LaMET and Weinberg's EFT

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Outline

- Weinberg's EFT
- Partons as EFT
- Quasi-PDF as Euclidean EFT
- LaMET as Euclidean EFT for partons
- Conclusion

Weinberg's EFT

Weinberg's EFT

- Weinberg got a Nobel prize for establishing electroweak unification theory based on renormalizability constraints.

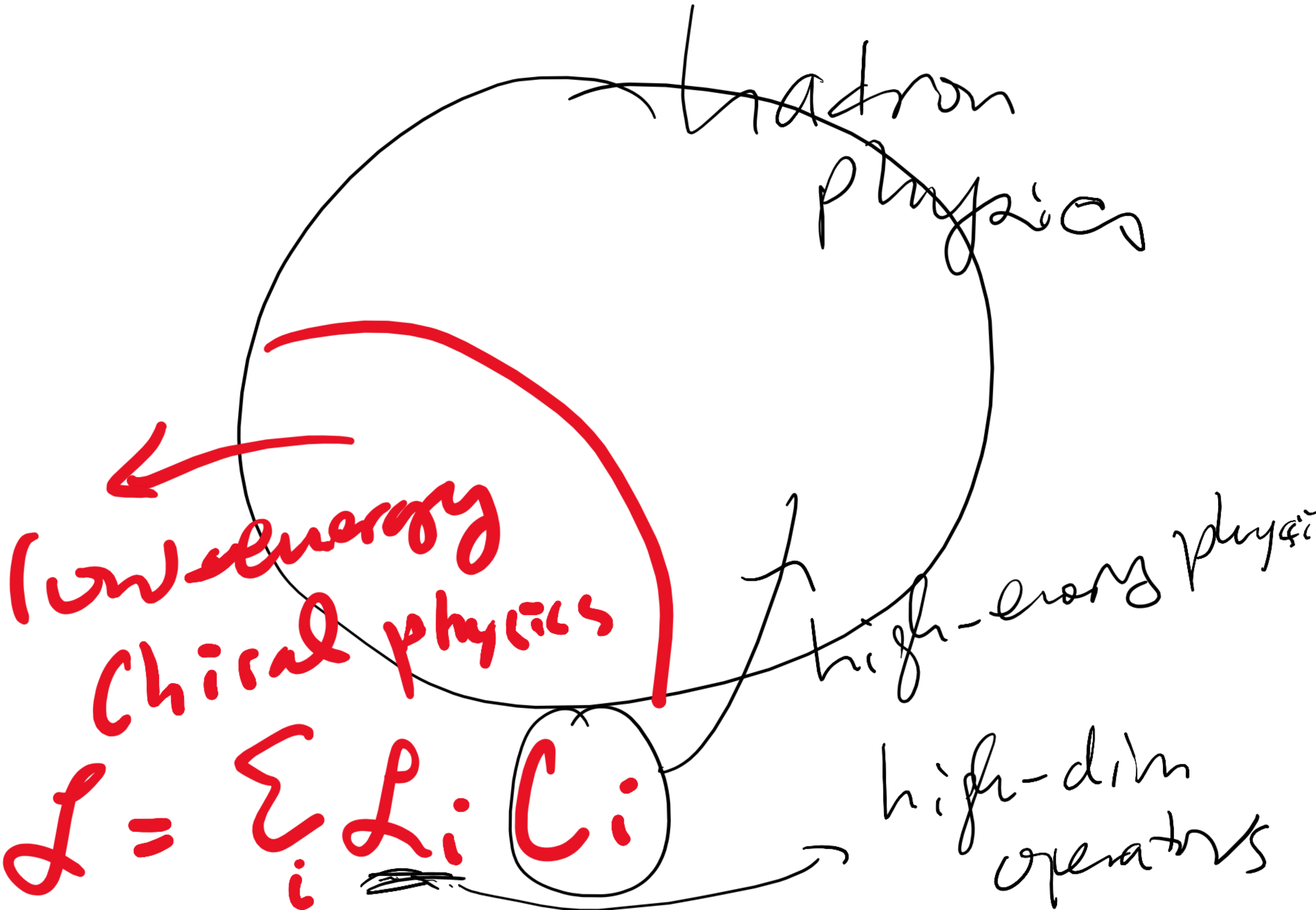
  --- t Hooft provide the actual proof

- However, after winning the prize, he turned around about renormalizability: Non-renormalizable field theory can also make sense!

