Determination of $\mathcal{CP}$-violating HZZ interaction with polarised beams at the ILC

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Motivation

1. The CP violation in HVV interaction can be a possible source of the baryogenesis
2. Achieving highest precision for determination the CP properties of HZZ coupling via Z decay at the future e^+e^- collider.
3. Polarised e^+e^- beams can be used to improve the sensitivity to the CP properties of HZZ coupling, by enhancing the cross-section or introducing additional observables

CP violation in Higgs to gauge bosons interaction

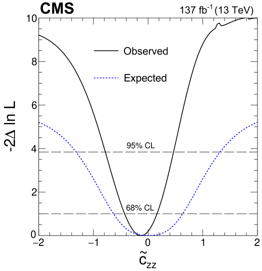
We only take the leading-order CP-odd terms into account

$$\mathcal{L}_{\mathrm{EFF}} = c_{\mathrm{SM}}\, Z_\mu Z^\mu H - \frac{c_{HZZ}}{v} Z_{\mu\nu} Z^{\mu\nu} H - \frac{\widetilde{c}_{HZZ}}{v} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H \tag{1}$$

(2)

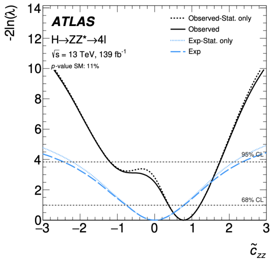
At LHC: $H \to 4\ell$ measurement:

[*CERN-EP-2021-054*]



$(\tilde{c}_{ZZ})_{\mathrm{CMS}} \sim [-0.66, 0.51]$

[*CERN-EP-2023-030*]

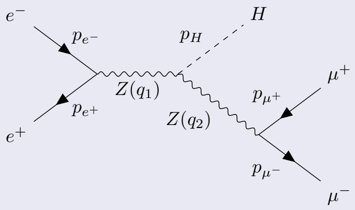


$(\tilde{c}_{ZZ})_{\mathrm{ATLAS}} \sim [-1.2, 1.75]$

Probing the CP violation at e^+e^- collider

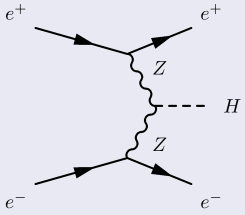
- Probe the CP-violation of *HZZ* at e^+e^- collider via Z decay from Higgs strahlung process or Z-fusion process

Higgs Strahlung



- Unpolarised study at CEPC [Q. Sha et al. 22']
- The effect of the initial polarized electrons is carried by the Z boson and transferred to the $\mu^+\mu^-$ pair by the Z decay

Z fusion



- Z-fusion study at CLIC [I. Bozovic et al. 23']
- Z-fusion process **cannot** carry the spin information of initial transversely polarised beams, since the final state electron and positron are unpolarised

polarisation matix for the initial beams:

$$\frac{1}{2}(1-\sigma\cdot P)_{\lambda\lambda'} = \frac{1}{2}\begin{pmatrix} 1-P^3 & P^1-iP^2 \\ P^1+iP^2 & 1+P^3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-f\cos\theta_P & f\sin\theta_P e^{-i\phi_P} \\ f\sin\theta_P e^{i\phi_P} & 1+f\cos\theta_P \end{pmatrix} \tag{3}$$

Bouchiat-Michel formula:

$$u(p,\lambda')\bar{u}(p,\lambda) = \frac{1}{2}(1+2\gamma_5)\not{p}\delta_{\lambda\lambda'} + \frac{1}{2}\gamma_5(\not{s}_-^1\sigma^1_{\lambda\lambda'} + \not{s}_-^2\sigma^2_{\lambda\lambda'})\not{p} \tag{4}$$

$$v(p,\lambda')\bar{v}(p,\lambda) = \frac{1}{2}(1-2\gamma_5)\not{p}\delta_{\lambda\lambda'} + \frac{1}{2}\gamma_5(\not{s}_+^1\sigma^1_{\lambda\lambda'} + \not{s}_+^2\sigma^2_{\lambda\lambda'})\not{p} \tag{5}$$

