Quantifying EFT Truncation Uncertainties at the LHC

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The Problem

- EFT = low energy expansion

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{BSM}}$$

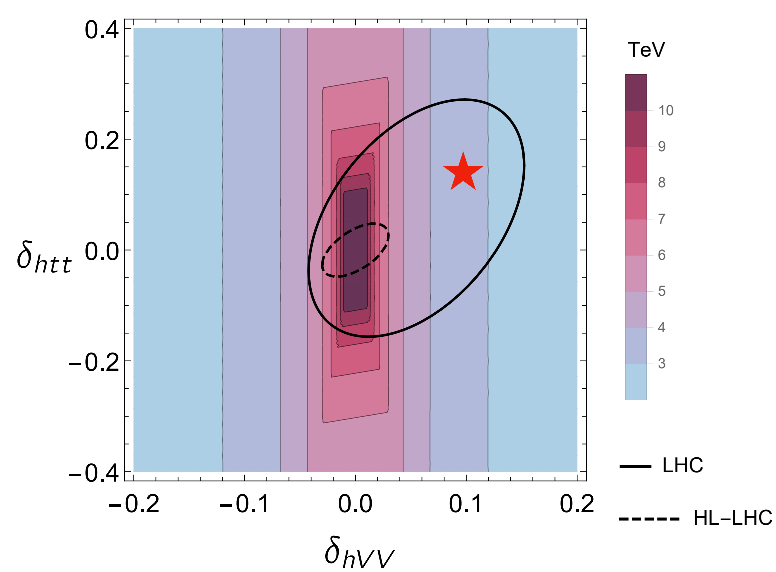
$$\mathcal{M}_{\text{BSM}} = \mathcal{M}_6 + \mathcal{M}_8 + \cdots \qquad \mathcal{M}_n \sim \frac{E^n}{\Lambda^n}$$

- $\mathcal{M}_{\text{BSM}}$ grows with energy

  $\Rightarrow$ signals at high energy

  $\Rightarrow$ EFT breaks down at high energy

- No guaranteed scale separation at LHC



Abu-Ajamieh, Chang, Chen, ML (2020)

Statement of Principles

- Constraining coefficients in a truncated expansion requires assumptions about the size of neglected terms

- Priors should be explicit and have a clear physical meaning

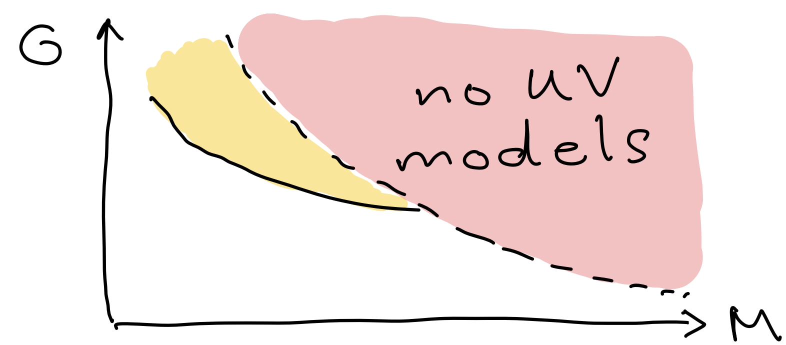
- Truncation error is a theory uncertainty that should be treated quantitatively

Outline of Proposal

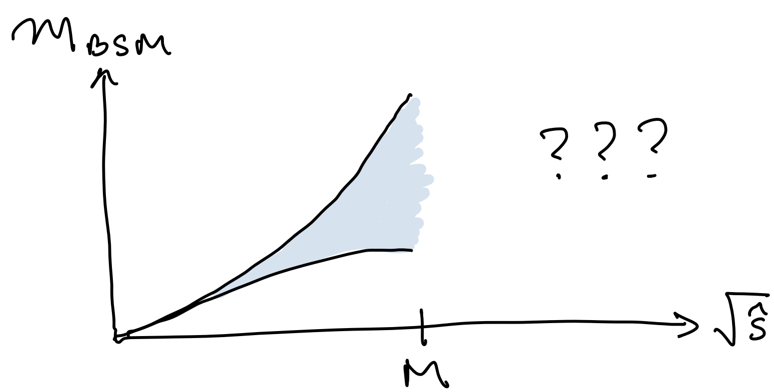
$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + G(\bar{\psi}\psi)^2$

