Higgs self-coupling:
Precise predictions in arbitrary models

Johannes Braathen (DESY)

ECFA Mini-workshop on the Higgs self-coupling focus topic | 15 May 2024

anyH3

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

DESY.

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Outline of the talk

- Calculating λ_{hhh} in BSM theories and mass-splitting effects
- How large can λ_{hhh} become in realistic/allowed scenarios?
- Automating calculations of λ_{hhh} with `anyH3`
- BSM trilinear scalar couplings

What I will not cover (→ backup)

- *Why investigate λ_{hhh} ?*
- *Current experimental determination and future prospects (including at lepton colliders)*
- *Could BSM show up first in λ_{hhh} ? (turns out the answer is "Yes!!")*

Calculating λ_{hhh} in models with extended scalar sectors

The Two-Higgs-Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{\rm 2HDM} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

$$v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2$$

- **Mass eigenstates**:
  h, H: CP-even Higgs bosons *(h → 125-GeV SM-like state);* A: CP-odd Higgs boson;
  $H^\pm$: charged Higgs boson

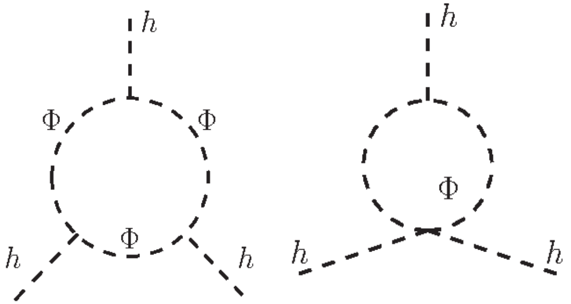
- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale M (defined by $M^2 \equiv 2m_3^2/s_{2\beta}$), angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta = v_2/v_1$)

- **BSM-scalar masses** take form $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2\,, \quad \Phi \in \{H, A, H^\pm\}$

- We take the **alignment limit α = β - π/2** → all Higgs couplings are SM-like at tree level
  → compatible with current experimental data

Mass-splitting effects in λ_{hhh}

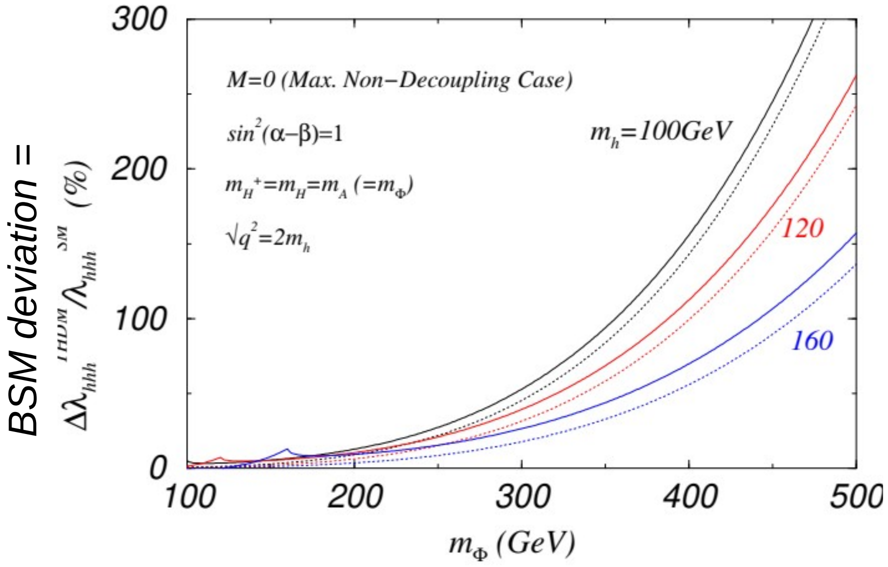
- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM: [Kanemura, (Kiyoura), Okada, Senaha, Yuan '02, '04]



$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$$

$$(\Phi \in \{H, A, H^\pm\})$$

$$[m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2]$$



- **Deviations of tens/hundreds of % from SM possible,** for **large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings**
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

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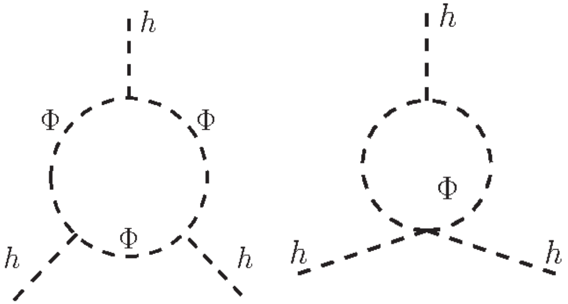

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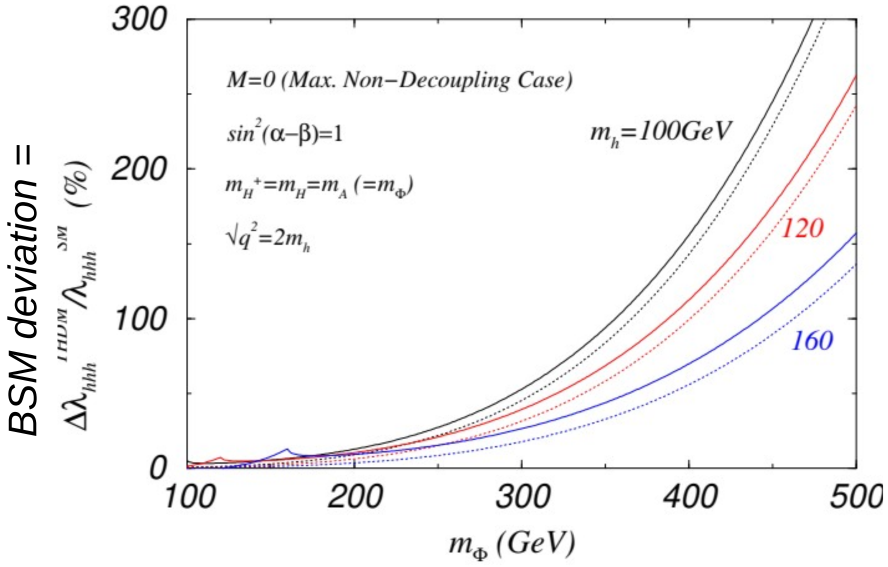

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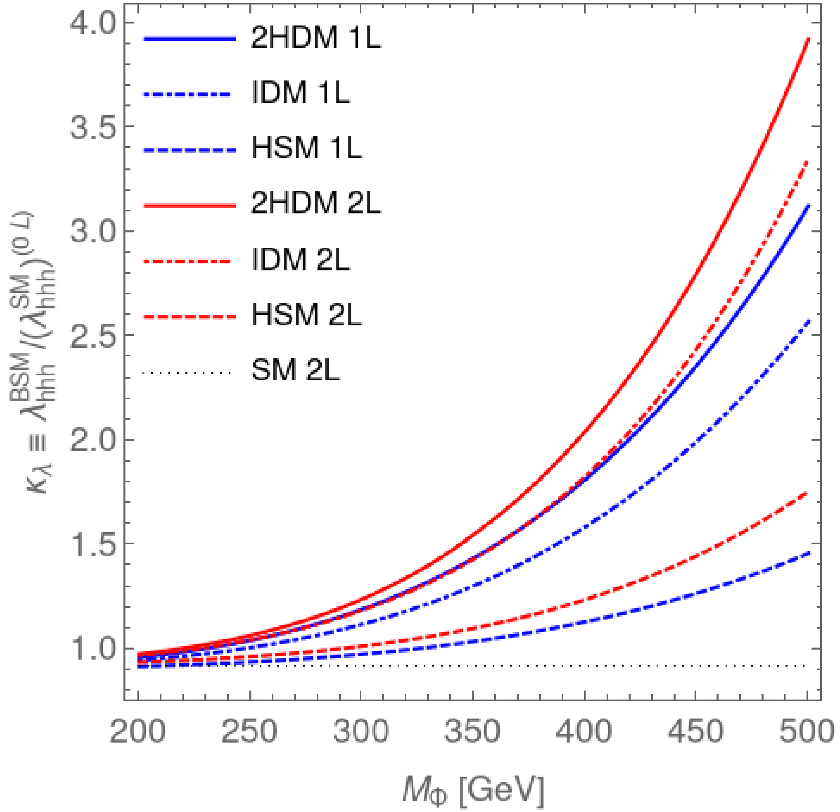
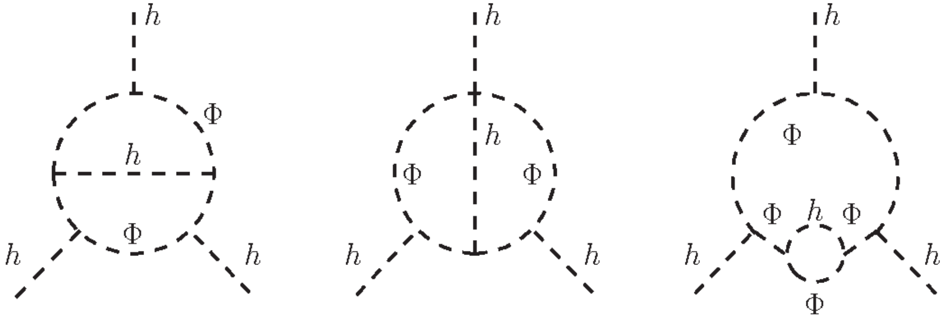
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

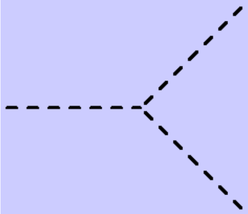
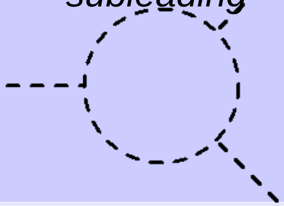
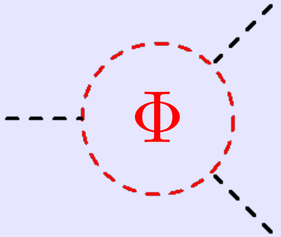
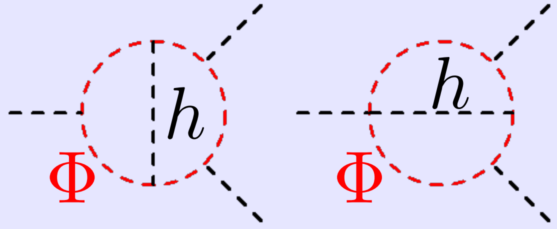
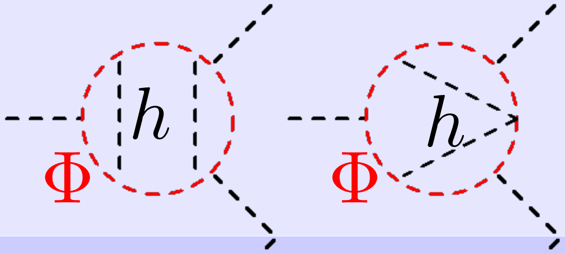
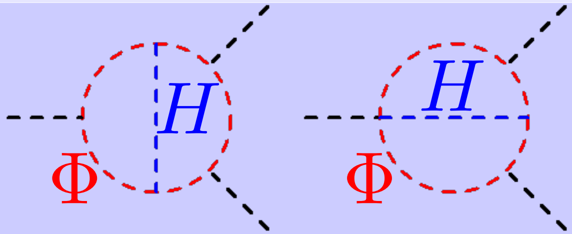
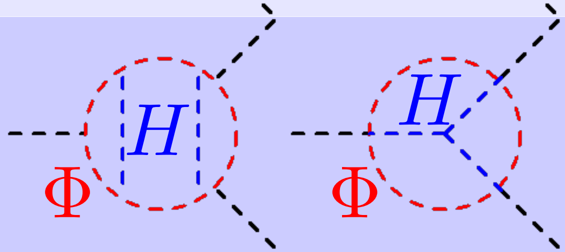
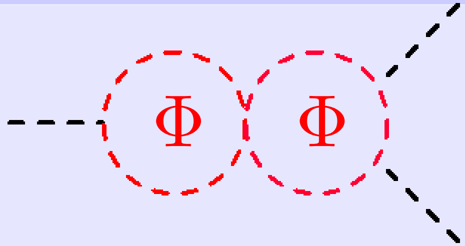
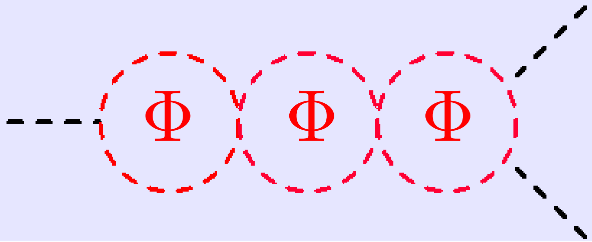
- Large effects **confirmed at 2L** in [JB, Kanemura '19] → **leading 2L corrections involving BSM scalars (H,A,$H^\pm$) and top quark**, computed in effective potential approximation





Examples of scalar contributions to λ_{hhh}

$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2} \qquad \Phi \in \{H, A, H^\pm\}$$
$$m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$$

Coupling/Order	0L	1L	2L	3L
g_{hhhh}		*subleading* 	*subleading*	*subleading*
$g_{(h)h\Phi\Phi}$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$	-	-		

NB: 1 h can be replaced by a VEV!

→ *no further type of coupling entering after 2L*

→ *for each class of diagrams, perturbative convergence was checked!* [Bahl, JB, Weiglein PRL '22]

Constraining BSM models with λ_{hhh}

i. *Can we apply the limits on κ_λ, extracted from experimental searches for di-Higgs production, for BSM models?*

ii. *Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?*

As a concrete example, we consider an aligned 2HDM

Based on

arXiv:2202.03453 **(Phys. Rev. Lett.) in collaboration with Henning Bahl and Georg Weiglein**

Can we apply di-Higgs results for the aligned 2HDM?

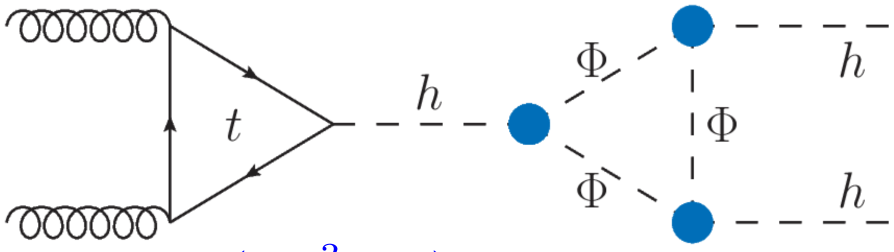
- Current strongest limit on κ_λ are from ATLAS double- (+ single-) Higgs searches

  $-0.4 < \kappa_\lambda < 6.3$ [ATLAS PLB, 2211.01216]

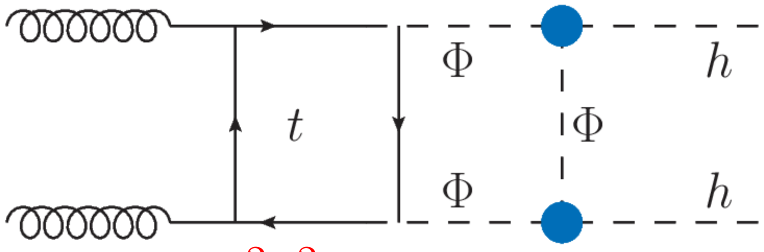
  [where $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$]

- What are the *assumptions* for the ATLAS limits?
  - All other Higgs couplings (to fermions, gauge bosons) are SM-like

    → this is **ensured by the alignment** ✓ (can also be relaxed → with κ_t floating + single-Higgs: $-1.4 < \kappa_\lambda < 6.1$)
  - The modification of λ_{hhh} is the only/main source of deviation of the *non-resonant di-Higgs production cross section* from the SM



$\propto \mathcal{O}(y_t g_{hh\Phi\Phi}^3)$ ***included***

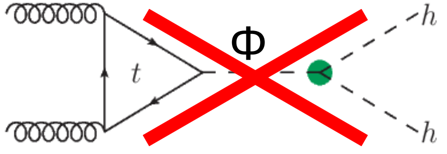


$\propto \mathcal{O}(y_t^2 g_{hh\Phi\Phi}^2)$ ***not included***

→ We **correctly include all leading BSM effects to di-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

- **We can apply the ATLAS limits to our setting!**

(Note: BSM resonant di-Higgs production cross section also suppressed at LO, thanks to alignment)



A parameter scan in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

- Our strategy:
  1. **Scan BSM parameter space**, keeping only points passing various theoretical and experimental constraints (*see below*)
  2. Identify regions with **large BSM deviations in λ_{hhh}**
  3. Devise a **benchmark scenario** allowing large deviations and investigate impact of experimental limit on λ_{hhh}
- *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types, or other BSM models with extended Higgs sectors
- Constraints in our parameter scan:

experimental
  - 125-GeV Higgs measurements with `HiggsSignals`
  - Direct searches for BSM scalars with `HiggsBounds`
  - b-physics constraints, using results from [Gfitter group 1803.01853]

  (Checked with `ScannerS` [Mühlleitner et al. 2007.02985])

  - EW precision observables, computed at two loops with `THDM_EWPOS` [Hessenberger, Hollik '16, '22]

theoretical
  - Vacuum stability
  - Boundedness-from-below of the potential

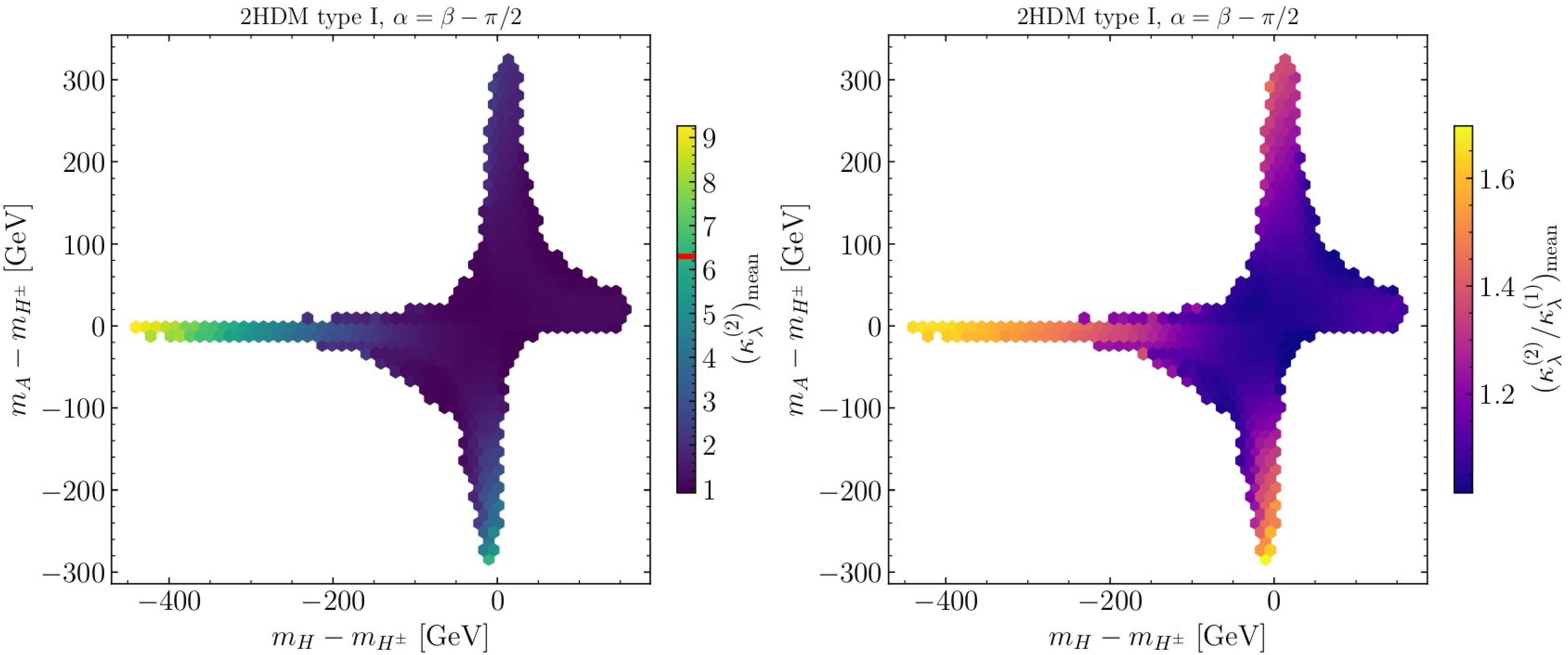
  (Checked with `ScannerS`)

  - NLO perturbative unitarity, using results from [Grinstein et al. 1512.04567], [Cacchio et al. 1609.01290]
- For points passing these constraints, we **compute κ_λ at 1L and 2L**, using results from [JB, Kanemura '19]

Parameter scan results

[Bahl, JB, Weiglein PRL '22]

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane



NB: all previously mentioned constraints are fulfilled by the points shown here

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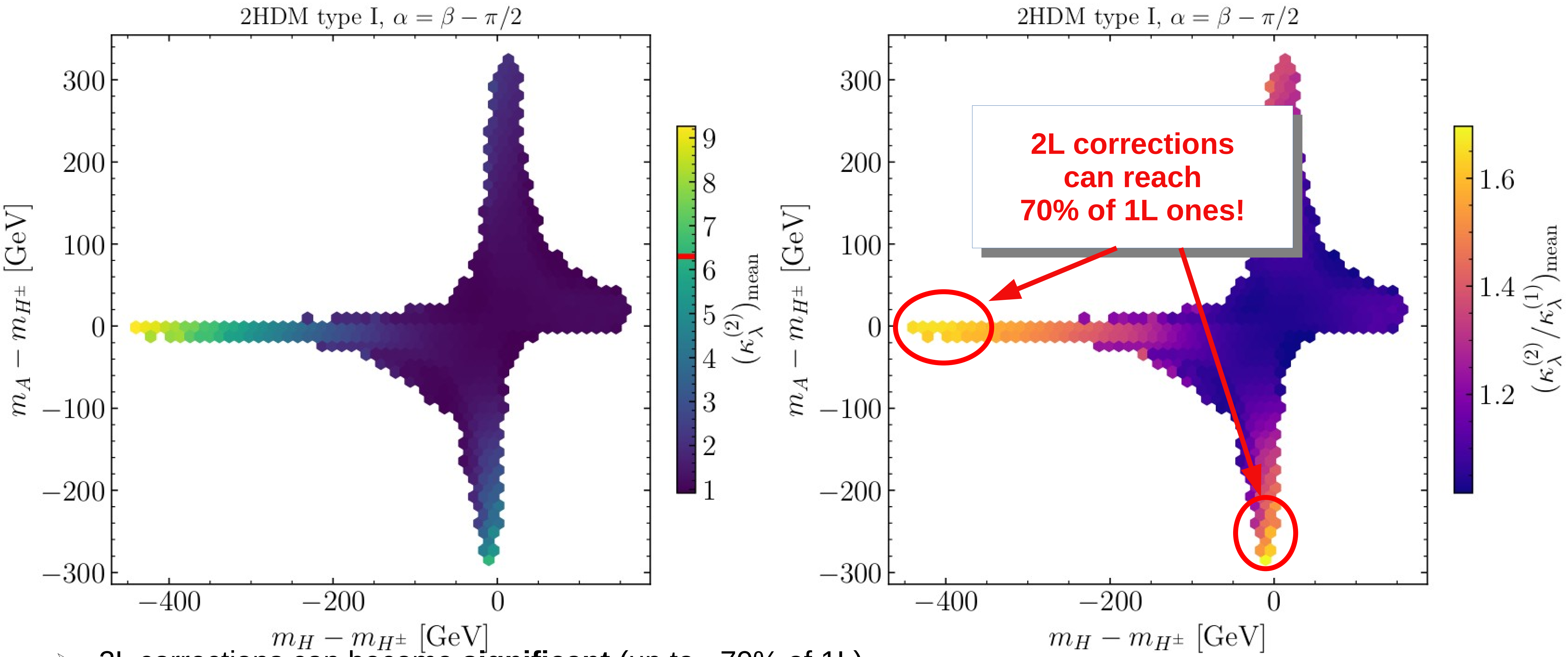

- 2L corrections can become **significant** (up to ~70% of 1L)