Spin density matrix for Higgs strahlung:

$$\begin{aligned}\rho^{ii'}(e^+e^-\to ZH) =& \frac{1}{2}(\delta_{\lambda_r\lambda_r'} + P_-^m\sigma^m_{\lambda_r\lambda_r'})\frac{1}{2}(\delta_{\lambda_u\lambda_u'} + P_+^n\sigma^n_{\lambda_u\lambda_u'})M^i_{\lambda_r\lambda_u}{M^*}^{i'}_{\lambda_r'\lambda_u'} \\ =& (1-P_-^3P_+^3)A^{ii'} + (P_-^3-P_+^3)B^{ii'} + \sum_{mn}^{1,2} P_-^m P_+^n C^{ii'}_{mn}\end{aligned} \tag{6}$$

where C_{mn} is the part with transversely polarised beams.

- Note that, one would not see any transverse polarisation effect when only one beams transversely polarised

Amplitude and CP-violation contribution

In order to simplify the analysis and get the idea of CP-violation effect, we only consider the additional contribution from the CP-odd term $\tilde{c}_{HZZ}$

$$\begin{aligned} |\mathcal{M}|^2 =& |c_{\rm SM}\mathcal{M}_{\rm SM} + \tilde{c}_{HZZ}\widetilde{\mathcal{M}}_{HZZ}|^2 \\ =& |c_{\rm SM}\mathcal{M}_{\rm SM}|^2 + |c_{\rm SM}\tilde{c}_{HZZ}\mathcal{M}_{\rm SM}\widetilde{\mathcal{M}}_{HZZ}| + |\tilde{c}_{HZZ}\widetilde{\mathcal{M}}_{HZZ}|^2 \end{aligned} \tag{7}$$

where

$$c_{\rm SM} \propto \cos\xi_{CP}, \qquad \tilde{c}_{HZZ} \propto \sin\xi_{CP} \tag{8}$$

Concerning the beam polarisation

$$\begin{aligned} |\mathcal{M}|^2 =& (1 - P_-^3 P_+^3)(\cos^2\xi_{CP}\,\mathcal{A}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{A}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{A}}_{\text{CP-even}}) \\ &+ (P_-^3 - P_+^3)(\cos^2\xi_{CP}\,\mathcal{B}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{B}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{B}}_{\text{CP-even}}) \\ &+ \sum_{mn}^{1,2} P_-^m P_+^n \left(\cos^2\xi_{CP}\,\mathcal{C}^{mn}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{C}^{mn}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{C}}^{mn}_{\text{CP-even}}\right) \end{aligned} \tag{9}$$

Only the interference term is CP-odd, which yield the CP-violation via triple-product correlations

$$\mathcal{A}_{\text{CP-odd}}, \mathcal{B}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\alpha\beta}[p^\mu_{e^-} p^\nu_{e^+} p^\alpha_{\mu^+} p^\beta_{\mu^-}] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{p}_{e^-} \tag{10}$$

$$\mathcal{C}^{mn}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\rho\sigma}[(p_{e^-} + p_{e^+})^\mu p^\nu_{\mu^+} p^\rho_{\mu^-} s^\sigma_{e^-}] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{s}_{e^-} \tag{11}$$

- The idea of using transverse polarisation to probe the CP properties of *HZZ* coupling see also [*S. Biswal et al. '09*]

arXiv: 2405.08494

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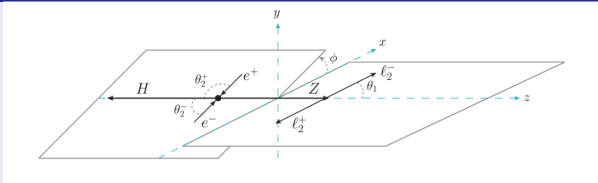
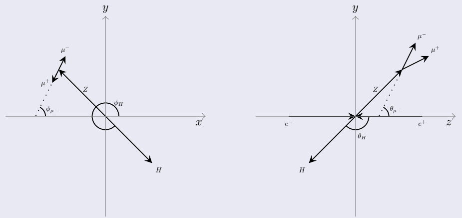
CP-sensitive observables

Coordinate systems with unpolarised or longitudinal polarised beams



- The ϕ is the azimuthal angle difference between the μ^--μ^+ plane and the Z-H plane

