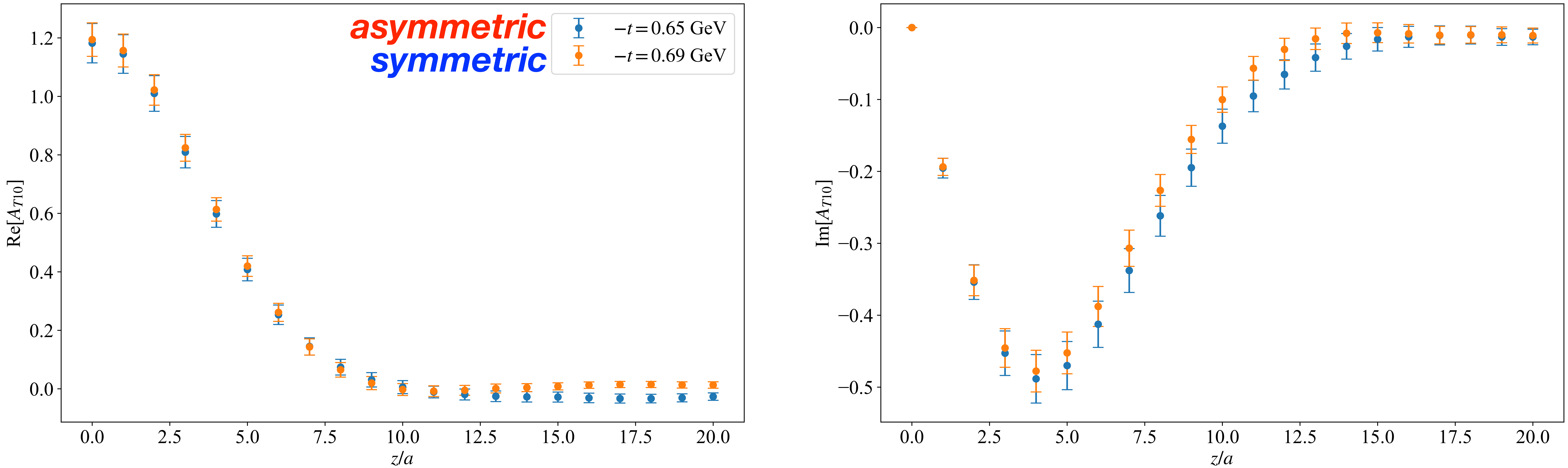
Averaging across 8 kinematic cases reduces the error by $1/\sqrt{8}$

Amplitude Comparison for different frames



- $-t = 0.65$ GeV corresponds to $|P_3| = 3$ and $\overrightarrow{\Delta} = (2{,}0{,}0)$+permutations in ***asymmetric*** frame
- $-t = 0.69$ GeV corresponds to $|P_3| = 3$ and $\overrightarrow{\Delta} = (2{,}0{,}0)$+permutations in ***symmetric*** frame
- Negligible difference between frames despite the 5% difference between $-t^s$ and $-t^a$.
  (Only $z > 15a$ for real part)

Amplitude Comparison for different frames



- $-t = 0.65$ GeV corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2,0,0)$+permutations in ***asymmetric*** frame
- $-t = 0.69$ GeV corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2,0,0)$+permutations in ***symmetric*** frame
- Negligible difference between frames despite the 5% difference between $-t^s$ and $-t^a$.
  (Only $z > 15a$ for real part)

Amplitudes are frame independent!

Quasi-GPDs

SF: Symmetric Frame

AF: Asymmetric Frame

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

Quasi-GPDs

SF: Symmetric Frame

AF: Asymmetric Frame

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

↕

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

Quasi-GPDs

SF: Symmetric Frame

AF: Asymmetric Frame

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

↕

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_{T2}\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_{T4} + A_{T10}$$

$$\longrightarrow \quad E_T = 2A_{T2} - A_{T4}$$

$$\longrightarrow \quad \widetilde{H}_T = -A_{T2} - \frac{m^2}{P_3}zA_{T12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_{T6} - \frac{2E^2}{P}zA_{T8}$$

GPDs defined in SF with A_i in AF

$$\longrightarrow H_T = -2A_{T2}\left(1 - \frac{(E_i+E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_{T4} - \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_{T8} + A_{T10}$$

$$\longrightarrow E_T = 2A_{T2} - A_{T4} + \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_{T8}$$

$$\longrightarrow \widetilde{H}_T = -A_{T2} - \frac{m^2}{P_3}zA_{T12}$$

$$\longrightarrow \widetilde{E}_T = -2A_{T6} - \frac{(E_i+E_f)^2}{2P}zA_{T8}$$

Quasi-GPDs

SF: Symmetric Frame

AF: Asymmetric Frame

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

$$\Updownarrow$$

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_{T2}\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_{T4} + A_{T10}$$

$$\longrightarrow \quad E_T = 2A_{T2} - A_{T4}$$

$$\longrightarrow \quad \widetilde{H}_T = -A_{T2} - \frac{m^2}{P_3}zA_{T12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_{T6} - \frac{2E^2}{P}zA_{T8}$$

GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_{T2}\left(1 - \frac{(E_i+E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_{T4} - \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_{T8} + A_{T10}$$

$$\longrightarrow \quad E_T = 2A_{T2} - A_{T4} + \frac{(E_f+E_i)(E_f-E_i)}{2P_3}zA_{T8}$$

$$\longrightarrow \quad \widetilde{H}_T = -A_{T2} - \frac{m^2}{P_3}zA_{T12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_{T6} - \frac{(E_i+E_f)^2}{2P}zA_{T8}$$

Amplitude decomposition matches results from previous results **[ETMC, PRD 105, 034501 (2022)]**