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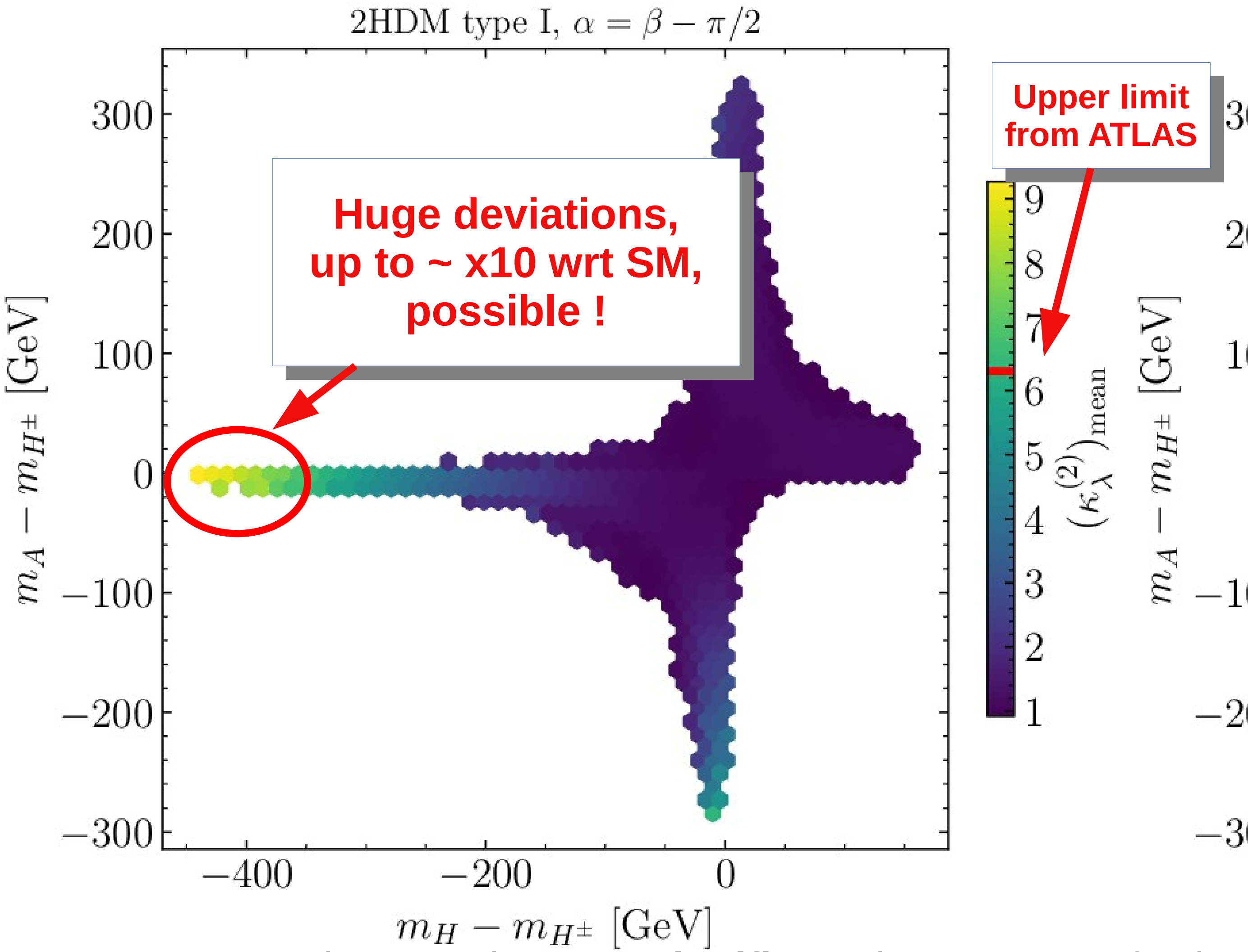
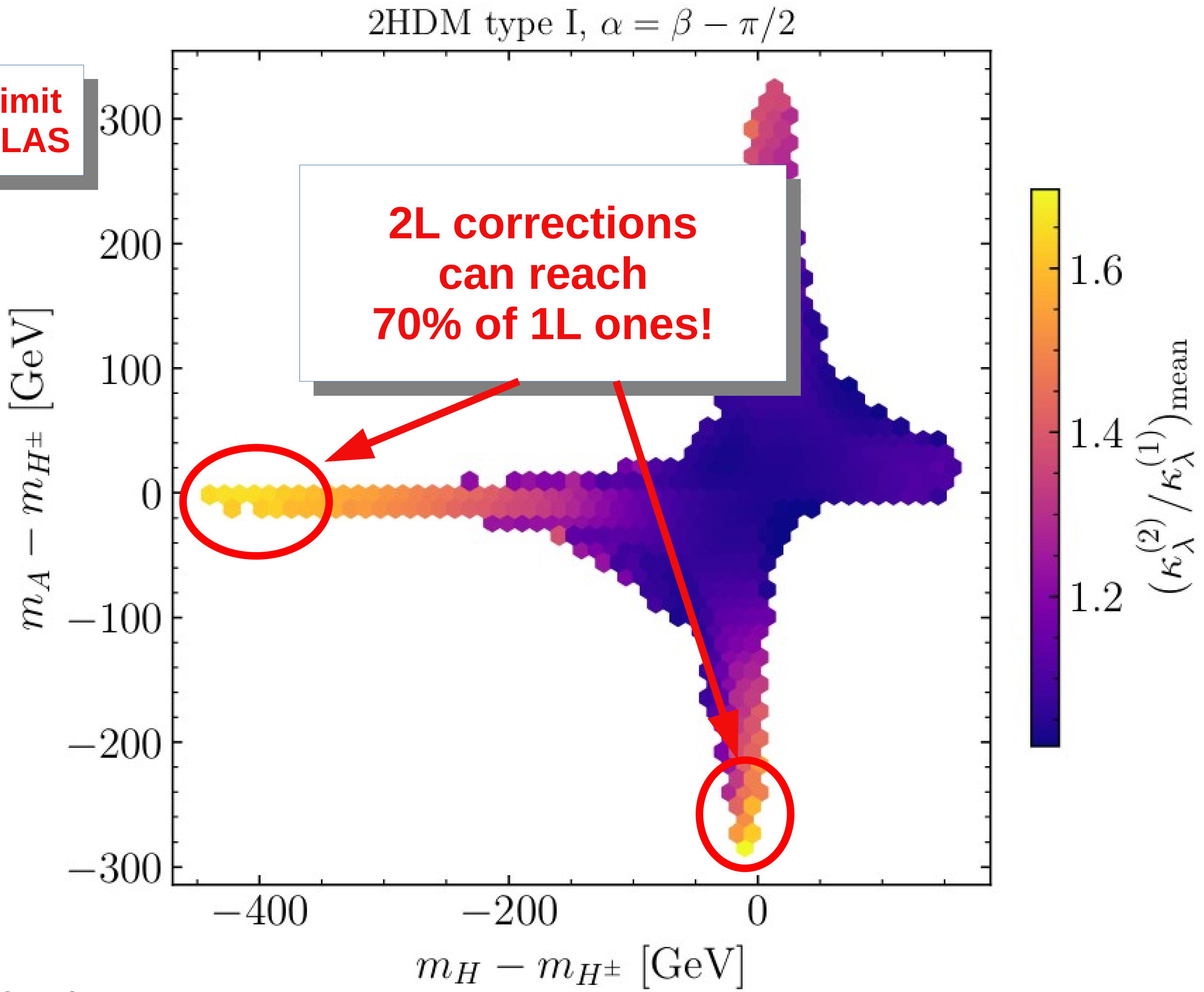




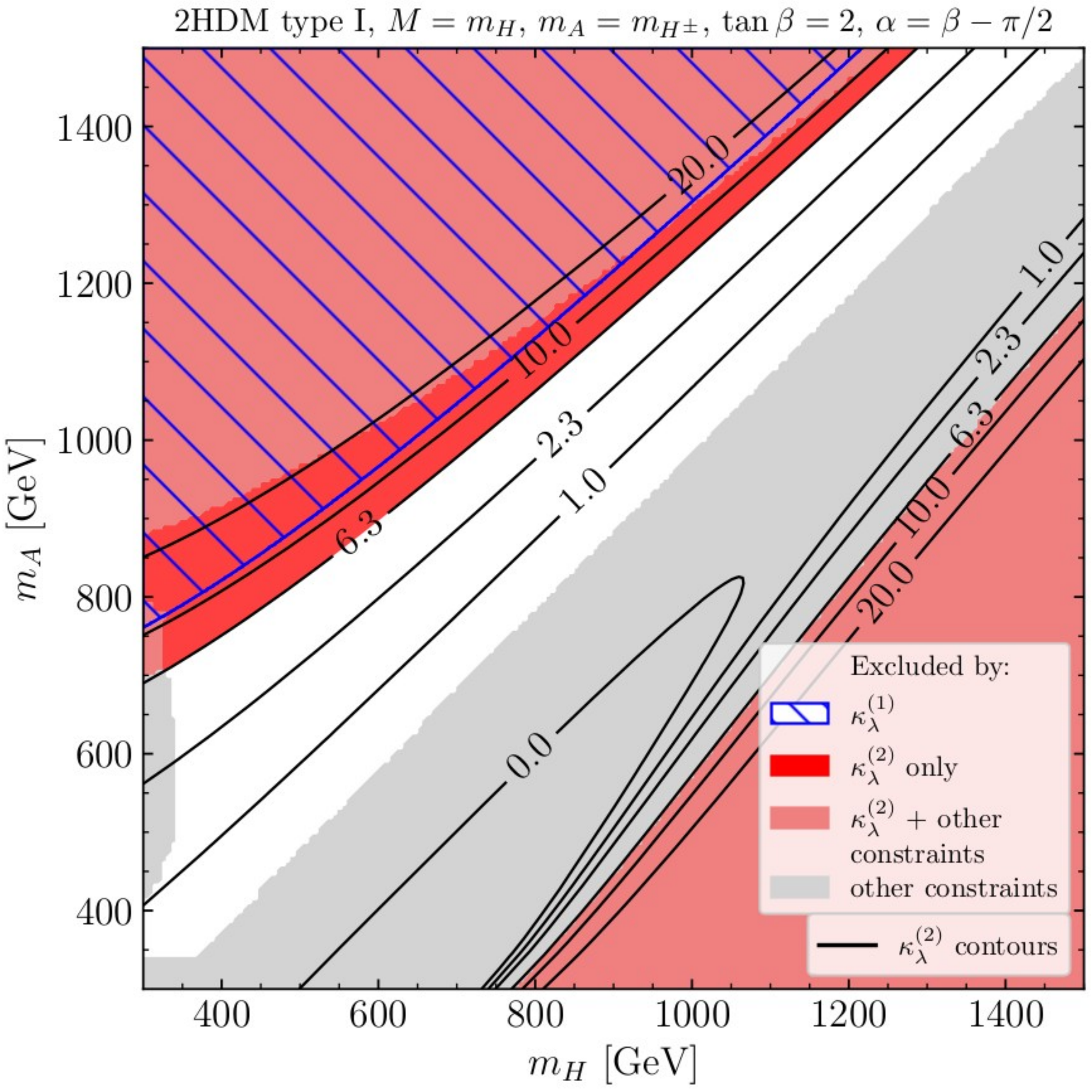
- 2L corrections can become **significant** (up to ~70% of 1L)
- **Huge enhancements** (by a factor ~10) of λ_{hhh} possible for $m_A \sim m_{H\pm}$ and $m_H \sim M$

A benchmark scenario in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$



- ***Grey area:*** area excluded by other constraints, in particular BSM Higgs searches, boundedness-from-below (BFB), perturbative unitarity
- ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

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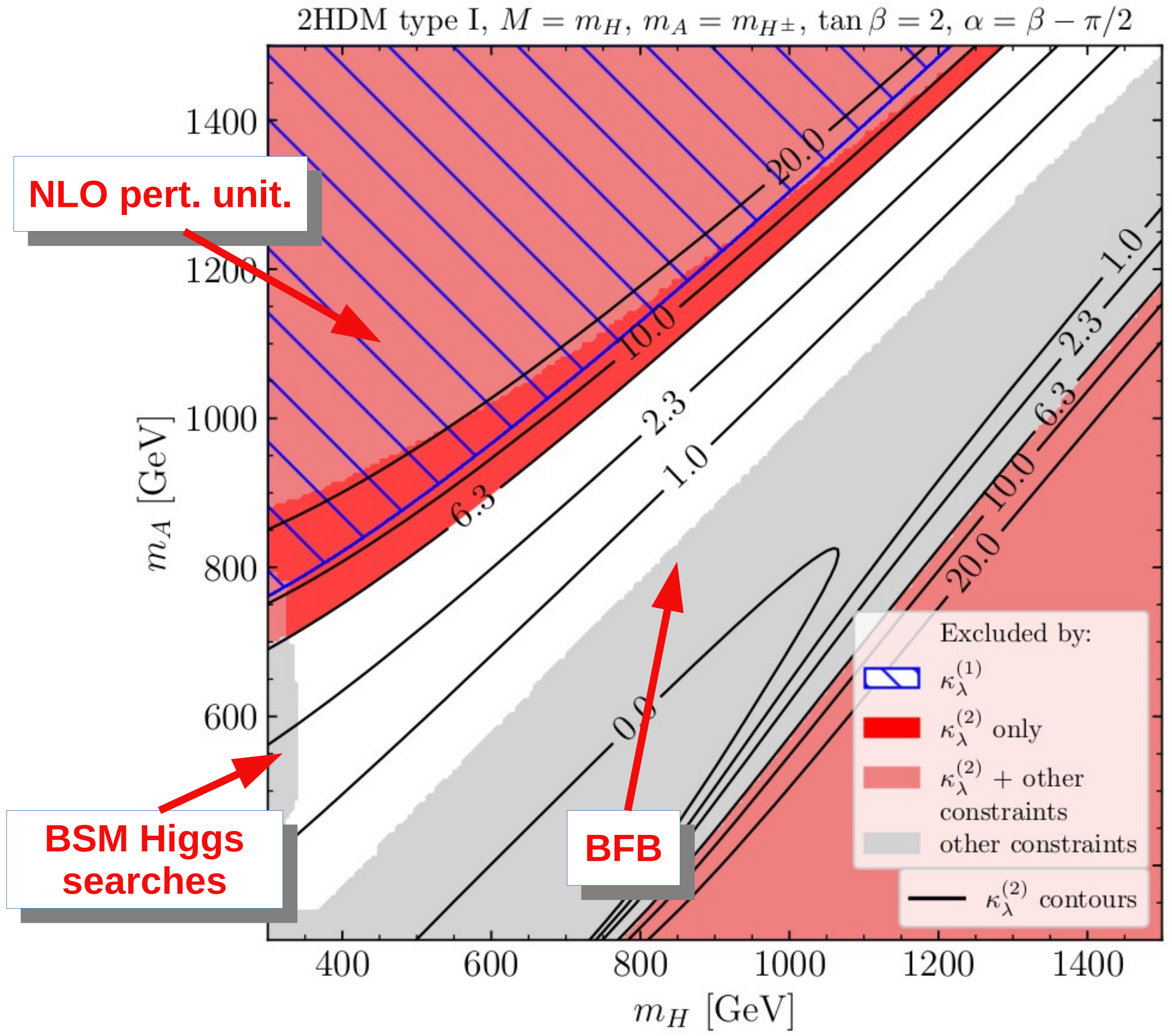

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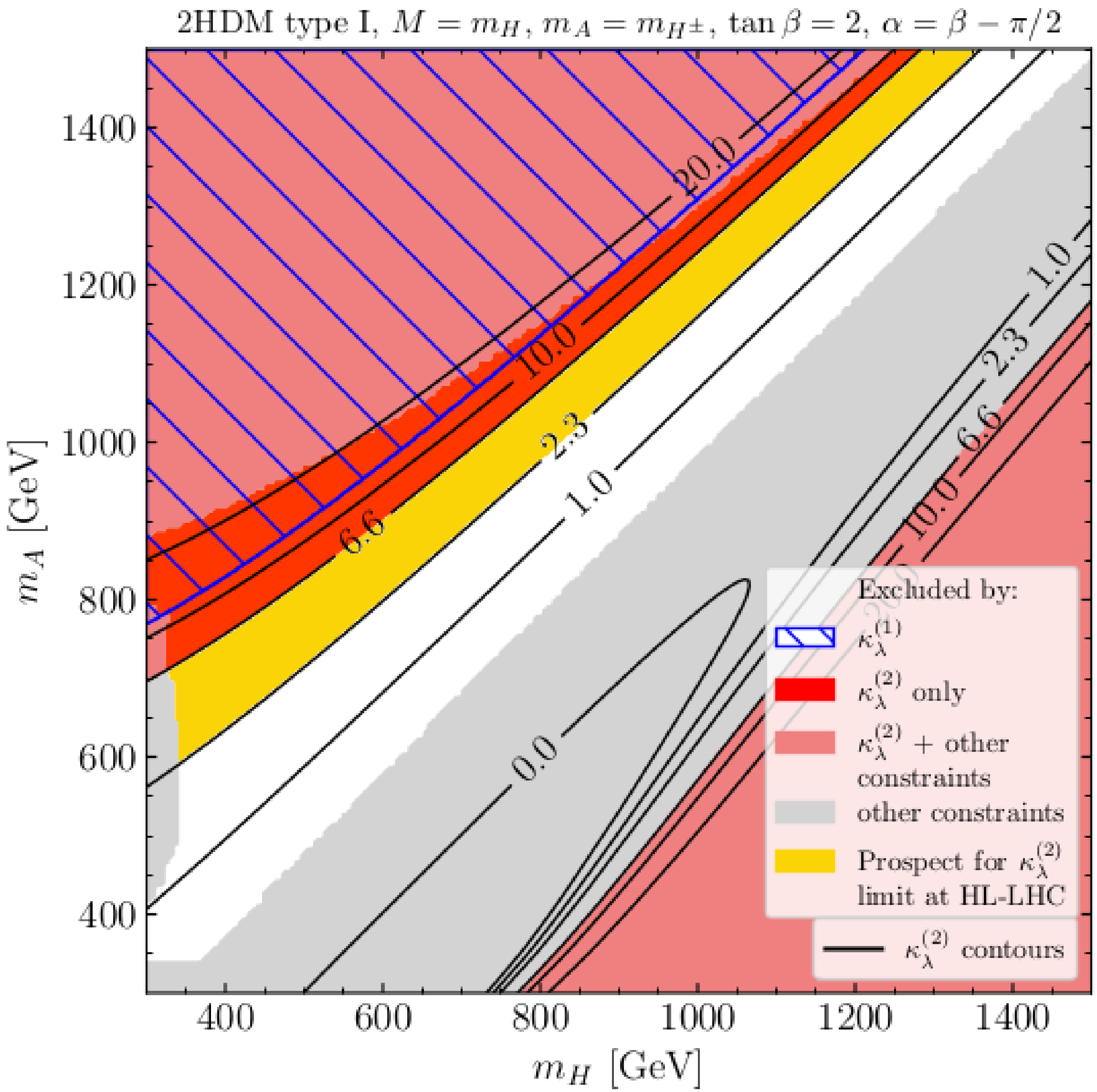
A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$

[Bahl, JB, Weiglein '23]



- ***Golden area:*** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)
- Of course, **prospects even better with an e^+e^- collider!**
- Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

Generic predictions for λ_{hhh}

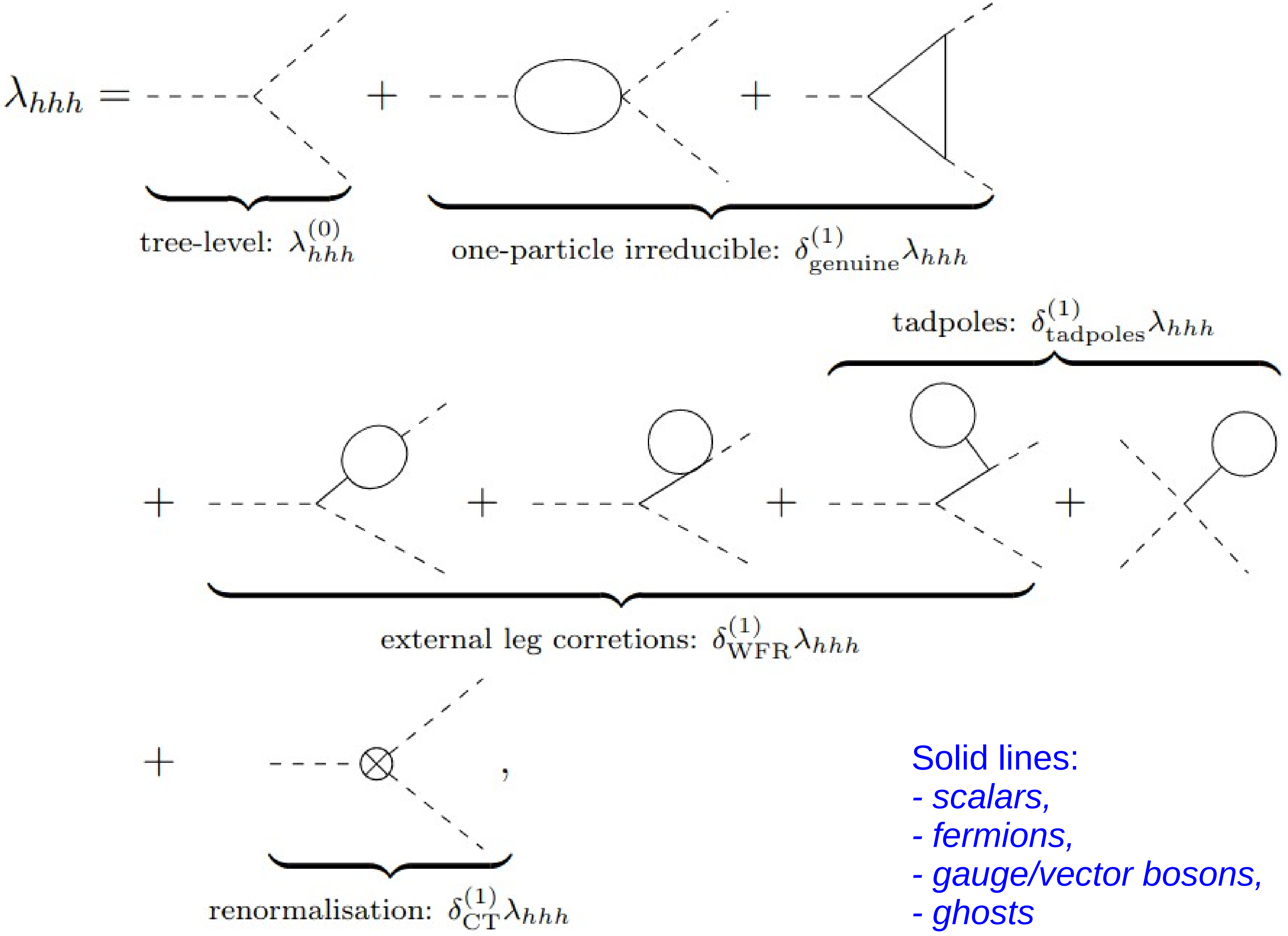
Based on

`arXiv:2305.03015` (EPJC) + WIP

in collaboration with Henning Bahl, Martin Gabelmann, Kateryna Radchenko Serdula and Georg Weiglein

Full one-loop calculation of λ_{hhh} with anyH3: how does it work?

