CP and entanglement in $H \to VV$ decays

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Relevant work:
JAAS et al. 2209.13441
JAAS 2209.14033
JAAS 2403.13942

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$H \to VV$ from first principles

Framework

For a pair of spin-1 particles *V=W,Z* the full spin density operator reads

$$\rho = \frac{1}{9}\left(1_{9\times9} + A^1_{LM} T^L_M \otimes 1_{3\times3} + A^2_{LM} 1_{3\times3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 polarisations for V_1

8 polarisations for V_2

64 spin correlations

where $T^L{}_M$ [L = 1,2] are irreducible tensors

$$T^1_1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_0 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T^2_2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^2_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^1_{-1} = -(T^1_1)^\dagger$$
$$T^2_{-2} = -(T^2_2)^\dagger$$
$$T^2_{-1} = -(T^2_1)^\dagger$$

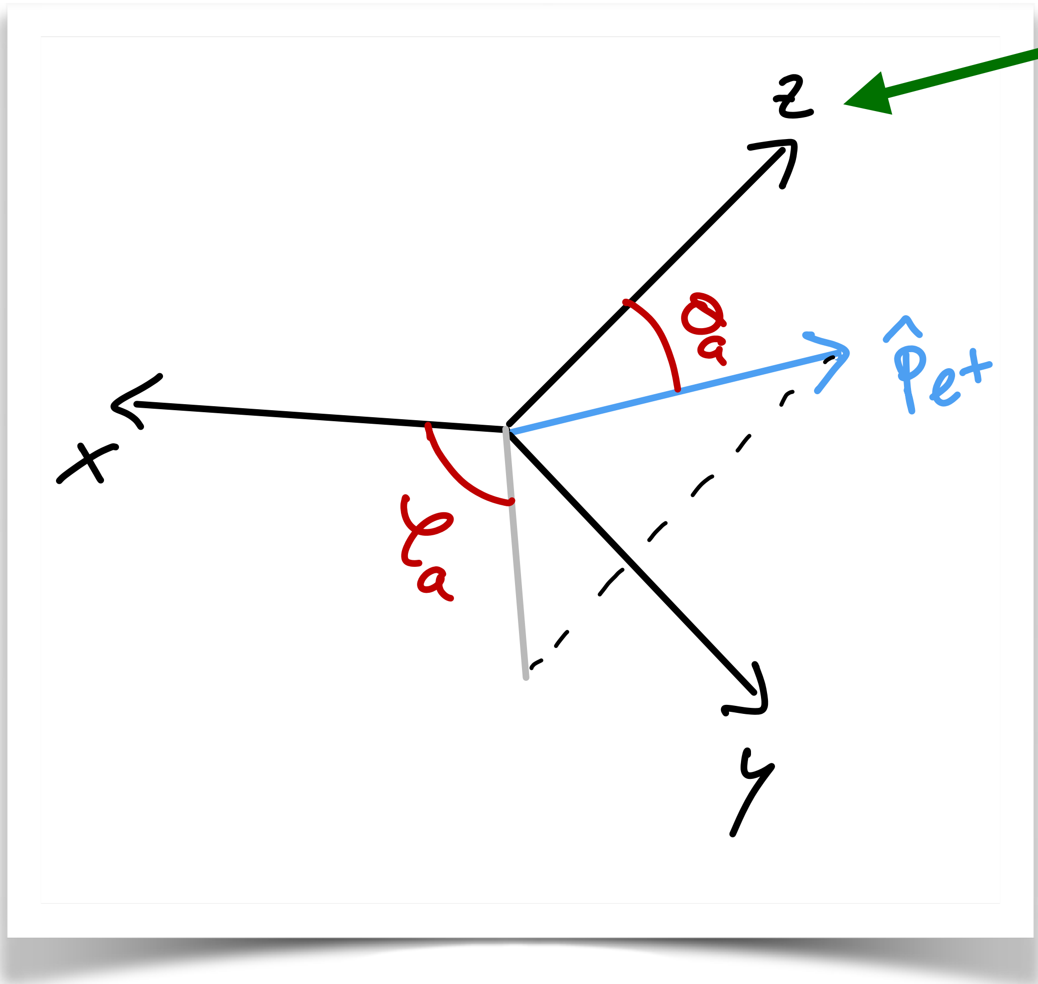
Alternative: Gell-Mann matrices

Framework

These *A* and *C* coefficients characterising the spin state of a *VV* pair can be measured from the charged lepton distributions, fixing a reference system

[the same for both bosons]

Usually, the ẑ axis is aligned with the weak boson momenta in *H* rest frame



$$\Omega_1 = (\theta_1, \varphi_1)$$
$$\Omega_2 = (\theta_2, \varphi_2)$$

rest-frame polar coordinates of the charged leptons (negative lepton for *Z*)

Framework

… and the density opeator translates into a 4D distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$\eta_\ell = \begin{cases} \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} & Z \\ 1 & W^- \\ -1 & W^+ \end{cases}$$

$$\Omega_1 = (\theta_1, \varphi_1)$$

$$\Omega_2 = (\theta_2, \varphi_2)$$

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}$$

Especially simple because spherical harmonics are orthogonal functions

constants you can calculate

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)^* Y^{M_2}_{L_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

data follow this distribution

calculate the average of this quantity on your data

the quantity you want

Framework

The terms of our expansion in Y_L^M has easier interpretation than equivalent expressions, e.g.