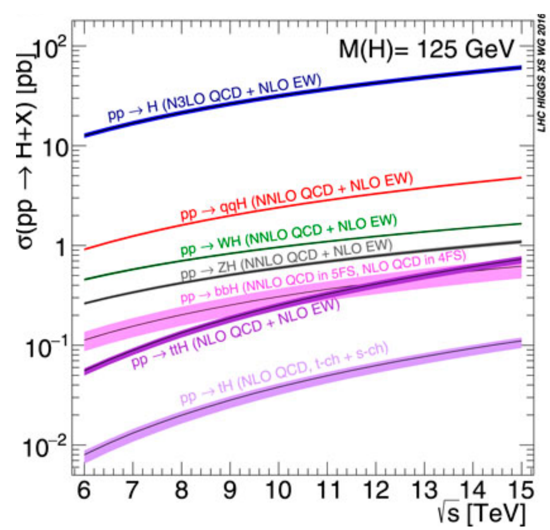
M = energy scale where EFT breaks down

Constrain G for given M:



Truncation uncertainty = theory uncertainty:

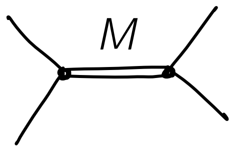




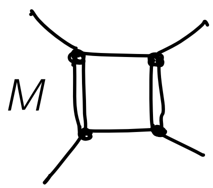
EFT Corrections

$$\mathcal{L}_{\mathrm{BSM}} = G(\bar{\psi}\psi)^2$$

UV complete models will generate EFT operators + 'descendants'



$$= \underbrace{-\frac{g^2}{M^2}}_{=\,G}(\bar{u}u)^2\left[1 + \underbrace{\frac{\hat{s}}{M^2} + \frac{\hat{s}^2}{M^4} + \cdots}_{\text{descendants}}\right]$$



$$= \underbrace{\frac{g^2}{16\pi^2 M^2}}_{=\,G}(\bar{u}u)^2\left[1 + \underbrace{c_1\frac{\hat{s}}{M^2} + c_2\frac{\hat{t}}{M^2} + c_3\frac{\hat{s}\hat{t}}{M^4} + \cdots}_{\text{descendants}}\right]$$

M = mass of heavy particle

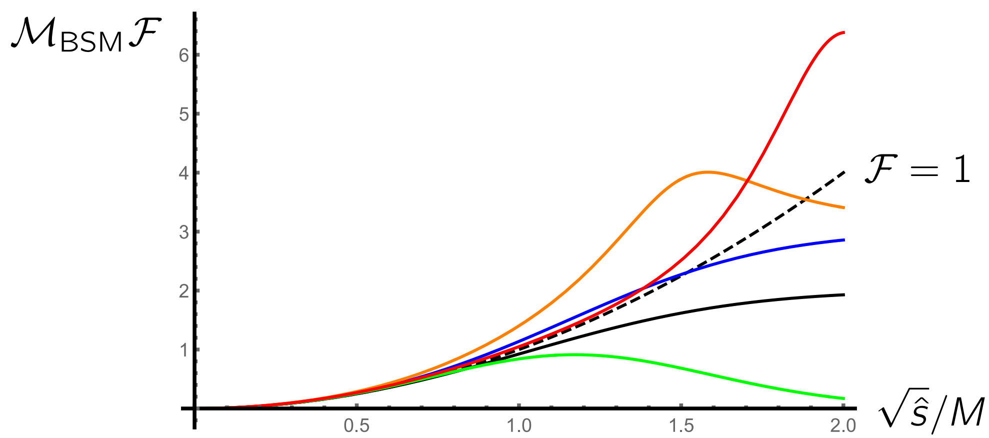
$$c_1, c_2, c_3, \ldots \sim 1$$

Use descendants to parameterize higher derivative effects in EFT

Form Factors

EFT model must make predictions for $\hat{s} > M$

$$\mathcal{M}_{\rm BSM}(\hat{s}, \hat{t}) \to \mathcal{M}_{\rm BSM}(\hat{s}, \hat{t})\mathcal{F}(x, y) \qquad x = \frac{\hat{s}}{M^2} \quad y = \frac{\hat{t}}{M^2} \quad z = \frac{\hat{u}}{M^2}$$