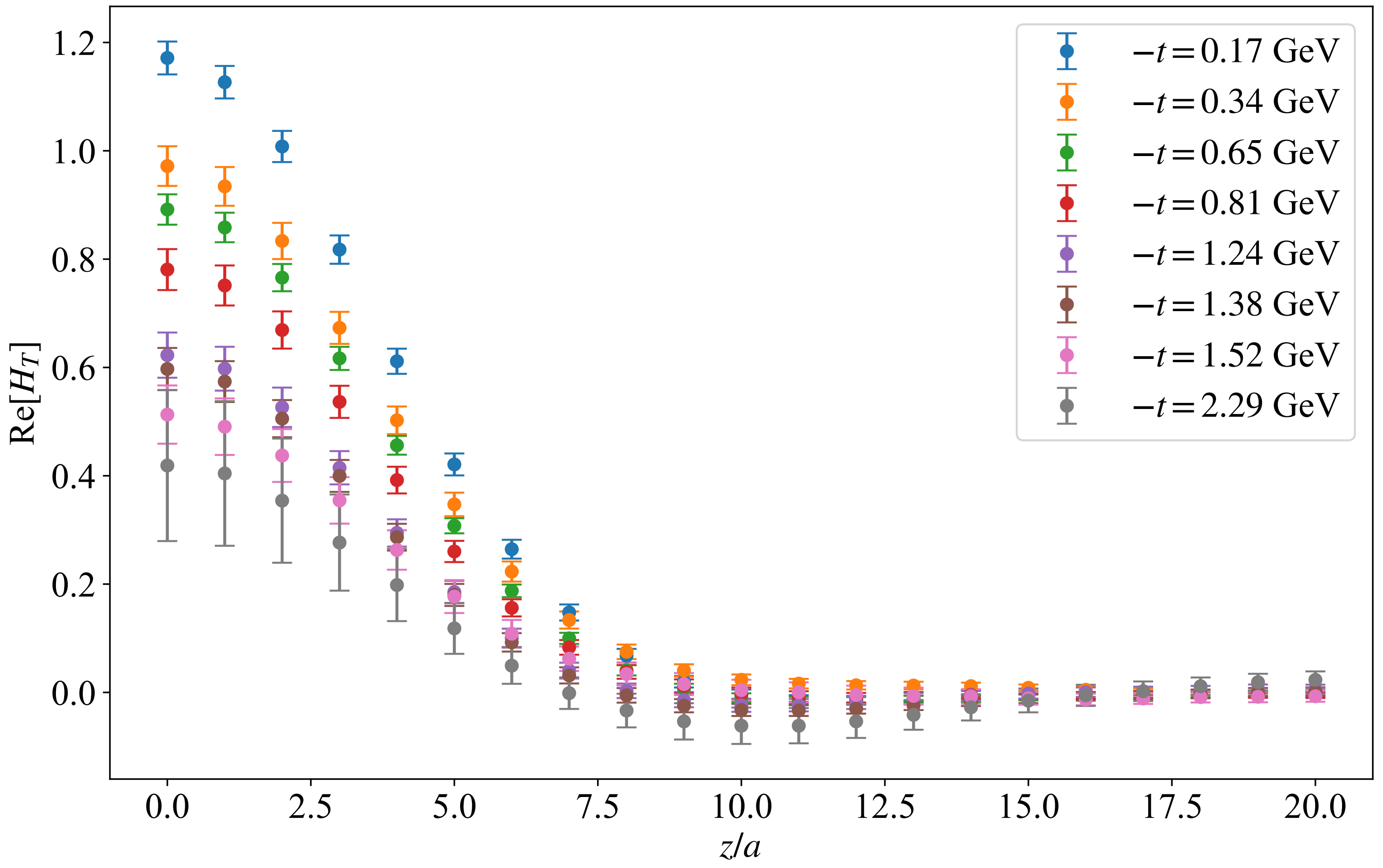
Quasi-GPDs

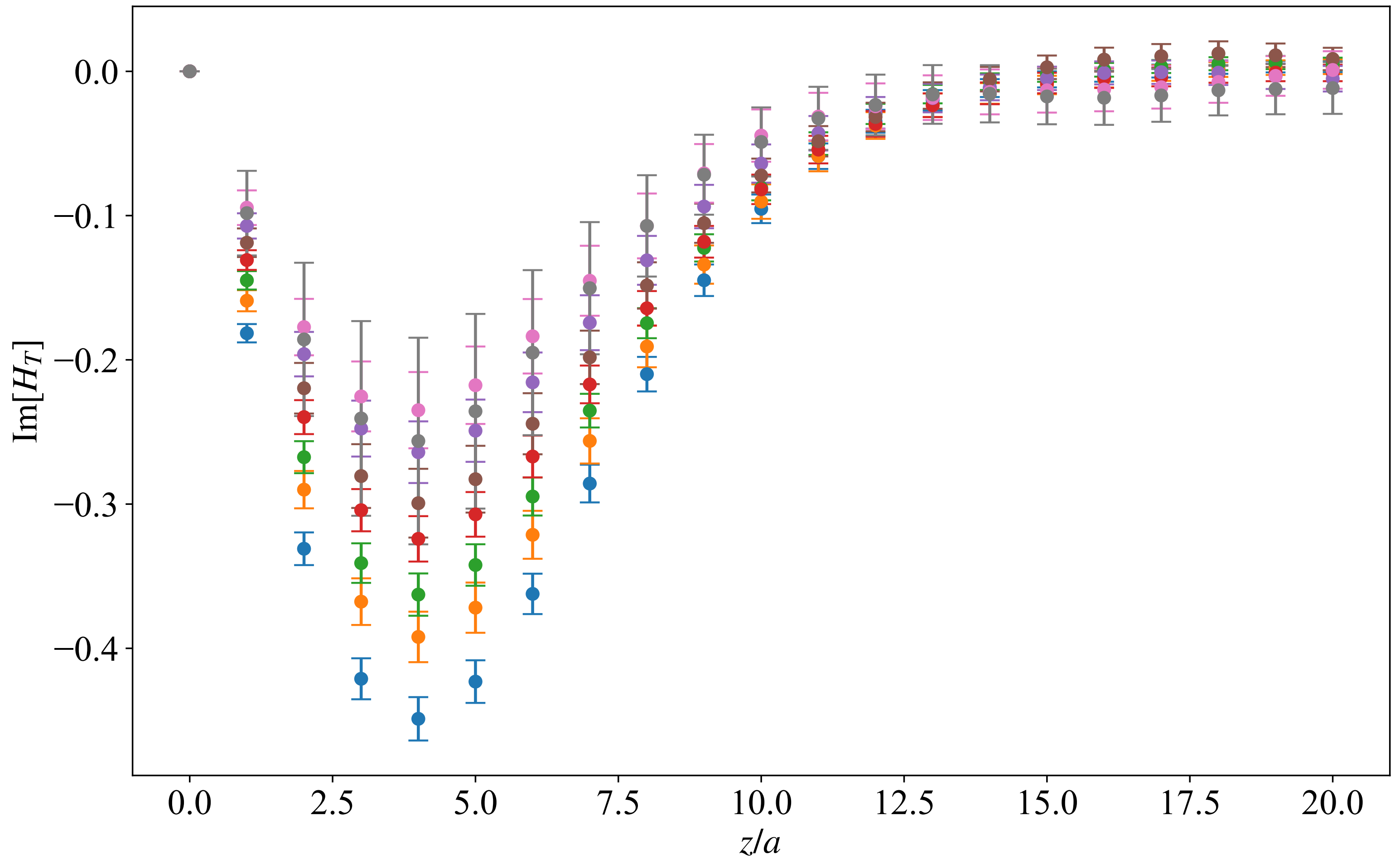
❖ Results in agreement between asymmetric and symmetric frames!

❖ Asymmetric frame is **computationally much more efficient** to produce data for a dense range of $-t$

$$H_T = -2A_{T2}\left(1 - \frac{(E_i+E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_{T4} - \frac{(E_f+E_i)(E_f-E_i)}{2P_3} z A_{T8} + A_{T10}$$