- Generic results applied to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., '11], [Darmé et al. '23]
- Loop functions evaluated via `COLLIER` [Denner et al '16] interface, `pyCollier`
- Restrictions on **particles** and/or **topologies** possible
- **Analytical results** (Python, Mathematica) & **fast numerical results** (with caching)
- **Renormalisation performed automatically** (*more in backup*)



Solid lines:
- *scalars,*
- *fermions,*
- *gauge/vector bosons,*
- *ghosts*

A cross-check: the decoupling limit

- Consider the decoupling limit in several BSM models

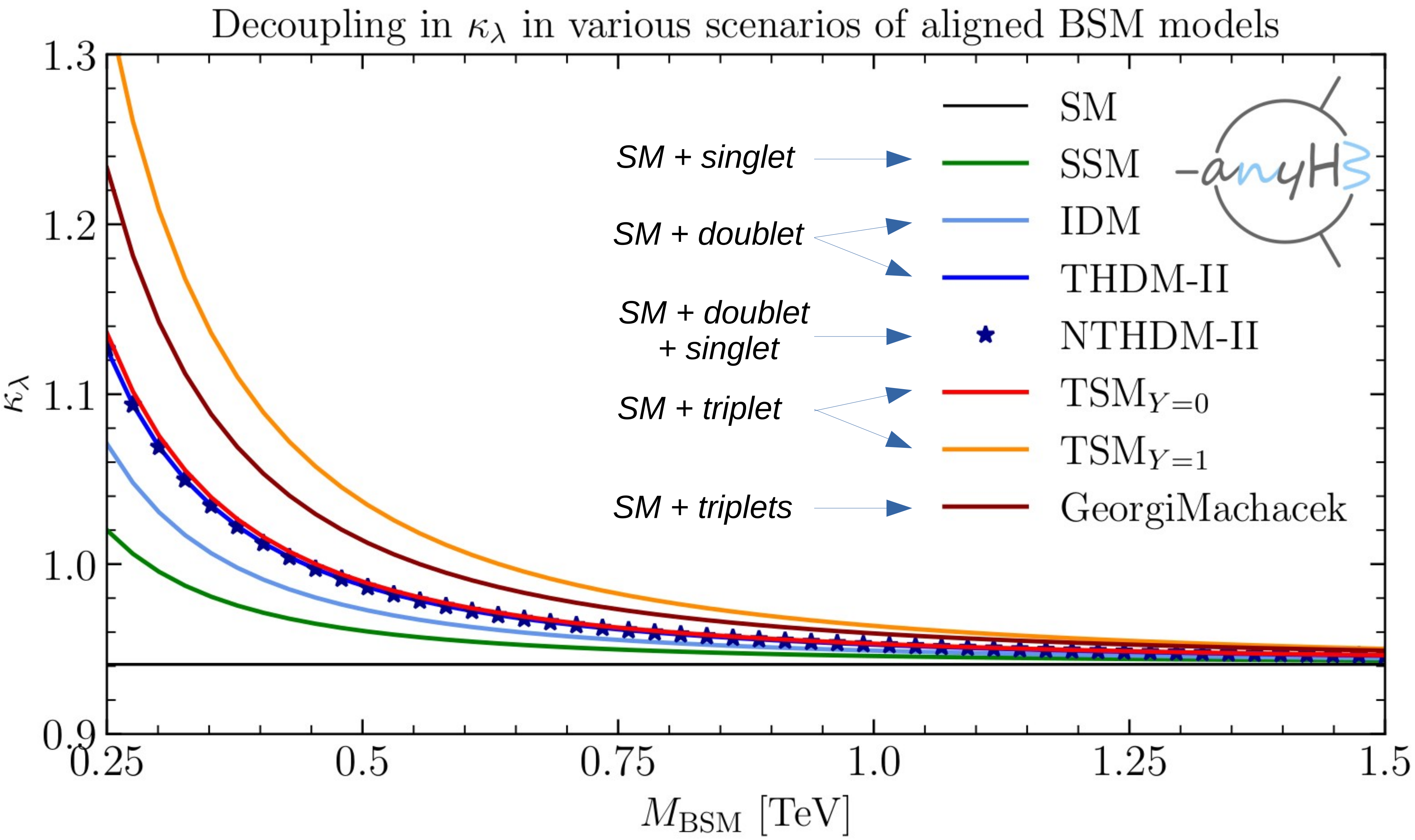
$$M_{\rm BSM}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

$\mathcal{M}$: BSM mass scale
$\tilde{\lambda}$: Quartic couplings

- Increase BSM mass scale

$$\mathcal{M} \to \infty$$

- BSM corrections to should vanish (c.f. decoupling theorem [Appelquist, Carrazone '75])



New results I: mass-splitting effects in various BSM models

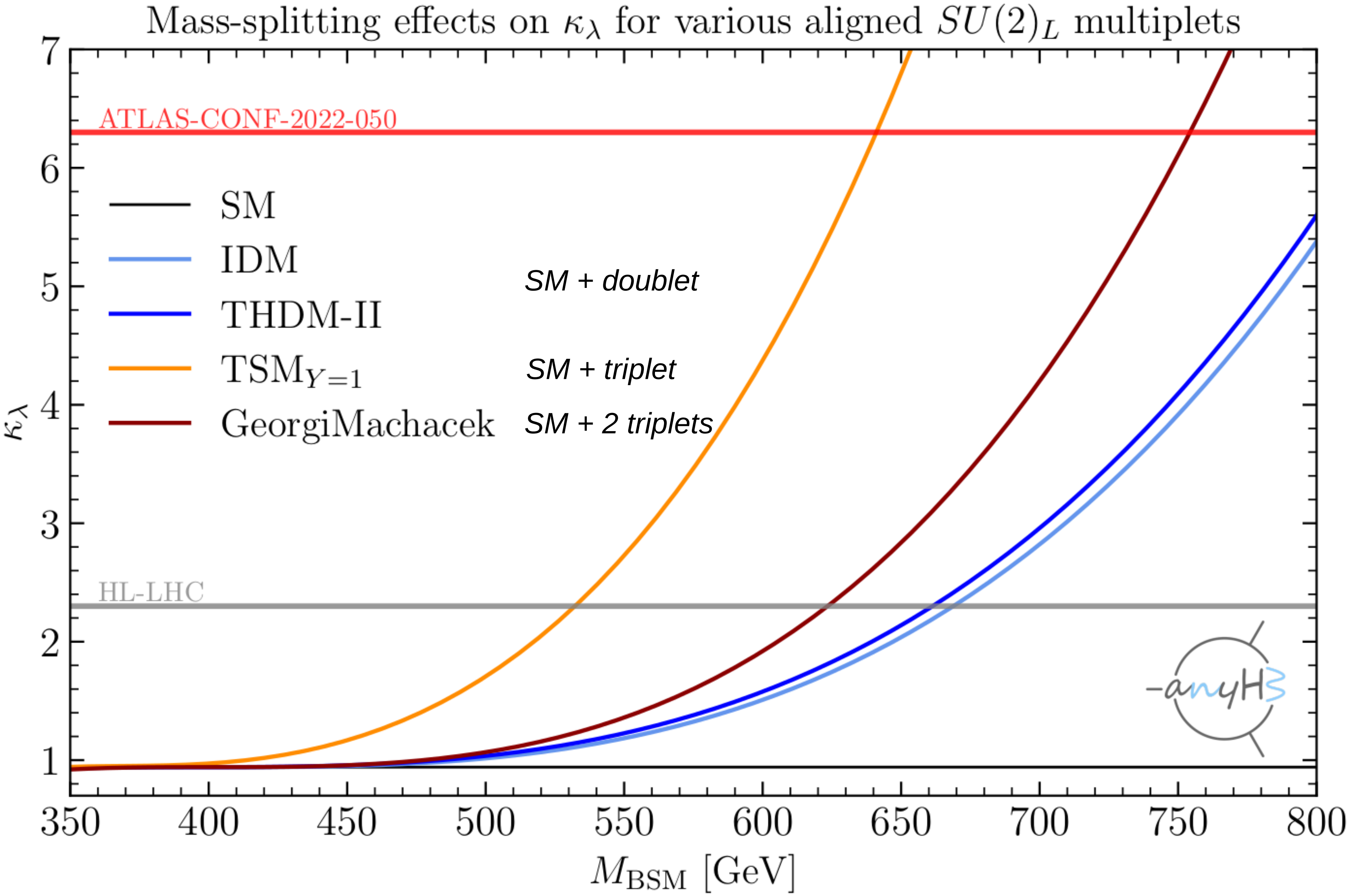
- Consider the non-decoupling limit in several BSM models

$$M^2_{\rm BSM} = \mathcal{M}^2 + \tilde{\lambda} v^2$$

- Increase $M_{\rm BSM}$, keeping $\mathcal{M}$ fixed
  → large mass splittings
  → **large BSM effects!**

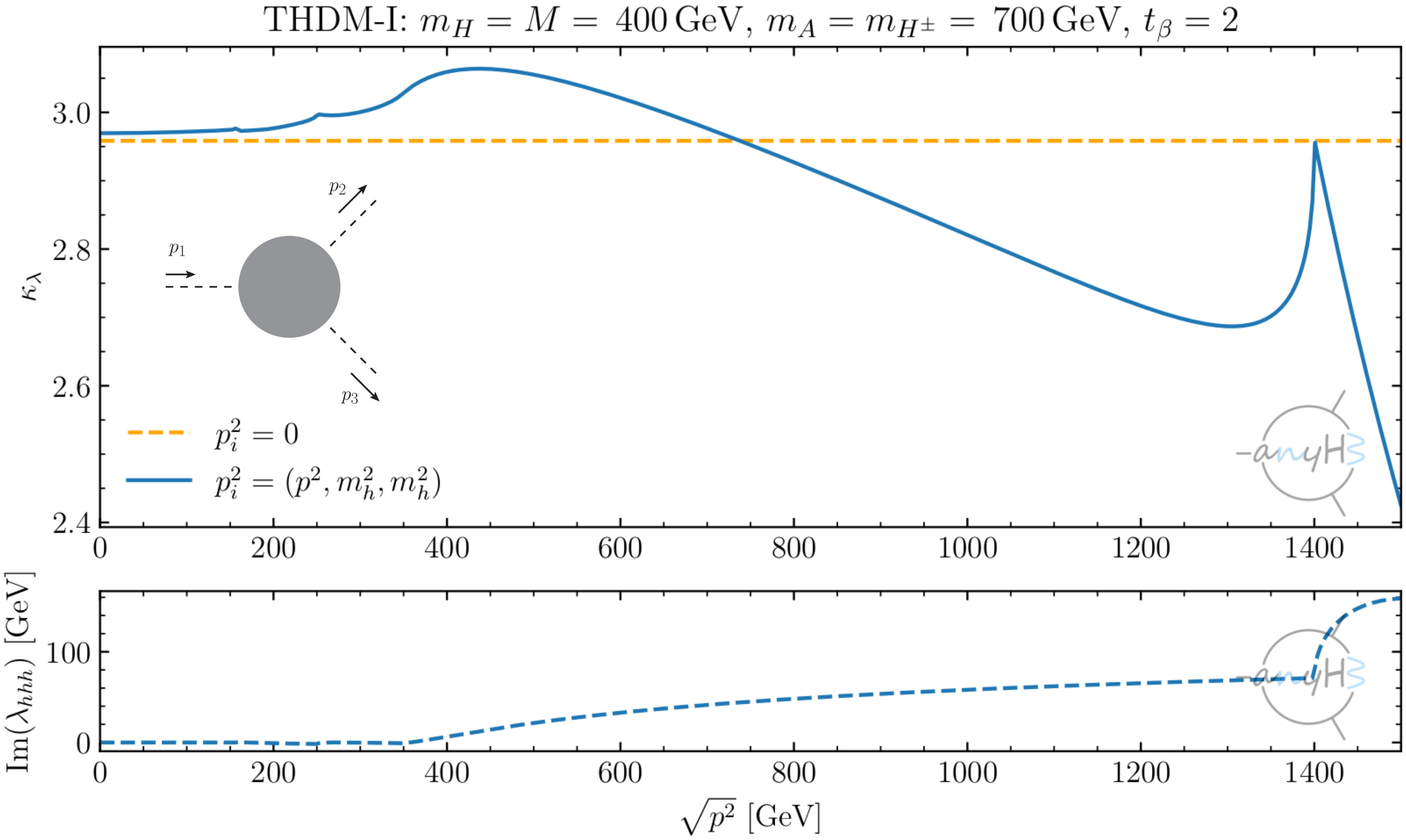
- Perturbative unitarity checked with `anyPerturbativeUnitarity`

- **Constraints on BSM parameter space!**



Here: scenarios with lightest BSM scalar mass + BSM mass param. at 400 GeV; other BSM scalar masses = M_{BSM}

New results II: momentum dependence in the 2HDM



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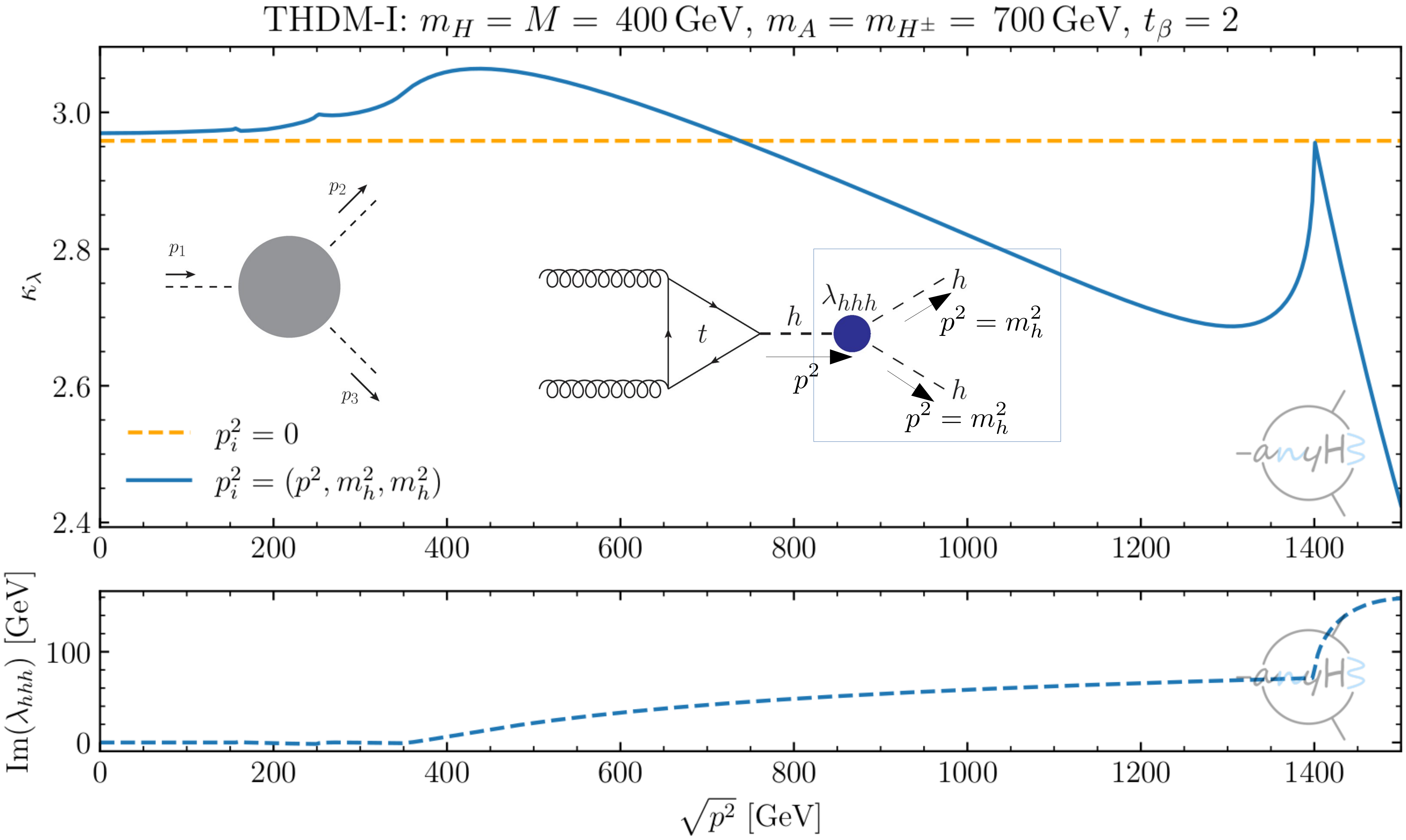

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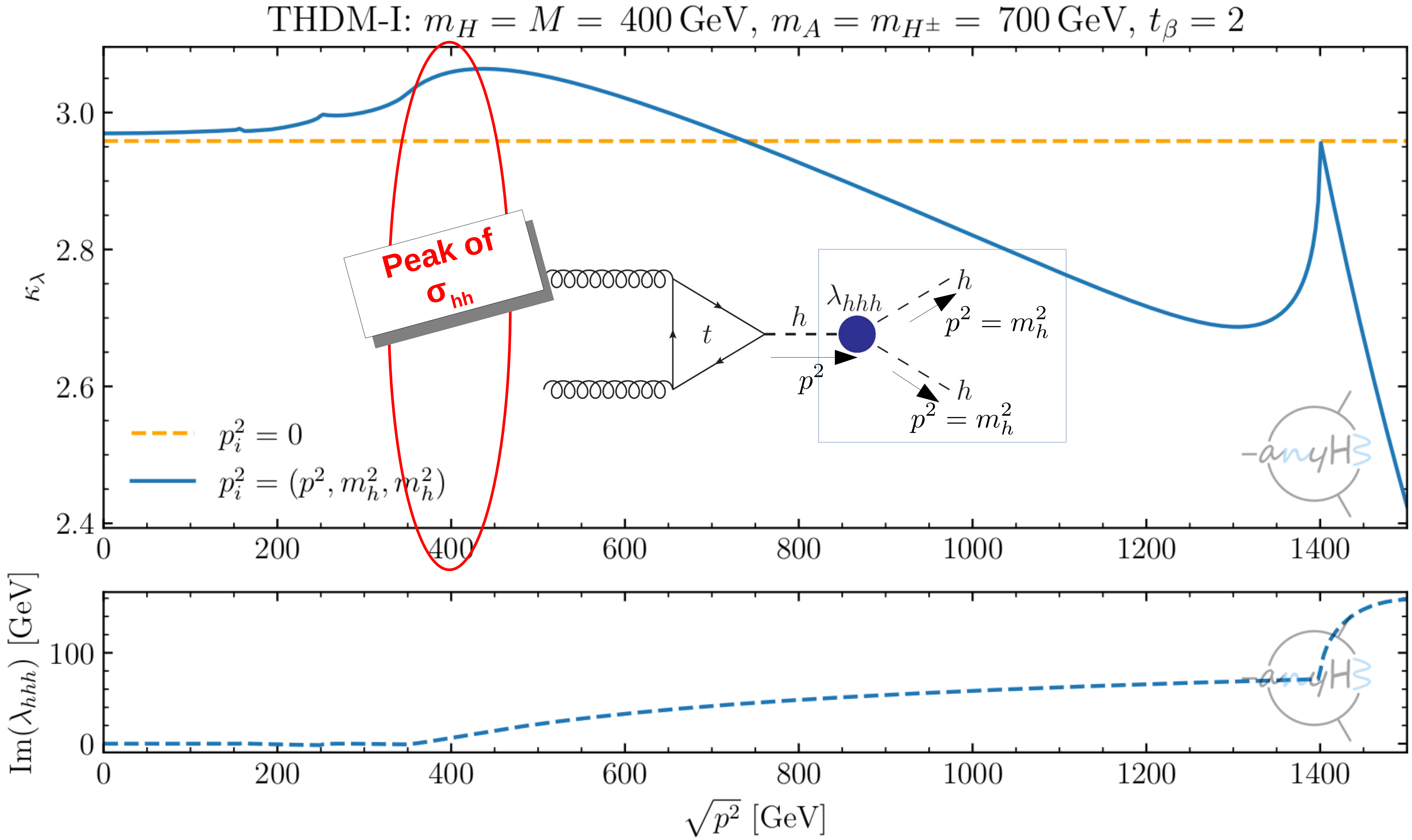

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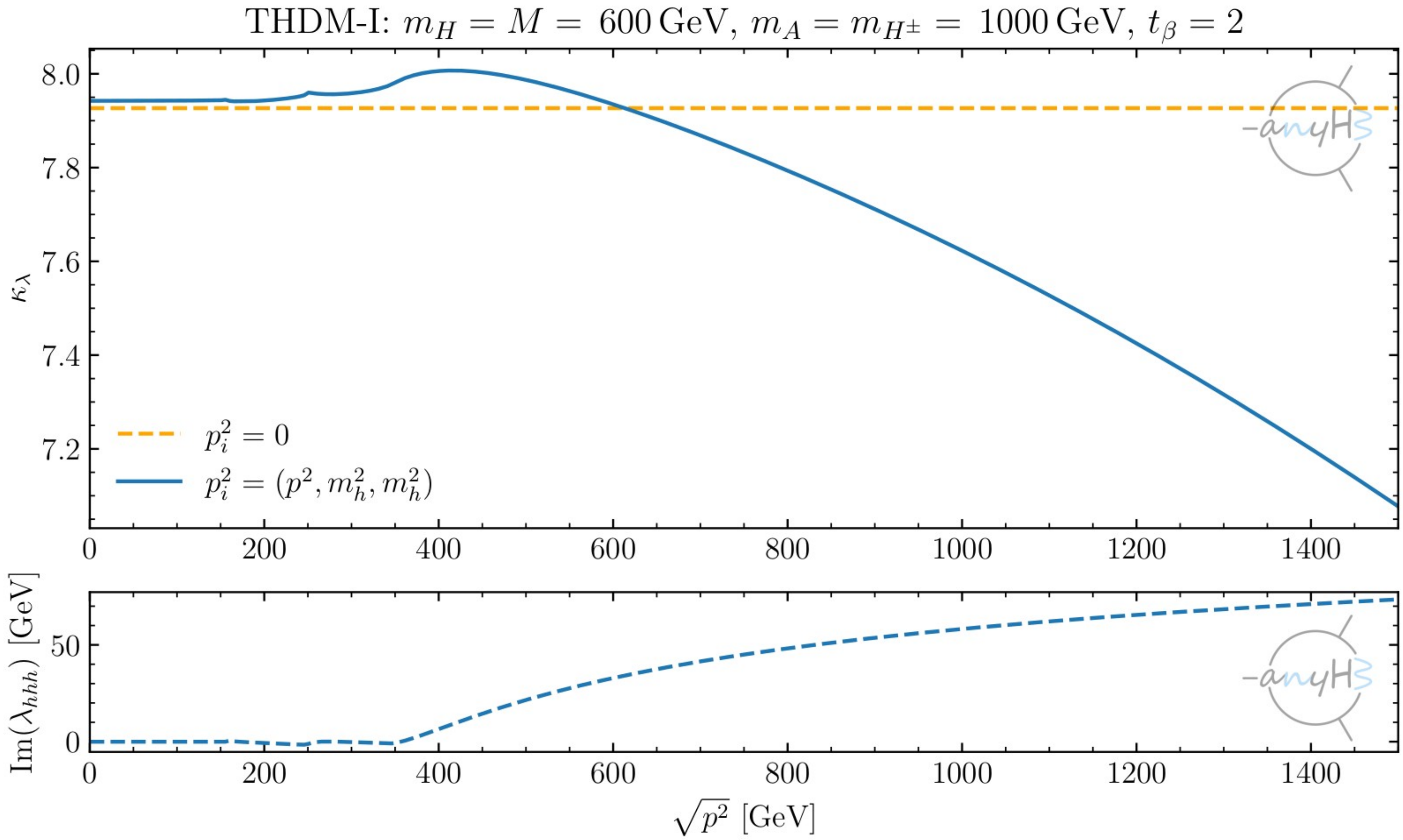
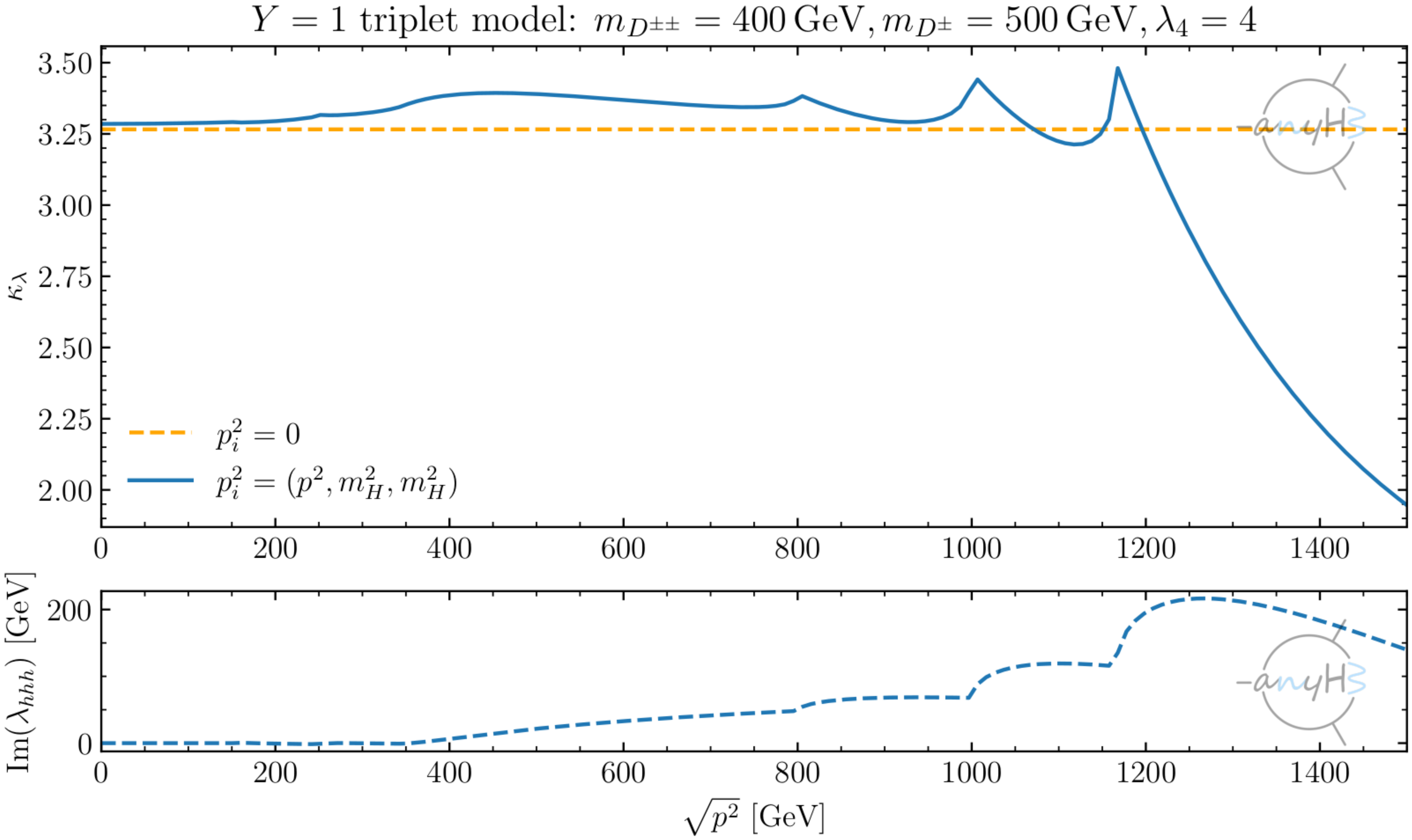


