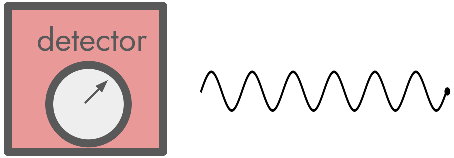
Entanglement and Bell Inequality Violation at the LHC in Top-Anti-Top Events

Matthew Low

arXiv:2310.17696
arXiv:2311.09166

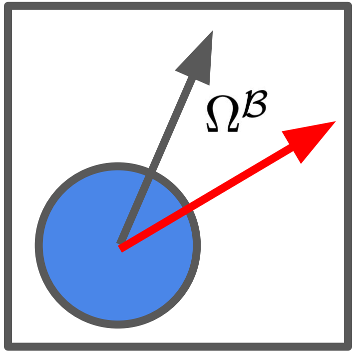
with Kun Cheng, Tao Han, Arthur Wu

Outline

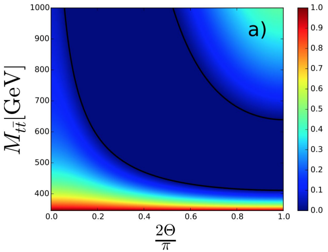


Big Picture

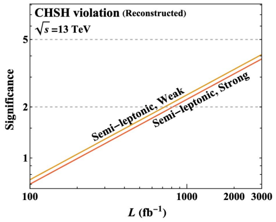
Quantum Review



Collider Review



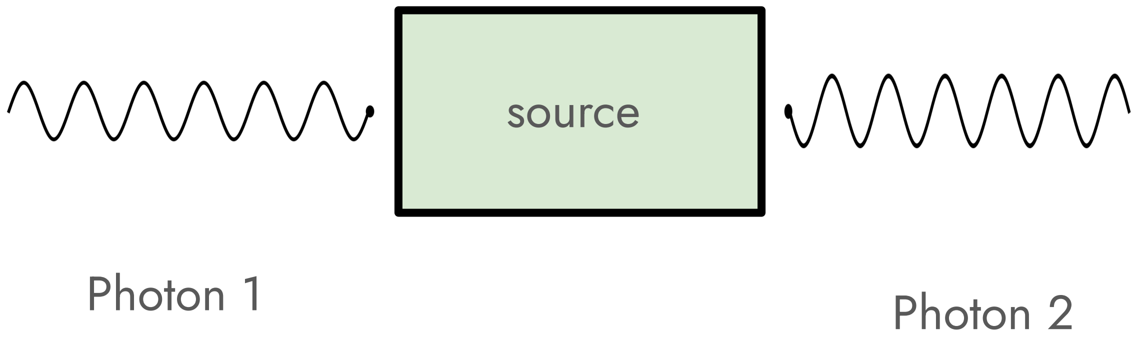
Phenomenology



Semi-Leptonic Channel

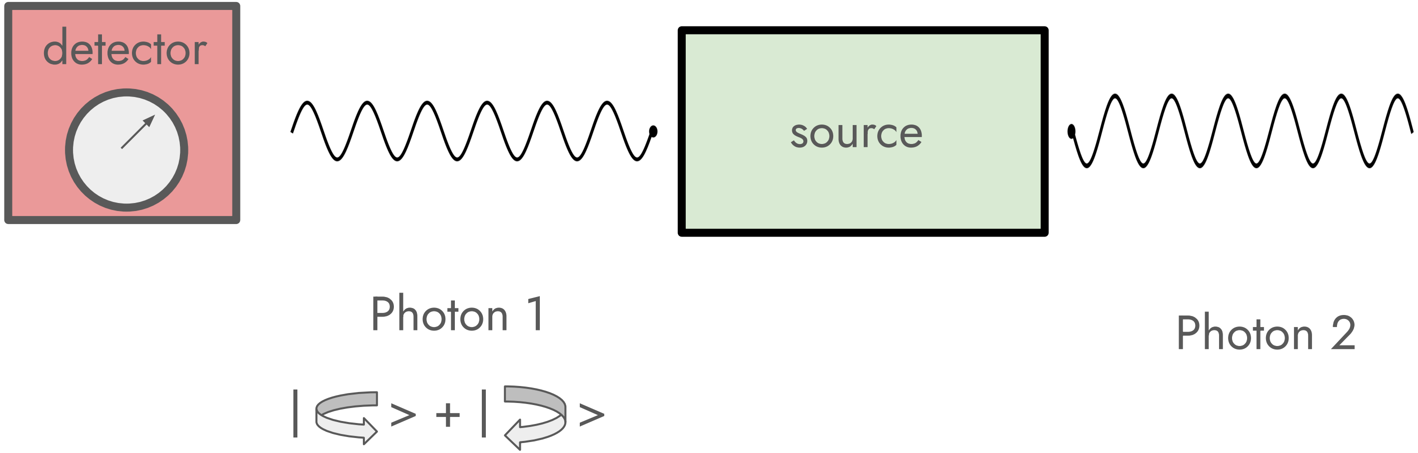
Big Picture

- Entanglement is a correlation between two qubits



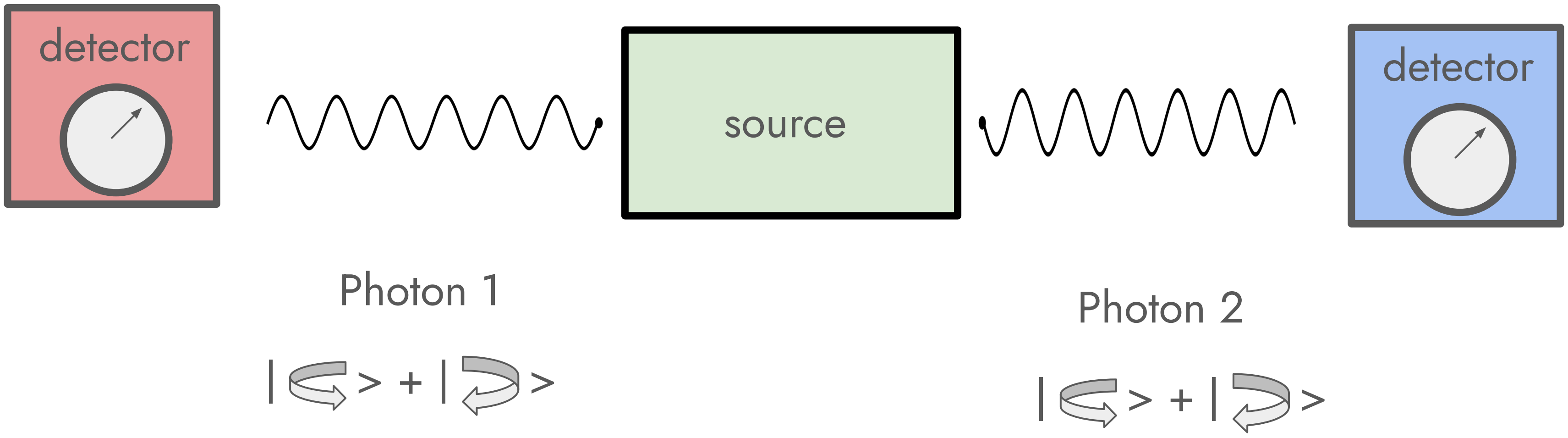
Big Picture

- Entanglement is a correlation between two qubits



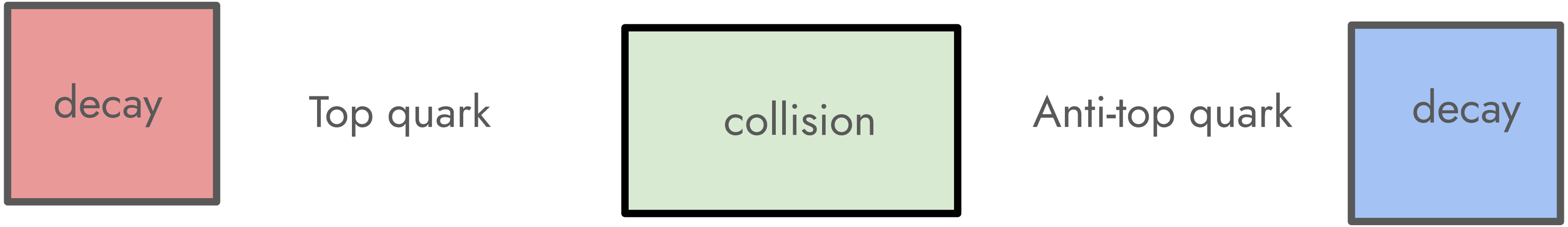
Big Picture

- Entanglement is a correlation between two qubits



Big Picture

- Entanglement is a correlation between two qubits



- Let the top and anti-top be qubits, perform the same quantum experiments!
  - Measuring entanglement (ATLAS has done this already!)
  - Looking for Bell inequality violation

Big Picture

Research Highlight | Published: 24 January 2024

Editors' picks 2023

Entanglement between a pair of top quarks

Iulia Georgescu

Nature Reviews Physics **6**, 85 (2024) | Cite this article

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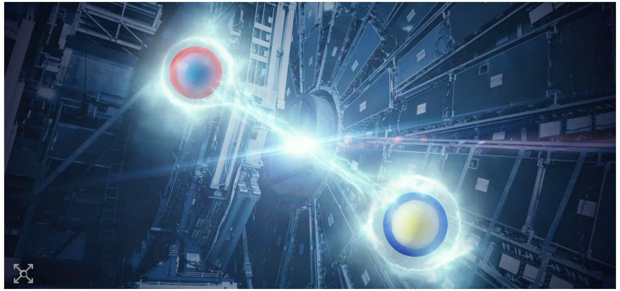
Entanglement is a purely quantum phenomenon that has been studied extensively in low-energy systems to explore the foundations of quantum mechanics and for applications in quantum technologies. Would entanglement manifest at very high energies, in a relativistic regime with exotic interactions and symmetries? There is no reason to suspect that it wouldn't, but so far there has not been any experiment able to test this assumption. Now, the ATLAS Collaboration at CERN has used data from 13 TeV proton–proton collisions at the

QUANTUM | RESEARCH UPDATE

Quantum entanglement observed in top quarks

11 Oct 2023



Top result: An artist's impression of top-quark entanglement. The line between the particles emphasizes the non-separability of the top-quark pair, which is produced by LHC collisions and recorded by ATLAS. (Courtesy: Daniel Dominguez/CERN)

Quantum Mechanics: States

- Quantum state $|\psi\rangle$ is a vector in Hilbert space
- *Example*: spin-½ has two states

$$|\psi\rangle = \alpha \mid \uparrow\rangle + \beta \mid \downarrow\rangle = \alpha|1\rangle + \beta|0\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- An observable A is $a = \langle\psi|A|\psi\rangle$
- *Example*: observable σ_z

$$\langle\psi|\sigma_z|\psi\rangle = |\alpha|^2 - |\beta|^2$$

spin-up = +1

spin-down = -1

Quantum Mechanics: States

- Bra-ket formalism only for pure states
- Generally, for mixed states requires a density matrix $\rho = |\psi\rangle\langle\psi|$
- An observable A is $a = \mathrm{tr}(\rho A)$
- *Example*:

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \mathrm{tr}(\rho\sigma_z) = 1$$

