In Memory of Igor Batalin

Green-Schwarz Superstring: BV

- Strings propagating in superspace $X^\mu(\sigma, \tau), \theta^a(\sigma, \tau)$
- Fermionic Siegel symmetry: infinitely reducible, on-shell algebra
- BV quantisation
- Infinite sequence of ghosts-for-ghosts

Green +CH

Kallosh et al

Siegel et al

Ghost Structure in BV

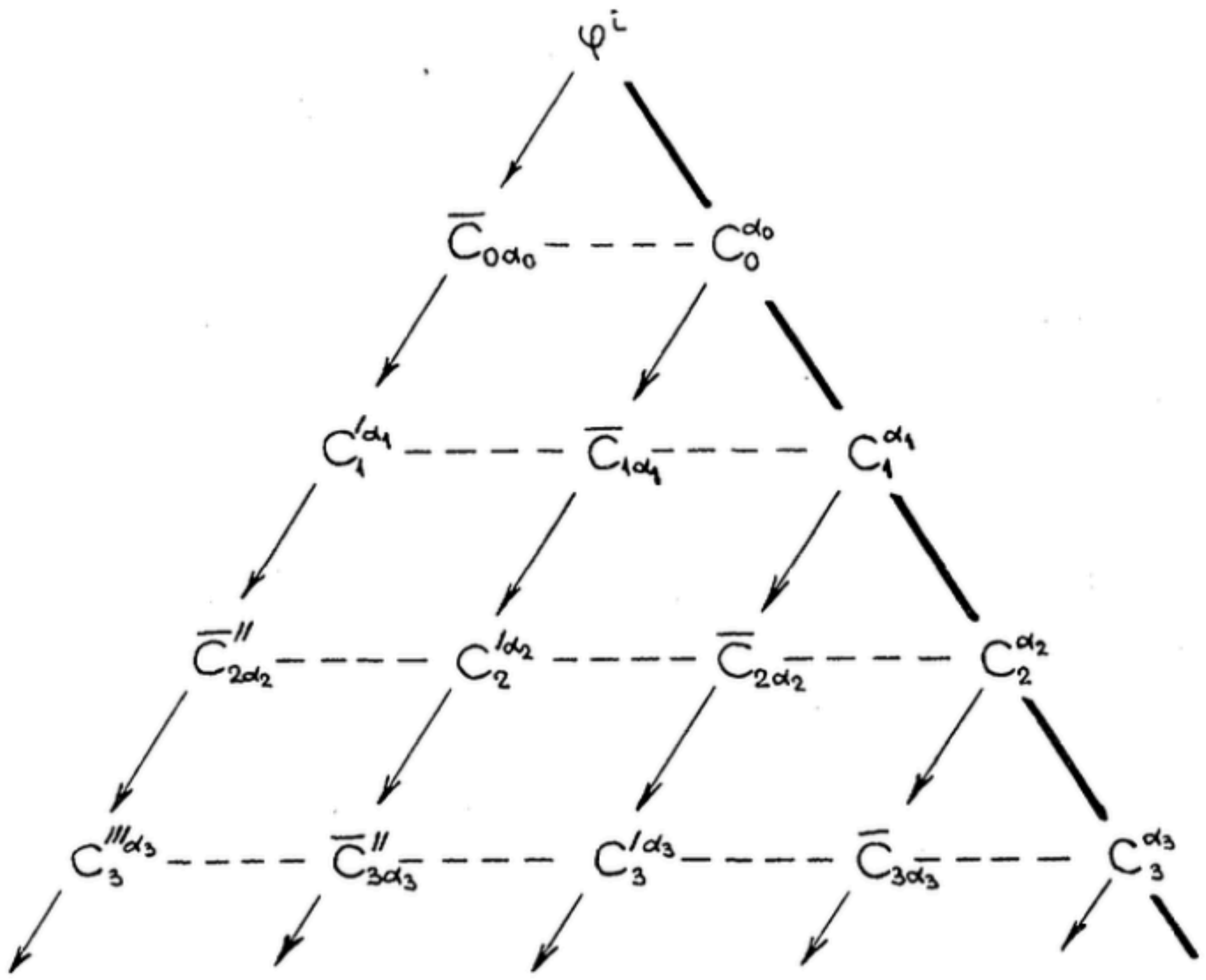


FIG. 3. The diagram of proliferation of ghosts. Each horizontal dashed line contains all ghosts which first arise at the given stage. The right bold line contains ghosts of the minimal (algebraic) sector. To each arrow there corresponds a nondegeneracy condition on the gauge fermion.

