Alexandria Quantum Computing Group (AleQCG)

CENTER OF EXCELLENCE FOR QUANTUM COMPUTERS

Basics of Quantum Computing Day2

Ahmed Younes

Vice Dean of Education and Student Affairs

Professor of Quantum Computing
Department of Mathematics and Computer Science
Faculty of Science, Alexandria University, Egypt

Founder & Leader of Alexandria Quantum Computing Group (AleQCG)

2nd ArPS summer School on Advanced Physics

Aug 25, 2024, 12:20 AM → Aug 29, 2024, 6:40 PM Africa/Cairo

Zewail City of Science and Technology

shaaban Khalil

Description

Outline

- Quantum Data – qubit
- Quantum Superposition.
- Bloch Sphere
- Dirac Notations.
- Linear Algebra for QC.
- Entanglement.
- Measurements.
- No Cloning Theory.

Quantum Data-qubit

A quantum bit of data is represented by a single atom that is in one of two states denoted by |0> and |1>. A single bit of this form is known as a ***qubit***

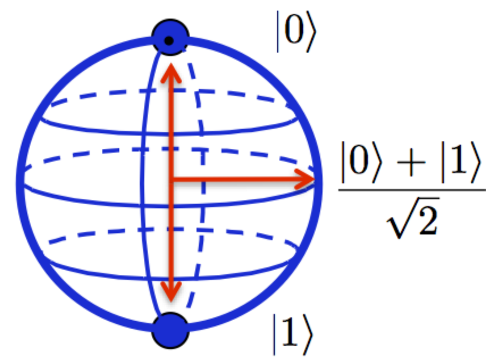
Quantum Superposition

A single qubit can be forced into a ***superposition*** of the two states denoted by the addition of the state vectors:

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Classical Bit



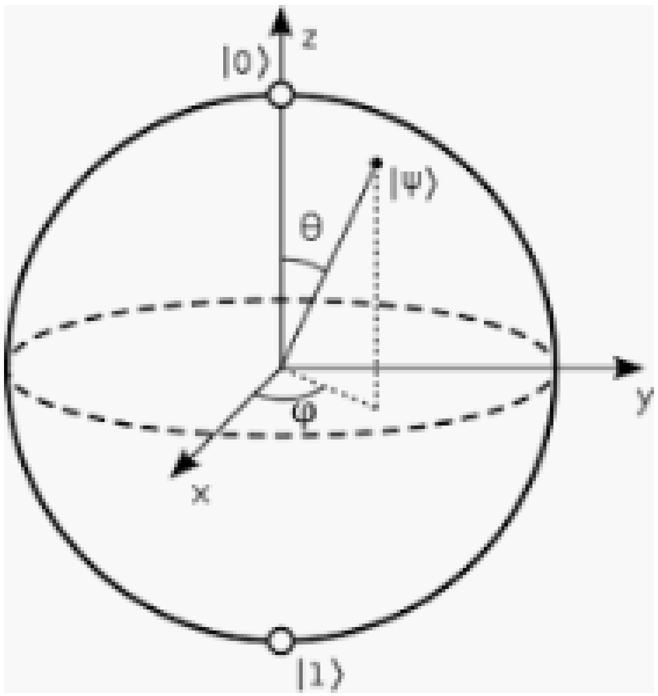
Qubit

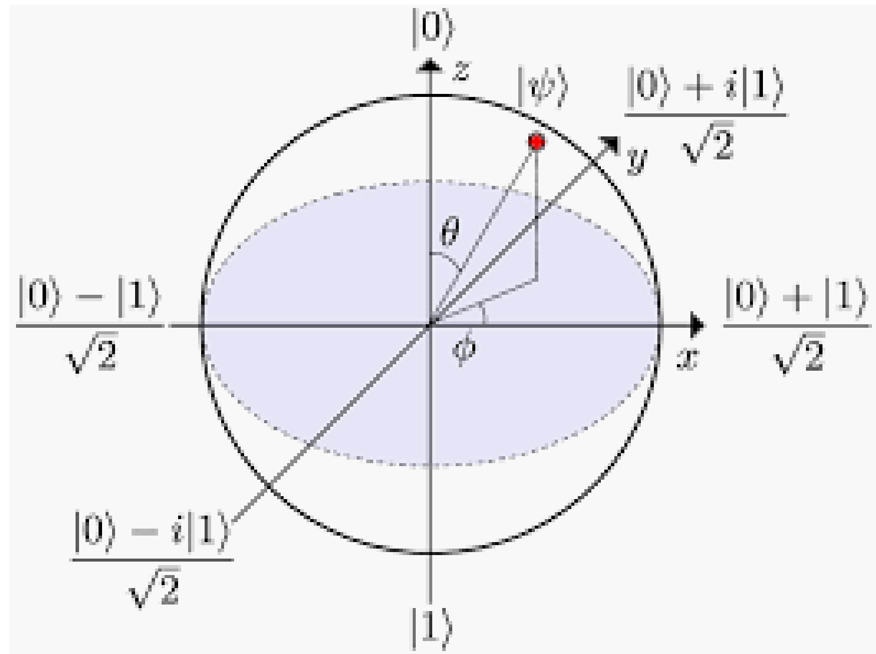
where ***a* and *b*** are complex numbers and $|a|^2 + |b|^2 = 1$

and $|a|^2 = a^*a^*$

A qubit in superposition is in both of the states |1> and |0> at the same time

Bloch Sphere





$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$

where $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$.

Dirac Notations

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \langle 0| = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\langle \psi| = a^*\langle 0| + b^*\langle 1| = \begin{bmatrix} a^* & b^* \end{bmatrix}$$

Inner Product

- Inner product between two vectors $|\psi_1>$ and $|\psi_2>$ is defined as follows:

$$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$$

$$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$$

$$\langle\psi_1|\psi_2\rangle = a_1 * a_2 + b_1 * b_2 (scaler)$$

$$\langle 0|0\rangle = \langle 1|1\rangle = 1$$

$$\langle 0|1\rangle = \langle 1|0\rangle = 0$$

Outer Product

- Outer product between two vectors $|\psi_1>$ and $|\psi_2>$ is defined as follows:

$$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$$

$$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$$

$$|\psi_1\rangle\langle\psi_2| = a_1a_2*|0\rangle\langle 0| + a_1b_2*|0\rangle\langle 1| + b_1a_2*|1\rangle\langle 0| + b_1b_2*|1\rangle\langle 1|$$

$$= \begin{bmatrix} a_1a_2* & a_1b_2* \\ b_1a_2* & b_1b_2* \end{bmatrix}$$

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, |0\rangle\langle 1| = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$|1\rangle\langle 0| = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiple Qubits

- For 2-qubit systems we have four states, so the system is described as either its individual components (if possible) or as a single system.
- Given the components of the system, we can combine the components using Tensor product.

$$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$$

$$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$$

$$\begin{aligned}|\psi\rangle &= |\psi_1\rangle \otimes |\psi_2\rangle \\ &= \left(a_1|0\rangle + b_1|1\rangle\right) \otimes \left(a_2|0\rangle + b_2|1\rangle\right) \\ &= a_1a_2\left(|0\rangle \otimes |0\rangle\right) + a_1b_2\left(|0\rangle \otimes |1\rangle\right) + b_1a_2\left(|1\rangle \otimes |0\rangle\right) + b_1b_2\left(|1\rangle \otimes |1\rangle\right) \\ &= \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle \\ &= \sum_{j=0}^{3} \alpha_j|j\rangle, \ \sum_{j=0}^{3} |\alpha_j|^2 = 1,\end{aligned}$$

