Search for the critical point in NA61/SHINE

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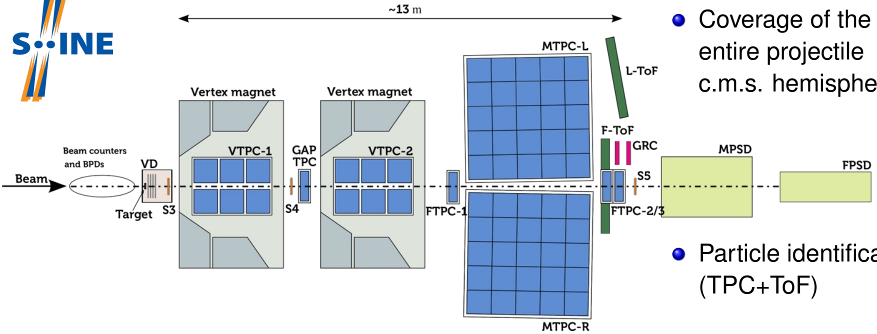
NCSR “Demokritos”, Athens, Greece

Outline

The NA61/SHINE experiment

NA61/SHINE
at CERN SPS

⟹ Multipurpose fixed target spectrometer with unique capabilities



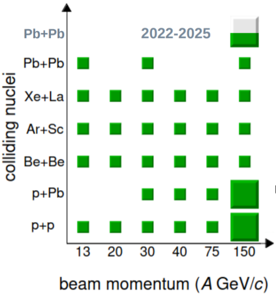
- Coverage of the entire projectile c.m.s. hemisphere
- Particle identification (TPC+ToF)

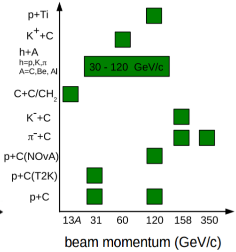
- **Strangeness in quark matter:** K^+, K^-, K^0_s, K^*, Λ, ϕ
- **Heavy quarks:** D^0 and $\bar{D}^0$
- **Correlations, fluctuations, HBT, intermittency...**

NA61/SHINE – Research program $\sqrt{s_{NN}}$ = 5-17/27 GeV

Strong interactions

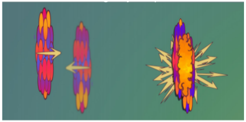
- study the onset of deconfinement
- **search for the critical point**
- measurement of open charm

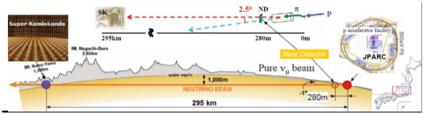


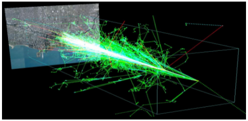


Neutrino and cosmic ray physics

- measurements for neutrino programs (J-PARC, Fermilab)
- measurements for cosmic-ray physics (Pierre-Auger, KASCADE, satellite experiments)

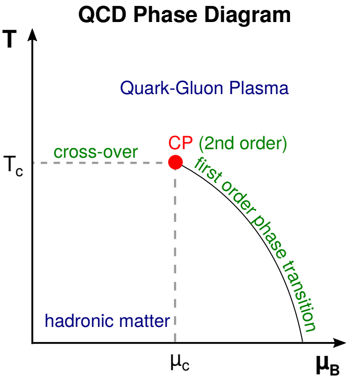


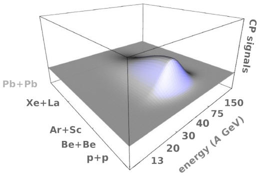




The NA61/SHINE critical point search

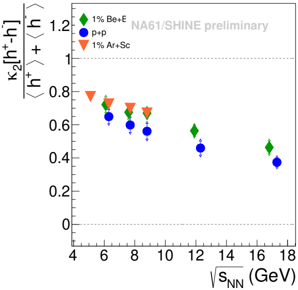
- **Critical point (CP)** — a hypothetical end point of **first order** phase transition line (QGP-HM) that has properties of **second order** phase transition
- **2nd order** phase transition → **scale invariance** → **power-law form** of **correlation function**
- Expectations for **enhanced fluctuations and correlations** in **configuration space** → projected to **momentum space** via quantum statistics and/or collective flow

