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Nuclear Reactions and Nuclear Fusion

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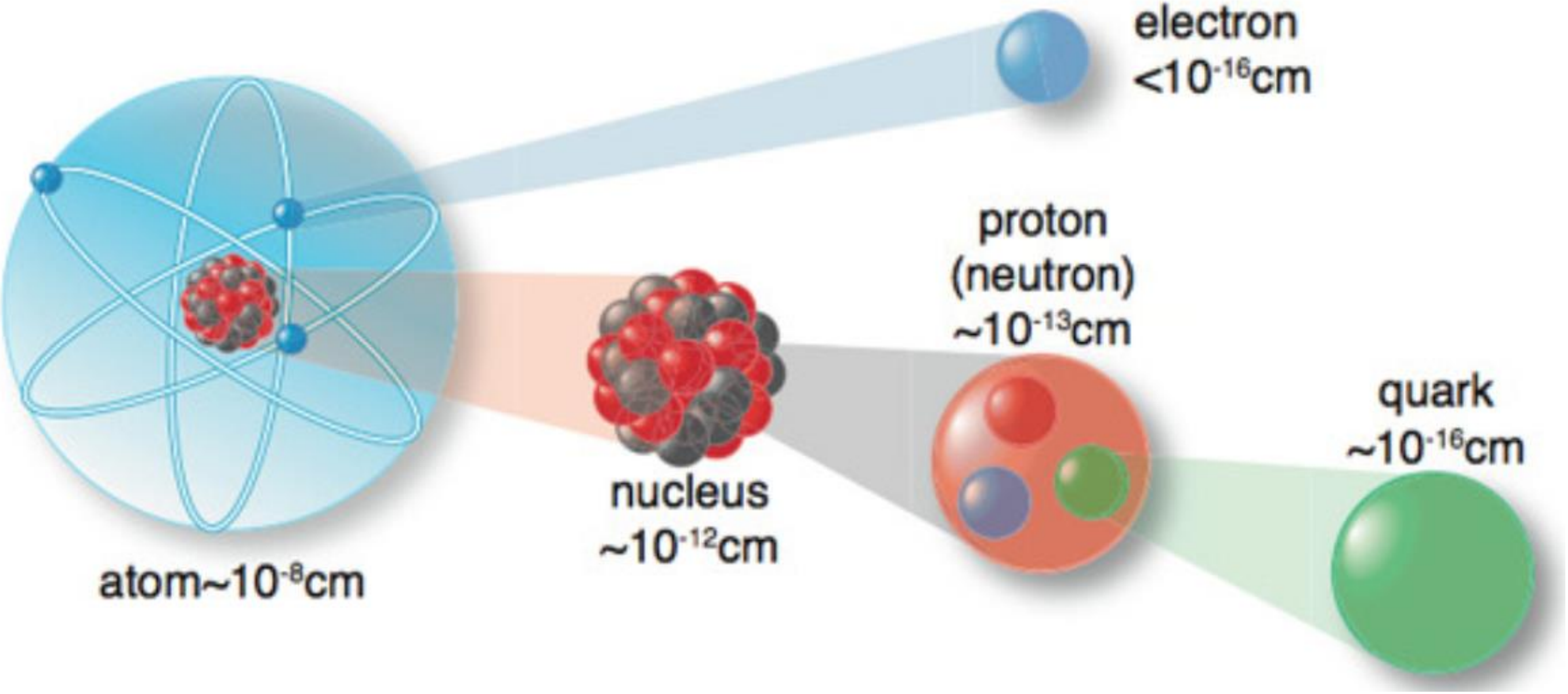
Introduction



Introduction: nuclear Binding Energy

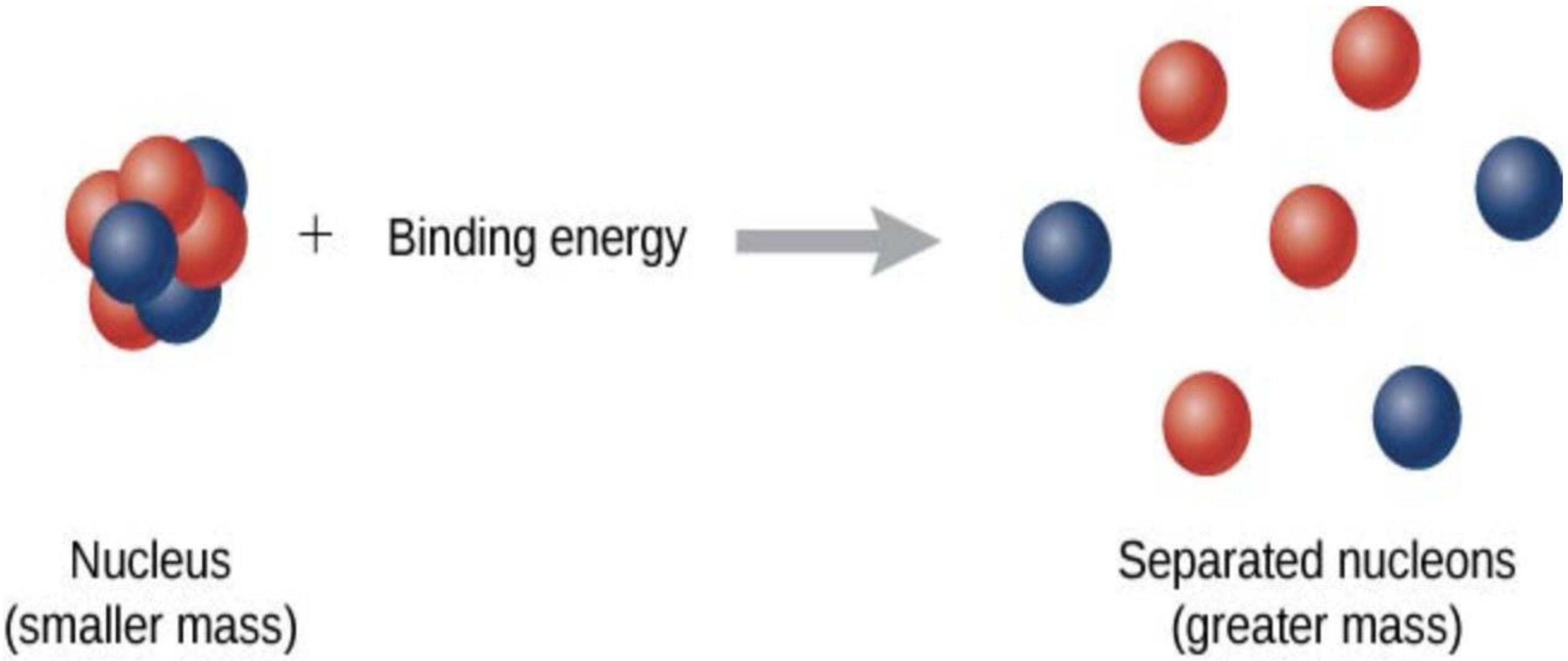
Mass-Energy Equivalence :

$$E = mc^2$$

This fundamental equation relates mass (m) to energy (E), where c is the speed of light in a vacuum (~*3×10⁸ m/s*).

Nuclear Binding Energy (BE) :

BE is the energy required to break up a nucleus into its constituent protons and neutrons.



BE is given by the difference in mass energy between the nucleus and its constituents

$$\mathrm{BE(Z,A)} = \left[Zm({}^1_1H) + Nm_n - M(Z,A)\right]C^2$$

Z: Number of protons, N: Number of neutrons, $m({}^1_1H)$: Mass of a hydrogen, m_n : Mass of a neutron, $M(Z,A)$:Mass of the nucleus.

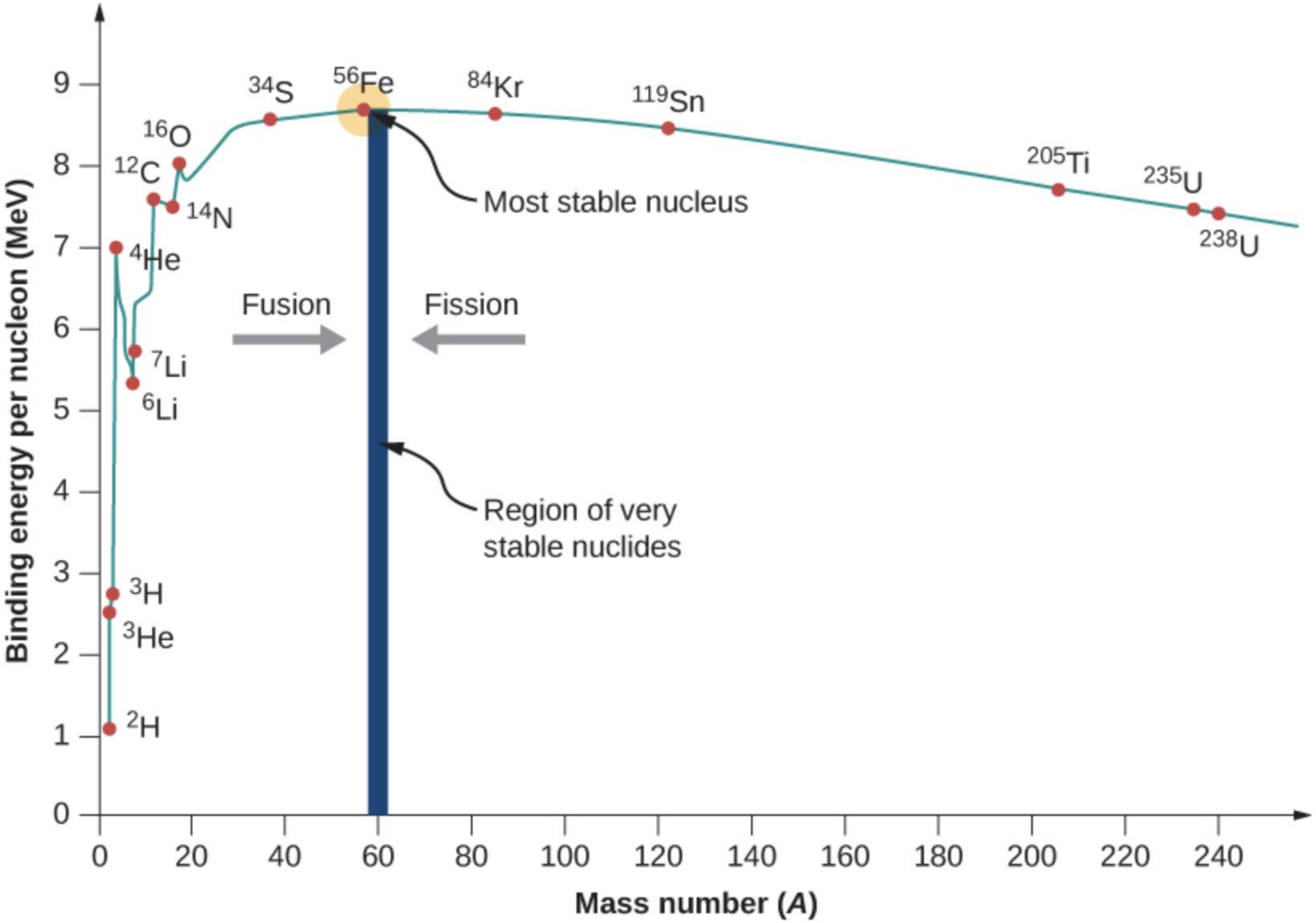
Introduction: Binding Energy per nucleon $\frac{BE}{N}$ and nuclear stability

- **Most stable** : the highest $\frac{BE}{N}$, mass numbers around 56 (like Iron-56).
- **Nuclear reaction:**
- **Fission:** a heavy nucleus (like uranium-235) splits into smaller nuclei, the binding energy per nucleon of the resulting fragments is higher than that of the original nucleus.

$$ {}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\,{}_{0}^{1}n + \mathrm{Q} $$

- **Fusion:** When light nuclei (like carbon-12 and helium-4) combine to form a heavier nucleus (like oxygen), the binding energy per nucleon of the resulting nucleus is higher than that of the original nuclei..