Multiple Qubits

$$\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle = \begin{vmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{vmatrix}$$

where,

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$|\underbrace{00\ldots00}_{n}\rangle \ , \ |00\ldots01\rangle \ , \ \ldots \ , \ |11\ldots10\rangle \ , \ |11\ldots11\rangle.$$

The standard way to associate column vectors corresponding to these basis vectors is as follows:

$$|00\ldots00\rangle \iff \left.\begin{pmatrix}1\\0\\0\\\vdots\\0\\0\end{pmatrix}\right\} 2^n, \quad |00\ldots01\rangle \iff \begin{pmatrix}0\\1\\0\\\vdots\\0\\0\end{pmatrix}, \quad \cdots$$

$$\cdots \quad , \quad |11\ldots10\rangle \iff \begin{pmatrix}0\\0\\0\\\vdots\\1\\0\end{pmatrix}, \quad |11\ldots11\rangle \iff \begin{pmatrix}0\\0\\0\\\vdots\\0\\1\end{pmatrix}.$$

Computation with Qubits

- Computation in quantum systems must be *reversible*, so that *no loss in energy* during the computation process.
- Quantum gates are represented as square matrices U that satisfy the *unitary* condition:

$$UU^{\dagger} = I$$

Quantum Operators

- For a 1-qubit system, the quantum gate must be 2x2.
- For a 2-qubit system, the quantum gate must be 4x4.
- For a *n*-qubit system, the quantum gate must be $2^n \times 2^n$.

Linear Transformations

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$U = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$$

$$|\psi'\rangle = U|\psi\rangle = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} x_0 a + x_1 b \\ x_2 a + x_3 b \end{bmatrix}$$

$$= (x_0 a + x_1 b)|0\rangle + (x_2 a + x_3 b)|1\rangle$$

Consider a two-qubit system $|\psi\rangle \otimes |\xi\rangle$. Applying U on $|\psi\rangle$ and V on $|\xi\rangle$ in parallel can be written as follows,

$$U \otimes V \left(|\psi\rangle \otimes |\xi\rangle \right) = U |\psi\rangle \otimes V |\xi\rangle .$$

where $U \otimes V$ can be combined in a single matrix of size 4×4 as follows,

$$\begin{aligned} U \otimes V &= \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix} \otimes \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \\ &= \begin{bmatrix} u_{00} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} & u_{01} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \\ u_{10} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} & u_{11} \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix} \end{bmatrix} \\ &= \begin{bmatrix} u_{00}v_{00} & u_{00}v_{01} & u_{01}v_{00} & u_{01}v_{01} \\ u_{00}v_{10} & u_{00}v_{11} & u_{01}v_{10} & u_{01}v_{11} \\ u_{10}v_{00} & u_{10}v_{01} & u_{11}v_{00} & u_{11}v_{01} \\ u_{10}v_{10} & u_{10}v_{11} & u_{11}v_{10} & u_{11}v_{11} \end{bmatrix} . \end{aligned}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Suppose we have a 2-qubit composite system, and we apply X to the first qubit. I to the second qubit at the same time.
Thus the 2-qubit input $|\psi_1\rangle \otimes |\psi_2\rangle$ gets mapped to
$X|\psi_1\rangle \otimes I|\psi_2\rangle = (X \otimes I)(|\psi_1\rangle \otimes |\psi_2\rangle)$.
That is, the linear operator describing this operation on the composite system has the matrix representation

$$X \otimes I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Quantum Measurement

- Quantum system can be transformed to a classical system using measurement.
- The superposition is collapsed to one it's possible states in a probabilistic way.

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

- Probability to find the 1st qubit in state |0> is $|\alpha_0|^2+|\alpha_1|^2$.
- Probability to find the 2nd qubit in state |1> is $|\alpha_1|^2+|\alpha_3|^2$.

Quantum Measurement

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

- If the 1st qubit of this system is measured and the outcome is |1>, then the system will be transformed to the following system

$$|\psi'\rangle = \frac{1}{\sqrt{|\alpha_2|^2 + |\alpha_3|^2}}\left(\alpha_2|10\rangle + \alpha_3|11\rangle\right)$$

- The amplitudes are re-normalized and the superposition of the second qubit is not affected by the measurement.

Entanglement

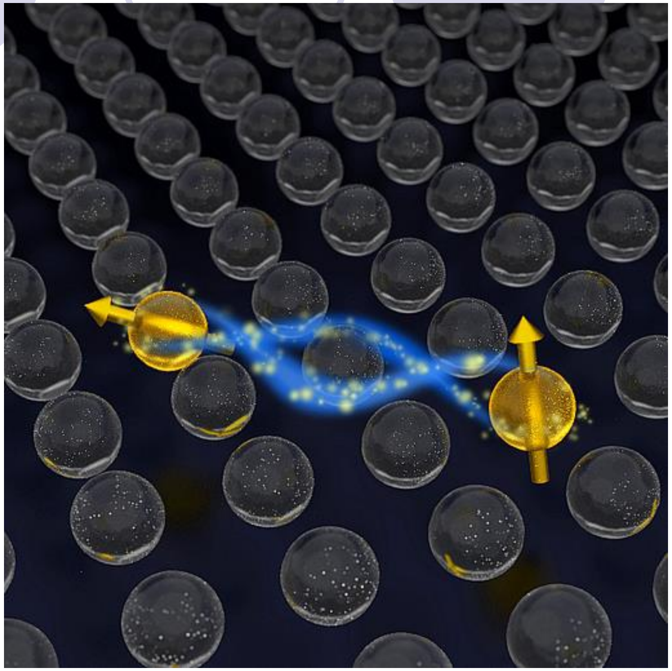
- ***Entanglement*** is the ability of quantum systems to exhibit correlations between states within a superposition.

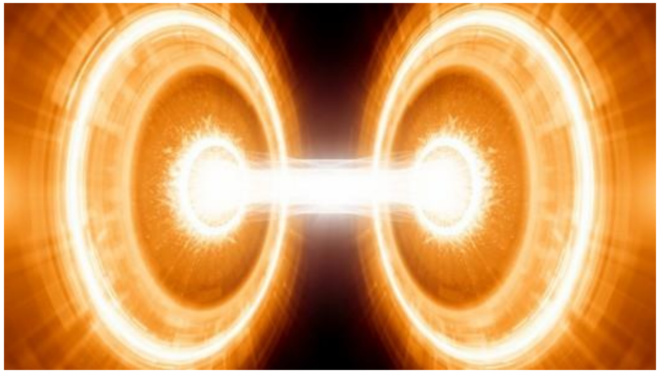
- Imagine two qubits, each in the state (a superposition of the 0 and 1.)

$$(a_1|0> + b_1|1>) \otimes (a_2|0> + b_2|1>)$$

- We can entangle the two qubits such that the measurement of one qubit is always correlated to the measurement of the other qubit.

$$a|00> + b|11>$$





Entanglement

- An arbitrary 2-qubit system can be represented as follows:

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

where α_j's can take any value as long as

$$\sum_{j=0}^{3}|\alpha_j|^2 = 1,$$

If α_0=0 and α_1=0:

$$\alpha_2|10\rangle + \alpha_3|11\rangle = |1\rangle \otimes \left(\alpha_2|0\rangle + \alpha_3|1\rangle\right)$$

If α_1=0 and α_3=0.

$$\alpha_0|00\rangle + \alpha_2|10\rangle = \left(\alpha_0|0\rangle + \alpha_2|1\rangle\right) \otimes |0\rangle$$

Entanglement

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

If α_1=0 and α_2=0:

$$\alpha_0|00\rangle + \alpha_3|11\rangle$$

If α_0=0 and α_3=0.

$$\alpha_1|01\rangle + \alpha_2|10\rangle$$

- These two systems are entangled and cannot be represented using their individual components.
- A measurement on one qubit affects the state of the other qubit.