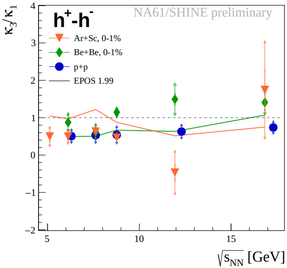


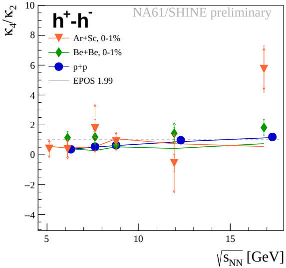


- **Scan** in the **experimentally controlled parameters** (collision energy, nuclear mass number, centrality). Conjecture is that, by varying them, we vary **freeze-out conditions** (T, μ_B)

Net electric charge fluctuations







$$\kappa_1 = \langle N \rangle$$

$$\kappa_2 = \left\langle (\delta N)^2 \right\rangle \qquad \langle N_2 \rangle \sim \xi^2$$

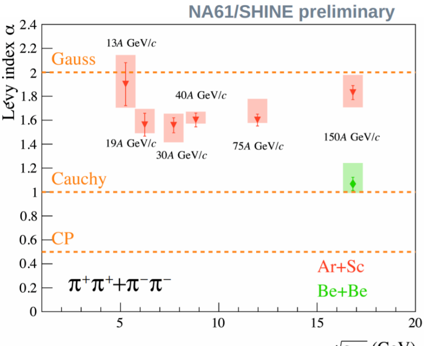
$$\kappa_3 = \left\langle (\delta N)^3 \right\rangle \qquad \langle N_4 \rangle \sim \xi^7$$

$$\kappa_4 = \left\langle (\delta N)^4 \right\rangle$$

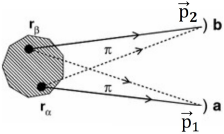
No significant non-monotonic signal observed

[NA61/SHINE, PoS(PANIC2021)238]
[NA61/SHINE, Status Report 2022]

Bose-Einstein (HBT) correlations (femtoscopy)



[B. Porfy, arXiv:2410.13975]



Correlation function from Lévy source:

$$C(q) = 1 + \lambda e^{-|qR|^{\alpha}}$$
$$q = |\vec{p}_1 - \vec{p}_2|$$

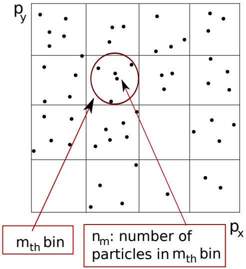
- Bose-Einstein correlations (femtoscopy) reveal the space-time structure of hadron production
- The Lévy parameter α describes the shape of the source and is sensitive to the system freezing out at the CP
- The new Ar+Sc results are close to Gaussian, and **far from the CP**

[Csörgő, Hegyi, Novák, Zajc, AIP Conf. Proc. 828 (2006) 525]

Ar+Sc, 0-10% central, NA61/SHINE preliminary

Be+Be, 0-20% central, NA61/SHINE, EPJC 83 (2023) 919

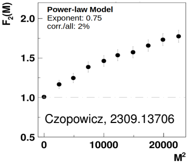
Proton intermittency – scaled factorial moments $F_r(M)$

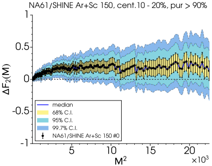


- When the system **freezes out at CP**, the **scaled factorial moments $F_r(M)$** are expected to follow a **power-law** behaviour:

$$F_r(M) \sim \left(M^2\right)^{\phi_r}$$

- For **protons** and $r = 2$, $\phi_2 = 5/6$ is expected
- Either **correlated**[1] or **statistically independent data points**[2] can be used





- **Cumulative variables**[3] or **mixed-event moment subtraction**[4] handle **baseline correlations**

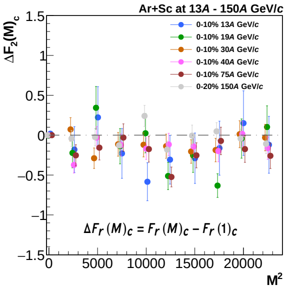
$$F_r(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{m=1}^{M^2} n_m(n_m-1)\ldots(n_m-r+1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{m=1}^{M^2} n_m \right\rangle^r}$$

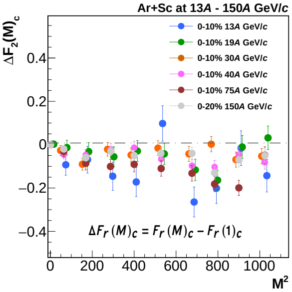
M^2 – number of bins; $\langle \ldots \rangle$ – averaging over events