$$
\begin{aligned}
\frac{d^3\Gamma}{dc_{\theta_1}\, dc_{\theta_2} d\phi} \sim a^2 \Bigg[& s^2_{\theta_1} s^2_{\theta_2} - \frac{1}{2\gamma_a} s_{2\theta_1} s_{2\theta_2} c_\phi + \frac{1}{2\gamma_a^2} \left[(1+c^2_{\theta_1})(1+c^2_{\theta_2}) + s^2_{\theta_1} s^2_{\theta_2} c_{2\phi} \right] \\
& - \frac{2\eta_1\eta_2}{\gamma_a} \left(s_{\theta_1} s_{\theta_2} c_\phi - \frac{1}{\gamma_a} c_{\theta_1} c_{\theta_2} \right) \Bigg] \\
& + |b|^2 \frac{\gamma_b^4}{\gamma_a^2}\, x^2\, s^2_{\theta_1} s^2_{\theta_2} \\
& + |c|^2 \frac{\gamma_b^2}{\gamma_a^2}\, 4x^2 \left[1 + c^2_{\theta_1} c^2_{\theta_2} - \frac{1}{2} s^2_{\theta_1} s^2_{\theta_2} (1 + c_{2\phi}) + 2\eta_1\eta_2 c_{\theta_1} c_{\theta_2} \right] \\
& - 2a \Im m(b) \frac{\gamma_b^2}{\gamma_a^2}\, x\, s_{\theta_1} s_{\theta_2} s_\phi \left[\eta_2 c_{\theta_1} + \eta_1 c_{\theta_2} \right] \\
& - 2a \Re e(b) \frac{\gamma_b^2}{\gamma_a^2}\, x \left[-\gamma_a s^2_{\theta_1} s^2_{\theta_2} + \frac{1}{4} s_{2\theta_1} s_{2\theta_2} c_\phi + \eta_1\eta_2 s_{\theta_1} s_{\theta_2} c_\phi \right] \\
& - 2a \Im m(c) \frac{\gamma_b}{\gamma_a}\, 2x \Bigg[- s_{\theta_1} s_{\theta_2} c_\phi (\eta_1 c_{\theta_2} + \eta_2 c_{\theta_1}) \\
& \qquad + \frac{1}{\gamma_a} \left(\eta_1 c_{\theta_1} (1 + c^2_{\theta_2}) + \eta_2 c_{\theta_2} (1 + c^2_{\theta_1}) \right) \Bigg] \\
& - 2a \Re e(c) \frac{\gamma_b}{\gamma_a}\, 2x\, s_{\theta_1} s_{\theta_2} s_\phi \left[- c_{\theta_1} c_{\theta_2} + \frac{s_{\theta_1} s_{\theta_2} c_\phi}{\gamma_a} - \eta_1\eta_2 \right] \\
& + 2 \Im m(b^* c) \frac{\gamma_b^3}{\gamma_a^2}\, 2x^2\, s_{\theta_1} s_{\theta_2} c_\phi \left[\eta_2 c_{\theta_1} + \eta_1 c_{\theta_2} \right] \\
& + 2 \Re e(b^* c) \frac{\gamma_b^3}{\gamma_a^2}\, 2x^2\, s_{\theta_1} s_{\theta_2} s_\phi \left[c_{\theta_1} c_{\theta_2} + \eta_1\eta_2 \right] \ ,
\end{aligned}
$$

Framework

$H \to VV$ has many symmetries, which impose restrictions and relations among the 80 coefficients. In terms of the three general helicity amplitudes a_{11}, a_{00}, a_{-1-1}, and for fixed m_{V^*},

$$A^1_{20} = A^2_{20} = \frac{1}{\sqrt{2}}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 + |a_{-1-1}|^2 - 2|a_{00}|^2\right]$$

$$\mathcal{N} = |a_{11}|^2 + |a_{-1-1}|^2 + |a_{00}|^2$$

$$C_{1010} = -\frac{3}{2}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 + |a_{-1-1}|^2\right]$$

$$C_{2020} = \frac{1}{\sqrt{2}}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 + |a_{-1-1}|^2 + 4|a_{00}|^2\right]$$

$$C_{222-2} = C^*_{2-222} = 3\frac{1}{\mathcal{N}}a_{11}a^*_{-1-1}$$

$$C_{111-1} = -C_{212-1} = C^*_{1-111} = -C^*_{2-121} = -\frac{3}{2}\frac{1}{\mathcal{N}}\left[a_{11}a^*_{00} + a_{00}a^*_{-1-1}\right]$$

CP-conserving

$$A^1_{10} = -A^2_{10} = \sqrt{\frac{3}{2}}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 - |a_{-1-1}|^2\right]$$

$$C_{1020} = -C_{2010} = \frac{\sqrt{3}}{2}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 - |a_{-1-1}|^2\right]$$

$$C_{1-121} = -C_{2-111} = C^*_{112-1} = -C_{211-1} = \frac{3}{2}\frac{1}{\mathcal{N}}\left[a_{00}a^*_{11} - a_{-1-1}a^*_{00}\right]$$

CP-violating

Framework

Coefficients with L = 1 have a suppressed effect on the distribution for *ZZ*

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2)$$
$$+ B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$B_1 = -\sqrt{2\pi}\eta_\ell \longrightarrow \eta_\ell = \pm 1\ (W)\ ;\ 0.13\ (Z)$$

- Coefficients A_{1M}, $C_{1M2M'}$ have a suppression 1/10 → Δ_{stat} 3x penalty
- Coefficients $C_{1M1M'}$ have a suppression 1/100 → Δ_{stat} 10x penalty

Therefore, it pays off to use relations between coefficients to extract observables from the most precisely measured ones.

In addition, coefficients with L = 2 are even in (θ,ϕ) and do not require to distinguish fermions in $V \rightarrow f_1 f_2$ [can use hadronic decays for example]

Measuring the full density operator

Measuring the full density operator

If one is able to experimentally measure

- initial spin state [trivial for *H*]
- decay amplitudes

then, one is able to determine the full density operator ρ_{LS1S2} that fully describes the Higgs decay, also including OAM!

For the Higgs boson decay to *VV*, the determination of ρ_{LS1S2} has to be performed in bins of m_{V^*} with subsequent [weighted] sum over bins

Enough accuracy with 20 GeV bins, even for HL-LHC statistics

Note: within *narrow* bins of m_{V^*}, the *VV* pair is produced in an almost pure state.

Measuring the full density operator

For the Higgs boson decay to *VV*,

$$(\rho_{LS_1S_2})^{s_1's_2';l'm'}_{s_1s_2;lm} = A_{s_1s_2;lm}A^*_{s_1's_2';l'm'}$$

with

$$A_{11;2-2} = A_{-1-1;22} = -\sqrt{\frac{2\pi}{15}}(a_{11} + 2a_{00} + a_{-1-1})$$

$$A_{10;2-1} = A_{01;2-1} = A_{0-1;21} = A_{-10;21} = \sqrt{\frac{\pi}{15}}(a_{11} + 2a_{00} + a_{-1-1})$$

$$A_{1-1;20} = A_{-11;20} = -\sqrt{\frac{2\pi}{45}}(a_{11} + 2a_{00} + a_{-1-1})$$

$$A_{00;20} = -\sqrt{\frac{4\pi}{45}}(a_{11} + 2a_{00} + a_{-1-1})$$

$$A_{10;1-1} = -A_{1-1;10} = -A_{01;1-1} = A_{0-1;11} = A_{-11;10} = -A_{-10;11} = \sqrt{\frac{\pi}{3}}(a_{11} - a_{-1-1})$$

$$A_{1-1;00} = -A_{00;00} = A_{-11;00} = -\sqrt{\frac{4\pi}{9}}(a_{11} - a_{00} + a_{-1-1})$$