Bell States

- Entangled states are considered as the heart for many quantum algorithms.
- For example, quantum teleportation, dense coding and quantum searching.
- Two-qubit entangled states are usually referred to as *Bell states, EPR states, EPR pairs* or *Bell basis*.

$$\frac{(|00\rangle \pm |11\rangle)}{\sqrt{2}}, \quad \frac{(|01\rangle \pm |10\rangle)}{\sqrt{2}}.$$

No Cloning Theory

- It is not possible to clone an unknown quantum state

No Cloning Assume we have a unitary operator U_{cl} and two quantum states $|\phi\rangle$ and $|\psi\rangle$ which U_{cl} copies, i.e.,

$$|\phi\rangle \otimes |0\rangle \xrightarrow{U_{cl}} |\phi\rangle \otimes |\phi\rangle$$
$$|\psi\rangle \otimes |0\rangle \xrightarrow{U_{cl}} |\psi\rangle \otimes |\psi\rangle$$

Proof : Suppose there exists a unitary operator U_{cl} that can indeed clone an unknown quantum state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$. Then

$$\begin{aligned} |\phi\rangle|0\rangle \xrightarrow{Ucl} |\phi\rangle|\phi\rangle &= (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle) \\ &= \alpha^2|00\rangle + \beta\alpha|10\rangle + \alpha\beta|01\rangle + \beta^2|11\rangle \end{aligned}$$

But now if we use U_{cl} to clone the expansion of $|\phi\rangle$, we arrive at a different state:

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle \xrightarrow{U_{cl}} \alpha|00\rangle + \beta|11\rangle .$$

Here there are no cross terms. Thus we have a contradiction and therefore there cannot exist such a unitary operator U_{cl}.

Quantum Gates and Circuits

Outline

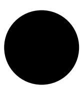
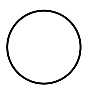
- Quantum gates.
- Quantum circuit model.
- Quantum truth table.
- Boolean quantum circuits.
- Quantum Simulation

Computation with Qubits

How does the use of qubits affect computation?

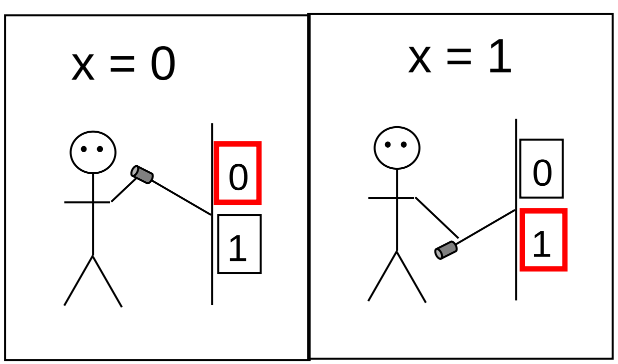
Classical Computation

Data unit: bit

 = '1'  = '0'

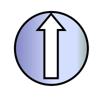
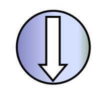
Valid states:

x = '0' or '1'



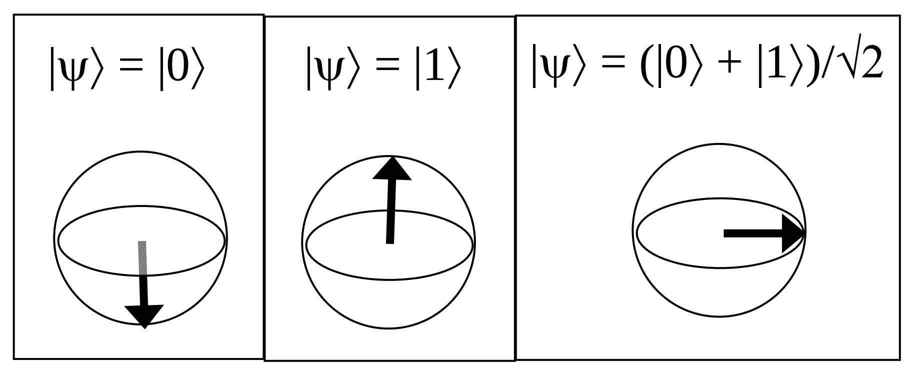
Quantum Computation

Data unit: qubit

 $=|1\rangle$  $=|0\rangle$

Valid states:

$|\psi\rangle = c_1|0\rangle + c_2|1\rangle$



Computation with Qubits

How does the use of qubits affect computation?

Classical Computation

Operations: logical

Valid operations:

NOT = (in across the top, out below)

	0	1
	1	0

1-bit

AND = (in across the top, in down the side, out in the body)

	0	1
0	0	0
1	0	1

2-bit

Quantum Computation

Operations: unitary

Valid operations:

1-qubit

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

2-qubit

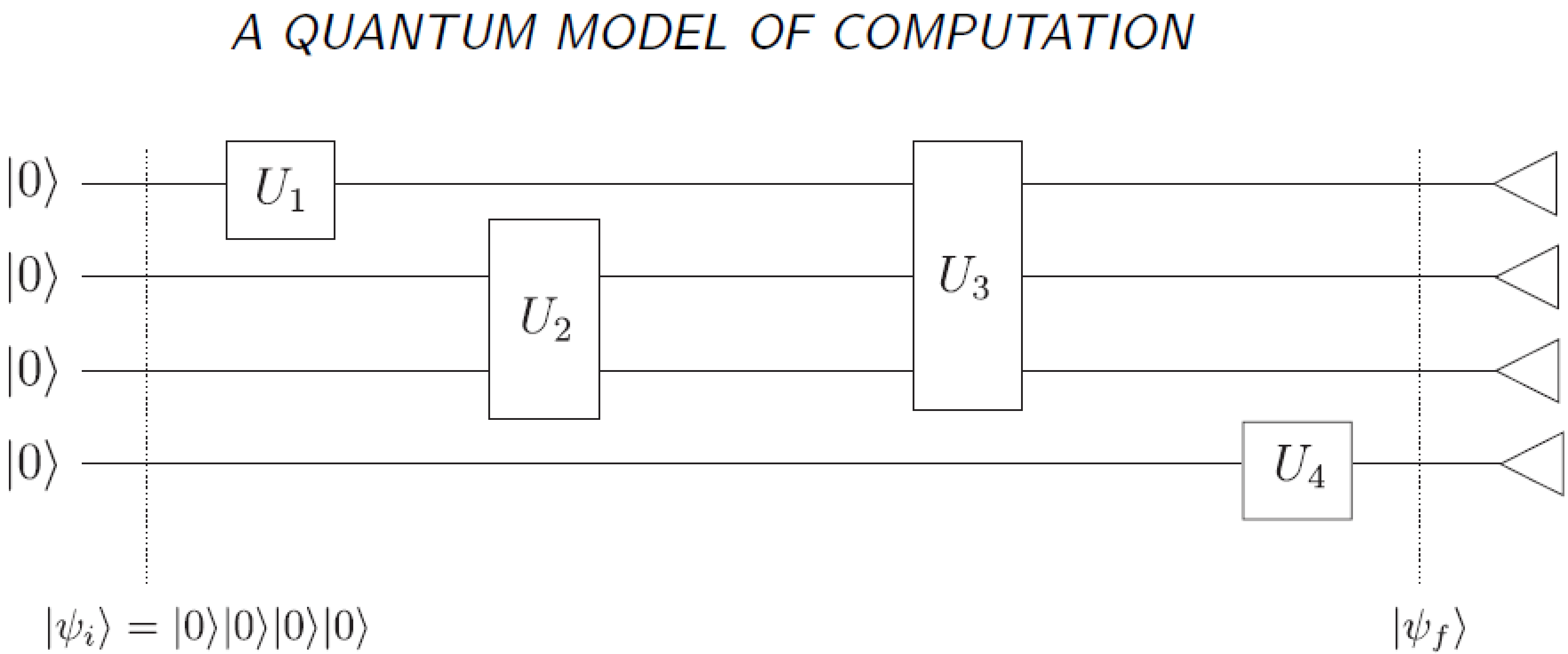
$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Computation with Qubits

- Computation in quantum systems must be *reversible*, so that *no loss in energy* during the computation process.
- Quantum gates are represented as square matrices U that satisfy the *unitary* condition:

$$UU^{\dagger} = I$$

Quantum Circuit Model



Single-qubit Quantum Gates

Identity Gate (I gate)

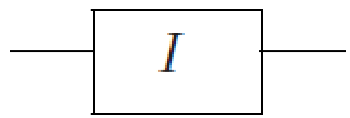
Unitary matrix representation, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Diagonal representation, $I = |0\rangle\langle 0| + |1\rangle\langle 1|$.

