

Improved Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

Lotta Jokiniemi (she/her)

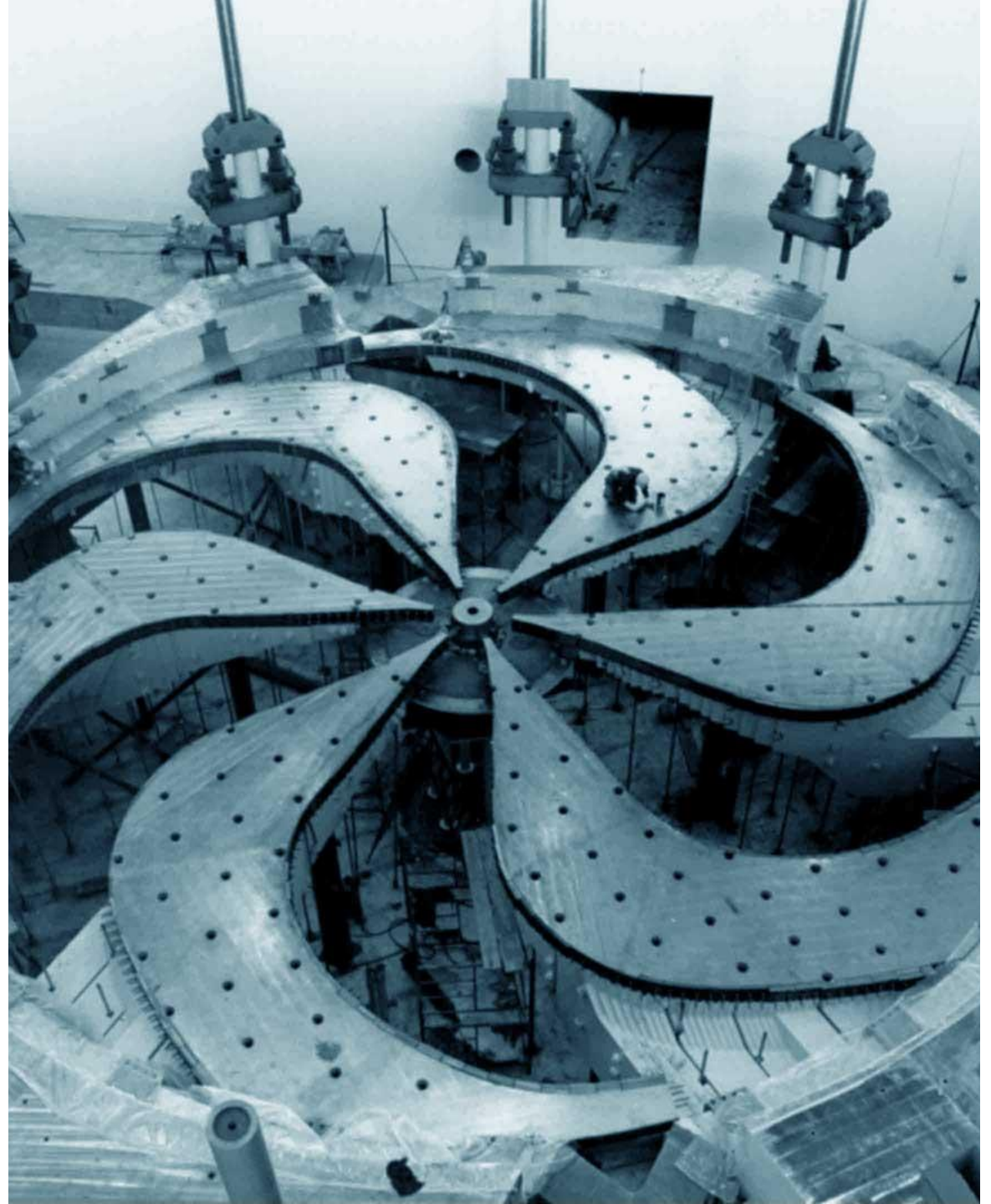
Postdoctoral Fellow, Theory Department,
TRIUMF

McDonald Institute Annual National Meeting,
Queen's University, Kingston
08/08/2024



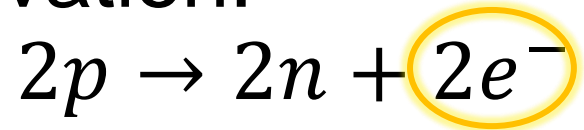
Arthur B. McDonald
Canadian Astroparticle Physics Research Institute

2024-08-08



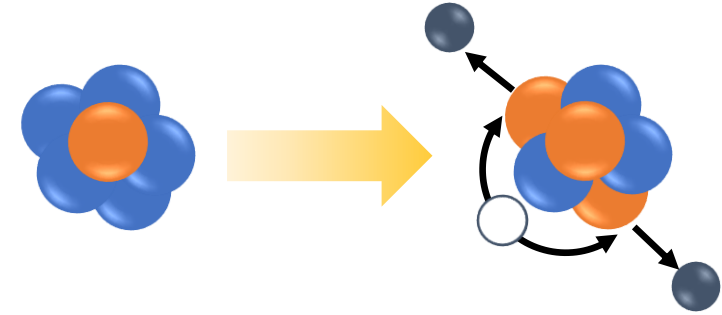
Neutrinoless double-beta ($0\nu\beta\beta$) decay

- ❑ Violates lepton-number conservation:



- ❑ Requires that neutrino is its own antiparticle (*Majorana particle*)

- ❑ If observed, $t_{1/2}^{0\nu\beta\beta} \gtrsim 10^{25}$ years
($t_{1/2}^{2\nu\beta\beta} \gtrsim 10^{20}$ years)



1935



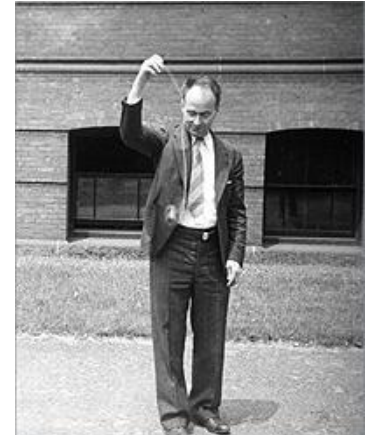
M. Göppert Mayer

1937



E. Majorana

1939



W. Furry

Half-life of neutrinoless double-beta decay

$$\frac{1}{t_{1/2}^{0\nu\beta\beta}} = g_A^4 G^{0\nu\beta\beta} |M^{0\nu\beta\beta}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

Half-life

Observable to be measured

Axial-vector coupling

Known from β -decay of neutrons; $g_A \approx 1.27$

Phase-space factor

Physics of the emitted electrons \rightarrow solved from atomic physics

Nuclear matrix element (NME)

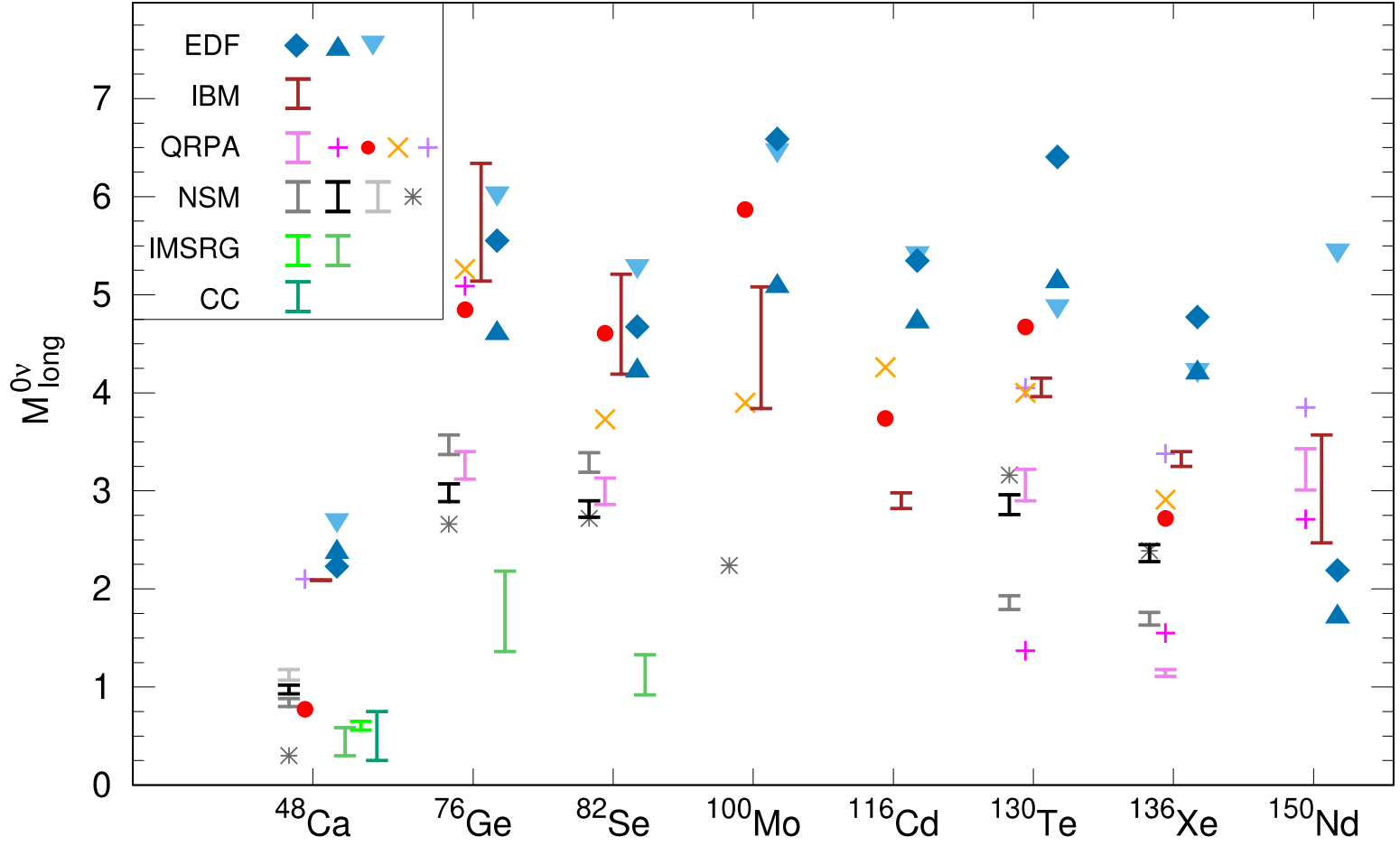
From nuclear theory

Majorana mass

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

New physics we want to extract

Nuclear matrix elements currently poorly known



M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

Nuclear matrix element

Operator ($2n \rightarrow 2p + 2e^-$)

$$M^{0\nu\beta\beta} = \langle 0_f^+ || \mathcal{O}^{0\nu\beta\beta} || 0_i^+ \rangle$$