Coordinate systems with transversely polarised beams ($\vec{n}_y \propto \vec{s}_{e^-}$, $\vec{n}_x \propto \vec{s}_{e^-} \times \vec{p}_{e^-}$, $\vec{n}_z \propto \vec{p}_{e^-}$)



- The ϕ_{μ^-} is the azimuthal angle of the μ^--μ^+ plane with fixing the y-axis orientation to $\vec{s}_{e^-}$

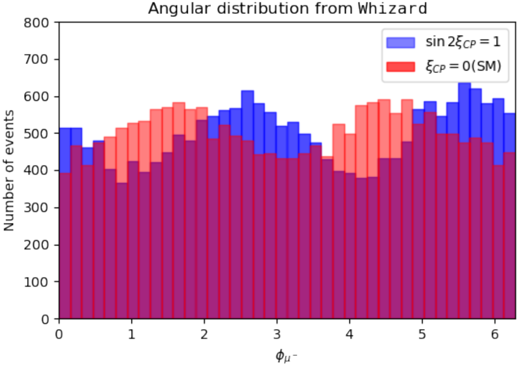
Angular distribution

Monte Carlo simulation by Whizard [1]

- We fix the total cross-section to the SM tree-level cross-section, and use 100% transversely polarized beams

$$\sigma_{\text{tot}} = \cos^2\xi_{CP}\,\sigma_{\text{SM}} + \sin^2\xi_{CP}\tilde{\kappa}^2_{HZZ}\,\tilde{\sigma}_{\text{HZZ}} = \sigma_{\text{SM}}, \quad (12)$$

$$P_-^2 = P_+^2 = 100\% \quad (13)$$



- The angular distribution of muon azimuthal angle is sensitive to the CP-violation

[1] http://whizard.hepforge.org

Azimuthal asymmetry

Construct the observables sensitive to CP-violation:

$$\mathcal{O}^{T}_{CP} \propto \cos\theta_H \sin 2\phi_{\mu^-}, \qquad \mathcal{O}^{UL}_{CP} \propto \cos\theta_\mu \sin\phi \tag{14}$$

We can define the following asymmetries:

$$\mathcal{A}^{T}_{CP} = \frac{N(\mathcal{O}^{T}_{CP} < 0) - N(\mathcal{O}^{T}_{CP} > 0)}{N_{\text{tot}}} \tag{15}$$

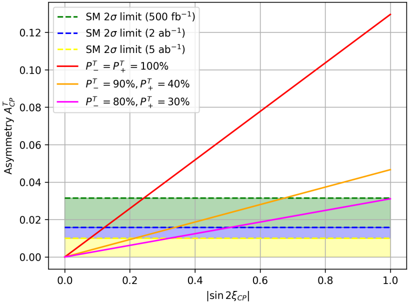
$$\mathcal{A}^{UL}_{CP} = \frac{N(\mathcal{O}^{UL}_{CP} < 0) - N(\mathcal{O}^{UL}_{CP} > 0)}{N_{\text{tot}}} \tag{16}$$

Statistical uncertainty (based on binomial distribution) of the Asymmetry:

$$\Delta\mathcal{A} = \sqrt{\frac{1 - \mathcal{A}^2}{N_{\text{tot}}}} \tag{17}$$

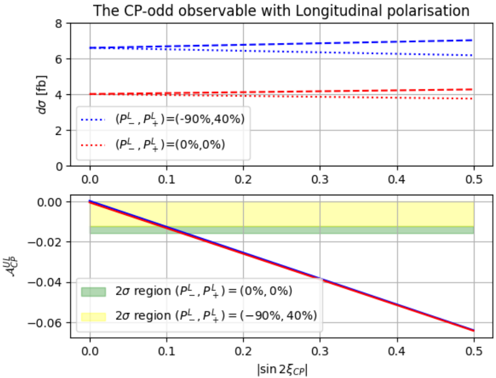
Variation of CP-mixing angle

We fix the total cross-section, and vary the CP-mixing angle ξ_{CP}



- This $\mathcal{A}_{CP}^T$ is linearly depending on the CP-mixing angle $\sin 2\xi_{CP}$
- The stronger transverse polarisation leads to larger $\mathcal{A}_{CP}^T$.
- For $(P_{e^-}^T, P_{e^+}^T) = (80\%, 30\%)$ and $L = 500\ \mathrm{fb}^{-1}$, one cannot distinguish the CP-violating case from CP-conserving case for any CP-mixing angle ξ_{CP} with only using $\mathcal{A}_{CP}^T$ observable.