➡ The problem is reduced to a binned measurement of the *a*'s

Measuring the full density operator

Use helicity basis to <u>measure</u> a_{11}, a_{00}, a_{-1-1} in data

→ model-independent →

obtain canonical amplitudes from a_{11}, a_{00}, a_{-1-1}

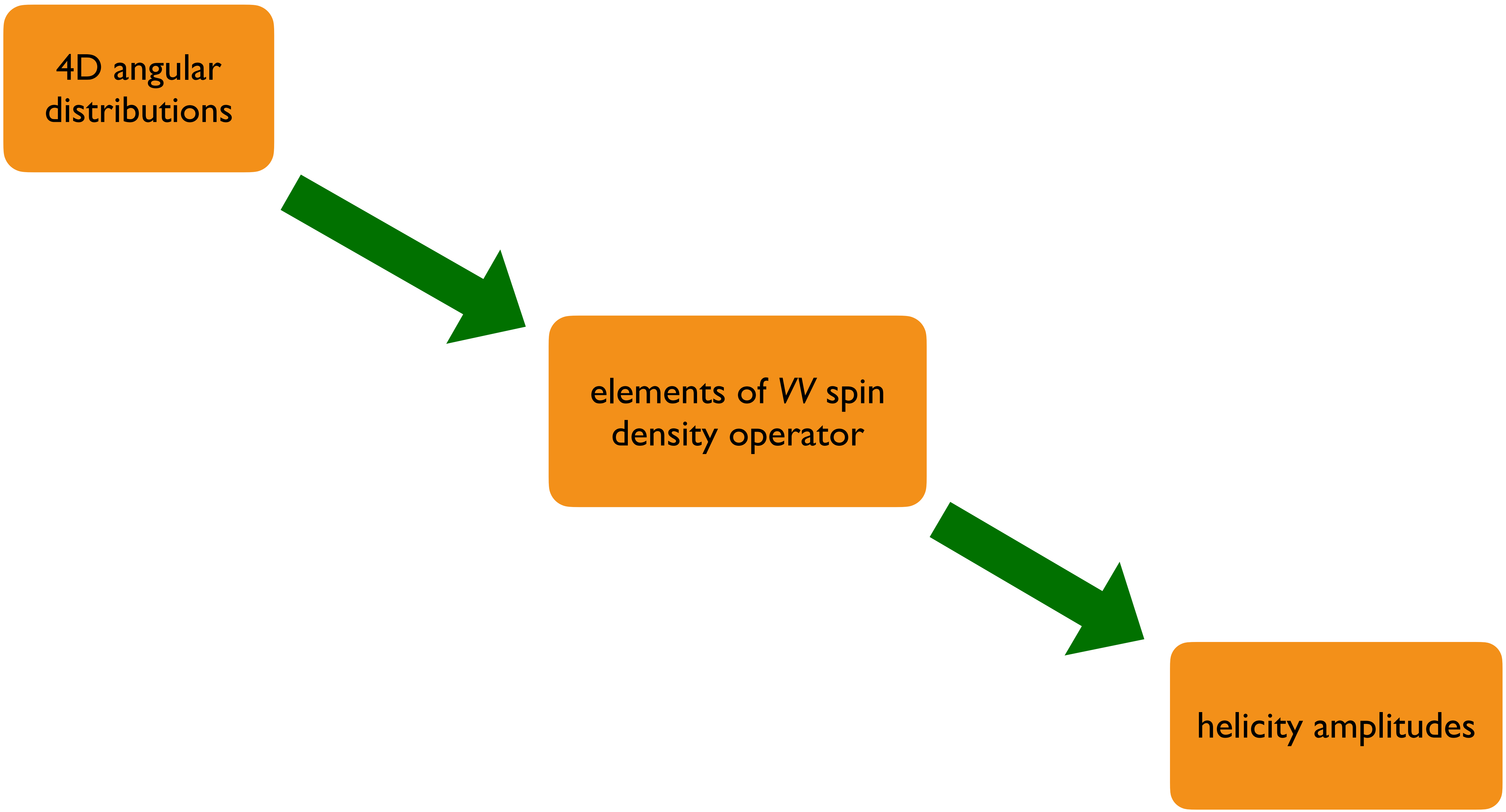
→ model-independent →

obtain ρ_{LS1S2} from data

Measuring helicity amplitudes

Measuring helicity amplitudes

To obtain helicity amplitudes a_{11}, a_{00}, a_{-1-1} we ignore OAM for a moment and work in the helicity basis [of course!]



Measuring helicity amplitudes

One can determine the amplitudes a_{11} , a_{00} , a_{-1-1} under two frameworks

Model independence is cool, but gives awful sensitivity.

On the other hand, CP-violating effects in $H \to VV$ are at the 10^{-5} level in the SM...

It may not be such a bad idea to assume CP conservation when testing entanglement

Measuring helicity amplitudes

If we assume CP conservation

- $a_{11} = a_{-1-1}$
- We have the condition $|a_{11}|^2 + |a_{00}|^2 + |a_{-1-1}|^2 = \mathcal{N}$ [say 1]
- there is only one independent parameter [relative sign with a_{00} fixed]

Then, the amplitudes a_{11}, a_{00}, a_{-1-1} can all be determined from binned measurements of $A_{20}^1 \equiv A_{20}^2$, where

$$B_2 = \sqrt{\frac{2\pi}{5}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\left[1 + A_{20}^1 B_2 Y_2^0(\theta_1, \phi_1) + A_{20}^2 B_2 Y_2^0(\theta_2, \phi_2) + \ldots\right]$$

Bipartite and tripartite entanglement

Bipartite and tripartite entanglement

Entanglement measurements involving OAM are rare — and never done in HEP!

HOME > SCIENCE > VOL. 361, NO. 6407 > QUANTUM ENTANGLEMENT OF THE SPIN AND ORBITAL ANGULAR MOMENTUM OF PHOTONS USING METAMATERIALS

REPORT

Quantum entanglement of the spin and orbital angular momentum of photons using metamaterials

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nature > letters > article

Letter | Published: 19 July 2001

Entanglement of the orbital angular momentum states of photons

Alois Mair, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Nature **412**, 313–316 (2001) | Cite this article

Bipartite and tripartite entanglement

There are several things one can address from ρ_{LS1S2}:

- Is tripartite entanglement genuine?
- Are *A*, *B* subsystems entangled when *C* is marginalised?