  Key word: power counting

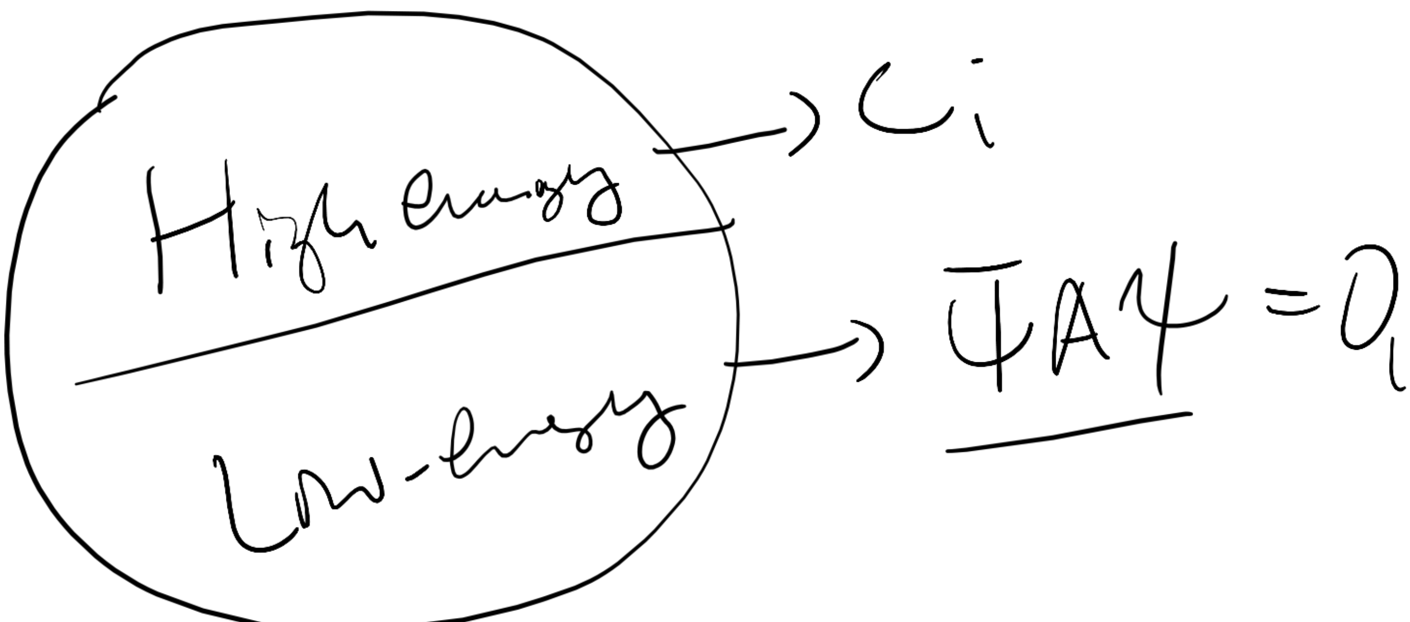
Chiral perturbation theory

- Given degrees of freedom (pion & nucleon) and symmetry, one can write down the most general lagrangian for low-energy chiral dynamics (EFT).
- The lagrangian is not renormalizale in the sense that it requires infinite number of counter terms.
- However, at a fixed order in chiral power counting, there are only a finite number of “low-energy constants” paramerizing high-energy physics.
- All physical observables can be calculated to a particular order using these LEC’s.


hadron physics
high-energy physics
low-energy chiral physics
$\mathcal{L} = \sum_i \mathcal{L}_i C_i$
high-dim operators

Wilson's OPE and QCD factorization

- For QCD, we have Wilson's OPE or factorization
- High-energy physics-> coefficient function
- Low-energy physics-> local operators



QCD Factorization as EFT: HEP view

- We can write fields in terms of low-energy and high-energy part

$$\psi = \psi_{high} + \psi_{low}$$

$$A = A_{high} + A_{low}$$

- For HEP interested in pQCD, we have an EFT with ψ_{high} plus "high-energy constants"

$$\langle \bar{\psi}_{low} .. A_{low} \ldots \psi_{low} \rangle$$

which parametrizes the low-energy physics.