Final state nuclear
wave function

Initial state nuclear
wave function

$$H\Psi^{(A)} = E\Psi^{(A)} \leftarrow H = \sum_i \frac{p^2}{2m} + \sum_{i \neq j} V^{2N} + \sum_{i \neq j \neq k} V^{3N}$$

Ab initio vs. phenomenological nuclear methods

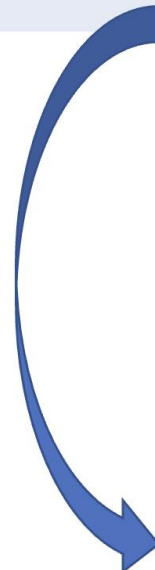
Ab initio (lat. 'from the beginning')



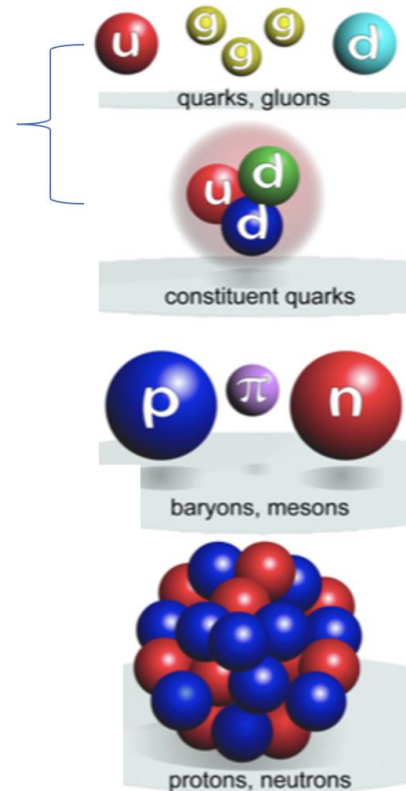
Phenomenological



Quantum Chromodynamics (QCD)



Genuine Ab Initio



Solve

$$H^{\text{eff}}\Psi(A) = E\Psi(A)$$

with H^{eff} adjusted to nuclear data (energies, decays, ...)

- Different models (nuclear shell model, random-phase approximation, ...)
- Not systematically improvable

What can we do with *ab initio* methods?

Exponentially scaling methods (couple cluster (VS-MP2, G, aM, GCM))

7

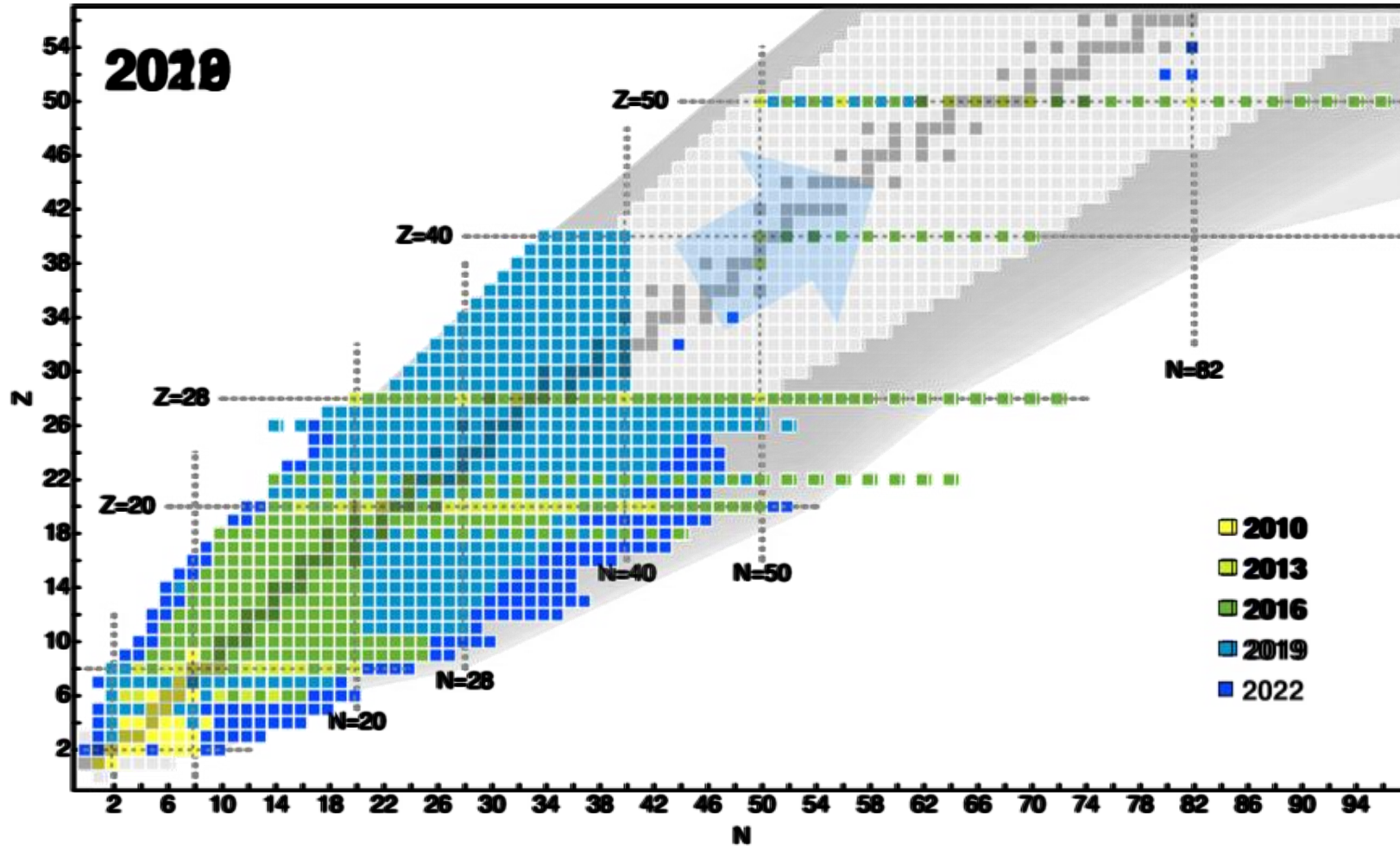
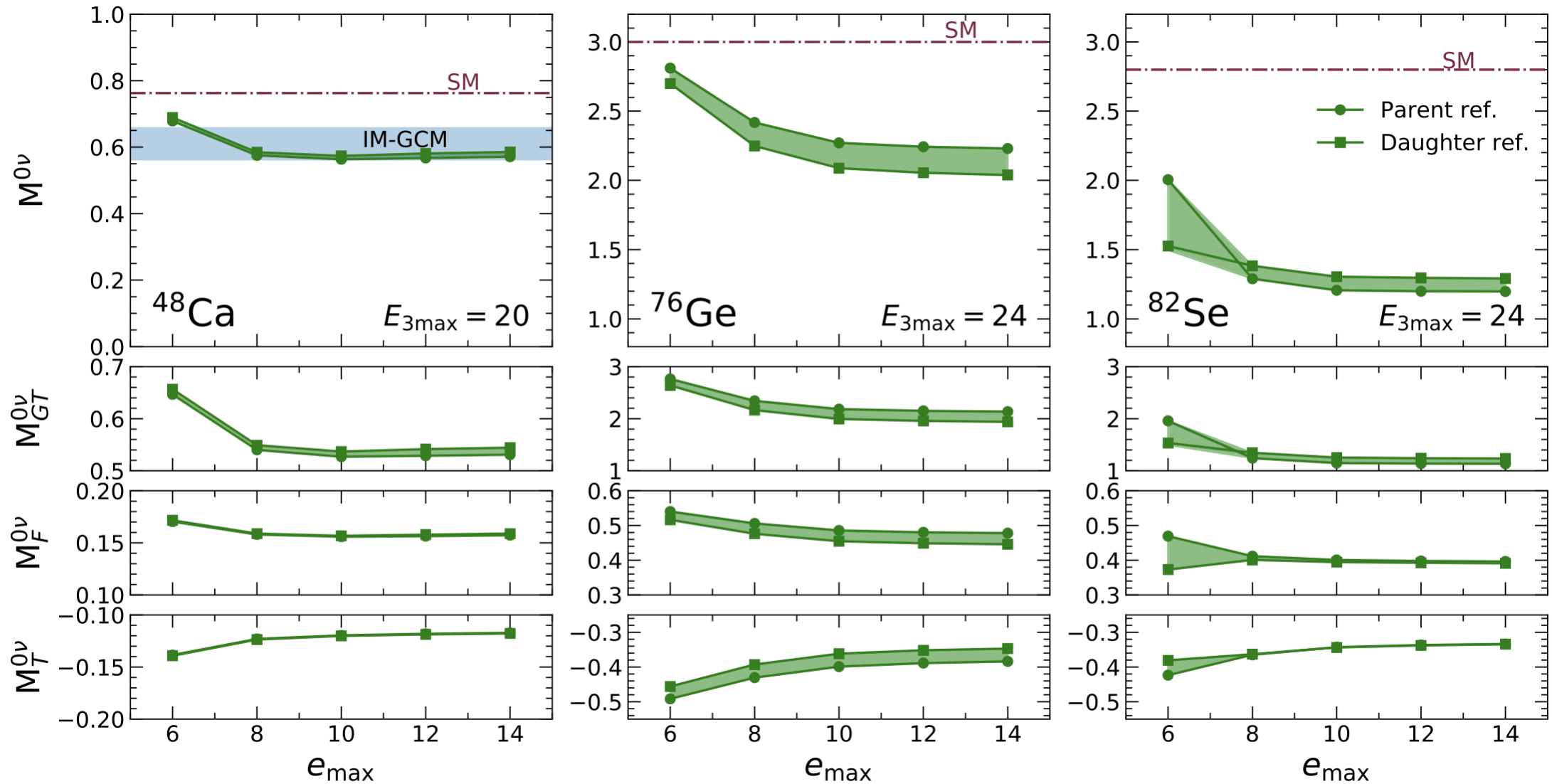
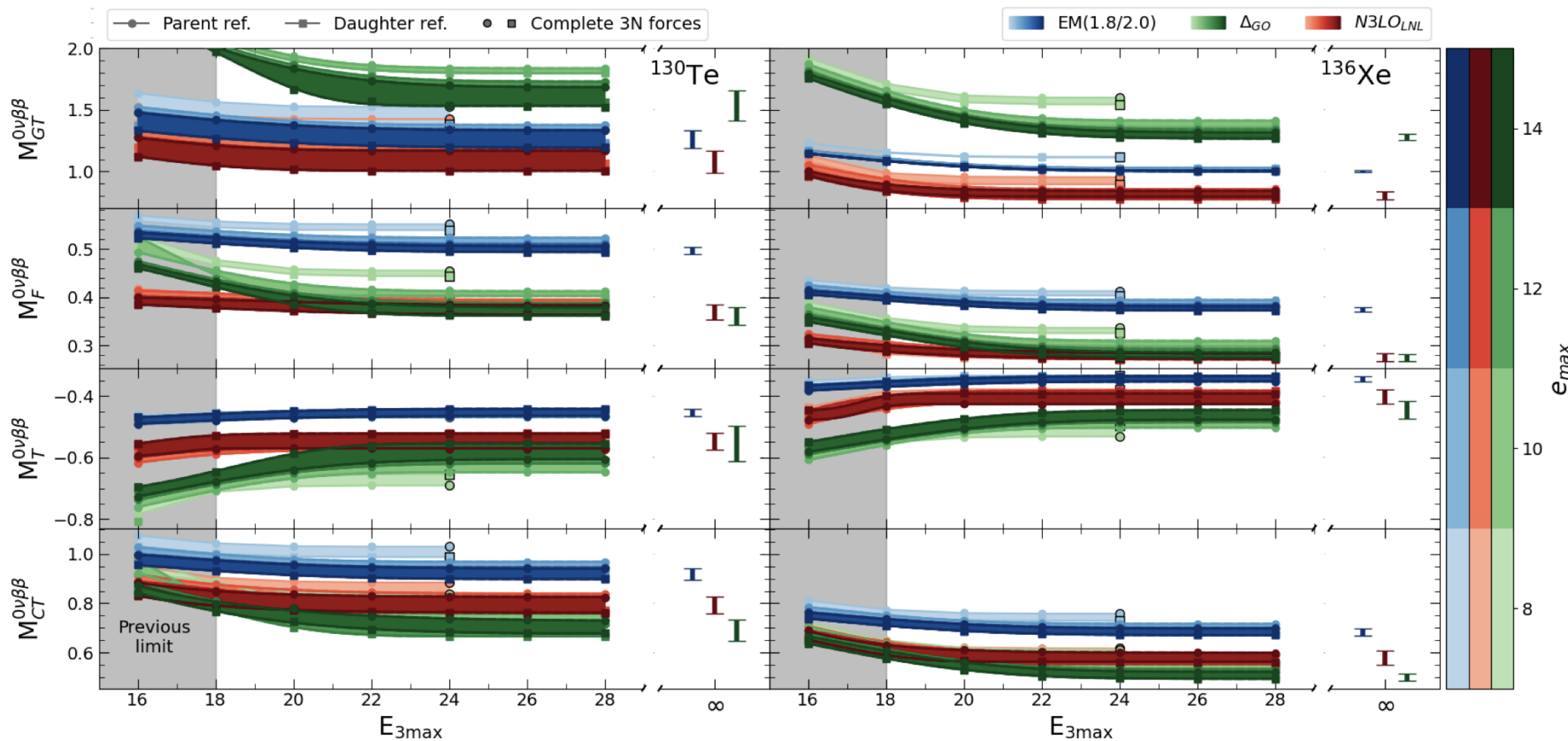