$$|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \rho = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \mathrm{tr}(\rho\sigma_z) = 0$$

- A mixed state is $\rho = \sum_{i=1} p_i \rho_i \qquad \sum_{i=1} p_i = 1$

Quantum Mechanics: States

- Qubit state can be decomposed

$$\rho = \frac{1}{2}\left(\mathbf{I}_2 + b_i\sigma_i\right)$$

- State described by b_1, b_2, b_3
- *Example*:

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

(b_1, b_2, b_3) = (0, 0, 1)

$$\rho = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

(b_1, b_2, b_3) = (-1, 0, 0)

Quantum Mechanics: States

- Bipartite qubit system is two qubits
- Label two sub-systems as A and B

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

- Common basis:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

- *Example*:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Quantum Mechanics: States

- Density matrix for two qubits is 4 x 4
- Example:

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

From state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- Decomposition for two qubits:

$$\rho = \frac{1}{4}\left(\mathbf{I}_4 + B_i^{+}\sigma_i \otimes \mathbf{I}_2 + B_i^{-}\mathbf{I}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j\right)$$

Quantum Mechanics: States

$$\rho = \frac{1}{4}\left(\mathbf{I}_4 + B_i^+ \sigma_i \otimes \mathbf{I}_2 + B_i^- \mathbf{I}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j\right)$$

B_i^+ Polarization vector of qubit A

B_i^- Polarization vector of qubit B

C_{ij} Spin correlation matrix

- Example:

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad B_i^+ = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad B_i^- = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Quantum Mechanics: Entanglement

- Einstein's "spooky action at a distance"
- Can only describe sub-system A with knowledge of sub-system B
- By contrast, a separable state can be written

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

separable

$$|\psi\rangle = |00\rangle = |0\rangle \otimes |0\rangle$$

entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- Entangled is non-separable

Quantum Mechanics: Entanglement

- Given a state, how to tell if it's entangled?
  - Compute **concurrence** $C[\rho]$ = 0 (separable)

    = 1 (maximally entangled)

    - For pure states $C[\rho] = \sqrt{2(1 - \mathrm{tr}_A(\rho))}$

      partial trace

    - Two qubits has a closed-form solution (even for mixed states)

Wootters quant-ph/9709029

Quantum Mechanics: Entanglement

- Given a state, how to tell if it's entangled?

Peres quant-ph/9604005

Horodecki, Horodecki, Horodecki quant-ph/9605038

  - Positive Partial Transpose (PPT)
    - Apply transpose to sub-system B to form ρ^{T_B}

    If ρ^{T_B} is a valid state => **separable**

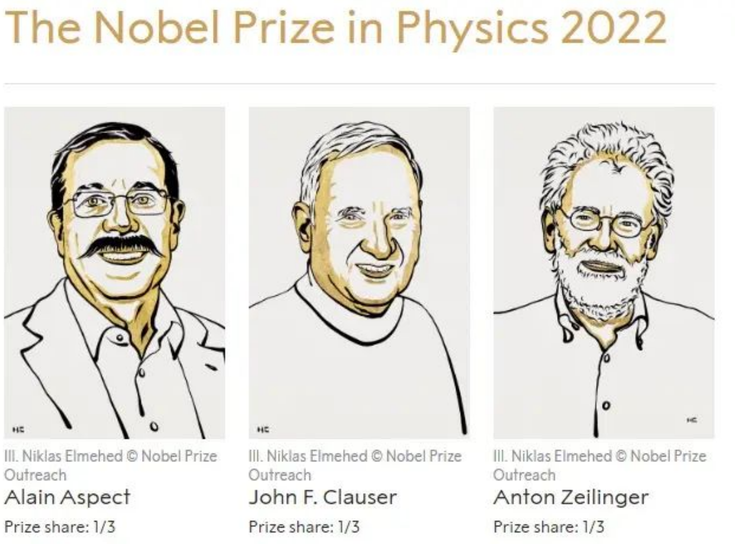
    If ρ^{T_B} is not a valid state => **entangled**