Level n+1:

Ghost C_n, anti-ghost $\bar{C}_n$,

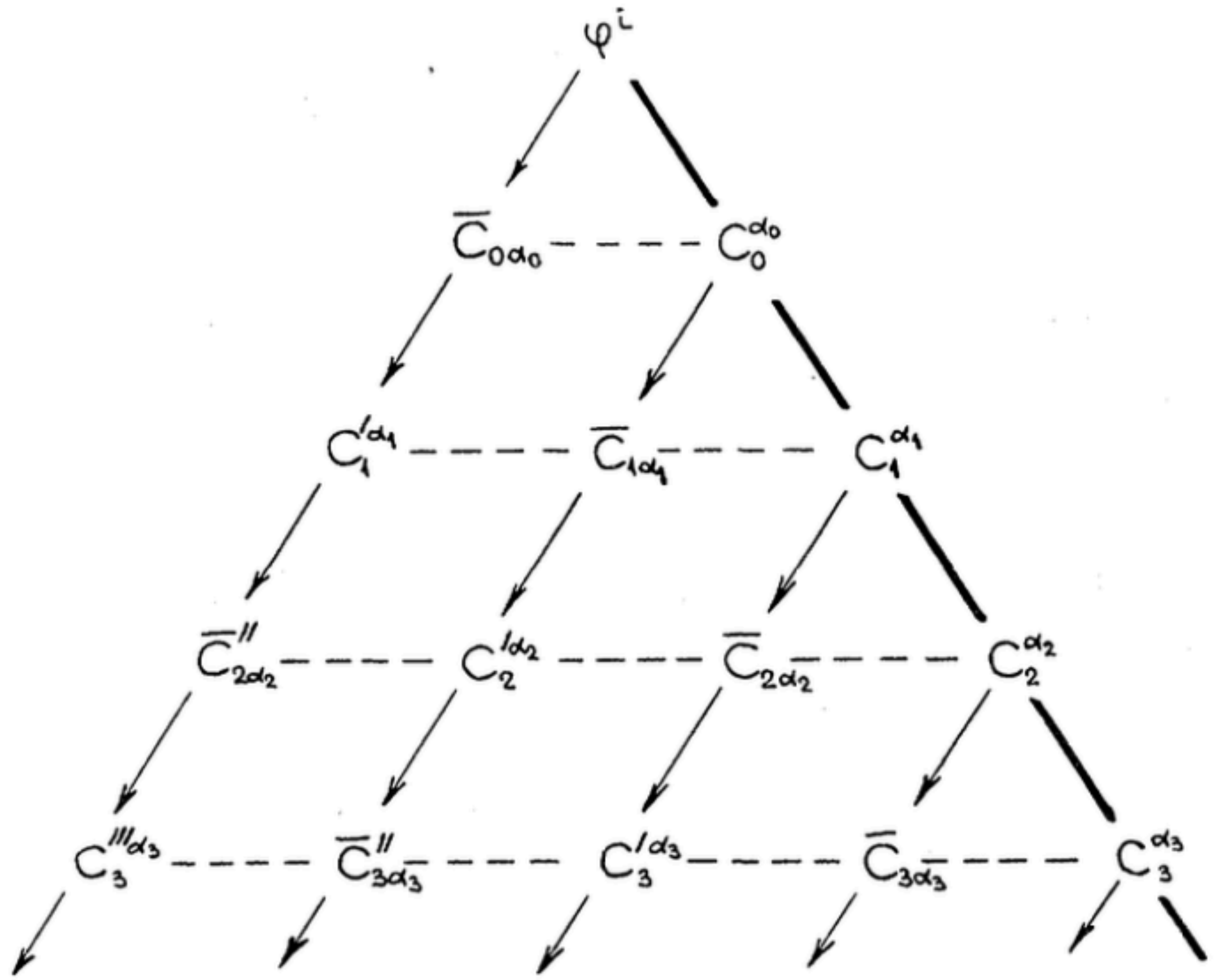
n extra ghosts $C'_n, C''_n, \dots$

Anti-BRST

In special gauges, there is an anti-BRST symmetry $\tilde{Q}$ as well as BRST Q

Sp(2) symmetry $Q^\alpha = (Q, \tilde{Q})$

Ghosts at level n: Sp(2) symmetric tensor $C^{\alpha_1 \alpha_2 \ldots \alpha_n}$



