Towards the HEFT-hedron: the complete set of positivity bounds on longitudinal gauge-Higgs scattering at NLO

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Motivation

1. More than a decade has passed since Higgs boson Discovery
2. No new particles seen in LHC ! New Physics soon or far in the future?
3. New Physics may come from precision measurements rather than new on-shell states.
4. Motivates us to understand the new Physics effects by utilizing standard EFT techniques to Standard Model(SM).
5. EFT Lagrangian built out of higher dimensional operators $\mathcal{O}_i$ s with coupling c_i s,

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda^{[\mathcal{O}_i]-4}} \mathcal{O}_i$$

6. Experimental goal: To constrain the space of the Wilson coefficients (WC), c_i s.

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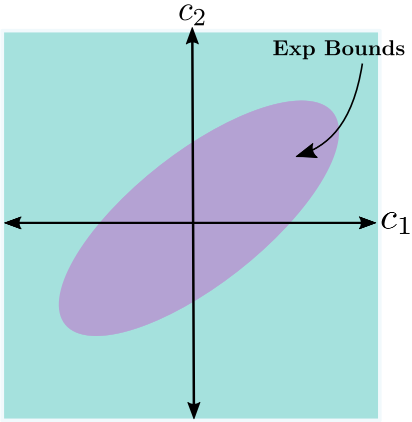
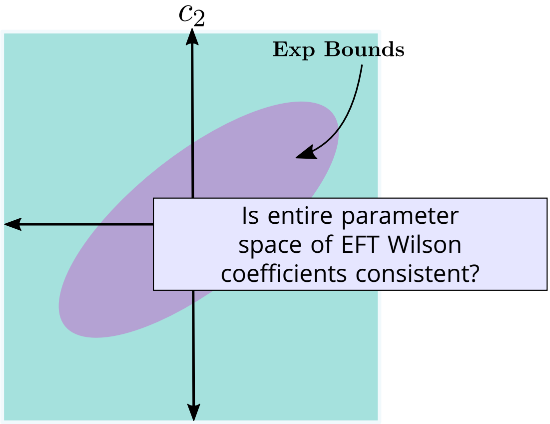
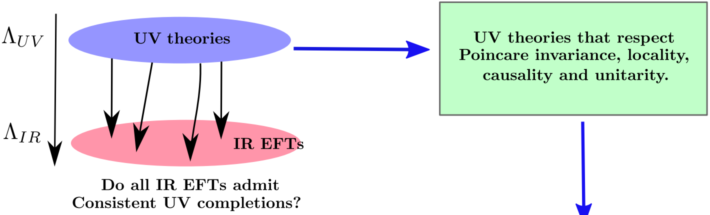
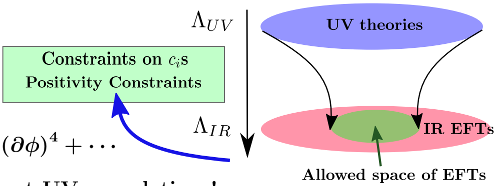


- Experimental goal: To constrain the space of the Wilson coefficients, c_i s.







Example:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \cdots$$

Only $c > 0$ admit consistent UV-completions!

Fig. Concept: S.Chattopadhyay

Ref. Pham, Truong'85; B. Ananthanarayan et al. '95
A. Adams et al. '06

$$M(s_*) = \frac{1}{2\pi i}\oint_{C_1}\frac{ds'\, M(s')}{(s'-s_*)} = \frac{1}{2\pi i}\oint_{C_2}\frac{ds'\, M(s')}{(s'-s_*)}$$

$$\frac{1}{2}\frac{\partial^2}{\partial s_*^2}M(s_*) = \frac{1}{2\pi i}\oint_{C_2}\frac{ds'\, M(s')}{(s'-s_*)^3}$$

$$= \frac{1}{\pi}\int_{4m^2}^{\infty} ds'\,\mathrm{Im}M(s')\left(\frac{1}{(s'-s_*)^3} + \frac{1}{(s'+s_*-4m^2)^3}\right) > 0$$

Froissart-Martin Bound:

$\lim_{s\to\infty}|\mathcal{M}(s, t\to 0)|/s^2 = 0$

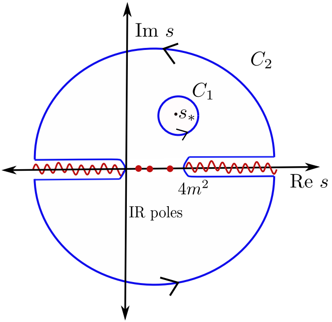


Figure: Analytic structure of $\mathcal{M}(s)$ in complex s plane.