Note: events can be generated by reweighting

- $x, y, z \ll 1$: descendants

  $$\mathcal{F}(x, y) = 1 + c_1 x + c_2 y + c_3 xy + \cdots$$

- $x, y, z \sim 1$: general behavior

- $x, y, z \gg 1$: $\mathcal{M}_{\rm BSM}\mathcal{F} \to$ constant (scale invariant)

Form Factors

$$\mathcal{F}(x,y,z) = \left(\frac{F(x)}{x}\right)^{\alpha}\Big[1 + c_1 F(x) + c_2 F(y) + c_3 F(x)F(y) + \cdots$$
$$+ \text{resonances}\Big]$$

$$F(x) = \begin{cases} x & x \ll 1, \\ 1 & x \gg 1 \end{cases}$$

- $c_1, c_2, \ldots$ parameterize theory uncertainty
- Treat $c_1, c_2, \ldots$ as nuisance parameters in likelihood analysis

$$L_{\text{EFT}} = \underbrace{P(\boldsymbol{\nu})}_{\text{theory prior}} P(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\nu})$$

$\boldsymbol{x}$ = data

$\boldsymbol{\theta} = (G, M)$

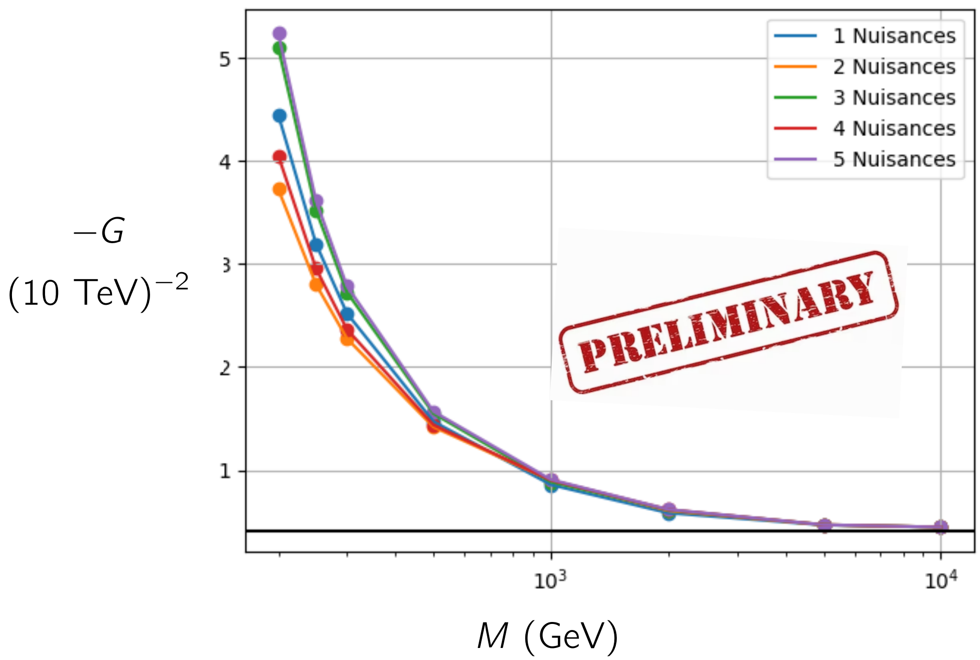
$\boldsymbol{\nu}$ = nuisance parameters

Results

Toy model: $\mathcal{L} = \mathcal{L}_{\text{QED}} + G(u_L^\dagger \gamma^\mu u_L)(e_R^\dagger \gamma_\mu e_R)$