These can be addressed using Peres-Horodecki sufficient condition for entanglement, and using the entanglement measure

$$N(\rho) = \frac{\|\rho^{T_B}\| - 1}{2}$$

for *A*, *B* any subsystems of $\mathcal{H}_L \otimes \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}$

Bipartite and tripartite entanglement

Tripartite entanglement is genuine if the *VV* state is entangled under any bipartition of $\mathcal{H}_L \otimes \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}$

$N > 0 \Rightarrow$ entanglement

A	B	N(ρ)
$\mathcal{H}_L$	$\mathcal{H}_{S1} \otimes \mathcal{H}_{S2}$	0.757
$\mathcal{H}_{S1}$	$\mathcal{H}_L \otimes \mathcal{H}_{S2}$	0.998
$\mathcal{H}_{S2}$	$\mathcal{H}_L \otimes \mathcal{H}_{S1}$	0.998

- Entanglement is very large in all cases.
- Values close to unity have a nice explanation...
- These are theoretical values for *ZZ*; for *WW* they are quite close.

Bipartite and tripartite entanglement

Given the three subsystems *A*, *B*, *C*, we can marginalise *C* [trace over its space] and obtain the entanglement between *A* and *B*

A	B	N(ρ)
$\mathcal{H}_L$	$\mathcal{H}_{S1}$	0.105
$\mathcal{H}_L$	$\mathcal{H}_{S2}$	0.105
$\mathcal{H}_{S1}$	$\mathcal{H}_{S2}$	0.843

- *C* is the unlisted subsystem in all cases.
- These are theoretical values for *ZZ*; for *WW* they are quite close.

This is different from what is done in helicity basis, e.g. 2209.13441

Bipartite and tripartite entanglement

10^6 *ZH* events expected, vs

- 1.7 10^7 $gg \to H$ events in Run2+3
- 1.6 10^8 $gg \to H$ events at HL-LHC

	BR	Sensitive to L=1	Sensitive to L=1
$ZZ \to 4\ell$	1.24 10^{-4}	✔	✔
$ZZ \to 2\ell\, 2\tau$	1.17 10^{-4}	✔	✔
$ZZ \to 2\ell\, 2q$	2.44 10^{-3}	✔	✗
$WW \to \ell\nu\ell\nu$	1.05 10^{-2}	✔	✔
$WW \to \ell\nu qq$	3.13 10^{-2}	✔	✗

➡ *ZZ* does not seem competitive

Probing new interactions

Probing new interactions

I will now use this framework to probe new physics. The general *HVV* interactions are

$$\mathcal{L}_{HWW} = \frac{g^2 v}{2}\left[a\, g_{\mu\nu} + b\,\frac{p_\mu p_\nu}{M_W^2} + c\,\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{M_W^2}\right] W_\mu^+ W_\nu^- H$$

$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

$$\mathcal{L}_{HZZ} = \frac{g^2 v}{4c_W^2}\left[a'\, g_{\mu\nu} + b'\,\frac{p_\mu p_\nu}{M_Z^2} + c'\,\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{M_Z^2}\right] Z_\mu Z_\nu H$$

SM: $a=a'=1$

anomalous, CP-conserving

anomalous, CP-breaking

Probing new interactions

The amplitude a_{00} can be measured very precisely from A_{20}

$$A_{20}^1 = A_{20}^2 = \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} \frac{|a_{00}|^2}{\mathcal{N}}$$

At the kinematical limit $m_{V^*} = m_H - m_V$, the state is a spin singlet with

$$-a_{00} = a_{11} = a_{-1-1}$$

But departures from SM prediction are possible for lower m_{V^*}, and are testable.

- A_{20} probes the presence of derivative CP-conserving terms.
- Can use hadronic decays as well as leptonic ones.

... no estimates as yet ... probably equivalent to previous work ...

Probing new interactions

The polarisation A_{10} and spin correlation coefficients C_{1M2-M} are CP-violating

$$A^1_{10} = -A^2_{10} = \sqrt{\frac{3}{2}}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 - |a_{-1-1}|^2\right]$$

$$C_{1020} = -C_{2010} = \frac{\sqrt{3}}{2}\frac{1}{\mathcal{N}}\left[|a_{11}|^2 - |a_{-1-1}|^2\right]$$

$$C_{1-121} = -C_{2-111} = C^*_{112-1} = -C_{211-1} = \frac{3}{2}\frac{1}{\mathcal{N}}\left[a_{00}a^*_{11} - a_{-1-1}a^*_{00}\right]$$

- A_{10} and C_{1M2-M} probe derivative CP-violating terms.
- They can be measured in semileptonic decays too: select leptonic *V* for L=1 and hadronic *V* for L=2

... no estimates as yet ... probably equivalent to previous work ...

Ideas to keep in mind

- ☑ $H \to ZZ$ at e^+e^- colliders is probably not competitive with LHC and HL-LHC.
- ☑ Except for the possibility of post-decay entanglement in $ZZ \to 2\ell\, 2\tau$ [no time to mention what this is].
- ☑ $H \to WW \to \ell\nu\ell\nu$ is competitive, because at e^+e^- colliders the two neutrinos can be reconstructed.
- ☑ $H \to WW \to \ell\nu qq$ is competitive too, because of smaller background
- ☑ $H \to WW$ offers a better opportunity to probe CP-violating interactions because L=1 correlations are not suppressed.
- ☑ If you want to work on it, let me know!

End

Rescuing *L* from oblivion

We have been hearing about measuring spin of top, *W*, *Z*, even τ, for decades.

But probably, last time you heard about orbital angular momentum (OAM) was in your degree. Why is it so?

OAM cannot be directly measured from angular distributions!

Yet, it is there. Consider for example $H \rightarrow VV$, with $V = W,Z$

Initial state J = 0 → Final state J = 0

total spin of *VV* pair: 0,1,2 ⇒ $\ell = 0,1,2$

Rescuing *L* from oblivion

Let's introduce a reference system (x,y,z) in the *H* rest frame. Decay amplitudes using a fixed spin quantisation axis $\hat{z}$ [whatever] look like

$$A^c_{11} = [\cdots]\, Y_2^{-2}(\Omega)$$

$$A^c_{10} = [\cdots]\, Y_1^{-1}(\Omega) + [\cdots]\, Y_2^{-1}(\Omega)$$

$$A^c_{1-1} = [\cdots]\, Y_0^{0}(\Omega) + [\cdots]\, Y_1^{0}(\Omega) + [\cdots]\, Y_2^{0}(\Omega)$$

$$\ldots$$

s_1, s_2: 3rd spin components for V_1, V_2 in $\hat{z}$ axis

There are 9 amplitudes. c superscript stands for \`canonical´ as opposed to the commonly-used helicity amplitudes

with Ω=(θ,ϕ) the angles of [say] V_1 in *H* rest frame

Note:

- spherical harmonics up to $\ell = 2$
- $s_1 + s_2 + m = 0$

OAM is there! And there it is, just like it should be!