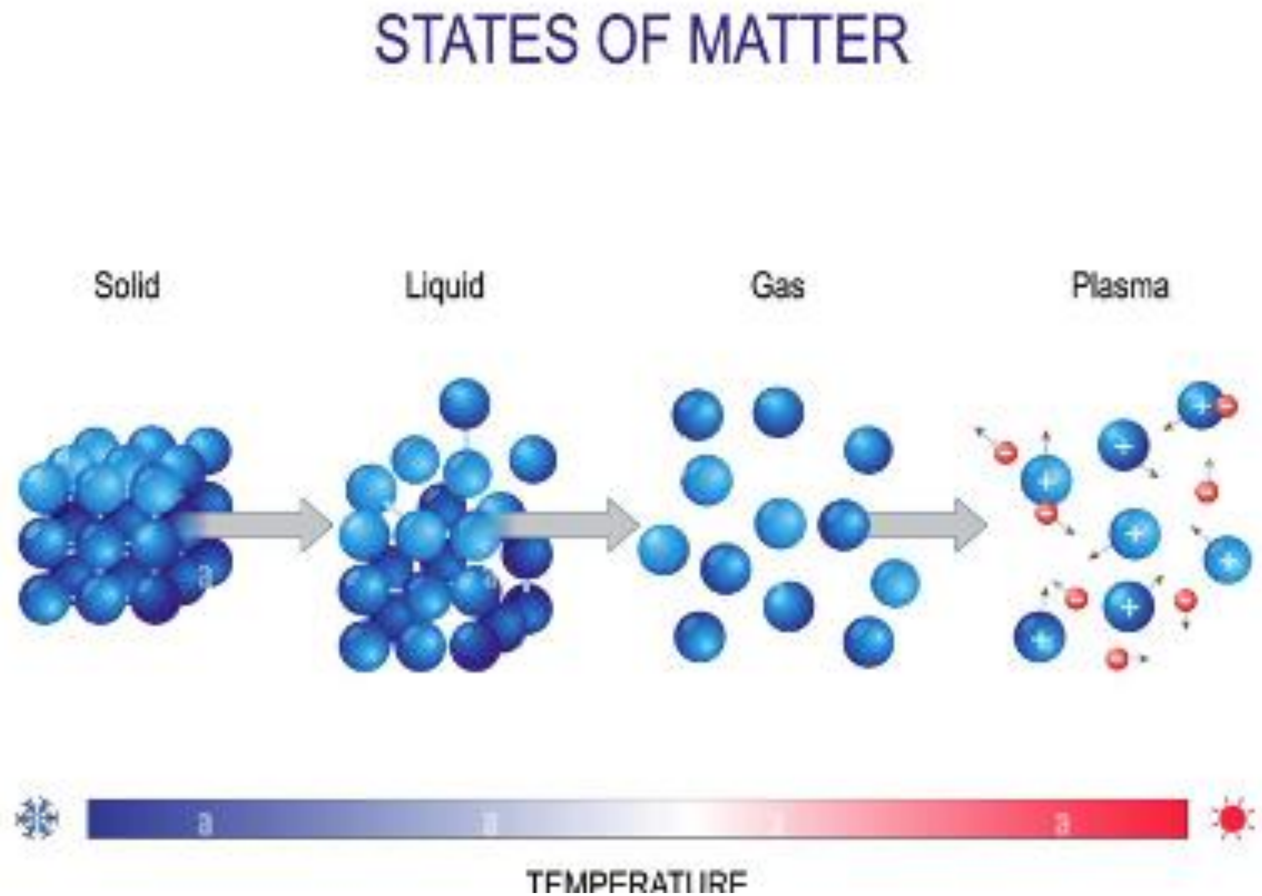
$$ {}_{6}^{12}C + {}_{2}^{4}He \rightarrow {}_{8}^{16}O + \mathrm{Q} $$



The plot shows binding energy per nucleon versus atomic mass number A

Introduction: Plasma

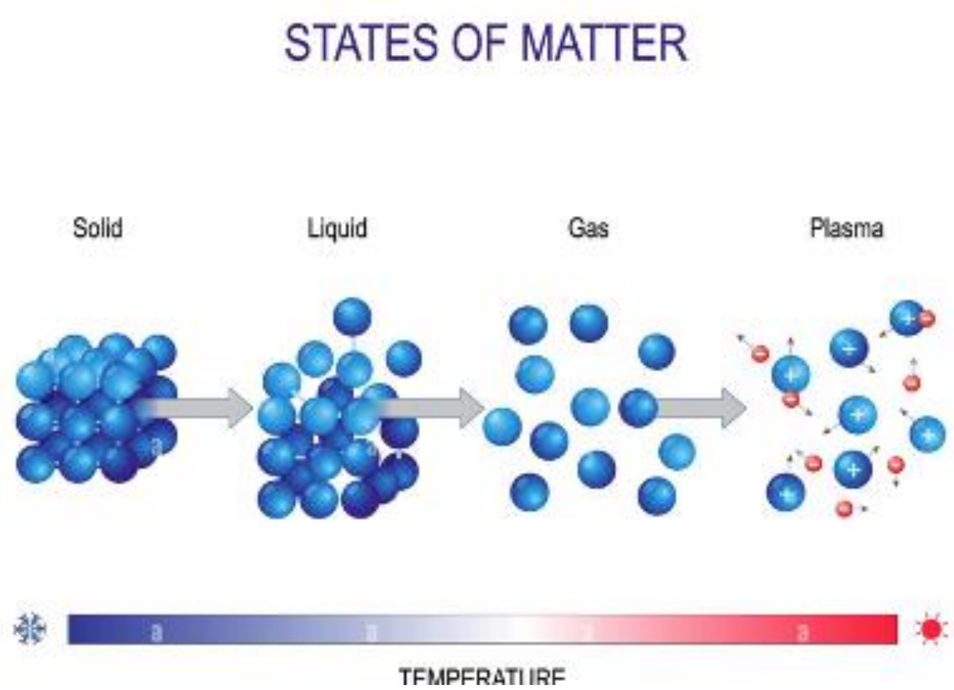
- **Plasma** is fourth state of matter, It is an **ionized fluid,** consisting of free electrons and ions
- Interact over long distances through electromagnetic forces.
- Exists at very high temperatures, where thermal energy is sufficient to ionize atoms.
- Temperatures greater than 10^3 degrees Celsius.



Introduction: Plasma

- The plasma is a dominate state of matter in the Sun.
- The core’s temperature is extremely high approximately 15 million degrees Celsius

- The density in the core is about 150 (g/cm³), which is around 150 times the density of liquid water.
- Plasma is fuel of fusion reaction
- In experimental fusion reactors (tokamaks) gases like deuterium and tritium are heated to millions of Kelvin, turning them into plasma



Nuclear Reactions

- The nuclear reaction is two nuclei or nuclear particles collide, to produce different products than the initial particles.

$$\mathbf{a} + \mathbf{X} \rightarrow \boldsymbol{b} + \boldsymbol{Y}$$

- a : the accelerated projectile,
- X : the target,
- Y and b : the reaction products
- **Kinds of reaction:**
- **Scattering:** the incident and outgoing particles are the same
    - Elestic : the incident and target particles remain in their ground states with conserved kinetic energy.
    - Inelastic: : the incident and/or target particles end up in excited states, resulting in a loss of kinetic energy.
- **Direct reactions,** only very few nucleons take part in the reactions, the others being just spectators
- **Compound reaction :**The incoming particle is fully absorbed by the nucleus, forming an intermediate compound nucleus before it decays by emitting particles.
- **Resonance reactions,** the incoming particle forms a “quasibound” state before the outgoing particle is ejected.

Energy of nuclear reaction

- The energy conservation for the nuclear reaction:

$$\mathbf{a+X \rightarrow \boldsymbol{b}+\boldsymbol{Y}}$$

$$\mathbf{m_X c^2 + T_X + m_a c^2 + T_a = m_Y c^2 + T_Y + m_b c^2 + T_b}$$

where the T ’s are the kinetic energies.

- We also define the reaction Q value

$$\boldsymbol{Q = \left(m_{iniial} - m_{final}\right)}\mathbf{c^2 = (m_X + m_a - m_Y - m_b)c^2}$$

And

$$\boldsymbol{Q = T_{final} - T_{iniial}} \mathbf{= T_Y + T_b - T_X - T_a}$$

- If $\boldsymbol{Q > 0}$, the reaction is exothermic or exoergic. Nuclear mass or binding energy is released as kinetic energy of the final products.
- If $\mathbf{Q < 0}$, the reaction is endothermic or endoergic and initial kinetic energy is converted into nuclear mass or binding energy.

Nuclear Reaction: Cross section

- The **cross section** is the probability that any interaction occurs between the incoming particle and the target nucleus.

- **The nuclear Cross-Section:** $\boldsymbol{\sigma = \frac{R}{\Phi\, n}}$

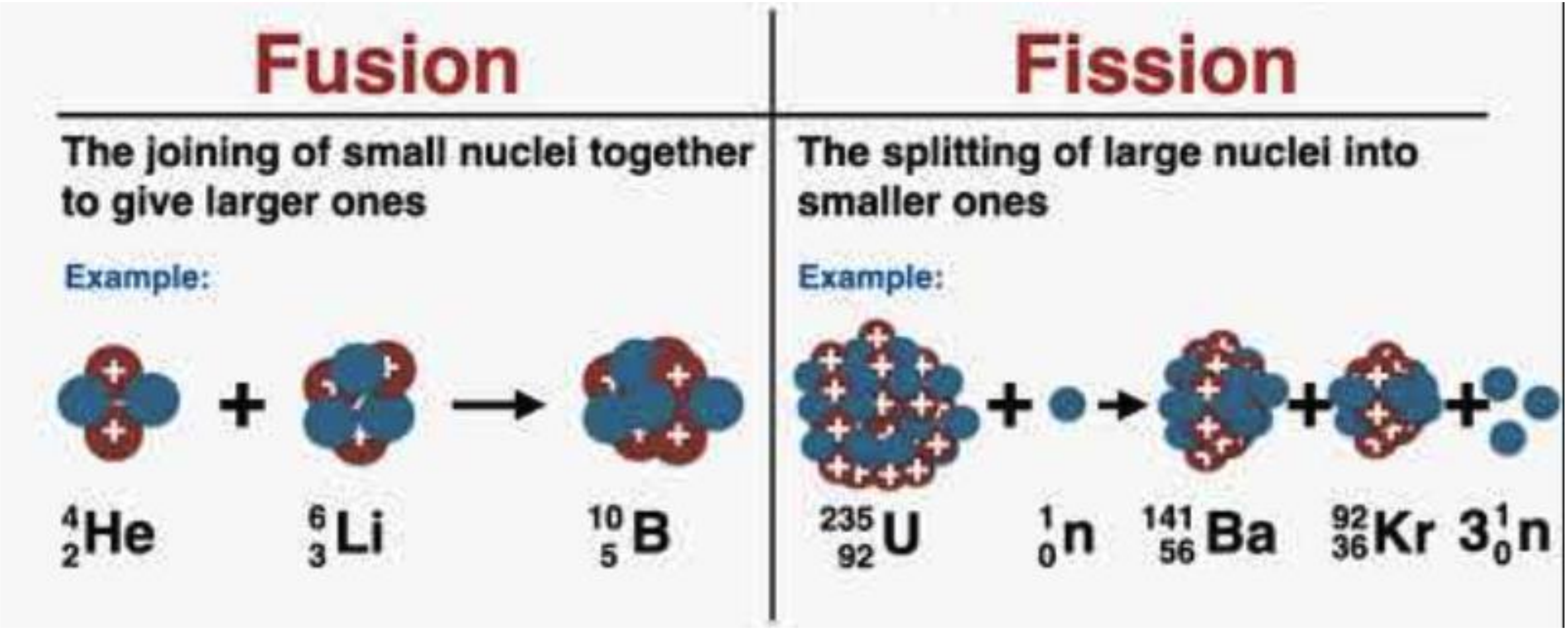
$\boldsymbol{R}$: number of reactions per unit time,