Exclusion with data = SM

LHC with 100 fb^{-1}



- Nuisance parameters weaken the bound

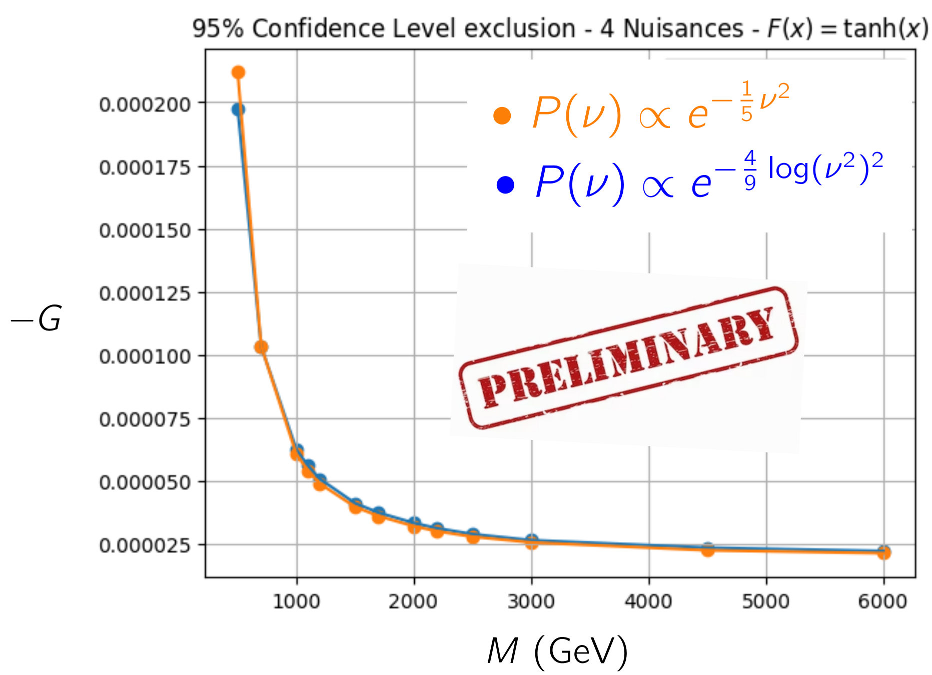
  (EFT prediction for $\hat{s} \gtrsim M^2$ has high uncertainty)

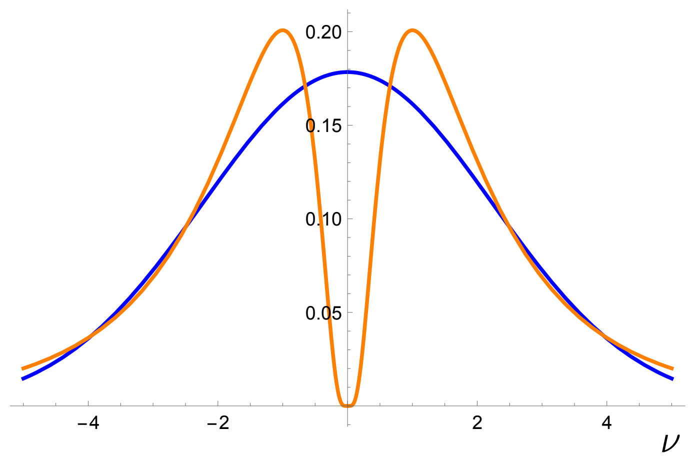
- Converges with few nuisance parameters

Nuisance Prior

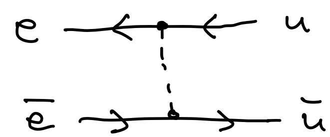
Results are insensitive to $P(\nu)$ if we normalize tail of distribution