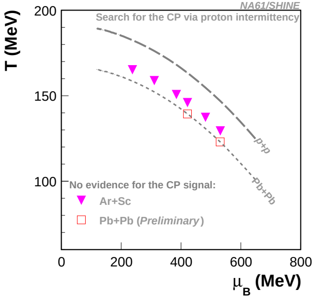
[Białas, Peschanski, NPB 273 (1986) 703]
[Wosiek, APPB 19 (1988) 863]
[Asakawa, Yazaki, NPA 504 (1989) 668]
[Barducci et al., PLB 231 (1989) 463]
[Satz, NPB 326 (1989) 613]
[Antoniou et al., PRL 97 (2006) 032002]
1 [NA61/SHINE, APP.Supp. 13 (2020) 637]
2 [NA61/SHINE, EPJC 83 (2023) 881]
3 [Białas, Gazdzicki, PLB 252 (1990) 483]
4 [NA49, EPJC 75 (2015) 587]

SHINE ^{40}Ar + ^{45}Sc independent bin proton intermittency

- **No signal** indicating the **critical point** in **cumulative independent bin** analysis







$1^2 \leq M^2 \leq 150^2$

$1^2 \leq M^2 \leq 32^2$

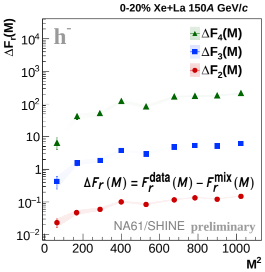
number of subdivisions in cumulative transverse momentum space

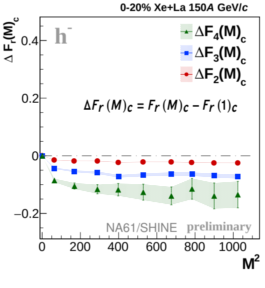
[NA61/SHINE, EPJC 83 (2023) 881]

[NA61/SHINE, EPJC 84 (2024) 7, 741]

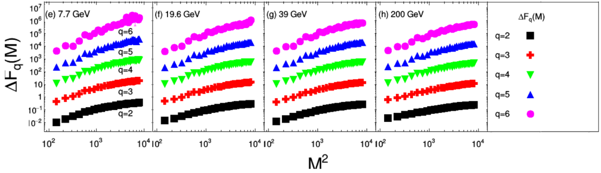
Intermittency of negatively charged hadrons

[V. Reyna, New Trends in HEP, Sfantu Gheorghe, Romania]



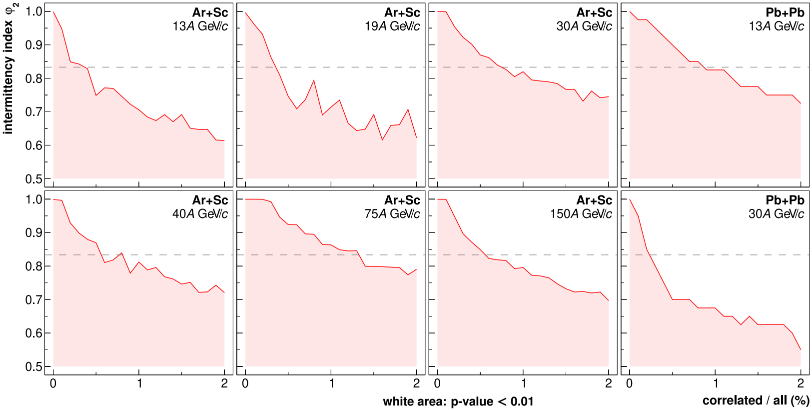


[STAR collaboration, Phys.Lett.B 845 (2023)]



- **STAR collaboration:** intermittency results of $\Delta F_q(M) \equiv F_q^{\text{data}}(M) - F_q^{\text{mixed}}(M),\ q = 2-6$ of **all charged hadrons** in **0-5% Au+Au collisions**
- **NA61/SHINE:** $\Delta F_r(M),\ r = 2-4$ for **non-cumulative** and **cumulative transformed p_T binning**, of **negatively charged (h^-) hadrons** in **0-20% Xe + La collisions**
- $\Delta F_q(M)$ **increases with** M^2 up to $M^2 \sim 4000$ in **STAR** results; interpretation of this increase was **unclear**, with **no specific theoretical prediction** given for **$h^{\pm}$ critical scaling**
- **Cumulative transform and/or short-range correlation cut removes** corresponding effect in **SHINE h^- analysis**, indicating a **systematic effect** as the origin of observed **STAR** scaling