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$\|1\rangle$

truth table

And its circuit takes the form,

$(a|0\rangle + b|1\rangle)$  $(a|0\rangle + b|1\rangle)$

NOT Gate (Pauli–X gate)

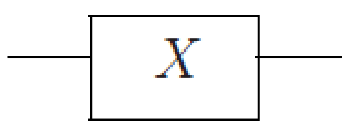
Unitary matrix representation, $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Diagonal representation, $X = |0\rangle\langle 1| + |1\rangle\langle 0|$.

Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

truth table

And its circuit takes the form,

$(a|0\rangle + b|1\rangle)$  $(a|1\rangle + b|0\rangle)$

Pauli–Y Gate

Unitary matrix representation, $Y = \begin{bmatrix} 0 & -\underline{i} \\ \underline{i} & 0 \end{bmatrix}$.

Input	Output
$\vert 0\rangle$	$\underline{i}\vert 1\rangle$
$\vert 1\rangle$	$-\underline{i}\vert 0\rangle$

truth table

Diagonal representation, $Y = -\underline{i}\left(|0\rangle\langle 1| - |1\rangle\langle 0|\right)$.

And its circuit takes the form,

$$(a|0\rangle + b|1\rangle) \;-\!\!\boxed{\;Y\;}\!\!-\; (a\underline{i}|1\rangle - b\underline{i}|0\rangle)$$

Phase Shift Gate (Pauli–Z gate)

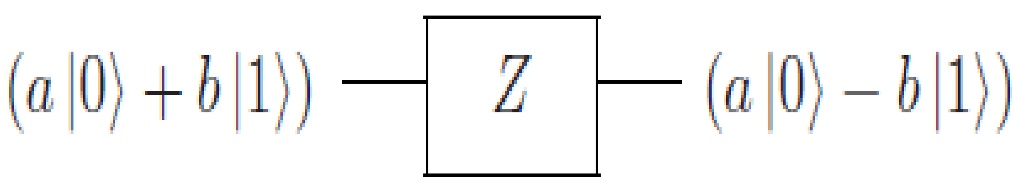
Unitary matrix representation, $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$

truth table

Diagonal representation, $Z = |0\rangle\langle 0| - |1\rangle\langle 1|.$

And its circuit takes the form,



Hadamard Gate (H gate)

Unitary matrix representation, $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$

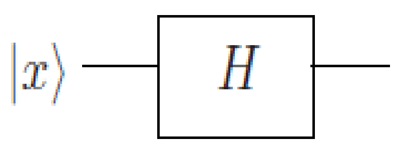
Input	Output
$\|0\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$
$\|1\rangle$	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$

truth table.

Diagonal representation,

$$H = \frac{1}{\sqrt{2}} \left(|0\rangle \langle 0| + |0\rangle \langle 1| + |1\rangle \langle 0| - |1\rangle \langle 1| \right).$$

And its circuit takes the form,

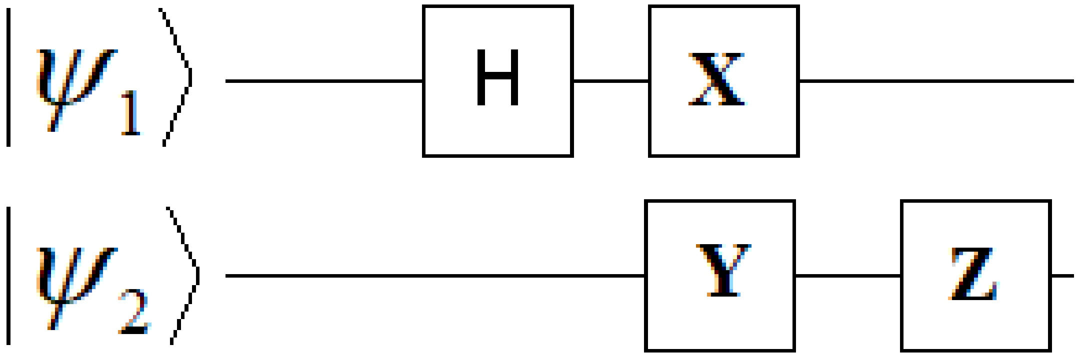


$$\frac{1}{\sqrt{2}} \left(|0\rangle + (-1)^x |1\rangle \right)$$

1- Qubit Gate Identities

- $Y = iXZ.$
- $H = (X + Z)/\sqrt{2}.$
- $S = T^2.$
- $HXH = Z.$
- $HYH = -Y.$
- $HZH = X.$
- $XY = -YX = iZ.$
- $ZX = -XZ = iY.$
- $YZ = -ZY = iX.$
- $XX = YY = ZZ = I.$

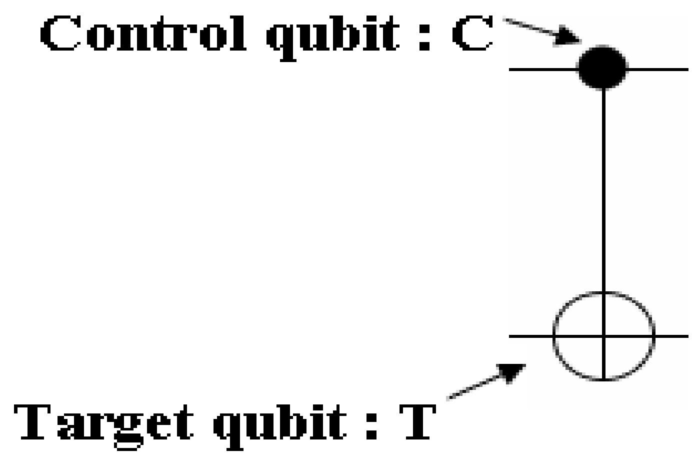
Tracing a Quantum Circuit



- What is the truth table?

Two qubit gates

- **The Controlled-NOT Gate (C_{not})**
- If C=0 then no change
- Else If C=1 then T is flipped



Diagonal representation,

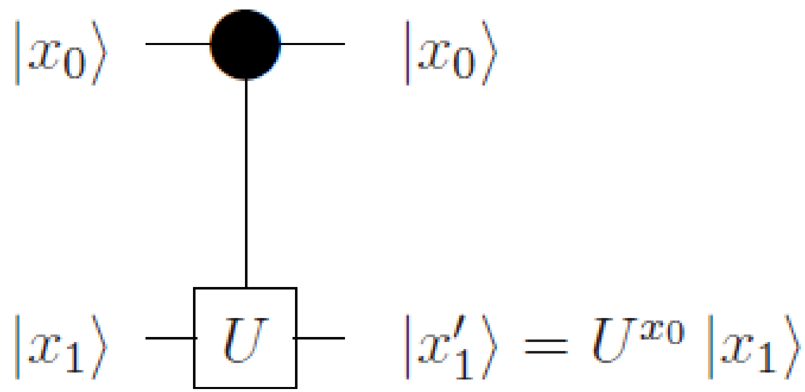
$$C_{not} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X.$$

$$C_{not} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

The C_{not} gate truth table.

