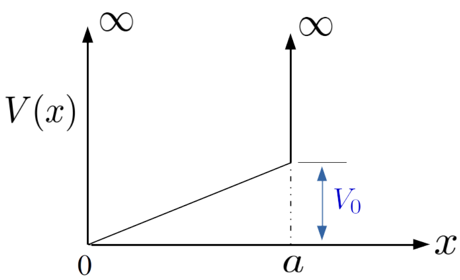
Advanced Quantum Mechanics

Assignment-II

1. Calculate the energies of the first three states of an infinite potential well whose one side is modified as shown in the following figure.



2. In a given orthonormal basis the Hamiltonian is given by the matrix,

$$H = H^{(0)} + H' = \text{diag}(1, 3, -2) + \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & \lambda \end{pmatrix},$$

where λ is a real constant and $\lambda \ll 1$.
(i) Find the exact eigenvalues of H.
(ii) Determine the eigenvalues again using perturbation theory up to second order.
(iii) Compare the results of (i) and (ii).

3. A simple harmonic oscillator is perturbed by an additional weak anharmonic force described by the potential $\Delta V = \lambda A \sin kx$, where A has the appropriate dimensions and λ is dimensionless. Find out the corrected ground state for this perturbed system.

4. A particle of mass m is confined to a circle of radius R, but is otherwise free. A perturbing potential $H' = A \sin\theta \cos\theta$ is applied, where θ is the angular position on the circle. Find the correct zero-order wave functions for the two lowest states of this system and calculate their perturbed energies to second order.

5. During the 1930s, it was noticed that there was a radio noise that varied on a daily cycle and appeared to be extraterrestrial in origin. After initial suggestions that this was due to the Sun, it was observed that the radio waves seemed to propagate

from the centre of the Galaxy. These discoveries were published in 1940 and were noted by Jan Oort who knew that significant advances could be made in astronomy if there were emission lines in the radio part of the spectrum. He referred this to Hendrik van de Hulst who, in 1944, predicted that neutral hydrogen could produce radiation at a frequency of 1420.4058 MHz due to two closely spaced energy levels in the ground state of the hydrogen atom. (Source: Wikipedia)
The origin of this spectrum is the spin-spin coupling between the proton and the electron in the hydrogen atom. Remember that while discussing spin-orbit interaction we neglected the proton spin. The respective magnetic dipole moments are given by,

$$\vec{\mu}_p = \frac{g_p e}{2m_p}\vec{S}_p, \qquad \vec{\mu}_e = -\frac{e}{m_e}\vec{S}_e,$$

The g-factor in case of proton is 5.59 (it is 2 for electron). From classical electrodynamics we know that a magnetic dipole with moment μ produces a magnetic field $\vec{B}$ as,

$$\vec{B} = \frac{\mu_0}{4\pi r^3}[3(\vec{\mu}.\hat{r})\hat{r} - \vec{\mu}] + \frac{2\mu_0}{3}\vec{\mu}\delta(\vec{r}).$$

Write down the Hamiltonian of the electron in the magnetic field due to the proton's magnetic dipole moment. Calculate the first order correction to the energy due to this Hamiltonian.
In the presence of spin-spin coupling, individual $\vec{S}_e$ or $\vec{S}_p$ is not conserved, but the total $\vec{S} = \vec{S}_e + \vec{S}_p$ is conserved. Since both electron and proton are spin-1/2 particles, the total spin $\vec{S}^2$ can have eigenvalues $2\hbar^2$ (i.e., total spin 1, triplet state, spins "parallel") and 0 (i.e., total spin 0, singlet state, spins "anti-parallel"). Calculate the difference between the energies (ΔE) of triplet and singlet states. If a photon is emitted in a transition from the triplet to the singlet state, what would be the frequency of that electron? Also find out the wavelength of the corresponding radiation.