Optical theorem:

$\mathrm{Im}M(s) = \sqrt{s(s-4m^2)}\,\sigma_T(s)$

$$\frac{1}{2}\frac{\partial^2}{\partial s^2}M(s,t)|_{t\to 0} = c > 0$$

Constrains WCs of irrelevant operators of EFTs that give rise to at least s^2 growth in $\mathcal{M}(s, t\to 0)$.

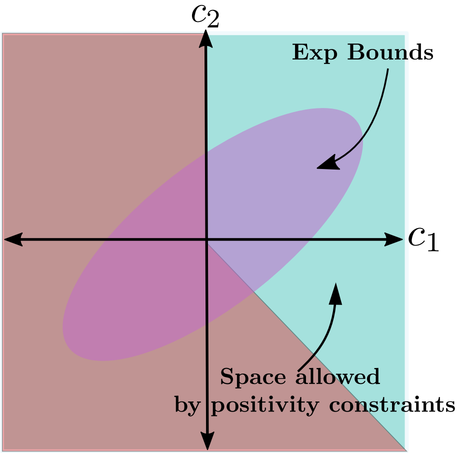


Figure: Positivity constraints puts theoretical priors in the space of WCs

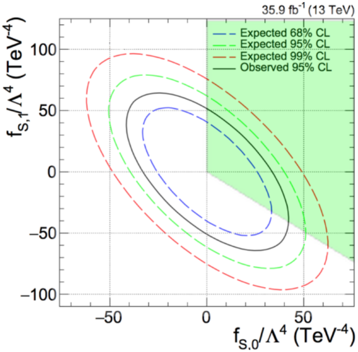


Figure: Positivity Bounds on WCs of SMEFT Dim-8 operators. Fig courtesy: Q. Bi et al. (2019)

- Positivity constraints for Dim-8 SMEFT operators are well studied.
- For Dim-8 bosonic SMEFT operators including aQGCs see Remmen, Rodd'19; Q. Bi, Zhang, Zhou'19, Zhang, Zhou'19, D. Ghosh et al.'22.
- For Dim-8 fermionic SMEFT operators see Remmen, Rodd'20.

- In this work, we explore positivity constraints on space of WCs of HEFT operators contributing to longitudinal gauge-Higgs scattering process such as the following processes,
  $V_L V_L \to V_L V_L$, $V_L V_L \to hh$, $V_L V_L \to V_L h$, $hh \to hV_L$ and $hh \to hh$ $\left(V \in \{W^{\pm}, Z\}\right)$.
- Most general parametrization of s^2 piece of these amplitudes given by HEFT, not SMEFT!

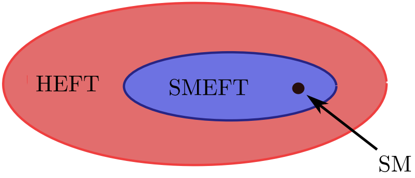


Figure: SMEFT $\subset$ HEFT for a given order of EFT expansion.

For UV perspective: Ref. Falkowski, Rattazzi '19; Alonso, Jenkins, Manohar '15, '16; Cohen, Craig, Lu, Sutherland '20

$\mathcal{O}_S$		CP
$\mathcal{O}_{S,0}$	$\left[(D_\mu H)^\dagger (D_\nu H)\right]\left[(D^\mu H)^\dagger (D^\nu H)\right]$	$+$
$\mathcal{O}_{S,1}$	$\left[(D_\mu H)^\dagger D^\mu H\right]^2$	$+$
$\mathcal{O}_{S,2}$	$\left[(D_\mu H)^\dagger (D_\nu H)\right]\left[(D^\nu H)^\dagger (D^\mu H)\right]$	$+$

Table: List of SMEFT dim-8 operators.

- In SMEFT at Dim-8, only 3 operators contribute to s^2-piece of gauge-Higgs scattering process.
- HEFT lagrangian contains 15 operators that give rise to s^2 pieces. (identified using Goldstone boson equivalence theorem.)
- These HEFT operators are more important than SMEFT as they already appear at NLO.