Rescuing *L* from oblivion

Hmm... ok... but how do the amplitudes get an angular dependence?

$$A^c_{11} = [\cdots]\, Y_2^{-2}(\Omega)$$

$$A^c_{10} = [\cdots]\, Y_1^{-1}(\Omega) + [\cdots]\, Y_2^{-1}(\Omega)$$

$$A^c_{1-1} = [\cdots]\, Y_0^{0}(\Omega) + [\cdots]\, Y_1^{0}(\Omega) + [\cdots]\, Y_2^{0}(\Omega)$$

$$\cdots$$

Setting a particular value for the third spin component along a particular direction $\hat{z}$ breaks isotropy in the Higgs decay.

This is in contrast with helicity amplitudes which are just numbers

$$A^h_{11} = a_{11}$$

$$A^h_{00} = a_{00}$$

$$A^h_{-1-1} = a_{-1-1}$$

Only 3 amplitudes

Rescuing *L* from oblivion

But wait… the Higgs is a scalar, and scalar decays are isotropic, how is this possible?

density operator for *L*

$\ell = 2$ …

$$\rho_L = \begin{pmatrix} 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.021 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.021 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.895 \end{pmatrix}$$

$\ell = 0$

$$\sum_{m=-2}^{m=2} |Y_m^2(\theta,\phi)|^2 = \frac{5}{4\pi}$$

→ all (θ,ϕ) dependence is lost

OAM cannot be directly measured from angular distributions!

[of course, you can calculate it, from initial state and decay amplitudes]

Quantum Entanglement: basics

Quantum entanglement: basics

The state of a system composed by two sub-systems A and B is separable if it can be written as

$$|\psi\rangle = |a\rangle_A \otimes |b\rangle_B$$

Otherwise, it is entangled, e.g. something like

$$|\psi\rangle = |a_1\rangle_A \otimes |b_1\rangle_B + |a_2\rangle_A \otimes |b_2\rangle_B$$

A typical example of entanglement is the combination of two spin-1/2 systems in the spin-0 configuration

$$|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B$$

General systems are not described by pure states $|\psi\rangle$ but by density operators ρ.

Quantum entanglement: basics

Any operator cannot be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

A density operator describing a composite system is separable if it can be written as

$$\rho_{\text{sep}} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Note: in general, one has something like

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\psi_i\rangle\langle\psi_j| \otimes |\psi_k\rangle\langle\psi_l|$$

Quantum entanglement: basics

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the partial transpose in subspace of B [for example] the resulting density operator is valid.

→ it has non-negative eigenvalues [unit trace and hermicity automatic]

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \quad \longrightarrow \quad \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

(n∗m) × (n∗m) matrix

Quantum entanglement: basics

To take away:

- It is quite complicated to prove [analytically] that a composite system is in a separable state.
- However, we are interested in showing that the system is entangled.
- To prove that, in some systems there are simple sufficient conditions that do the work
  - two spin-1/2 particles
  - $H \to VV$ [bipartite]
- Otherwise, use directly the counter-reciprocal of Peres-Horodecki necessary condition

$$\rho^{T2} \text{ non-positive} \Rightarrow \rho^{T2} \text{ not valid} \Rightarrow \text{system entangled}$$

Bell inequalities

Bell inequalities

Bell-like inequalities hold for classical systems. Their violation implies quantum mechanics.

In particular, the violation implies that the quantum system is not described by hidden variables.

Bell-like inequalities are based on measurements for two separate sub-systems A [Alice] and B [Bob]. Experiments usually performed measuring spins.

Bell inequalities

A useful formulation of Bell-like inequalities for spin-1/2 is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables A, A'. Bob measures two spin observables B, B'. [Both normalised to unity]. Then, clasically:

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \leq 2$$

these are spin correlation observables!

One can show violation of CHSH inequalities if one finds spin observables A, A' for Alice and B, B' for Bob such that the inequality is violated.

in a given quantum state!

Bell inequalities

For spin-1 systems there is an inequality that is stronger than CHSH. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B] CGLMP PRL '02

$$
\begin{aligned}
I_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\
& - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2
\end{aligned}
$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}} (| + -\rangle - |00\rangle + | - +\rangle)$$

and can be conveniently written in terms of a `Bell operator´

$$\mathcal{O}_{\text{Bell}} = \frac{4}{3\sqrt{3}}(T^1_1 \otimes T^1_{-1} + T^1_{-1} \otimes T^1_1) + \frac{2}{3}(T^2_2 \otimes T^2_{-2} + T^2_{-2} \otimes T^2_2)$$

However, this is not optimal for the mixed spin state of the *VV* pair resulting from *H* decay...

OAM strikes again! The *VV* pair is produced in a state of zero total angular momentum. But besides spin, there is OAM.