QCD Factorization as EFT: hadron structure

- For QCD theorists interested in hadron structure, we integrate out the high-energy degrees of freedom, $\psi_{high,}$ and the remainder is a low-energy lagrangian

$$L(\psi_{low}) = \sum_i \quad L_i$$

- This is an EFT for "high-energy constants"

Partons as EFT

Partons

- Partons are infinite-momentum collinear modes which are part of low-energy QCD DOF.
- Parton EFT
  - H-formulation: infinite momentum frame

    light-front quantization
  - Lagrangian formulation:

    Soft-collinear effective theory (SCET)

Non-local EFT

- Parton EFT is non-local

  contains operators $\frac{1}{iD^{+}}$
- Power counting-> twist counting
- Infinite number of counter terms-> infinite number of "high-energy constant"
- This is the PDF

Inverse problem

- Infinite number of constants require infinite number of experimental data to fix.
- But we don't have!
- This is the inverse problem: there is an optimal solution but there is no unique solution: "global analysis"
- We assume there are correlations among infinite constants

$$f(x) = \sum_i C_i \, x^{\alpha_i} (1-x)^{\beta_i}$$

The hope?

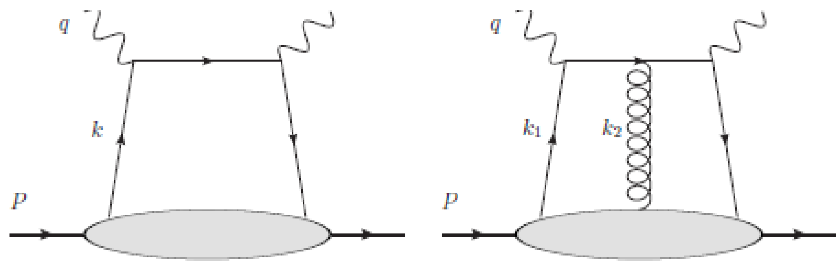
- Solve parton EFT, not fit!
- Light-front quantization
  - Light-cone singularity
  - Non-perturbative solutions
  - Critical theory
- Soft-Collinear EFT
  - Time-dependent correlation function!
  - Quantum computer

Quasi-PDF as Euclidean EFT

Quasi partons

- Partons are idealized objects, which do not exist in nature
- In reality, we have quasi-partons in a large-momentum hadrons
- Quasi-PDF are PHYSICAL momentum distribution in a large-P hadron, more physical than partons.

DIS Factorization in quasi-PDF



IG. 1: Tree-level diagrams for DIS process.

$$q^\mu = (0,0,0,-Q) \ ,$$
$$P^\mu = M\gamma v^\mu \ , \quad \gamma = \sqrt{1+\frac{Q^2}{4x_B M^2}} \ ,$$

respectively, where $v^\mu = (1,0,0,v)$ and $v^\mu v_\mu = 1/\gamma^2$. In the Bjorken limit, $0 < x_B < 1$, $\gamma \sim Q \to \infty$ and $v \to 1$.

Fig. 1, in which the hadron tensor is,

$$W^{\mu\nu}(x_B, Q^2) = \frac{1}{2\pi}\text{Im}\int i\frac{d^4k}{(2\pi)^4}\text{Tr}\left[\gamma^\mu S(k+q)\gamma^\nu M(k)\right] + \text{ crossing} \quad (3)$$

where $S(k)$ is the single quark propagator of four-momentum k^μ, and $M(k)$ is the single quark Green's function in the hadron,

$$M(k)^{\alpha\beta} = \int d^4\xi e^{i\xi\cdot k}\langle P|T\overline{\psi}^{\beta}_{\text{low}}(0)\psi^{\alpha}_{\text{low}}(\xi)|P\rangle \quad (4)$$

where $|P\rangle$ is the hadron state.