Variation of CP-mixing angle

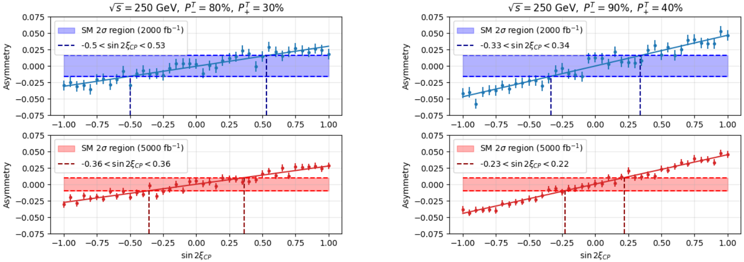


- The $\mathcal{A}^{UL}_{CP}$ linearly depends on the $\sin 2\xi_{CP}$ as well, while the beams polarisation cannot change the $\mathcal{A}^{UL}_{CP}$.
- One can also simultaneously measure the $\mathcal{A}^{UL}_{CP}$ when initial beams are transversely polarised.

Determination of the CP-mixing angle

We made a linear fit for the asymmetries with respect to the $\sin 2\xi_{CP}$

$$\mathcal{A}_i = a \sin 2\xi_{CP} + b \tag{18}$$



- The fitting results for Monte-Carlo simulation data are basically match to the analytical calculation.

Determination of the CP-mixing angle

- Simply combine the two asymmetries

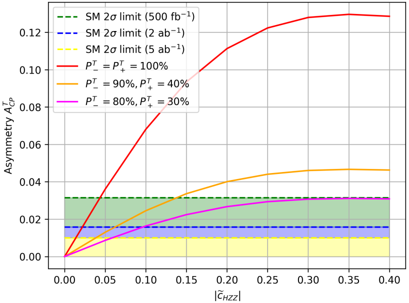
$$\chi^2_{\mathcal{A}_{\mathcal{CP}}} = (\frac{\mathcal{A}^T_{CP}}{\Delta\mathcal{A}^T_{CP}})^2 + (\frac{\mathcal{A}^{UL}_{CP}}{\Delta\mathcal{A}^{UL}_{CP}})^2 < 3.81 \quad (19)$$

(P_-, P_+)	$\mathcal{L}$ [ab^{-1}]	sin $2\xi_{CP}$ limit (95% C.L.) $\mathcal{A}^T_{CP}$	Combine $\mathcal{A}^T_{CP}$ & $\mathcal{A}^{UL}_{CP}$	$\mathcal{A}^{UL}_{CP}$
Observables				
Transverse polarisation				
$(80\%, 30\%)$	2.0	[-0.50, 0.53]	[-0.113, 0.125]	
$(80\%, 30\%)$	5.0	[-0.36, 0.36]	[-0.068, 0.079]	
$(90\%, 40\%)$	2.0	[-0.33, 0.34]	[-0.118, 0.110]	
$(90\%, 40\%)$	5.0	[-0.23, 0.22]	[-0.066, 0.077]	
$(100\%, 100\%)$	5.0	[-0.082, 0.069]	[-0.056, 0.051]	
Longitudinal polarisation				
$(-80\%, 30\%)$	2.0			[-0.119,0.082]
$(-80\%, 30\%)$	5.0			[-0.066,0.063]
$(-90\%, 40\%)$	2.0			[-0.085,0.106]
$(-90\%, 40\%)$	5.0			[-0.059,0.062]
$(-100\%, 100\%)$	5.0			[-0.047,0.053]

* The systematic uncertainties can be cancelled out by the CP-odd asymmetry, since the background contribution is basically CP-even.