Independent bin proton intermittency – exclusion plots



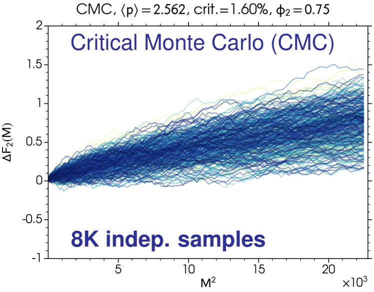
Exclusion plots for parameters of simple power-law model:

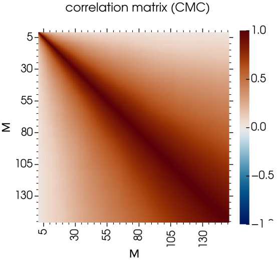
- power-law exponent ϕ in $|\Delta\vec{p}_T|$ correlation function $\rho\left(|\Delta\vec{p}_T|\right) = |\Delta\vec{p}_T|^{-\phi}$, $\varphi_2 = (\phi + 1)/2$
- fraction of correlated particles

Expected intermittency index: $\varphi_2 = 5/6$ (3D Ising universality class)

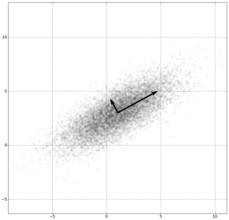
Handling bin correlations through PCA

- $F_2(M)$ **values** for different **M-bin sizes** are **correlated**, if the **same events** are used to calculate **different bins**; this **invalidates** fitting & model comparison
- **Independent points** can be used, but **point uncertainties increase!**



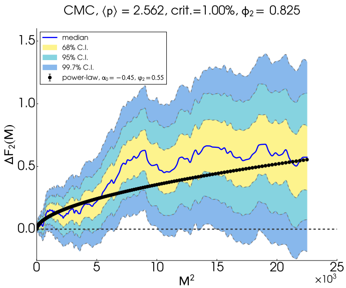


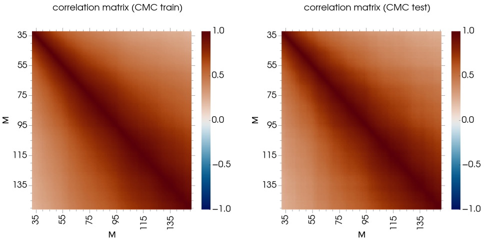
- An alternative is to **untangle correlations**

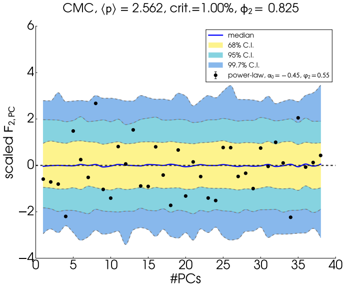


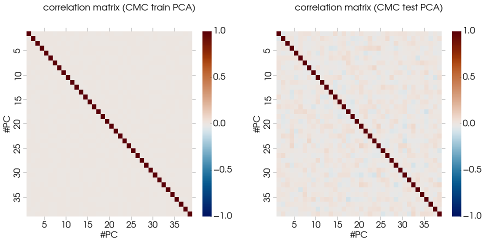
- We can do this via **Principal Component Analysis (PCA)**: **center and scale** sample points in M**-space**, then **rotate** the axes to make **independent linear combinations** of M**-bins**. Finally, keep **only few** significant components [N. Davis, arXiv:2409.14185]

Performing a scan in power-laws with PCA (CMC "data")





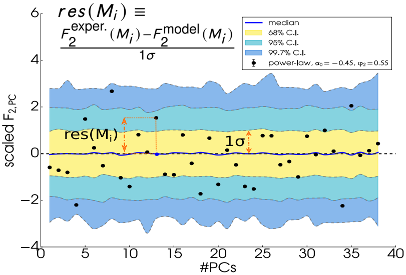


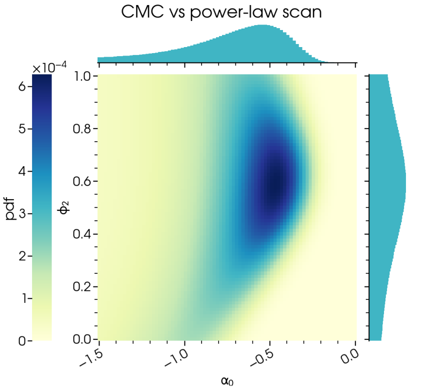


[N. Davis, arXiv:2409.14185]

- **Original sample** becomes **PC baseline**; **all power-laws** compared to it.