in helicity basis

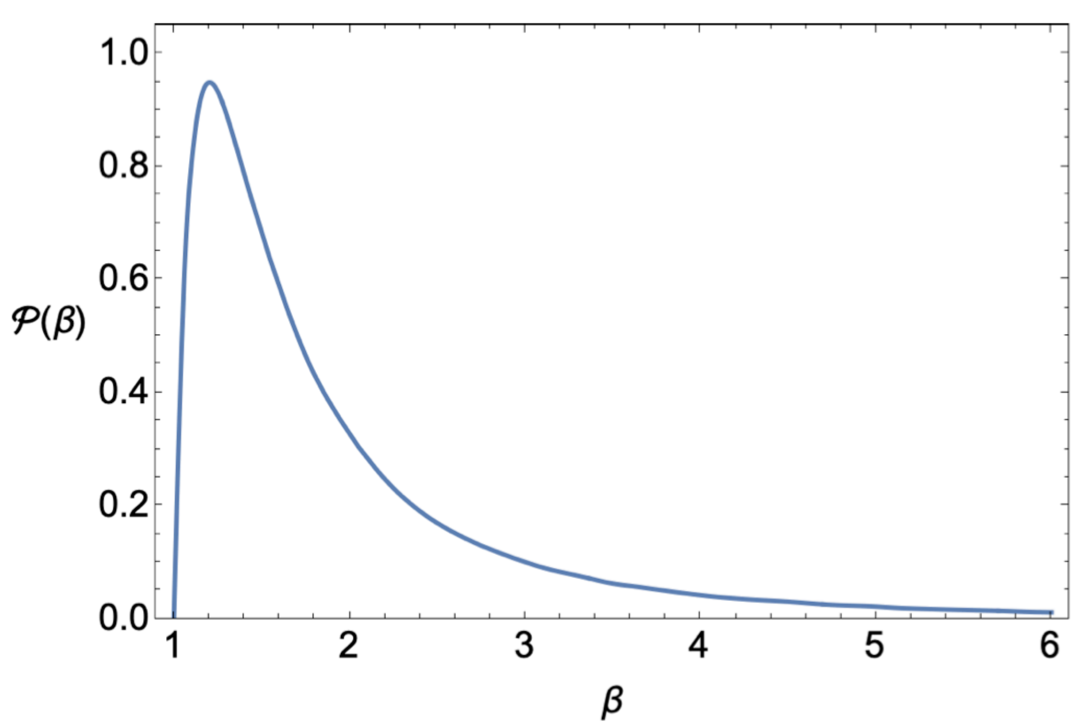
V at rest in *H* c.m. frame → L = 0 ; S = 0

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|+-\rangle - |00\rangle + |-+\rangle\right)$$

V not at rest but yet angular momentum conservation →

$$|\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$

p.d.f. for *H* → *ZZ*

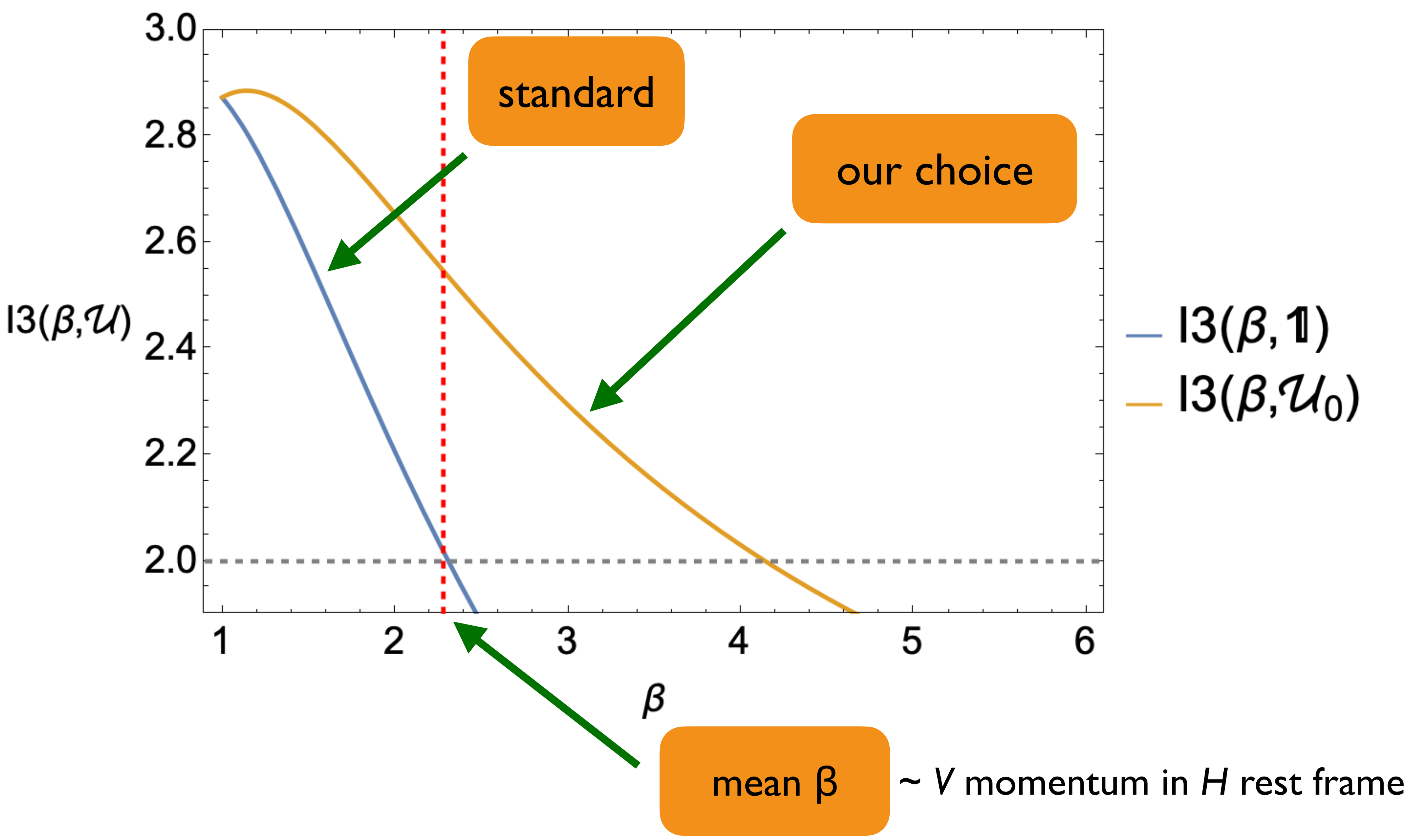


$$\rho = \int d\beta\ \mathcal{P}(\beta)|\psi_\beta\rangle\langle\psi_\beta|$$

mixed state

For $H \to VV$ a Bell operator that has larger expected value can be written

$$\mathcal{O}'_{\text{Bell}} = -\frac{4}{3\sqrt{3}} T^1_0 \otimes T^1_0 + \frac{1}{2} T^2_0 \otimes T^2_0 + \frac{2}{3\sqrt{3}} \left(T^1_1 \otimes T^1_{-1} + T^1_{-1} \otimes T^1_1 \right) - \frac{1}{3} \left(T^2_1 \otimes T^2_{-1} + T^2_{-1} \otimes T^2_1 \right) + \frac{1}{12} \left(T^2_2 \otimes T^2_{-2} + T^2_{-2} \otimes T^2_2 \right)$$