Figure courtesy of A. Belley, (adapted from H. Hergert, *Front. Phys.* 8 (2020))

Ab initio $0\nu\beta\beta$ -decay NMEs for ^{48}Ca , ^{76}Ge and ^{82}Se



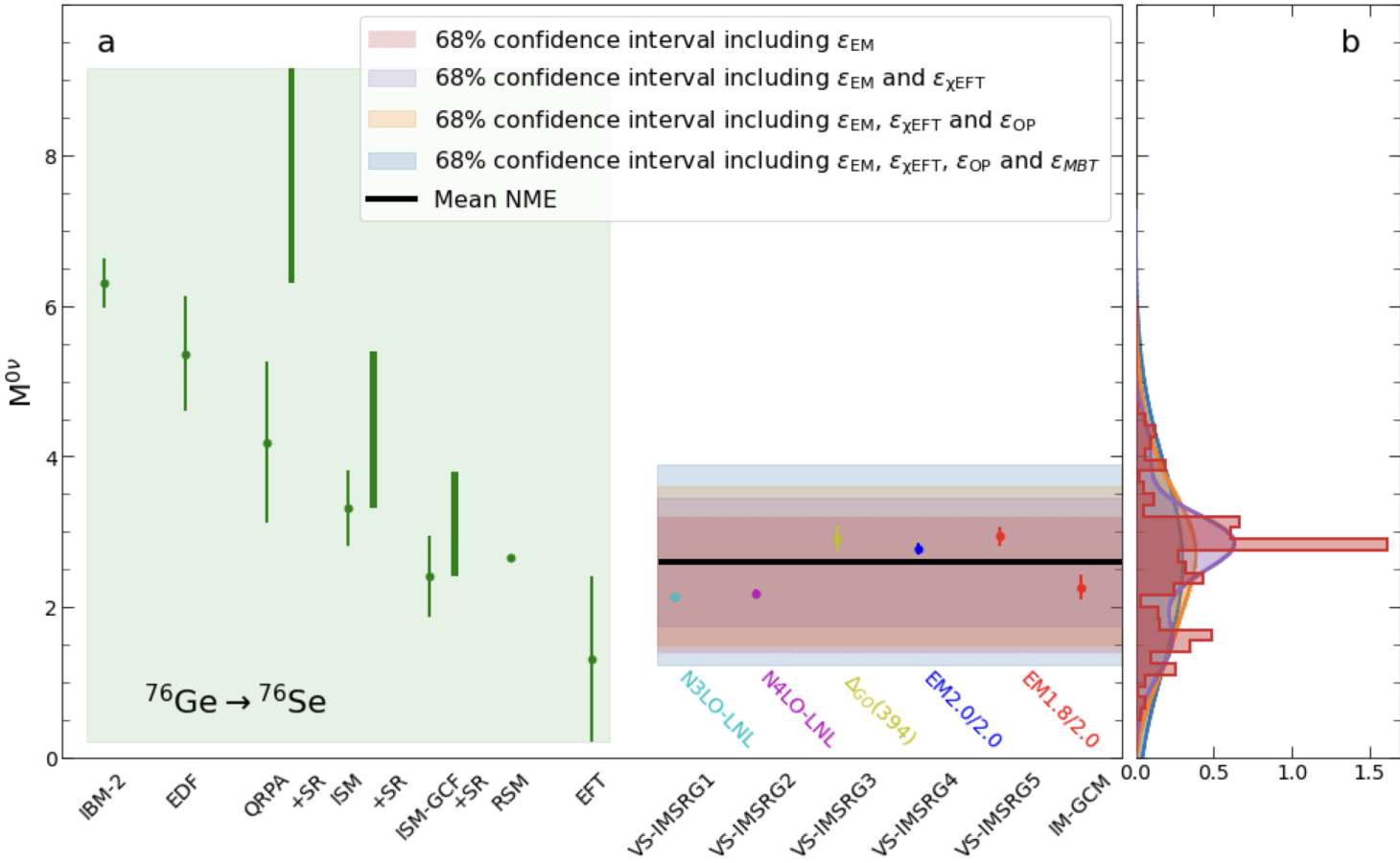
Ab initio $0\nu\beta\beta$ -decay NMEs for ^{130}Te and ^{136}Xe