Type	$\mathcal{O}^{UhD^4}$	CP
VVVV	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2$	$+$
VVVV	$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle$	$+$
VVVV	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle$	$+$
VVVV	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \langle \mathbf{V}^\nu \mathbf{V}_\nu \rangle$	$+$
VVVV	$(\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle)^2$	$+$
VVVh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$i \langle \mathbf{T}\mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \frac{D^\nu h}{v}$	$+$
VVhh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \frac{h D^\mu D^\nu h}{v^2}$	$+$
VVhh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \frac{D_\nu h D^\nu h}{v^2}$	$+$
VVhh	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \frac{h D^\mu D^\nu h}{v^2}$	$+$
VVhh	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \frac{D_\nu h D^\nu h}{v^2}$	$+$
Vhhh	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \frac{h D^\nu h D_\nu D^\mu h}{v^3}$	$-$
hhhh	$\frac{1}{v^4} h^2 (D_\mu D_\nu h)(D^\mu D^\nu h)$	$+$

Table: List of HEFT operators at NLO giving rise to s^2 growth.

Complete set of positivity bounds for HEFT at NLO

- We will obtain positivity constraints on 15-dimensional space of these WCs.
- Dim-8 SMEFT is a 3-dimensional sub-space.
- Gauge-Higgs scattering is the only process that can grow as s^2 in HEFT at NLO.
- This is the complete set operators that are subject to positivity bounds.

Type	$\mathcal{O}^{UhD^4}$	CP
VVVV	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2$	$+$
VVVV	$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle$	$+$
VVVV	$\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle$	$+$
VVVV	$\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}^\mu \rangle \langle \mathbf{V}^\nu \mathbf{V}_\nu \rangle$	$+$
VVVV	$(\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}^\mu \rangle)^2$	$+$
VVVh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \langle \mathbf{TV}_\nu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{TV}^\mu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}^\mu \rangle \langle \mathbf{TV}_\nu \rangle \frac{D^\nu h}{v}$	$-$
VVVh	$i \langle \mathbf{TV}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{TV}^\mu \rangle \frac{D^\nu h}{v}$	$+$
VVhh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \frac{hD^\mu D^\nu h}{v^2}$	+
VVhh	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \frac{D_\nu h D^\nu h}{v^2}$	+
VVhh	$\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}_\nu \rangle \frac{hD^\mu D^\nu h}{v^2}$	+
VVhh	$\langle \mathbf{TV}_\mu \rangle \langle \mathbf{TV}^\mu \rangle \frac{D_\nu h D^\nu h}{v^2}$	+
Vhhh	$\langle \mathbf{TV}_\mu \rangle \frac{hD^\nu hD_\nu D^\mu h}{v^3}$	$-$
hhhh	$\frac{1}{v^4} h^2 (D_\mu D_\nu h)(D^\mu D^\nu h)$	+

Table: List of HEFT operators at NLO giving rise to s^2 growth.

- These 15 HEFT operators can be mapped to the following 15 anomalous couplings.
- So, constraints derived on WCs of 15 HEFT operators can be translated to space of these 15 anomalous couplings.[1]