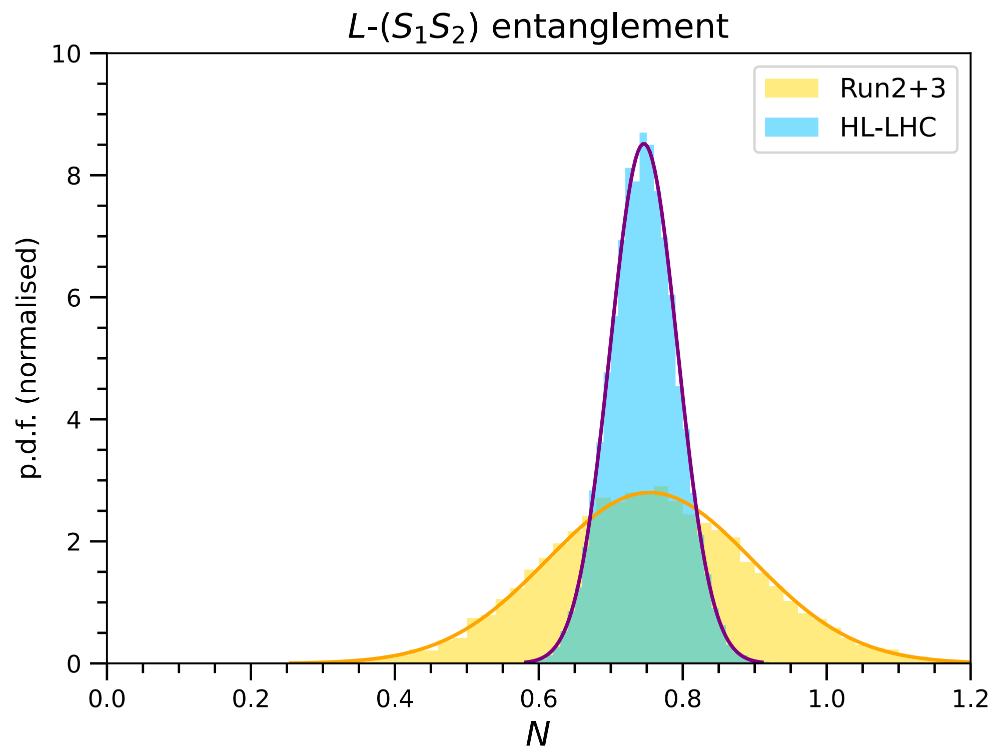


Expected statistical uncertainty

Expected statistical uncertainty in *ZZ*

Pseudo-experiments performed assuming 490 events

[xsec × BR × lumi × eff=0.25]