    - State is non-negative operator (all eigenvalues >= 0)
      - list of inequalities
      - >= 1 inequality violated means entangled

Quantum Mechanics: Bell's Inequality

Bell 1964

- An inequality that is satisfied by all local theories (including hidden variable theories)



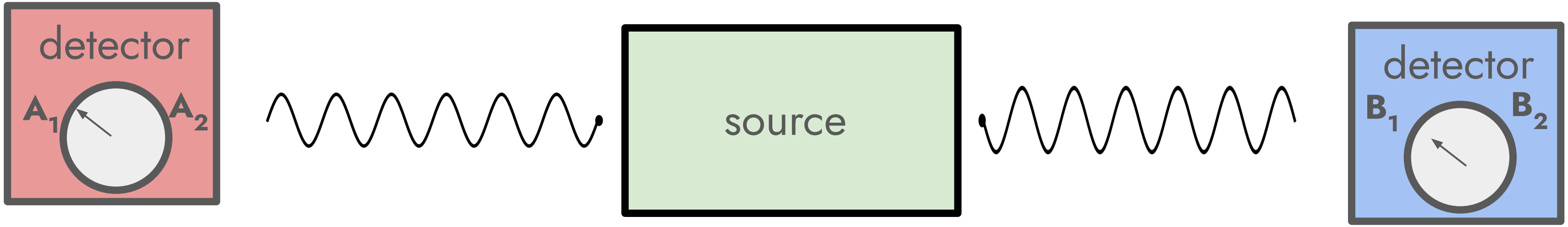
- "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science."

Quantum Mechanics: Bell's Inequality

Clauser, Horne, Shimony, Holt, 1969

- For two qubits the Clauser-Horne-Shimony-Holt (CHSH) inequality is

$$|\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle| \leq 2$$



- Example: $A_1 = \sigma_x$, $B_1 = \sigma_y$ $\quad \langle A_1 B_1 \rangle = \mathrm{tr}[(\sigma_x \otimes \sigma_y)\rho]$

Quantum Mechanics: Bell's Inequality

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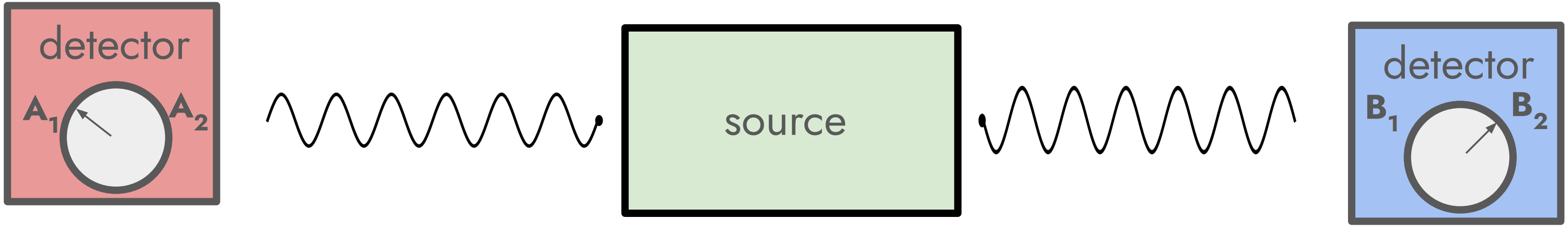


- Example: $A_1 = \sigma_x$ $B_2 = \sigma_z$ $\langle A_1 B_2 \rangle = \mathrm{tr}[(\sigma_x \otimes \sigma_z)\rho]$

Quantum Mechanics: Bell's Inequality

Clauser, Horne, Shimony, Holt, 1969

- For two qubits the Clauser-Horne-Shimony-Holt (CHSH) inequality is

$$|\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle| \leq 2$$

- Assemble measurements and test inequality
- *Example*:

$$A_1 = \sigma_x \qquad \langle \sigma_x \otimes \sigma_z \rangle = \mathrm{tr}[(\sigma_x \otimes \sigma_z)\rho]$$

$$B_2 = \sigma_z \qquad = C_{13}$$

- Depends on choices of $A_{1,2}$ and $B_{1,2}$

Collider Physics

- Cross-section for two-body scattering

$$\sigma(\mathcal{X}\mathcal{Y} \to \mathcal{A}\mathcal{B}) = \int \mathrm{d}\Pi \overline{\sum_{\text{initial}}} \sum_{ab,\bar{a}\bar{b}} \mathcal{M}(\mathcal{X}\mathcal{Y} \to \mathcal{A}\mathcal{B})_{a\bar{a}} \mathcal{M}^*(\mathcal{X}\mathcal{Y} \to \mathcal{A}\mathcal{B})_{b\bar{b}},$$

  - Average over initial spin, colors, etc.
  - Sum over final spins, colors, etc.

- Leave final spins unsummed

$$R_{ab,\bar{a}\bar{b}} = \overline{\sum_{\text{initial}}} \mathcal{M}(\mathcal{X}\mathcal{Y} \to \mathcal{A}\mathcal{B})_{a\bar{a}} \mathcal{M}^*(\mathcal{X}\mathcal{Y} \to \mathcal{A}\mathcal{B})_{b\bar{b}},$$

  - **Production spin density matrix** (4 x 4 matrix for two spin-½ outgoing particles)

Collider Physics

- Same formalism applies to decays

$$\Gamma^{\mathcal{A}}_{ab} = \mathcal{M}(\mathcal{A} \to a_1 a_2 a_3)_a \mathcal{M}^*(\mathcal{A} \to a_1 a_2 a_3)_b,$$

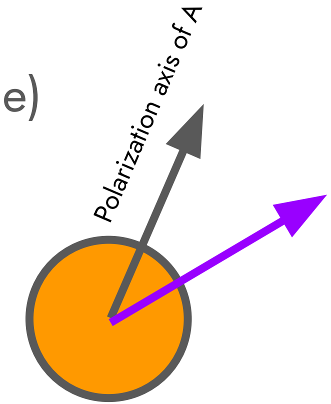
  - **Decay spin density matrix** (2 x 2 matrix for spin-½ particle)

- Consider one of the decay products

polarization

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_v} = \frac{1}{2}\left(1 + |\vec{B}|\kappa_v \cos\theta_v\right),$$

Spin analyzing power



Mother A

Daughter a_1

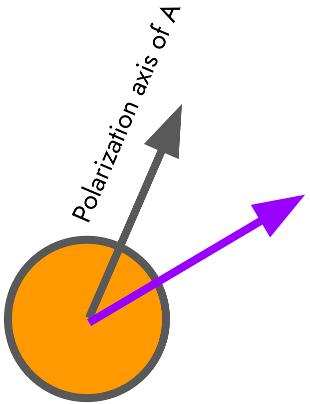
Collider Physics

- Spin analyzing power quantifies how spin of mother is preserved by direction distribution of daughter
- *Example*: top decays

Spin Analyzer	Power
lepton/down-quark	1.00
neutrino/up-quark	-0.34
b-quark or W	∓ 0.40
soft-quark	0.50
optimal hadronic	0.64

(Lepton is maximally correlated)

(For hadronic decays, cannot distinguish jets)