C_0^{α}

$C_1^{\alpha\beta}$

$C_2^{\alpha\beta\gamma}$

$C_3^{\alpha\beta\gamma\delta}$

Generalisation of BV

"The BRST and Anti-BRST Invariant Quantization of General Gauge Theories"

CH, 1990

"An Sp(2) covariant version of generalized canonical quantization of dynamical systems with linearly dependent constraints"

Batalin, Lavrov, Tyutin, 1990

Later papers

Batalin, Lavrov, Tyutin, Marnelius, Semikhatov,…

SO(n,1)xSp(2)→OSp(n,1|2)

p-form gauge fields + ghosts, anti-ghosts, extra ghosts

p=3 $A_{mnp}, C_{mn\alpha}, C_{m\alpha\beta}, C_{\alpha\beta\gamma} \quad \rightarrow \quad A_{MNP}$

$M = (m, \alpha)$ Vector of OSp(n,1|2)

Superstring $X^M = (X^m, c^\alpha)$ Vector of OSp(9,1|2)

$\Theta^A = (\theta^a, \kappa^a_\alpha, \kappa^a_{\alpha\beta}, \ldots)$ Spinor/metaplectic of OSp(9,1|2)

Green +CH

Monopoles, Dirac Strings and Generalised Symmetries

Magnetic Monopoles

- Gauge group $\to U(1)$. 't Hooft-Polyakov monopole
- Effective field theory: Maxwell theory with Dirac monopoles
- Dirac (1931): monopoles $\implies$ quantisation of charge
- Dirac (1948): electric charges + magnetic monopoles + EM field. Action + Quantum theory
- Strange non-locality. Dirac string attached to each monopole

- Dirac veto: position of strings arbitrary, so long as they do not intersect worldlines of electrically charged particles

- Veto $\Longrightarrow$ field equations independent of positions of strings

- Dirac quantisation condition $\Longrightarrow$ quantum theory well-defined and independent of positions of strings

- Singularities on Dirac strings

- Generalisation to p-form gauge fields coupling to branes

Deser, Gomboroff, Henneaux, Teitelboim

Wu-Yang Monopoles

- Heavy monopole: treat as (semi-) classical background
- Quantise electrons and EM field in monopole background
- Remove location of monopole from spacetime, leaving space of non-trivial topology. EM field : U(1) bundle over this
- Gauge connection A_α in each patch U_α, no Dirac strings
- Magnetic charge is topological

Light monopoles

- If monopoles are light and dynamical, Wu-Yang approach problematic
- Quantum monopole doesn’t have definite location. Instead there is a wave function
- No clear choice of which points to exclude from spacetime for Wu-Yang bundle
- In path integral, sum over all monopole trajectories. Different bundle for each trajectory?

N=4 Super Yang-Mills

- Gauge symmetry broken to U(1). Gauge coupling g, Higgs VEV v
- W-bosons: $m_W \sim v$. Monopoles: $m_M \sim v/g^2$
- If $g \sim 1$, then $m_W \sim m_M$ and if v small, both are light
- Dyons also light. All massless as $v \to 0$
- Light W-bosons, monopoles and dyons, all dynamical quantum particles

Dirac Approach Revisited

- Quantises system of dynamical electrons and monopoles plus EM field
- Dirac: field equations independent of positions of strings
- Independence of actions and path integral on positions of strings: generalised symmetries
- These generalised symmetries are anomalous. Restriction to configurations in which anomaly is zero = Dirac veto

Maxwell with Sources

$$d^\dagger F = j \qquad\qquad dF = *\tilde{j}$$

Electric 1-form current j, Magnetic 1-form current $\tilde{j}$

Conserved: $d^\dagger j = 0$ $\qquad d^\dagger \tilde{j} = 0$

Maxwell with Sources

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Dirac: introduce 2-form $\tilde{J}$ with