The General Controlled–*U* Gate (*C*-*U* gate)



The Controlled–U gate.

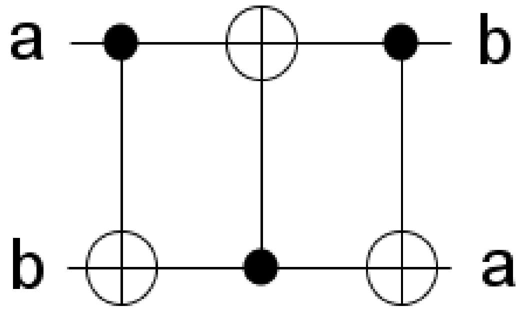
It works as follows: U will be applied on the target qubit $|x_1\rangle$ if and only if the control qubit $|x_0\rangle$ is set to $|1\rangle$

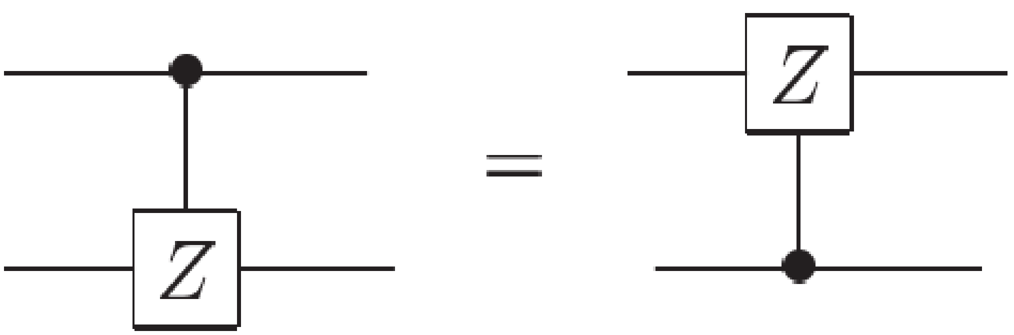
$$C - U = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U.$$

Examples

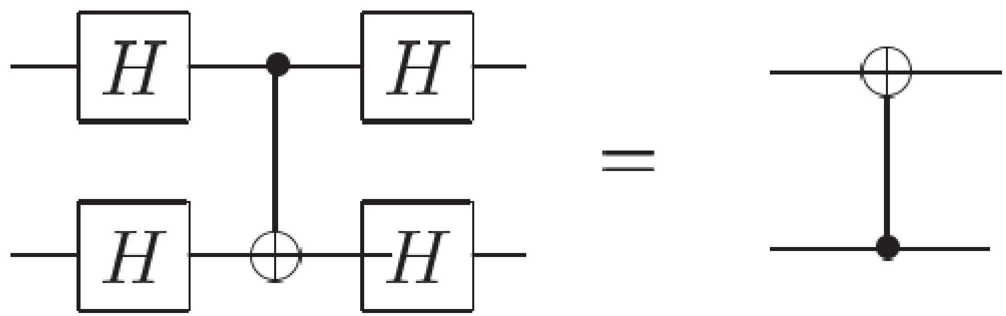
Swap Circuit:

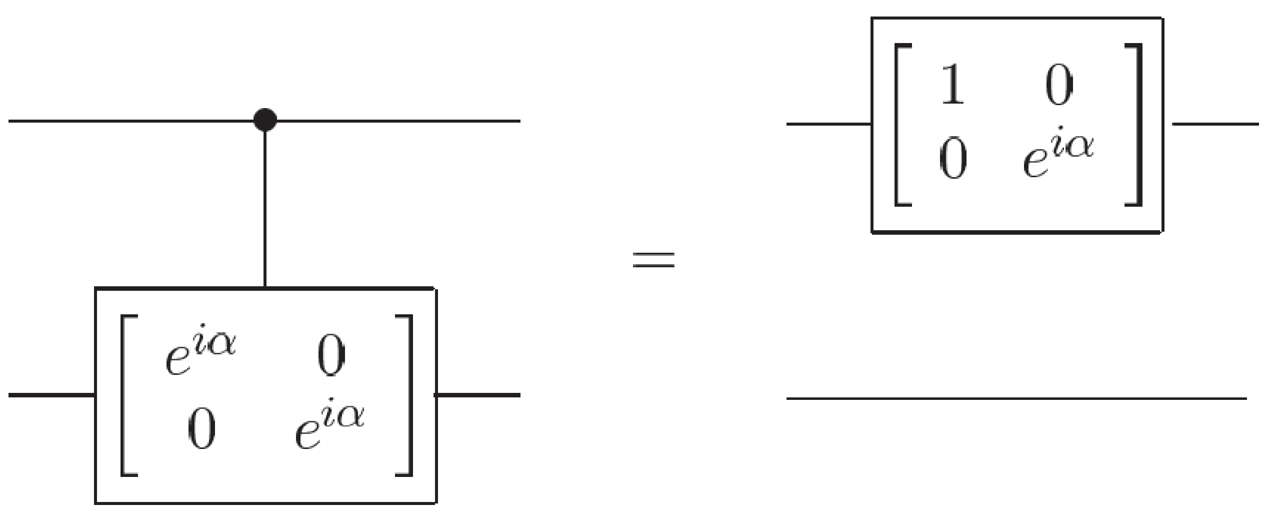
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





Examples


H
H
H
H
=

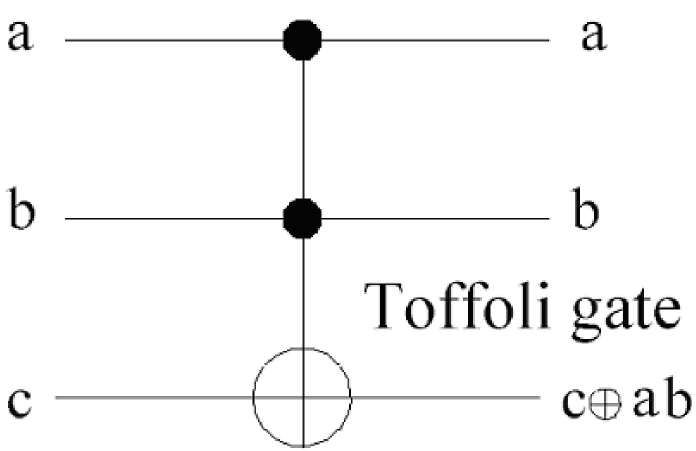

$\begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$
=
$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$

Three qubit gates

Toffoli gate :

Is considered to be universal…

Setting C=1 will convert it to classical *NAND* gate which is universal from classical point of view.