A. Belley, T. Miyagi, S.R. Stroberg, and J. D. Holt, arXiv:2307.15156

Uncertainty quantification of $0\nu\beta\beta$ decay of ^{76}Ge

Ab initio + machine learning + Bayesian statistics



A. Belley et al., Phys. Rev. Lett. 132, 182502 (2024)

Combined constraints for Majorana masses

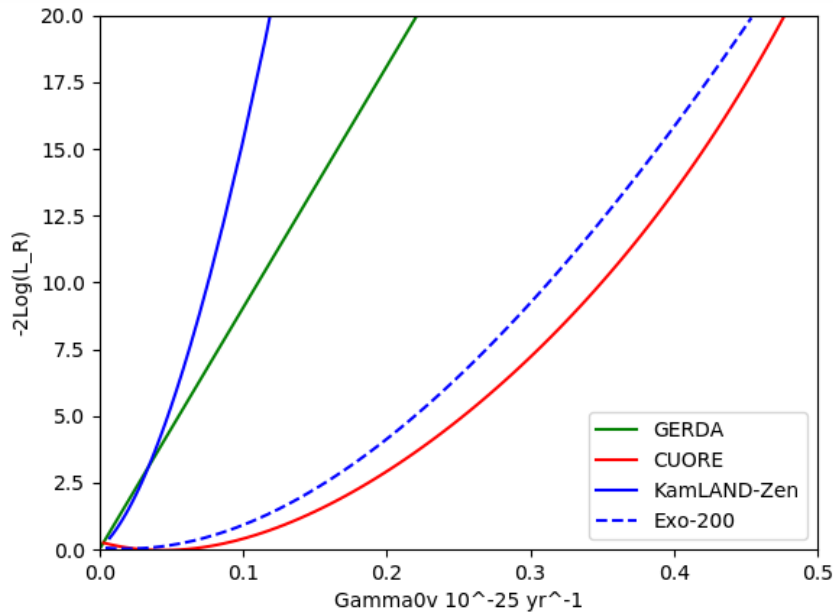
Combined constraints on Majorana masses from neutrinoless double beta decay experiments

Steven D. Biller
Phys. Rev. D **104**, 012002 – Published 6 July 2021

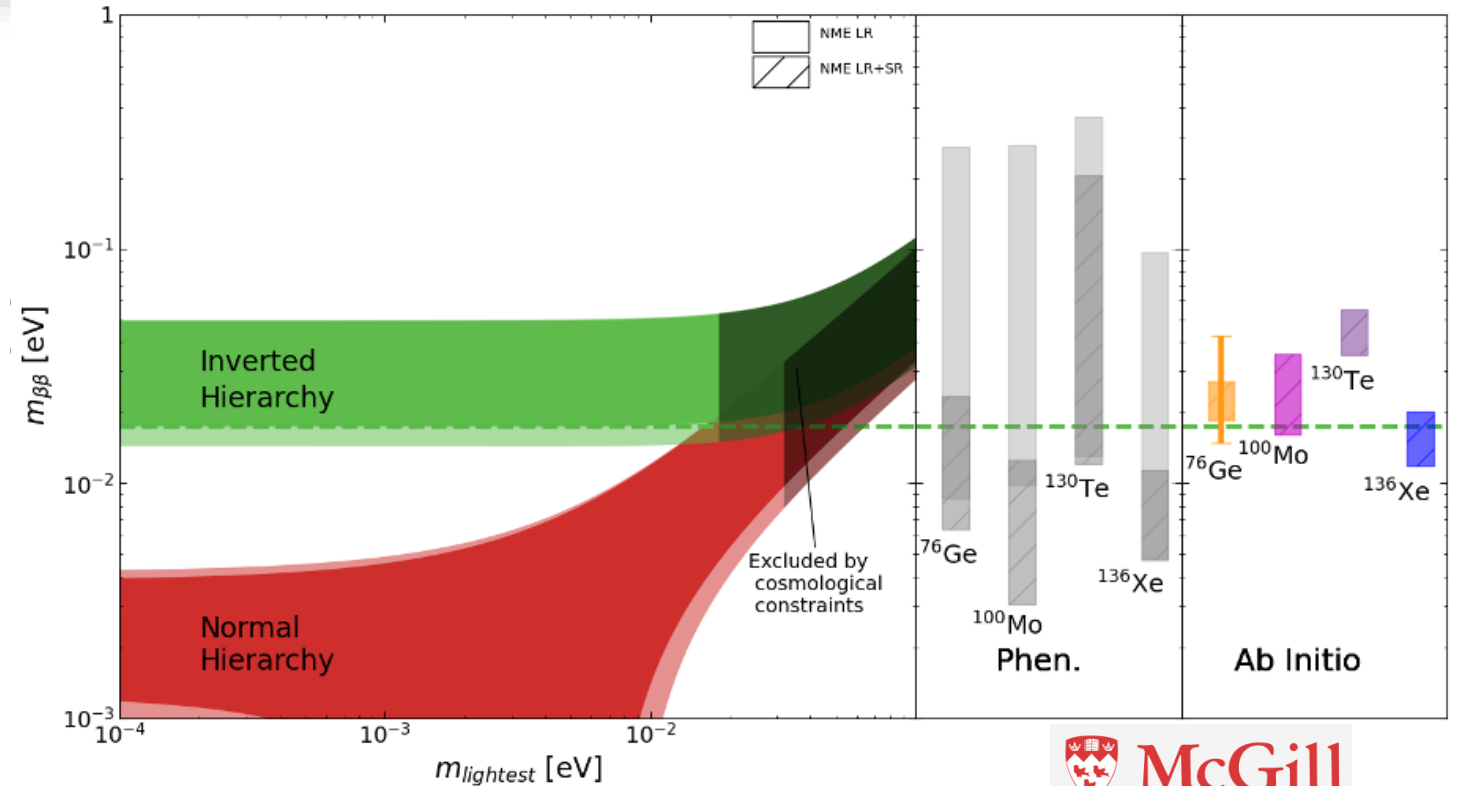


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T. Shickele, LJ, A. Belley, et al., in progress.



GERDA: [PRL 125, 252502 \(2020\)](#)
 CUORE: [arXiv:2404.04453](#)
 KamLAND-Zen: [arXiv:2406.11438](#)
 EXO-200: [PRL 123, 161802 \(2019\)](#)



→ Constraints for heavy neutrinos (A. Todd)
 → Constraints for next-generation experiments

Improvements to the operator of $0\nu\beta\beta$ decay

$$\mathcal{O}^{0\nu} = \mathcal{O}_{\text{LO}}^{0\nu} + \mathcal{O}_{\text{N}^2\text{LO}}^{0\nu} + \dots$$