❖ Large contribution from A_{T10}



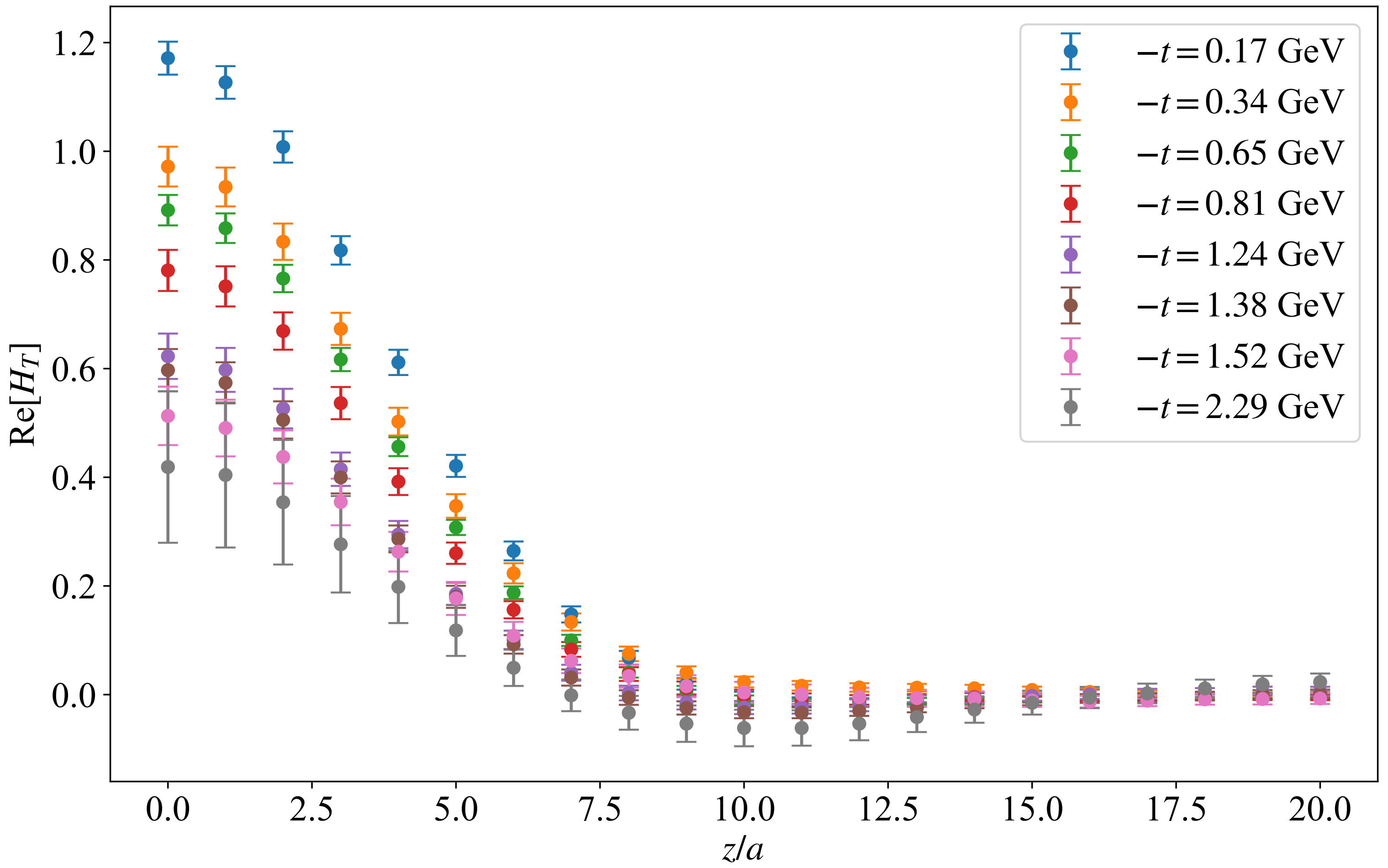


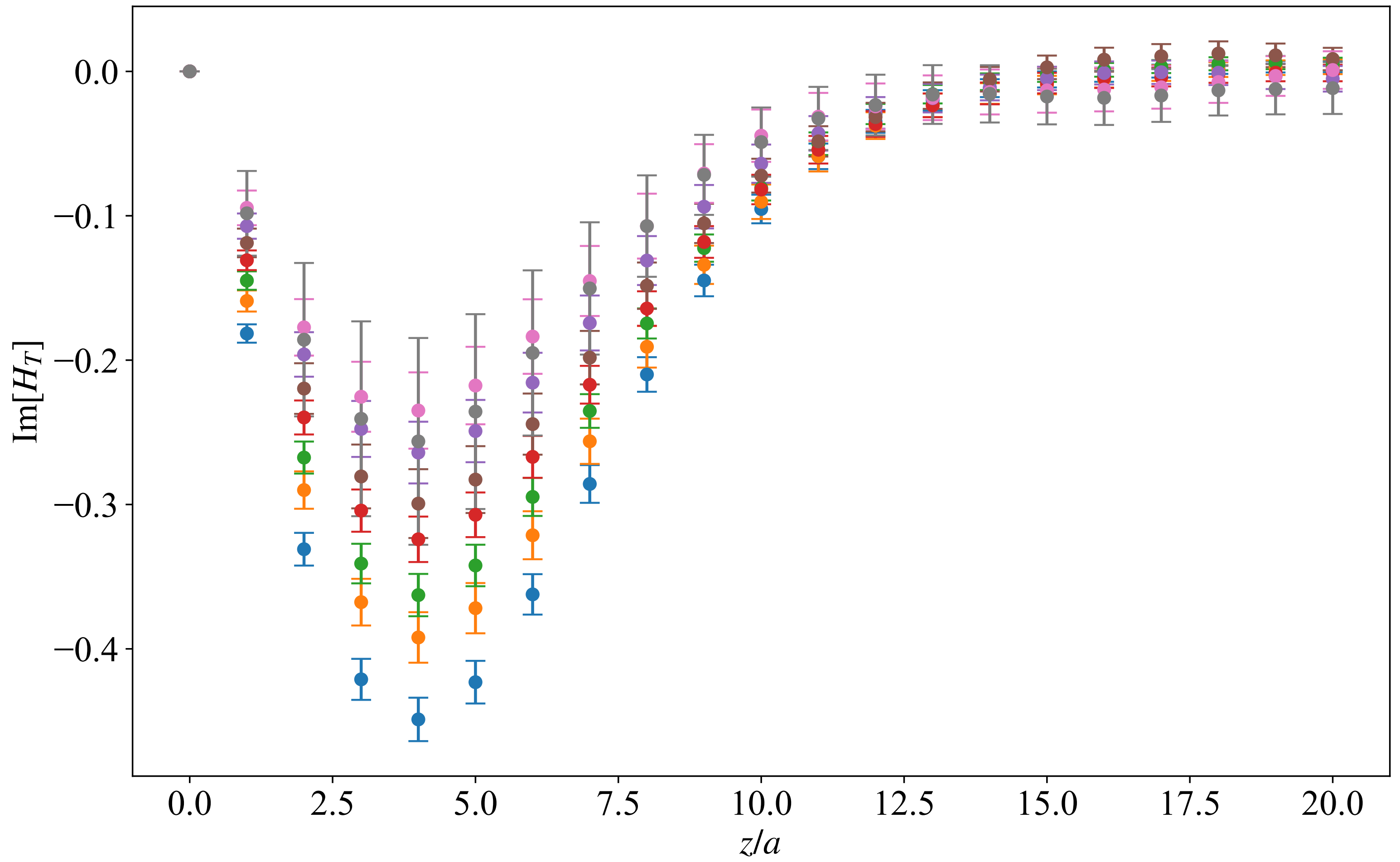
Quasi-GPDs

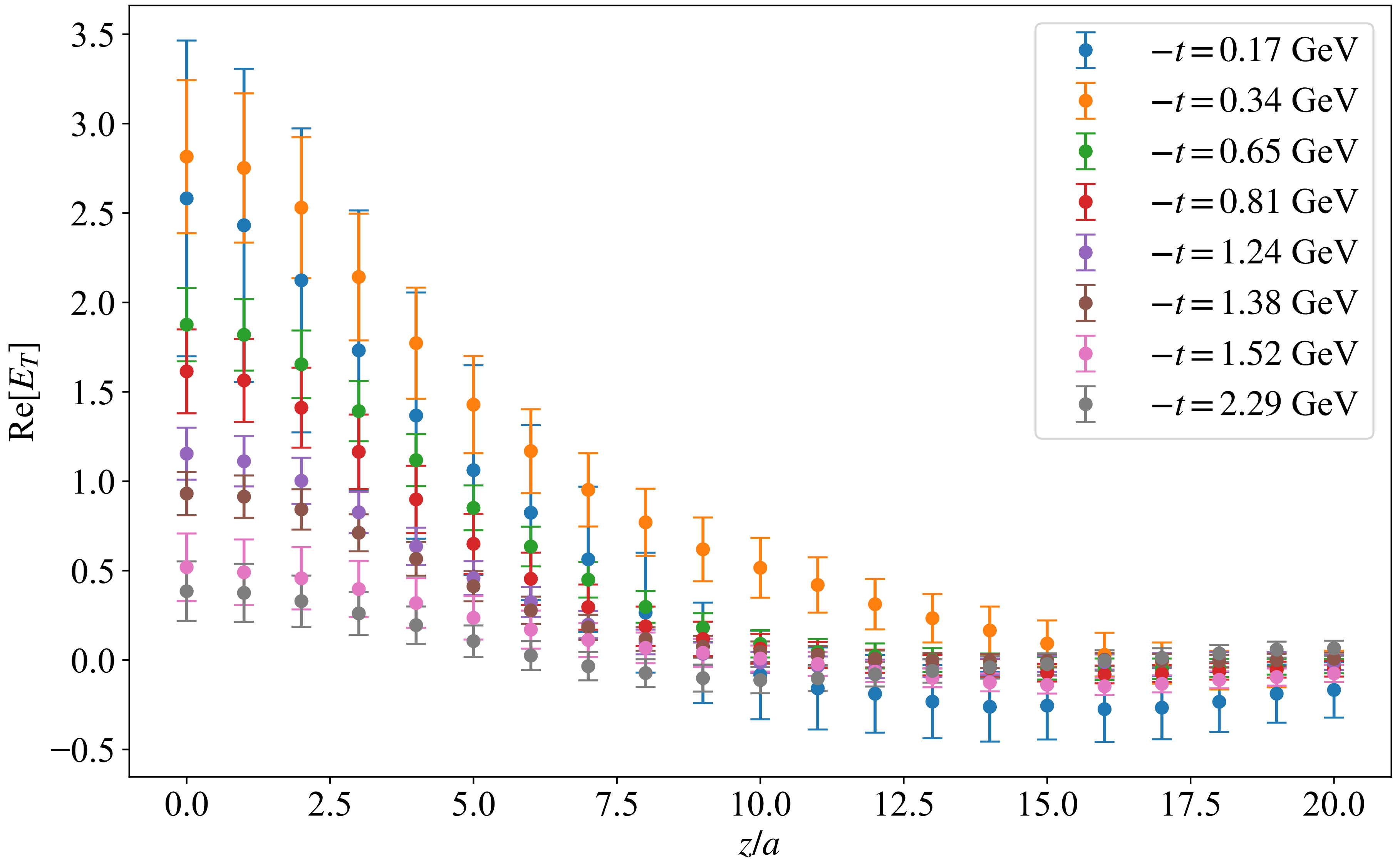
- ❖ Results in agreement between asymmetric and symmetric frames!
- ❖ Asymmetric frame is **computationally much more efficient** to produce data for a dense range of $-t$

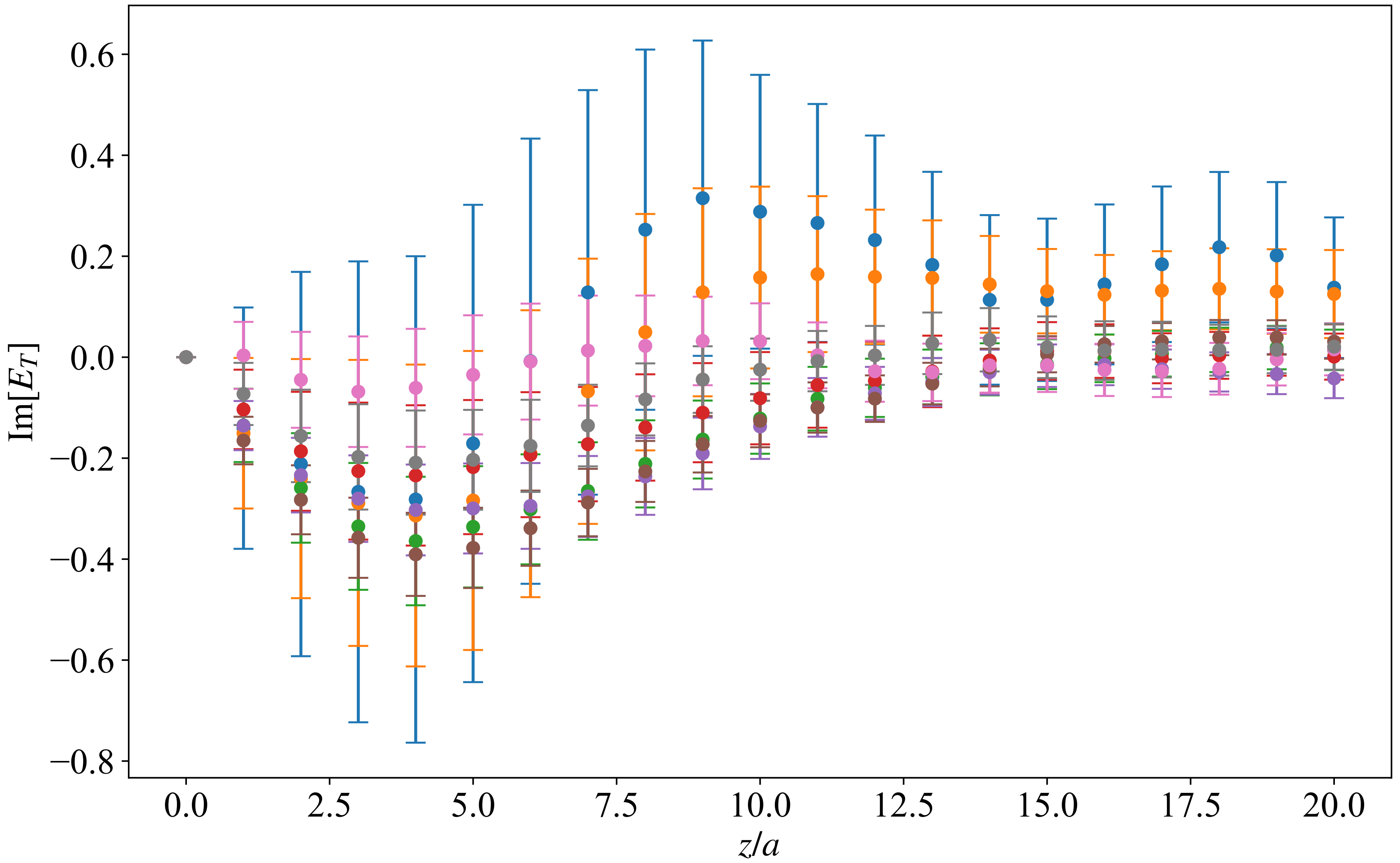
$$H_T = -2A_{T2}\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_{T4}$$
$$- \frac{(E_f + E_i)(E_f - E_i)}{2P_3} z A_{T8} + A_{T10}$$