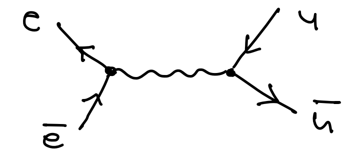
$\mathrm{Prob}(c > 3) = 0.05$

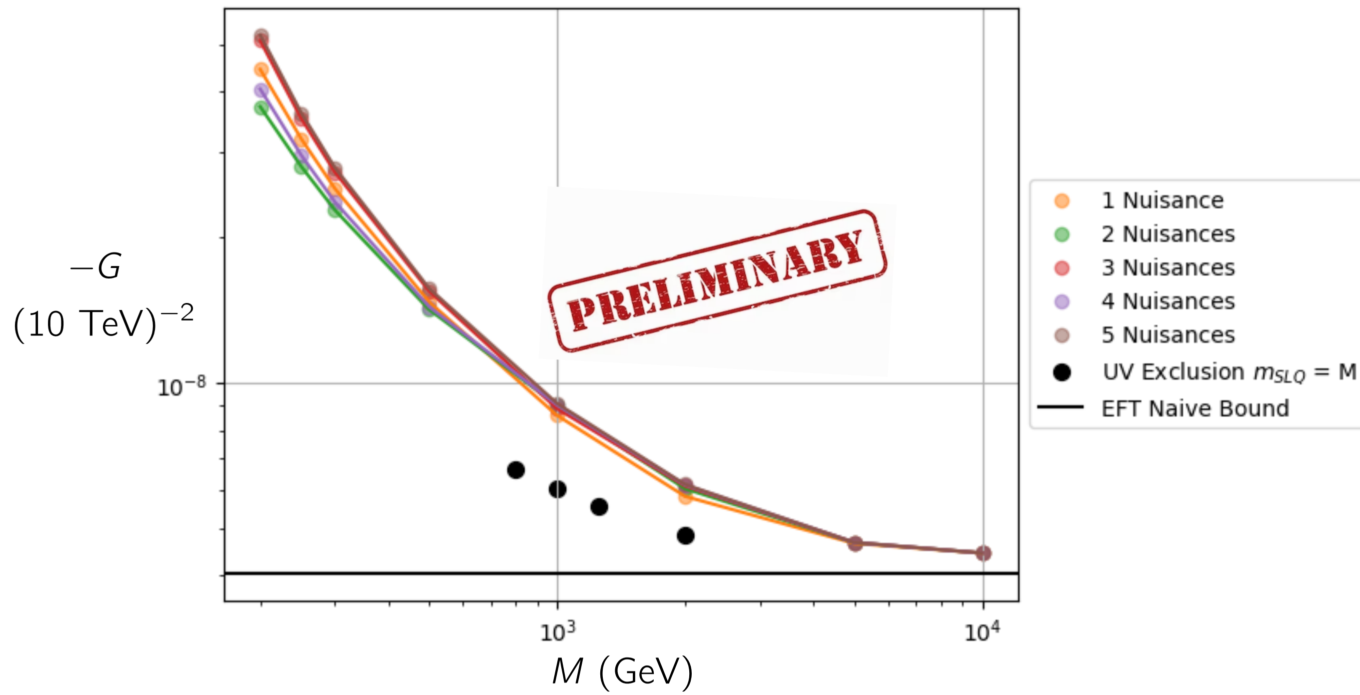




Comparison with UV Models

Scalar leptoquark model: 

Z' model: 