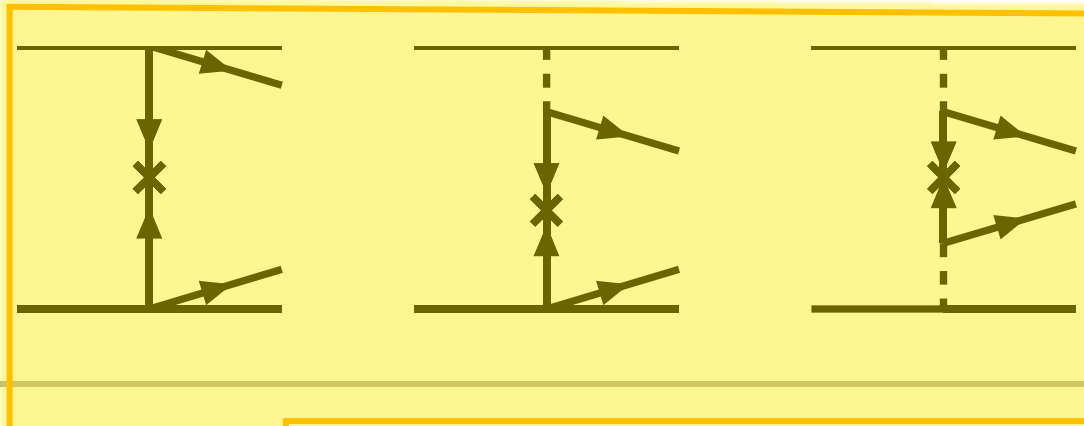
What is normally included

Leading order

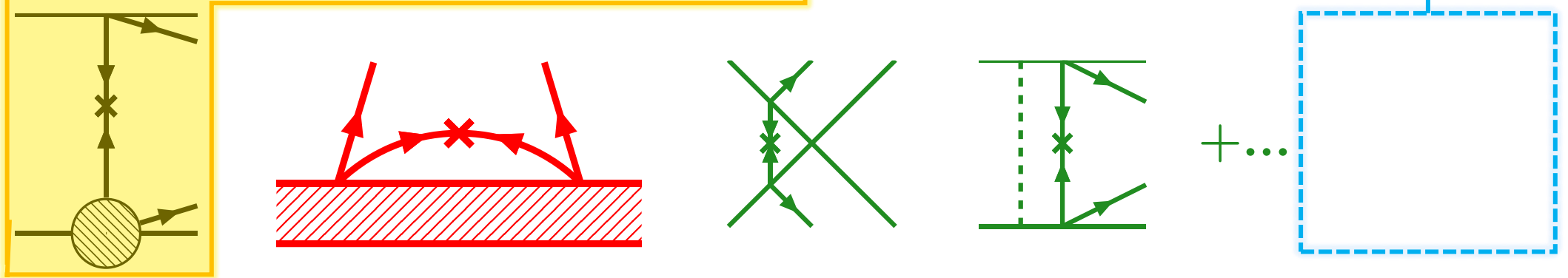
Next-to-next-to-leading order

What needs to be included

LO



N²LO



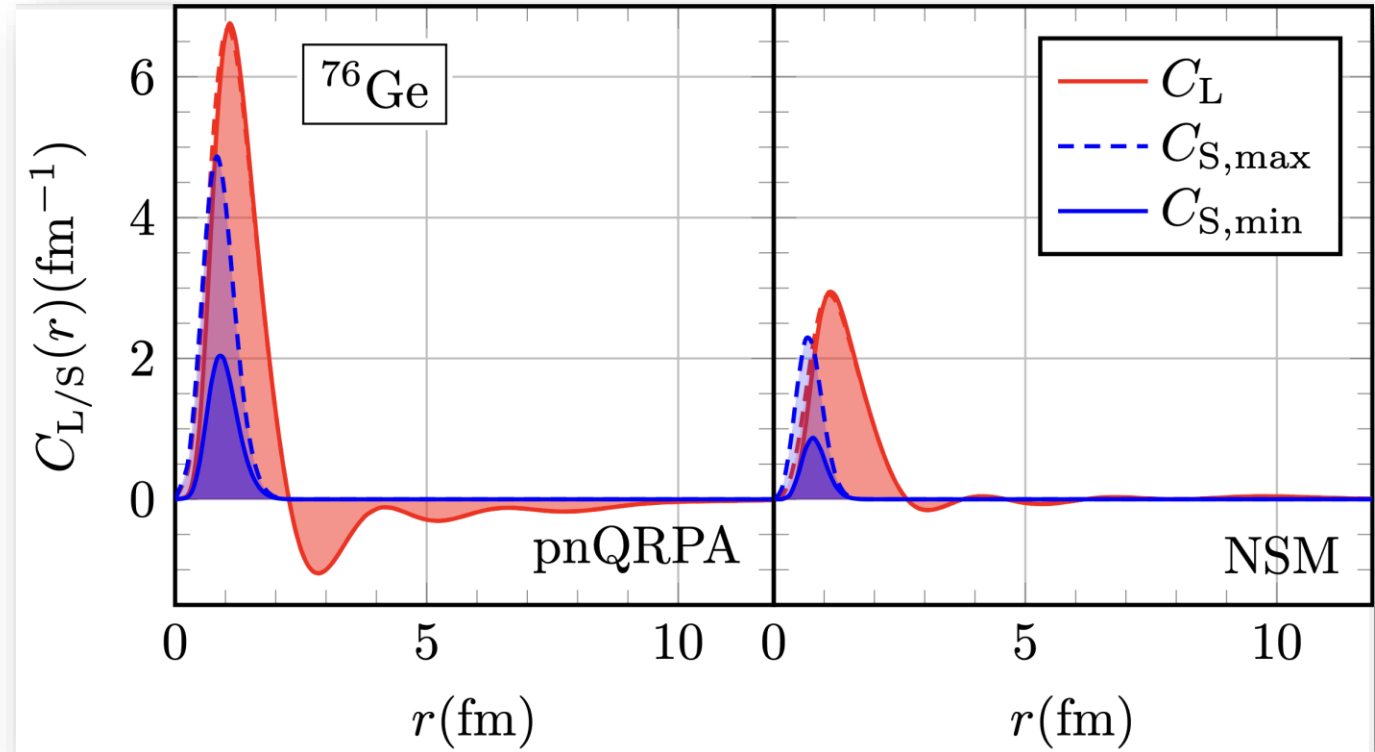
Contact term in pnQRPA and nuclear shell model (NSM)

$$O_S^{0\nu} = 2g_\nu^{NN} \tau_a^- \tau_b^-$$

$$\int C_{L/S}(r) dr = M_{L/S}^{0\nu}$$

- $\frac{M_S^{0\nu}}{M_L^{0\nu}} \approx 30\% - 80\%$ (pnQRPA)
- $\frac{M_S^{0\nu}}{M_L^{0\nu}} \approx 15\% - 50\%$ (NSM)
- $\frac{M_S^{0\nu}}{M_L^{0\nu}} \approx 30\% - 90\%$ (*ab initio*)

R. Wirth et al., PRL 127, 242502 (2021)
 A. Belley et al., PRL 132, 182502 (2024)
 A. Belley et al., arXiv:2307.15156



LJ, P. Soriano, J. Menéndez, Phys. Lett. B 823, 136720 (2021)

Effective-field theory corrections to the operator

$$\mathcal{O}^{0\nu} = \mathcal{O}_{\text{LO}}^{0\nu} + \mathcal{O}_{\text{N}^2\text{LO}}^{0\nu} + \dots$$

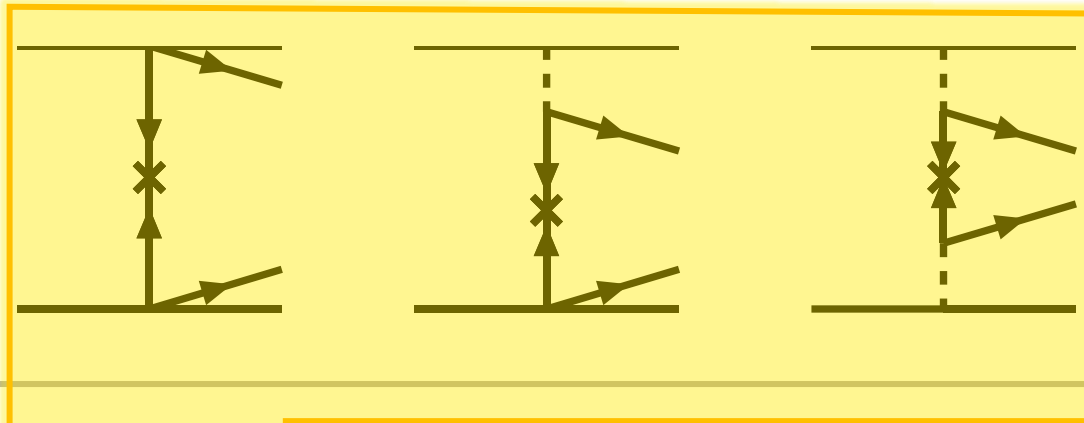
What is normally included

Leading order

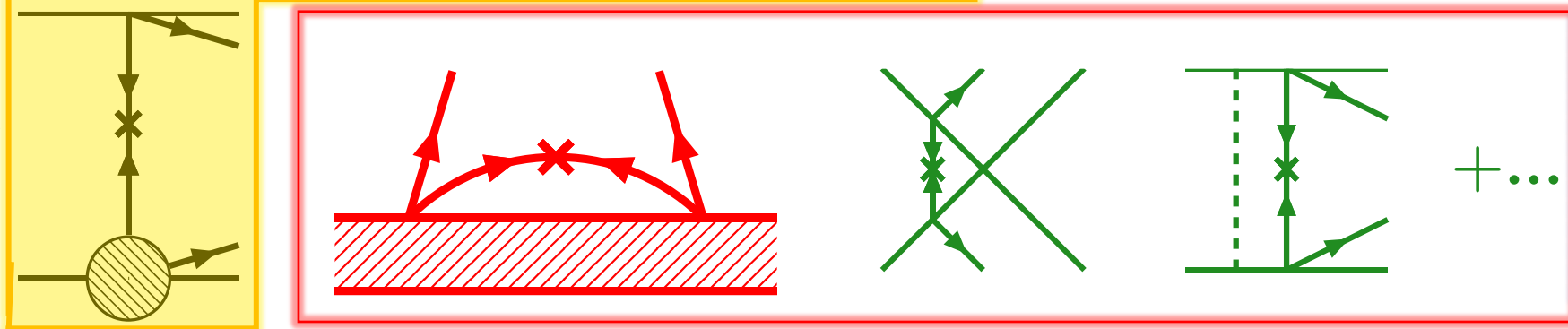
Next-to-next-to-leading order

What needs to be included

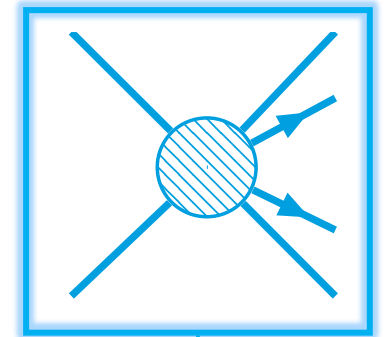
LO



N²LO

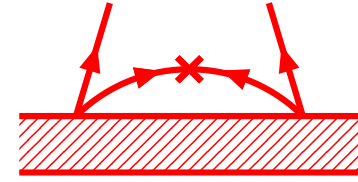


How about these?