$\boldsymbol{\Phi}$: Flux of incoming particles, number of particles passing through a unit area per unit time

$\boldsymbol{n}$: the number of target nuclei per unit area.

- The unit of cross sections is **barn (b)**

Types of Nuclear Reactions



Fusion

The joining of small nuclei together to give larger ones

Example:

$${}^{4}_{2}\text{He} + {}^{6}_{3}\text{Li} \rightarrow {}^{10}_{5}\text{B}$$

Fission

The splitting of large nuclei into smaller ones

Example:

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$$

Nuclear Fission

- Refers to the splitting of an atomic nucleus into two or lighter nuclei.

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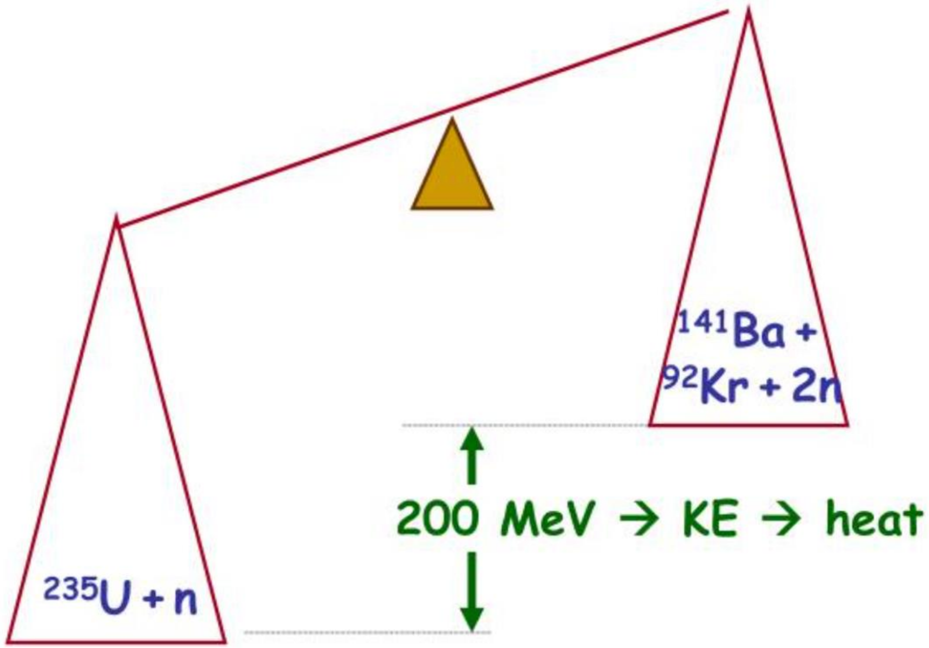
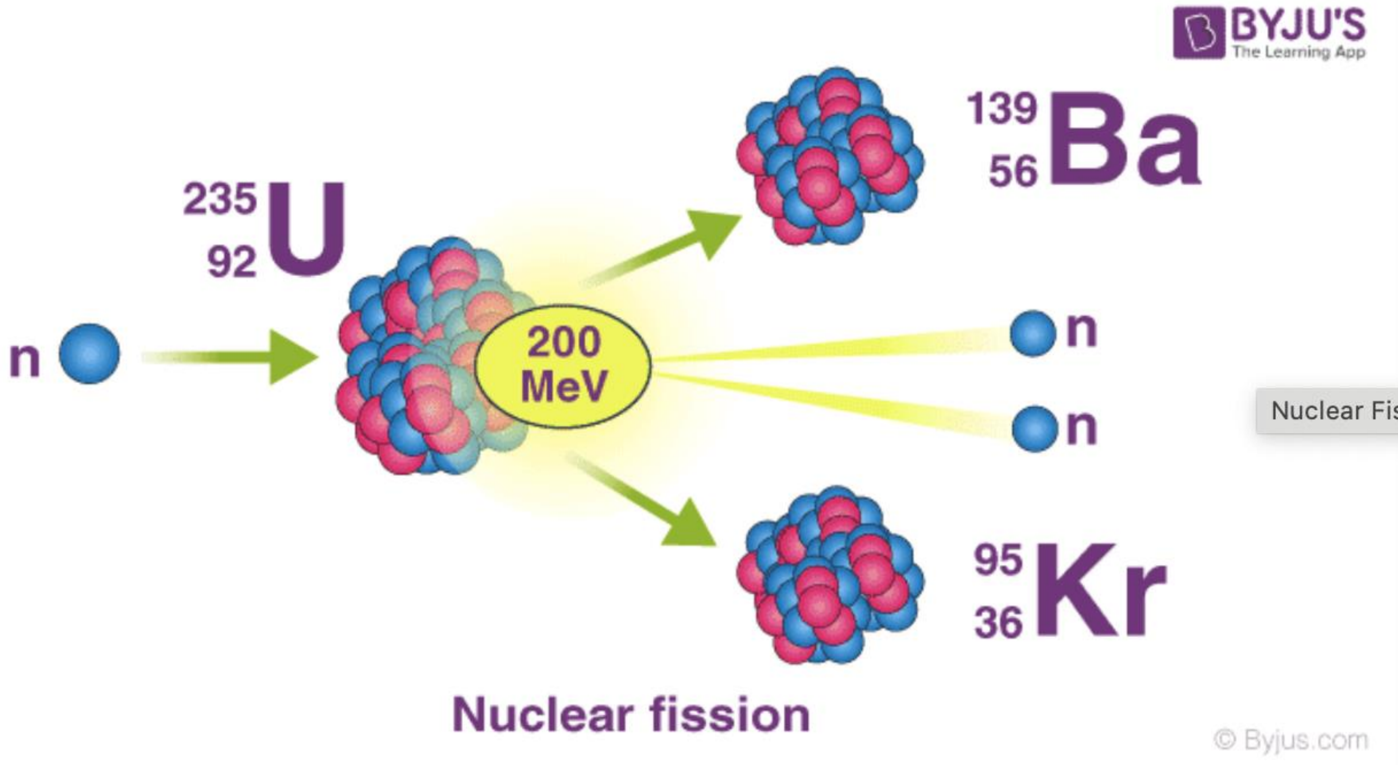
- Is accompanied by the emission of neutrons and gamma rays



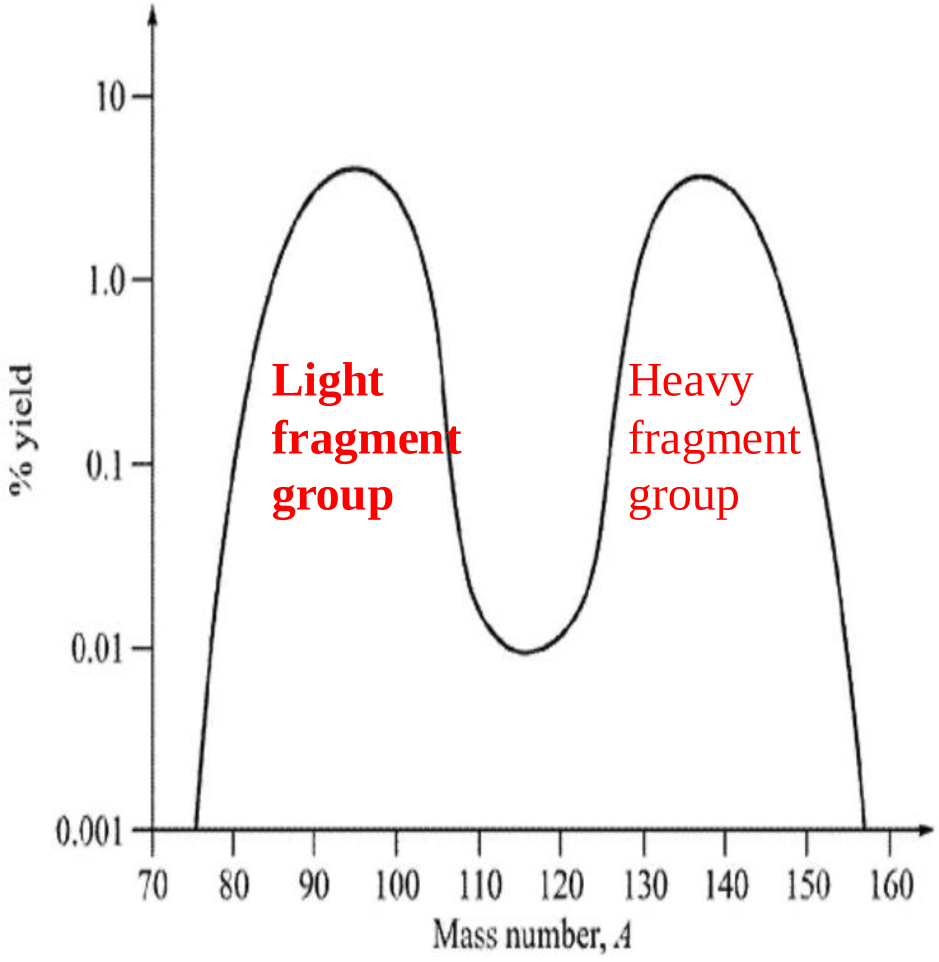


- Nuclear fission was first discovered by the German chemists Otto Hahn and Fritz Strassmann in the year 1938.