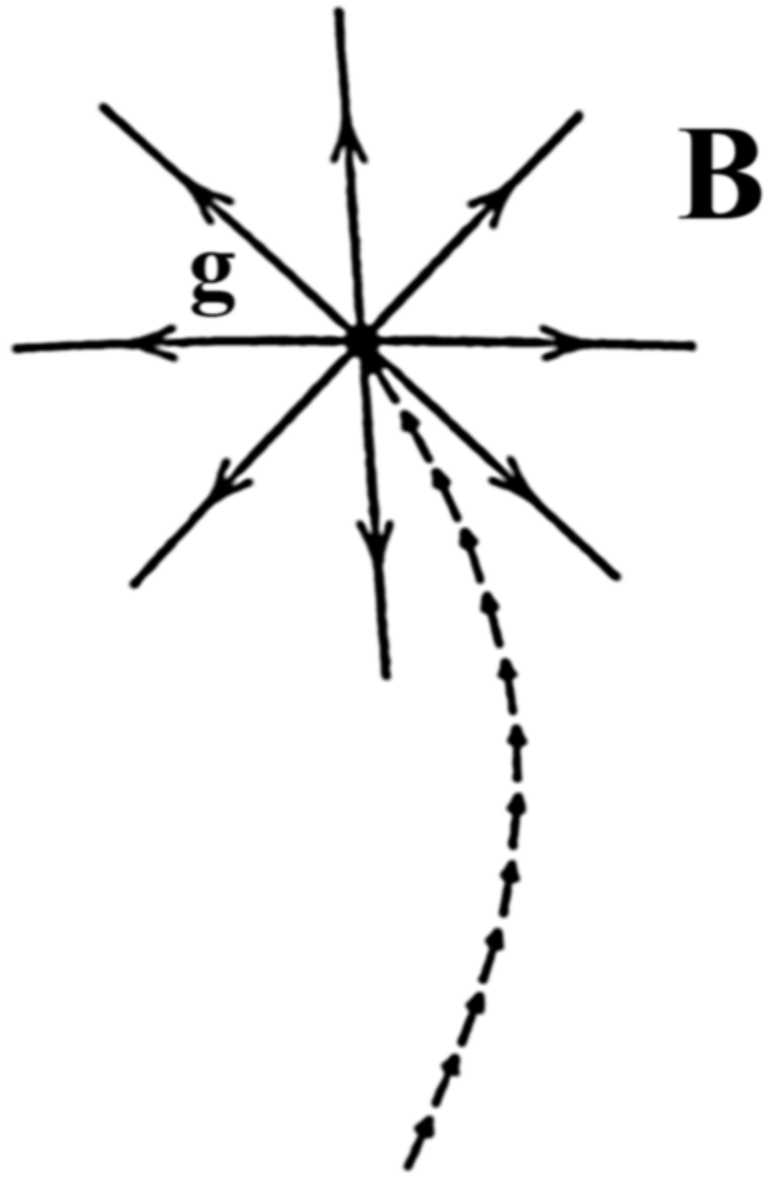
$$d^\dagger \tilde{J} = \tilde{j}$$

Dirac string is attached to each magnetic monopole, going to infinity.

2-form $\tilde{J}$ is the current density for these strings:

if $\tilde{j}$ localised on the world-line of a magnetic monopole,

then $\tilde{J}$ is localised on the world-sheet of the corresponding Dirac string.



The Dirac Formalism

$$dF = *\tilde{j} \qquad\qquad d^\dagger \tilde{J} = \tilde{j}$$

$$\implies \qquad d(F - *\tilde{J}) = 0$$

$$F = *\tilde{J} + dA$$

1-form potential A

$$S[A] = S_{particles} + \int \frac{1}{2} F \wedge *F - A \wedge *j$$

$$S_{particles} = \sum (mass) \times (length\ of\ worldline)$$

Also: Dirac strings for electric charges, 2-form current J

$$d^\dagger J = j$$

$$-\int A \wedge *j = \int F \wedge *J$$

$$S[A] = S_{particles} + \int \frac{1}{2} F \wedge *F + F \wedge *J$$

Dual formulation

Solve $$d^\dagger F = j$$

$$* F = * J + d\tilde{A}$$

Dual potential $\tilde{A}$ $$\tilde{F} = * F$$

$$S[\tilde{A}] = \int \frac{1}{2} \tilde{F} \wedge *\tilde{F} - \tilde{A} \wedge *\tilde{j}$$

p-form delta-function

For p-dimensional submanifold $\mathscr{N} \subset \mathscr{M}$ define **p-form delta-function:** $\delta_{\mathscr{N}}$

For any p form ω on $\mathscr{M}$

$$\int_{\mathscr{N}} \omega = \int_{\mathscr{M}} \omega \wedge {}^{*}\delta_{\mathscr{N}}$$