The ultrasoft-neutrino term

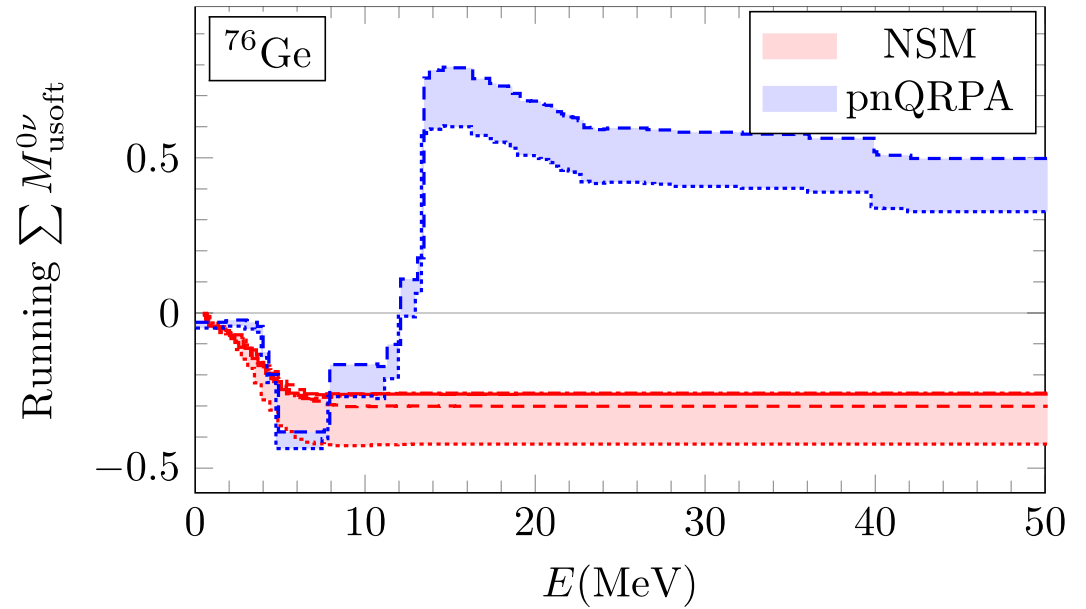
$$M_{\text{usoft}}^{0\nu} = \sum_n f(E_n) \left\langle 0_f^+ \left\| \sum_a \sigma_a \tau_a^- \right\| 1_n^+ \right\rangle \left\langle 1_n^+ \left\| \sum_b \sigma_b \tau_b^- \right\| 0_i^+ \right\rangle$$



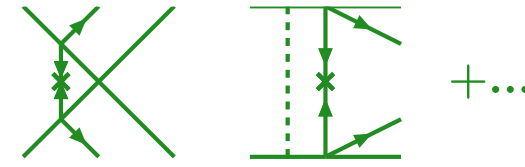
□ Contribution from “Ultrasoft” neutrinos ($|\mathbf{k}| \ll k_F \sim 100 \text{ MeV}$)

□ $\frac{M_{\text{usoft}}^{0\nu}}{M_L^{0\nu}} \simeq 5\% - 10\%$ (pnQRPA)

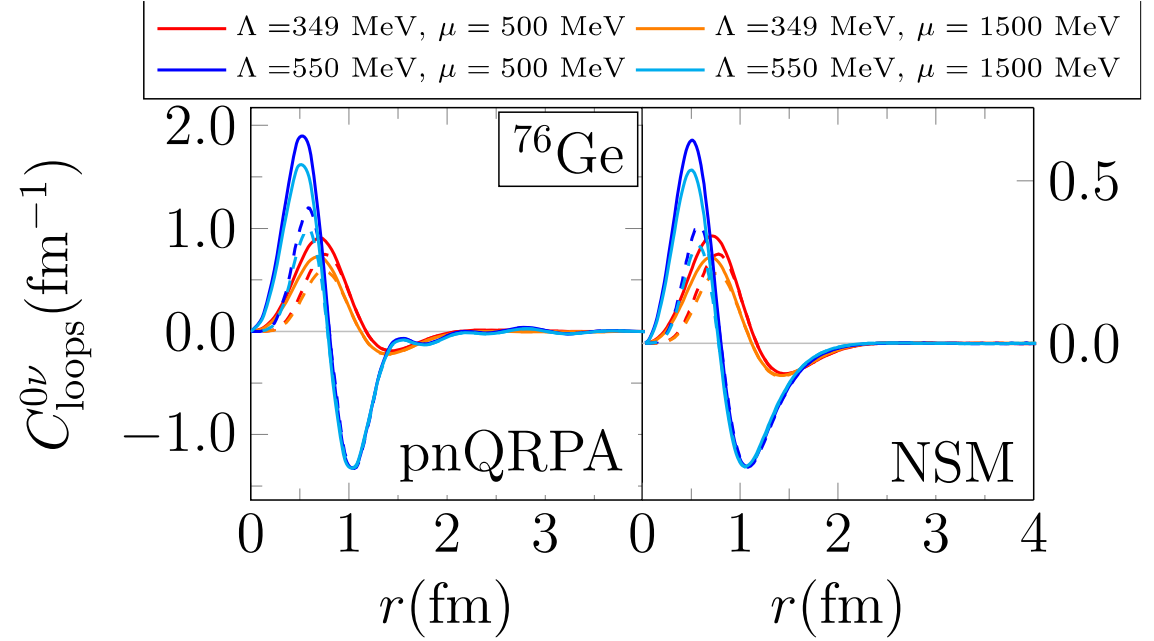
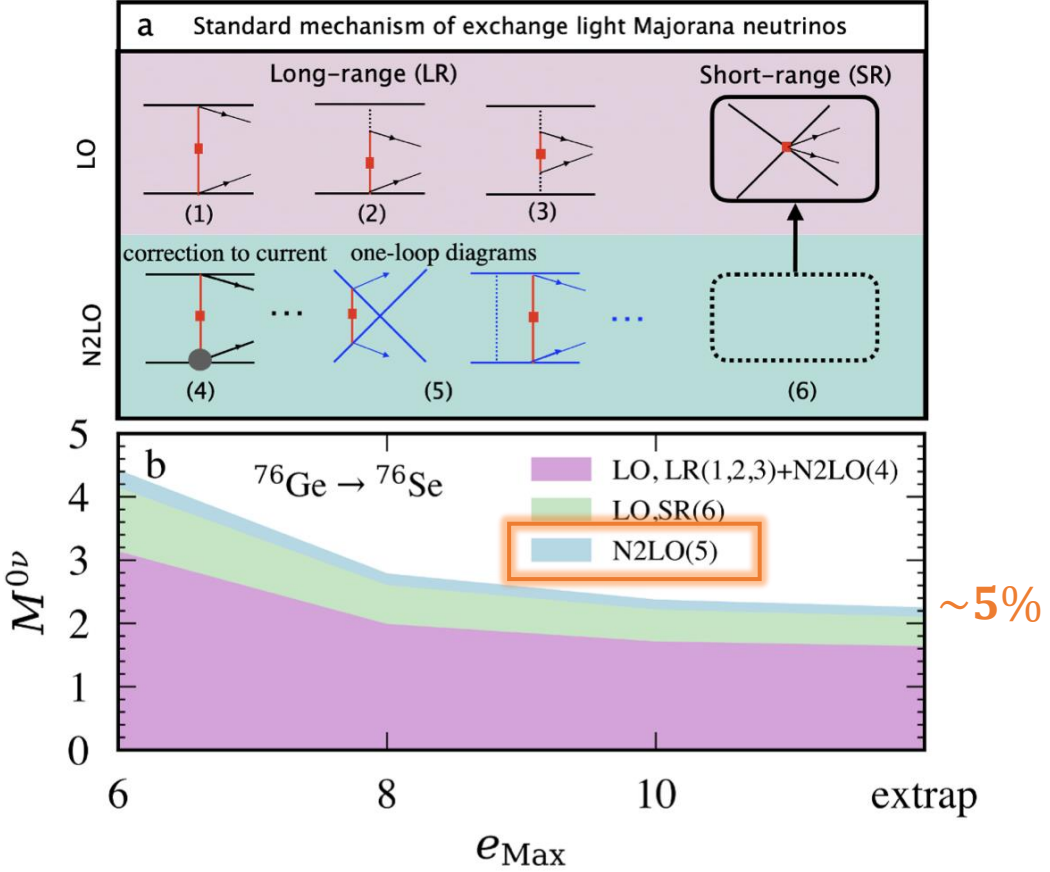
□ $\frac{M_{\text{usoft}}^{0\nu}}{M_L^{0\nu}} \approx -5\%$ (NSM)



N²LO loop corrections



$$O_{\text{loops}}^{0\nu} = e^{-q^2/(2\Lambda^2)} (O_{\text{VV}}^{(m,n)} + O_{\text{AA}}^{(m,n)} + O_{\text{US}}^{(m,n)} + O_{\text{CT}}^{(m,n)})$$

