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Evanescent operators at the one-loop level

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Outline

1. Introduction
2. Origin of Evanescent operators
3. Application
4. Evanescent-free schemes
5. Summary

based on: 2208.10513, 2211.01379, 2401.16904
in collaboration with Marko Pesut and Zach Polonsky

Outline

Motivation

Matching

One-loop level

RGE running

At two-loop level

Basis changes

At one-loop

Outline

Example: Fierz transformations

Four-fermion (4F) operators

$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F}$

Example

$(\overline{q}_1^{\alpha}\gamma_{\mu}P_L q_2^{\beta})(\overline{q}_3^{\beta}\gamma^{\mu}P_R q_4^{\alpha}) = -2(\overline{q}_1 P_R q_4)(\overline{q}_3 P_L q_2)$

$d = 4 - 2\epsilon$

$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F} + E_{\mathcal{O}}$

Evanescent operators

Definition

$E_{\mathcal{O}} = \mathcal{O}_{4F} - \mathcal{F}\mathcal{O}_{4F}$

Evanescent

$E_{\mathcal{O}} \overset{d \to 4}{\to} 0$

Basis

$\{\mathcal{O}_j, E_i\}$

Evanescent operators: Complication

E_i

Finite contributions from one-loop insertions

Scheme dependence

ADMs and matching

Solution

Interpret finite contributions as one-loop shifts in $\mathcal{F}$

Traditional way

Basis

$\{\mathcal{O}_j, E_i\}$

Fierz identities

$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + E_i$

Finite contributions

Resulting from E_i

Novel way: One-loop Fierz identities

JA/Pesut: 2208.10513

Basis

$\{\mathcal{O}_j\}$

Fierz identities

$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + \frac{\alpha_s}{4\pi}\sum_j a_j \mathcal{O}_j$

One-loop shifts

Expressed in physical basis

Computation

One-loop corrections

$L\mathcal{O}, \quad L\mathcal{F}\mathcal{O}$

Taking difference

$L\mathcal{O} - L\mathcal{F}\mathcal{O} = LE$

Finite shifts

$LE = \frac{\alpha_s}{4\pi} \sum_i a_i \mathcal{O}_i$

Operators

Four-fermion operators

4q, SL, 4l

Dirac structures

vector, scalar, tensor

Colour

singlet and crossed

Four-fermi operators

JA/Pesut: 2208.10513

Basis

$\gamma_\mu P_A \otimes \gamma^\mu P_B\,, \quad P_A \otimes P_B\,, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \qquad A, B = L, R$

Contributions

QCD, QED

General scheme

$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho P_L = 4(4 - a_1 \epsilon) \gamma_\mu P_L \otimes \gamma^\mu P_L$

Dipole operators

JA/Pesut/Polonsky: 2211.01379

Basis

$$\gamma_\mu P_A \otimes \gamma^\mu P_B\,, \quad P_A \otimes P_B\,, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \qquad A, B = L, R$$

Contributions

QCD, QED

Dipoles

$$D^B_{q_1 q_2 G} = \frac{1}{g_s} m_q (\bar{q}_1 \sigma^{\mu\nu} P_B T^A q_2) G^A_{\mu\nu}$$

$$D^B_{f_1 f_2 \gamma} = \frac{1}{e} m_f (\bar{f}_1 \sigma^{\mu\nu} P_B f_2) F_{\mu\nu}$$

Outline

LQ matching

Scalar leptoquark

$$\mathcal{L}_{q\ell}^{LQ} = \bar{q}\left(\Gamma_L^S P_L + \Gamma_R^S P_R\right)\ell\,\Phi^* + \text{h.c.}$$

Matching

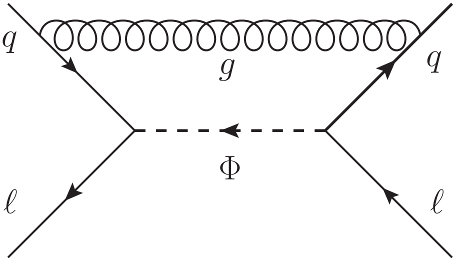
SL operators

QCD corrections

$\sim 10\ \%$ corrections

JA/Crivellin/Greub: 1811.08907

Matching: Example



Issue

Matching: LQ basis

$$\widetilde{O}_S^{AB} = (\overline{q} P_A \ell)(\overline{\ell} P_B q)$$
$$\widetilde{O}_T^{A} = (\overline{q} \sigma_{\mu\nu} P_A \ell)(\overline{\ell} \sigma^{\mu\nu} P_A q)$$

Running: SM basis

$$O_S^{AB} = (\overline{q} P_A q)(\overline{\ell} P_B \ell)$$
$$O_T^{A} = (\overline{q} \sigma_{\mu\nu} P_A q)(\overline{\ell} \sigma^{\mu\nu} P_A \ell)$$