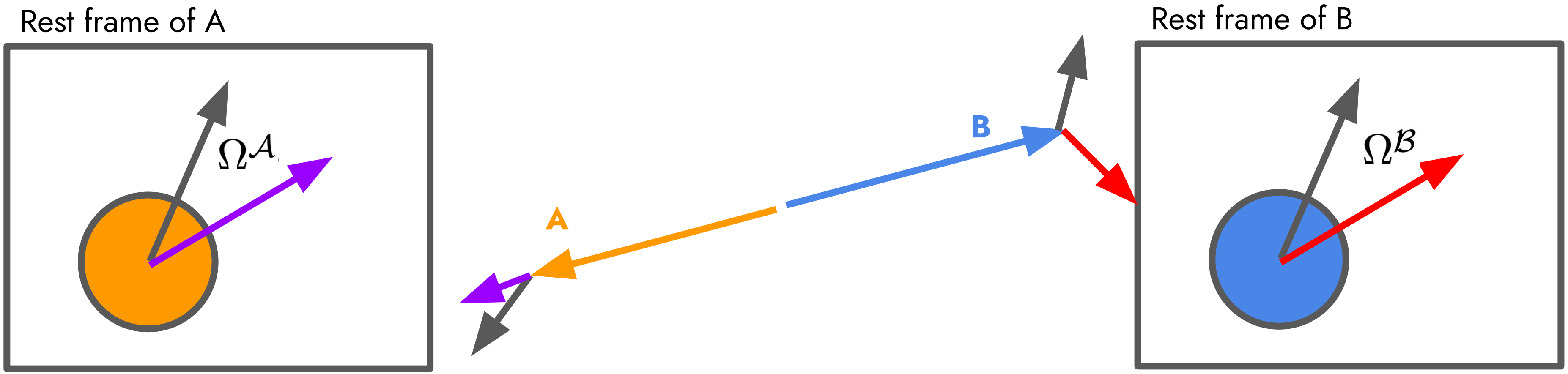
Mother A

Daughter a_1

Tweedie 1401.3021

Collider Physics

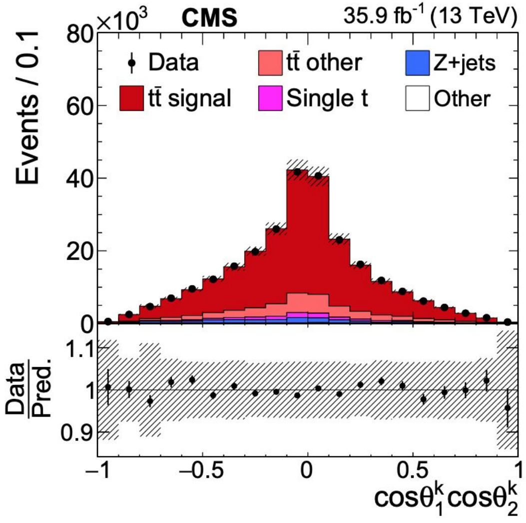
- Decay angle distribution determined by spins of (mother) particles



$$\frac{1}{\sigma}\frac{d^4\sigma}{d^2\Omega^{\mathcal{A}}d^2\Omega^{\mathcal{B}}} = \frac{1}{(4\pi)^2}\left(1+\sum_i\left(\kappa^{\mathcal{A}}\,B_i^{\mathcal{A}}\Omega_i^{\mathcal{A}}+\kappa^{\mathcal{B}}\,B_i^{\mathcal{B}}\Omega_i^{\mathcal{B}}\right)+\sum_{i,j}\kappa^{\mathcal{A}}\kappa^{\mathcal{B}}\,\Omega_i^{\mathcal{A}}C_{ij}\Omega_j^{\mathcal{B}}\right),$$

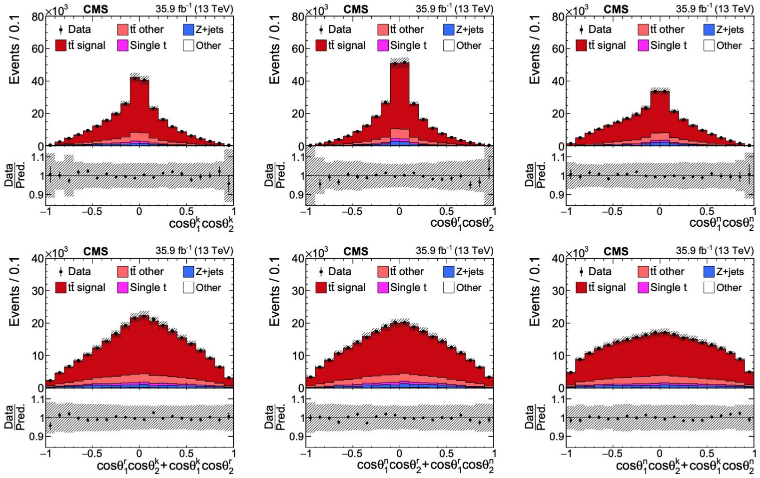
Collider Physics

- While individual spins not measured, can statistically measure
- Measuring angles of decay products will measure the parameters **B**$_i$ and **C**$_{ij}$



CMS 1907.03729

- Can reconstruct the full production spin density matrix



Collider Physics

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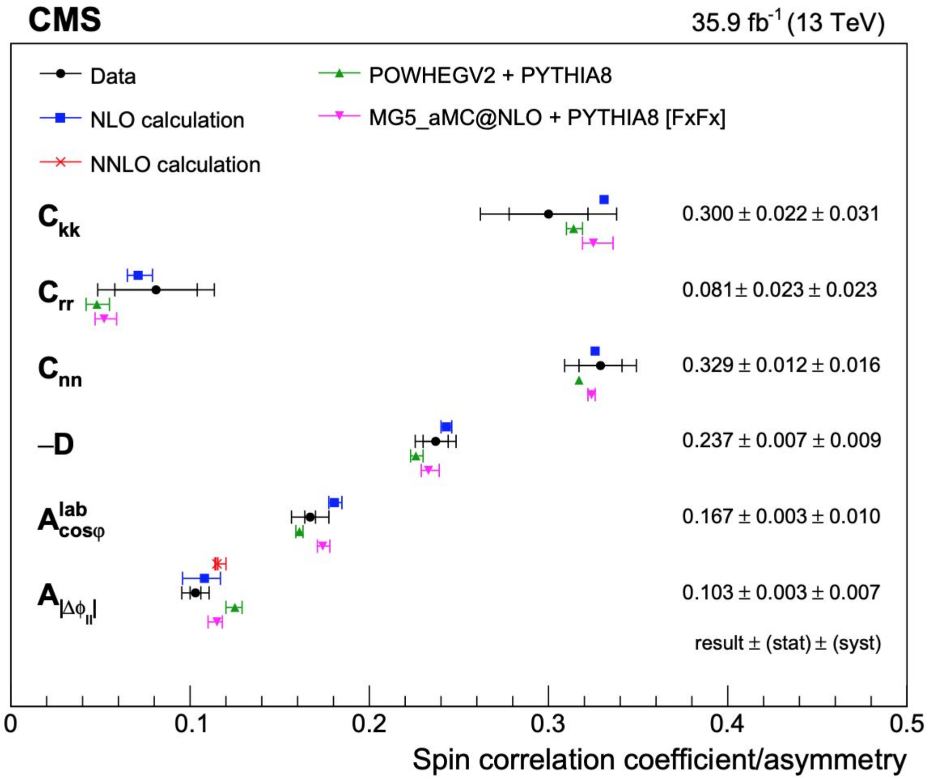
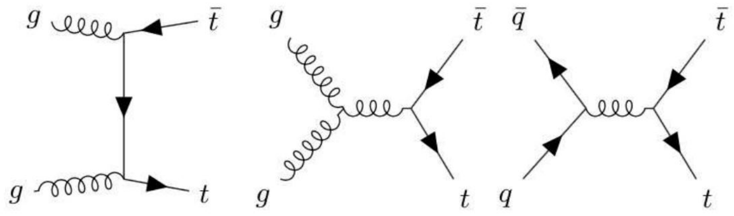