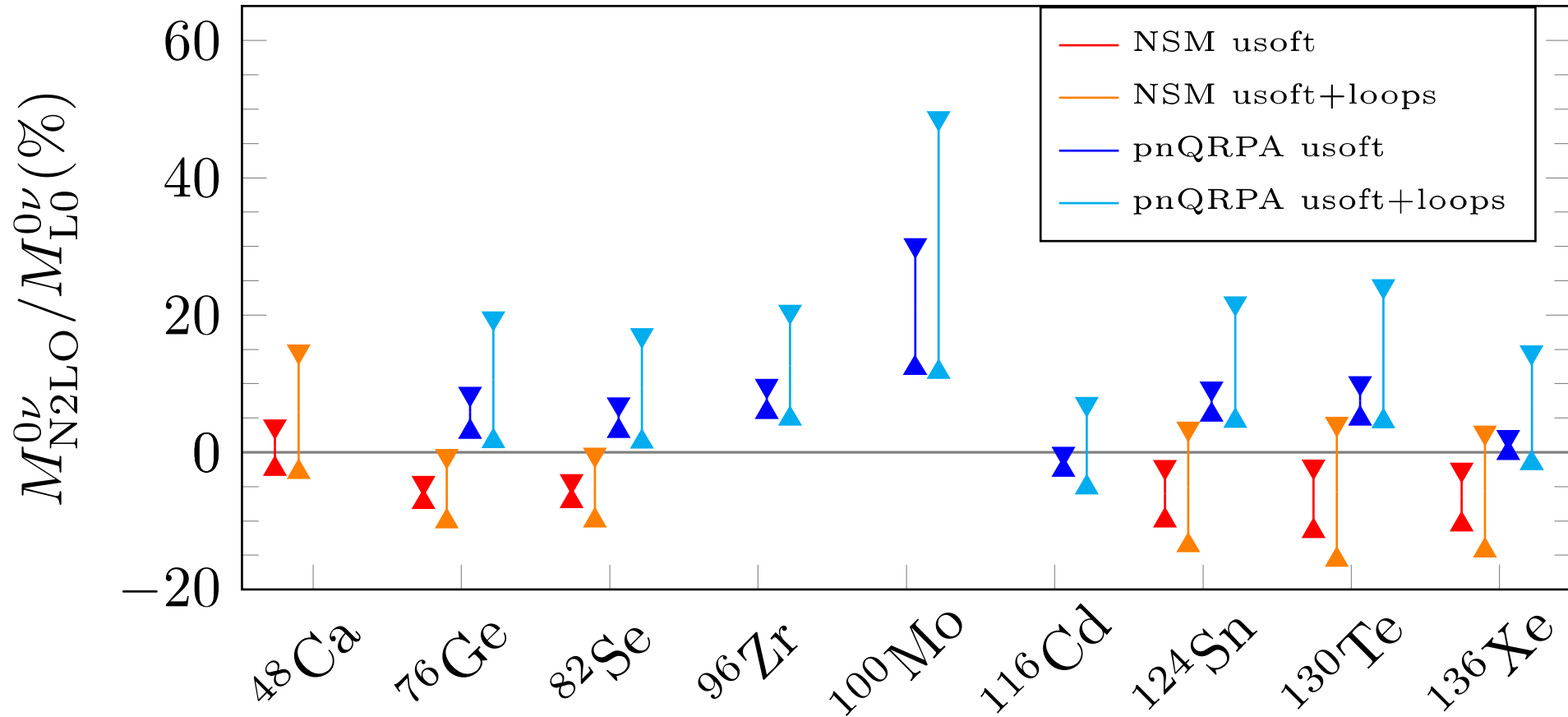


$$\frac{M_{\text{loops}}^{0\nu}}{M_{\text{L}}^{0\nu}} \lesssim 10\% \quad \frac{M_{\text{loops}}^{0\nu}}{M_{\text{L}}^{0\nu}} \lesssim 5\%$$

A Belley et al., Phys. Rev. Lett. 132, 182502 (2024)

D. Castillo, LJ, P. Soriano, J. Menéndez, arXiv:2408:03373

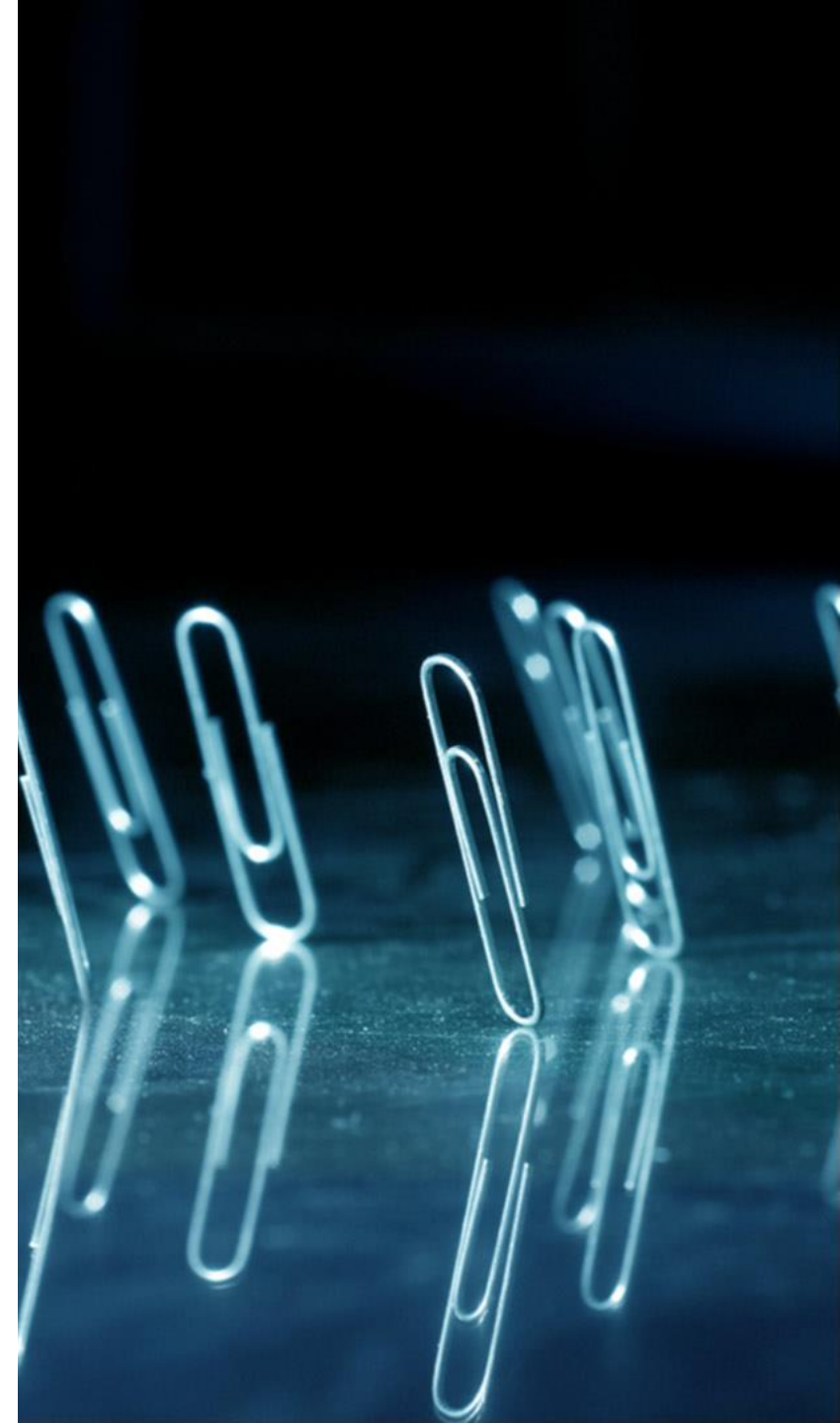
Combined N²L0 corrections



D. Castillo, LJ, P. Soriano, J. Menéndez, arXiv:2408:03373

→ The N2LO corrections should be included in any $0\nu\beta\beta$ calculations

Could We Learn from Other Nuclear Processes?



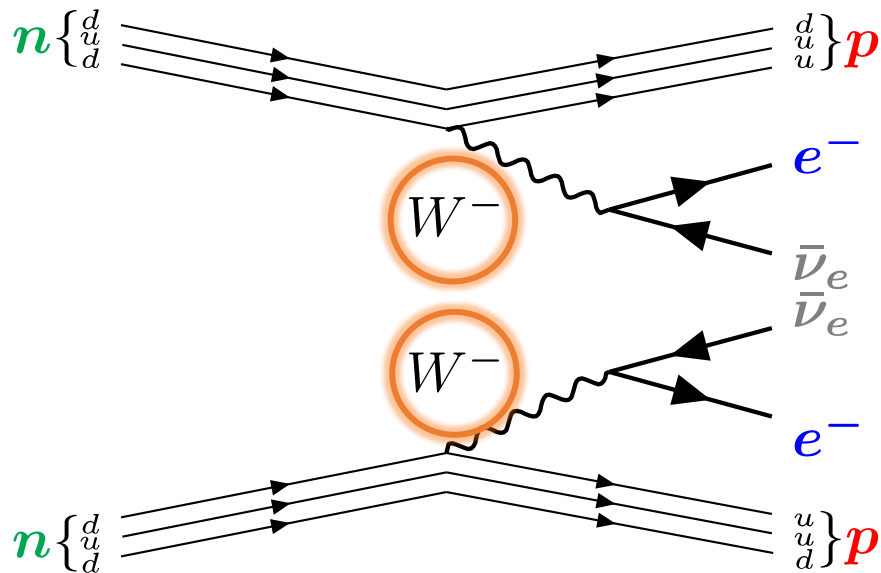
Probing $0\nu\beta\beta$ decay by $2\nu\beta\beta$ decay

Similarities:

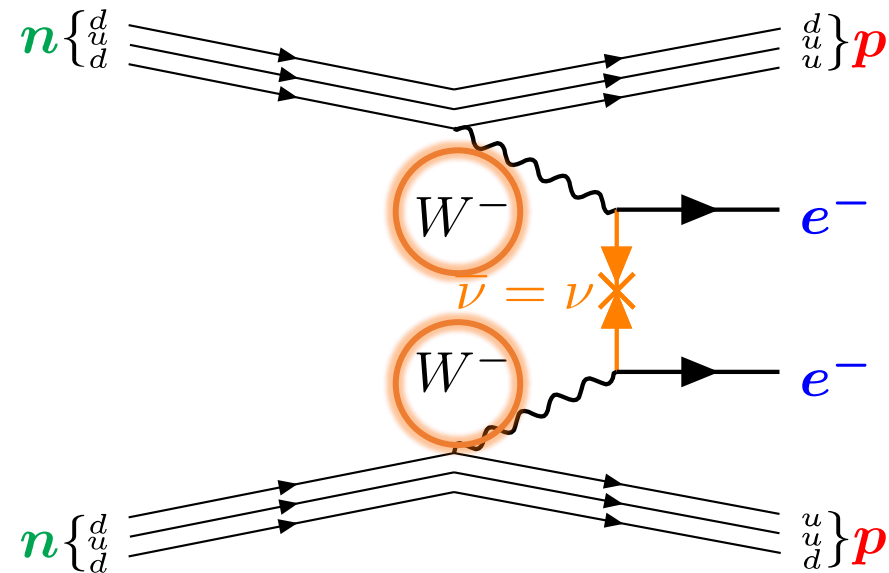
- Same initial and final nuclei
- Weak-interaction processes

Differences:

