"Non-local nucleon matrix elements in the rest frame"

J. Karpie, C. Monahan, A. Radyushkin, arXiv: 2407.16577

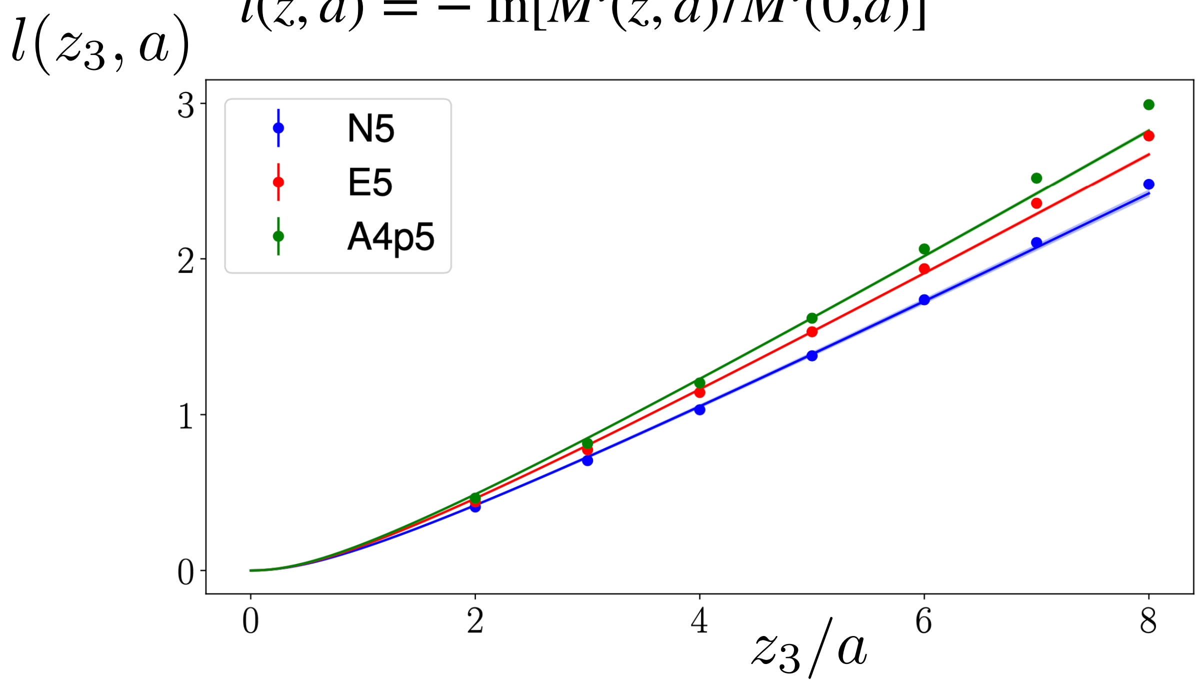
Comments, X. Gao, J. He, Y. Su, R. Zhang and YZ, arXiv: 2408.04674

$$M^t(z,a) = \langle P=0 \,|\, \bar{\psi}(z)\gamma^t W(z,0)\psi(0) \,|\, P=0 \rangle$$