If $\mathscr{N} \subset \mathscr{M}$ specified by $x^{\mu} = X^{\mu}(\sigma^{a})$

$$\delta_{\mathscr{N}}(x) = \int d^{p}\sigma \quad \varepsilon^{a_1 a_2 \ldots a_p} \frac{\partial X^{\mu_1}}{\partial \sigma^{a_1}} \frac{\partial X^{\mu_2}}{\partial \sigma^{a_2}} \ldots \frac{\partial X^{\mu_p}}{\partial \sigma^{a_p}} \delta(x - X(\sigma))$$

For submanifolds with boundary:

$$\delta_{\partial\mathscr{N}} = d^{\dagger}\delta_{\mathscr{N}}$$

Currents

Particle of electric charge q, world-line is a curve $\mathscr{C}$, $x^\mu = X^\mu(\tau)$

$$q\int_{\mathscr{C}} A = q\int d\tau\, A_\mu(X(\tau))\frac{dX^\mu}{d\tau} = \int_{\mathscr{M}} A \wedge *j$$

$$j^\mu(x) = q\int d\tau \frac{dX^\mu}{d\tau}\delta(x - X(\tau))$$

$$j = q\delta_{\mathscr{C}}$$

$d^\dagger J = j$ $J = q\delta_{\mathscr{D}}$ $\mathscr{D}$ is a 2-surface with boundary $\mathscr{C}$

Wilson Loops

$\mathscr{C}$ a closed curve bounding 2-surface $\mathscr{D}$

$$q\int_{\mathscr{C}} A = q\int_{\mathscr{D}} F \qquad \Longrightarrow$$

$$\int_{\mathscr{M}} A \wedge *j = \int_{\mathscr{M}} F \wedge *J \qquad J = q\delta_{\mathscr{D}} \qquad d^{\dagger}J = j$$

For two surfaces $\mathscr{D}, \mathscr{D}'$ with same boundary $\mathscr{C}$

$$q\int_{\mathscr{D}'} F - q\int_{\mathscr{D}} F = q\int_{\mathscr{D}'-\mathscr{D}} F = 2\pi pq$$

Then $\qquad W(\mathscr{C}) = e^{\frac{i}{\hbar}q\int_{\mathscr{D}} F}$

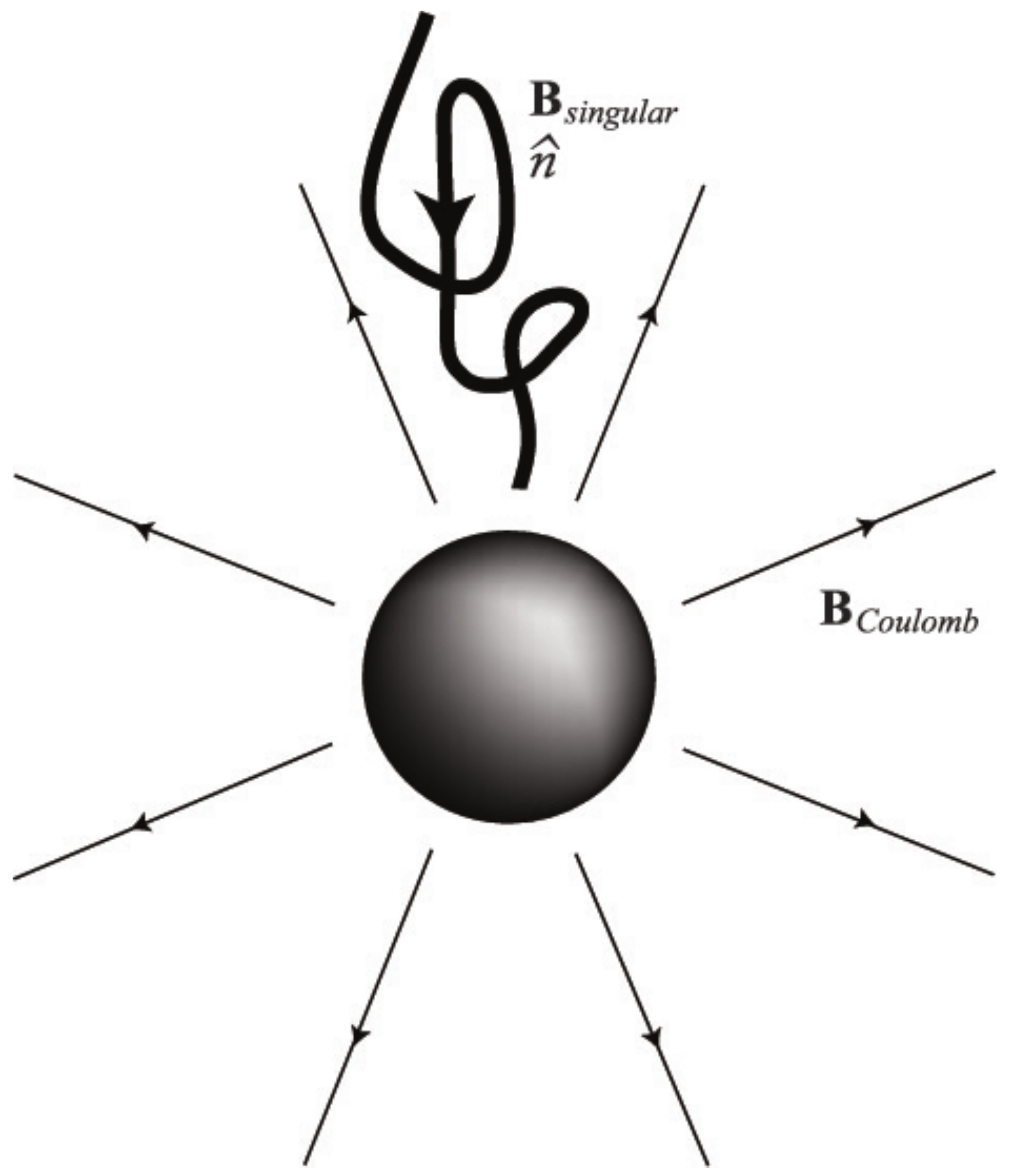
well-defined if $\qquad pq = 2\pi n, \quad n \in \mathbb{Z}$ $\qquad$ Dirac quantisation