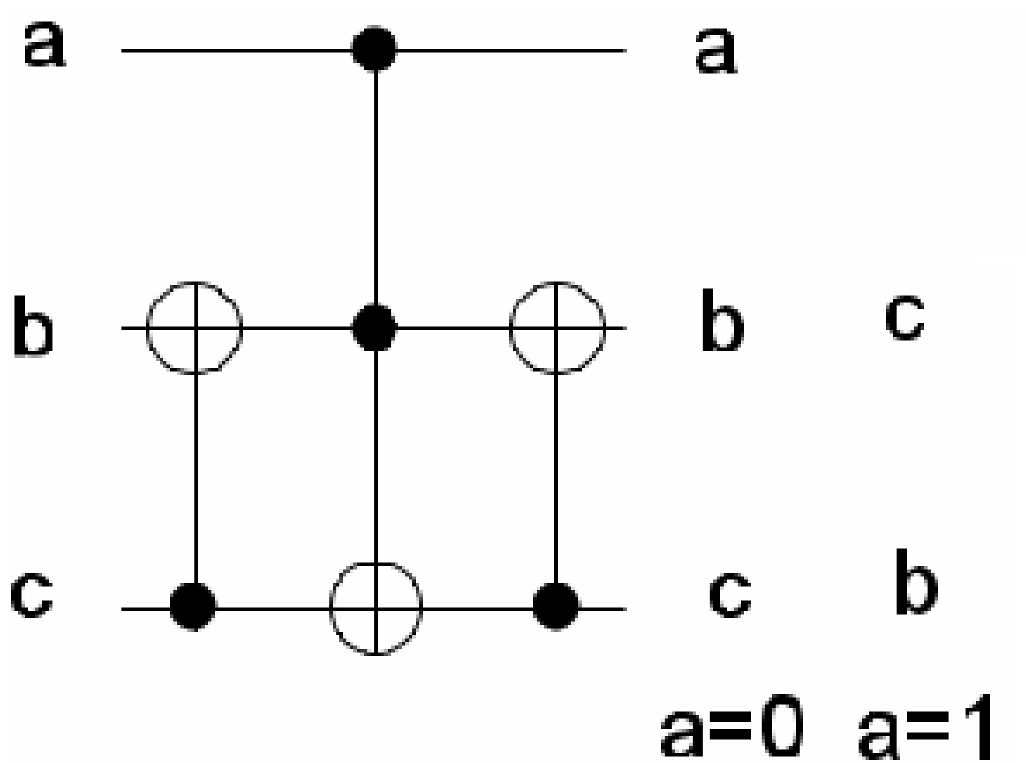


Input	Output
$\|000\rangle$	$\|000\rangle$
$\|001\rangle$	$\|001\rangle$
$\|010\rangle$	$\|010\rangle$
$\|011\rangle$	$\|011\rangle$
$\|100\rangle$	$\|100\rangle$
$\|101\rangle$	$\|101\rangle$
$\|110\rangle$	$\|111\rangle$
$\|111\rangle$	$\|110\rangle$

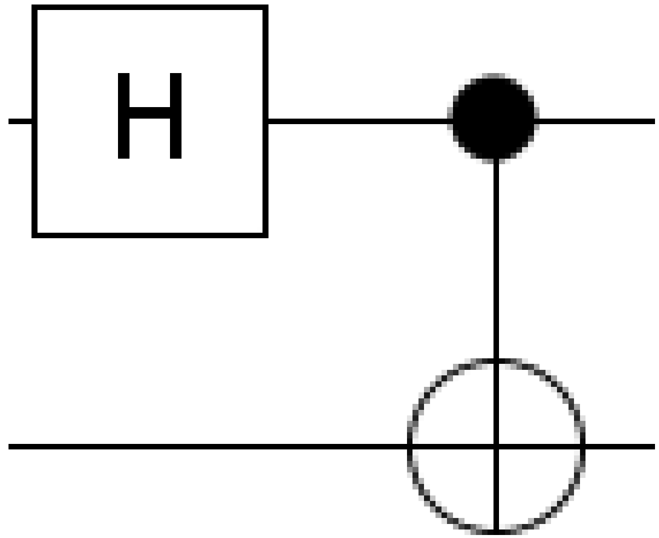
truth table.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

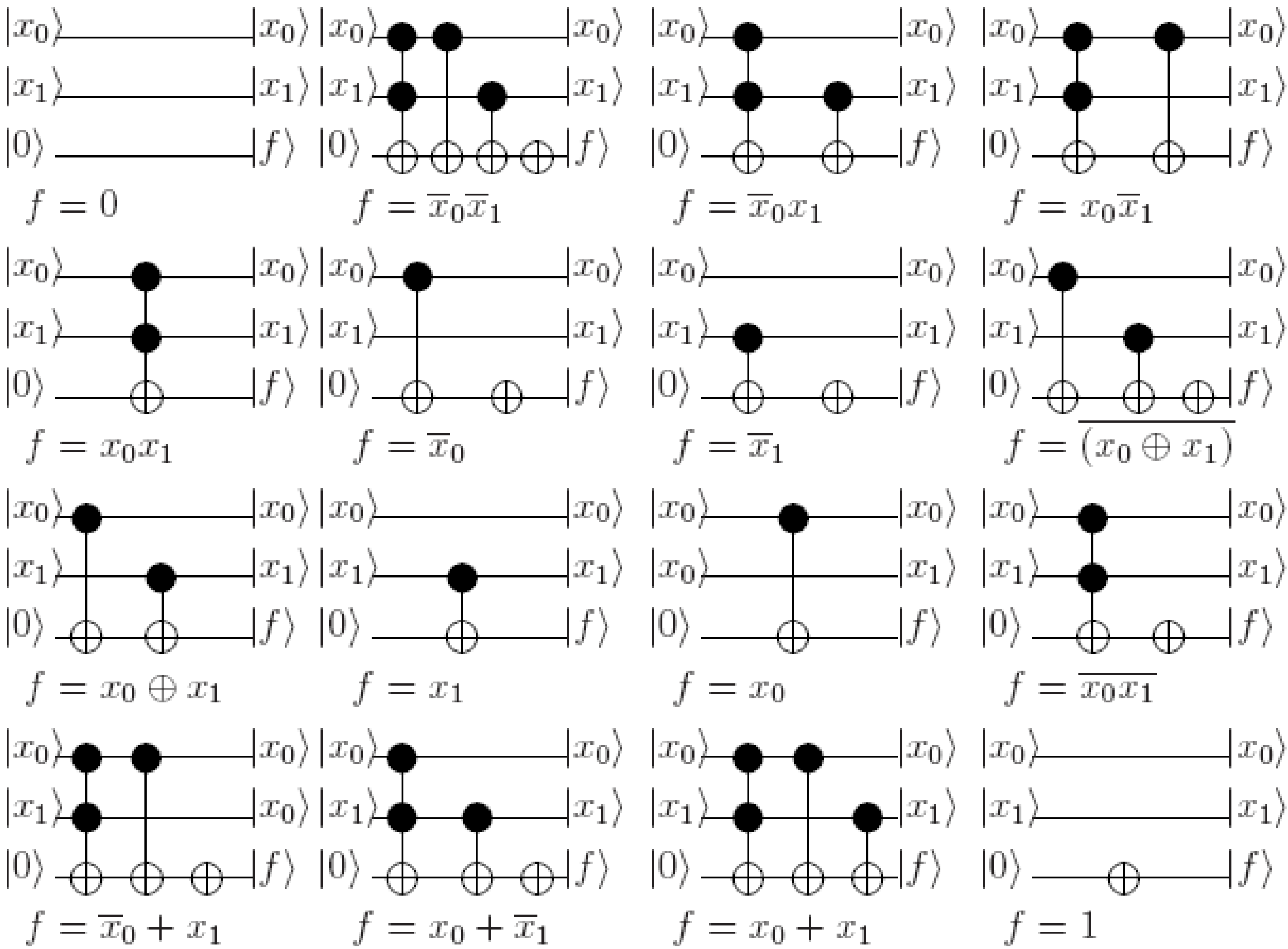
Controlled Swap Circuit (Fredkin Gate)



???