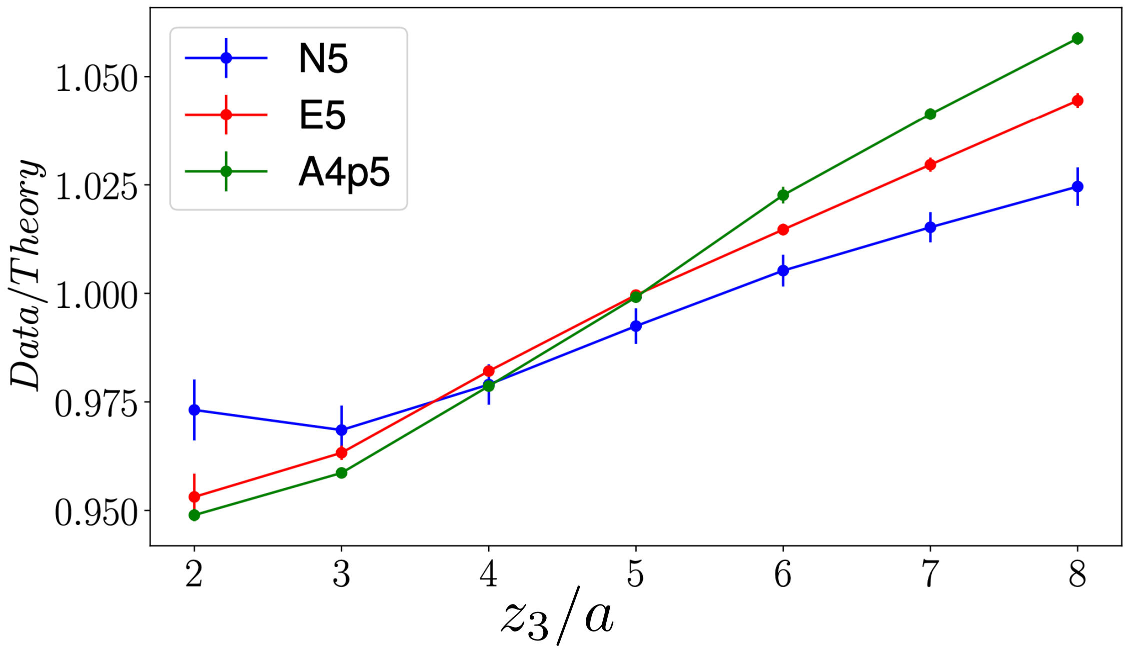
Lattice (n_f=2, CLS):

Ens. ID	a(fm)	M_π(MeV)	$L^3 \times T$	$N_{\rm cfg}$
A4p5	0.0749(8)	446(1)	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	$48^3 \times 96$	477

$$l(z,a) = -\ln[M^t(z,a)/M^t(0,a)]$$



$\Lambda_{\rm QCD} \sim 330$MeV, χ^2/dof $\gtrsim$ O(10)



Theory:

Polyakov regulator in coordinate space

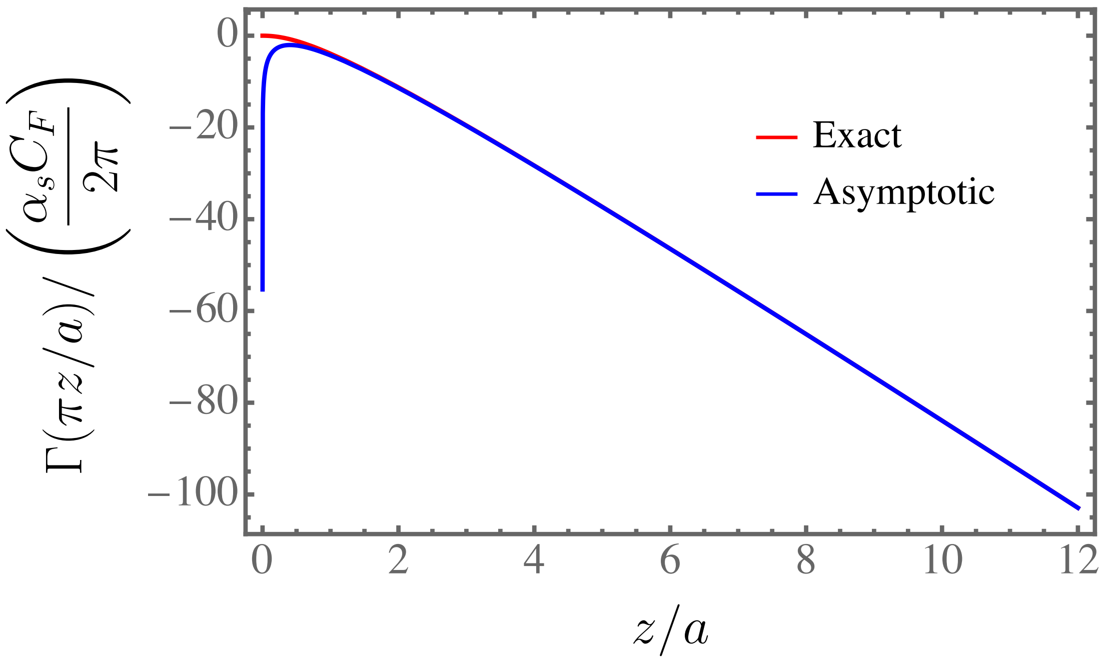
$$\exp\left(\Gamma(z/a_{\rm PR})\right) \qquad a_{\rm PR} = a/\pi$$

$$\Gamma(\zeta) = \frac{\alpha_s C_F}{2\pi}\left[\left(2+\frac{1}{\zeta^2}\right)\ln(1+\zeta^2) - 2\zeta\tan^{-1}\zeta - 1\right]$$

$$\alpha_s = \alpha_s(\frac{\pi}{a}) = -\frac{2\pi}{(11-n_f/3)\ln(a\Lambda_{\rm QCD}/\pi)}$$