- C_{kk}, C_{rr}, C_{nn} are diagonal components of spin correlation matrix
- $D = (-⅓)(C_{kk} + C_{rr} + C_{nn})$
- Will show that D measures entanglement
- CMS found that there was no entanglement

Phenomenology

Afik, de Nova 2003.02280

- Afik and de Nova described formalism to go from spin correlations to a quantum state for top-anti-top system



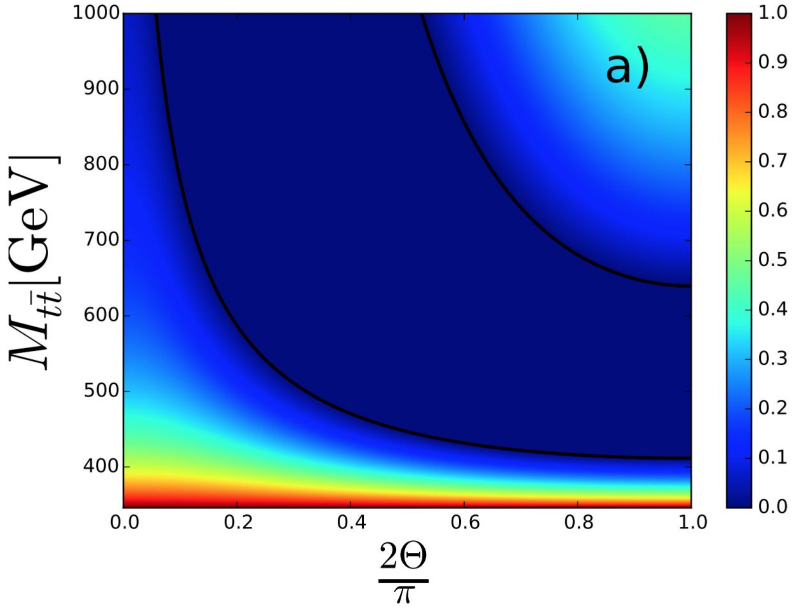
- Qubit A = spin of top
- Qubit B = spin of anti-top
- Each phase space point is a different quantum state
  - Invariant mass $m_{\bar{t}t}$
  - Scattering angle θ
- When using a fixed basis, can integrate over phase space for a total quantum state

Phenomenology

Afik, de Nova 2003.02280

- Production spin density matrix for $q\bar{q} \to t\bar{t}$ identified with quantum density matrix
  $\rho_{q\bar{q}}(m_{t\bar{t}}, \theta)$
- Similar for initial state $gg \to t\bar{t}$
  $\rho_{gg}(m_{t\bar{t}}, \theta)$

$C[\rho_{gg}]$ →



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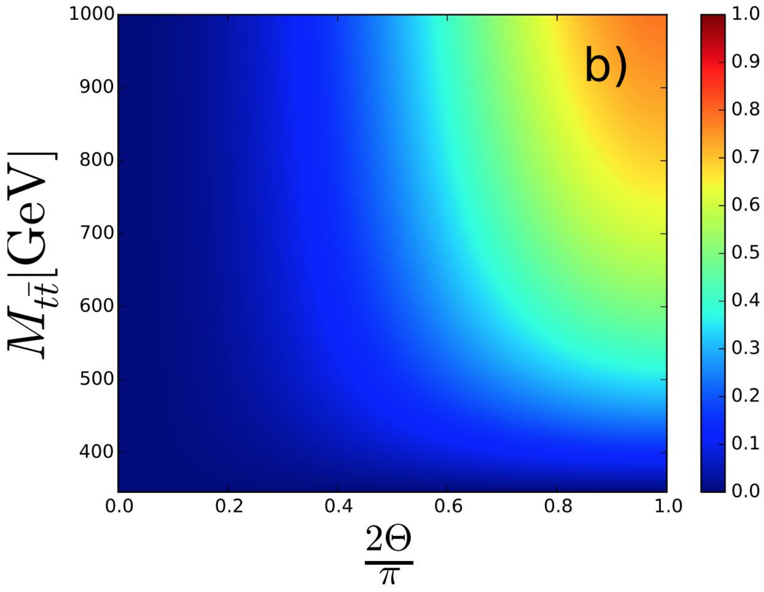
$C[\rho_{q\bar{q}}]$ →



Phenomenology

Afik, de Nova 2003.02280

- Total quantum state is a mixed state

$$\rho_{t\bar{t}} = p_{gg}\rho_{gg} + p_{q\bar{q}}\rho_{q\bar{q}}$$

- Concurrence in ttbar system is

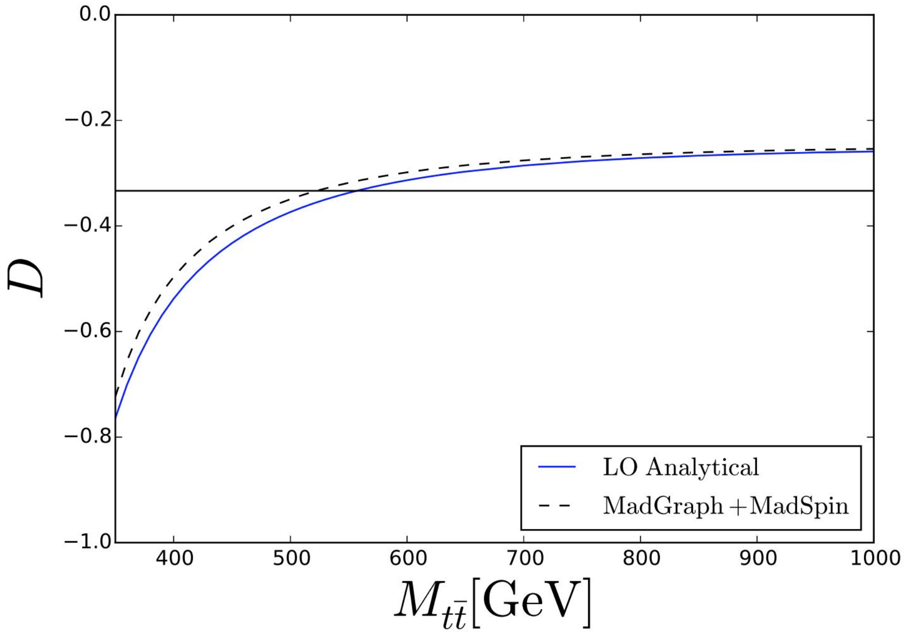
$$C[\rho_{\bar{t}t}] = \frac{-1 - 3D}{2}$$

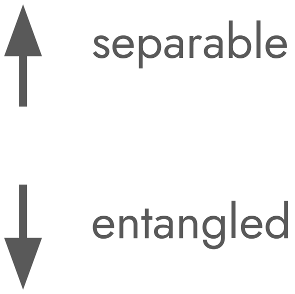
- D < -⅓ (entangled), D>-⅓ (separable), where D is
    - Angle between lepton and anti-lepton
    - Trace of spin correlation matrix