Estimating power-law model likelihoods





[N. Davis, arXiv:2409.14185]

Power-law model

$$\Delta F_2(M) \equiv 10^{\alpha_0} \left(\frac{M^2}{10^4}\right)^{\varphi_2}$$

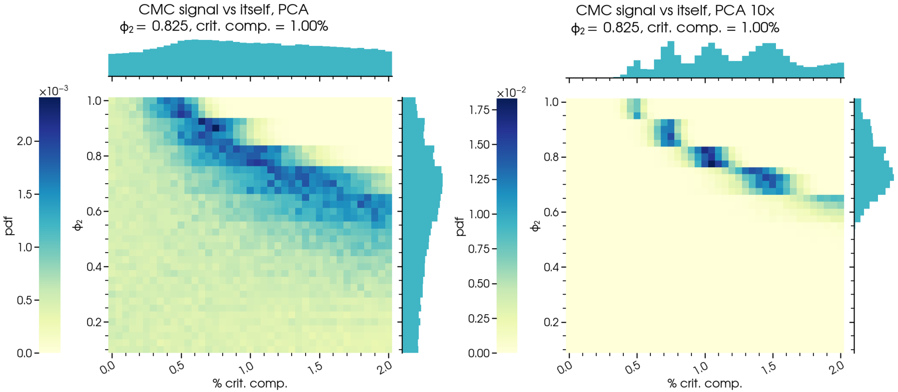
α_0 : power-law strength,
φ_2: power-law exponent

$$\chi^2 = \sum_i res^2(M_i) \Rightarrow \text{Model Weight} \sim e^{-\frac{\chi^2}{2}}$$

- **Scan in power-law parameters** $\Rightarrow$ **best-fitting power laws** to the data
- **Critical component** can be estimated by **power-law strength α_0**
- **PCA transformation** ensures **valid model weighting**

The role of event statistics

- **Event statistics** (number of analyzed events) **greatly affects predictive power** of model scans!



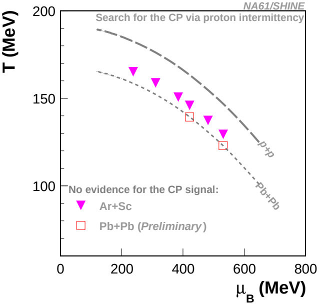
- **Critical Monte Carlo (CMC)** model simulations indicate that a $\sim$ **10× increase** in event statistics for **SHINE** could detect **as weak as a 1%** critical component signal! The **upgraded NA61/SHINE detector** is expected to provide **sufficient data** [M. Maćkowiak-Pawłowska *et al.* [NA61/SHINE], CERN-SPSC-2023-022]

Summary

- Results on **net-charge fluctuations** in p+p, Be+Be and Ar+Sc energy scans show **no non-monotonic signal**
- Obtained exponents from the Lévy-shaped source fit in the **HBT analyses** of pions produced in Be+Be at $\sqrt{s_{NN}} \approx 17$ GeV and Ar+Sc energy scan are **far from the values predicted for the critical point**
- Results on the dependence of **proton scaled factorial moments** of multiplicity distribution on cumulative momentum bin size, analyzed using independent data points for:
  - protons in Pb+Pb at $\sqrt{s_{NN}} \approx 5$ GeV
  - protons in Pb+Pb at $\sqrt{s_{NN}} \approx 7.5$ GeV
  - protons in Ar+Sc at $\sqrt{s_{NN}} \approx 5 - 17$ GeV

  show **no indication of a power-law increase**

Status of NA61/SHINE CP search via proton intermittency



Points indicate analyzed reactions with no evidence for CP. They are placed at T - μ_B values calculated from Becattini, Manninen, Gazdzicki, Phys. Rev. C73 (2006)

- **No indication of a power-law increase** in **negatively charged hadron factorial moments** in Xe+La ($\sqrt{s_{NN}} \approx 17$ GeV) when **cumulative p_T bins** are used
- Please also see **F. Diakonos talk** for a discussion of the **STAR h^{+-} intermittency** result!