“Perturbation theory can describe data up to ~ 0.6 fm”?

Existing knowledge suggests that NP effects become important for $z \gtrsim 0.3$ fm, e.g., instanton size, QCD sum rules, gluon mass, static potential, etc.



Issues in the ansatz:

- Simple exponentiation of NLO correction not justified
- Large logs need resummation, which runs α_s from c/z, $c \sim 1$
- Renormalon due to regularization of linear divergence not treated.

$$\Gamma(\zeta) = \frac{\alpha_s C_F}{2\pi}\left[\left(2+\frac{1}{\zeta^2}\right)\ln(1+\zeta^2)-2\zeta\tan^{-1}\zeta-1\right]$$

$$\zeta = \pi z/a \qquad \lim_{\zeta\to\infty}\Gamma(\zeta) = \frac{\alpha_s C_F}{2\pi}\left(-\pi|\zeta|+2\ln\zeta^2+1\right)$$

$\zeta \to \infty$ limit is saturated quickly for $z \geq 2a$ ($\lesssim 1\%$)

Self-renormalization scheme:

Y.-K. Huo, Y. Su et al. (LPC), arXiv: 2103.02965

$$\Gamma_{\rm SR}(z,a) = \frac{kz}{a\ln(a\Lambda_{\rm QCD})} + m_0 z + Z_{\overline{\rm MS}}(z) + \ldots$$

log + finite terms

But the agreement among multiple ensembles is still nontrivial.

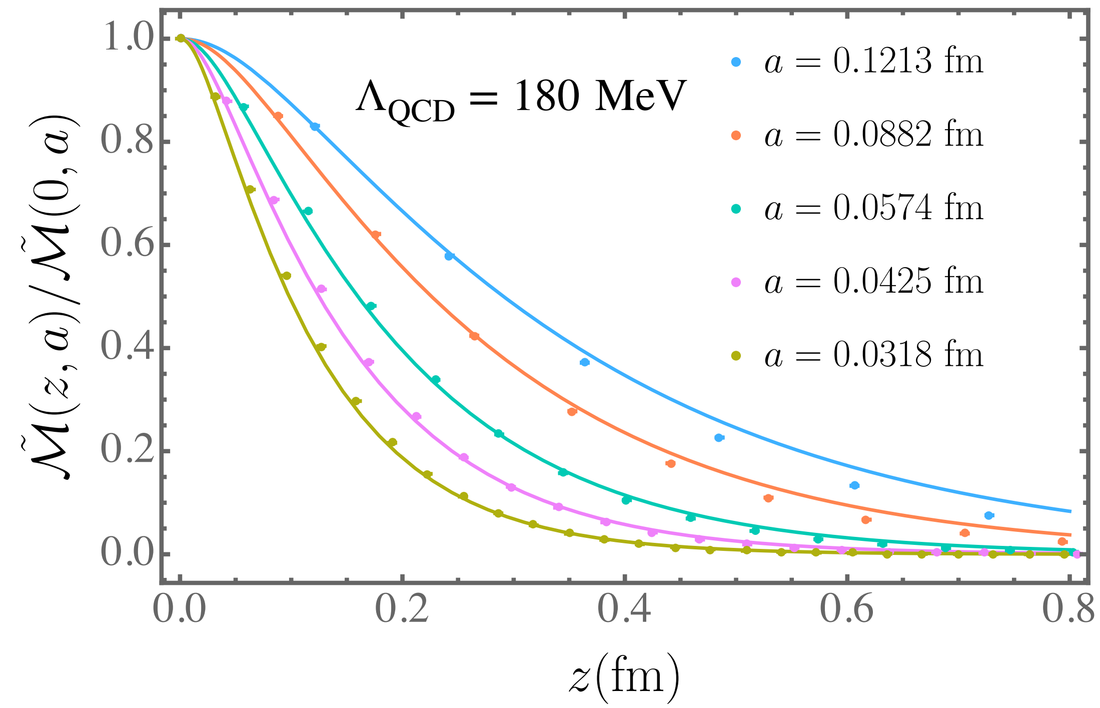
Why?