Phenomenology

Afik, de Nova 2003.02280

- Parton-level study



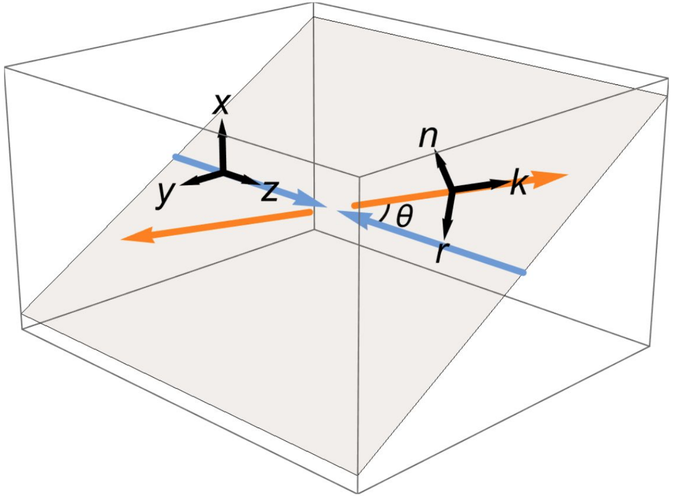


Phenomenology

Cheng, Han, ML [2311.09166](2311.09166)

Afik, de Nova [2203.05582](2203.05582)

- Spin quantization axis (basis) matters for quantum states



- Beamline basis (x, y, z) leads to quantum state
- Helicity basis (k, r, n) leads to fictitious state
- Fictitious states still show there **exists** entanglement but the measured value is not meaningful

Phenomenology

Fabbrichesi, Floreanini, Panizzo 2102.11883

Severi, Boschi, Maltoni, Sioli 2110.10112

- **Bell inequality violation** measured via spin correlation matrix

  $$m_1 + m_2 > 1$$

  The m_i are the 1st and 2nd eigenvalues of $C^T C$

  This is the optimal choice of $A_{1,2}$ and $B_{1,2}$

- Better to fix $A_{1,2}$ and $B_{1,2}$ rather than optimizing

  $$\sqrt{2}\,| - C_{rr} + C_{nn}| \leq 2,$$

- Studies found just less than 2σ for 3 ab^{-1}

Phenomenology

Aguilar-Saavedra, Casas 2205.00542

- Rather than measuring full spin correlation matrix and extracting the relevant entries, can select observable directly related to combination

$$\varphi_+ = \frac{1}{2}(\varphi_a + \varphi_b)\,, \quad \varphi_- = \frac{1}{2}(\varphi_a - \varphi_b)\,.$$

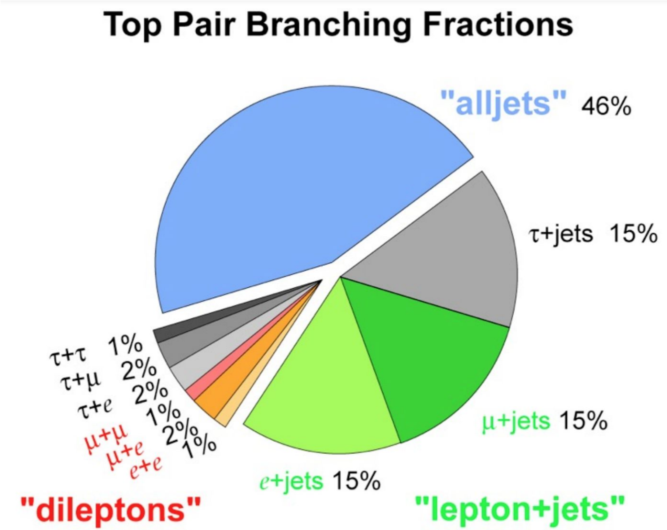
$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi_+ d\varphi_-} = \frac{1}{2\pi^2} + \frac{\alpha_a\alpha_b}{32}\left[\frac{C_{11}+C_{22}}{2}\cos 2\varphi_- + \frac{C_{11}-C_{22}}{2}\cos 2\varphi_+ + \frac{C_{12}+C_{21}}{2}\sin 2\varphi_+ + \frac{C_{21}-C_{12}}{2}\sin 2\varphi_-\right].$$

- Reduces uncertainty by ~30%

Semi-Leptonic Channel

Han, ML, Wu 2205.00542

- Leptonic channel benefits from lepton spin analyzing power
- Semi-leptonic channel benefits from more events



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optimal hadronic	0.64

Tweedie 1401.3021

Semi-Leptonic Channel

Han, ML, Wu [2205.00542](2205.00542)

- Measuring the concurrence

$$C_{\text{entangled}} \pm \Delta C$$

- Want to keep $C_{\text{entangled}}$ as large as possible and ΔC as small as possible

$$\Delta C_{\text{semi-leptonic}} = \sqrt{\frac{\text{BR}_{\text{leptonic}}}{\text{BR}_{\text{semi-leptonic}}}} \Delta C_{\text{leptonic}}$$

$$= (0.38) \Delta C_{\text{leptonic}}$$

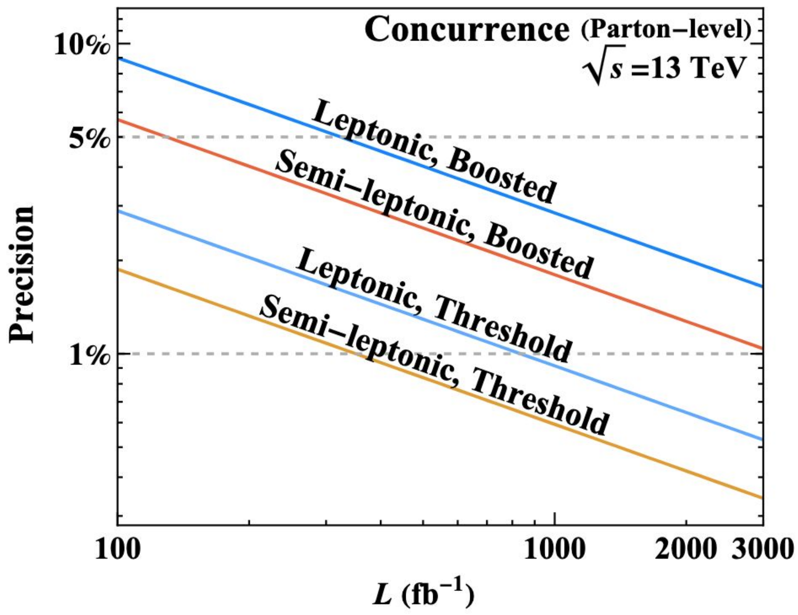
- But $C_{\text{entangled}}$ is lower for hadronic decay rather than leptonic decay

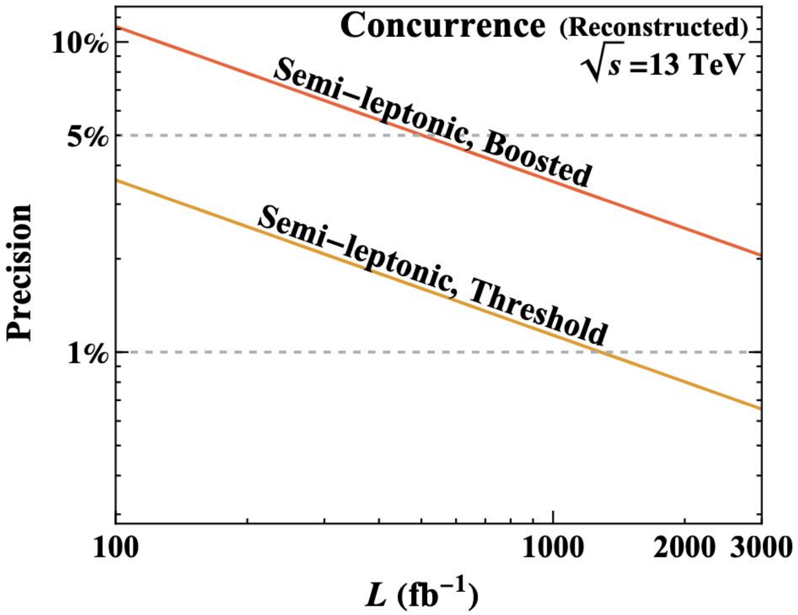
$$C_{\text{semi-leptonic}} = (0.64) C_{\text{leptonic}}$$