We will now restrict ψ_{low} to those collinear fields making up the hadron with velocity v,

$$\psi_{\text{low}}(x) = \psi_v(x) + \ldots\ , \quad (5)$$

then

$$M(k)^{\alpha\beta} = \int d^4\xi e^{i\xi\cdot k}\langle P|T\overline{\psi}^{\beta}_{v}(0)\psi^{\alpha}_{v}(\xi)|P\rangle\ . \quad (6)$$

The effective Fourier components of $\psi_v(x)$ have momentum k^μ, with the following decomposition,

$$k^\mu = \alpha v^\mu + \beta \bar{v}^\mu + k_\perp^\mu, \quad k^2 \sim \Lambda_{\rm QCD}^2, \tag{7}$$

where $\bar{v}^\mu = (v, -1)$, $\bar{v}_\mu^2 = -1/\gamma^2$, and $v \cdot \bar{v} = 2v$; $\alpha \sim \gamma\Lambda_{\rm QCD}$ and $\beta \sim \Lambda_{\rm QCD}/\gamma$. Thus the coefficient of $\bar{v}^\mu$ are suppressed by $1/\gamma$. Moreover, ψ_v satifies,

$$\not{v}\psi_v = 0 \ , \tag{8}$$

following from the leading order equations of motion (EOM) in $1/\gamma$.

The leading contribution to the hadron tensor comes from transverse polarization of the photon and thus $i, j = \perp$. In light of the trace in Eq.(3), the quark propagator can be simplified,

$$S(k+q) = \frac{i(\not{k} + \not{q})}{(k+q)^2 + i\epsilon} = \frac{i\not{q}}{2k \cdot q - Q^2 + i\epsilon} = \frac{i\gamma^z}{2k^z - Q + i\epsilon} \ , \tag{9}$$

where in the second equality, we use Eq. (8) in the numerator to eliminate $\not{k}$ and neglected $k^2 \sim \Lambda_{\rm QCD}^2$ in the denominator. Defining $k^z = Qy/2x_B$, the integration over k^0 and $k_\perp$ in Eq. (3) can be carried out,

$$
\begin{aligned}
W^{\mu\nu} &= -g_\perp^{\mu\nu} {\rm Im} \int_\infty^\infty \frac{dy}{2\pi} \tilde{f}(y) \frac{1}{y/x_B - 1 + i\epsilon} + \text{crossing} \\
&= -g^{\mu\nu} \frac{1}{2} \left(\tilde{f}(x_B) + \tilde{f}(-x_B) \right) ,
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{f}(y, P^z) &= \frac{1}{2} \int dz e^{izk^z} \langle P | \bar{\psi}_v(z) \gamma^z \psi_v(0) | P \rangle \\
&= \frac{1}{2P^z} \int d\lambda e^{iy\lambda} \langle P | \bar{\psi}_v(z) \gamma^z \psi_v(0) | P \rangle , \quad (10)
\end{aligned}
$$

with dimensionless $\lambda \equiv zP^z$. The above result is identical to the standard QCD factorization result, except that the distribution $\tilde{f}(y, P^z)$ replaces the light-cone distribution

$$
f(x) = \frac{1}{2P \cdot n} \int_{-\infty}^\infty d\lambda e^{ix\lambda} \langle P | \bar{\psi}(\lambda n) \gamma^+ W(\lambda n, 0) \psi(0) | P \rangle . \quad (11)
$$

The key of the derivation is that the k^0 components of the quark four-momentum can be eliminated through the equation of motion (EOM) of the effective field $\not{v}\psi_v$=0. And therefore, k^0 integration can be carried out in the

$$A_v^\mu = \alpha v^\mu + \beta \bar{v}^\mu + A_\perp^\mu \ , \tag{12}$$

and at leading order, only the α component dominates. For example, when there is one interaction with the gauge potential (see Fig. 1), we replace the quark propagator in Eq. (3) by

$$S(k_1 + q)\gamma^\alpha S(k_1 + k_2 + q) \ , \tag{13}$$

and $M(k)$ by $M^\alpha(k_1, k_2)$ where an additional $A_v^\alpha(k_2)$ appears. Both S can be simplified by the EOM of the effective field,

$$\begin{aligned} S(k_1+q) &\sim \frac{i\gamma^z}{2k_1^z - Q + i\epsilon}, \\ S(k_1+k_2+q) &\sim \frac{i\gamma^z}{2k_1^z + 2k_2^z - Q + i\epsilon} \ , \end{aligned} \tag{14}$$