Outlook

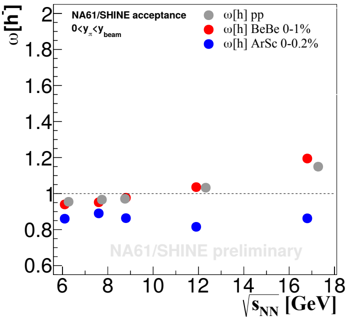
- Long-standing **bin-by-bin correlation problem** now **effectively solved**; **Principal Component Analysis (PCA)** methodology allows **direct handling** of **factorial moment bin correlations** using **the full event statistics**
- **Critical Monte Carlo** (CMC) model simulations indicate that a $\sim$ **10× increase** in event statistics for **SHINE** could detect **as weak as a 1%** critical component signal. The **upgraded NA61/SHINE detector** is expected to provide **sufficient data**

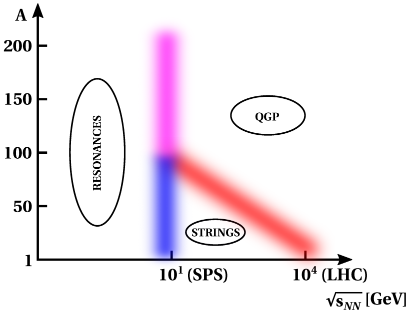
Thank You!

Backup Slides

Backup Slides Outline

Multiplicity fluctuations





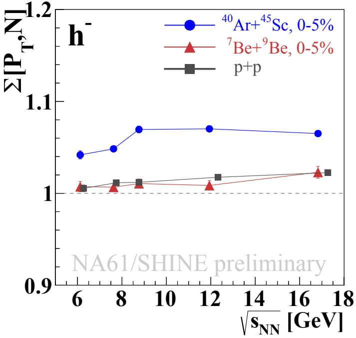
$$\omega[N] = \frac{Var[N]}{\langle N \rangle}$$

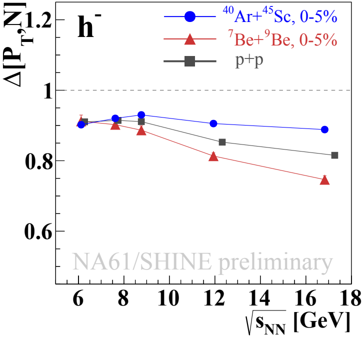
Be+Be similar to p+p, Ar+Sc different ⟶ onset of fireball (?).

No collision energy dependence that could be related to the critical point observed in Ar+Sc

[NA61/SHINE, PoS CPOD2017 (2018) 012]
[Andronov, Kuich, Gazdzicki, Universe 9 (2023) 2, 106]

Multiplicity-transverse momentum fluctuations





$$\Sigma[A,B] = \frac{1}{C_\Sigma}\left[\langle B\rangle\omega[A]+\langle A\rangle\omega[B]-2\Big(\langle AB\rangle-\langle A\rangle\langle B\rangle\Big)\right]$$

$$\Delta[A,B] = \frac{1}{C_\Delta}\Big[\langle B\rangle\omega[A] - \langle A\rangle\omega[B]\Big]$$

$$P_T = \sum^{N} p_{T_i}, \quad C_\Delta = C_\Sigma = \langle N\rangle\omega[p_T]$$

Be+Be similar to p+p, Ar+Sc different ⟶ onset of fireball (?).

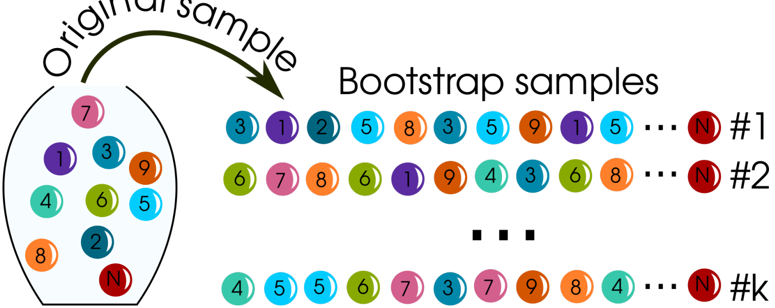
No collision energy dependence that could be related to the critical point observed in Ar+Sc

[NA61/SHINE, Acta Phys.Polon.Supp. 10 (2017) 449]

Intermittency analysis tools: the bootstrap

- Random **sampling** of events, **with replacement**, from the original set of events;
- *k* bootstrap samples ($k \sim 1000$) of the **same number of events** as the original sample;
- Each **statistic** ($\Delta F_2(M)$, ϕ_2) **calculated for bootstrap** samples as for the **original**; [B. Efron, *The Annals of Statistics* **7**,1 (1979)]
- **Variance of bootstrap values** estimates **standard error** of **statistic**.