- Different momentum exchange
- $2\nu\beta\beta$ decay measured, $0\nu\beta\beta$ not



$$q_{2\nu\beta\beta} \approx 2 \text{ MeV}$$



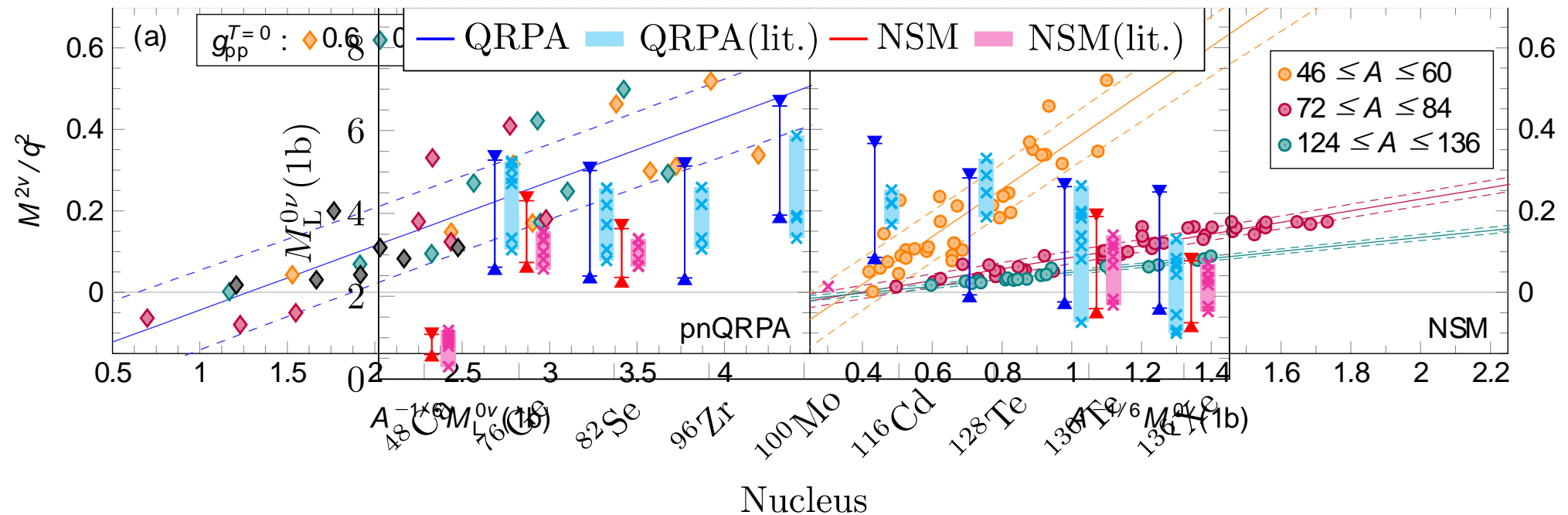
$$q_{0\nu\beta\beta} \approx 100 \text{ MeV}$$

Probing $0\nu\beta\beta$ decay by $2\nu\beta\beta$ decay

- Nuclear matrix elements of $0\nu\beta\beta$ and $2\nu\beta\beta$ decays correlated
- We can derive $M^{0\nu}$ from the correlations and measured $2\nu\beta\beta$ -decay half-lives

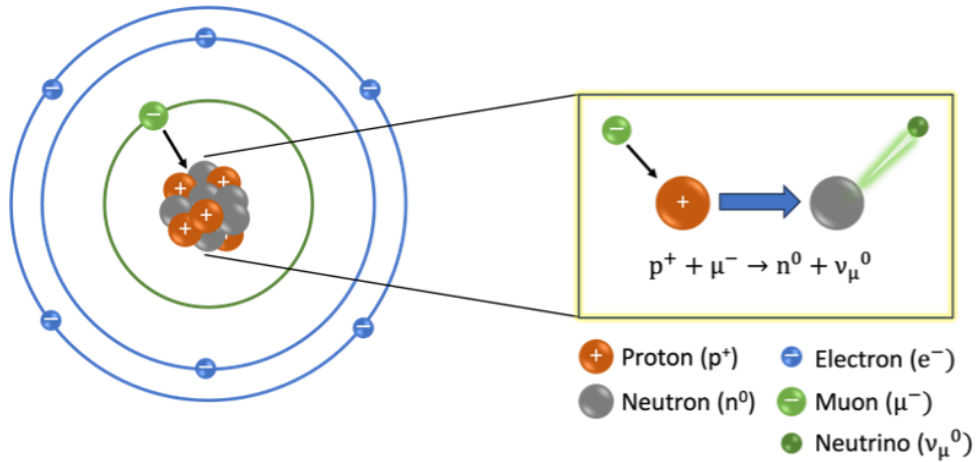
→ **Theoretical uncertainties based on systematics**

- Will *ab initio* methods find similar correlations? (D. Sedghi)

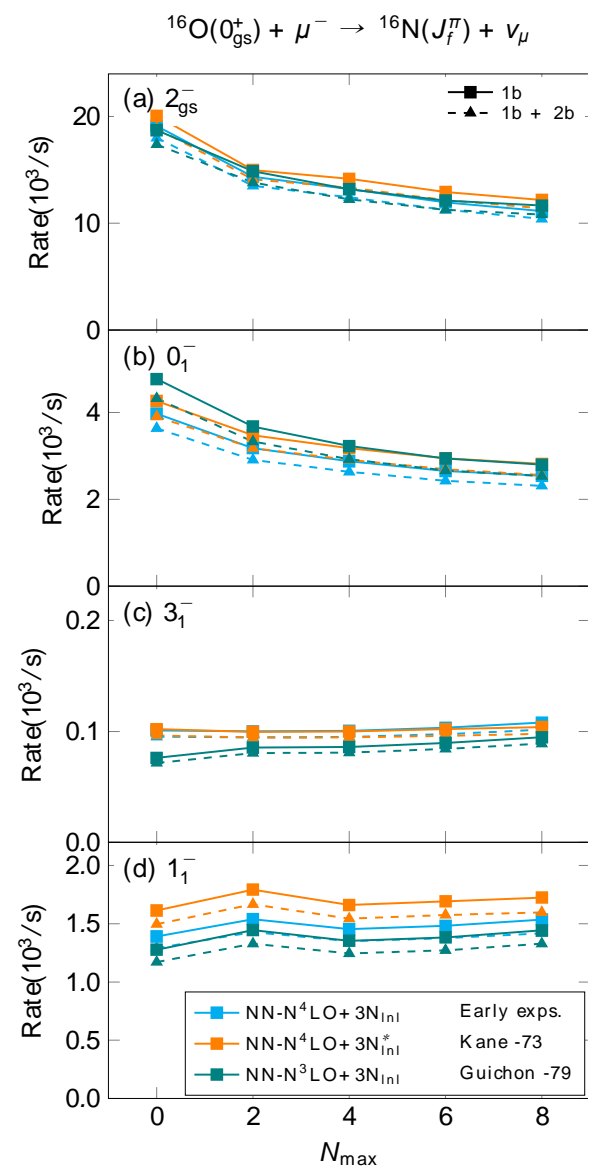


LJ, B. Romeo, P. Soriano, J. Menéndez, *Phys. Rev. C* 107, 044305 (2023)

Muon capture on ${}^6\text{Li}$, ${}^{12}\text{C}$, ${}^{16}\text{N}$ from *ab initio* nuclear theory



- ❑ Momentum exchange $q = m_\mu + E_i - E_f \approx 100 \text{ MeV}$
 - ❑ Involves vector, axial-vector, magnetic and pseudoscalar nuclear-weak currents
- Can be used as a probe of $0\nu\beta\beta$ decay



PHYSICAL REVIEW C **109**, 065501 (2024)

Muon capture on ${}^6\text{Li}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ from *ab initio* nuclear theory

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❑ *Ab initio* no-core shell-model calculations in good agreement with experiments

Summary and Outlook

- ❑ *Ab initio* methods now capable of computing $0\nu\beta\beta$ -decay nuclear matrix elements
- ❑ Corrections to the $0\nu\beta\beta$ -decay operators evaluated up to N²LO
- ❑ Related processes, such as $2\nu\beta\beta$ decay and muon capture can help further constrain $0\nu\beta\beta$ decay

Outlook

- ❑ $0\nu\beta\beta$ decay can be mediated by **other mechanisms** (ongoing *ab initio* studies by A. Todd et al.)
- ❑ *Ab initio* studies for **$2\nu\beta\beta$ decay** (D. Sedghi)
- ❑ **Two-body currents** would enter at N³LO, but effects may be larger (TODO to the community)
- ❑ How does nuclear deformation affect the *ab initio* predictions? (TODO to the community)

Thank you
Merci

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Baishan Hu (dark matter)

Antoine Belley ($0\nu\beta\beta$ decay)

Mathieu Bruneault (dark matter)

Taiki Shickele ($0\nu\beta\beta$ decay)

Alexander Todd ($0\nu\beta\beta$ decay)

Didar Sedghi ($2\nu\beta\beta$ decay)

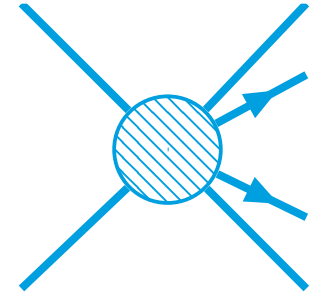


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New leading order short-range nuclear matrix element

$$\frac{1}{t_{1/2}^{0\nu\beta\beta}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$



- The contact term was missing from all calculations

V. Cirigliano et al., Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)

- The operator connects directly initial and final nuclei

$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} (0_f^+ || \sum_{a,b} \tau_a^- \tau_b^- \int j_0(qr) h_S(q^2) q^2 dq || 0_i^+),$$

$$h_S(q^2) = 2g_v^{NN} e^{-q^2/(2\Lambda^2)}$$

Unknown