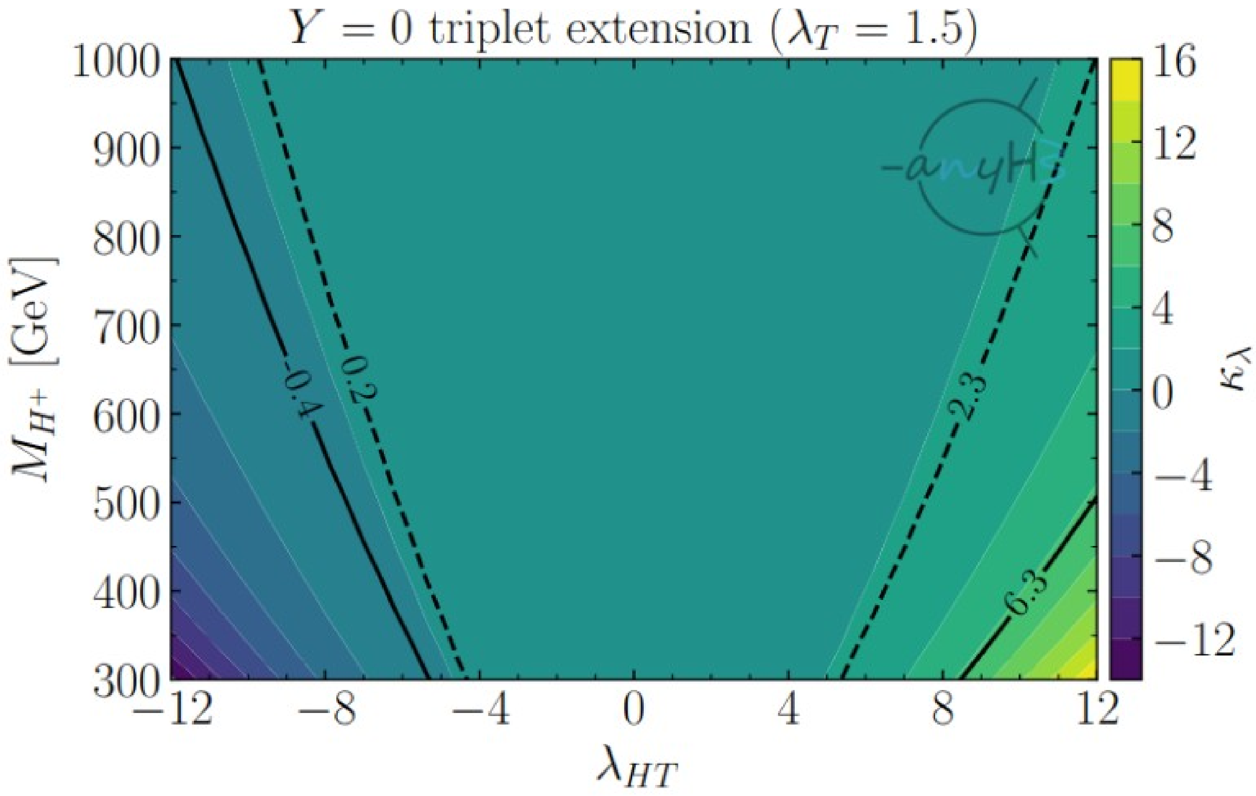
New results III: momentum dependence in a Y=1 triplet extension

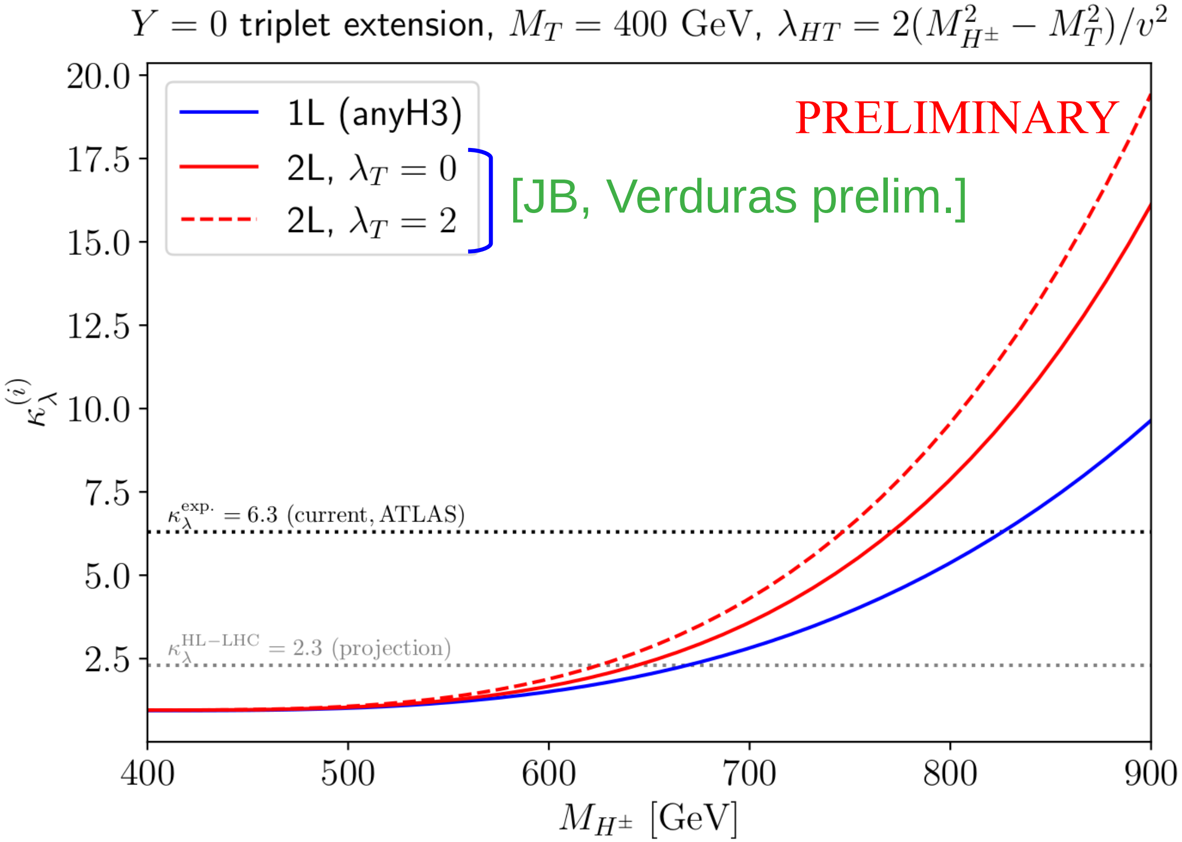


New results IV: probing scalar DM models with κ_λ

Real VEV-less triplet model:

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2, \ \langle T\rangle = 0, \ \langle \Phi \rangle = v_{SM}$$



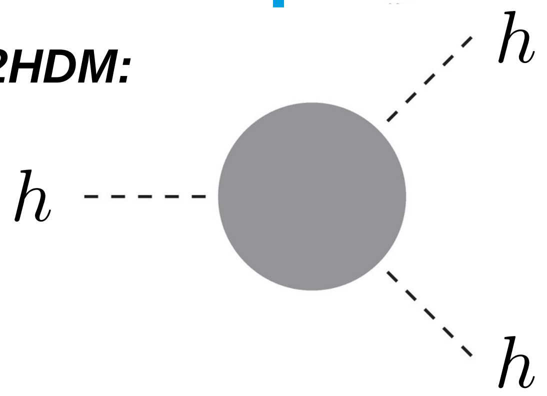


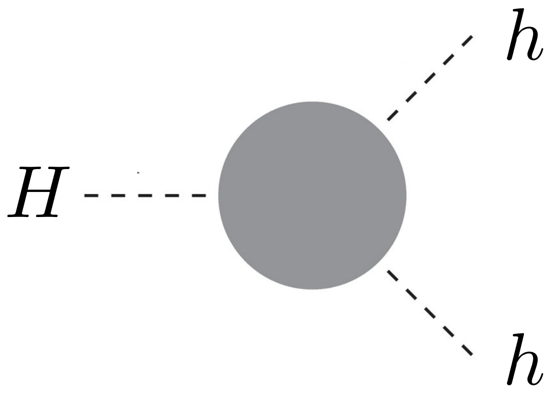
- *Left*: κ_λ @ 1L in plane of $M_{H\pm}$ and λ_{HT} (portal coupling) with anyH3
- *Right*: κ_λ @ 2L, with results from [JB, Verduras *WIP*]

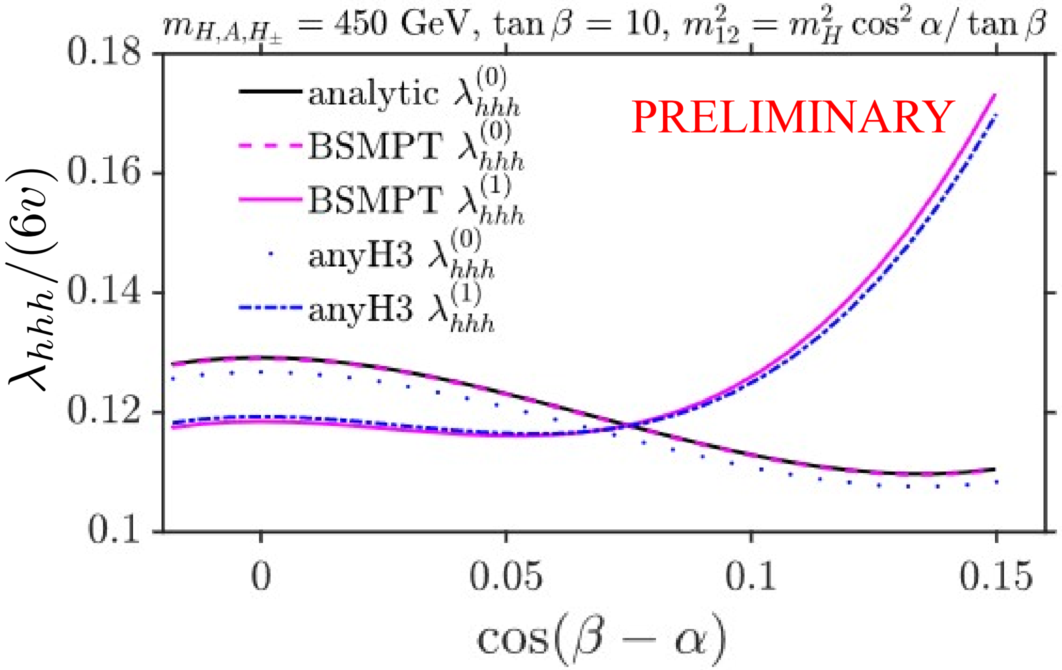
Ongoing development: `anyLamijk`

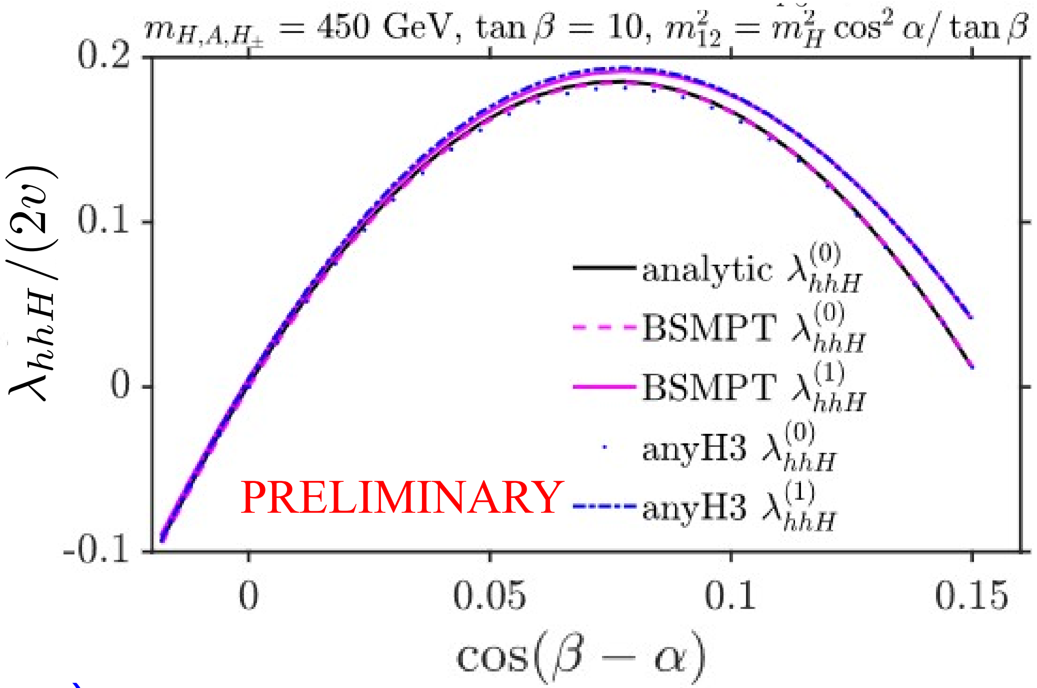
[Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein *WIP*]

Example in a 2HDM:









→ excellent agreement with BSMPT results (in eff. pot. approx.)

→ full 1L OS schemes for λ_{hhh} and λ_{hhH} couplings worked out in 2HDM [Bahl, JB, Gabelmann, Radchenko Serdula, Weiglein], SSM [JB, Heinemeyer, Verduras], and more [Bosse, JB, Gabelmann, Hannig, Weiglein]!

Summary

- λ_{hhh} plays a crucial role to probe the **shape of the Higgs potential**, and search indirect **signs of New Physics**
- λ_{hhh} can **deviate significantly from SM prediction** (by up to a factor **~10**), for otherwise theoretically and experimentally allowed points, due to **mass-splitting effects in radiative corrections involving BSM scalars** → current experimental bounds on λ_{hhh} can **already exclude significant parts of otherwise unconstrained BSM parameter space**, and future prospects even better!
- **Python package `anyH3` allows calculation of λ_{hhh} for arbitrary renormalisable theories** with
  - Full 1L effects including p^2 dependence
  - Highly flexible choices of renormalisation schemes → predefined or by user
  - Uses **UFO** model inputs (generated with SARAH, FeynRules or using custom ones)
  - Part of wider **anyBSM framework**, including calculation λ_{ijk}, under development
  - Currently 14 models included (publicly), easy inclusion of further models → **new ideas/requests welcome!**

Get started at https://anybsm.gitlab.io/
or directly in terminal with

`pip install anyBSM` **&** `anyBSM --help` **!**

Thank you very much for your attention!

Contact

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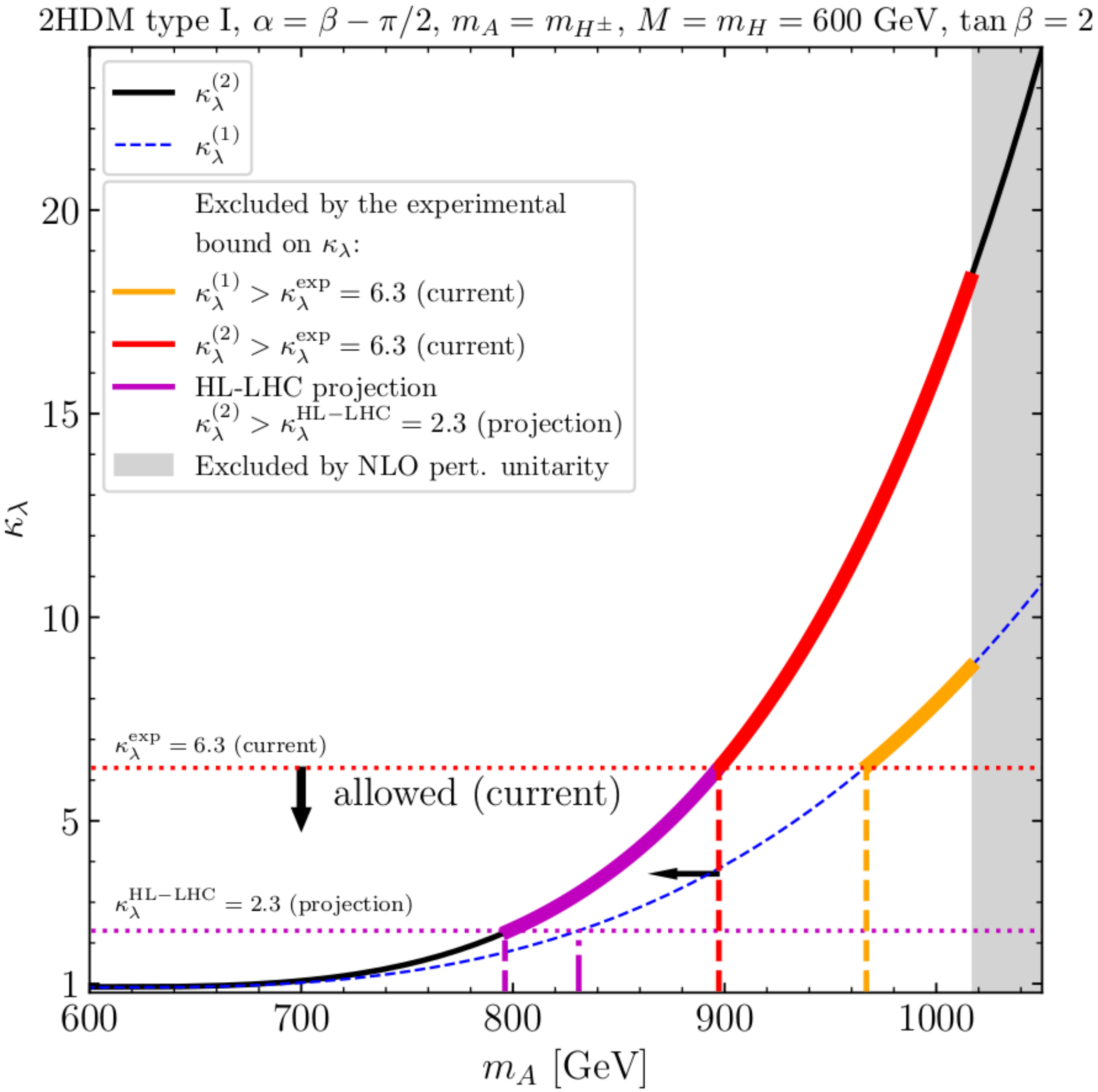
Backup

A benchmark scenario in the aligned 2HDM – 1D scan

Within the previously shown plane, we fix $M=m_H=600$ GeV, and vary $m_A=m_{H\pm}$

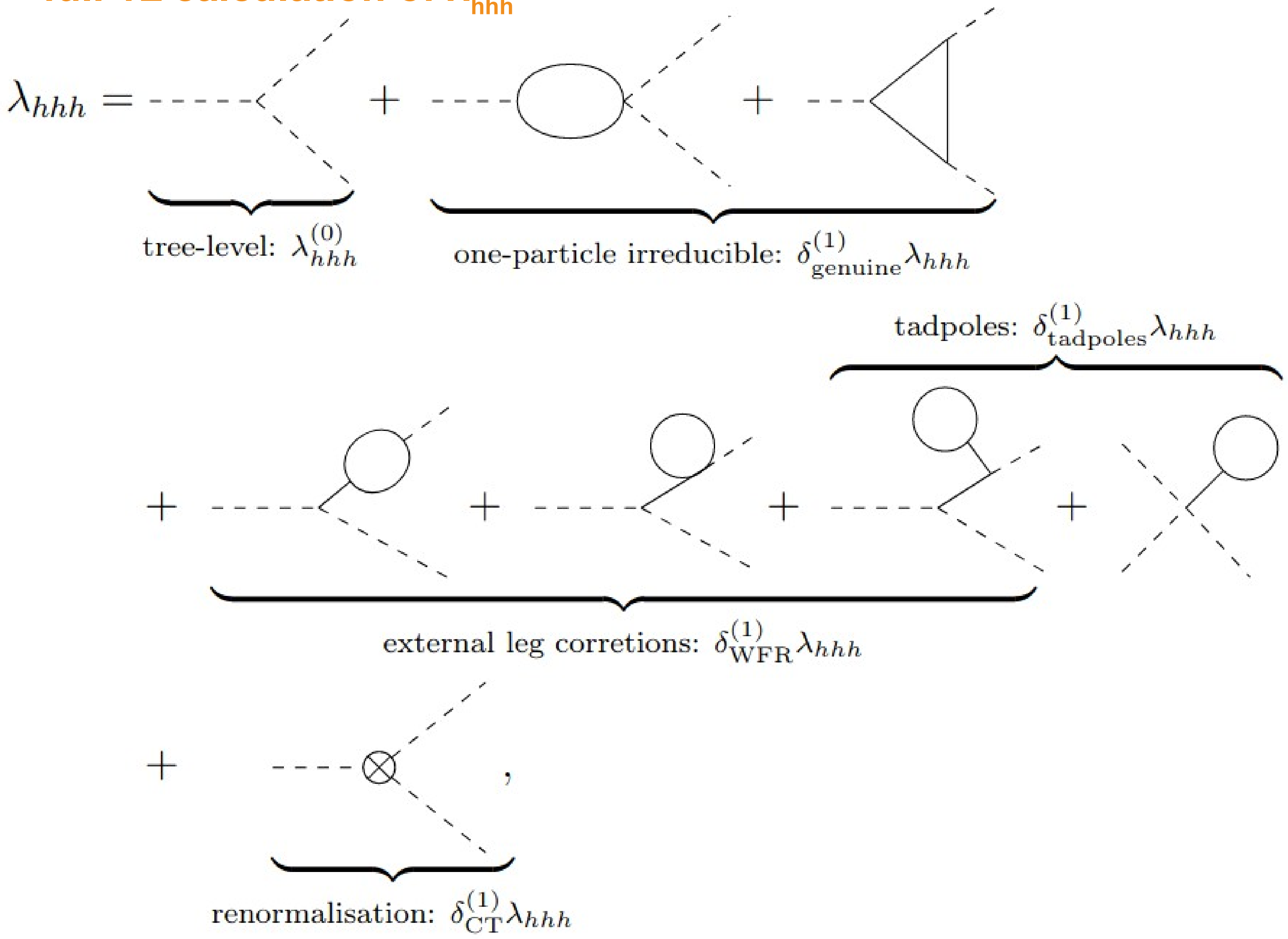
[Bahl, JB, Weiglein PRL '22]



- Illustrates the significantly improved reach of the experimental limit when including **2L corrections** in calculation of κ_λ
- A stricter choice for the perturbative unitarity constraint (grey) does not significantly change the region excluded by $\kappa_\lambda^{(2)}$

Computing λ_{hhh} in general renormalisable theories: ingredients

anyH3 → full 1L calculation of λ_{hhh}

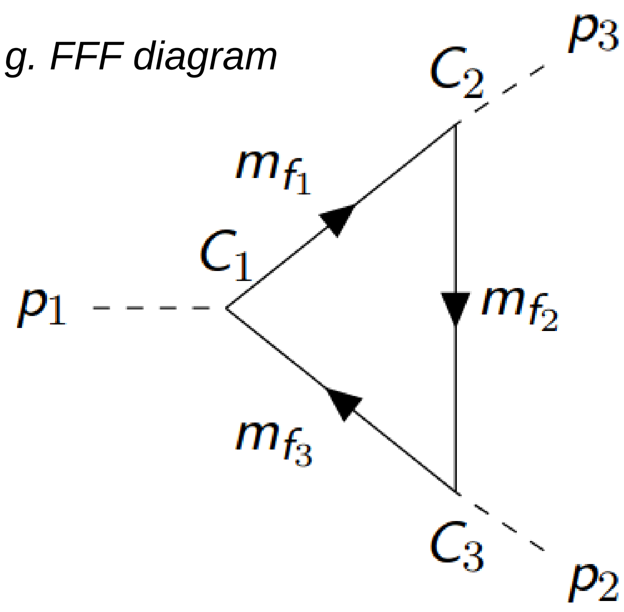


- Solid lines:
  - *scalars,*
  - *fermions,*
  - *gauge/vector bosons,*
  - *ghosts*
- Restrictions on particles and/or topologies possible

Computing λ_{hhh} in general renormalisable theories: method

Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic

e.g. FFF diagram



- Couplings $C_i = C_i^L P_L + C_i^R P_R, \text{ where } P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$
- Masses on the internal lines m_{fi}, i=1,2,3
- External momenta p_i, i=1,2,3

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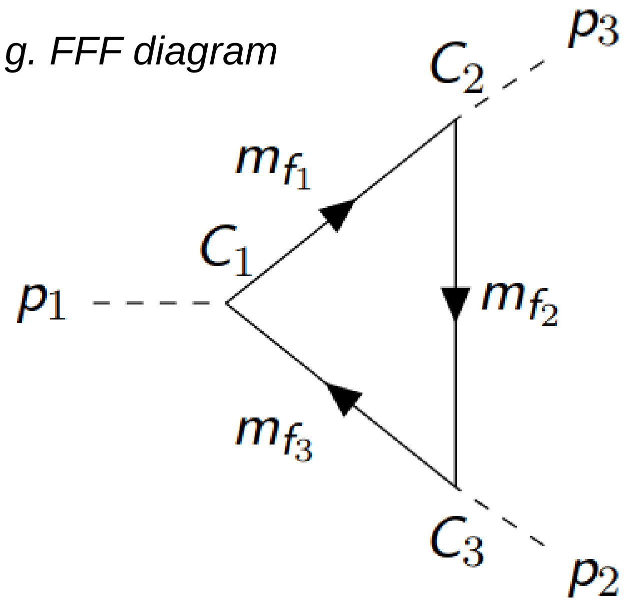

- Couplings $C_i = C_i^L P_L + C_i^R P_R, \text{ where } P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$
- Masses on the internal lines m_{f_i}, i=1,2,3
- External momenta p_i, i=1,2,3

$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

(**B0**, **C0**, **C1**, **C2**: loop functions)

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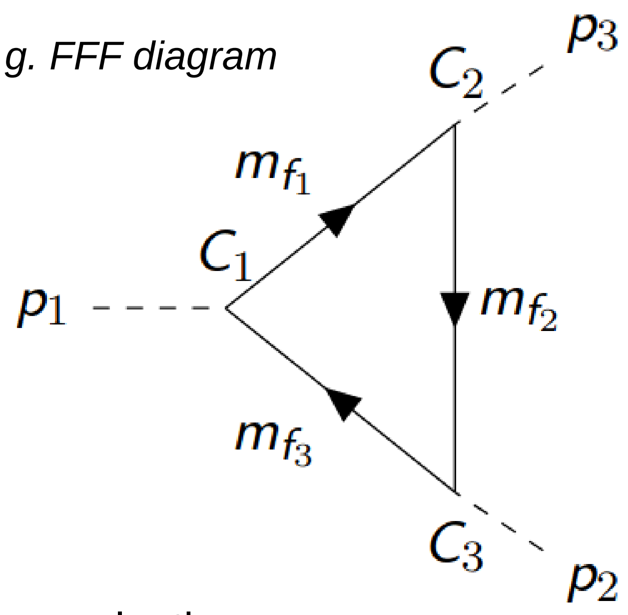
Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic

e.g. FFF diagram



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- Masses on the internal lines m_{f_i}, i=1,2,3
- External momenta p_i, i=1,2,3