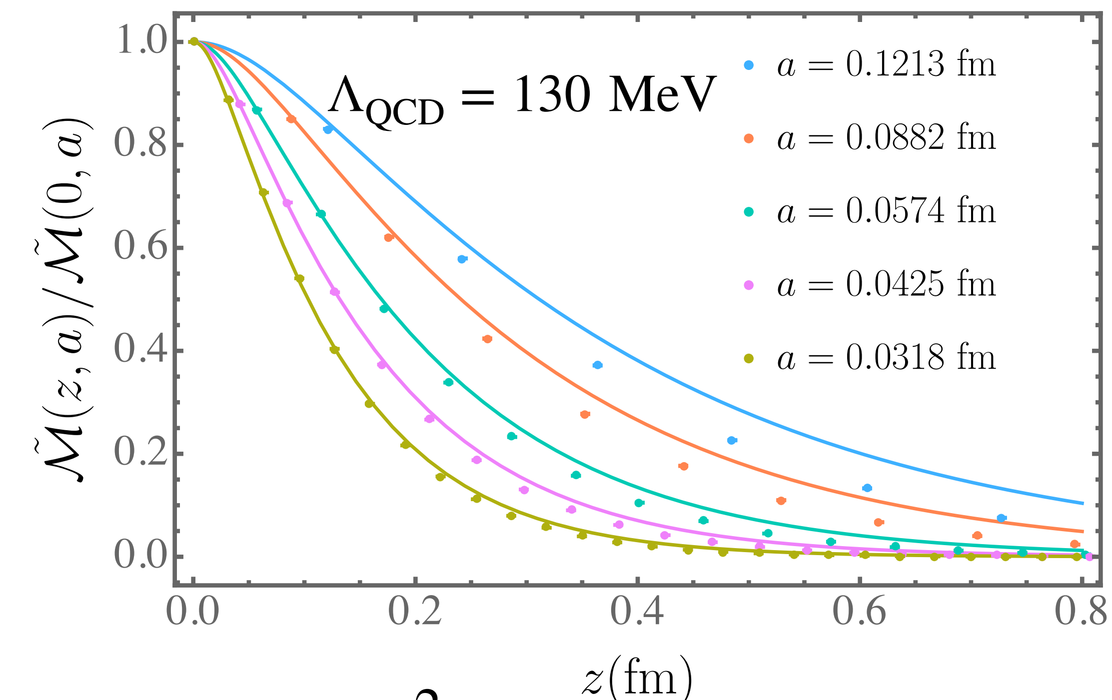
Pion matrix elements

$m_\pi = 310$ MeV, $n_f = 2+1+1$, MILC

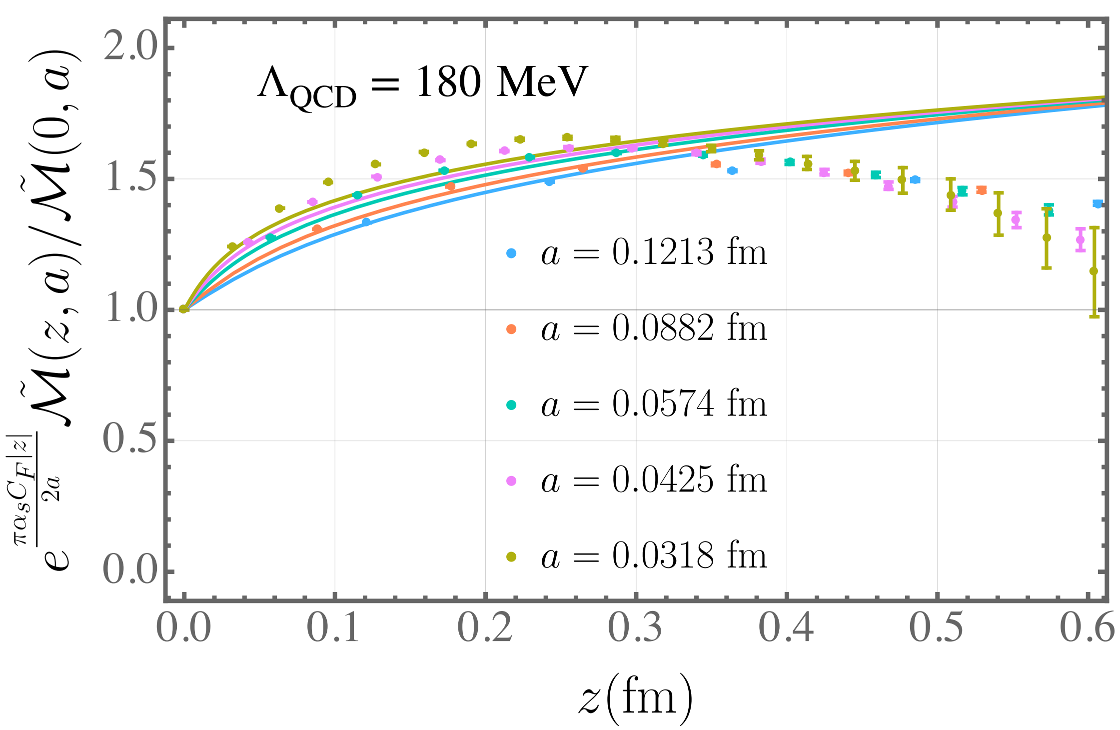
Y.-K. Huo, Y. Su et al. (LPC), arXiv: 2103.02965