- ❖ Large contribution from A_{T10}









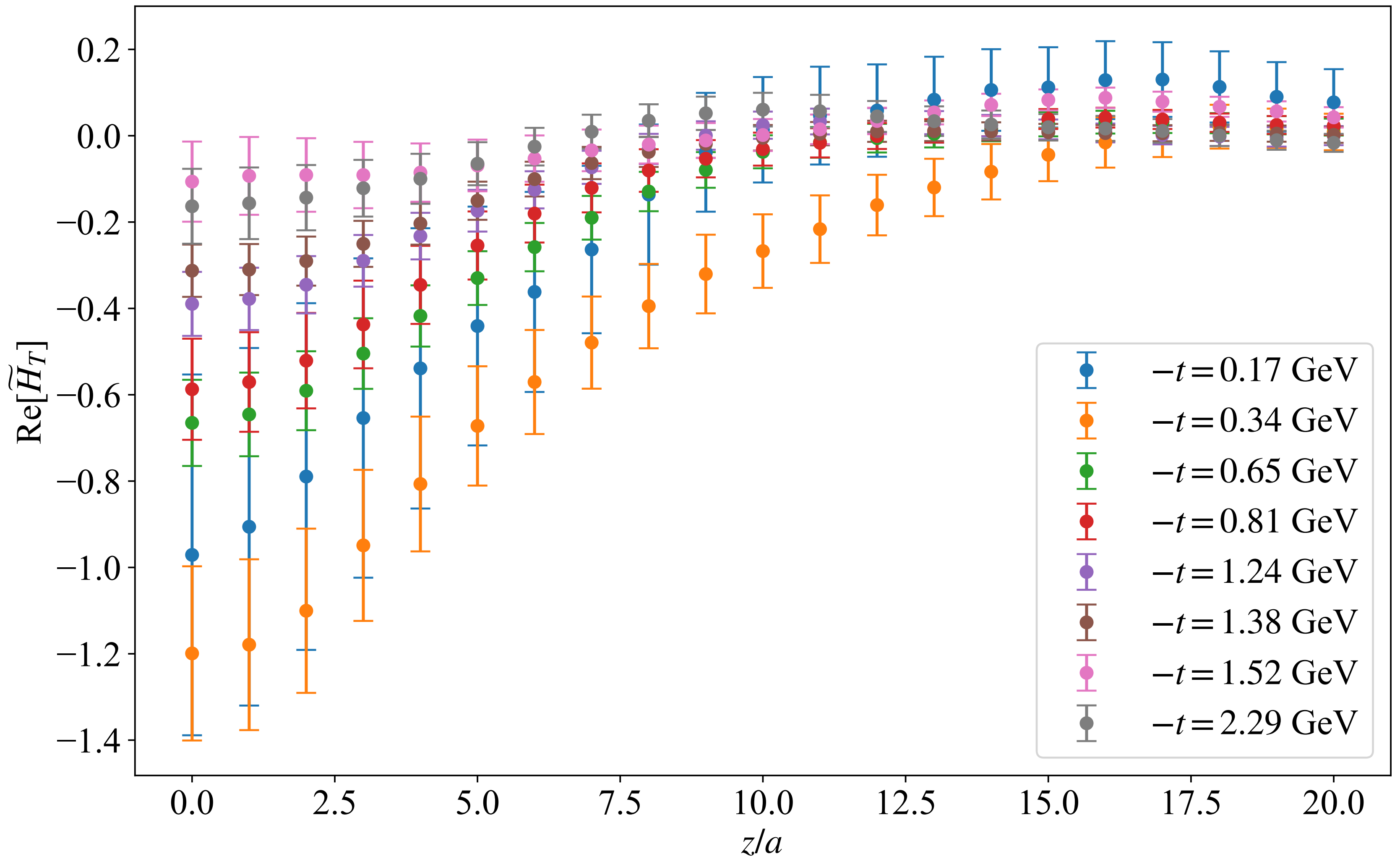
$$E_T = 2A_{T2} - A_{T4} + \frac{(E_f + E_i)(E_f - E_i)}{2P_3} z A_{T8}$$

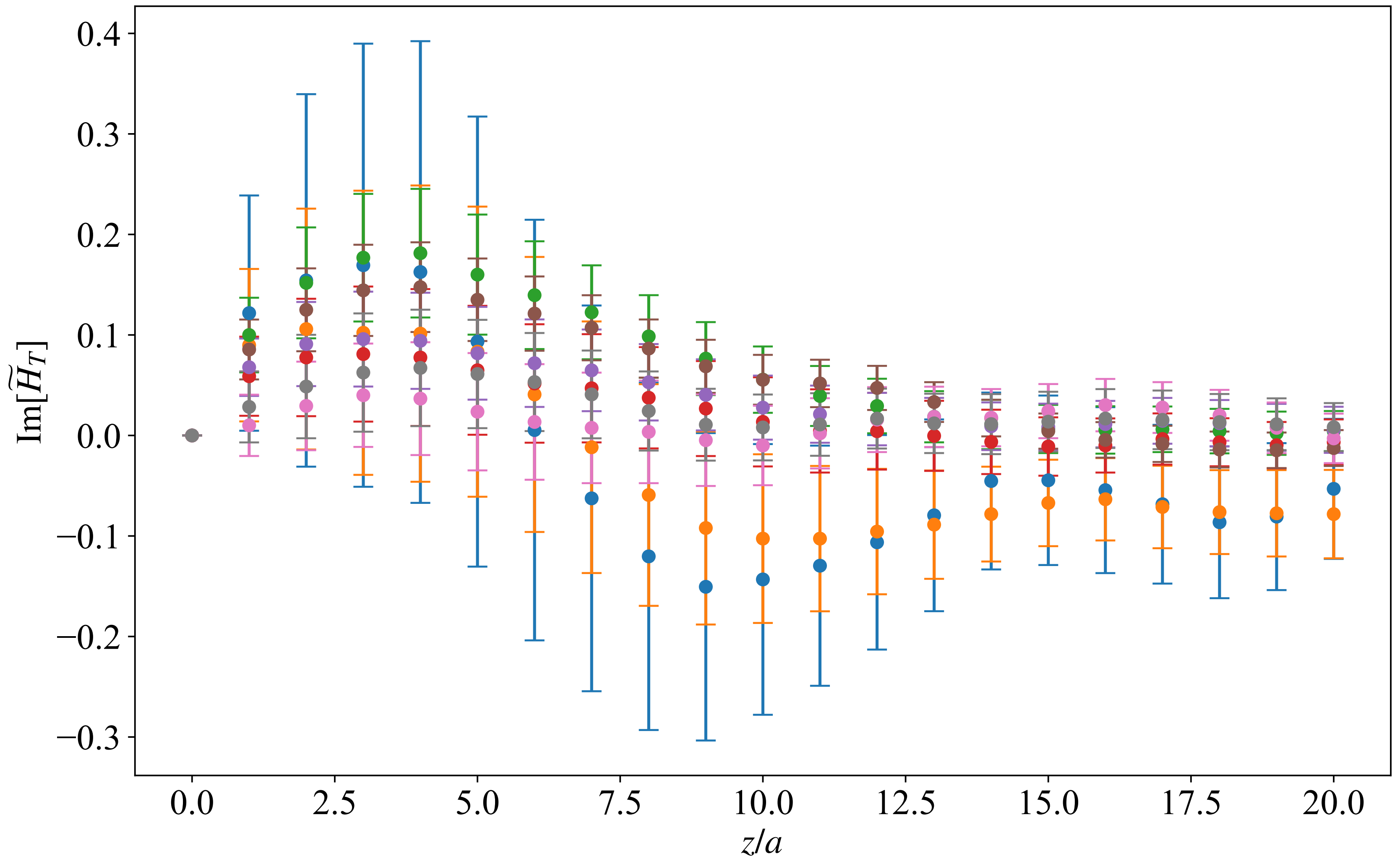
- ❖ Magnitude decreases as $-t$ increases