Nuclear Fission: products mass distribution

- The Mass distribution of fission products : light fragment and heavy fragment
- About 97% of the total fission products fall within range from **85 to 105** (lighter group) and range from **130 to 150** (heavier group).

$$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{141}\mathrm{Ba} + {}^{92}\mathrm{Kr} + 3\,{}^{1}\mathrm{n}$$
$$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{144}\mathrm{Xe} + {}^{90}\mathrm{Sr} + 2\,{}^{1}\mathrm{n}$$
$$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{146}\mathrm{La} + {}^{87}\mathrm{Br} + 3\,{}^{1}\mathrm{n}$$
$$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{137}\mathrm{Te} + {}^{97}\mathrm{Zr} + 2\,{}^{1}\mathrm{n}$$
$$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{137}\mathrm{Cs} + {}^{96}\mathrm{Rb} + 3\,{}^{1}\mathrm{n}$$



Per cent yield of different fission fragments in fission of ^{235}U.

Nuclear Fission: the energy released

For nuclear fission:

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{95}_{42}\mathrm{Mo} + {}^{139}_{57}\mathrm{La} + 7\,{}^{0}_{-1}\mathrm{e} + 2\,{}^{1}_{0}\mathrm{n} $$

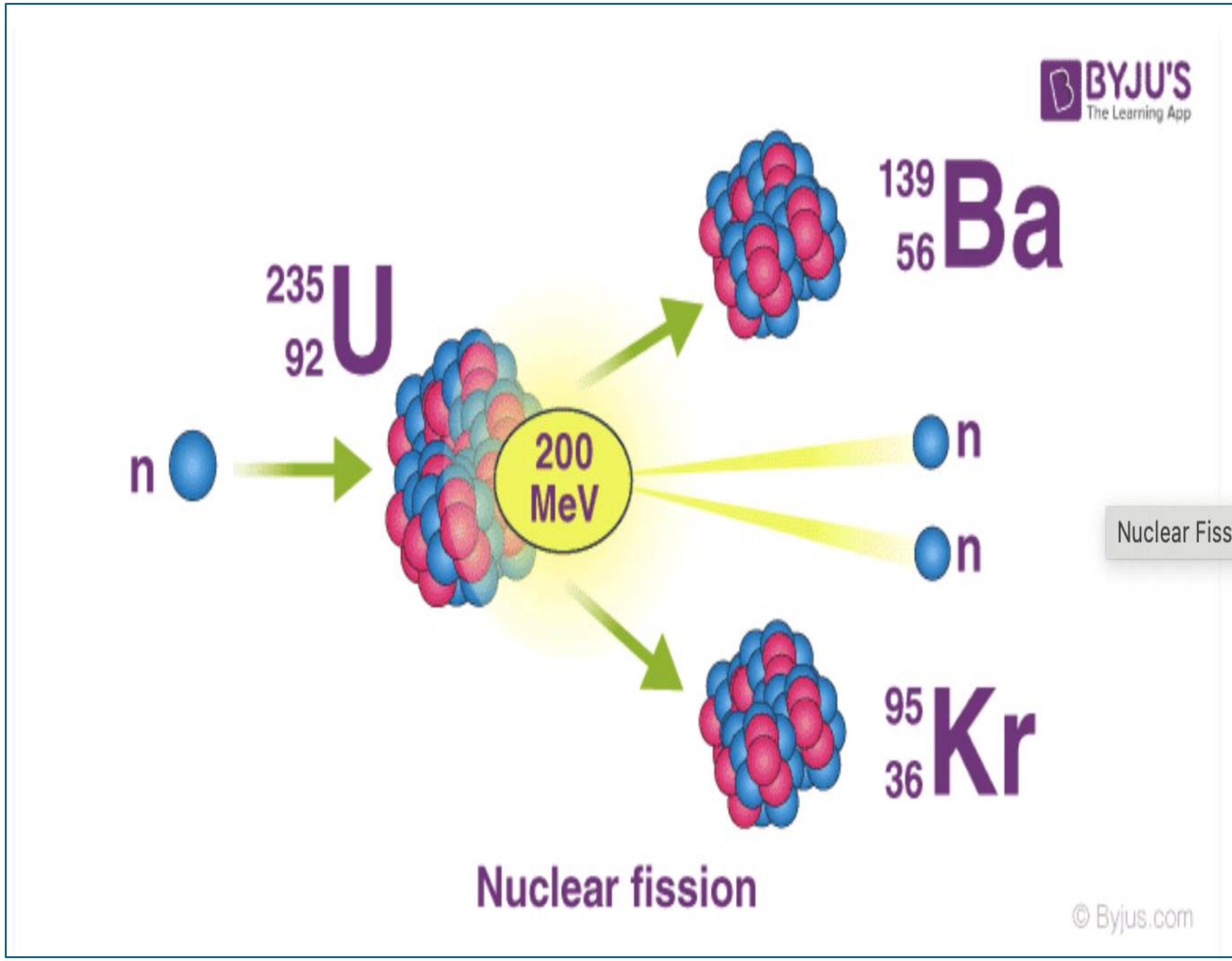
$$ Q = \Delta mc^2 = 0.219c^2 * 931.481/c^2 $$

$Q = 204\,MeV$ **(exothermic)**

For one gram:

The energy release 8.2 $X10^{10}$ **Joules.**

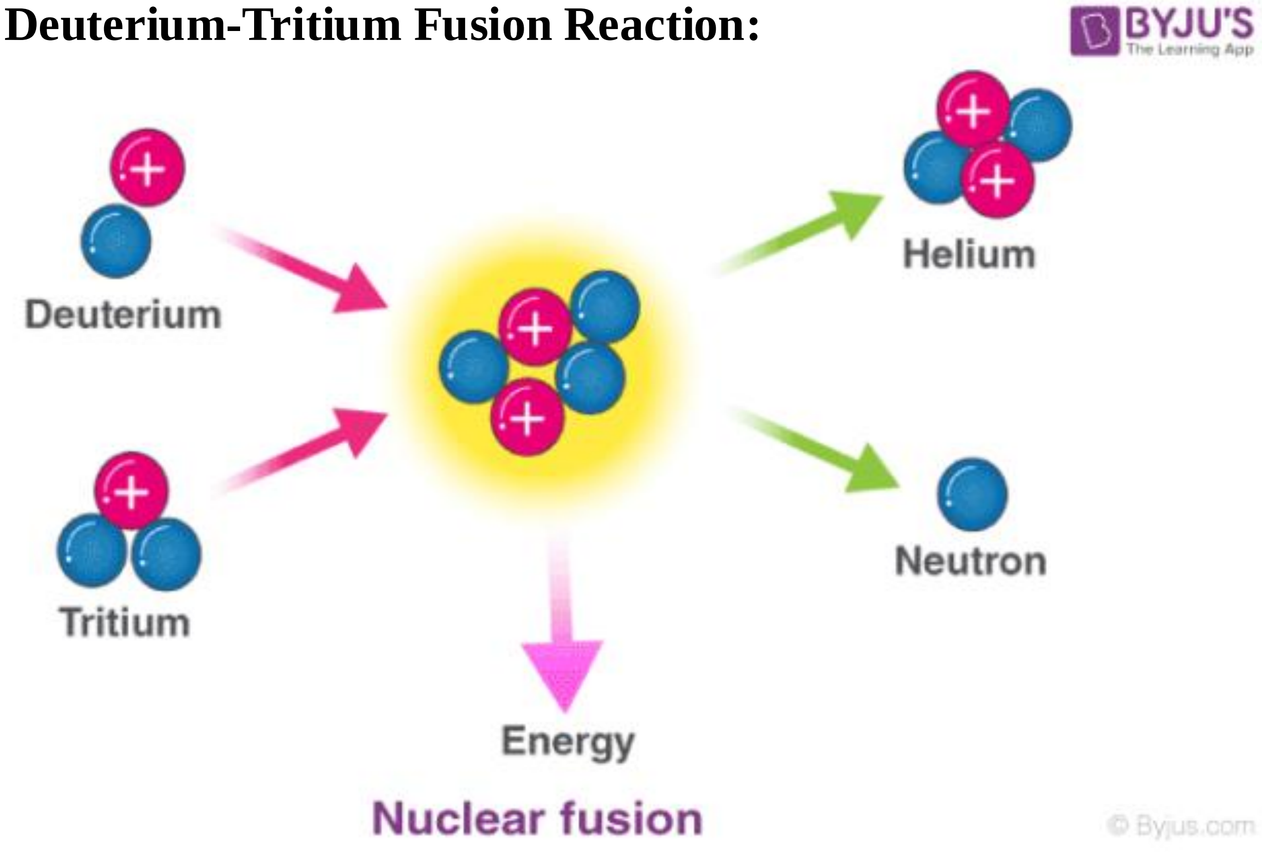
Burning approximately 3.4 tons of coal



Nuclear Fusion: introduction

- Fusion is the process by which two light nuclei combine to form a heavier nucleus.
- The two nuclei momentarily form a compound nucleus in an excited state, which then decays into the final product.
- fusion is not a natural process on earth
- Fusion was first observed by Mark Oliphant, Paul Harteck and Ernest Rutherford in 1933.

Deuterium-Tritium Fusion Reaction:



D +T → He + n + 17.6 MeV

This reaction releases 17.6 MeV of energy (Q>0, exothermic)

Nuclear Fusion: introduction

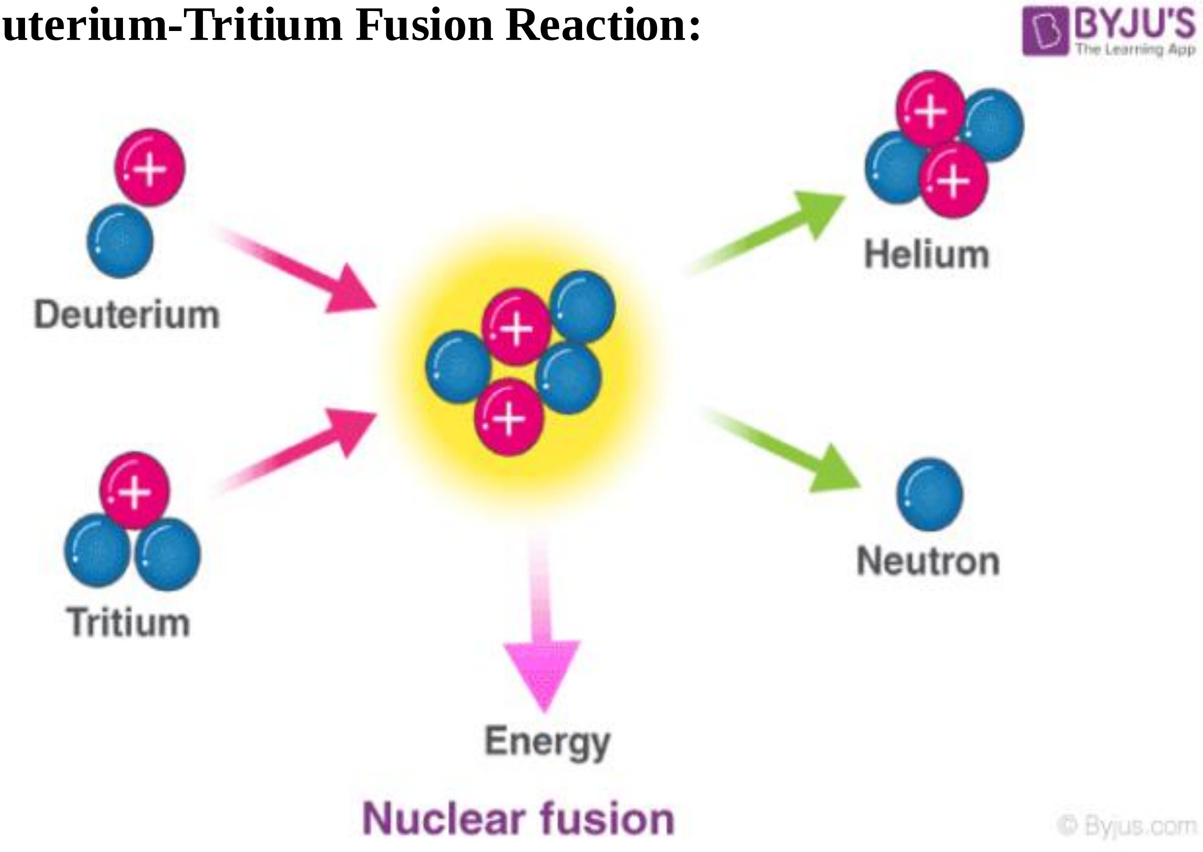
- In nuclear fusion:

$$D + T \rightarrow He + n + 17.6\ \text{MeV}$$

- $Q = \Delta mc^2$
  $Q = 0.0188\ \text{u} * 931.481/u$
  $Q = 17.6\ \text{MeV}$