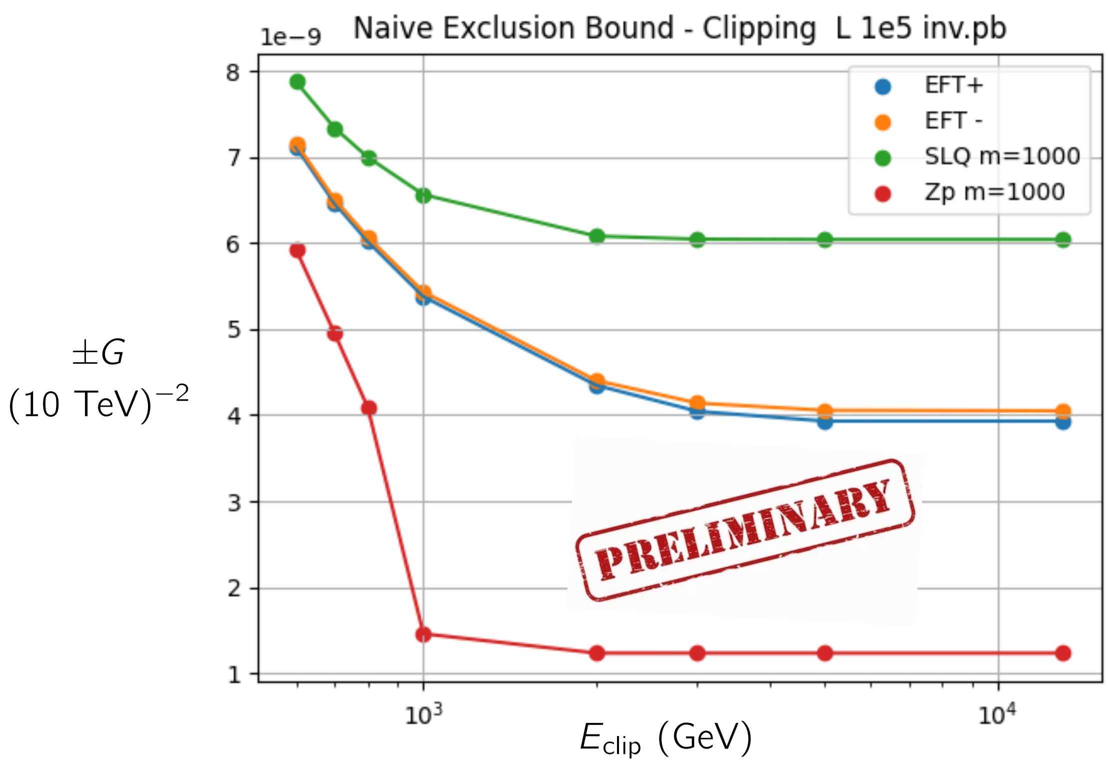


- Naive EFT bound is *stronger* than bound for leptoquark model (predicts high energy growth for $\hat{s} \gtrsim m_{\rm SLQ}^2$)
- EFT with theory errors is more conservative approaches UV model bound for $M = m_{\rm SLQ} \to \infty$

Data Clipping?

Ignore data with $\sqrt{s} > E_{\text{clip}}$

Compare clipped EFT exclusion with exclusion for UV models



Clipped EFT bound is still stronger than bound for leptoquark model...

BSM Discovery

Perform search optimized for SM exclusion

$$\text{likelihood ratio} = \frac{L_{\text{EFT}}(\boldsymbol{x}|\boldsymbol{\theta}, \boldsymbol{\nu})}{L_{\text{SM}}}$$

Nuisance parameters $\Rightarrow$ weaker bound

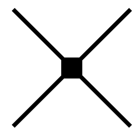
Better description of anomalous data $\Rightarrow$ stronger bound

Look Everywhere!

The method presented here can be used to test *any* BSM operator

Classify BSM operators using on-shell techniques:

$$\Delta\mathcal{L} = \sum_{\mathcal{O}} G_{\mathcal{O}}\mathcal{O} \qquad \longleftrightarrow$$



$$\Delta\mathcal{M}_{\text{tree}} = \sum_{\mathcal{O}=\text{primary}} G_{\mathcal{O}}\mathcal{M}_{\mathcal{O}}\left[1 + c_1\frac{s}{M^2} + c_2\frac{t}{M^2} + \cdots\right]$$

Finitely many primary operators with ≤ 4 external particles
$\Rightarrow$ complete classification of all BSM operators

G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi, Y. Weiss, arXiv:2008.09652
S. Chang, M. Chen, D. Liu, ML, arXiv:2212.06215
L. Bradshaw, S. Chang, arXiv:2304.06063
C. Arzate, S. Chang, G. Jacobo, arXiv:2312.03821