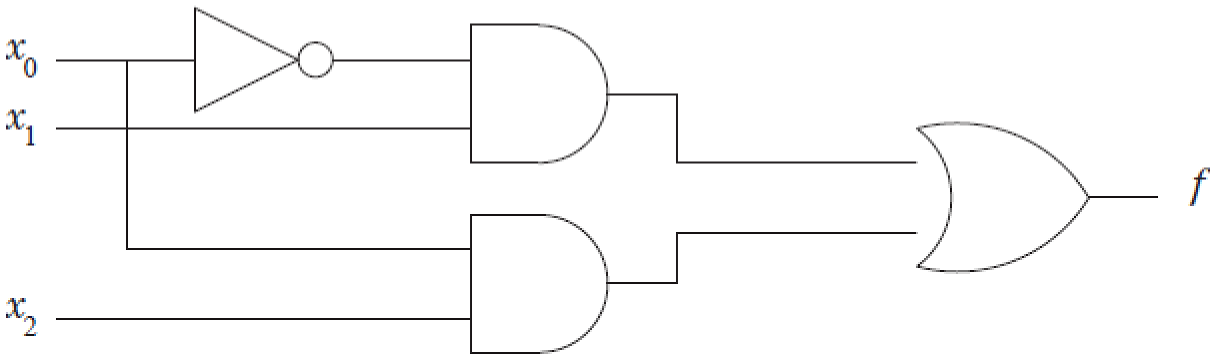

H

Two-qubits Boolean Circuits

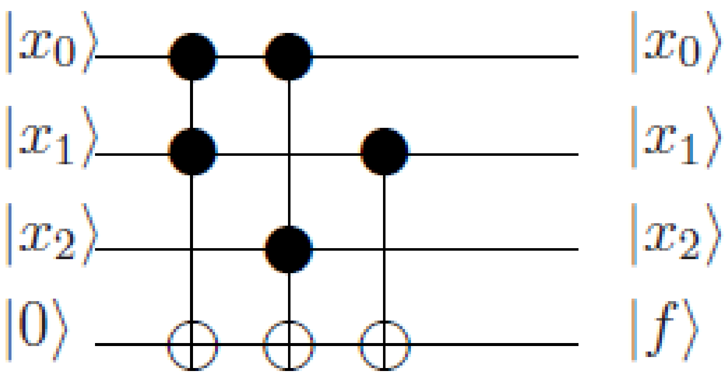


Boolean Quantum Circuits

$$f = \overline{x_0}x_1 + x_0x_2$$



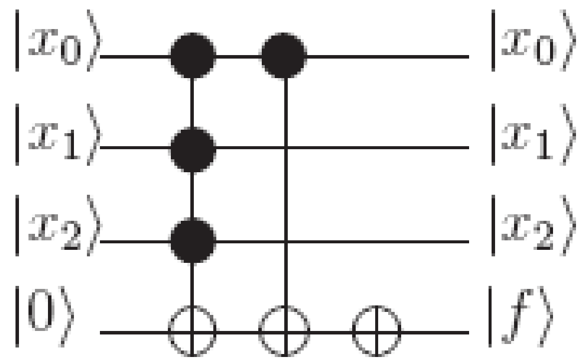
Digital circuit



Quantum circuit

x_0	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

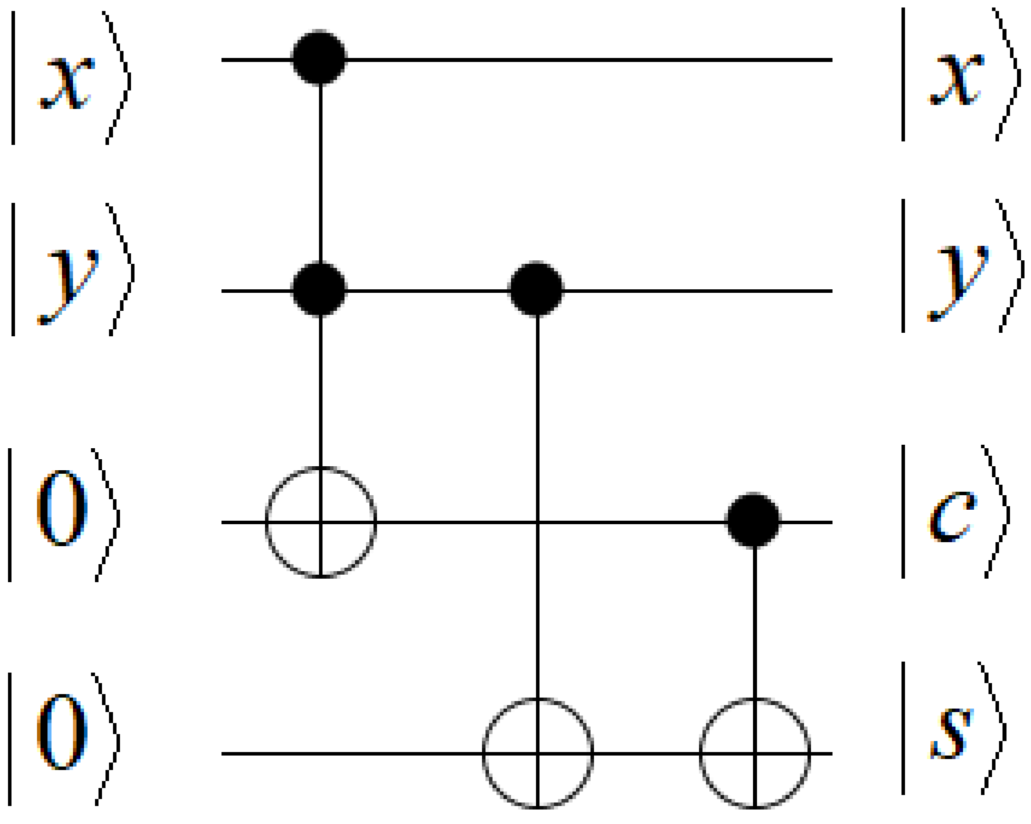
Boolean Quantum Circuits



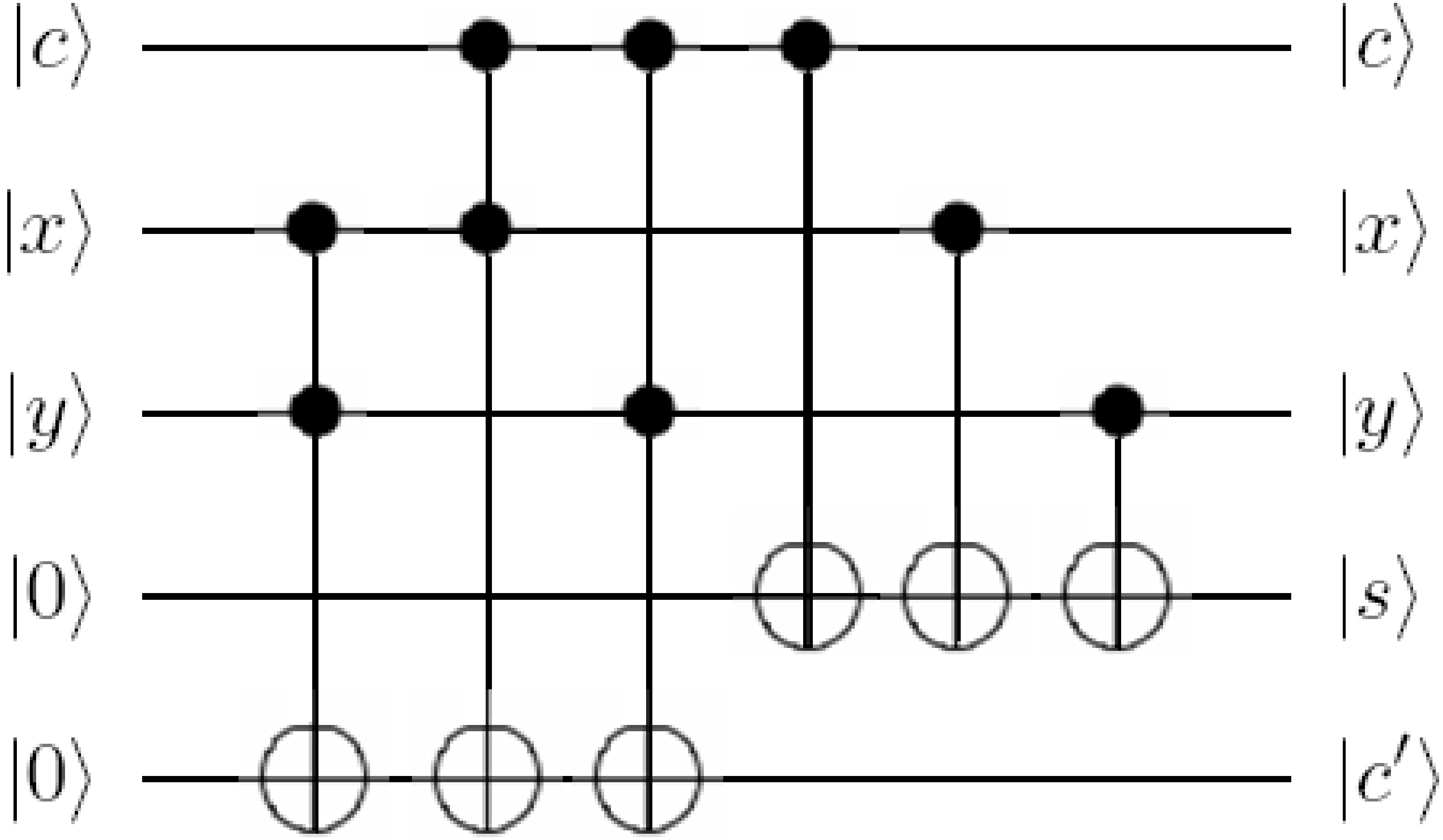
Quantum circuit implementation for $f(x_0, x_1, x_2) = \overline{x_0} + x_1 x_2$.