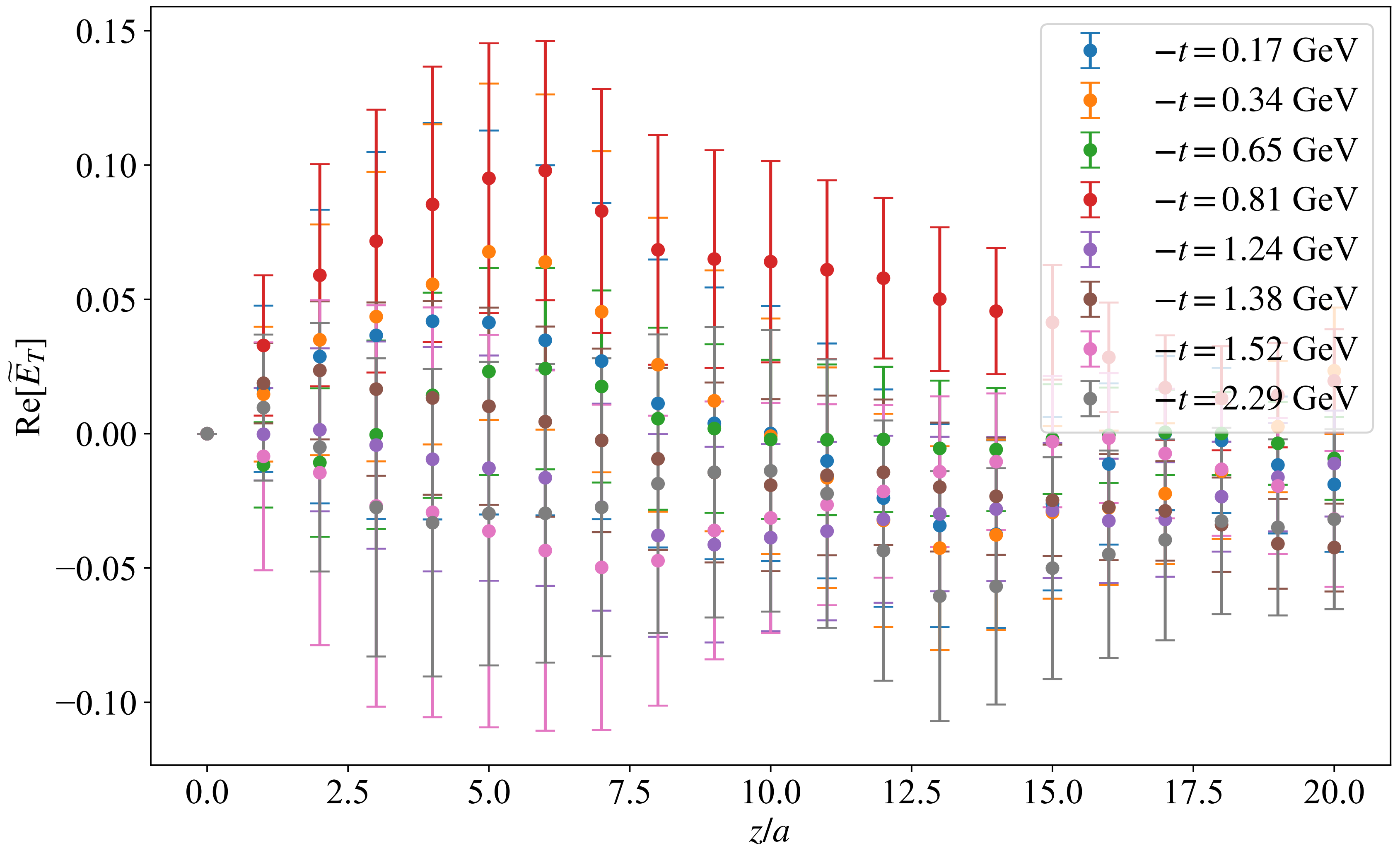
Quasi-GPDs

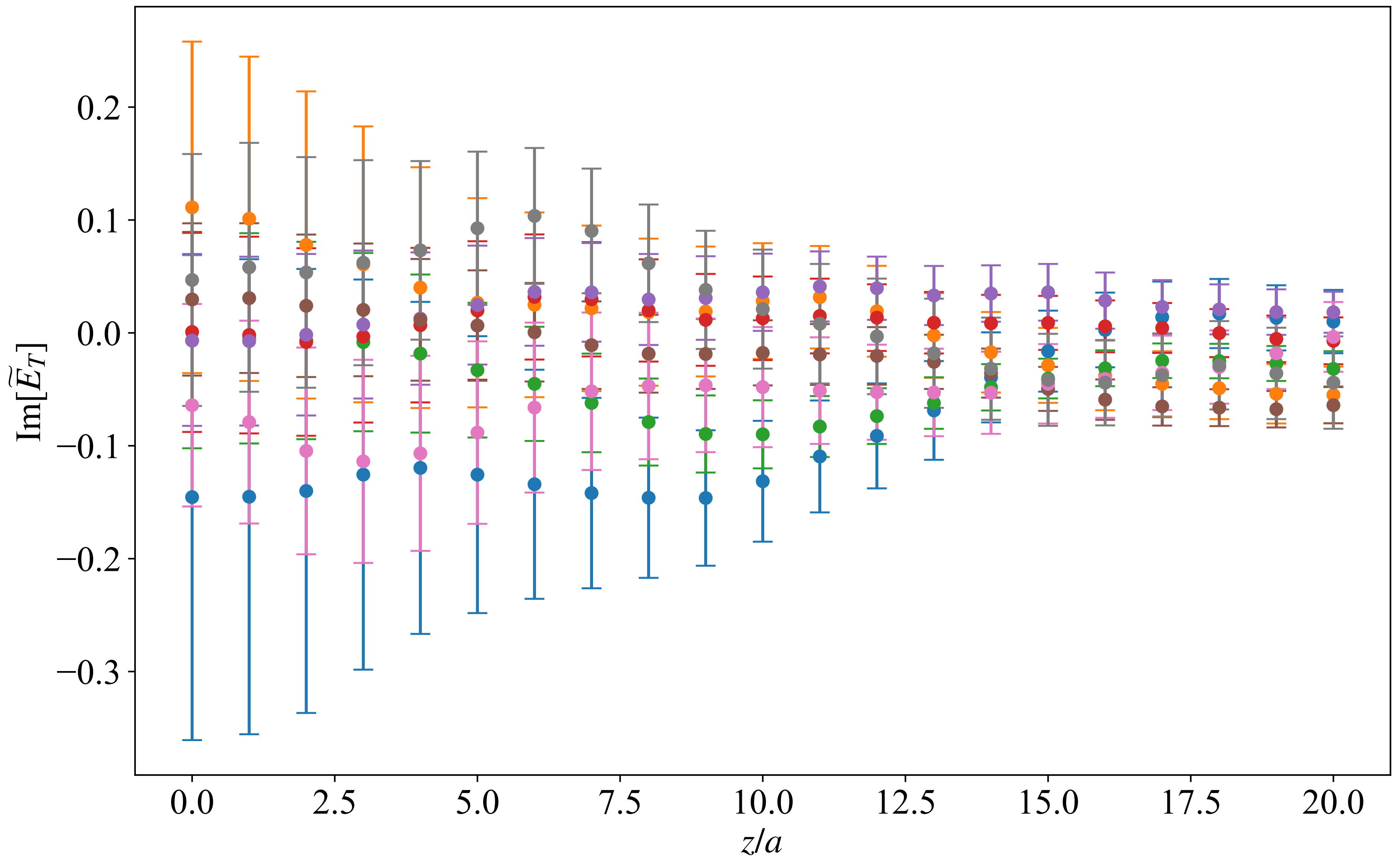
$$\widetilde{H}_T = -A_{T2} - \frac{m^2}{P_3} z A_{T12}$$

❖ Magnitude decreases as $-t$ increases









$$\widetilde{E}_T = -2A_{T6} - \frac{(E_i + E_f)^2}{2P} z A_{T8}$$

❖ Odd in $\xi \to -\xi$

❖ Zero in $\xi = 0$

[Meissner, et al., JHEP 08, 056 (2009)]

Position space to Momentum space

- Multiple methods to consider
  - Standard Fourier Transform

Position space to Momentum space

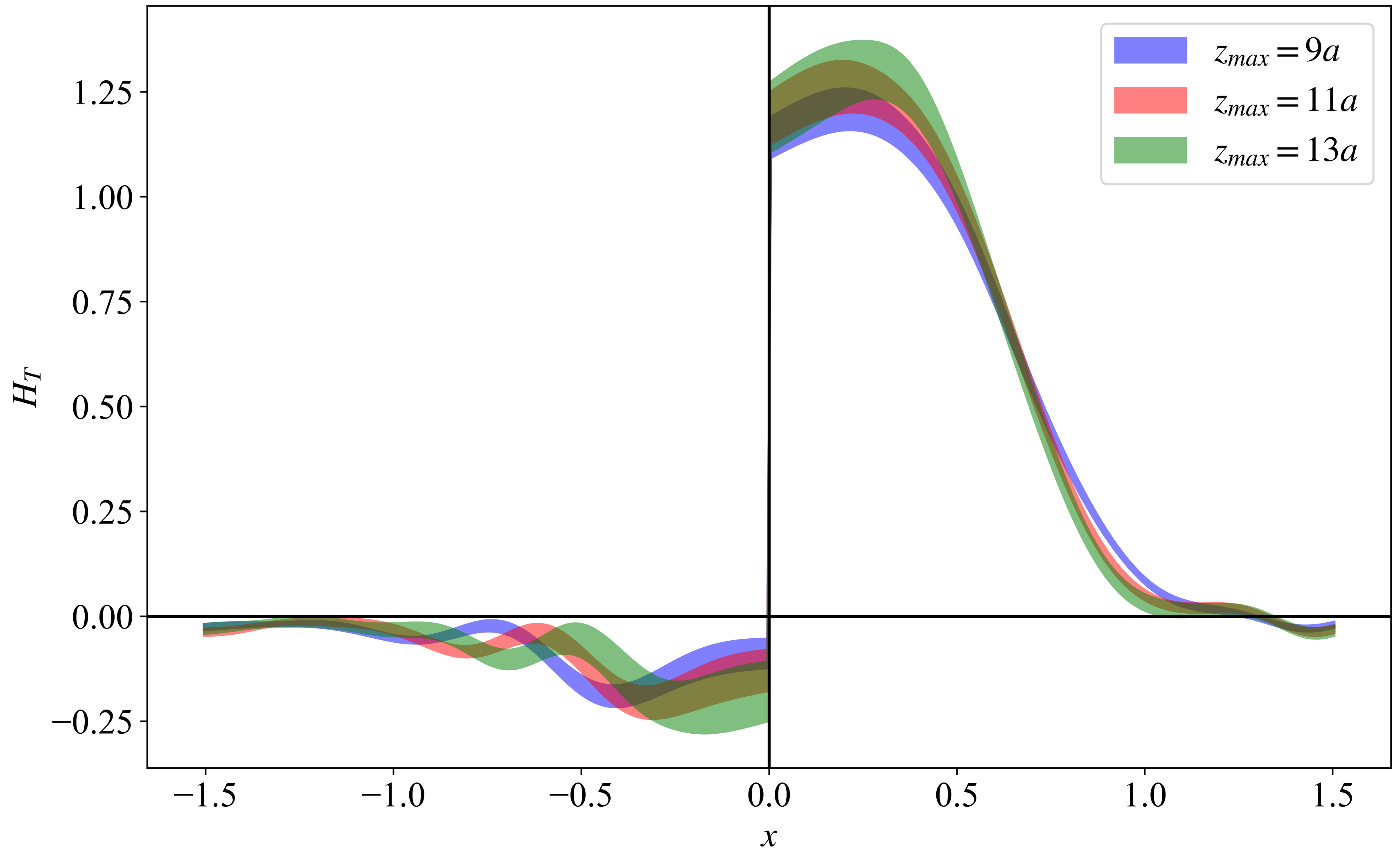
- Multiple methods to consider
  - Standard Fourier Transform ⟶ ***Abandoned***