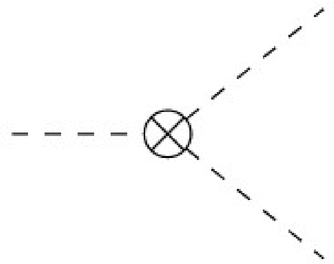
$$
\begin{aligned}
&= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + \\
&C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1} \mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + \\
&C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + \\
&2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
&C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
&C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + \\
&(C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - \\
&p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
&C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))
\end{aligned}
$$

For evaluation:

- Apply to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., '11], [Darmé et al. '23]
- Evaluate loop functions via `COLLIER` [Denner et al '16] interface, `pyCollier`
- All included in public tool `anyH3` [Bahl, JB, Gabelmann, Weiglein '23]

(**B0**, **C0**, **C1**, **C2**: loop functions)

Flexible choice of renormalisation schemes

$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} =$  $= \; ?$

- **1L calculation** → renormalisation of all parameters entering λ_{hhh} at tree-level
- In general:

$$(\lambda^{(0)}_{hhh})^{\mathrm{BSM}} = (\lambda^{(0)}_{hhh})^{\mathrm{BSM}}(\underbrace{m_h \simeq 125\text{ GeV}, v \simeq 246\text{ GeV}}_{\text{SM sector}}, \underbrace{m_{\Phi_i}}_{\substack{\text{BSM}\\ \text{masses}}}, \underbrace{\alpha_i}_{\substack{\text{BSM}\\ \text{mixing angles}}}, \underbrace{v_i}_{\substack{\text{BSM}\\ \text{VEVs}}}, \underbrace{g_i}_{\substack{\text{indep.}\\ \text{BSM coups.}}})$$

- Most automated codes: $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$ only
- **`anyH3`**: much more flexibility, following **user choice**:
  - **SM sector** (m_h, v): fully OS or $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - **BSM masses**: OS or $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - **Additional couplings/vevs/mixings**: by default $\overline{\mathrm{MS}}$, but **user-defined ren. conditions** also possible!

$$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} = \sum_x \left(\frac{\partial}{\partial x}(\lambda^{(0)}_{hhh})^{\mathrm{BSM}}\right)\delta^{\mathrm{CT}}x\,, \qquad \text{with } x \in \{m_h, v, m_{\Phi_i}, v_i, \alpha_i, g_i, \text{etc.}\}$$

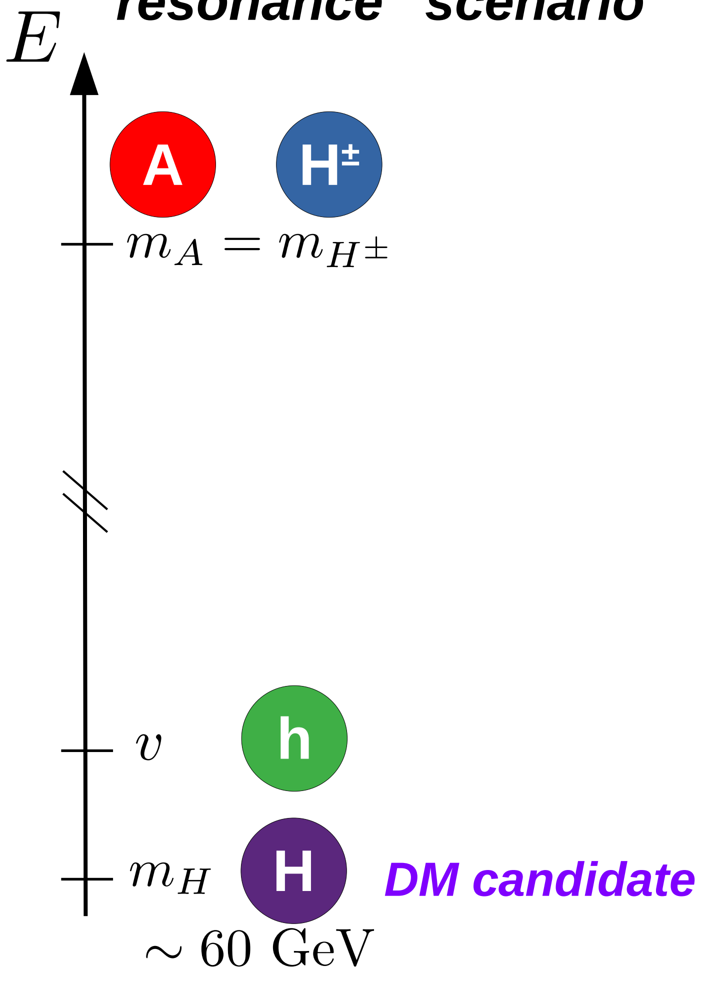
Renormalised in $\overline{\mathrm{MS}}$, OS, in custom schemes, etc.

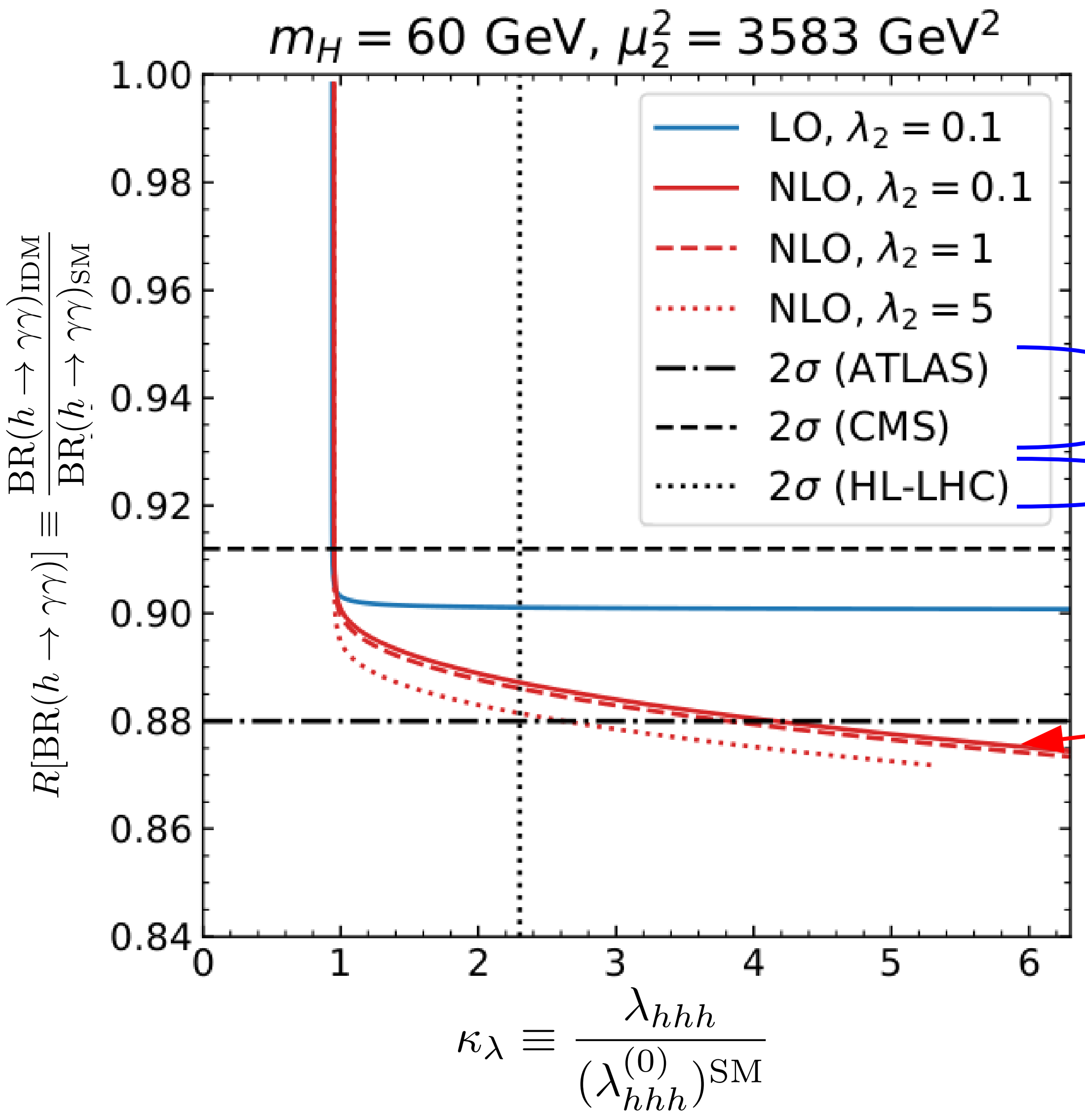
Correlation between κ_λ and BR($h \to \gamma\gamma$) at one and two loops

Could BSM Physics be found first in κ_λ ?

[Aiko, JB, Kanemura '23 + WIP] + [JB, Kanemura '19]

Inert Doublet Model in DM-inspired "Higgs resonance" scenario





[λ_2 : inert doublet self-coupling]

Expected bounds on R[BR($h \to \gamma\gamma$)] at HL-LHC

Expected bound on κ_λ at HL-LHC

$m_{H\pm} = m_A$ varied along the curves (until limit from pert. unit.)

Correlation between κ_λ and BR($h\to\gamma\gamma$) at one and two loops

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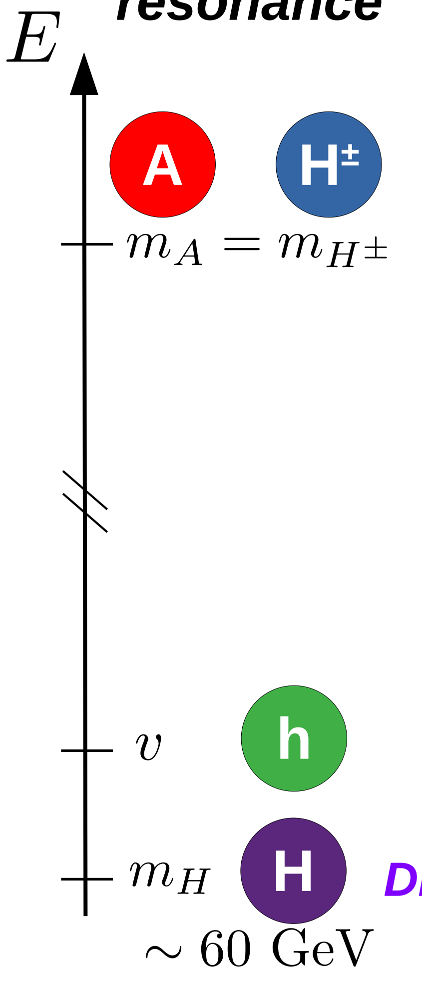
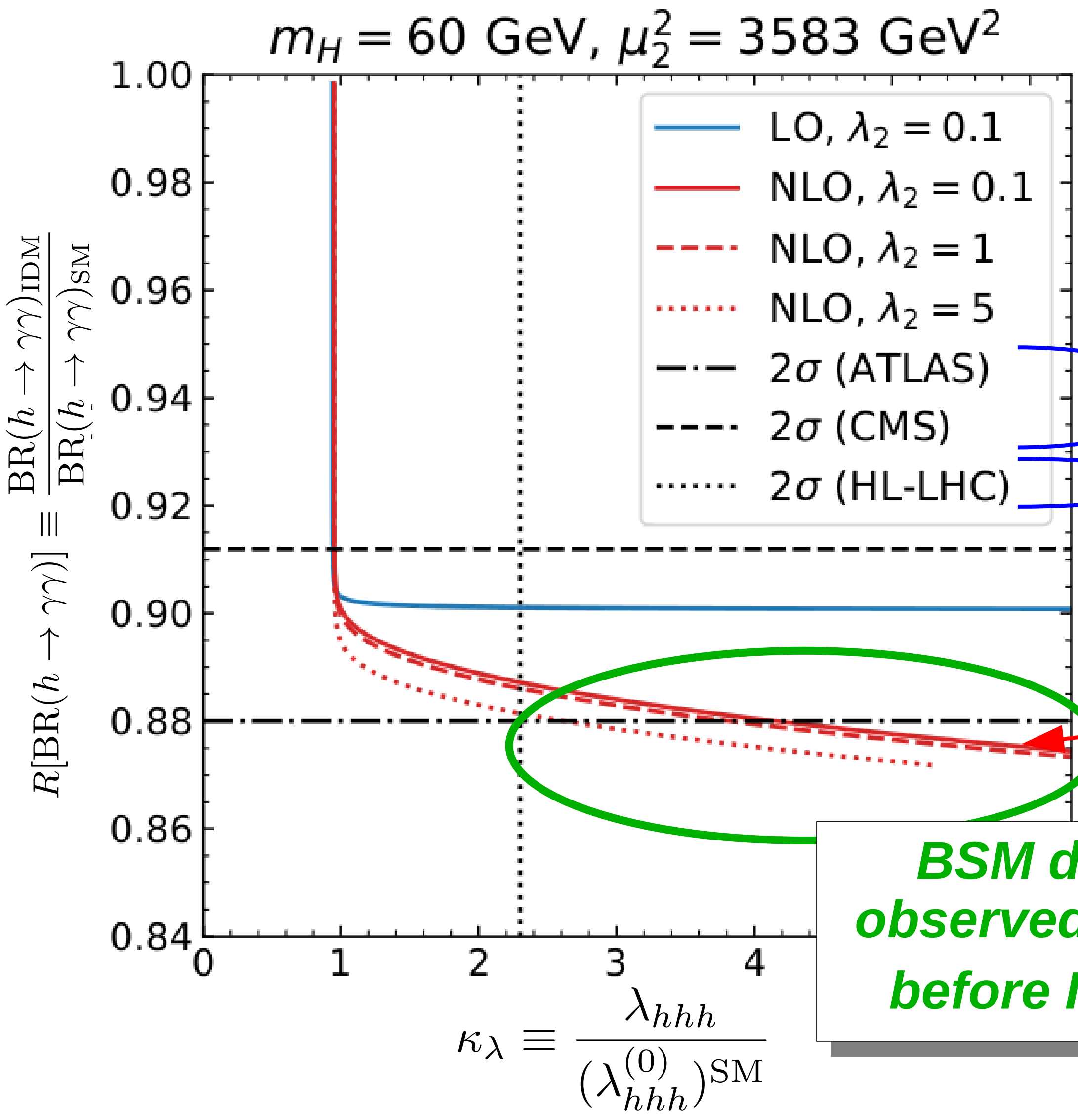
Inert Doublet Model in DM-inspired "Higgs resonance" scenario



DM candidate



[λ_2 : inert doublet self-coupling]

Expected bounds on R[BR($h\to\gamma\gamma$)] at HL-LHC

Expected bound on κ_λ at HL-LHC

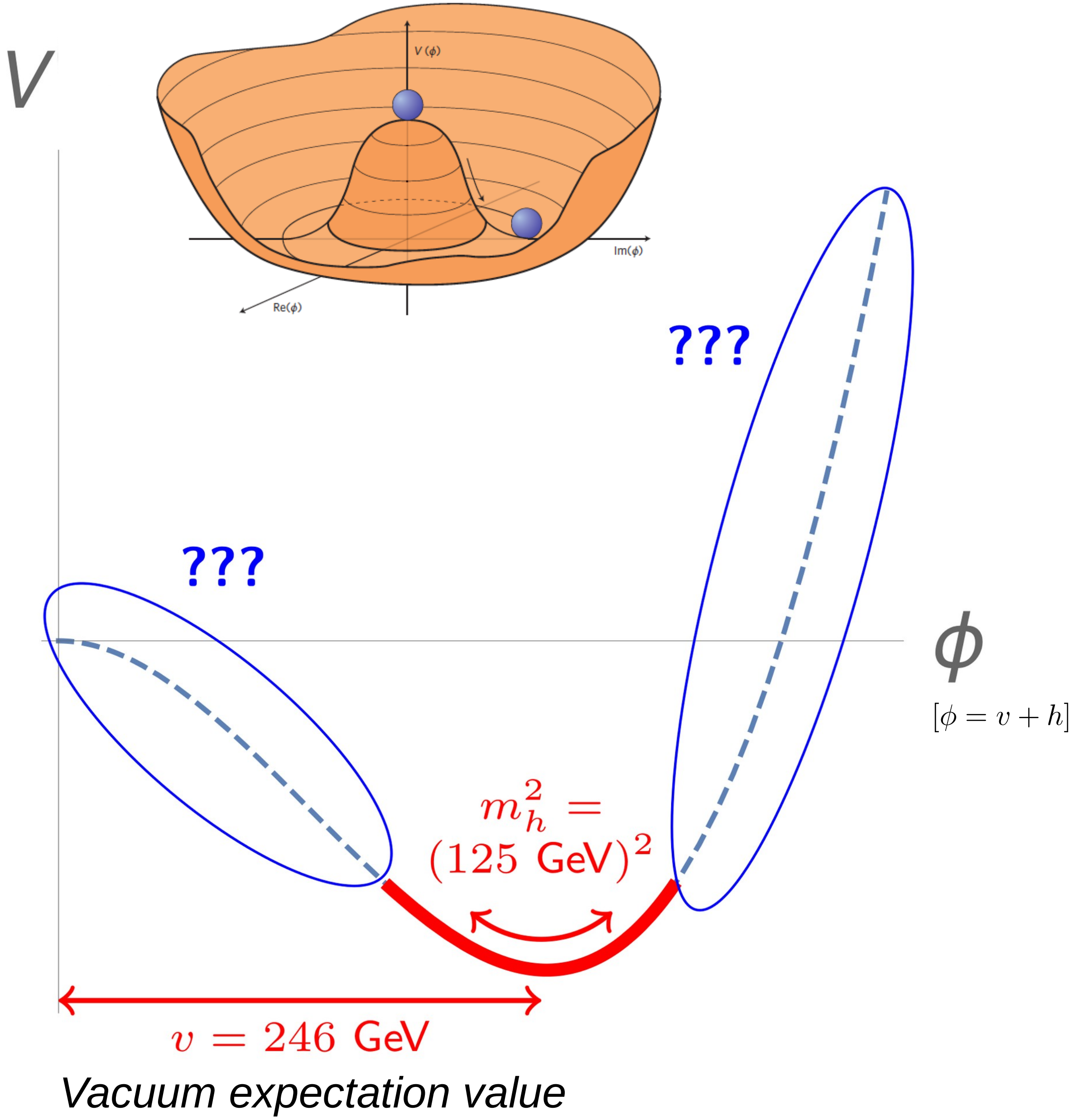
$m_{H\pm} = m_A$ varied along the curves (until limit from pert. unit.)

BSM deviation observed first in κ_λ, before $\Gamma(h\to\gamma\gamma)$!

Why investigate λ_{hhh} ?

Form of the Higgs potential and trilinear Higgs coupling

- Brout-Englert-Higgs mechanism = **origin of electroweak symmetry breaking** ...
  … but very little known about the **Higgs potential** causing the phase transition



Form of the Higgs potential and trilinear Higgs coupling

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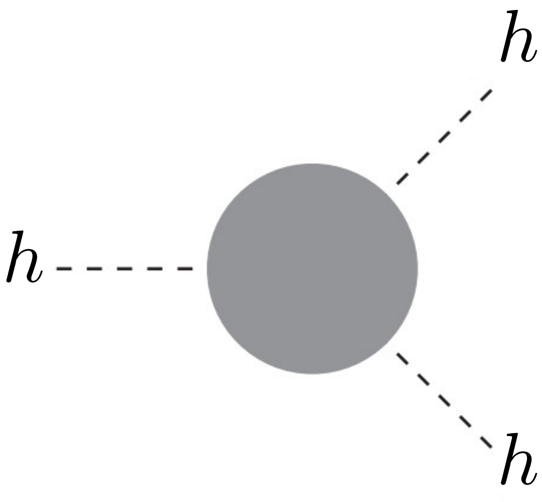
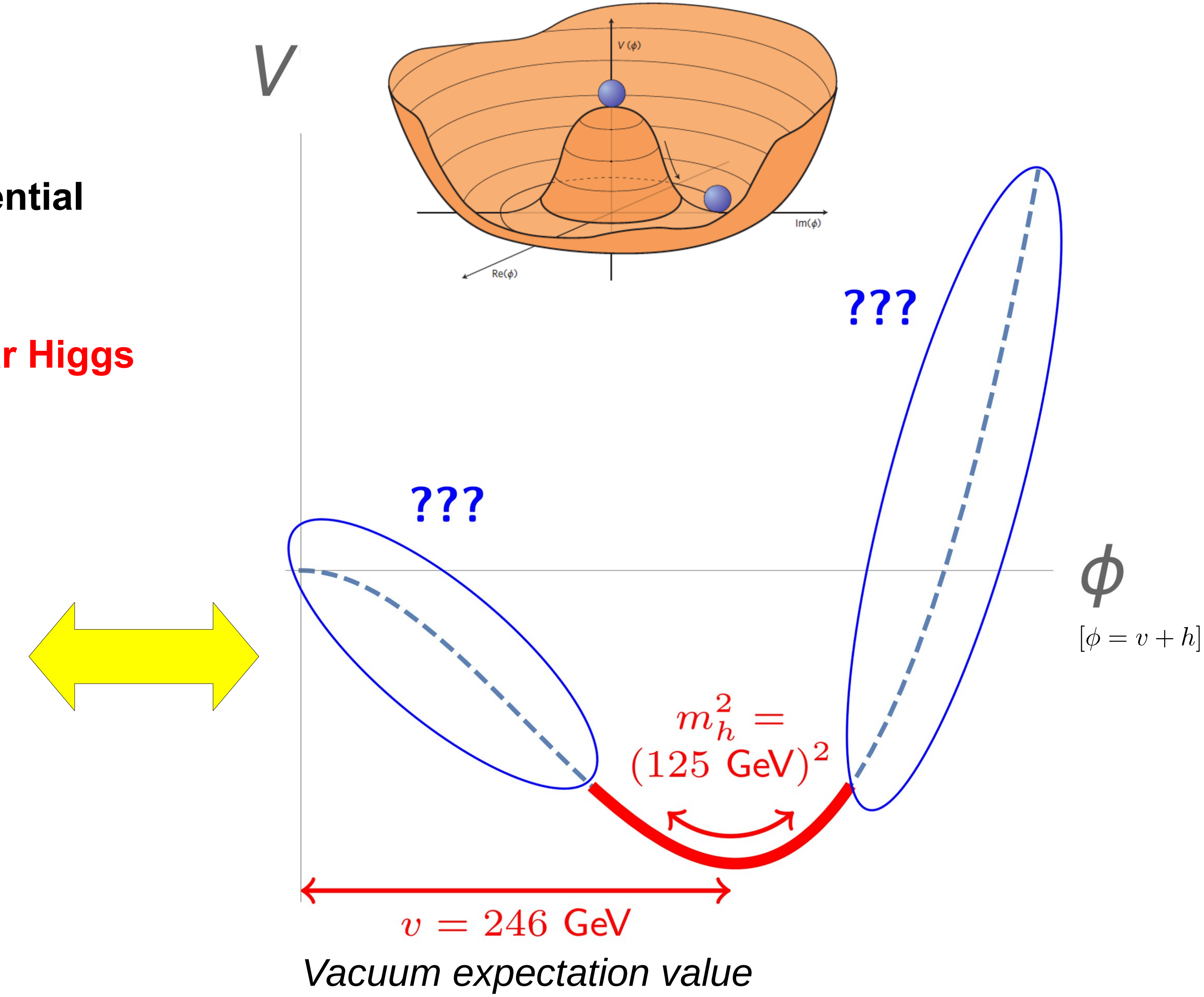
- Shape of the potential determined by **trilinear Higgs coupling λ_{hhh}**