- For one gram of helium : the energ release 4.24×10^{11} J
- Burning approximately 17.7 tons of coal

Deuterium-Tritium Fusion Reaction:



$D + T \rightarrow He + n + 17.6\ \text{MeV}$

This reaction releases 17.6 MeV of energy (Q>0, exothermic)

Characteristics of fusion : Energy Release Q

- For the nuclear reaction

$$\mathbf{a+X=\boldsymbol{b}+\boldsymbol{Y}}$$

- Total final energy of the product particles will be $\boldsymbol{Q}$

$$\boldsymbol{Q=T_b+T_Y=\frac{1}{2}m_b v_b^2+\frac{1}{2}m_Y v_Y^2}$$

- if we neglect the initial motions, the final momenta are equal and opposite

$$\vec{p}_b=\vec{p}_Y$$

$$m_b\ v_b = m_Y\ v_Y$$

Characteristics of fusion : Energy Release Q

The lighter product particle takes the largest part of the energy

$$T_b = \frac{Q}{1 + \frac{m_b}{m_Y}}$$

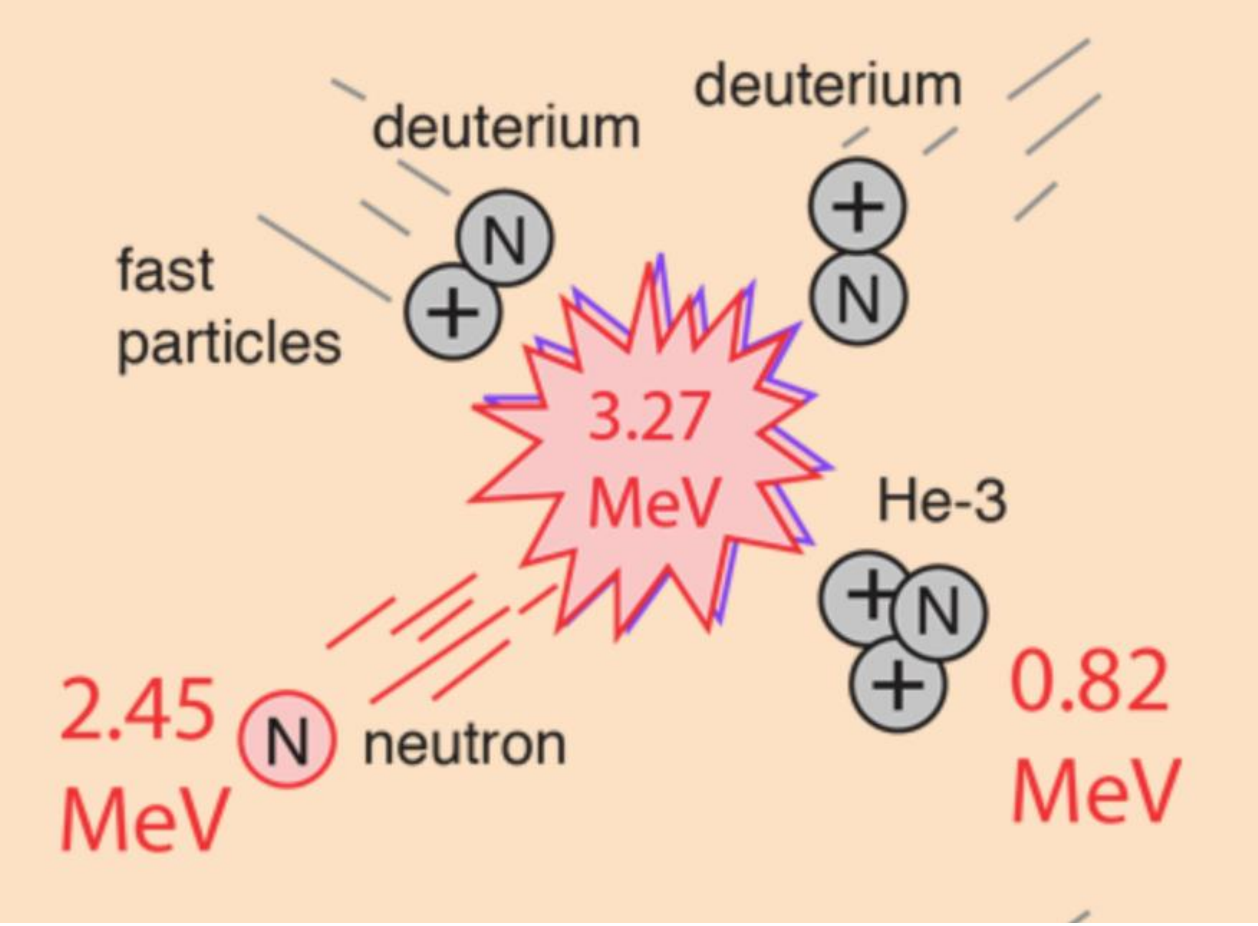
$$\mathrm{T_Y} = \frac{\mathrm{Q}}{1 + \frac{\mathrm{m_Y}}{\mathrm{m_b}}}$$

Characteristics of fusion : Energy Release

- **Deuterium-Deuterium Fusion Reactions:**

$$\mathbf{D + D \rightarrow He^{3} + n + 3.27\ MeV}$$

- The reaction releases **Q = 3.27 MeV** for production Helium-3 and a neutron,
- **~2.45MeV** is carried by neutron and
- ~**0.82 MeV** is carried by Helium.

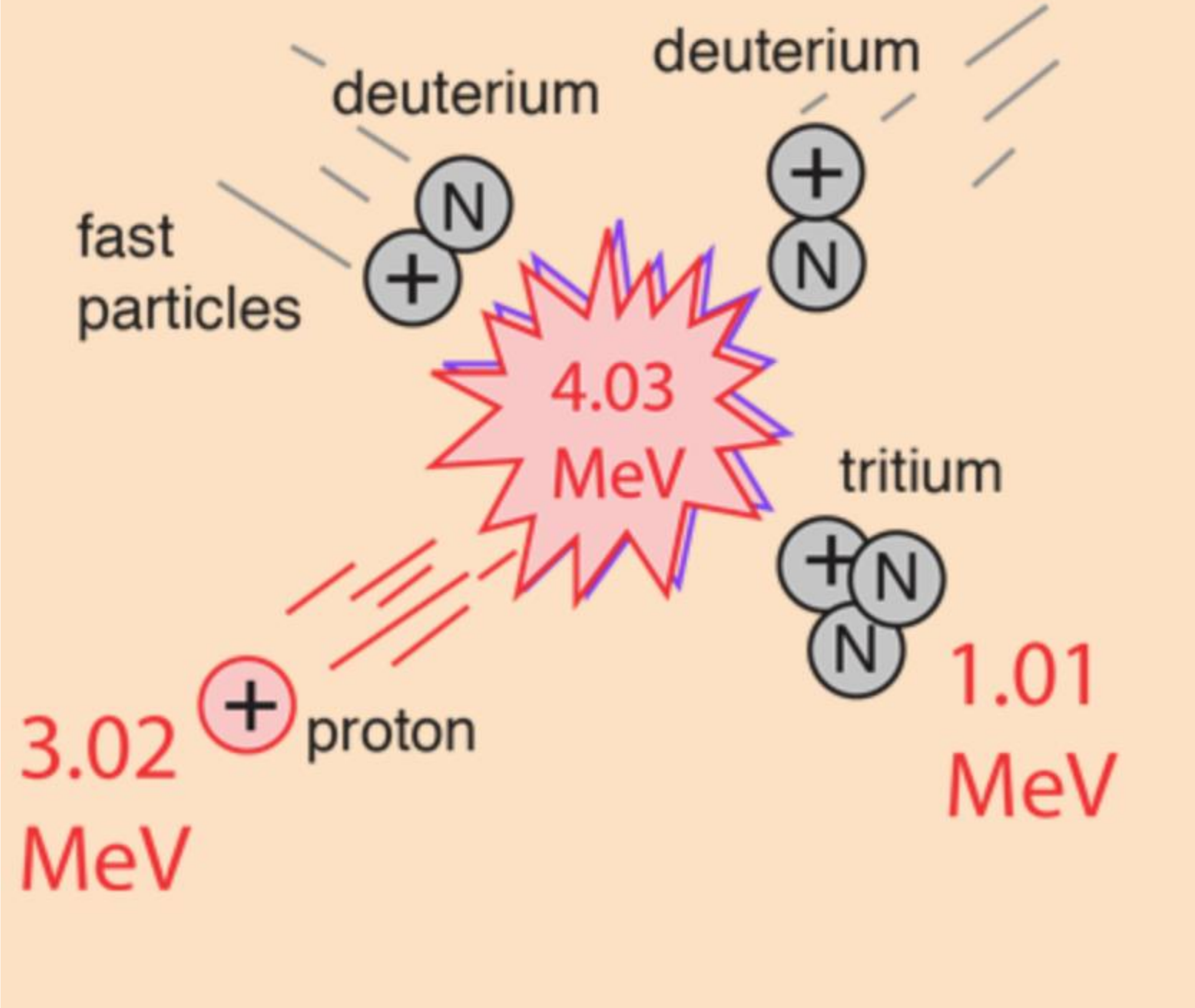


Characteristics of fusion : Energy Release

- **Deuterium-Deuterium Fusion Reactions:**

$$\mathbf{D + D \rightarrow T + p + 4.03\ MeV}$$

- The reaction releases **Q= 4.03 MeV** for production Tritium and a proton
- **~3.02MeV** is carried by proton
- ~**1.01 MeV** is carried by Tritium.