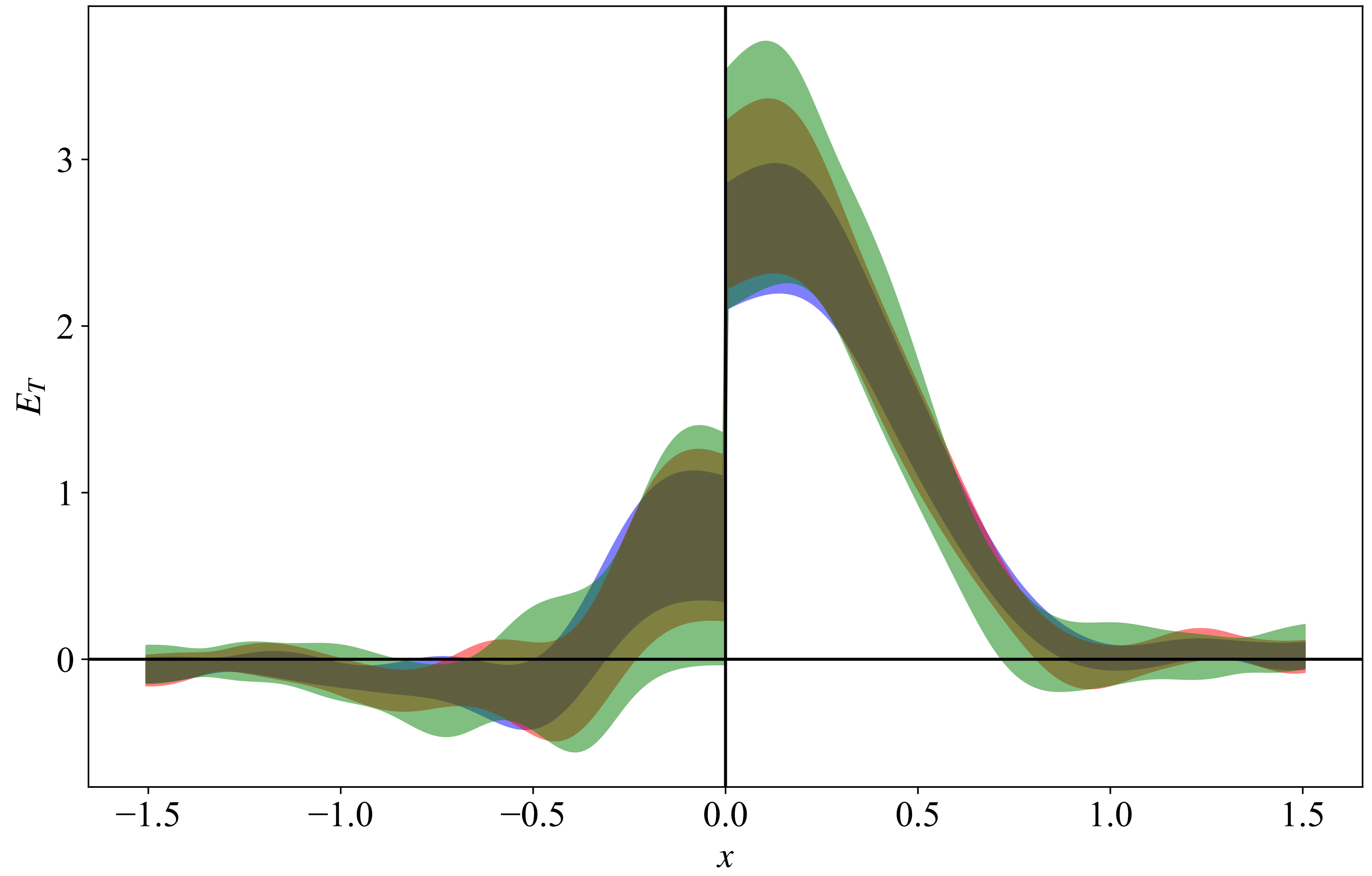
Position space to Momentum space

- Multiple methods to consider
  - Standard Fourier Transform → ***Abandoned***
  - Backus-Gilbert method → ***Why?*** **[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]**

Position space to Momentum space

- ❖ Multiple methods to consider
  - ❖ Standard Fourier Transform ⟶ ***Abandoned***
  - ❖ Backus-Gilbert method ⟶ ***Why?*** **[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]**
- ❖ Backus-Gilbert:
  - ❖ Model independent
  - ❖ Criterion: variance of solution with respect to statistical variation of input data is minimal
- ❖ Test the dependence on z_{max} in reconstruction





All calculations use $z_{max} = 11a$ to match to light-cone

Matching to the Light-Cone

[Liu, et al., Phys. Rev. D 100, 034006 (2019)]

$$GPD(x,\xi,t) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C_\Gamma \left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z} \right) qGPD(y,\xi,t)$$

$$C_{i\sigma^{z\perp}}\left(x, \xi=0, \frac{p^a}{\mu}\right) = \frac{\alpha_s C_F}{2\pi} f_1\left(x, \xi=0, \frac{p^z}{\mu}\right)_+ + 2\delta(1-x)\frac{\alpha_s C_F}{4\pi}\left[\frac{-1}{\epsilon_{UV}} + \ln\left(\frac{\mu^2}{\mu'^2}\right)\right] + \left[\left|\frac{p^z}{p_R^z}\right| \frac{\alpha_s C_F}{2\pi} f_2\left(\frac{p^z}{p_R^z}(x-1)+1, r\right)\right]_+$$

Matching to the Light-Cone

[Liu, et al., Phys. Rev. D 100, 034006 (2019)]

$$GPD(x,\xi,t) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C_\Gamma\left(\frac{x}{y},\frac{\xi}{y},r,\frac{yP^z}{\mu},\frac{yP^z}{p_R^z}\right) qGPD(y,\xi,t)$$

$$C_{i\sigma^{z\perp}}\left(x,\xi=0,\frac{p^a}{\mu}\right) = \frac{\alpha_s C_F}{2\pi} f_1\left(x,\xi=0,\frac{p^z}{\mu}\right)_+ + 2\delta(1-x)\frac{\alpha_s C_F}{4\pi}\left[\frac{-1}{\epsilon_{UV}} + \ln\left(\frac{\mu^2}{\mu'^2}\right)\right] + \left[\left|\frac{p^z}{p_R^z}\right| \frac{\alpha_s C_F}{2\pi} f_2\left(\frac{p^z}{p_R^z}(x-1)+1,r\right)\right]_+$$

$x<0:$

$$f_1(x,\xi=0) = \frac{-2x}{x-1}\ln\left(\frac{x-1}{x}\right)$$

$$f_2(x,\xi=0) = \frac{-3}{2(1-x)} - \frac{r-2x}{(r-1)(r-4x+4x^2)} - \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)}\tan^{-1}\left(\frac{\sqrt{r-1}}{2x-1}\right)$$

$0<x<1:$

$$f_1(x,\xi=0) = \frac{2x}{1-x}\left[\ln\left(\frac{4x(1-x)(p^z)^2}{\mu^2}\right) - 1\right]$$

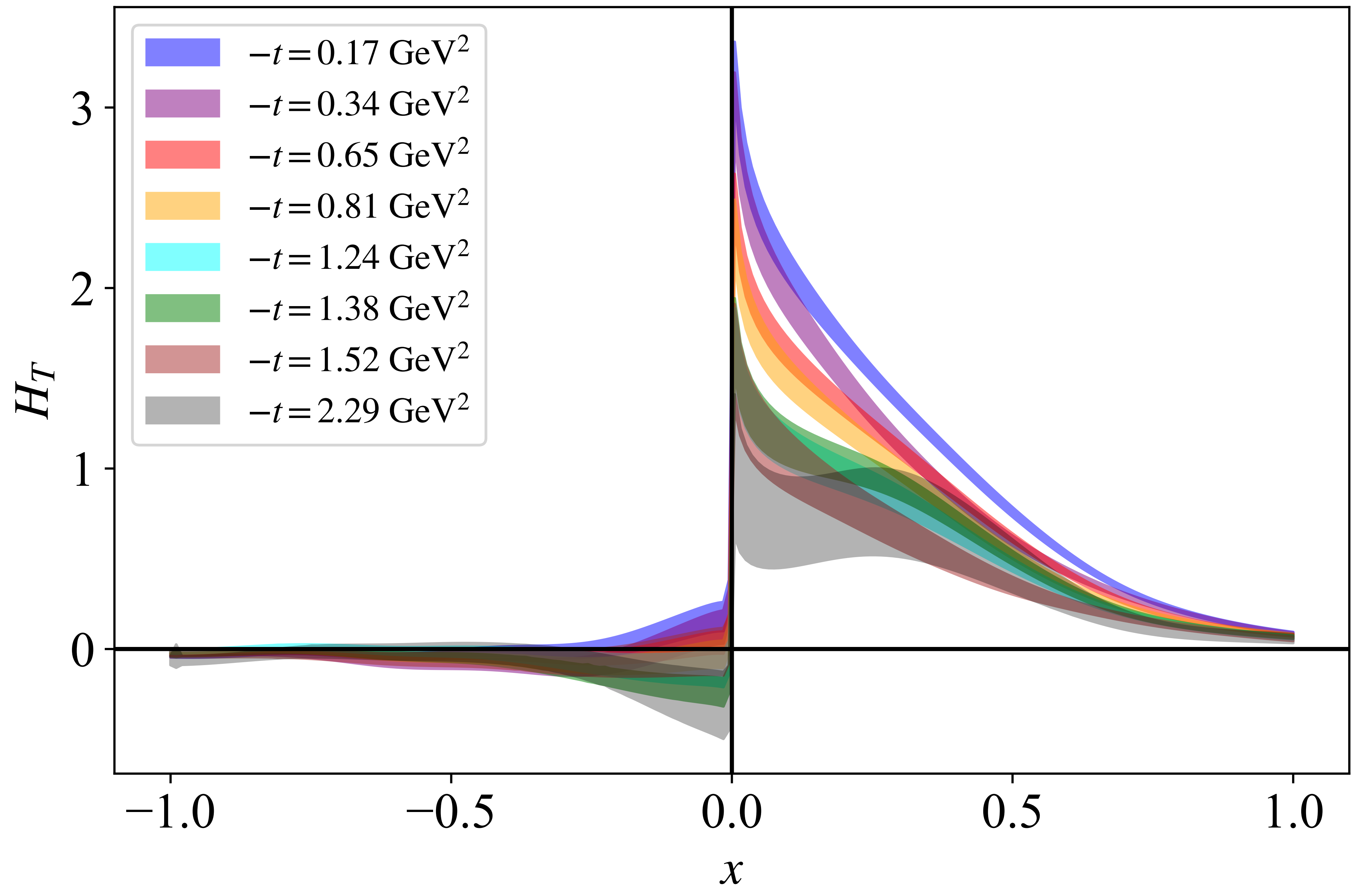
$$f_2(x,\xi=0) = \frac{1-3r+2x}{2(r-1)(1-x)} + \frac{r-2x+rx}{(r-1)^{3/2}(1-x)}\tan^{-1}\left(\sqrt{r-1}\right)$$