- Naively expect the semi-leptonic channel is **60% better**

Semi-Leptonic Channel

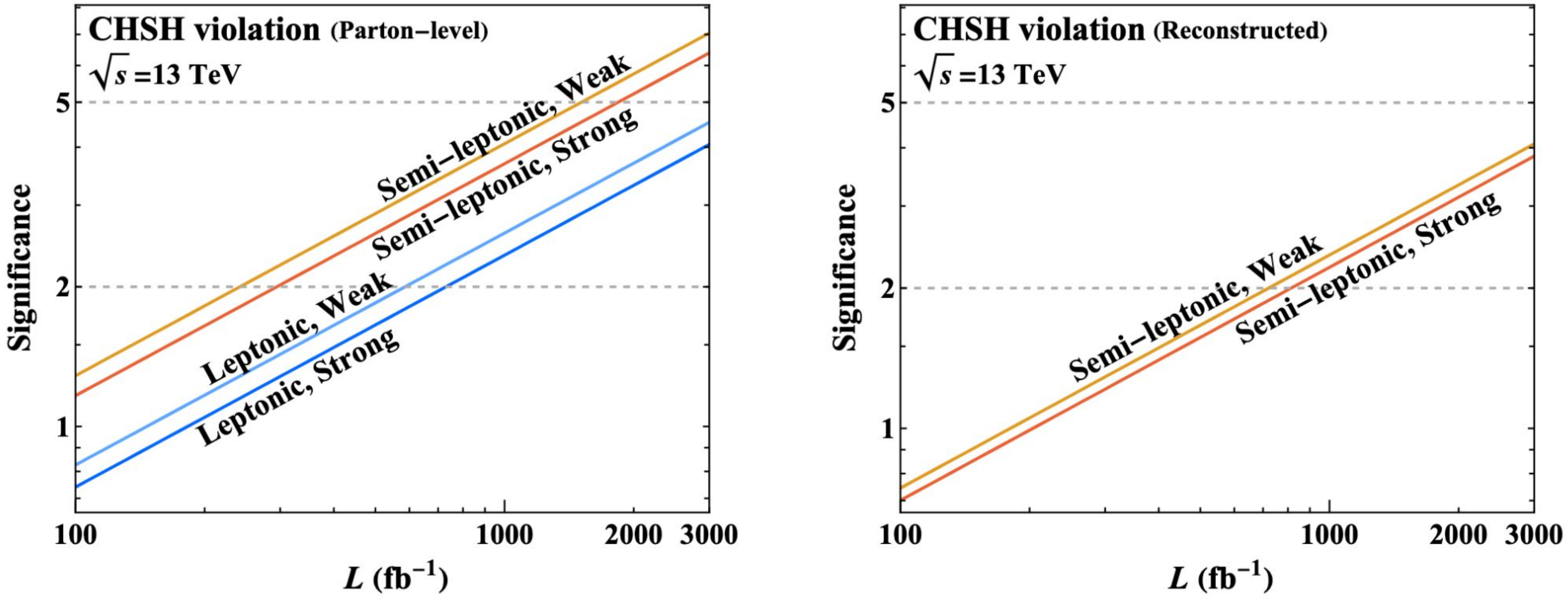
Han, ML, Wu [2205.00542]





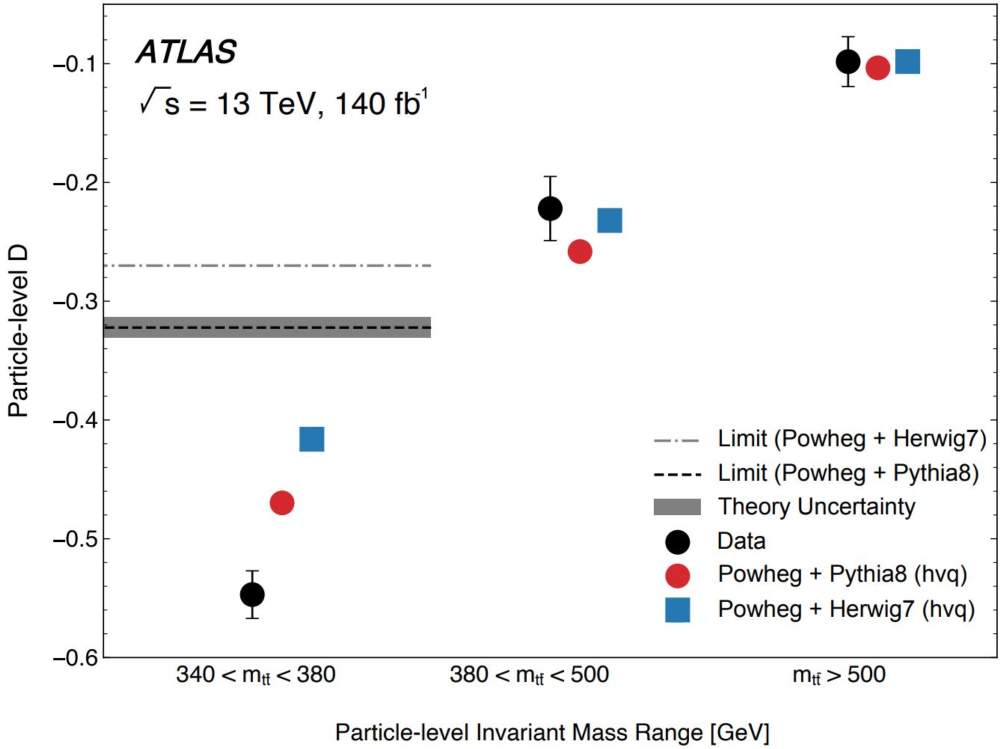
Semi-Leptonic Channel

Han, ML, Wu [2205.00542](2205.00542)

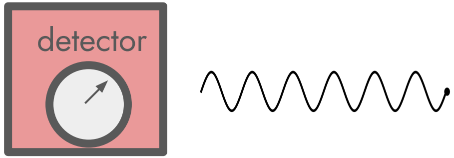


Leptonic Channel

ATLAS [2311.07288](2311.07288)