$$\begin{aligned}
\Delta\mathcal{L}_{QGC} &= g^2 c^2_{\theta W}\left[\delta g^Q_{ZZ1} Z^\mu Z^\nu W^-_\mu W^+_\nu - \delta g^Q_{ZZ2} Z^\mu Z_\mu W^{-\nu} W^+_\nu\right] \\
&+ \frac{g^2}{2}\left[\delta g^Q_{WW1} W^{-\mu} W^{+\nu} W^-_\mu W^+_\nu - \delta g^Q_{WW2}\left(W^{-\mu} W^+_\mu\right)^2\right] + \frac{g^2}{4c^4_{\theta W}} h^Q_{ZZ}(Z^\mu Z_\mu)^2 \\
\Delta\mathcal{L}_{(\partial h)^2 V^2} &\supset g^{hh}_{Z1}\frac{g^2}{c^2_{\theta_w}}\frac{(\partial_\nu h)^2 Z_\mu Z^\mu}{v^2} + g^{hh}_{Z2}\frac{g^2}{c^2_{\theta_w}}\frac{\partial_\mu h \partial_\nu h}{2v^2} Z^\mu Z^\nu + g^{hh}_{W1} g^2 \frac{(\partial_\nu h)^2}{v^2} W^{+\mu} W^-_\mu \\
&+ g^{hh}_{W2} g^2 \frac{\partial_\mu h \partial_\nu h}{2v^2}(W^{+\mu} W^{-\nu} + h.c.) \\
\Delta\mathcal{L}^{hV^3} &\supset i g^{\partial h V^3}_{W1}\frac{g^3}{2c_{\theta W}}\frac{\partial_\mu h}{v} Z_\nu\left(W^+_\mu W^{-\nu} - h.c.\right) + g^{\partial h V^3}_{W2}\frac{g^3}{2c_{\theta W}}\frac{\partial_\mu h}{v} Z_\nu (W^+_\mu W^{-\nu} + h.c.) \\
&+ g^{\partial h V^3}_{W3}\frac{g^3}{2c_{\theta W}}\frac{\partial_\mu h}{v} Z_\mu W^+_\mu W^{-\mu} + g^{\partial h V^3}_{Z}\frac{g^3}{2c^3_{\theta W}}\frac{\partial_\mu h}{v} Z_\mu Z_\nu Z^\nu \\
\Delta\mathcal{L}^{h^3 V} &= g^{\partial h^3 V}\frac{g}{2c_{\theta W} v^3}\partial_\nu h \partial^\nu h \partial_\mu h Z^\mu, \qquad \Delta\mathcal{L}^{(\partial h)^4} = \frac{g^{(\partial h)^4}}{v^4}, \partial_\mu h \partial^\mu h \partial_\nu h \partial^\nu
\end{aligned}$$

[1] also TGCs.

Positivity bounds on HEFT at NLO

- **We calculate contributions from HEFT operators at NLO to longitudinal gauge-Higgs scattering amplitude.**
- **We present coefficients of s^2 terms in these amplitudes using the following matrix equation.**

$$\mathcal{K}_{ijkl} = \frac{v^4}{2}\frac{\partial^2 \mathcal{M}_{ijkl}(s, t \to 0)}{\partial s^2} =$$

	$\phi_1\phi_1$	$\phi_1\phi_2$	$\phi_1\phi_3$	$\phi_2\phi_1$	$\phi_2\phi_2$	$\phi_2\phi_3$	$\phi_3\phi_1$	$\phi_3\phi_2$	$\phi_3\phi_3$	$\phi_1 h$	$\phi_2 h$	$\phi_3 h$	$h\phi_1$	$h\phi_2$	$h\phi_3$	hh
$\phi_1\phi_1$	a_1	0	0	0	a_2	0	0	0	a_3	0	0	a_9	0	0	a_9	a_{12}
$\phi_1\phi_2$	0	a_4	0	a_2	0	0	0	0	0	0	0	$-a_{16}$	0	0	a_{16}	0
$\phi_1\phi_3$	0	0	a_6	0	0	0	a_3	0	0	a_{10}	0	0	a_9	a_{16}	0	0
$\phi_2\phi_1$	0	a_2	0	a_4	0	0	0	0	0	0	0	a_{16}	0	0	$-a_{16}$	0
$\phi_2\phi_2$	a_2	0	0	0	a_1	0	0	0	a_3	0	0	a_9	0	0	a_9	a_{12}
$\phi_2\phi_3$	0	0	0	0	0	a_6	0	a_3	0	0	a_{10}	0	$-a_{16}$	a_9	0	0
$\phi_3\phi_1$	0	0	a_3	0	0	0	a_6	0	0	a_9	a_{16}	0	a_{10}	0	0	0
$\phi_3\phi_2$	0	0	0	0	0	a_3	0	a_6	0	$-a_{16}$	a_9	0	0	a_{10}	0	0
$\phi_3\phi_3$	a_3	0	0	0	a_3	0	0	0	a_5	0	0	a_7	0	0	a_7	a_8
$\phi_1 h$	0	0	a_{10}	0	0	0	a_9	$-a_{16}$	0	a_{14}	0	0	a_{12}	0	0	0
$\phi_2 h$	0	0	0	0	0	a_{10}	a_{16}	a_9	0	0	a_{14}	0	0	a_{12}	0	0
$\phi_3 h$	a_9	$-a_{16}$	0	a_{16}	a_9	0	0	0	a_7	0	0	a_{13}	0	0	a_{15}	a_{11}
$h\phi_1$	0	0	a_9	0	0	$-a_{16}$	a_{10}	0	0	a_{12}	0	0	a_{14}	0	0	0
$h\phi_2$	0	0	a_{16}	0	0	a_9	0	a_{10}	0	0	a_{12}	0	0	a_{14}	0	0
$h\phi_3$	a_9	a_{16}	0	$-a_{16}$	a_9	0	0	0	a_7	0	0	a_{15}	0	0	a_{13}	a_{11}
hh	a_{12}	0	0	0	a_{12}	0	0	0	a_8	0	0	a_{11}	0	0	a_{11}	a_{15}