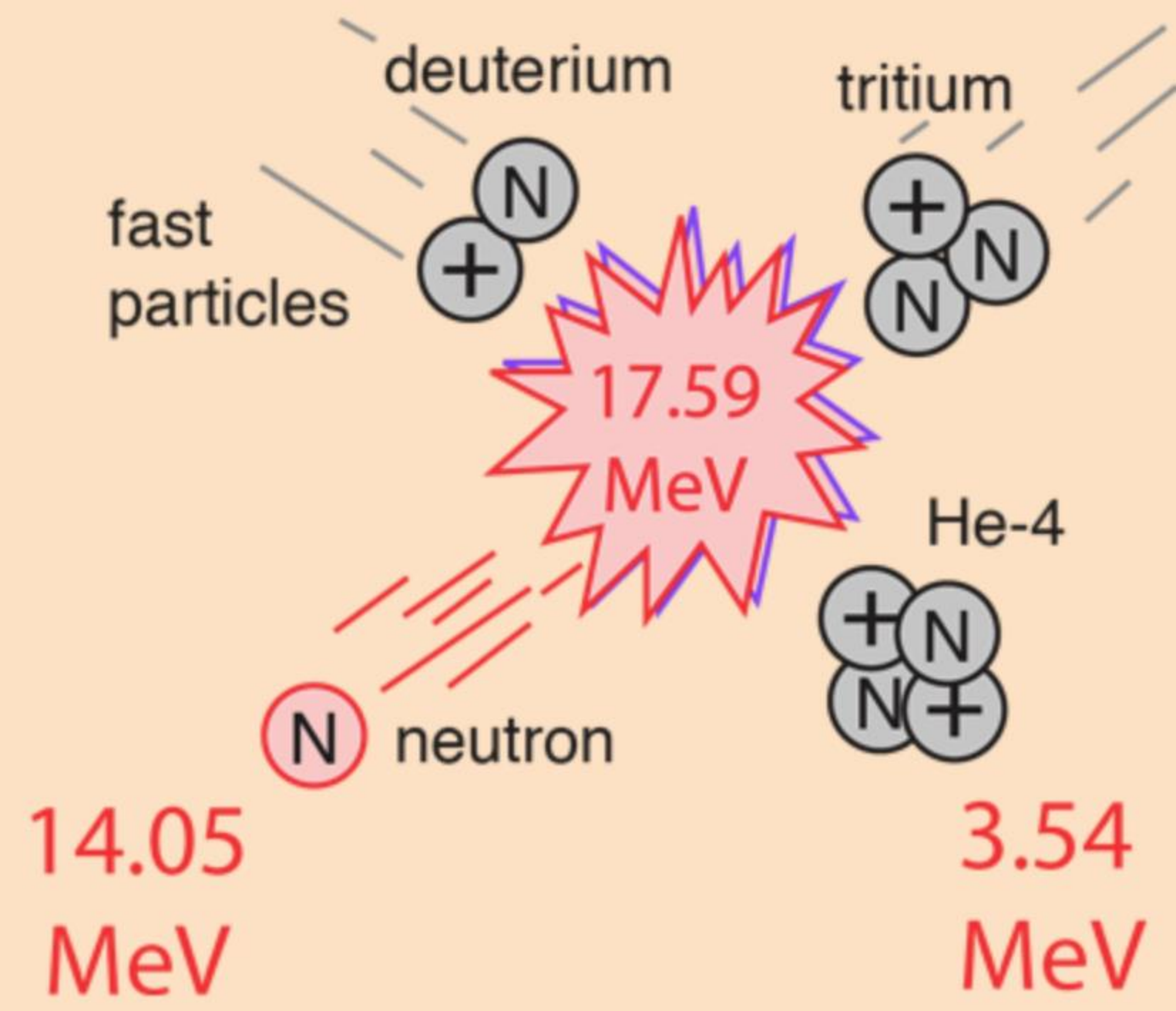


Characteristics of fusion : Energy Release

- **Deuterium-Tritium Fusion Reaction:**

  D +T → He + n + 17.6 MeV

- This reaction releases **Q=17.6 MeV** of energy,
- **14.05MeV** is carried by neutron
- **3.54 MeV** is carried by Helium



Conditions for Nuclear Fusion

- **Overcome Coulomb barrier.**

  Maintain kinetic energy by high temperature.
- **High collision rate**

  Maintain the density of nuclei by high pressure.
- **Containment**

  By magnetic confinement (e.g., tokamaks)

  By inertial confinement (e.g., laser fusion).

Characteristics of fusion: Coulomb barrier

- **Coulomb barrier**—the electrostatic repulsion between the positively charged protons in the nuclei.
- To **nuclear force** overcomes the **barrier**, the particles need sufficient **kinetic energy** to get close enough fo fuse
- If R_a and R_X are the radii of the reacting particles in

$$\mathrm{a} + \mathrm{X} \rightarrow b + Y$$

Coulomb Barrier Formula:

$$V_c = \frac{\mathbf{e^2}}{\mathbf{4\pi\epsilon_0}} \frac{\mathbf{Z_a Z_X}}{\mathbf{R_a + R_X}}$$

- The barrier for deuterium-tritium reactions:
  - $V_c = 9 \times 10^9 \frac{(1.6\times10^{-19})^2}{1.2fm(2^{1/3}+3^{1/3})} = 7.155 \times 10^{-14} J = 0.45$ MeV.
- Where $R = 1.2fm\, A^{1/3}$
- Note: at room temperature T=300K, kT is about 0.025 eV

Characteristics of fusion: Coulomb barrier

- **High Temperature:** Needed to provide kinetic energy to overcome Coulomb barrier

$$\boldsymbol{V_c = \frac{3}{2} k_B T}$$

- k_B is the Boltzmann constant $(8.617 \times 10^{-5} \frac{eV}{K})$
- T is the temperature in Kelvin.
- V_c is the Coulomb barrier energy.

- $T = \frac{2\times 0.45\times 10^{6} eV}{3\times 8.617\times 10^{-5} eV} = 3.5 \times 10^{9} K$
- **For deuterium-tritium fusion :**
- **Temperature Required**: Approximately **3.5 billion Kelvin** ($3.5 \times 10^{9} K$).
- **For Proton-proton fusion :.**
- **Temperature Required**: Approximately **110 million K (**$1.1 \times 10^{8} K$**)**.

Thermonuclear fusion reactor : Reaction Rate

- The reaction rate $\boldsymbol{R}$ depends on several factors, including the density of the reactants, the temperature, and the cross-section of the fusion reaction.

- **Fusion Reaction Rate:** $\boldsymbol{R = n_1 n_2 \langle \sigma v \rangle}$

  $\boldsymbol{n_1, n_2}$: number densities per unit volume,
  $\boldsymbol{v}$ the relative velocity of the reacting particles
  $\boldsymbol{\sigma}$ is the fusion cross-section as a function of relative velocity v,
  $\boldsymbol{\langle \sigma v \rangle}$: Average reaction rate .

- **Average reaction rate** $\boldsymbol{\langle \sigma v \rangle}$:

$$\boldsymbol{\langle \sigma v \rangle = \int \sigma v \, f(v) dv}$$

- $\boldsymbol{f(v)}$is the Maxwell-Boltzmann distribution function for the velocity of the particles.

Thermonuclear fusion reactor: Reaction Rate

- **Maxwell-Boltzmann Distribution:**

$$f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 exp\left(-\frac{mv^2}{2k_B T}\right)$$

The distribution of particle velocities in a gas at thermal equilibrium,

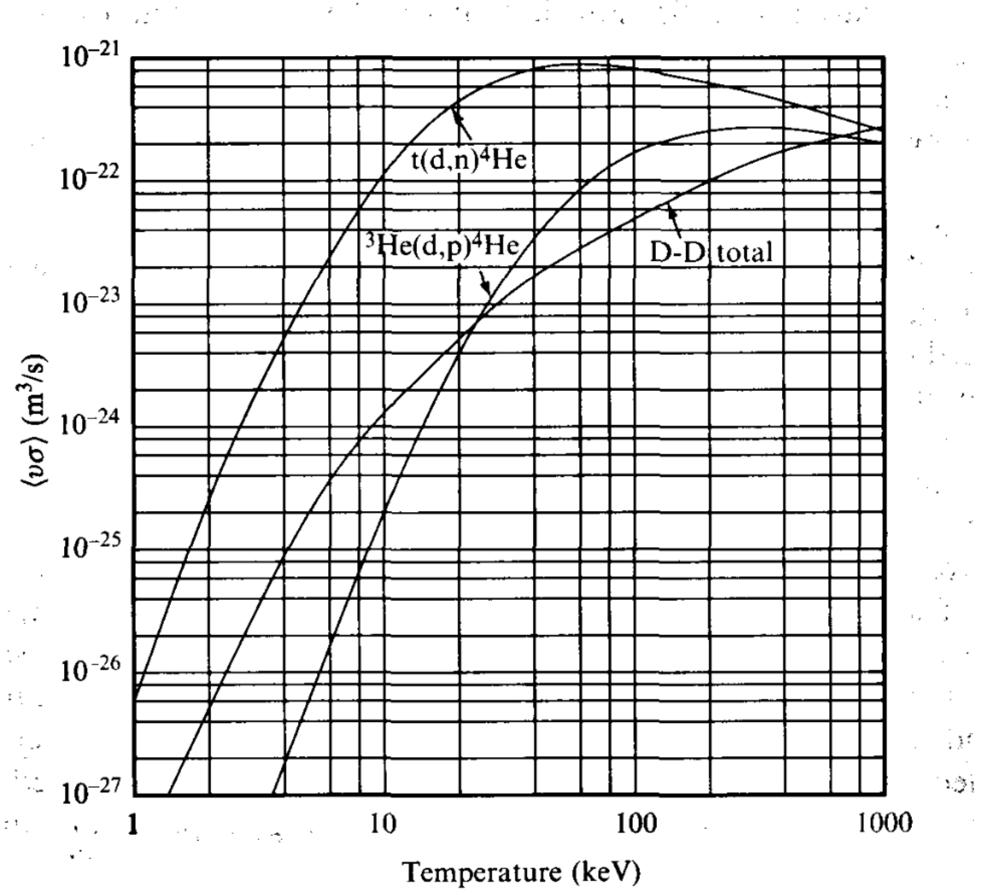
m: mass of a particle,

k_B : Boltzmann constant,

T : Temperature,

v : Velocity.

- the D-T reaction may become less favorable than others, but in the temperature region that is likely to be achievable in a thermonuclear fusion reactor (1-10 keV or T - 10^7-10^8 K) the D-T reaction is clearly favored.



Fusion reaction rate σv, averaged over the Maxwell-Boltzmann distribution, for several fusion reactions

Thermonuclear fusion reactor: Power Density

- **Power density $\boldsymbol{P}$:** produced by fusion reactions in a plasma,

$$\boldsymbol{P = n_i n_j \langle \sigma v \rangle Q}$$

$\boldsymbol{n_i}$, $\boldsymbol{n_j}$Number densities of interacting species,
$\boldsymbol{\langle \sigma v \rangle}$: Rate coefficient for the fusion reaction,
$\boldsymbol{Q}$: Energy released per reaction.

- For a D-T fusion plasma, with equal densities for deuterium and tritium:

$$\boldsymbol{P = n^2 \langle \sigma v \rangle Q}$$

power density depends strongly on both the plasma density and the reaction rate

Thermonuclear fusion reactor : Lawson Criterion

- **Lawson Criterion:** a balance between plasma density, temperature, and confinement time to achieve self-sustaining fusion reactions and net positive energy output.

$$\boldsymbol{nT\tau \geq \frac{12}{\langle \sigma v \rangle}}$$

$\boldsymbol{n}$: Particle density, $\boldsymbol{T}$: plasma Temperature, $\boldsymbol{\tau}$: Confinement time, $\boldsymbol{\langle \sigma v \rangle}$: Fusion reactivity.