For $\not{A}_v$, the situation is a bit involved, since $A_v^z \sim A_v^0$ as $v \to c$, we have the leading contributon,

$$\not{A}_v = A_v^0 \gamma^0 - A_v^z \gamma^z \tag{15}$$

However, after commutation with γ^z, we have

$$\gamma^z \not{A}_v = -(A_v^0\gamma^0 + A_v^z\gamma^z)\gamma^z \sim 2A_v^z \tag{16}$$

where again we have used quark's EOM and $A_v^z \sim A_v^0$. Therefore, effectively all $\not{A}_v = -2A_v^z\gamma^z$, which allows one to calculate diagrams with an arbitrary number of A_v^μ interactions.

Adding all the quark eikonal interactions, one has

$$\tilde{f}(y, P^z) = \frac{1}{2P^z}\int_{-\infty}^{\infty} d\lambda e^{iy\lambda} \langle P|\bar{\psi}_v(z) W(z,0)\gamma^z \psi_v(0)|P\rangle$$

QPDF factorization

- qPDF factorization can be carried out for more complicated Feynman diagrams and for other processes like Drell-Yan.
- Global analysis can be carried out for quasi-PDF
- QPDF can be calculated directly on lattice QCD, there is no inverse problem!

LaMET as Euclidean EFT for partons

EFT for partons

- High-energy physical observables can be factorized in terms of PDFs
- The same can be done in terms of qPDFs
- Therefore, PDFs can be entirely expressed in terms of qPDFs
- This is the large-momentum effective theory for partons: LaMET

Leading LaMET lagrangian

- LaMET starts with hadrons with large momentum P or velocity v,

of velocity v. The leading effective lagrangian for the quark collinear modes can be written as,

$$\mathcal{L}^{(0)}_{q,v} = \overline{\psi}_v \left[iv \cdot D + \frac{i\bar{v} \cdot D}{2\gamma^2} + (iD_\perp)\frac{1}{2i\bar{v} \cdot D}(iD_\perp) \right] \not{\bar{v}} \psi_v \tag{19}$$

where $\bar{v} = (v, 0, 0, -1)/2v$ and $v_\mu \bar{v}^\mu = 1$. One can also add the leading-order lagrangian for the gluon collinear modes. This effective theory formally converges to SCET or light-front quantization in the $v \to c$ limit. However,

Features of LaMET

- There is no light-cone singularities

  no extra renormalization/ zero mode problem
- It is a Euclidean theory, can be calculated on lattice or instanton liquid.
- P serves as a rapidity regulator, and evolution equation can be derived
  - For PDF, it is DGLAP
  - For TMDs, it is the Collins-Soper evolution.

There is no inverse problem

- Take parton observables as physical, one can a Weinberg-like systematic expansion

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{y} C_2\left(\frac{y}{x}, \frac{P^z}{\mu}\right) \tilde{f}\left(y, \frac{P^z}{\mu}\right) + \left(\frac{\Lambda_{\rm QCD}}{P^z}\right)^2 \sum_i \int_{-\infty}^{\infty} \frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{dy_3}{y_3} C_{4i}\left(\frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x}, \frac{P^z}{\mu}\right) \times \tilde{f}_i\left(y_1, y_2, y_3, \frac{P^z}{\mu}\right) + \ldots \qquad (22)$$

- So long as the expansion converges, PDF at any x can be computed with controlled errors.

x-dependence

- LaMET calcaluates/predicts the x-dependence without model-dependent fits (inverse problem)
- No other methods can do this.

Conclusion

- LaMET is an EFT in the sense of Weinberg