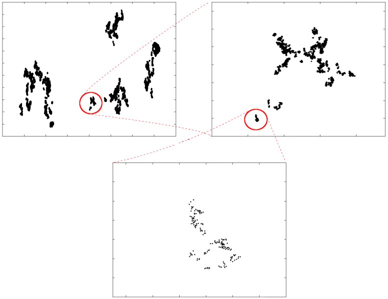
[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]



Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only **protons** produced;
  - **One cluster** per event, produced by sampling random Lévy walk of **adjustable dimension** d_F, e.g.: $d_F^B = 1/3 \Rightarrow \phi_2 = 1 - d_F^B/2 = 5/6$
  - **Lower / upper bounds** of Lévy walks $p_{min,max}$ plugged in;
  - Cluster center **adjustable** to **experimental set mean proton** p_T per event;
  - **Poissonian** proton multiplicity distribution.

Lévy walk example



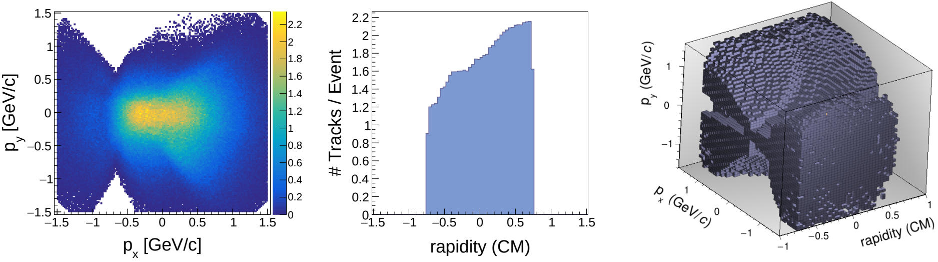
Input parameters (example)

Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}
Value	$0.1 \rightarrow 1$	$800 \rightarrow 1200$	$\langle p \rangle_{\text{non-empty}}$

*[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

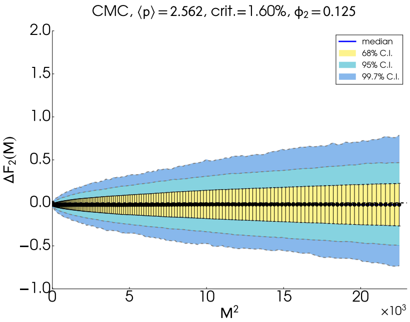
CMC – background simulation & detector effects

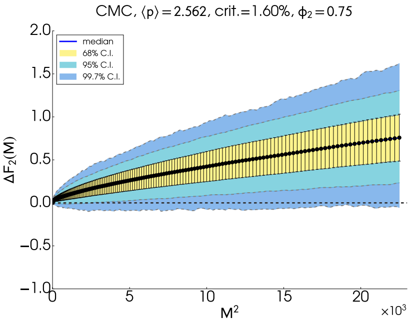
- **Non-critical background** simulation: **replace critical tracks** by **uncorrelated (random) tracks**, with **fixed probability:** $\mathcal{P}_{track} = 1 - \mathcal{P}_{crit}$, where $\mathcal{P}_{crit}$ is the **percentage** of **critical component**;
- p_T **distribution** of background tracks **plugged in** to **match experimental data**;
- y_{CM} **rapidity value** generated **orthogonal** to p_T, matching **experimental distribution**;
- p_T, y_{CM}, **quality & acceptance cuts** applied, same as in **NA61/SHINE data**;



CMC scan $\Delta F_2(M)$ – examples

- Results shown for **CMC $\Delta F_2(M)$**, with **$\langle p \rangle = 2.562$**, corresponding to **NA61/SHINE Ar+Sc @ 150A GeV/c, cent.10-20%**;
- **2 settings**:
  1. **$\phi_2 = 0.125$, crit.% = 1.60%**;
  2. **$\phi_2 = 0.750$, crit.% = 1.60%**;
- For each setting, **∼ 8K independent samples** of **∼ 400K events** are generated; event statistics selected to **match NA61/SHINE data**.