VVVV(aQGCs)

VVVh

VVVh

VVhh

Vhhh

hhhh

To derive the positivity constraints we utilize the following,

$$\alpha_i \beta_j \alpha_k^* \beta_l^* \mathcal{K}_{ijkl} > 0 \quad \text{for } \forall \alpha, \beta$$

See for example Remmen and Rodd'19,'20.

We extract positivity constraints on the WCs from the positive-definiteness of the matrix,

$$(\gamma_\beta)_{ij} = \beta_j \beta_l^* \mathcal{K}_{ijkl} \succ 0 \quad \text{for } \forall \beta$$

For example,with, $\beta_2, \beta_3 = 0,$ a 2×2 **principle sub-matrix of** γ_β **is given by,**

$$\begin{bmatrix} a_1|\beta_1|^2 + a_{11}|\beta_4|^2 & 2a_7\Re(\beta_1\beta_4^*) \\ 2a_7\Re(\beta_1\beta_4^*) & a_6|\beta_1|^2 + a_{13}|\beta_4|^2 \end{bmatrix}$$

Only CP-even WCs appear in the main diagonal.

All CP-odd WCs reside in the off-diagonal.

Positive definiteness of this matrix, gives us the following constraints,

$$a_1,\ a_6,\ a_{11},\ a_{13} > 0$$
$$4a_7^2 < \left(\sqrt{a_6 a_{11}} + \sqrt{a_1 a_{13}}\right)^2$$

In particular, if the WCs of the RHS is zero,then the WCs of LHS must vanish.

First set of positivity constraints: Some specific linear combinations of WCs of CP-even operators should have a definite sign.

$$c_1 + c_2 > 0$$
$$c_2 > 0$$
$$4(c_1 + c_2) + 2(c_3 + c_4) + c_5 > 0$$
$$4c_2 + c_3 > 0$$
$$c_{10} < 0$$
$$2c_{10} + c_{12} < 0$$
$$c_{15} > 0$$

Second set of positivity constraints: magnitudes of some WCs - including all CP-violating ones - must be smaller than product of CP-even WCs.

All CP-odd, all $VVVh$ and $Vhhh$ anomalous coupling

$$4a_3^2 < \left(a_6 + \sqrt{a_1 a_5}\right)^2$$
$$a_9^2 < |a_{11}| a_2$$
$$a_{12}^2 < |a_{13}| a_5$$
$$a_{15}^2 < |a_{13}| a_{16}$$
$$4a_7^2 < \left(\sqrt{a_6 a_{11}} + \sqrt{a_1 a_{13}}\right)^2$$
$$4a_{10}^2 < \left(\sqrt{a_6 a_{11}} + \sqrt{a_4 a_{13}}\right)^2$$
$$4a_8^2 < \left(a_{11} + \sqrt{a_1 a_{16}}\right)^2$$
$$\left(a_{12} + a_{15} + 2a_{14}\right)^2 < (a_5 + a_{13} + 2a_{12})(a_{13} + a_{16} + 2a_{15})$$

Using correlations between SMEFT couplings the constraints can be projected into the 3D SMEFT parameter space,reproducing the well-known bounds on SMEFT WCs[1].