$x>1:$

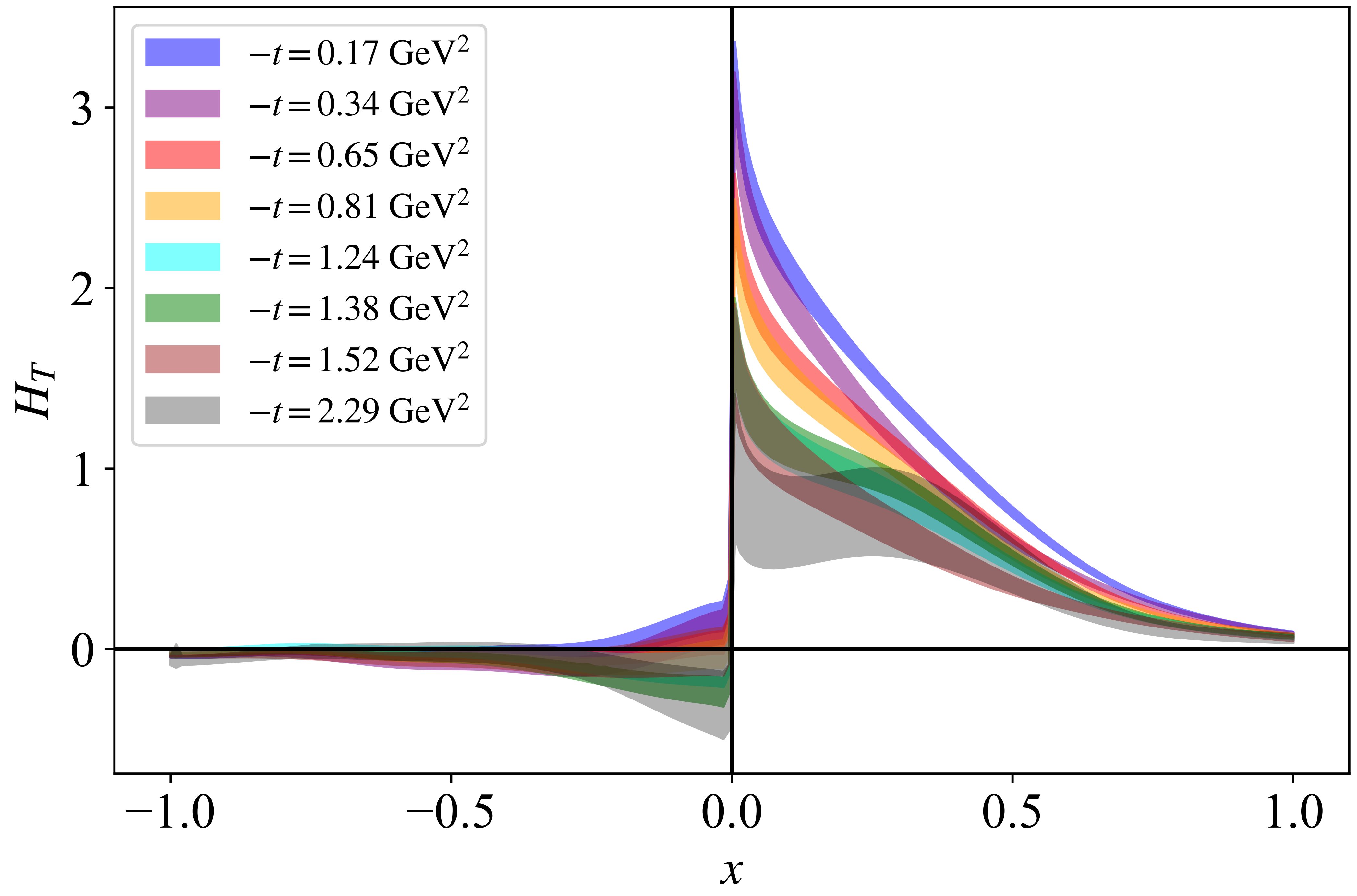
$$f_1(x,\xi=0) = \frac{2x}{x-1}\ln\left(\frac{x-1}{x}\right)$$

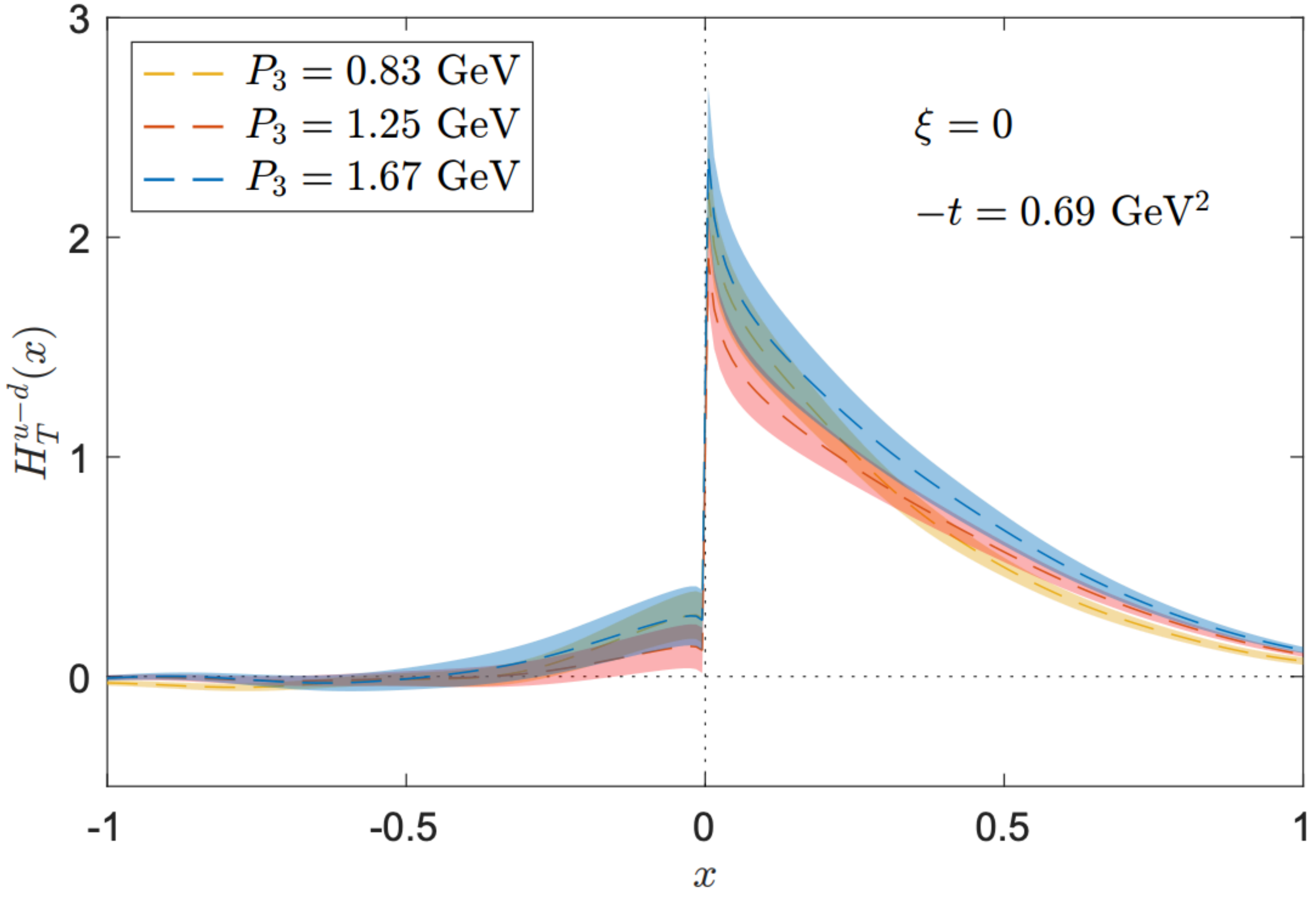
$$f_2(x,\xi=0) = \frac{3}{2(1-x)} + \frac{r-2x}{(r-1)(r-4x+4x^2)} + \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)}\tan^{-1}\left(\frac{\sqrt{r-1}}{2x-1}\right)$$