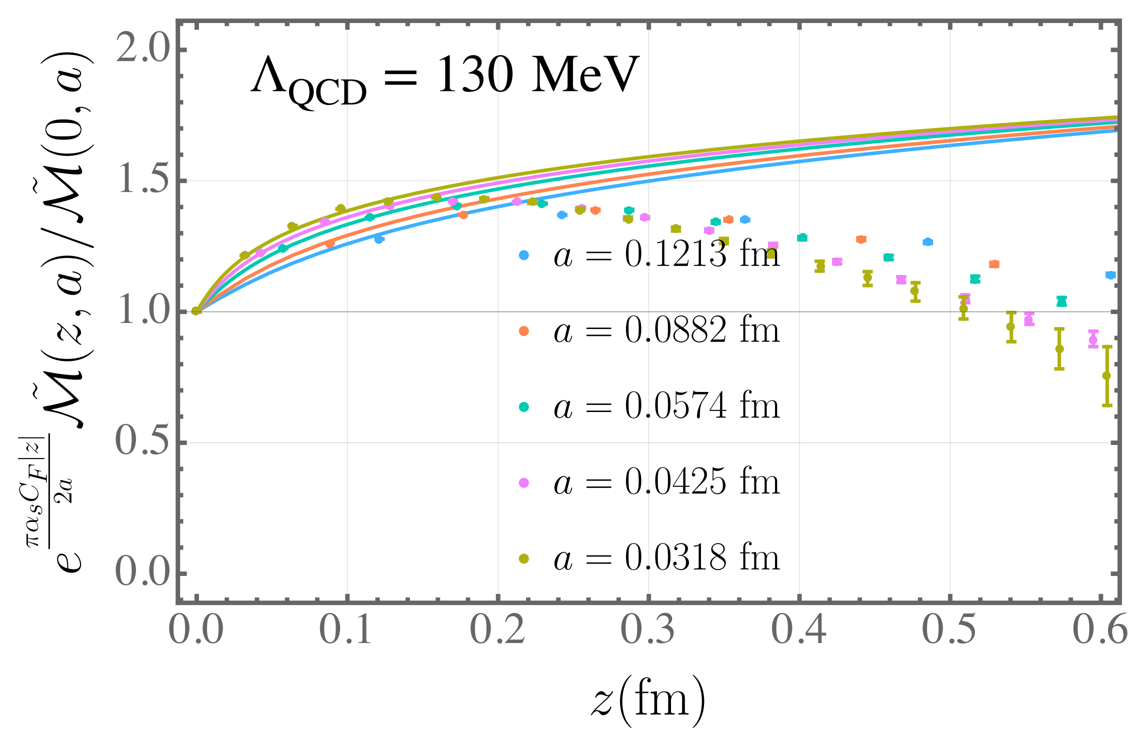
Bare matrix elements





Matrix elements with linear divergence, $\exp[-\frac{\alpha_s C_F}{2\pi}\frac{\pi^2 z}{a}]$, subtracted