Magnitude of WCs of all CP-odd operators are bounded by CP-even WCs.

a_i s are linear combinations of WCs, c_i s.

$$a_1 = 16(c_1 + c_2) \quad a_2 = 4(2c_1 + c_2) \quad a_3 = 8c_1 + 2c_2 + c_3 + 4c_4$$
$$a_4 = 8c_2 \quad a_5 = 16(c_1 + c_2) + 8(c_3 + c_4) + 4c_5 \quad a_6 = 2(4c_2 + c_3)$$
$$a_7 = c_6 + c_7/2 \quad a_8 = 2c_{11} \quad a_9 = c_7 \quad a_{10} = -c_8/4 \quad a_{11} = -2c_{10}$$
$$a_{12} = 2\left(c_6 + c_7 + c_9/2\right) \quad a_{13} = -\left(2c_{10} + c_{12}\right) \quad a_{14} = \left(2c_{11} + c_{13}\right)$$
$$a_{15} = -c_{14}/2 \quad a_{16} = 4c_{15}$$

[1]Remmen and Rodd'2019 and Q.Bi et al.'20.

DC, S.Chattopadhyay, R.S.Gupta (in prep)

Positivity bounds on HEFT operators at NLO

First set of positivity constraints: Some specific linear combinations of WCs of CP-even operators should have a definite sign.

$$c_1 + c_2 > 0$$
$$c_2 > 0$$
$$4(c_1 + c_2) + 2(c_3 + c_4) + c_5 > 0$$
$$4c_2 + c_3 > 0$$
$$c_{10} < 0$$
$$2c_{10} + c_{12} < 0$$
$$c_{15} > 0$$

Second set of positivity constraints: magnitudes of some WCs - including all CP-violating ones - must be smaller than product of CP-even WCs.

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$$4a_3^2 < \left(a_6 + \sqrt{a_1 a_5}\right)^2$$
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$$a_{12}^2 < |a_{13}| a_5$$
$$a_{15}^2 < |a_{13}| a_{16}$$
$$4a_7^2 < \left(\sqrt{a_6 a_{11}} + \sqrt{a_1 a_{13}}\right)^2$$
$$4a_{10}^2 < \left(\sqrt{a_6 a_{11}} + \sqrt{a_4 a_{13}}\right)^2$$
$$4a_8^2 < \left(a_{11} + \sqrt{a_1 a_{16}}\right)^2$$
$$\left(a_{12} + a_{15} + 2a_{14}\right)^2 < \left(a_5 + a_{13} + 2a_{12}\right)\left(a_{13} + a_{16} + 2a_{15}\right)$$

Only $\approx 5\%$ of EFT parameter space are consistent with this bounds.

$\approx 20\%$ of 5D EFT parameter space for aQGCs within exp bounds are valid.

a_i s are linear combinations of WCs, c_i s.

$a_1 = 16(c_1 + c_2)$ $\quad a_2 = 4(2c_1 + c_2)$ $\quad a_3 = 8c_1 + 2c_2 + c_3 + 4c_4$

$a_4 = 8c_2$ $\quad a_5 = 16(c_1 + c_2) + 8(c_3 + c_4) + 4c_5$ $\quad a_6 = 2(4c_2 + c_3)$

$a_7 = c_6 + c_7/2$ $\quad a_8 = 2c_{11}$ $\quad a_9 = c_7$ $\quad a_{10} = -c_8/4$ $\quad a_{11} = -2c_{10}$

$a_{12} = 2\left(c_6 + c_7 + c_9/2\right)$ $\quad a_{13} = -\left(2c_{10} + c_{12}\right)$ $\quad a_{14} = \left(2c_{11} + c_{13}\right)$

$a_{15} = -c_{14}/2$ $\quad a_{16} = 4c_{15}$

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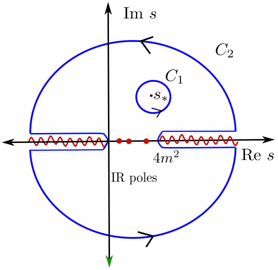
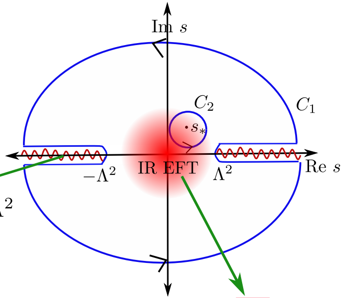


Substracting IR poles
and branch cuts for $s < \Lambda^2$



UV spectral density $s > \Lambda^2$

$\rho_l^{ijkl}(s')$ [1]

su**-symmetric fixed-**t **dispersion relation**

$\mathcal{M}^{EFT}_{ijkl}(s,t) = \sum_{m,n=0} \tilde{c}^{m,n}_{ijkl} v^m t^n$

$$\tilde{c}^{m,n}_{ijkl} = 16 \sum_l (2l+1) \int_{\Lambda^2}^{\infty} \mathrm{d}s' \left[\rho_l^{ijkl}(s') + (-1)^m \rho_l^{ilkj}(s')\right] \sum_{p=0}^{n} \frac{L_l^p H_{m+1}^{n-p}}{s'^{m+n+1}}$$