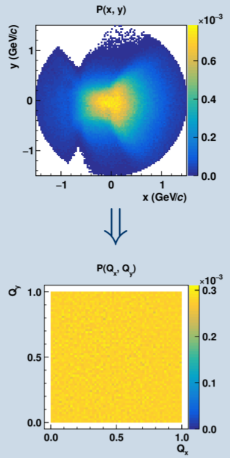
Independent bin analysis with cumulative variables

- **M-bin correlations** complicate uncertainties estimations for $\Delta F_2(M)$ & ϕ_2; one way around this problem is to use **independent bins** – a **different subset** of events is used to calculate $F_2(M)$ for **each** M;
- **Advantage:** correlations are no longer a problem; **Disadvantage:** we **break up statistics**, and can only **calculate** $F_2(M)$ for **a handful of bins**.
- Furthermore, instead of p_x and p_y, one can use **cumulative quantities:** [Bialas, Gazdzicki, PLB 252 (1990) 483]

$$Q_x(x) = \int_{min}^{x} P(x)dx \Bigg/ \int_{min}^{max} P(x)dx;$$

$$Q_y(x,y) = \int_{ymin}^{y} P(x,y)dy \Bigg/ P(x)$$

  - transform any distribution into **uniform** one $(0,1)$;
  - **remove the dependence** of F_2 on the shape of the **single-particle distribution**;
  - approximately **preserves ideal power-law** correlation function. [Antoniou, Diakonos, https://indico.cern.ch/event/818624/]

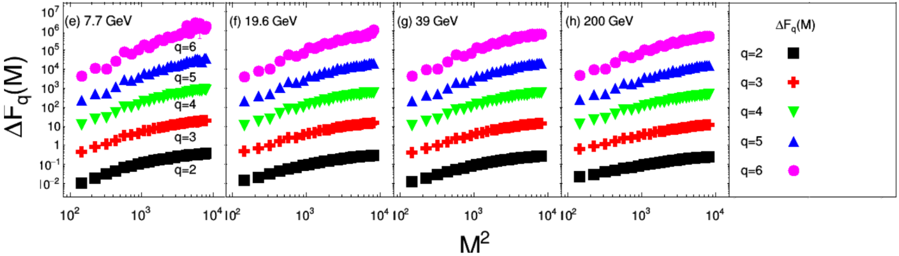


(example for 0-5% Ar+Sc at 150A GeV/c)

STAR $h^{\pm}$ intermittency analysis

- In March 2023, the **STAR collaboration** published intermittency results of ΔF_q of charged hadrons in **0-5% Au+Au collisions** at four example energies;

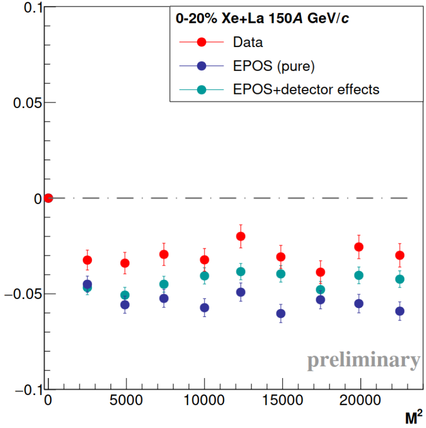
  [STAR collaboration, Phys.Lett.B 845 (2023)]



- Plots: $\Delta F_q(M) = F_q^{\text{data}}(M) - F_q^{\text{mixed}}(M)$ $(q = 2-6)$, in double-logarithmic scale;
- STAR reported that $\Delta F_q(M)$ **increases** with M^2 and **saturates** when M^2 is larger than $M^2 > 4000$;
- Interpretation of the **source** of this increase was **unclear**; **no specific theoretical prediction** is given for $h^{\pm}$ **critical scaling**.

SHINE Xe + La negatively charged hadrons intermittency

- Intermittency analysis performed on **negatively charged hadrons (h^-)** in **SHINE Xe + La collisions @ 150A GeV/c**; motivated by **corresponding STAR analysis;** [STAR collaboration, Phys.Lett.B 845 (2023)]



[V. Reyna, CPOD 2024, Berkeley, California]

- Results after **cumulative transform** and **short-range correlation Δp_T** cut ($\Delta p_T < 100$ MeV/c **removed**) **do not show any signal** indicating the **critical point**;
- Could the results of **STAR** (reported **increase of ΔF_2** with M) also be interpreted as due to **short-range correlations**?