Advanced Quantum Mechanics

Assignment-III

1. Calculate the differential and total cross section in the first Born approximation,

   (i) for a Coulomb potential $V(r) = Z_1 Z_2 e^2/r$, where $Z_1 e$ and $Z_1 e$ are the charges of the projectile and target particles, respectively.

   (ii) for the Yukawa potential $V(r) = V_0 e^{-r/R}/r$.

   (iii) for the potential $V(r) = Ae^{-r^2/a^2}$.

2. Consider an attractive delta-shell potential ($\lambda > 0$),

$$V(r) = -\frac{\hbar^2 \lambda}{2M}\delta(r-a).$$

   (i) Calculate the phase shift $\delta_l(k)$, where l is the angular momentum quantum number.

   (ii) In the case $l = 0$, investigate the existence of bound states by examining the analytic properties of the partial scattering amplitude. Are there any resonances?

   (Note that bound states will correspond to imaginary values of the wave number, i.e. negative energies. Resonances, on the other hand, correspond to positive energy values that make the partial cross section maximal.)

3. A diatomic molecule, such as H_2 or O_2, consisting of two identical atoms is modelled by two identical spherically symmetric scattering centres separated by a distance R. Suppose the scattering amplitude for scattering an electron off one of these centres is known to be $f(0)$. Find the scattering cross-section for scattering off the molecule. You can neglect the effect of multiple scattering.