Linear combination of WCs of EFT at tree level

express IR observables in terms of UV spectral densities

[1] $\mathrm{Im}\mathcal{M}^{UV}_{ijkl}(\mu,\, t) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l \left(1 + \frac{2t}{\mu}\right) \rho_l^{ijkl}(\mu)$

Ref: Q.Chen et al.'23

$$\mathcal{M}^{EFT}_{ijkl}(s,t) = \sum_{m,n=0} \tilde{c}^{m,n}_{ijkl} v^m t^n$$

$$\tilde{c}^{m,n}_{ijkl} = 16 \sum_l (2l+1) \int_{\Lambda^2}^{\infty} \mathrm{d}s' \left[\rho_l^{ijkl}(s') + (-1)^m \rho_l^{ilkj}(s')\right] \sum_{p=0}^{n} \frac{L_l^p H_{m+1}^{n-p}}{s'^{m+n+1}}$$

st-crossing

$$\mathcal{M}_{ijkl}(s,t) = \mathcal{M}_{ikjl}(t,s)$$

substitute $\tilde{c}^{m,n}_{ijkl}$ s in terms of ρ_l^{ijkl} s

$$\mathcal{F}(\tilde{c}^{m,n}_{ijkl}, \tilde{c}^{m,n}_{ikjl}) = 0$$

subject to Unitarity constraints

$$16 \sum_l (2l+1) \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s'}{(s')^{r+4}} \left(\tilde{\mathcal{F}}_{r,i_r}\left(\rho_l^{ijkl}(s'), \rho_l^{ikjl}(s'), \rho_l^{ilkj}(s')\right)\right) = 0$$

null-constraints on $\rho_l^{ijkl}(s')$

For example:

$$0 \le \rho_l^{iiii} \le 2$$

$$0 \le \rho_l^{ijij} \le \frac{1}{2}$$

$$\left|\rho_l^{iijj}\right| \le 1 - \left|1 - \frac{(\rho_l^{iiii} + \rho_l^{jjjj})}{2}\right|$$

$$\ldots$$

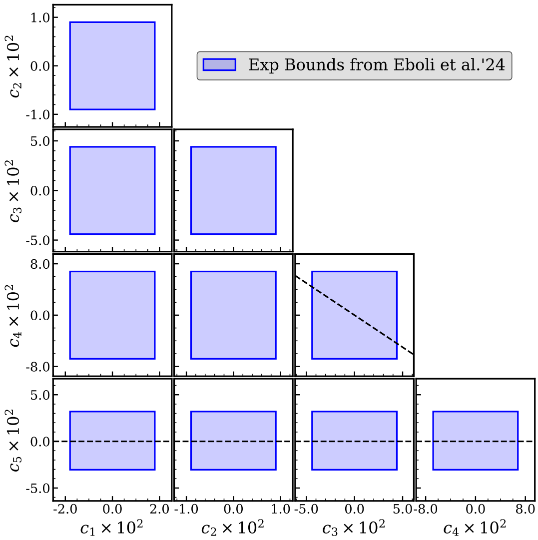
has been used to derive positivity constraints thus far

We obtain double-sided bounds on $\tilde{c}^{m=2,n=0}_{ijkl}$ s by varying ρ_l^{ijkl} s subject to unitarity and null constraints.

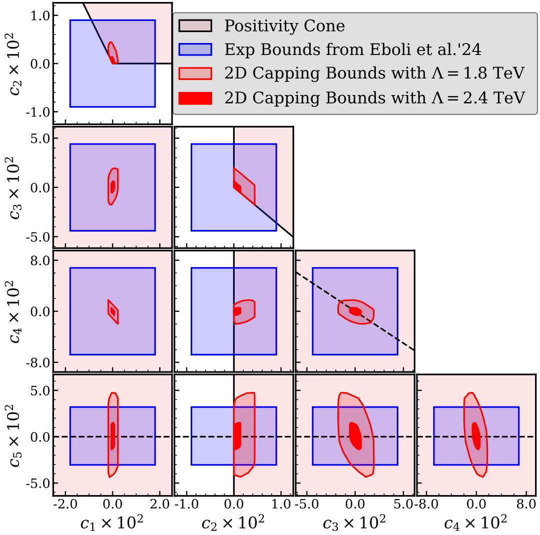
implies double-sided bounds on WCs of IR EFT operators.

implemented numerically by casting into a LPP.