$$f(x_0, x_1, x_2) = \overline{x_0} + x_1 x_2 = x_0 x_1 x_2 \oplus x_0 \oplus 1$$

1-bit Half Adder



1-bit Full Adder

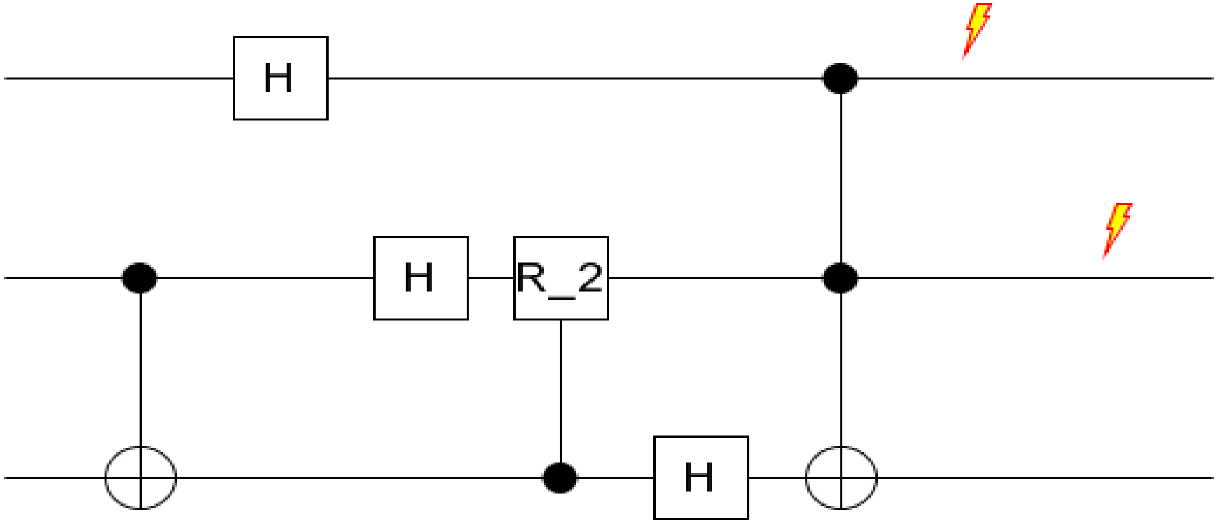


Let |c> = |1>, |x> = |0>, |y> = |1>
Then |s> = |0>, |c'> = |1>

Quantum Computation

- Quantum computation can be summarised as applying a sequence of transformations, called *quantum gates*, followed by a measurement.

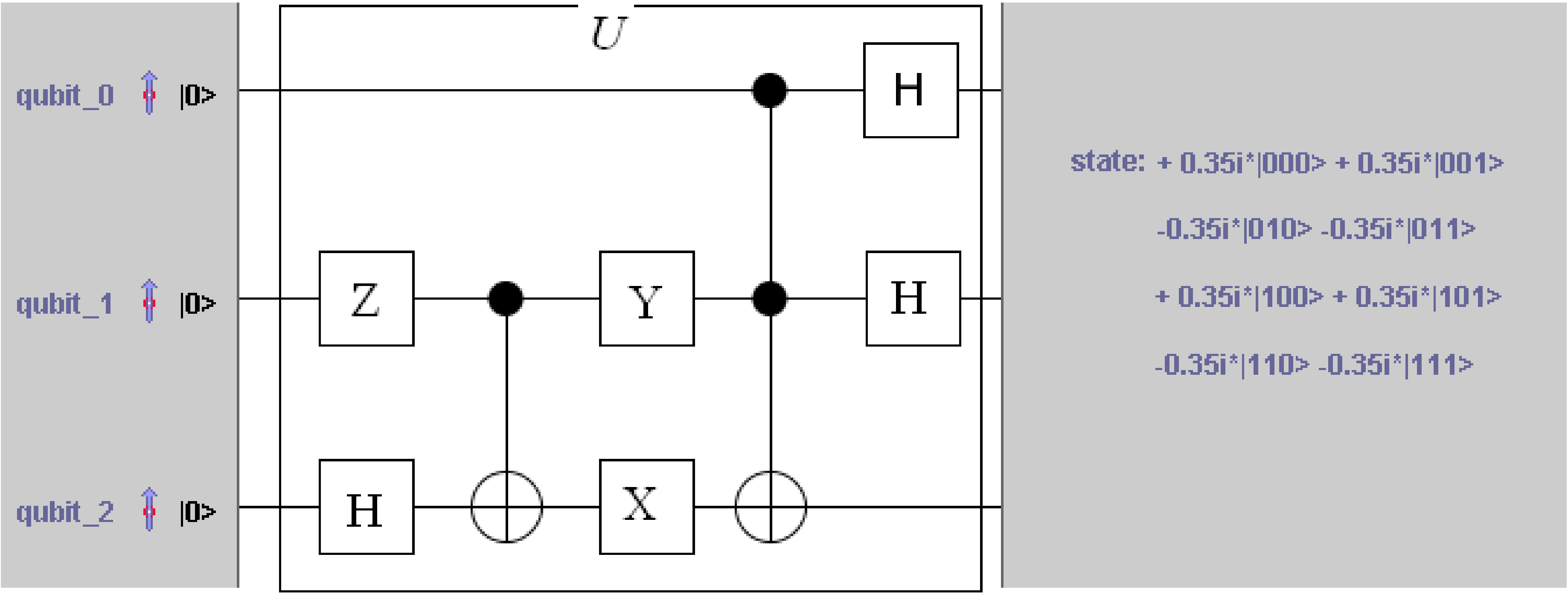
Input



Output

Set of Transformations (Gates)

Quantum Computation (cont.)



$$U = (H \otimes H \otimes I) T (I \otimes Y \otimes X)(I \otimes CNOT)(I \otimes Z \otimes H)$$

Thank you