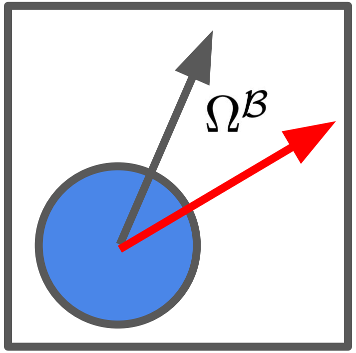


Summary

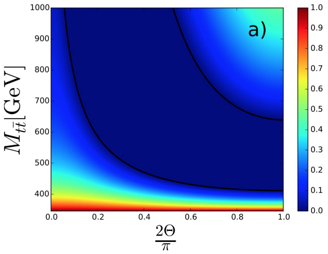


Big Picture

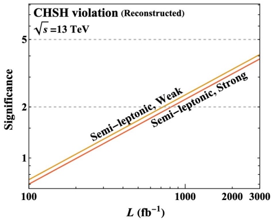
Quantum Review



Collider Review



Phenomenology

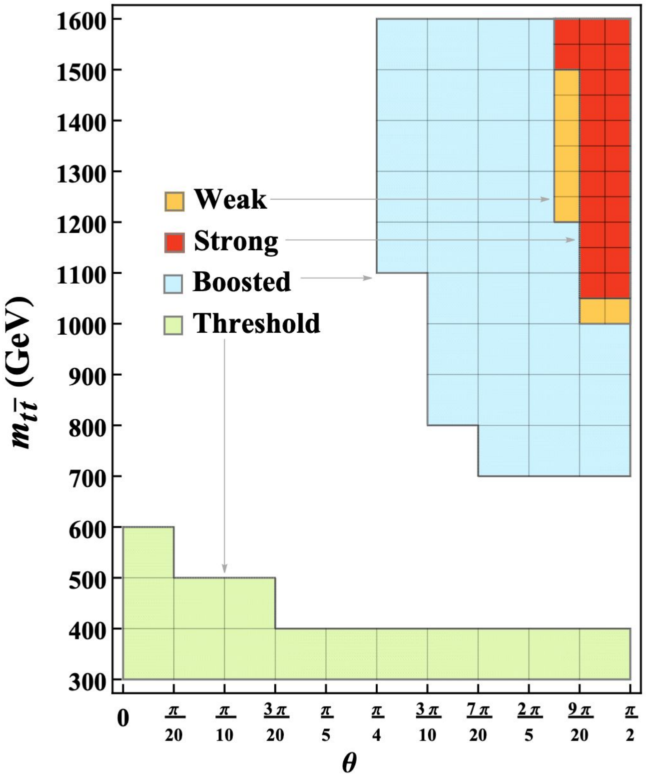


Semi-Leptonic Channel

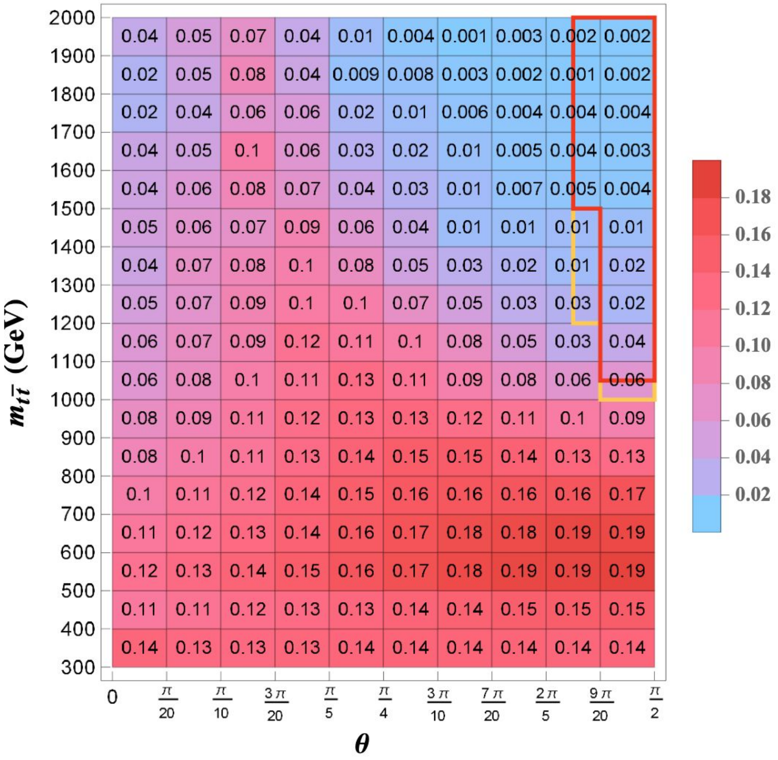
Outlook

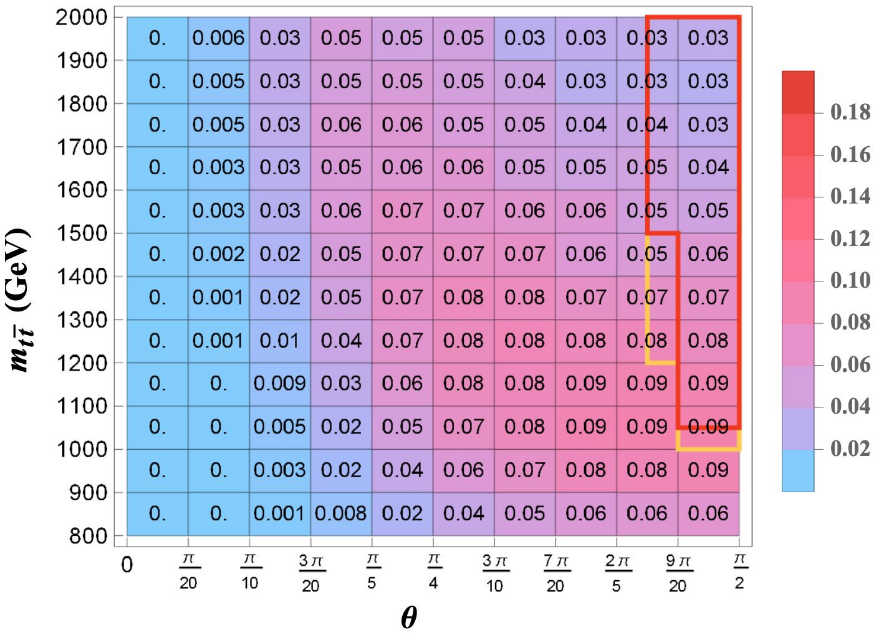
- ATLAS has measured entanglement in leptonic top-anti-top events
- Semi-leptonic should be more sensitive, but has not been measured yet
- Still awaiting CMS measurement
- (Experimental to do): Bell inequality measurements in top-anti-top
- (Experimental to do): Other channels: H to ZZ, etc.
- Other quantum correlations?
- Novel quantum experiments?

Back-Up: Signal Regions



Back-Up: Reconstruction Efficiency





Back-Up: Entanglement

Parton-level	Efficiency	$\epsilon N_{\text{parton}}$ (139 fb^{-1})	$2\mathcal{C}(\rho)$ (Individual)	$2\mathcal{C}(\rho)$ (Direct)	Precision
Threshold	0.16	1.26×10^6	0.518 ± 0.010	0.522 ± 0.008	1.6%
Boosted	0.13	1.15×10^5	0.576 ± 0.032	0.566 ± 0.027	4.8%

Reconstructed	N_{detected} (139 fb^{-1})	$2\mathcal{C}(\rho)$ (Individual)	$2\mathcal{C}(\rho)$ (Direct)	Precision
Threshold	1.26×10^6	0.523 ± 0.033	0.522 ± 0.016	3.0%
Boosted	1.15×10^5	0.549 ± 0.084	0.552 ± 0.052	9.5%

Back-Up: Bell Inequality Violation

Parton-level	Efficiency	$\epsilon N_{\text{parton}}$ (300 fb^{-1})	$B-\sqrt{2}$ (Individual)	$B-\sqrt{2}$ (Direct)	Significance (300 fb^{-1})	Significance (3000 fb^{-1})
Weak	0.080	6280	0.22 ± 0.11	0.22 ± 0.10	2.2σ	7.0σ
Strong	0.078	4127	0.26 ± 0.14	0.25 ± 0.12	2.0σ	6.4σ

Reconstructed	N_{detected} (300 fb^{-1})	$B-\sqrt{2}$ (Individual)	$B-\sqrt{2}$ (Direct)	Significance (300 fb^{-1})	Significance (3000 fb^{-1})
Weak	6280	0.23 ± 0.18	0.22 ± 0.22	1.3σ	4.1σ
Strong	4127	0.27 ± 0.22	0.25 ± 0.28	1.2σ	3.8σ