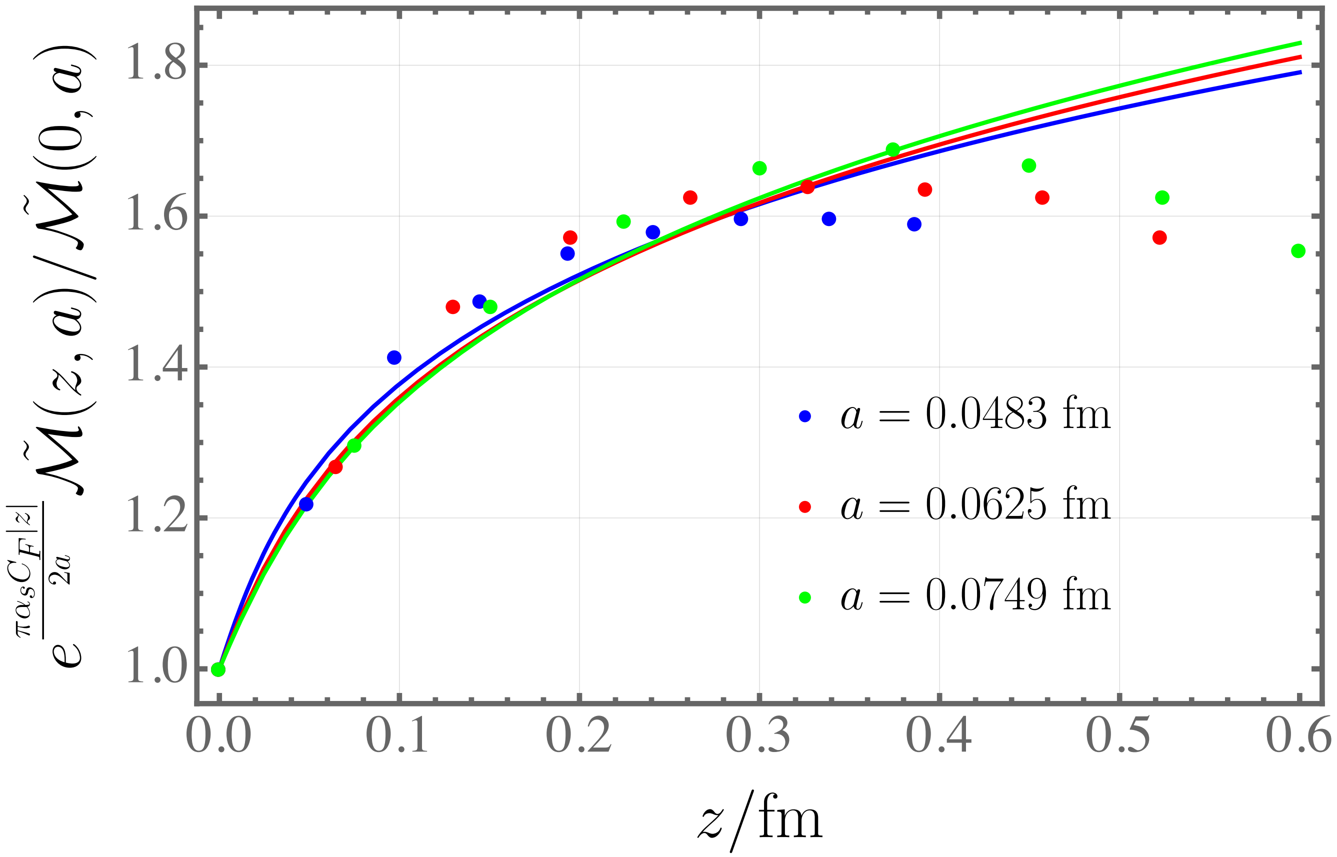




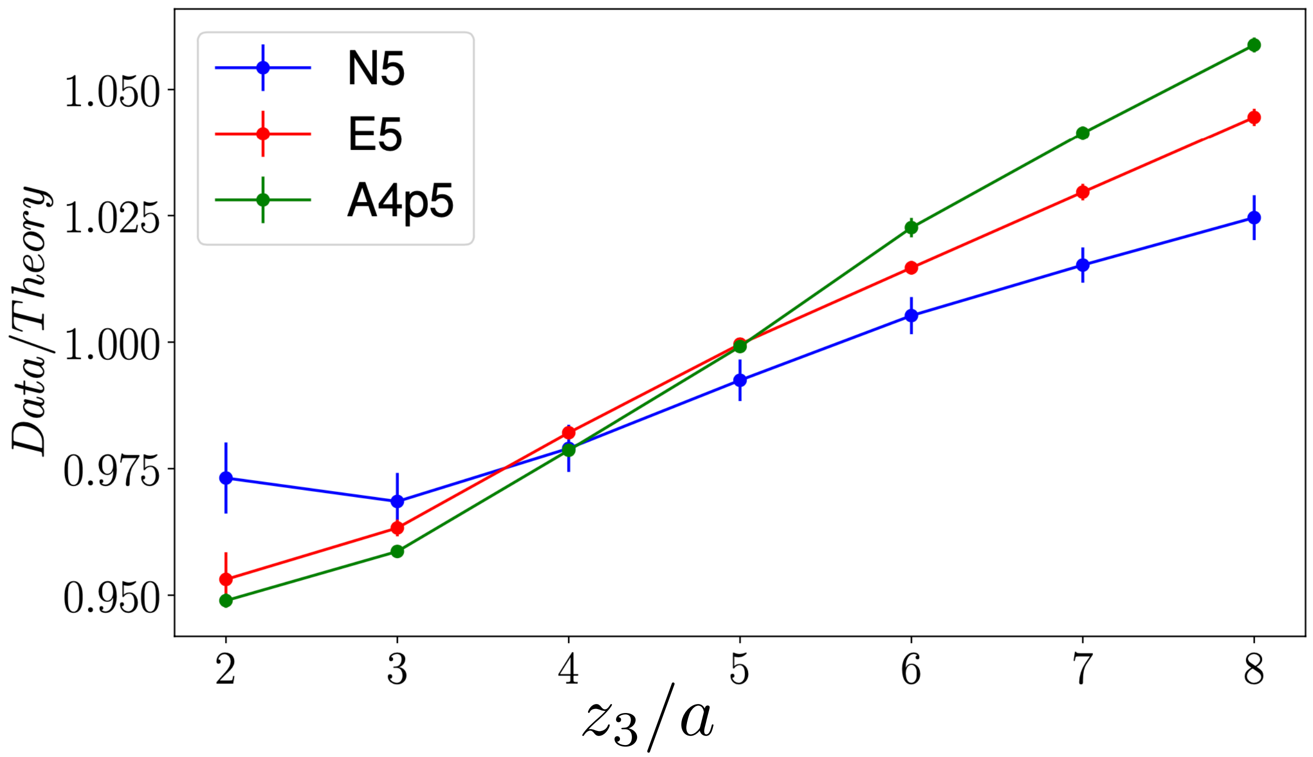
Nucleon matrix elements

$m_\pi = 445$ MeV, $n_f = 2$, CLS

J. Karpie, C. Monahan, A. Radyushkin, arXiv: 2407.16577



- Disagreement starts to show up at z ~ 0.35 fm
- Relative difference may be $\lesssim 15\,\%$, but the trends of theory and data are different.



- $\lesssim 5\,\%$ difference likely due to plot of **log**(matrix element)/**log**(theory)
- A slight linear dependence suggests the effect of linear renormalon

Conclusions

Linear divergence dominates the bare matrix elements

$$\exp\left(-\delta m(a)\,|z|\,\right) \qquad\qquad \delta m(a) = \frac{1}{a}\sum_{i=1}^{\infty}\alpha_s^i(a) + O(a\Lambda_{\mathrm{QCD}})$$

It only depends on the UV cutoff, which is why the NLO perturbative correction can approximate it to a good extent.

Apart from the linear divergence, the remaining contribution cannot be described by perturbation theory beyond 0.2~0.3 fm.

Beyond *z*~0.3 fm, one can add more parameters to the perturbative expression to improve the fit quality, but it is modeling.