- For **D-T fusion** (Deuterium-Tritium), the Lawson Criterion value

$$\boldsymbol{nT\tau \geq 3 \times 10^{21} s\, m^{-3} keV}$$

Thermonuclear fusion reactor : Lawson Criterion

- For a D-T plasma with a temperature of 10 keV: the product of density and confinement time

$$\boldsymbol{n\tau \geq 10^{20} s\, m^{-3}}$$

- For a D-T plasma : If the plasma temperature is 20 keV and the density is $\boldsymbol{10^{20}\, m^{-3}}$, the confinement time $\boldsymbol{\tau}$ must be at least 0.3 seconds to meet the Lawson Criterion.

Thermonuclear fusion reactor : energy loss

- Bremsstrahlung radiation (energy loss) in plasmas is proportional to $\sqrt{T}\ and\ Z^2$ **atomic number** and **plasma temperature**.
- **Temperature Requirements:**
- **D-T Plasma:** the temperature must be above about 4 keV (400 million K) to achieve a positive energy balance.
- **D-D Plasma:** temperatures need to exceed 40 keV (4 billion K) making it less favorable than D-T.

Artificial nuclear fusion: Magnetic Confinement Reactor.

- Plasma can be confined using a magnetic field. in a doughnut-shaped device .

- Confined plasma can be maintained for several seconds

- Achieve relatively low densities.

- ITER (International Thermonuclear Experimental Reactor):

Artificial nuclear fusion: Inertial Confinement Reactor

- Use of intense laser or ion beams to compress small fuel pellets
- Can achieve very high densities

- Only be maintained for the order of a microsecond.

- National Ignition Facility (NIF)

Nuclear Fusion in Stars

- The core of stars is of sufficiently high temperature and sufficiently dense for fusion to take place.

- There are the two main nuclear fusion cycles that power stars

1. **The Proton-Proton (PP) cycle** : in stars with masses similar to or less than that of the Sun.

2. **The Carbon-Nitrogen-Oxygen (CNO) cycle**:

   in stars more massive than the Sun (greater than about 1.3 solar masses

Nuclear Fusion in sun: (p-p) cycle

- The dominant nuclear fusion process in the Sun, about 99% of the Sun's energy
- Responsible for the conversion of hydrogen into helium.
- The process occurs in the Sun’s core, where temperatures reach about 15 million Kelvin.
- A chain of fusion and β-decay.
- β-decay the source of neutrinos emitted from stars.

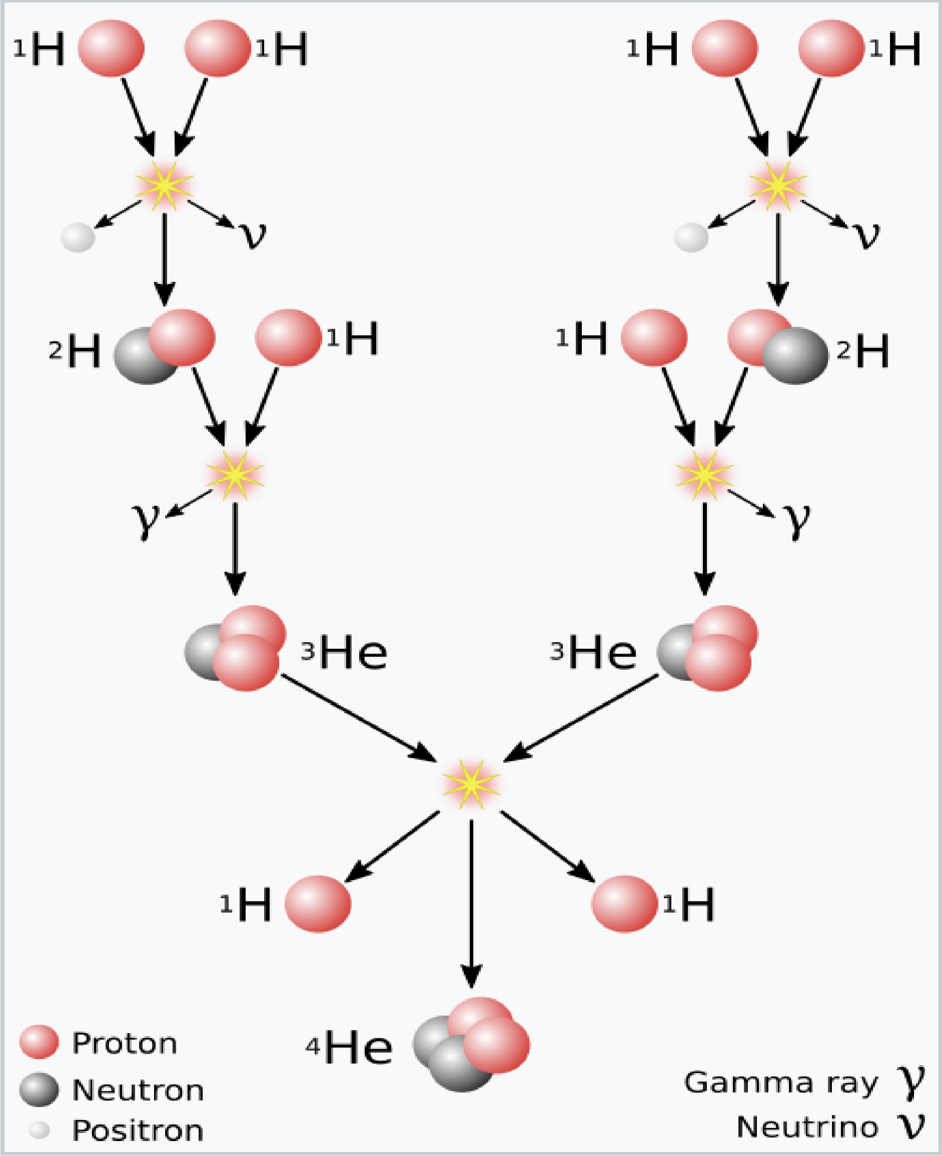
Nuclear Fusion in sun: (p-p) cycle

The P-P cycle involves three stages

- **Stage 1: Proton-Proton Fusion**
- $\boldsymbol{p + p \rightarrow D + e^{+} + v_{e} + 0.42\text{MeV}}$
- **Stage 2: Deuteron-proton Fusion**
- $\mathbf{D}+\boldsymbol{p \rightarrow {}_{2}^{3}He + \gamma + 5.49\ \text{MeV}}$
- **Stage 3: Helium-3 -Helium-3 Fusion**
- $\mathbf{{}_{2}^{3}He + {}_{2}^{3}He \rightarrow {}_{2}^{4}He + 2p + 12.86\ MeV}$

- **Net Reaction of the Proton-Proton cycle**

$$\boldsymbol{4p \rightarrow {}_{2}^{4}He + 2e^{+} + 2v_{e} + 26.7\ \text{MeV}}$$



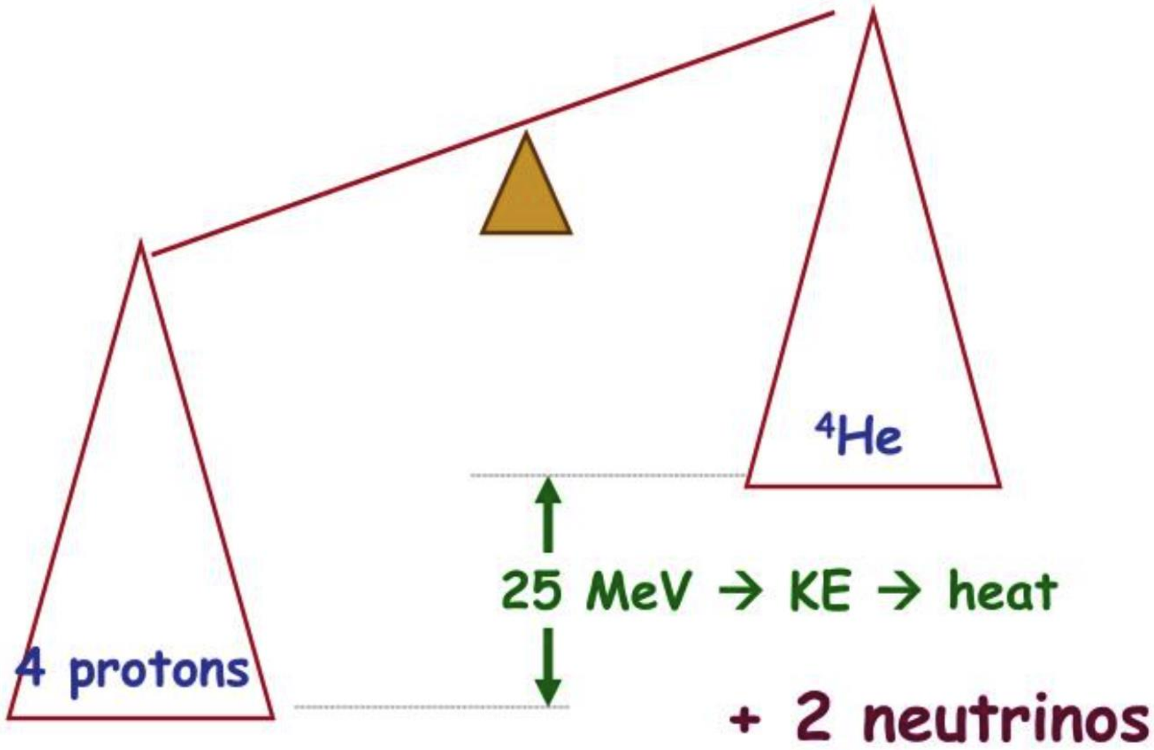
Nuclear Fusion in sun: (p-p) cycle

- Net Reaction of the Proton-Proton cycle

$$4p \rightarrow {}^{4}_{2}\mathrm{He} + 2e^{+} + 2\upsilon_{e} + 26.7\ \mathrm{MeV}$$