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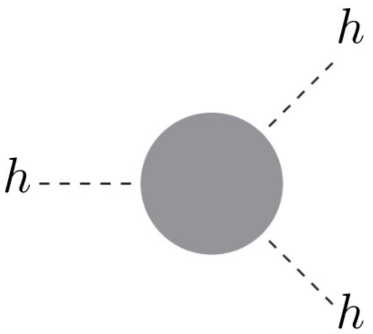
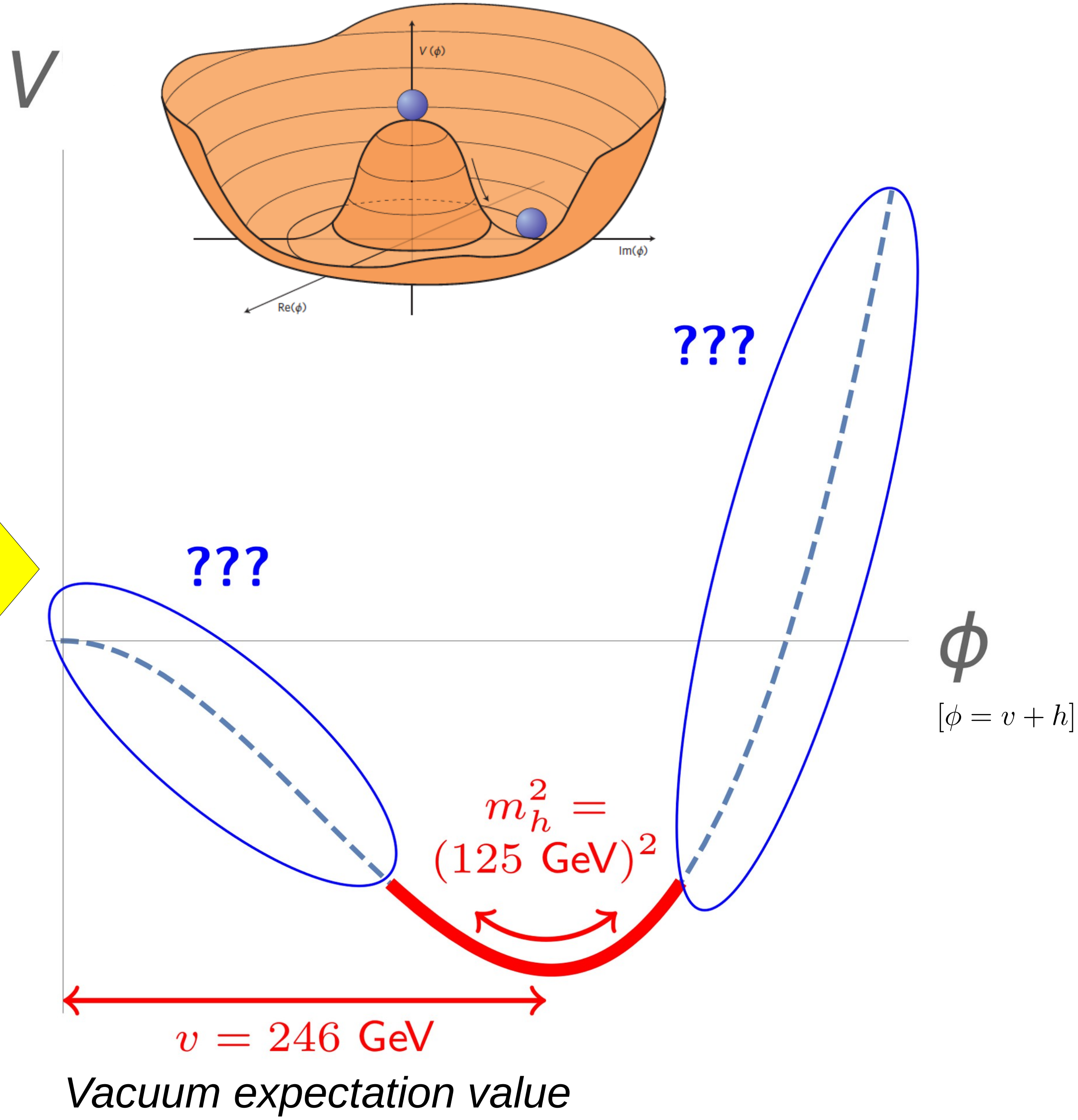




In the SM: $V_{\text{SM}}^{(0)} = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\underbrace{\left(\frac{3m_h^2}{v}\right)}_{\equiv(\lambda_{hhh}^{(0)})^{\text{SM}}} h^3 + \frac{1}{4!}\left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$

In general:

$$V^{(0)} = \frac{1}{2}m_h^2 h^2 + \frac{1}{3!}\overbrace{\kappa_\lambda\left(\frac{3m_h^2}{v}\right)}^{\equiv\lambda_{hhh}} h^3 + \frac{1}{4!}\kappa_{\lambda_4}\left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$$

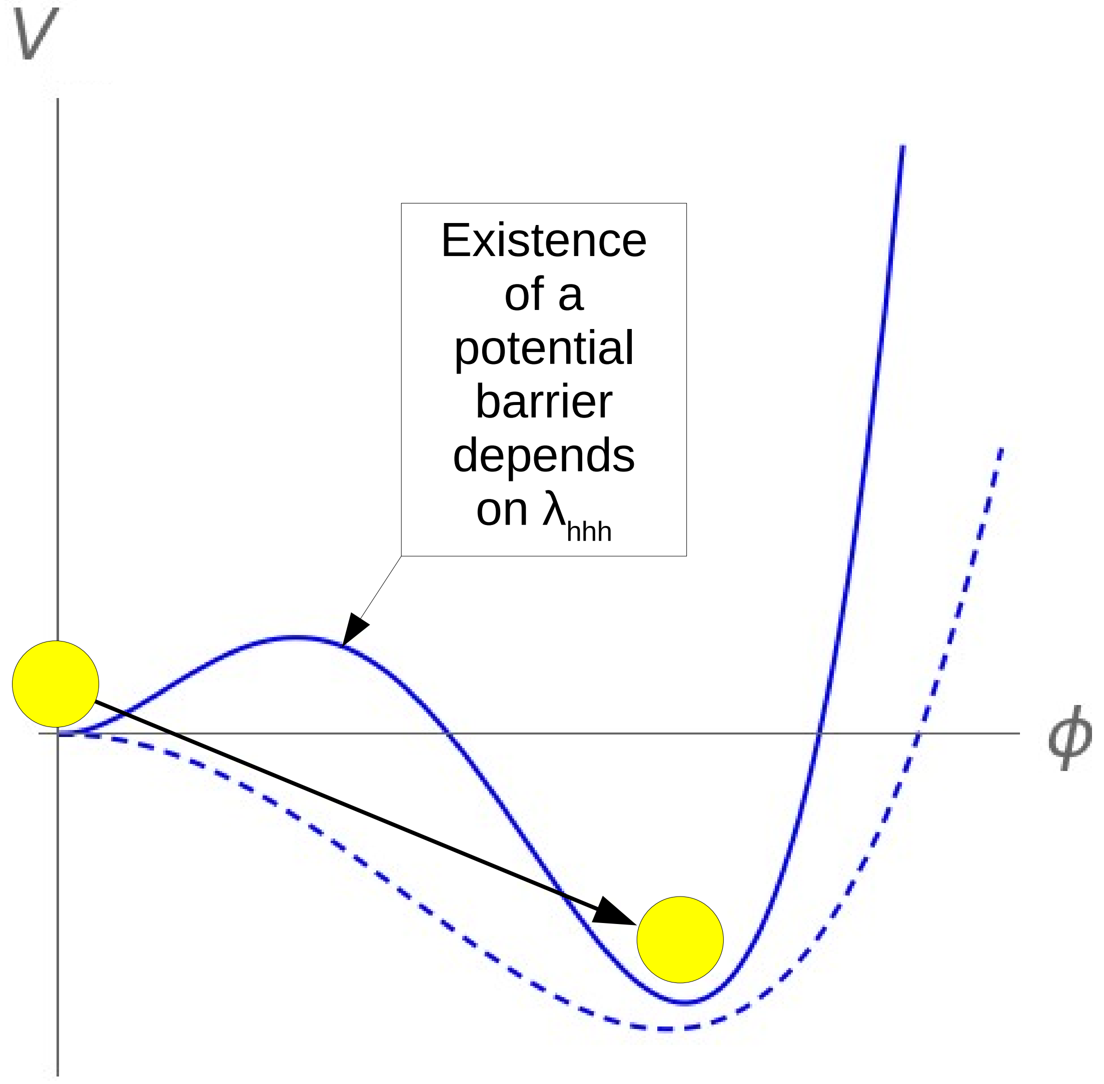
with $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{\text{SM}}$

Form of the Higgs potential and baryon asymmetry

- Brout-Englert-Higgs mechanism = **origin of electroweak symmetry breaking** ...
  … but very little known about the **Higgs potential** causing the phase transition

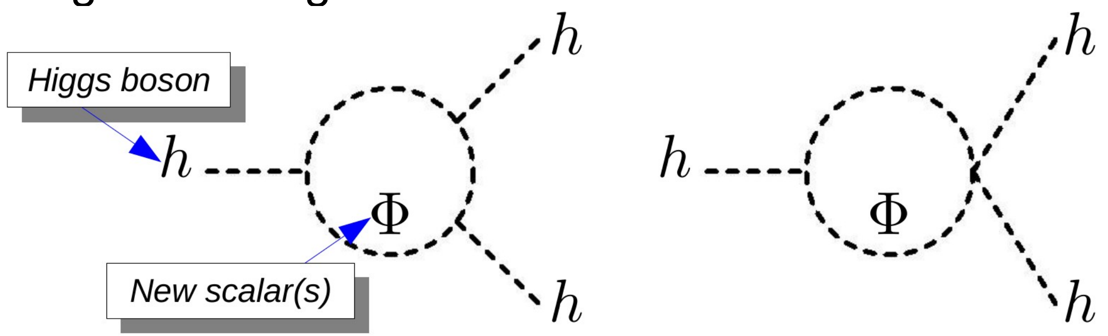
- Shape of the potential determined by **trilinear Higgs coupling λ_{hhh}**

- Among **Sakharov conditions** necessary to explain **baryon asymmetry via electroweak phase transition** (EWPT):
  - ***Strong first-order EWPT***
    → barrier in Higgs potential
    → typically significant deviation in λ_{hhh} from SM



Probing New Physics with the trilinear Higgs coupling

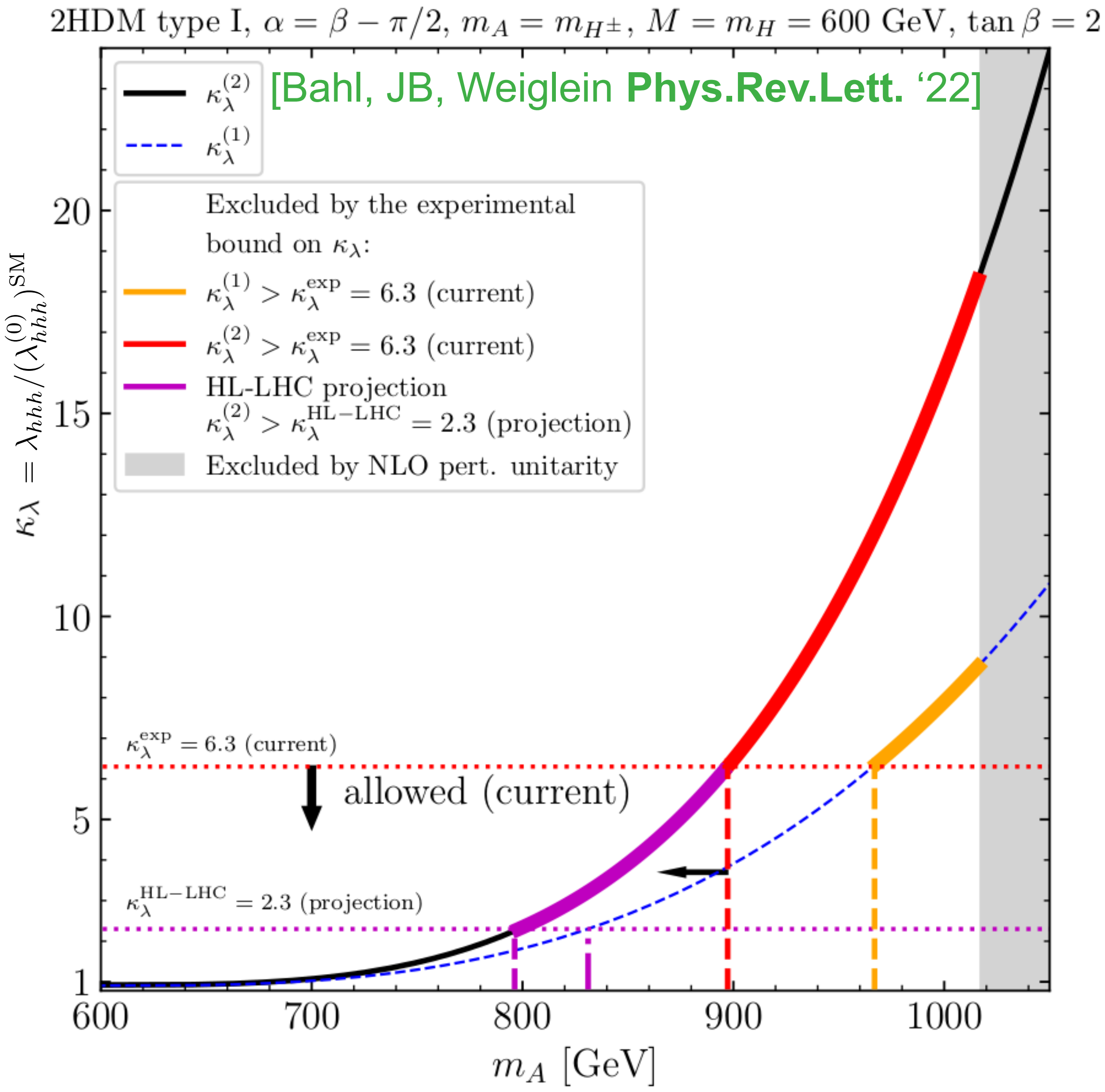
- **Large effects from New Physics possible in λ_{hhh},** due to radiative corrections from extra scalars, e.g. at leading order



- Comparing latest exp. bounds

$$-0.4 < \kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda^{(0)}_{hhh})^{\mathrm{SM}}} < 6.3$$

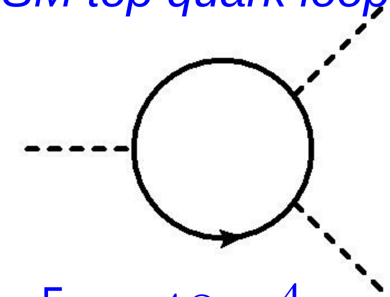
[ATLAS PLB 2023]
with precise theory predictions for λ_{hhh} provides a **powerful new tool to constrain BSM models**
[Bahl, JB, Weiglein **Phys.Rev.Lett.** '22]



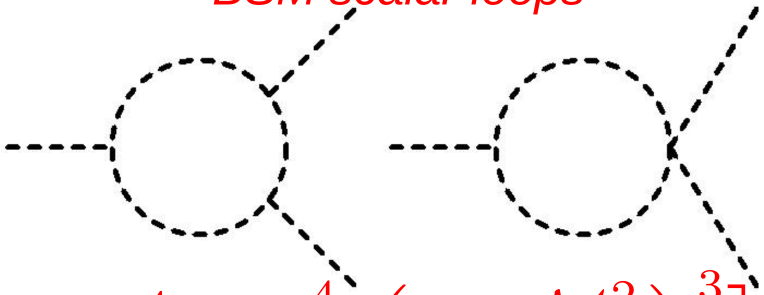
One-loop mass-splitting effects

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop



BSM scalar loops



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48 m_t^4}{v^3} + \sum_\Phi \frac{4 n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

First found in 2HDM:
[Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda} v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda} v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda} v^2 \end{cases}$$

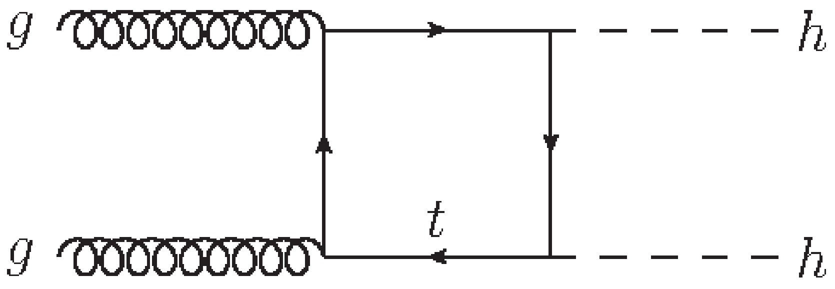
→ **Huge BSM effects possible!**

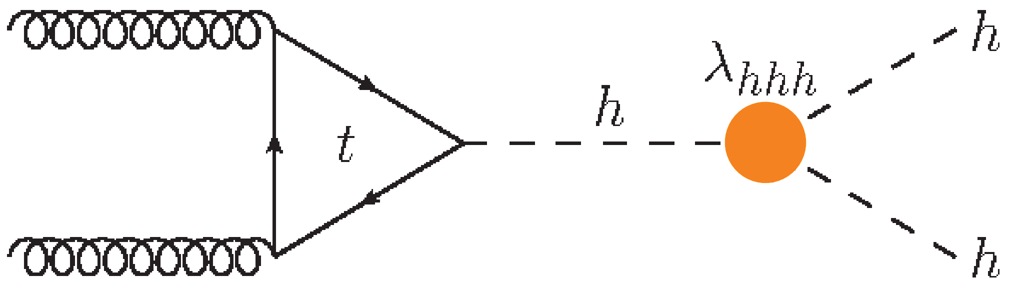
Probing λ_{hhh} at the (HL-)LHC

Experimental probes of λ_{hhh}

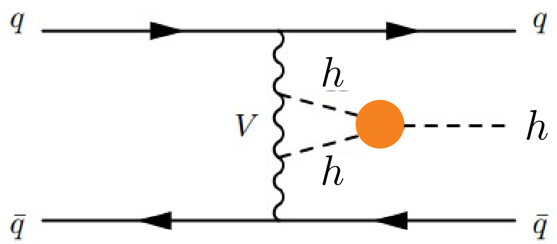
[NB: triple-Higgs production in a few slides]

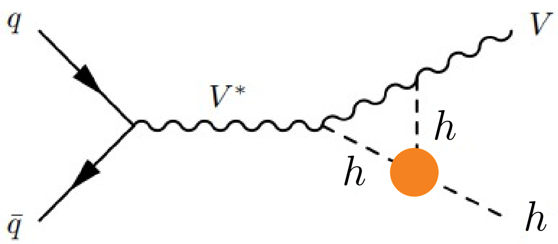
- **Double-Higgs production** → λ_{hhh} enters at <u>leading order (LO)</u> → **most direct probe!**

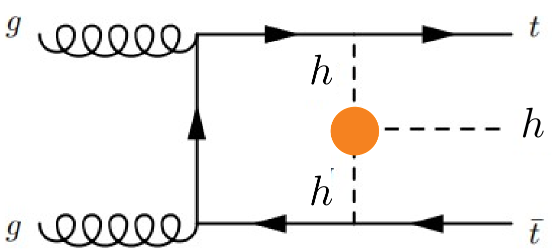


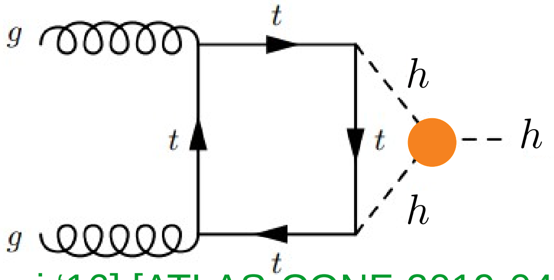


- **Single-Higgs production** → λ_{hhh} enters at <u>NLO</u> (i.e. indirect probe)



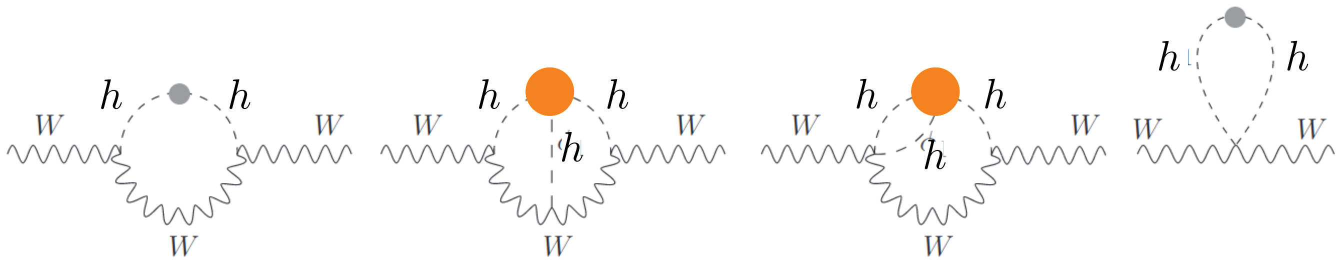


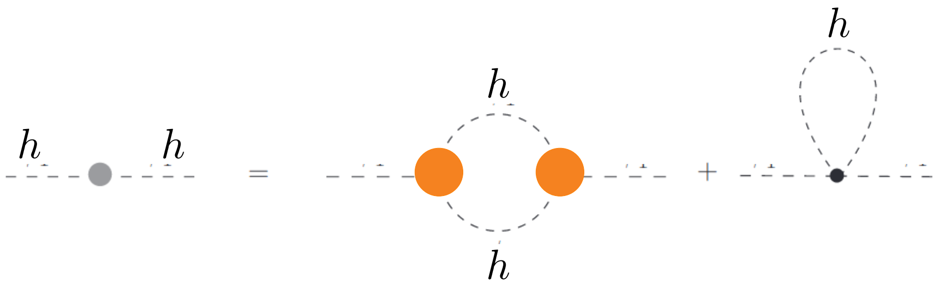




[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

- **Electroweak Precision Observables** (EWPOs) → λ_{hhh} enters at <u>NNLO</u> (i.e. indirect probe)



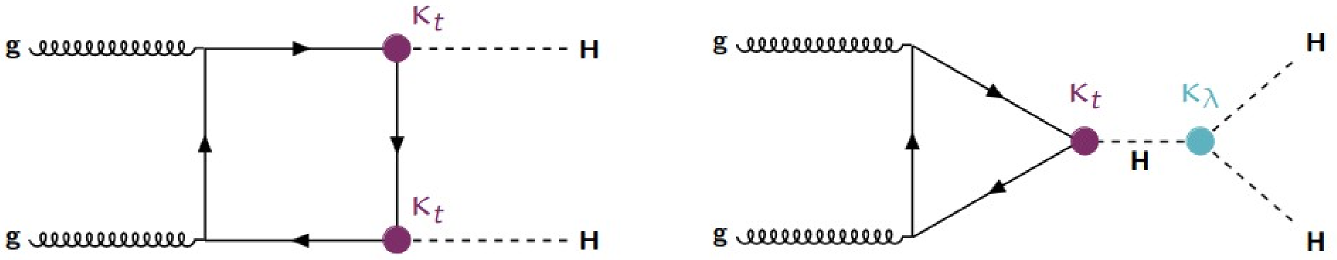


[Degrassi, Fedele, Giardino '17]

Probing λ_{hhh} via double-Higgs production

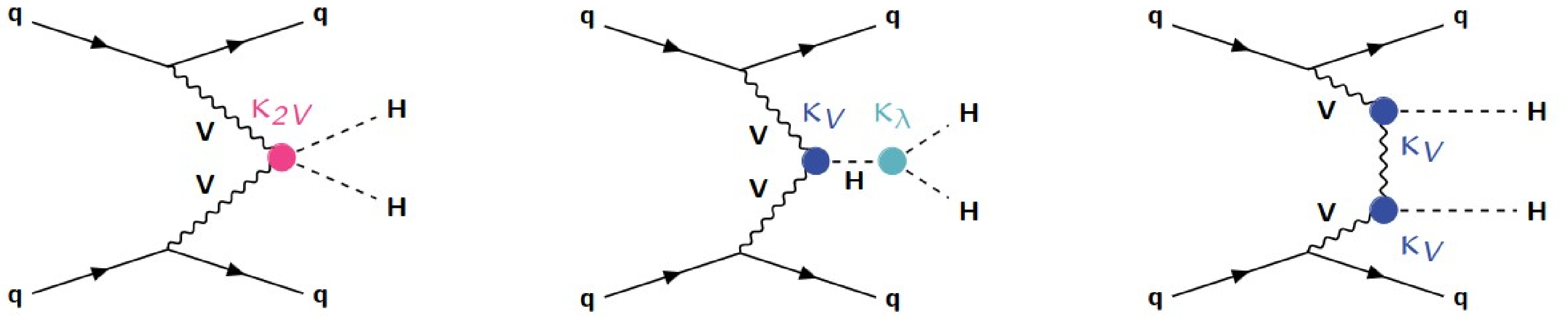
gluon-fusion

$\sigma_{ggF}(pp \rightarrow HH) = 31.05$ fb



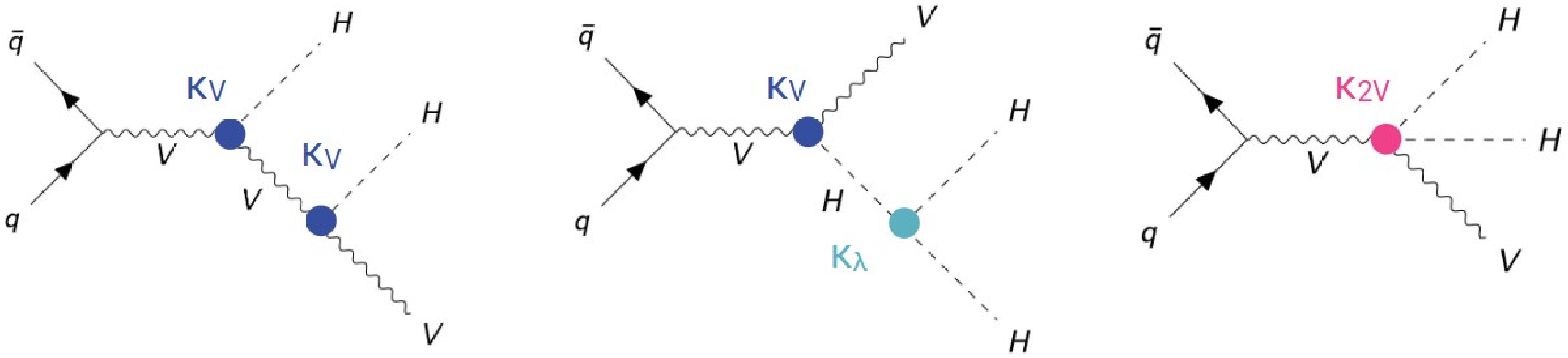
VBF

$\sigma_{VBF}(pp \rightarrow HH) = 1.726$ fb



VHH

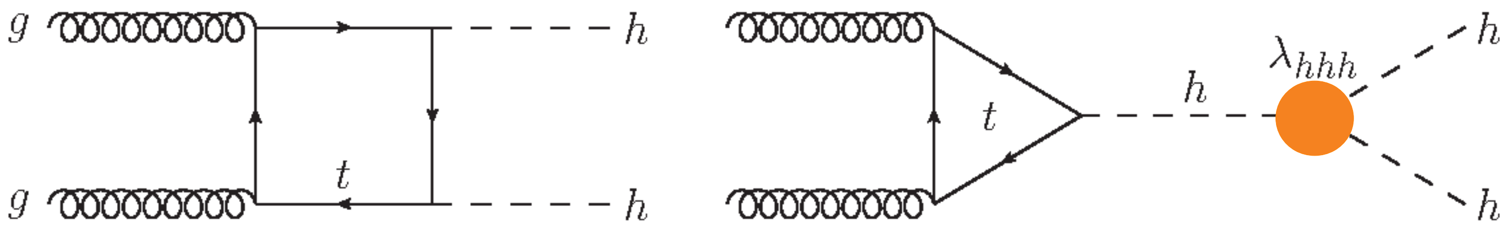
$\sigma_{VHH}(pp \rightarrow HH) = 0.86$ fb