Primary Operators

i	$\mathcal{O}_i^{hZ\bar{f}f}$	CP	$d_{\mathcal{O}_i}$	SMEFT Operator	c Unitarity Bound
1	$hZ^\mu \bar{\psi}_L \gamma_\mu \psi_L$	$+$	5	$i(H^\dagger \overleftrightarrow{D}_\mu H)\bar{Q}_L \gamma^\mu Q_L$	$\frac{0.6}{E_{\text{TeV}}^2}, \frac{5}{E_{\text{TeV}}^4}$
2	$hZ^\mu \bar{\psi}_R \gamma_\mu \psi_R$	$+$		$i(H^\dagger \overleftrightarrow{D}_\mu H)\bar{u}_R \gamma^\mu u_R$	
3	$hZ^{\mu\nu} \bar{\psi}_L \sigma_{\mu\nu} \psi_R + \text{h.c.}$	$+$	6	$\bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \widetilde{H} W^a_{\mu\nu} + \text{h.c.}$	$\frac{2}{E_{\text{TeV}}^2}, \frac{10}{E_{\text{TeV}}^4}$
4	$ih\widetilde{Z}_{\mu\nu} \bar{\psi}_L \sigma^{\mu\nu} \psi_R + \text{h.c.}$	$-$		$i\bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \widetilde{H} \widetilde{W}^a_{\mu\nu} + \text{h.c.}$	
5	$ihZ^\mu (\bar{\psi}_L \overleftrightarrow{\partial}_\mu \psi_R) + \text{h.c.}$	$+$	6	$(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L \overleftrightarrow{D}^\mu u_R)\widetilde{H} + \text{h.c.}$	$\frac{0.1}{E_{\text{TeV}}^3}, \frac{4}{E_{\text{TeV}}^6}$
6	$hZ^\mu \partial_\mu (\bar{\psi}_L \psi_R) + \text{h.c.}$	$-$		$i(H^\dagger \overleftrightarrow{D}_\mu H) D^\mu (\bar{Q}_L u_R)\widetilde{H} + \text{h.c.}$	
7	$ihZ^\mu \partial_\mu (\bar{\psi}_L \psi_R) + \text{h.c.}$	$+$		$(H^\dagger \overleftrightarrow{D}_\mu H) D^\mu (\bar{Q}_L u_R)\widetilde{H} + \text{h.c.}$	
8	$hZ^\mu (\bar{\psi}_L \overleftrightarrow{\partial}_\mu \psi_R) + \text{h.c.}$	$-$		$i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L \overleftrightarrow{D}^\mu u_R)\widetilde{H} + \text{h.c.}$	
9	$ihZ_{\mu\nu} (\bar{\psi}_L \gamma^\mu \overleftrightarrow{\partial}^\nu \psi_L)$	$+$	7	$i\|H\|^2 \widetilde{W}^{a\mu\nu} (\bar{Q}_L \gamma_\mu \sigma^a \overleftrightarrow{D}_\nu Q_L)$	$\frac{0.4}{E_{\text{TeV}}^3}, \frac{1}{E_{\text{TeV}}^4}$
10	$h\widetilde{Z}_{\mu\nu} \partial^\mu (\bar{\psi}_L \gamma^\nu \psi_L)$	$-$		$\|H\|^2 \widetilde{W}^{a\mu\nu} D_\mu (\bar{Q}_L \gamma_\nu \sigma^a Q_L)$	
11	$ih\widetilde{Z}_{\mu\nu} (\bar{\psi}_R \gamma^\mu \overleftrightarrow{\partial}^\nu \psi_R)$	$+$		$i\|H\|^2 \widetilde{B}^{\mu\nu} (\bar{u}_R \gamma_\mu \overleftrightarrow{D}_\nu u_R)$	
12	$h\widetilde{Z}_{\mu\nu} \partial^\mu (\bar{\psi}_R \gamma^\nu \psi_R)$	$-$		$\|H\|^2 \widetilde{B}^{\mu\nu} D_\mu (\bar{u}_R \gamma_\nu u_R)$	

Enables ‘bottom-up’ phenomenology

Conclusions

- EFT truncation uncertainty is a theory uncertainty
- Proposed physical parameterization of uncertainties

  M = scale where EFT breaks down
- Treat using standard statistical methods (nuisance parameters)