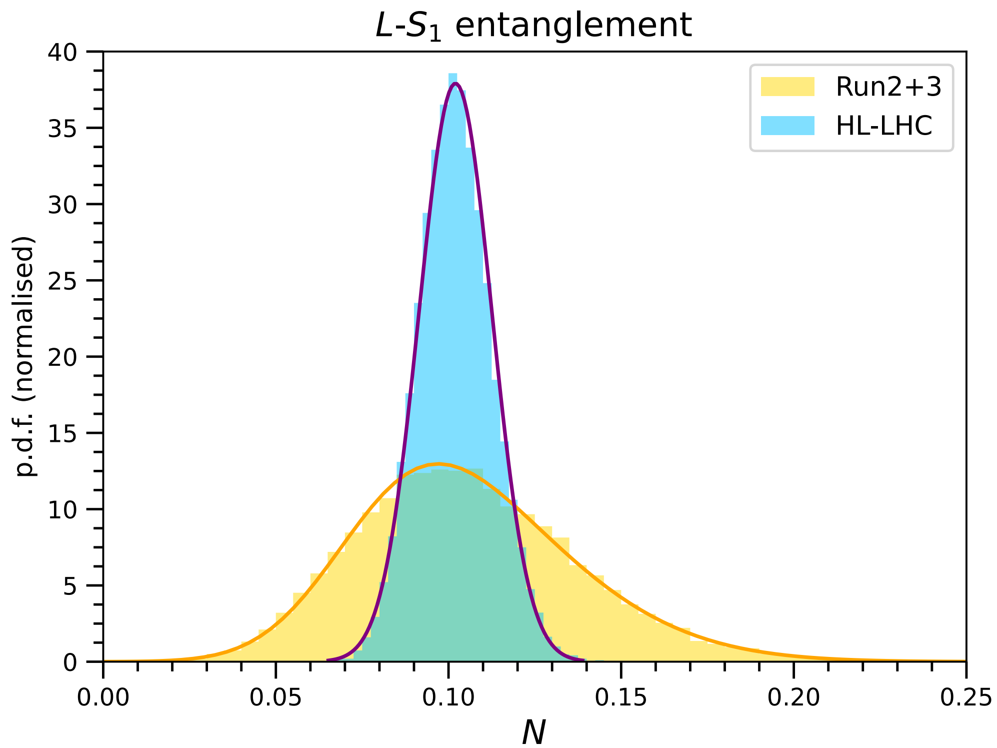


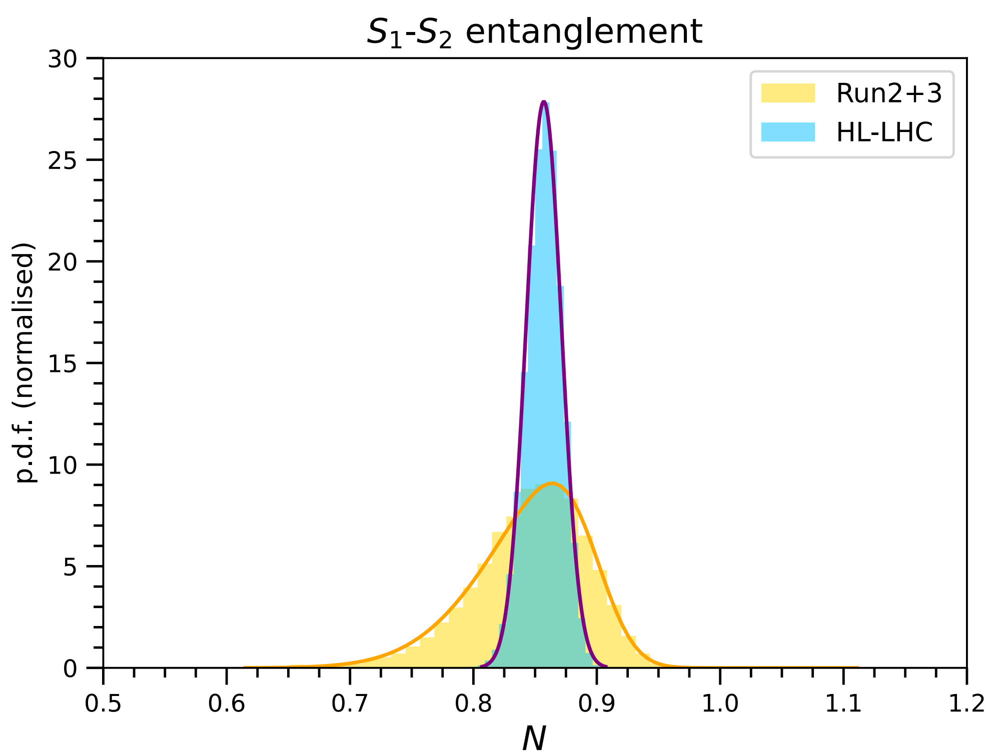
Tripartite entanglement

	Runs 2+3
L-(S_1S_2)	5.3σ
S_1-(LS_2)	≫5σ
S_2-(LS_1)	≫5σ

Likely, genuine tripartite entanglement can be established in Run 3

Expected statistical uncertainty in *ZZ*





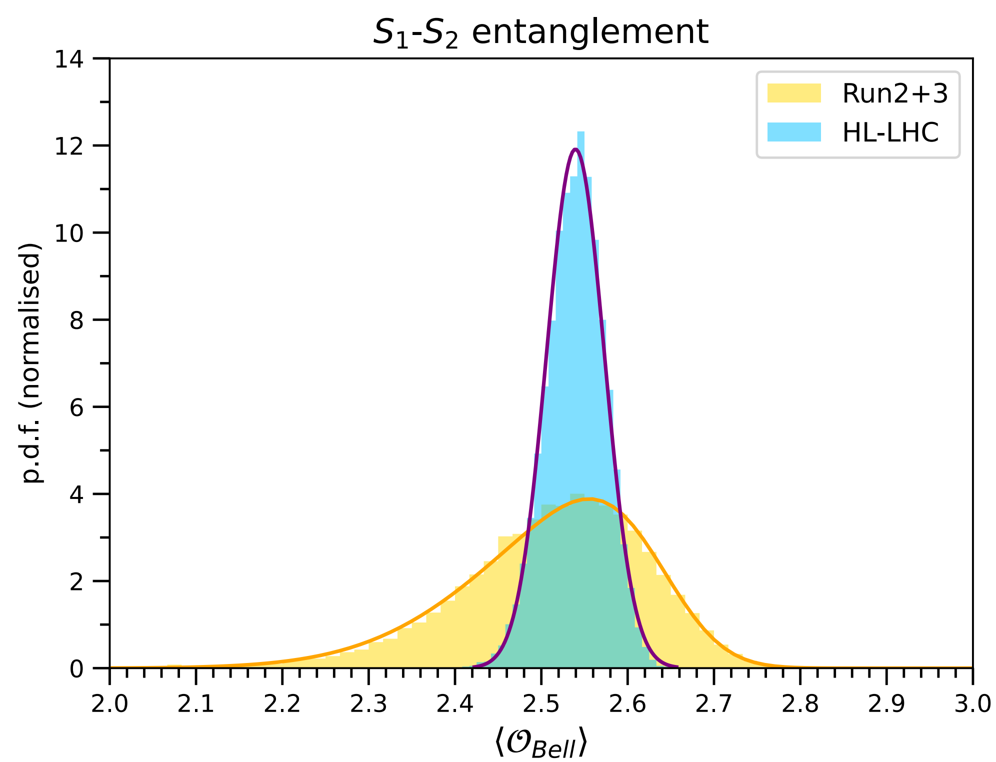
Bipartite entanglement

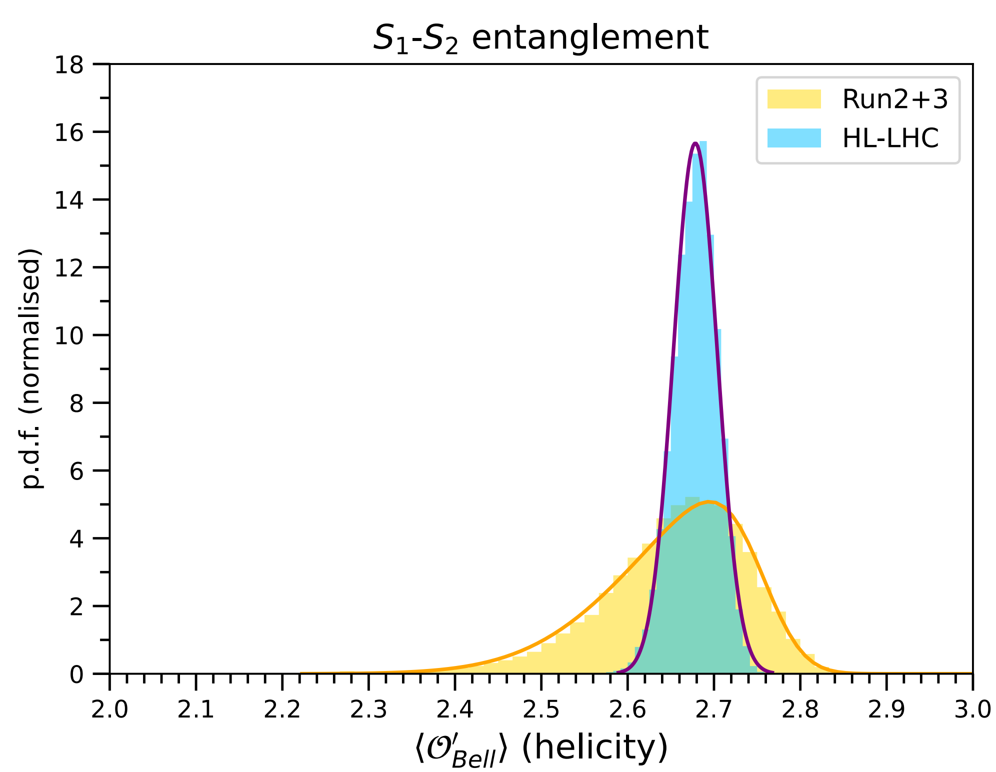
	Runs 2+3
L-S_1	4.3σ
L-S_2	4.3σ
S_1-S_2	≫5σ

The p.d.f. is not Gaussian for a small dataset but is well approximated by a skew-normal distribution.

Expected statistical uncertainty in *ZZ*

Also: Bell inequality violation





Bell inequality

	Runs 2+3
Canonical basis	3.8σ
Helicity basis	5.7σ

Bases are not equivalent. In both cases we integrate over decay angles in *H* rest frame, but in helicity basis the reference system moves too.