Light-Cone Results



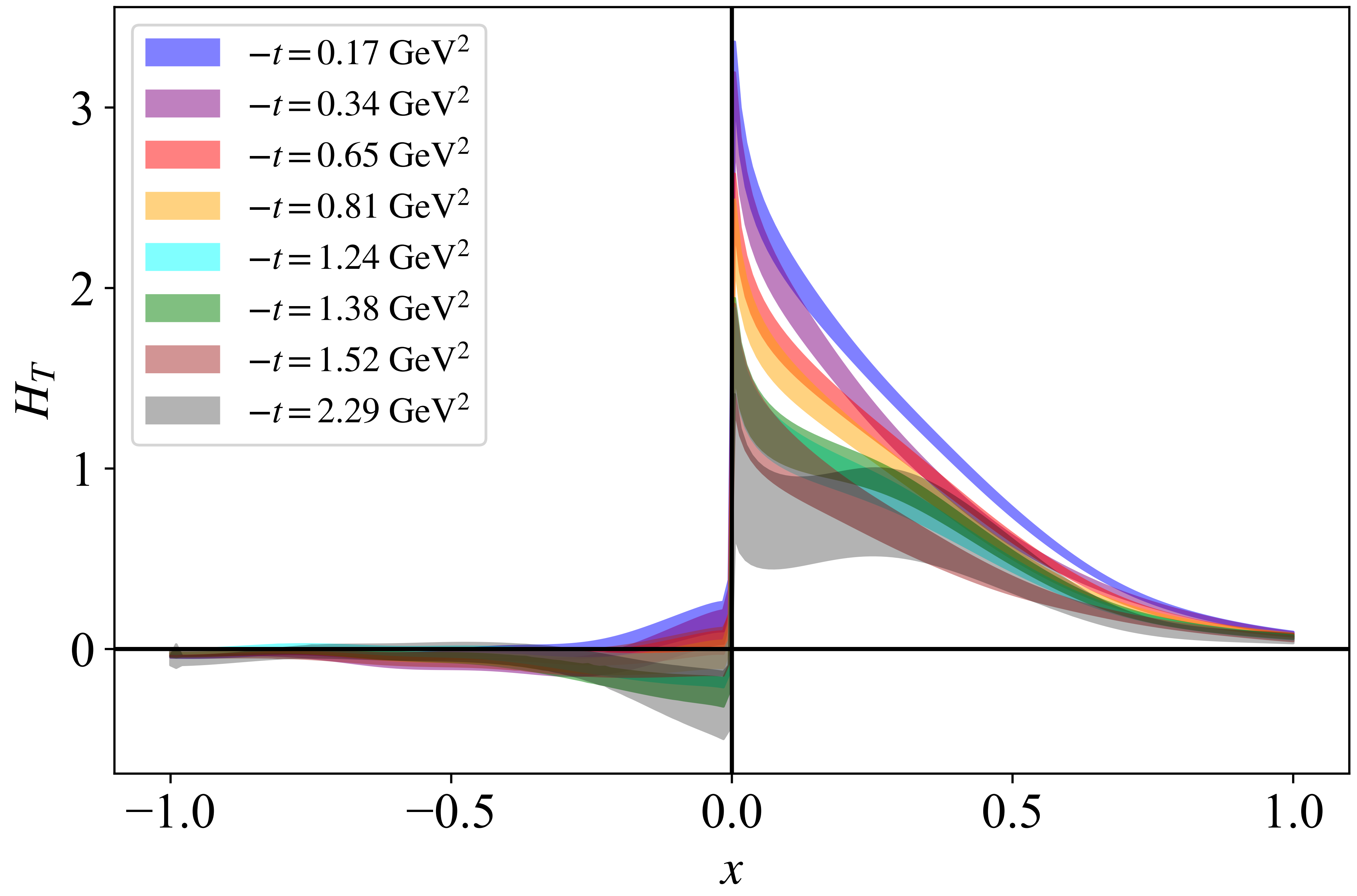
Light-Cone Results

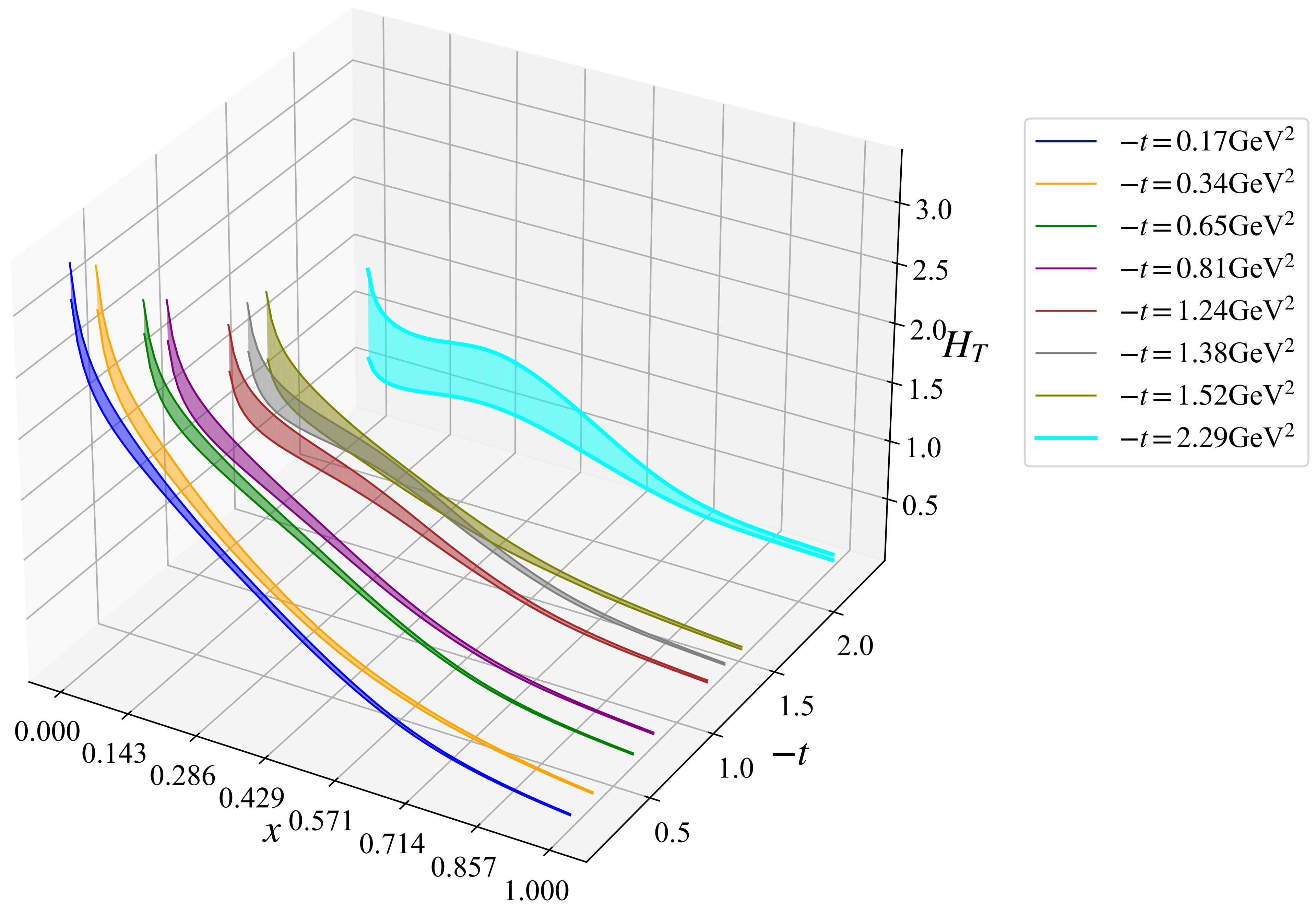




- ❖ Looks very similar to past results when extracting from symmetric frame!
- ❖ Very good signal!
- ❖ Asymmetric frame passes sanity checks!

Light-Cone Results

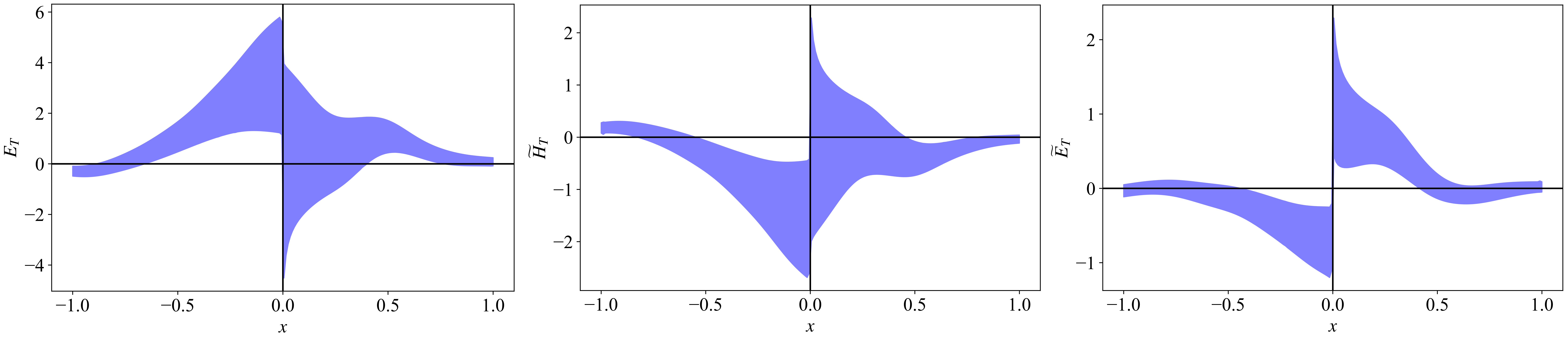




- ❖ Looks very similar to past results when extracting from symmetric frame!
- ❖ Very good signal!
- ❖ Asymmetric frame passes sanity checks!

Light-Cone Results

$|P_3| = 1.25\,\text{GeV} \quad -t = 0.65\ \text{GeV}$



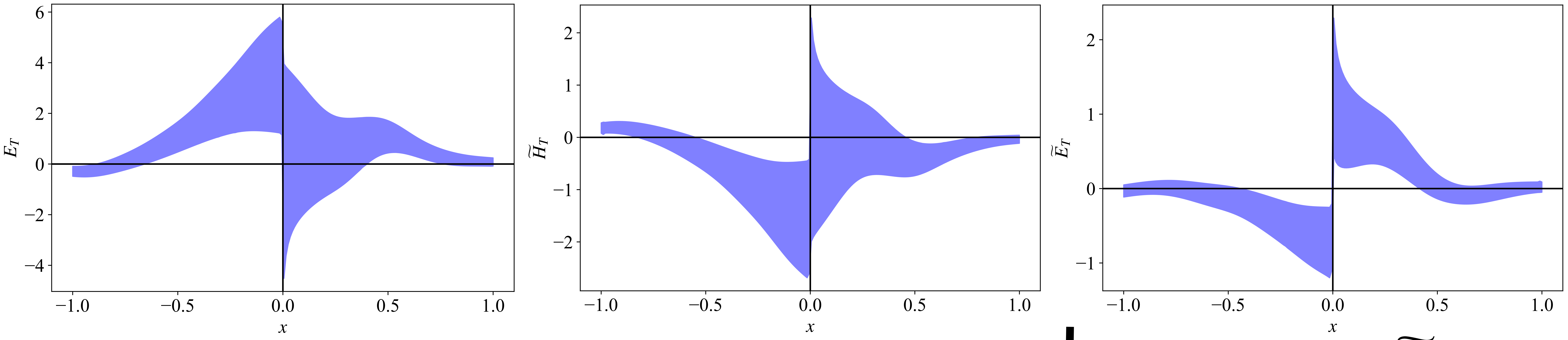
❖ Can combine $2\widetilde{H}_T + E_T$

❖ Related to transverse spin structure

❖ Interpretation is lateral deformation of transversely polarized quark in an unpolarized proton

Light-Cone Results

$|P_3| = 1.25\,\mathrm{GeV} \quad -t = 0.65\ \mathrm{GeV}$



- Can combine $2\widetilde{H}_T + E_T$
  - Related to transverse spin structure
  - Interpretation is lateral deformation of transversely polarized quark in an unpolarized proton

- Mellin Moments of $\widetilde{E}_T$
  - $\int_{-1}^{1} dx\ \widetilde{E}_T(x,\xi) = 0$
  - $\int_{-1}^{1} dx\ x\widetilde{E}_T(x,\xi) = 2\xi\widetilde{B}_{T21}(t)$

Summary and Future Work

❖ Implementation of asymmetric frame is a computationally less expensive method

❖ A dense range of $-t$ values obtained

Summary and Future Work

- ❖ Implementation of asymmetric frame is a computationally less expensive method
- ❖ A dense range of $-t$ values obtained
- ❖ Parametrization of $-t$ dependence
- ❖ Fourier Transform to impact parameter space
- ❖ Introduce non-zero skewness ($\Delta_3 \neq 0$)
- ❖ Include more ensembles for the investigation of systematic uncertainties (e.g., discretization effects, momentum boost effects)

Summary and Future Work

❖ Implementation of asymmetric frame is a computationally less expensive method

❖ A dense range of $-t$ values obtained

❖ Parametrization of $-t$ dependence

❖ Fourier Transform to impact parameter space

❖ Introduce non-zero skewness ($\Delta_3 \neq 0$)

❖ Include more ensembles for the investigation of systematic uncertainties (e.g., discretization effects, momentum boost effects)

Thank You!!!

Acknowledgements

❖ U.S. Department of Energy, Office of Nuclear Physics, Early Career Award under Grant No. DE-SC0020405

❖ This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231 using NERSC award ALCC-ERCAP0025953.

❖ PLGrid Infrastructure by Prometheus in Cracow

❖ Poznan Supercomputing and Networking Center by Eagle

❖ Interdisciplinary Centre for Mathematical and Computational Modeling of the Warsaw University by Okeanos

❖ Academic Computer Center in Gdańsk by Tryton