  $\Rightarrow$ EFT prediction is uncertain for large $\hat{s}$
- Makes physical assumptions (priors) explicit
- Results are conservative compared to UV model searches

  but sensitive for large M
- Method is general and practical

  $\Rightarrow$ enables 'bottom-up' phenomenology

Backup Slides

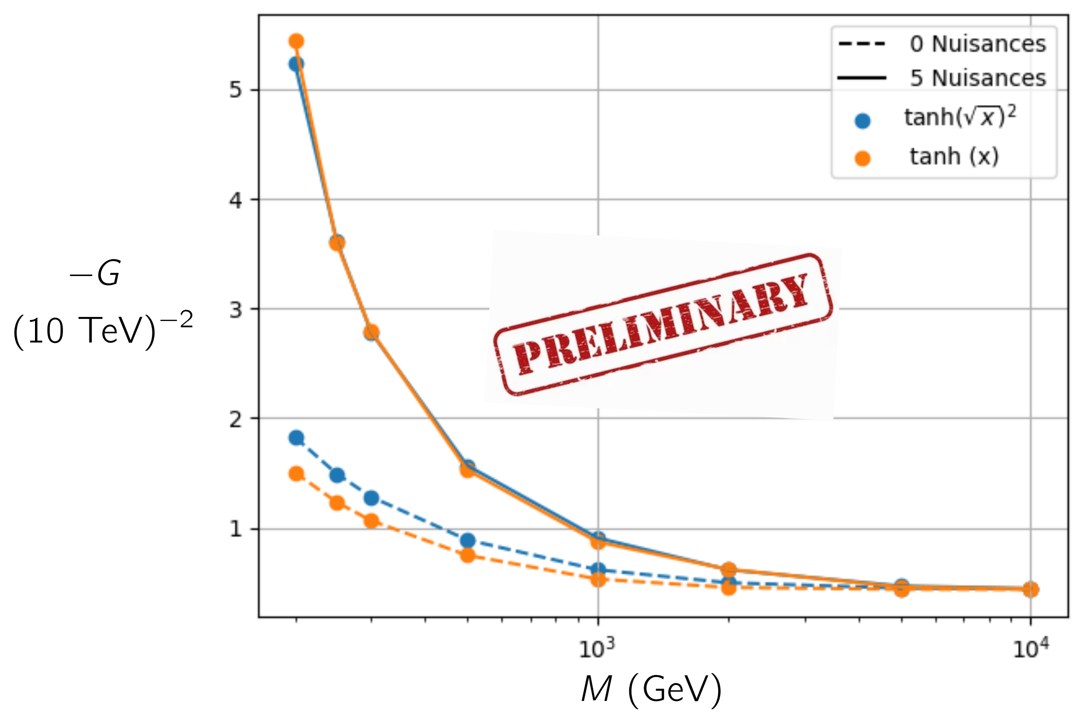
Form Factor Dependence

Dependence on the form of $F(x)$

$$\mathcal{F}(x,y,z) = \left(\frac{F(x)}{x}\right)^{\alpha}\Big[1 + c_1 F(x) + c_2 F(y) + c_3 F(x)F(y) + \cdots$$

$$+ \text{resonances}\Big]$$

$$F(x) = \begin{cases} x & x \ll 1, \\ 1 & x \gg 1 \end{cases}$$



Parameterizing Resonances

$$\mathcal{F}(x,y,z) = \left(\frac{F(x)}{x}\right)^{\alpha}\Bigg[1 + c_1 F(x) + c_2 F(y) + c_3 F(x) F(y) + \cdots$$
$$+ \sum_a S_a \left(\frac{1}{x-\sigma_a} + \frac{1}{\sigma_a}\right)$$
$$+ \sum_b T_a \left(\frac{1}{y-\tau_b} + \frac{1}{\tau_b}\right)$$
$$+ \sum_c U_c \left(\frac{1}{z-\mu_c} + \frac{1}{\mu_c}\right)\Bigg]$$

$$\sigma_a = \frac{m_a^2}{M^2}\left(1 - i\Gamma_a/m_a\right)$$

$m_a =$ resonance mass

$\Gamma_a =$ resonance width

$$\left|\frac{S_a M^2}{m_a^2}\right| \lesssim 1$$

tuning

$$\frac{S_a M^2}{m_a^2}\frac{G M^{d-4}}{16\pi} \lesssim \frac{\Gamma_a}{m_a} \lesssim 1$$

unitarity

Adding Resonances