Ref: Q.Chen et al.'23

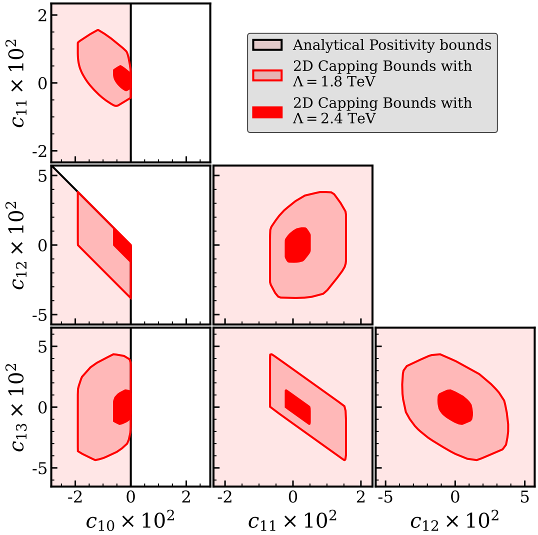


- Experimental Bounds: Based on Eboli et al. (2024). Process: $pp \to VVjj$ (VBS).

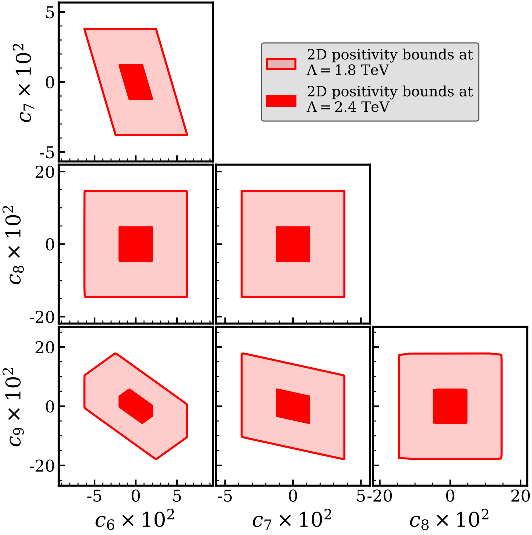


- Experimental Bounds: Based on Eboli et al. (2024). Process: $pp \to VVjj$ (VBS).
- Positivity Bounds: Exclude significant regions of the EFT parameter space.
- Capping Bounds: Found to be stronger than experimental limits for certain c_i values.
- Can obtain SMEFT bounds by projecting to SMEFT planes (dashed lines).

Bounds on VVhh



Bounds on VVVh



- Exp bounds on WCs for $VV \to VV$ and $VV \to hh$ processes provide indirect constraints on WCs for $VV \to Vh$ processes.

$$Vhhh: \ |c_{14}| < 0.0356$$

$$hhhh: \ 0 < c_{15} < 0.0066$$

$$\text{for } \Lambda = 2.4 \text{ TeV}$$

Summary and Outlook

- Poincare invariance + locality + causality + unitarity leads to positivity constraints and reduces allowed IR-parameter space significantly.
- we explore positivity constraints on space of WCs of HEFT operators contributing to longitudinal gauge-Higgs scattering process such as the following processes,
  $V_L V_L \to V_L V_L$, $V_L V_L \to hh$, $V_L V_L \to V_L h$, $hh \to hV_L$ and $hh \to hh$ $\left(V \in \{W^{\pm}, Z\}\right)$.
- $HEFT \supset SMEFT$, thus our bounds are more general than SMEFT.
- We derive positivity constraints on 15-D space of WCs of HEFT operators that contributes to these process.
  $\approx$ 20% of (5-dim) EFT parameter space for aQGCs within the Exp. bounds is valid.
- Utilizing null constraints and unitarity, we cap the positivity cone, obtaining double-sided bounds on these 15 WCs.
  In some cases, these bounds are tighter than the current experimental limits.