Variation of the CP-odd coupling

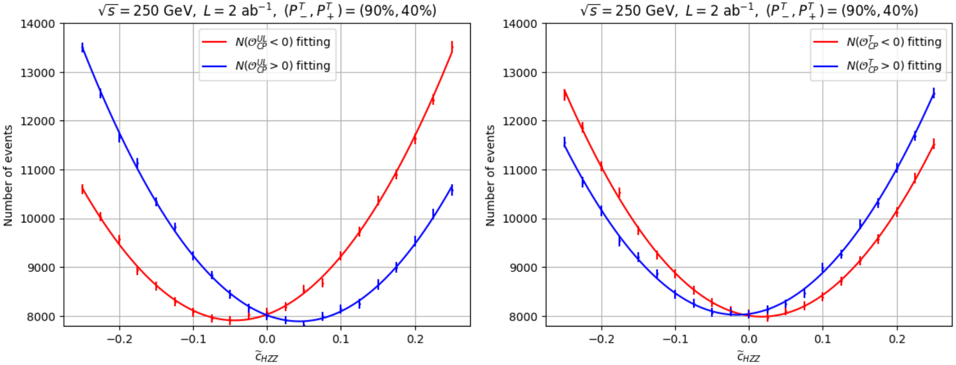
We fix $c_{\rm SM} = 1$ and vary $\tilde{c}_{HZZ}$, in this case $\sigma_{\rm tot}$ would be increased by $\tilde{c}_{HZZ}$



- The $\mathcal{A}_{CP}^T$ can reach to maximal when $\tilde{c}_{HZZ} \sim 0.35$, and asymmetry $\mathcal{A}_{CP}^T$ would decrease for much higher $\tilde{c}_{HZZ}$.
- For $(P_{e^-}^T, P_{e^+}^T) = (80\%, 30\%)$ and $L = 500\ {\rm fb}^{-1}$, one still cannot determine any CP-odd coupling $\tilde{c}_{HZZ}$.

Determination of the CP-odd coupling

Monte Carlo simulation by `Whizard`



- We made the quadratic function fit for the signal regions with varying $\tilde{c}_{HZZ}$

$$N_i = a\tilde{c}_{HZZ}^2 + b\tilde{c}_{HZZ} + c \tag{20}$$

Determination of the CP-odd coupling

- One can combine the signal regions

$$\chi^2_N = \sum_i \left(\frac{(N(\mathcal{O}_i < 0) - N^{\mathrm{SM}}(\mathcal{O}_i < 0))^2}{N(\mathcal{O}_i < 0)} + \frac{(N(\mathcal{O}_i > 0) - N^{\mathrm{SM}}(\mathcal{O}_i > 0))^2}{N(\mathcal{O}_i > 0)} \right) \tag{21}$$

(P_-, P_+) Observables	Luminosity [ab^{-1}]	$\tilde{c}_{HZZ}$ ($\times 10^{-2}$) limit (95% C.L.) $\mathcal{O}^T_{CP}$	Combine $\mathcal{O}^{UL}_{CP}$ & $\mathcal{O}^T_{CP}$	$\mathcal{O}^{UL}_{CP}$
Transverse polarisation				
(80%, 30%)	2.0	[-4.45,4.65]	[-2.26, 1.93]	
(80%, 30%)	5.0	[-3.55,3.85]	[-1.29, 1.06]	
(90%, 40%)	2.0	[-4.55,4.15]	[-2.24, 1.69]	
(90%, 40%)	5.0	[-2.65,3.75]	[-1.12, 0.98]	
Longitudinal polarisation				
$(-80\%, 30\%)$	2.0			[-1.55,1.96]
$(-80\%, 30\%)$	5.0			[-1.01,1.16]
$(-90\%, 40\%)$	2.0			[-1.73,1.53]
$(-90\%, 40\%)$	5.0			[-0.93,1.18]

* The explicit combined results can be obtained by the background simulation and log-likelihood estimation