- Four protons are consumed to produce one helium-4 (${}^{4}_{2}\mathrm{He}$) nucleus, along with two positrons, two neutrinos, and energy.
- The total energy released in one complete p-p cycle is about 26.7 MeV in the form of kinetic energy of the particles, gamma rays, and the annihilation of positrons with electrons.
- The energy transferred to the Sun's surrounding plasma and eventually radiated out as light and heat.
- The Sun fuses about 4×10^{38}protons per second
- The released energy per second is about

$$\mathbf{26.7 \times\ 10^{38}\ MeV \sim 4.28 \times\ 10^{26}\ J}$$



Nuclear Fusion in Star: (CNO) cycle

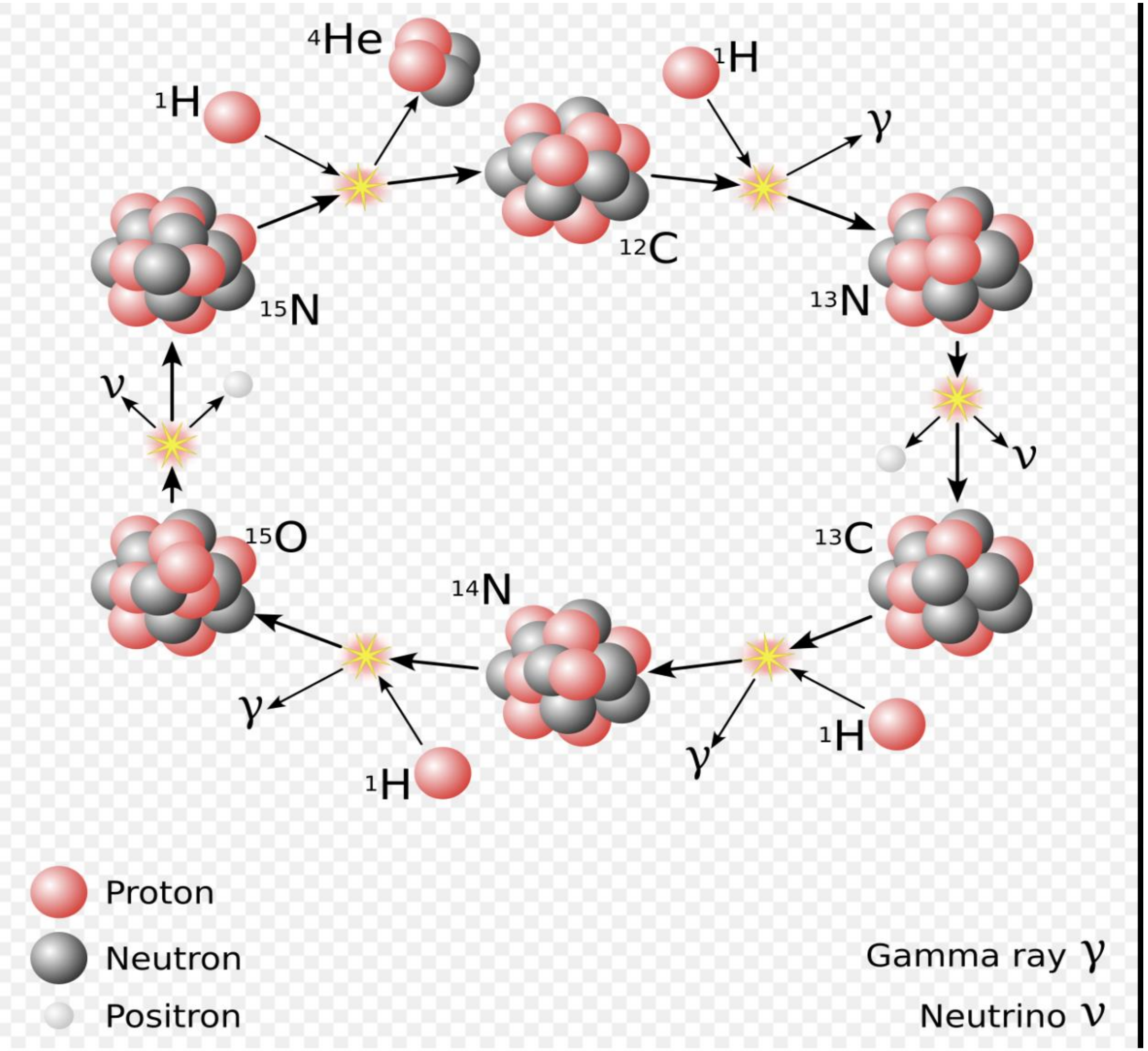
- The CNO cycle uses carbon, nitrogen, and oxygen as catalysts to convert hydrogen into helium.
- Dominant in stars with masses greater than that of the Sun. typically about 1.3 solar masses.
- **The core temperatures** in star exceeds 18 million Kelvin.

- **Energy Released** about 26.7 MeV per cycle, with gamma rays and neutrinos produced.

Nuclear Fusion in Star: (CNO) cycle

The CNO cycle **involves six stages**

- **Step 1: Proton Carbon-12 fusion**
- $^{12}_{6}C + p \rightarrow {}^{13}_{7}N + \gamma + 1.95\text{MeV}$
- **Step 2: Beta Decay of Nitrogen-13**
- $^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + v_e + 2.22\text{ MeV}$
- **Step 3: Proton Carbon-13 fusion**
- $^{13}_{6}C + p \rightarrow {}^{14}_{7}N + \gamma + 7.55\text{MeV}$
- **Step 4: Proton Nitrogen-14 fusion**
- $^{14}_{7}N + p \rightarrow {}^{15}_{8}O + \gamma + 7.35\text{ MeV}$
- **Step 5: Beta Decay of Oxygen-15**
- $^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + v_e + 1.73\text{ MeV}$
- **Step 6: Proton Nitrogen-15 fusion**
- $^{15}_{7}N + p \rightarrow {}^{12}_{6}C + {}^{4}_{2}He + 4.96\text{MeV}$



Nuclear Fusion in star: (CNO) cycle

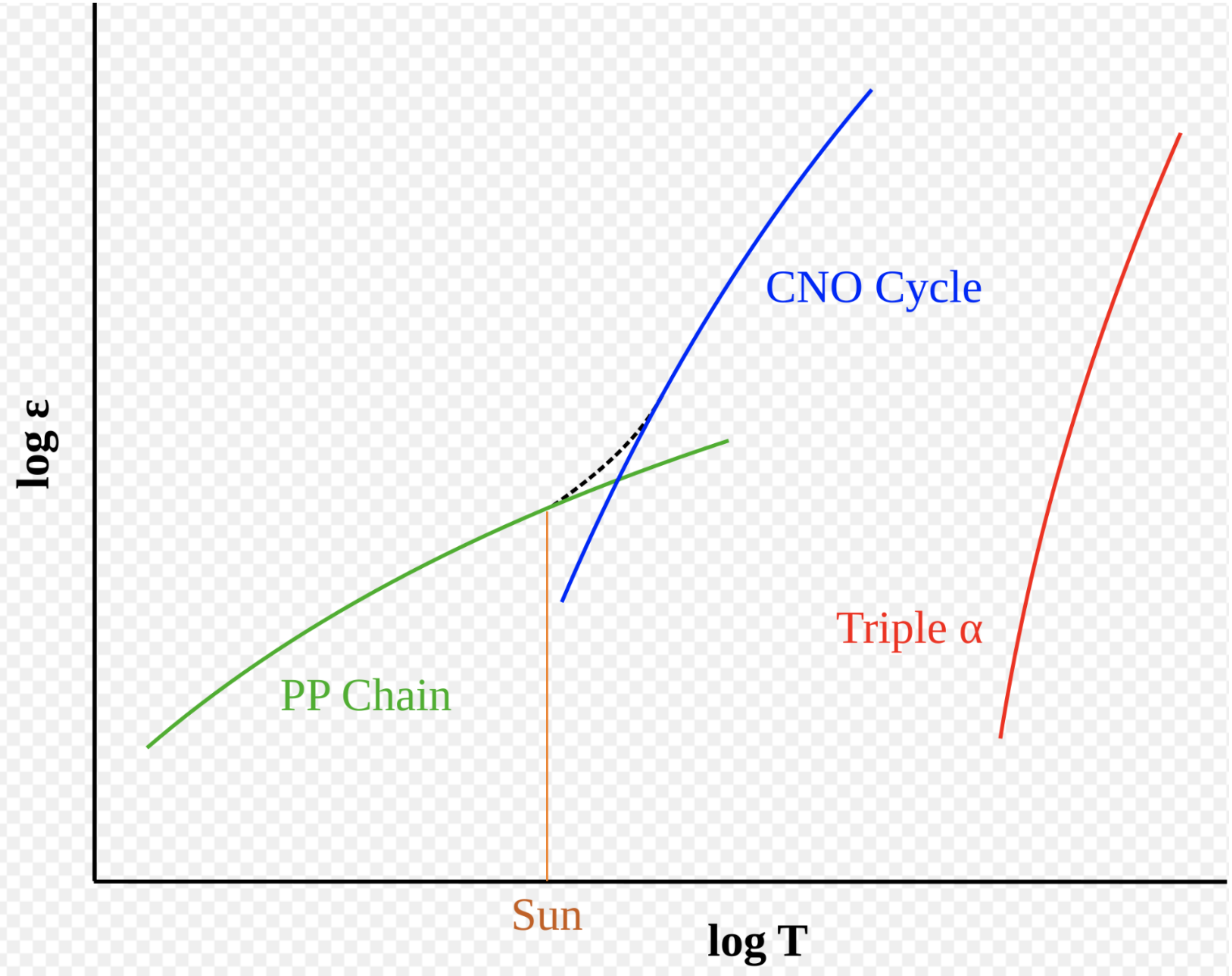
- **Net Reaction of the CNO Cycle**
- The overall result of the CNO cycle is the conversion of four protons (hydrogen nuclei) into one helium-4 nucleus:

$$\boldsymbol{4p \rightarrow {}_{2}^{4}He + 2e^{+} + 2\upsilon_{e} + 26.7\ \mathrm{MeV}}$$

- The cycle releases energy in the form of gamma rays and the kinetic energy of the particles involved.
- The positrons emitted annihilate with electrons, producing gamma rays.
- The neutrinos escape the star without interacting much, carrying away some energy.
- The net energy released per cycle is around **26.7 MeV**, similar to the proton-proton chain.
- A massive star where the CNO cycle is dominant, fuses about 4×10^{38} protons per second
- The released energy per second is about $\boldsymbol{26.7 \times 10^{38}\ \mathrm{MeV} \sim 4.28 \times 10^{26}\ \mathrm{J}}$

PP cycle and CNO cycle

- Logarithmic of the relative energy output (E) of (p–p) chain, CNO chain and triple-α fusion processes at different temperatures (T).
- The dashed line shows the combined energy generation of the p–p and CNO processes within a star.
- At the Sun's core temperature of 15.5 million K, The p-p process is dominant.
- The P-P process and the CNO process are equal at around 20 million kelvin.



Advantages of Extracting energy from a controlled fusion reactor

- **Abundant and cheap Fuel Supply:** Fusion fuels (e.g., deuterium and lithium) are abundant and a cheap .
- **No Long-Lived Radioactive Waste:** Fusion produces minimal long-lived radioactive waste.
- **High Energy Output:** Fusion reactions have a higher energy yield compared to fission.

References

- "Introduction to Nuclear Reactions" by C.A. Bertulani
- "Nuclear Physics: Principles and Applications" by John Lilley NASA –
- "How the Sun Shines" by John N. Bahcall
- "The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter" by Stefano Atzeni and Jurgen Meyer-ter-Vehn
- "Fusion: Science, Politics, and the Invention of a New Energy Source" by Garry McCracken and Peter Stott ITER Project Official Documentation National Ignition Facility (NIF) Documentation
- "An Indispensable Truth: How Fusion Power Can Save the Planet" by Francis F. Chen
- "The Physics of Plasma-Fueled Fusion: Theory and Applications" by W.M. Stacey
- "Nuclear Physics: Principles and Applications" by John Lilley.
- Clayton, D. D. (1983). *Principles of Stellar Evolution and Nucleosynthesis*. University of Chicago Press.
- Kippenhahn, R., & Weigert, A. (1990). *Stellar Structure and Evolution*. Springer-Verlag.
- Adelberger, Eric G.; et al. (12 April 2011). "Solar fusion cross sections. II. The pp chain and CNO cycles". *Reviews of Modern Physics*. **83** (1): 201

Q&A

Thank you