Asymptotic symmetries in the BRST formalism

Marc Henneaux
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Workshop:
Gauge invariance: quantization and geometry
Mons, 9 September 2024

Igor Anatoljewitsch Batalin (1945-2024)

The power of the antifield formalism

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This will be amply illustrated in the talks of this meeting and is extremely well summarized in the description of the meeting.

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"The principle of gauge invariance and the quantum paradigm are two cornerstones of modern fundamental physics and geometry. The intricate relation of the two underlies various challenges in the physics of fundamental interactions and modern developments in mathematics ranging from quantum gravity and topological effects in quantum physics to homological algebra, supergeometry and derived geometry. The by now classical works of Igor Batalin with E. Fradkin and G. Vilkovisky targeted the very compatibility of gauge invariance and quantisation, resulting in a very general mathematical formalism which by now is considered as a far reaching generalisation of the usual Lagrangian/Hamiltonian framework at both the classical and the quantum level. The applications of the approach of Batalin, Fradkin and Vilkovisky expanded far beyond the original scope of gauge theory quantisation and the approach itself is more and more considered as a proper language to define quantum gauge field theories, to look for new theories of fundamental interactions and to study geometrical structures."

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The presentation will rely on various ideas such as locality, equivalence of the Lagrangian and Hamiltonian BRST approaches, asymptotic conditions etc.

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End of the problem of deriving the correct Feynman rules... but starting point of many unanticipated developments given the remarkable structure that BV uncovered !

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(As we shall recall below, the two formalisms are equivalent and can be explicitly related.)

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of the extended master equation $(S', S') = 0$.

One can show that a solution is guaranteed to exist provided one includes all the rigid symmetries including the higher order ones corresponding to higher order conservation laws $\partial_\mu j^{\mu\mu_2\cdots\mu_k} = 0$.

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The derivation of the Ward identities proceeds then as usual.

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(These charges extend the phase space gauge-invariant charges $Q^{(0)}_\Delta$ by terms involving the ghosts and their conjugate momenta, $Q_\Delta = Q^{(0)}_\Delta + \mathscr{P}_a V^a_b \eta^b + \cdots$ so that $[Q_\Delta, \Omega] = 0$ holds. We denote the constraints by G_a and one has $[Q^{(0)}_\Delta, G_a] \approx 0$.)

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(which would not be the case had we imposed $\Omega'|\psi>=0$).

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How can one take these into account?

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$$S = \int d^D x \mathscr{L}(\phi^A, \phi^*_A, \partial_\mu \phi^A, \partial_\mu \phi^*_A \cdots, \partial^s_{\mu_1 \cdots \mu_s} \phi^A, \partial^s_{\mu_1 \cdots \mu_s} \phi^*_A) \text{ ???}$$

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(For instance, anomalies are BRST-exact if one allows non-local expressions but are non-exact in the space of local functionals.)

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[The asymptotic behaviour of the original fields is known from the "pre-BRST" theory.]

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Without loss of generality, one can analyse the problem in the Hamiltonian formulation.

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(where the integration over space is understood). Here, z^A are the canonical variables, λ^a the Lagrange multipliers for the constraints $G_a \approx 0$, B_A is the symplectic pre-potential and $H_0 = \int d^d x \mathcal{H}_0$ is the gauge invariant Hamiltonian.

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We recall that the momenta and the Lagrange multipliers can be viewed as auxiliary fields.

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The BRST charge reads

$$\Omega = G_a \eta^a + \cdots \qquad [\Omega, \Omega] = 0$$

while the BRST-invariant Hamiltonian is

$$H = H_0 + \cdots \qquad [H, \Omega] = 0$$

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$$S = \int dt \left(B_A \dot{z}^A + \lambda^*_a \dot{\eta}^a - H + z^*_a [z^A, \Omega] + \eta^*_a [\eta^a, \Omega] - [\Omega, \lambda^*_a] \lambda^a \right)$$

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A crucial feature is that the momentum $\mathscr{P}_a$ canonically conjugate to the ghost η^a in the Poisson bracket coincides with the antifield λ^*_a conjugate to the Lagrange multiplier λ^a in the antibracket.

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A crucial feature is that the momentum $\mathscr{P}_a$ canonically conjugate to the ghost η^a in the Poisson bracket coincides with the antifield λ^*_a conjugate to the Lagrange multiplier λ^a in the antibracket.

One can verify that S fulfills all the properties that the solution of the master equation should.

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A similar argument shows that surface terms are also absent in $[H, \Omega]$ under the same conditions.

This is fine if one considers only proper gauge symmetries, but we want to include also improper ones. In that case, the equations might fail to hold because of non-vanishing surface terms (the bulk terms remain of course ok).

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$$\epsilon^a = \epsilon^a_{proper} + \epsilon^a_{improper}$$

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and $\epsilon^a_{improper}$ is an improper gauge transformation determined from its non trivial asymptotic behaviour in some definite way, e.g., through gauge conditions that fix in a unique way the continuation " inside" of $\epsilon^a_{improper}$ from its asymptotic form ϵ^a_{∞}.

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One can similarly decompose the ghosts as

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where η^a_{proper} are the original ghosts and $\eta^a_{improper}$ can be viewed as “rigid ghosts".

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Marc Henneaux (Université Libre de Bruxelles and Collège de France)

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The quadratic terms in the ghosts are also automatically modified to take into account the algebra of the asymptotic symmetries.

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Since these coincide with the antifields conjugate to the Lagrangian multipliers, they are sources that can be assumed to go to zero at infiny, making equal to zero the surface integrals involving them.

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In the end, one gets an extended Ω and an extended H

that fulfill the necessary equations $[\Omega, \Omega] = 0$ and $[\Omega, H] = 0$

even if the ghosts are allowed to have a non trivial asymptotic behaviour at infinity.

Conclusions and comments

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THANK YOU !