Slide by K. Leney @ HiggsDays 23

Probing λ_{hhh} via double-Higgs production

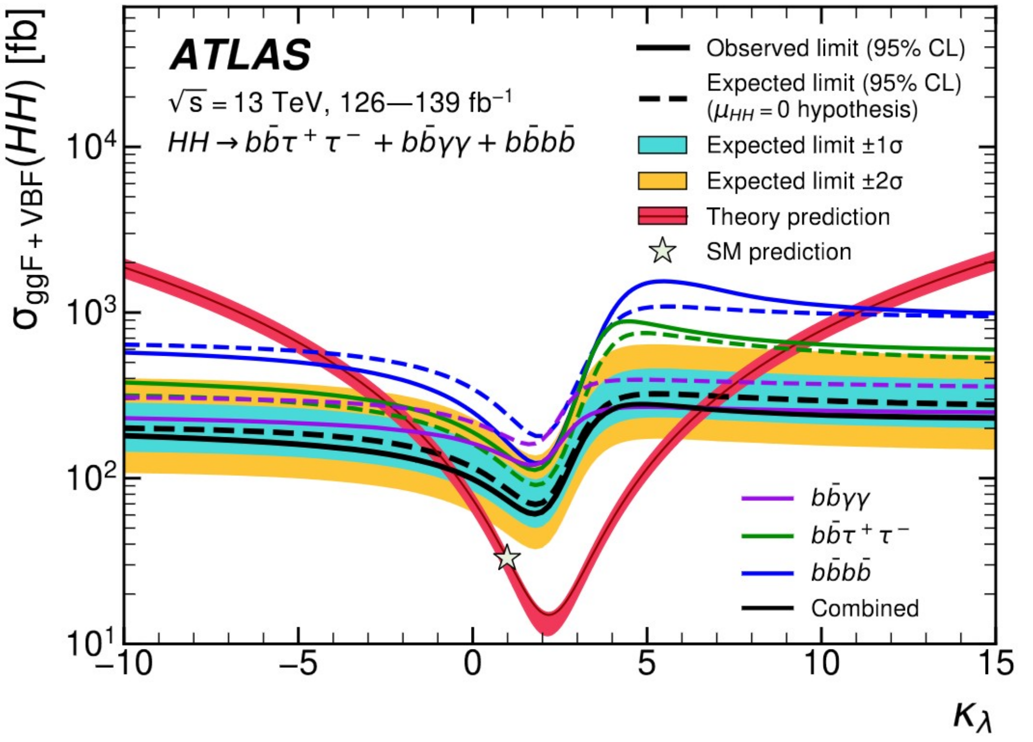
- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



- Box and triangle diagrams **interfere destructively**
→ small prediction in SM
→ BSM deviation in λ_{hhh} can **significantly enhance double-Higgs production!**

- Search limits on double-Higgs production
→ **limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- Current best limits: **$-0.4 < \kappa_\lambda < 6.3$** (95% CL) [ATLAS PLB '23]
(including information from single-Higgs production)
$-1.4 < \kappa_\lambda < 6.3$ (95% CL) [ATLAS PLB '23]
(including information from single-Higgs production + κ_t floating)
$-1.2 < \kappa_\lambda < 6.5$ (95% CL) [CMS '22]



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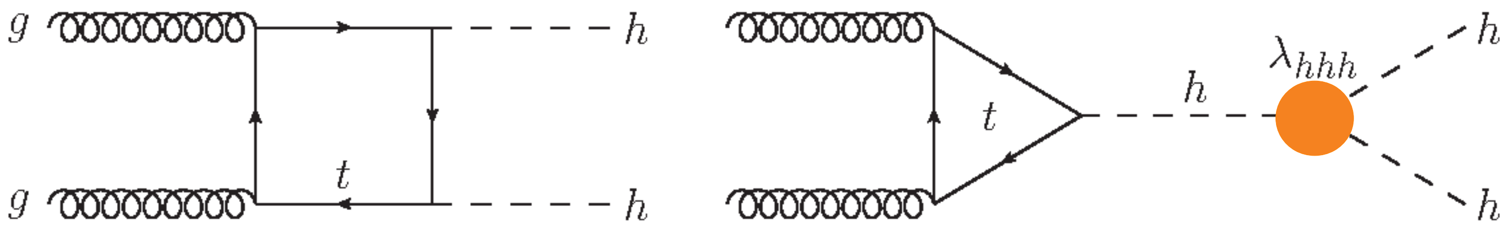

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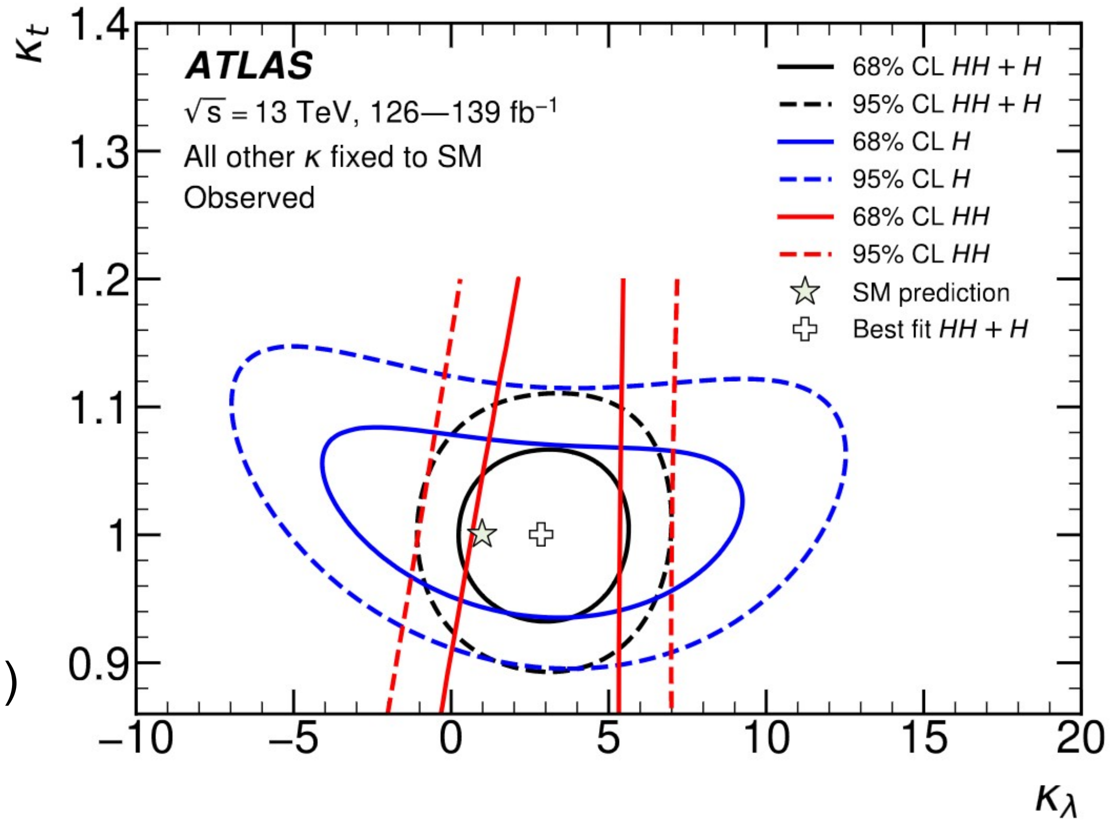

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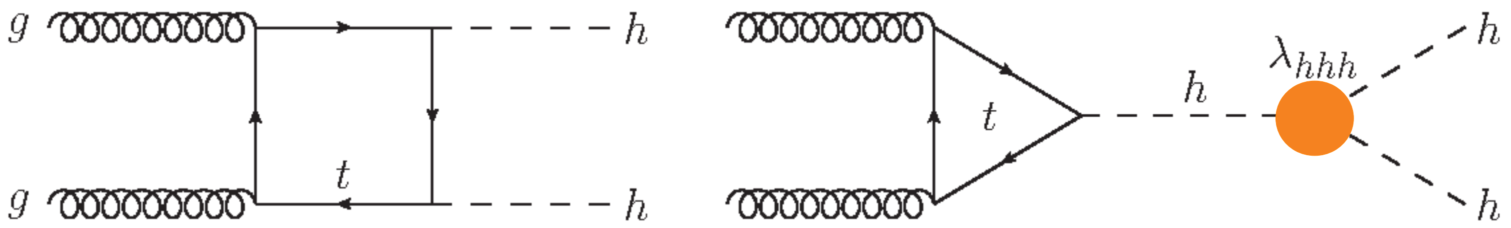

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- Search limits on double-Higgs production
  → **limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- Prospects at HL-LHC: **$0.1 < \kappa_\lambda < 2.3$** (95% CL) with ATLAS+CMS
  [Cepeda et al. '19]

  $0.0 < \kappa_\lambda < 2.7$ (95% CL) with ATLAS alone
  [ATL-PHYS-PUB-2022-053]

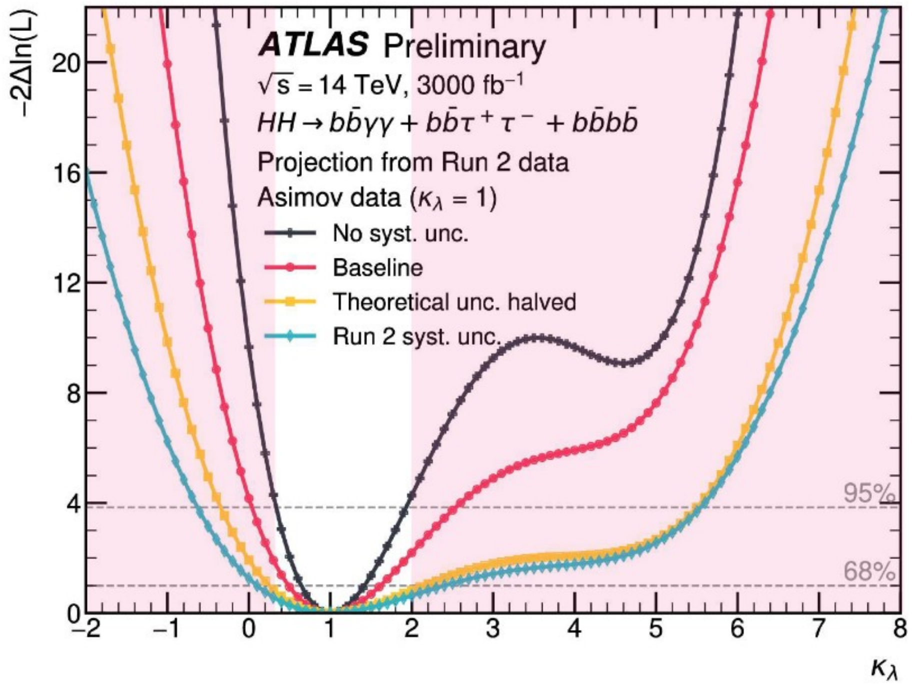


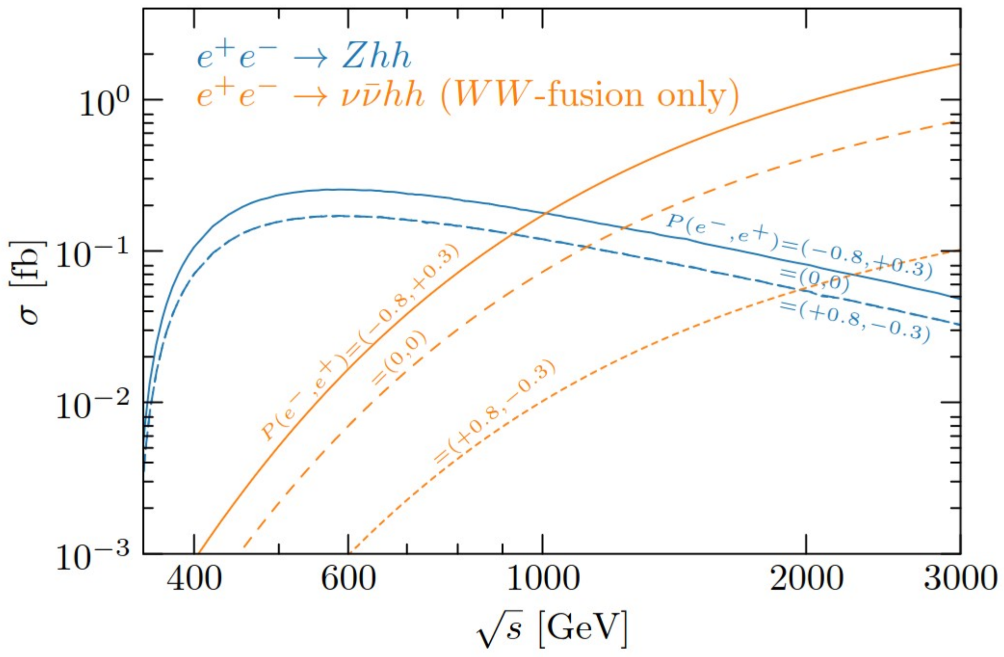
Figure adapted from [ATL-PHYS-PUB-2022-053]

Standard probes of λ_{hhh} at e^+e^- colliders

Direct probes of λ_{hhh} at e^+e^- colliders

- **Double-Higgs production, either in $e^+e^- \to Zhh$ or $e^+e^- \to \nu\bar{\nu}hh$**
- Relies however on being above the Zhh threshold!

Figure from [De Blas et al. 1812.02093]



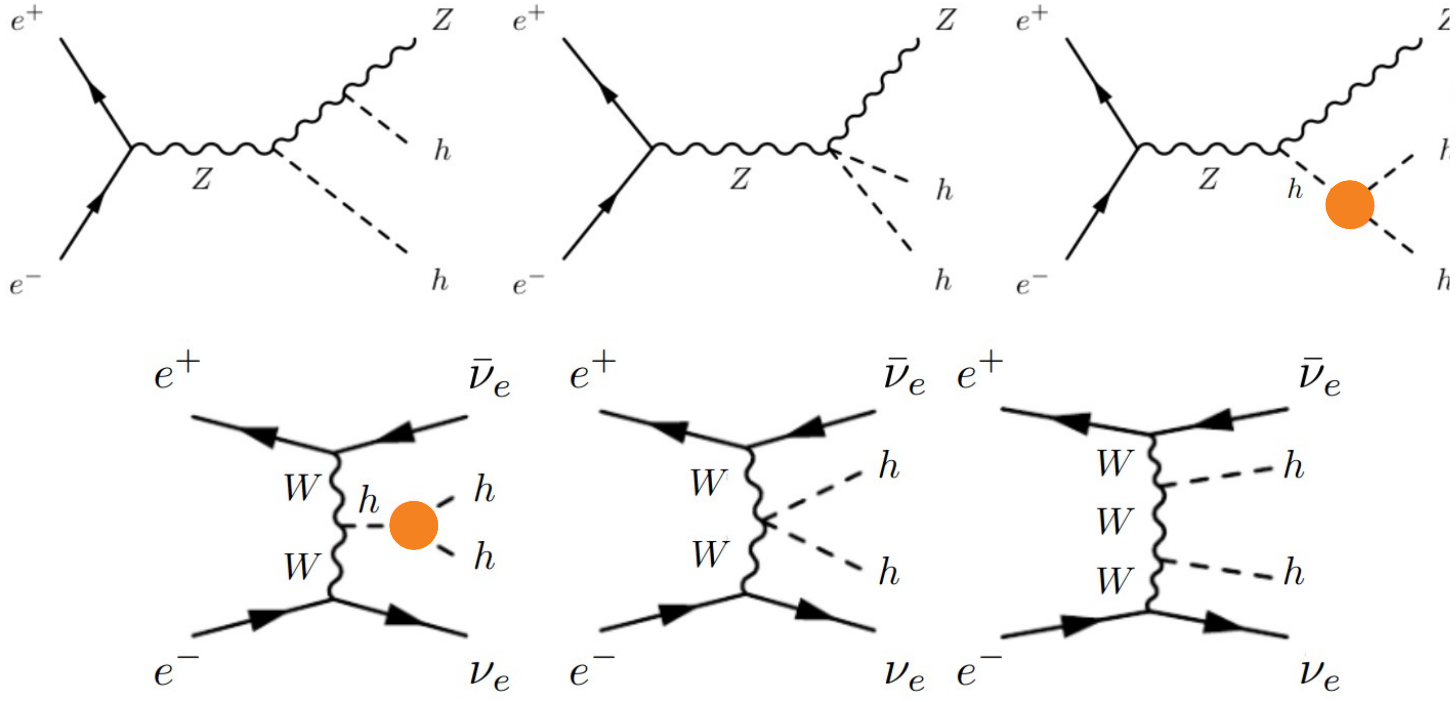


Figure from [De Blas et al. 1905.03764]

- $e^+e^- \to Zhh$ better at $\sqrt{s}$~500 GeV
- $e^+e^- \to \nu\bar{\nu}hh$ better for larger $\sqrt{s}$

Indirect probes of λ_{hhh} at e^+e^- colliders

- Below the Zhh threshold, λ_{hhh} can still be investigated through its (indirect) effect in quantum corrections to single-Higgs production

- In particular, λ_{hhh} enters NLO corrections to $e^+e^- \to Zh$
  First pointed out in [McCullough '13], numerous works since (also with global analyses in EFT setting)

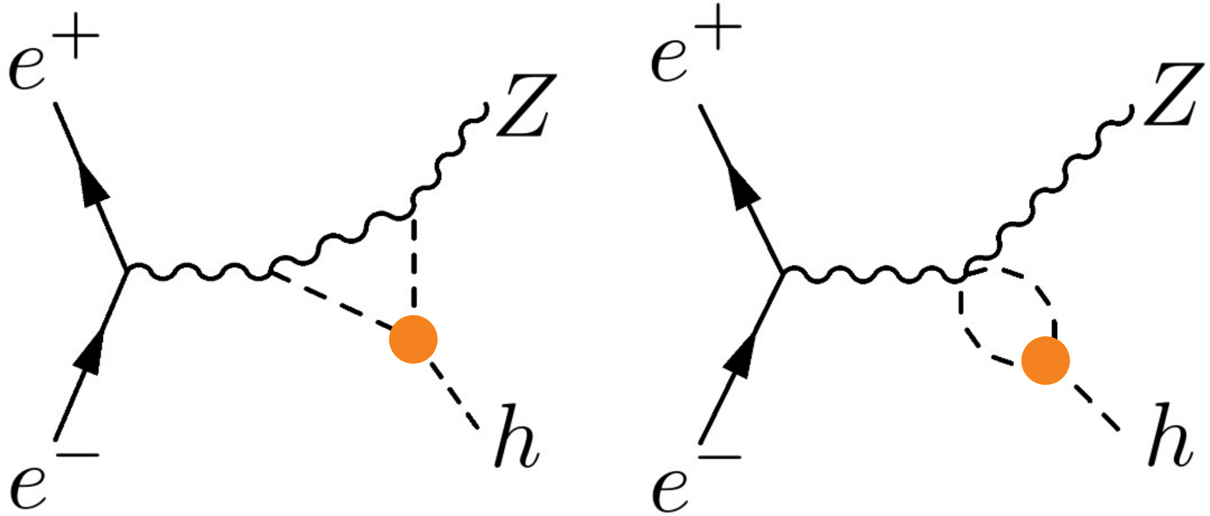


Figure adapted from [McCullough 1312.3322]

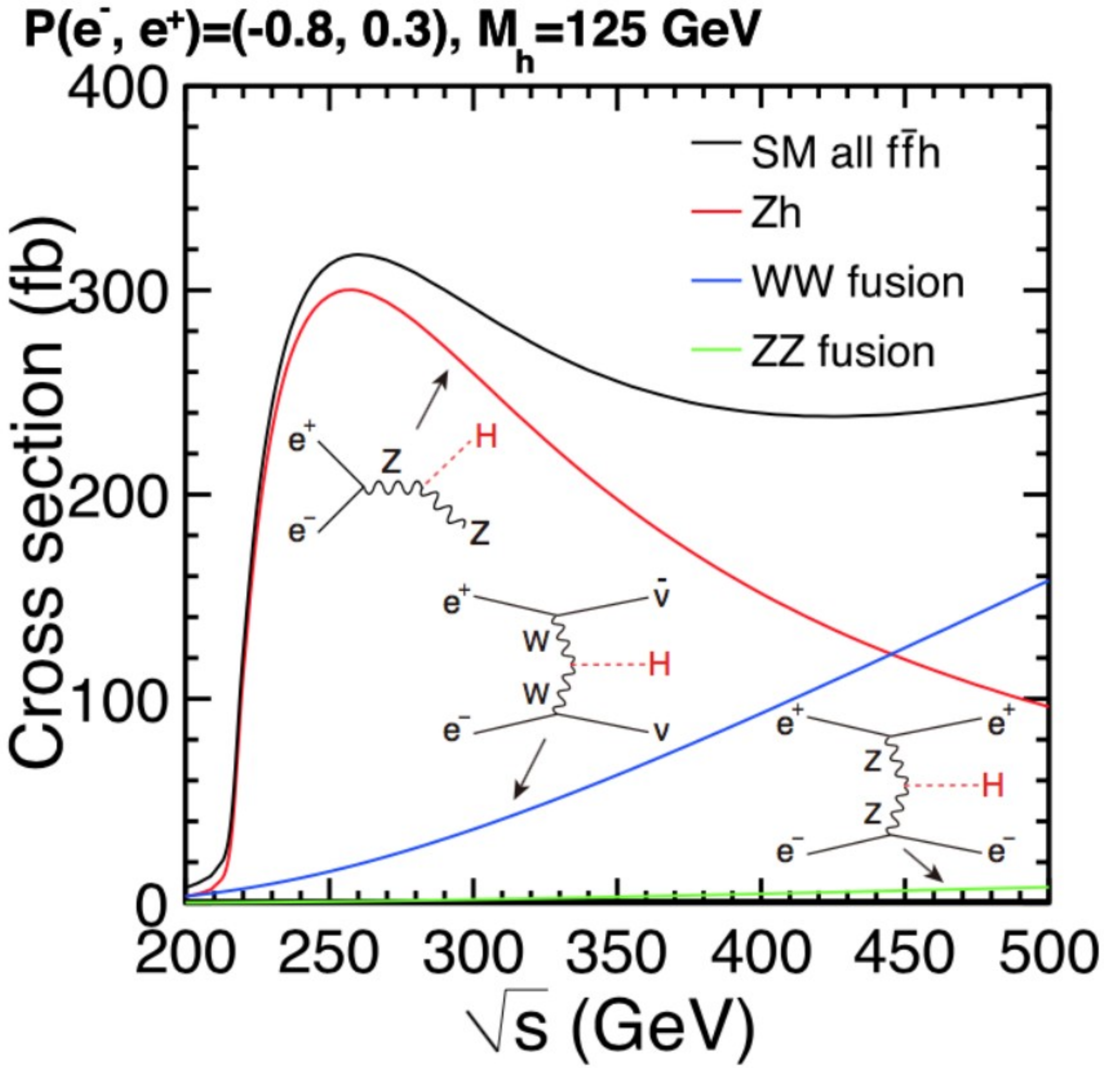


Figure from [Fujii et al. 1710.07621]

Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

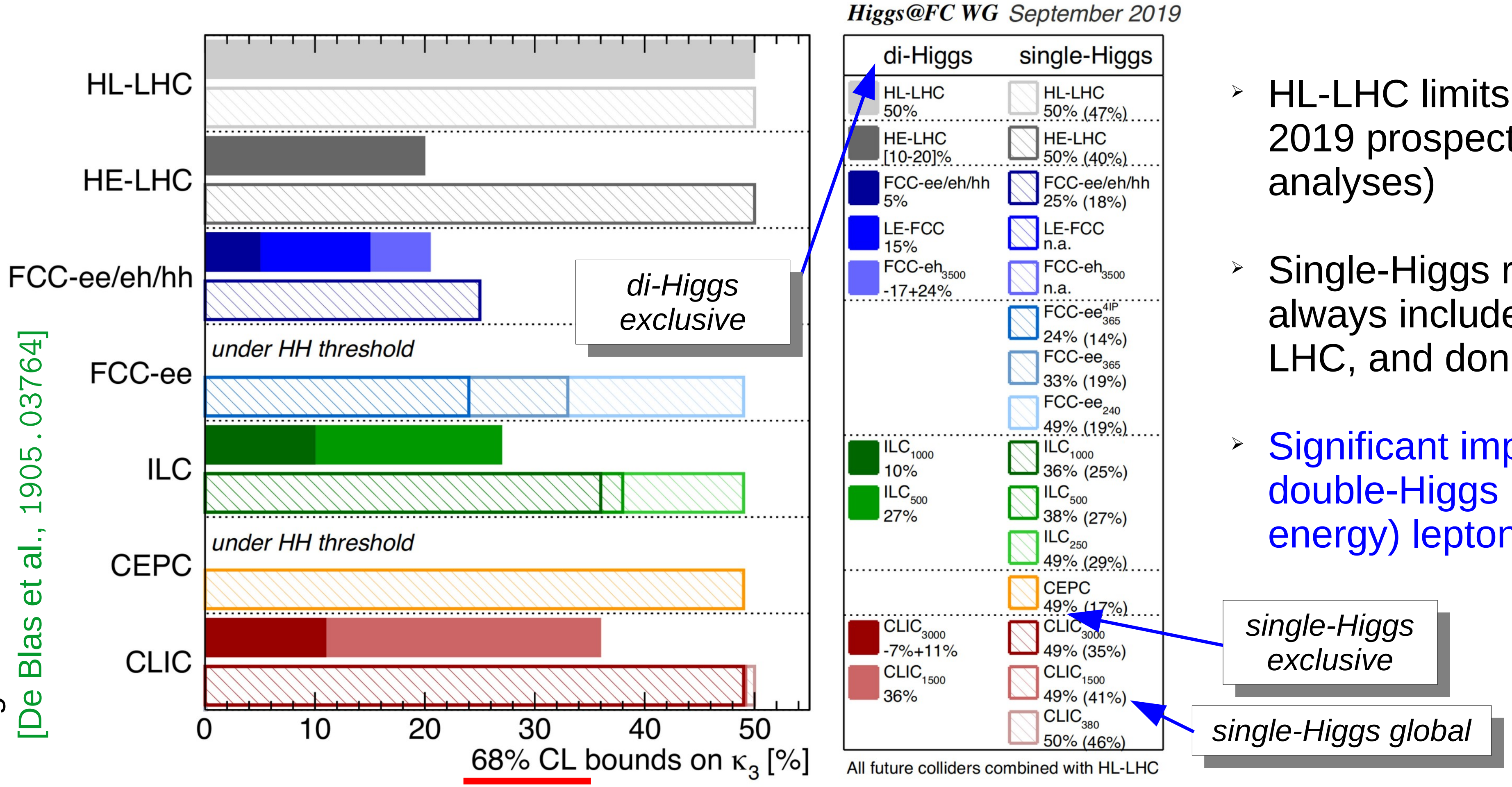


Figure from [De Blas et al., 1905.03764]

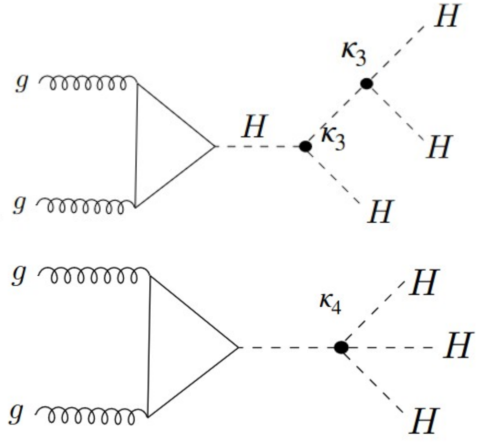
- HL-LHC limits will likely outperform 2019 prospects (even with global analyses)
- Single-Higgs results at lepton colliders always include information from HL-LHC, and don't improve much (if at all)
- Significant improvements only with double-Higgs production at (high-energy) lepton colliders or FCC-hh

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

New investigations via triple-Higgs production

Constraining the trilinear and quartic Higgs couplings at the same time

Hadron collider



$\kappa_3 = \kappa_\lambda$: trilinear coupling modifier
κ_4 : quartic coupling modifier

Lepton collider

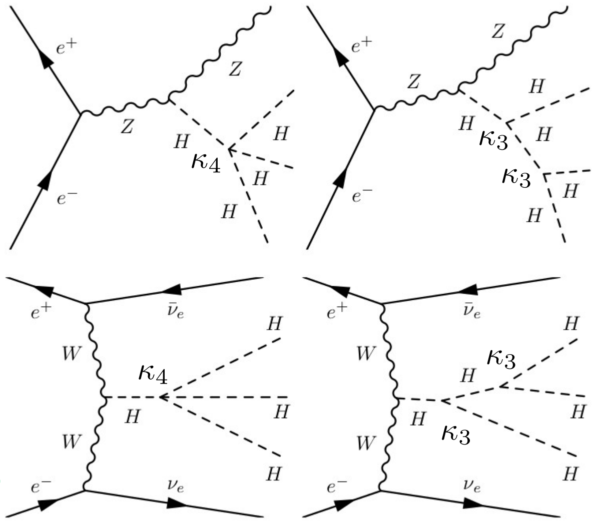
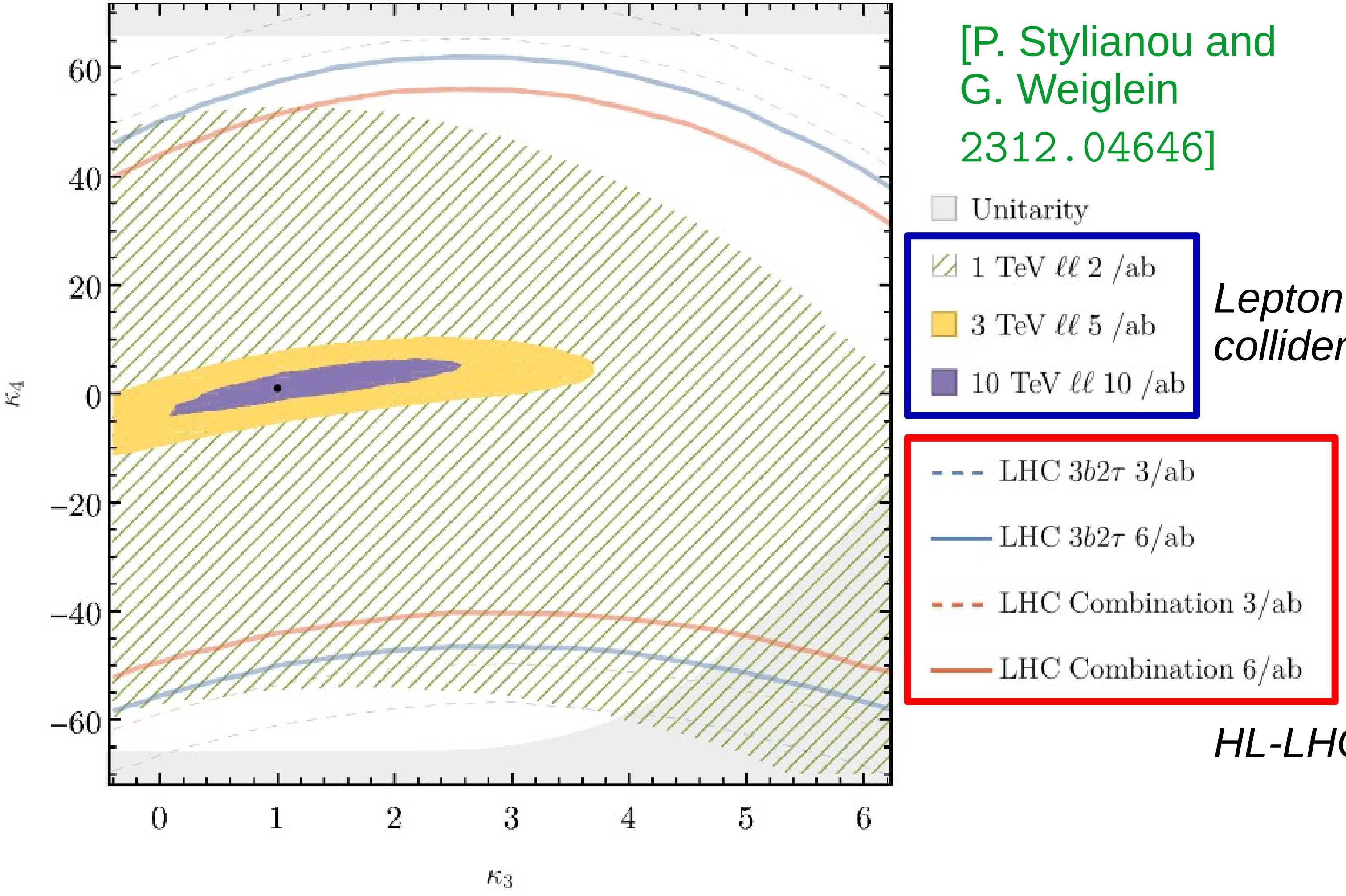


Figure adapted from [Maltoni, Pagani, Zhao 1802.07616]



[P. Stylianou and G. Weiglein 2312.04646]

Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}

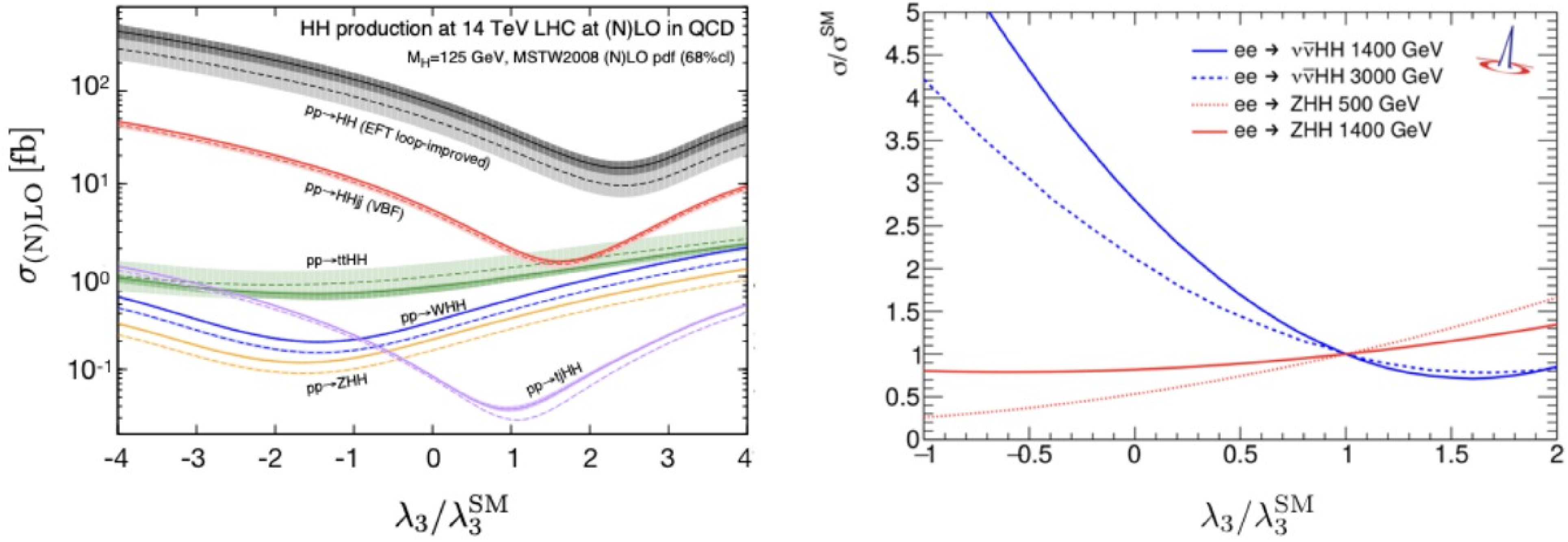


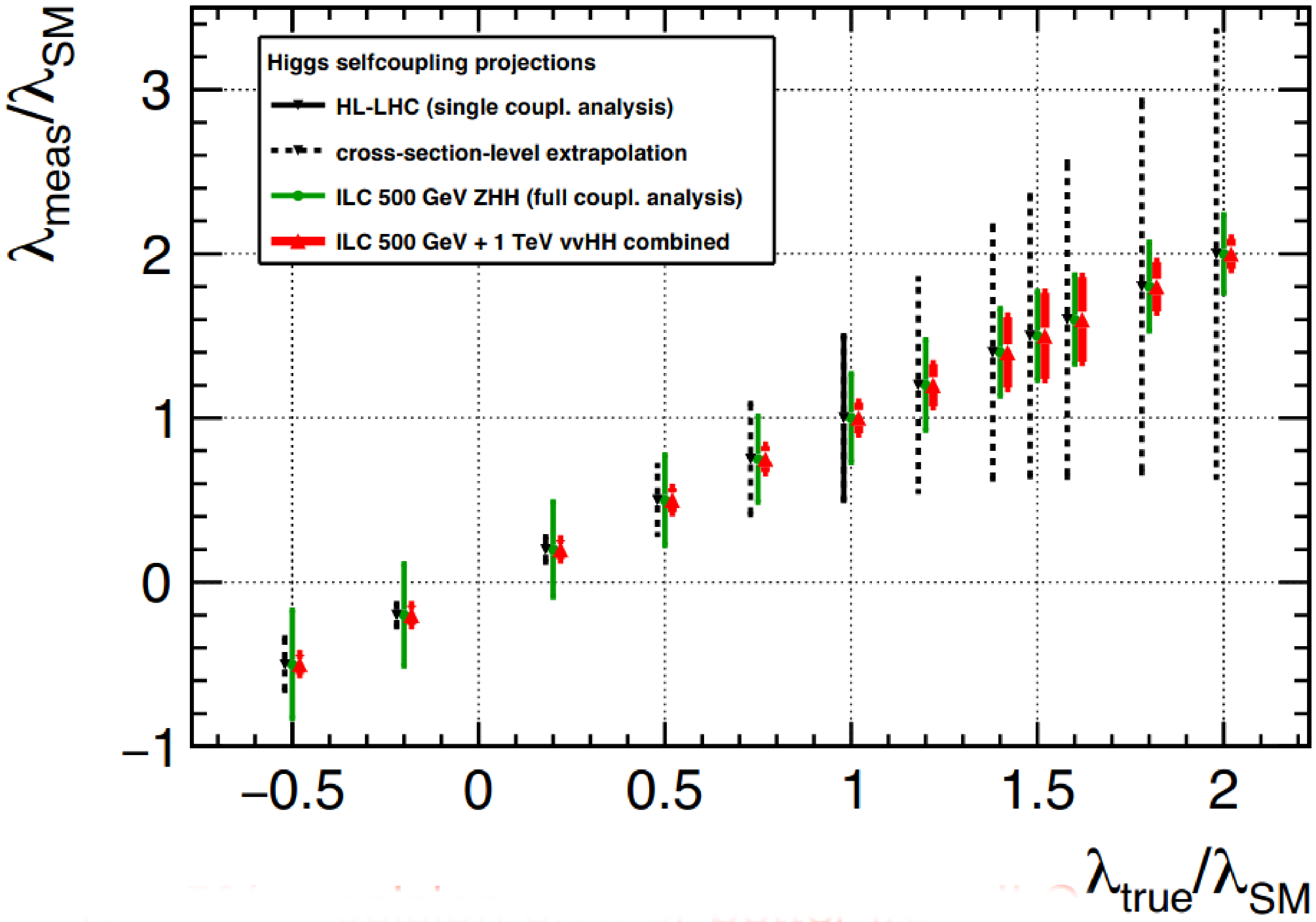
Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

[Frederix et al., 1401.7340]

Plots taken from
[de Blas et al., 1905.03764]

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}



[J. List et al. '21]

See also [Dürig, DESY-THESIS-2016-027]

Two-loop calculation of λ_{hhh}

Goal: How large can the two-loop corrections to λ_{hhh} become?

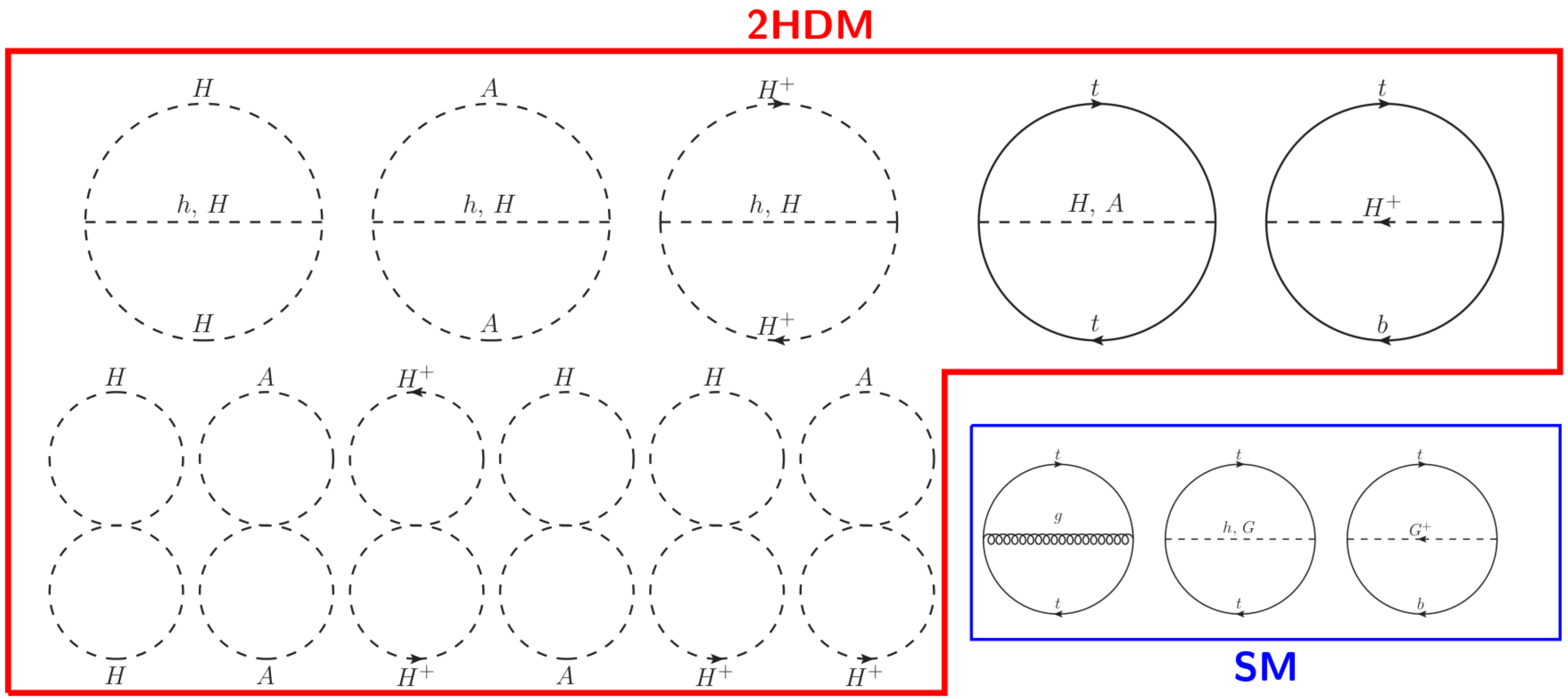
Based on

`arXiv:1903.05417` **(PLB) and** `arXiv:1911.11507` **(EPJC) in collaboration with Shinya Kanemura**

Our effective-potential calculation

[JB, Kanemura '19]

- **Step 1**: compute $V_{\rm eff} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}$ ($\overline{\rm MS}$ result)
  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to* $V^{(2)}$ = diagrams involving **heavy BSM scalars and top quark**
  - **Neglect masses of light states** (SM-like Higgs, light fermions, ...)



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  - → $V^{(2)}$: 1PI vacuum bubbles
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- **Step 2**: derive an effective trilinear coupling

$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$

($\overline{\text{MS}}$ result too)

Express tree-level result in terms of effective-potential Higgs mass

Our effective-potential calculation

[JB, Kanemura '19]

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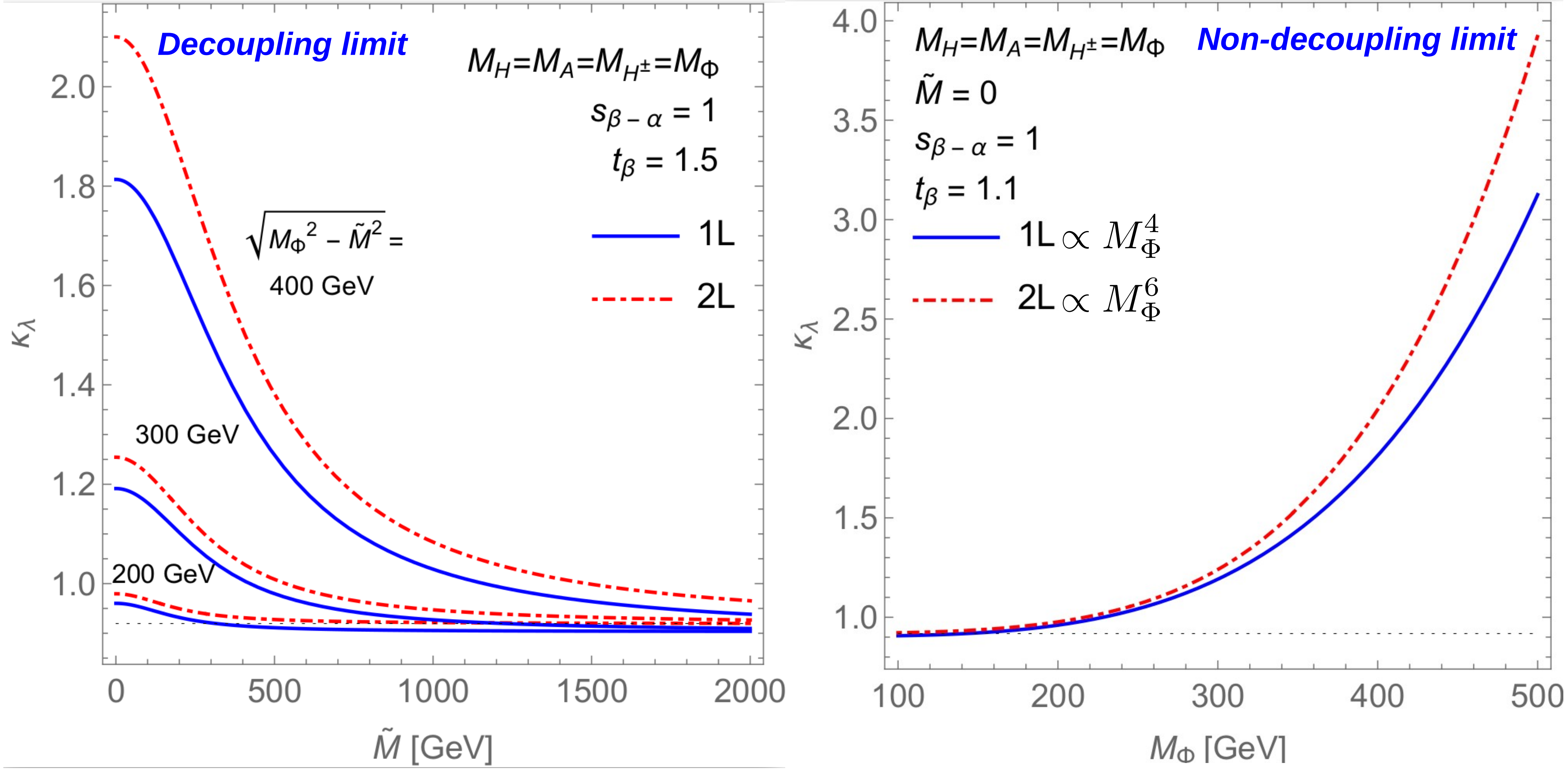
  ($\overline{\text{MS}}$ result too)

- **Step 3**: conversion from $\overline{\text{MS}}$ to OS scheme
  - Express result in terms of **pole masses**: M_t, M_h, M_Φ (Φ=H,A,H$^\pm$); OS Higgs VEV $v_{\text{phys}} = \frac{1}{\sqrt{\sqrt{2}G_F}}$
  - Include **finite WFR**: $\hat{\lambda}_{hhh} = (Z_h^{\text{OS}}/Z_h^{\overline{\text{MS}}})^{3/2}\lambda_{hhh}$
  - Prescription for M to ensure **proper decoupling** with $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

Our results in the aligned 2HDM

[JB, Kanemura ‘19]

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_{H^\pm}$



$\overline{\text{MS}}$ to OS scheme conversion

- V_{eff}: we use expressions in MS scheme hence results for λ_{hhh} also in $\overline{\text{MS}}$ scheme

- We include finite counterterms to express the Higgs trilinear coupling in terms of physical quantities

$$\underbrace{m_X^2}_{\overline{\text{MS}}} = \underbrace{M_X^2}_{\text{pole}} - \mathfrak{Re}[\Pi_{XX}^{\text{fin.}}(p^2 = M_X^2)], \qquad v^2 = \underbrace{(\sqrt{2}G_F)^{-1}}_{\equiv v_{\text{OS}}^2} + \frac{3M_t^2}{16\pi^2}\left(2\log\frac{M_t^2}{Q^2} - 1\right) + \cdots$$

- Also we include finite WFR effects → OS scheme

$$\underbrace{\hat{\lambda}_{hhh}}_{\text{OS}} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{finite WFR}} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}} = -\underbrace{\Gamma_{hhh}(0,0,0)}_{\text{3-pt. func.}}$$

$\overline{\text{MS}}$ to OS scheme conversion

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\text{MS}}}) + \kappa f^{(1)}(x^{\overline{\text{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

$\overline{\text{MS}}$ to OS scheme conversion

- OS result is obtained as

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then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x}\right]$$

$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x} + \cancel{\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2}\right]$$

because we neglect m_h in the loop corrections and $\lambda_{hhh}^{(0)} = 3m_h^2/v$ (in absence of mixing)