Combine results

One-loop Fierz

Basis change

Tree-level Fierz

$$R_0 = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ -6 & \frac{1}{2} \end{pmatrix}$$

One-loop Fierz

$$R_1 = \begin{pmatrix} 0 & \frac{N_c^2-1}{16N_c} \\ \frac{7-7N_c^2}{N_c} & 0 \end{pmatrix}$$

$$\begin{pmatrix} O_S^{AA} \\ O_T^A \end{pmatrix} = R_0 \begin{pmatrix} \widetilde{O}_S^{AA} \\ \widetilde{O}_T^A \end{pmatrix} + \frac{\alpha_s}{4\pi} R_1 \begin{pmatrix} \widetilde{O}_S^{AA} \\ \widetilde{O}_T^A \end{pmatrix}$$

Outline

Issue: EV-to-physical mixing

Evanescent operators

Needed for mapping to physical basis

EV insertions

generate physical operators

Renormalization

Subtract physical contributions

Prescription method

JA/Pesut/Polonsky: 2401.16904

Prescription

$$(\Gamma_{\mathcal{S}_i}) \otimes (\Gamma_{\mathcal{S}_j}) = C_{ij}^{k\ell}(\Gamma_{\mathcal{O}_k}) \otimes (\Gamma_{\mathcal{O}_\ell})$$

Coefficients

$$C_{ij}^{k\ell} = a_{ij}^{k\ell} + \varepsilon\, b_{ij}^{k\ell}$$

$a_{ij}^{k\ell}$ = four-dim. part

$b_{ij}^{k\ell}$ = arbitrary

Usage

Iterative application of relations $\leftrightarrow$ no EV-to-physical mixing

Proof

Physical Operator Insertion

$Q = (\Gamma_1) \otimes (\Gamma_2)$

$\Rightarrow \langle Q \rangle^{(1)} \sim (\Gamma' \Gamma_1 \Gamma') \otimes (\Gamma' \Gamma_2 \Gamma') = C_i (\Gamma_i) \otimes (\Gamma_i)$

EV definition

$E := (\Gamma' \Gamma_1 \Gamma') \otimes (\Gamma' \Gamma_2 \Gamma') - C_i (\Gamma_i) \otimes (\Gamma_i)$

$\Rightarrow \langle E \rangle^{(1)} \sim (\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - C_i (\Gamma'' \Gamma_i \Gamma'') \otimes (\Gamma'' \Gamma_i \Gamma'') =$
$(\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - C_i C_i' (\Gamma_i) \otimes (\Gamma_i)$

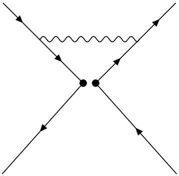
Vanishing mixing

$E' := (\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - K_i (\Gamma_i) \otimes (\Gamma_i)$

$\Rightarrow \langle E \rangle^{(1)} \sim \langle E' \rangle^{(1)} + (K_i - C_i C_i') (\Gamma_i) \otimes (\Gamma_i)$

$\rightarrow$ no mixing if $K_i = C_i C_i'$

Example: Scalar and Tensor 1L insertion

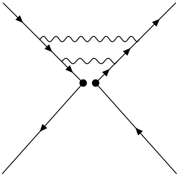


Prescription

$$(P_L\gamma^\mu\gamma^\nu)\otimes(\gamma_\nu\gamma_\mu P_L) = (4-2\epsilon)(P_L)\otimes(P_L)+(1+\epsilon b_{t1})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

$$(\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) =$$
$$(48+\epsilon b_{s2})(P_L)\otimes(P_L)+(12+\epsilon b_{t2})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

Scalar 2L insertion



More complicated structure

$$
\begin{aligned}
(P_L\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\gamma_\nu\gamma_\mu P_L) \\
&= (4-2\epsilon)(P_L\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha P_L) \\
&\quad + (1+\epsilon b_{t1})(\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta)\otimes(\sigma_{\mu\nu}\gamma_\beta\gamma_\alpha P_L) \\
&= \big(64-\{16+b_{s2}+48b_{t1}\}\epsilon\big)(P_L)\otimes(P_L) \\
&\quad + \big(16-\{2+16b_{t1}+b_{t2}\}\epsilon\big)(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)
\end{aligned}
$$

Advantages

Conceptually simpler

No insertions/renormalization of EVs

Independent of treatment of γ_5

Can be used in combination with HV

Algorithmic procedure

Automation

Results from literature

Two-Loop QCD ADM for $\Delta F = 1$ Herrlich/Nierste: hep-ph/9604330

four-quark operators

Two-Loop QCD ADM for $\Delta F = 2$ Herrlich/Nierste: hep-ph/9604330

Double-insertions from $\Delta F = 1$ operators

Various matching calculations

Fix scheme constants

Outline

Summary

Evanescent operators

Shifts in Fierz transformations

EV-free schemes

Prescription method

Simplifications

No EV renormalization, automation