Comparison

Determination of the CP-odd coupling

	95% C.L. (2σ)limit						
Experiments	ATLAS	CMS	HL-LHC	CEPC	CLIC	CLIC	ILC
Processes	$H \to 4\ell$	$H \to 4\ell$	$H \to 4\ell$	HZ	W-fusion	Z-fusion	$HZ,\ Z \to \mu^+\mu^-$
$\sqrt{s}$ [GeV]	13000	13000	14000	240	3000	1000	250
Luminosity [fb^{-1}]	139	137	3000	5600	5000	8000	5000
$(\lvert P_-\rvert, \lvert P_+\rvert)$							$(90\%, 40\%)$
$\tilde{c}_{HZZ}$ $(\times 10^{-2})$	[-16.4, 24.0]	[-9.0, 7.0]	[-9.1, 9.1]	[-1.6, 1.6]	[-3.3, 3.3]	[-1.1, 1.1]	[-1.1, 1.0]
$f_{CP}^{HZZ} (\times 10^{-5})$	[-409.82, 873.58]	[-123.78, 74.91]	[-126.54, 126.54]	[-3.92, 3.92]	[-16.66, 16.66]	[-1.85, 1.85]	[-1.85, 1.53]
$\tilde{c}_{ZZ}$	[-1.2, 1.75]	[-0.66, 0.51]	[-0.66, 0.66]	[-0.12, 0.12]	[-0.24, 0.24]	[-0.08, 0.08]	[-0.08, 0.07]

- The e^+e^- colliders can significantly improve the sensitivity to CP-odd HZZ coupling compared to the LHC or HL-LHC.
- The sensitivity with polarised beams is better than the analysis with unpolarised beams, where the center-of-mass energy and luminosity are similar.
- The Z-fusion process can have similar sensitivity but with much higher center-of-mass energy.

Summary

Conclusions

- The e^+e^- collider can achieve high precision to CP properties of HZZ interaction.
- The initial transversely polarised beams introduce additional CP-odd observables, which can be combined and improve the sensitivity to CP-odd structure.
- The longitudinally polarised beams enhance the total cross-section and suppress the statistical uncertainty, which can improve the CP-odd structure sensitivity as well.
- Both transverse and longitudinal polarisation improve compared to unpolarised case, where the transverse polarisation offers more observables
- Z-fusion process cannot get benefit from transverse polarisation, while Z-fusion process analysis at higher center-of-mass energy can be a complementary study for HZZ CP properties.

Thank you!

Back up

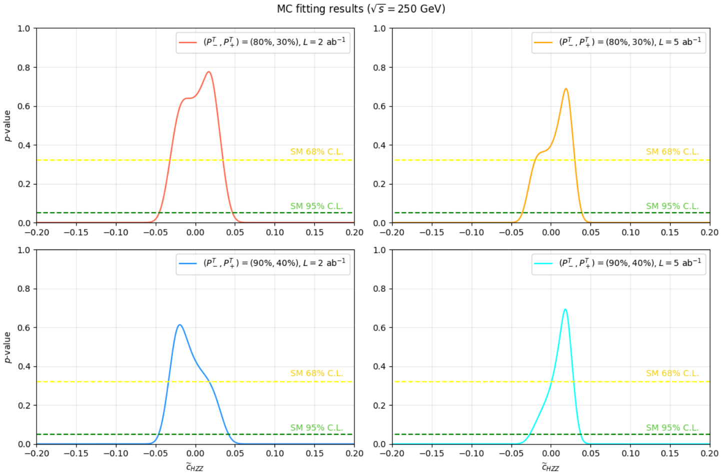
Matching conditions between different interpretations

$$f_{CP}^{HZZ} = \frac{\Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}{\Gamma_{H\to ZZ}^{CP-\mathrm{even}} + \Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}, \tag{22}$$

$$\frac{\Gamma_{H\to ZZ}^{CP-\mathrm{odd}}}{\Gamma_{H\to ZZ}^{CP-\mathrm{even}}} \sim \frac{\sigma_3}{\sigma_{\mathrm{SM}}}[pp \to H \to 4\ell(13\ \mathrm{TeV})] \sim 0.153. \tag{23}$$

$$\widetilde{c}_{HZZ} = \frac{g_1^2 + g_2^2}{4}\tilde{c}_{ZZ} = \frac{m_Z^2}{v^2}\tilde{c}_{ZZ}. \tag{24}$$

Back up


MC fitting results ($\sqrt{s}$ = 250 GeV)
$(P_-^T, P_+^T) = (80\%, 30\%)$, $L = 2$ ab^{-1}
$(P_-^T, P_+^T) = (80\%, 30\%)$, $L = 5$ ab^{-1}
$(P_-^T, P_+^T) = (90\%, 40\%)$, $L = 2$ ab^{-1}
$(P_-^T, P_+^T) = (90\%, 40\%)$, $L = 5$ ab^{-1}
SM 68% C.L.
SM 95% C.L.
p-value
$\tilde{c}_{HZZ}$