Strings

For each τ: Dirac string from the particle to infinity

specified by $Y^{\mu}(\tau, \sigma)$ with $Y^{\mu}(\tau, 0) = X^{\mu}(\tau)$

String world-sheet $\mathscr{D}$ specified by $x^{\mu} = Y^{\mu}(\tau, \sigma)$



N particles, $i = 1, \ldots, N$

Masses m_i, electric charges q_i and magnetic charges p_i moving

Worldlines $\mathscr{C}_i$ given by $x^\mu = X_i^\mu(\tau_i)$

$$j(x) = \sum_{i=1}^{N} q_i \delta_{\mathscr{C}_i}(x), \quad \tilde{j}(x) = \sum_{i=1}^{N} p_i \delta_{\mathscr{C}_i}(x) \qquad \tilde{J} = \sum_i p_i \delta_{\mathscr{D}_i}$$

$$S_{particles} = \sum_i m_i \int d\tau_i \sqrt{\dot{X}_i^2}$$

Field equations

$$m_i \frac{d^2 X_i^\mu}{d\tau_i^2} = (q_i F^{\mu\nu} + p_i * F^{\mu\nu}) \eta_{\nu\rho} \frac{dX_i^\rho}{d\tau_i} \qquad d^\dagger F = j$$

- Veto $\Longrightarrow$ field equations independent of positions of strings

$d^\dagger J = j$ $\quad$ $d^\dagger \tilde{J} = \tilde{j}$ $\quad$ Don't determine $J, \tilde{J}$ completely

$\delta J = d^\dagger \rho$ $\quad$ $\delta \tilde{J} = d^\dagger \tilde{\rho}$ $\quad$ SYMMETRY

$F = *\tilde{J} + dA$ $\quad$ $*F = *J + d\tilde{A}$ $\quad$ invariant if

$\delta A = *\tilde{\rho}$ $\quad$ $\delta \tilde{A} = *\rho$

3-form symmetries

Interpretation of Symmetry

Smooth deformation of Dirac string world-sheet $\mathscr{D}$ to $\mathscr{D}'$:

Family of Dirac string world-sheets $\mathscr{D}(\xi)$ parameterised by ξ with

$\mathscr{D}(0) = \mathscr{D}$ and $\mathscr{D}(1) = \mathscr{D}'$

Family of 2-dimensional world-sheets sweeps out a 3-dimensional surface $\mathscr{E}$

$$\tilde{J}' - \tilde{J} = p d^\dagger \delta_{\mathscr{E}}$$

Change in $\tilde{J}$ for infinitesimal deformation of $\mathscr{D}$, of the form $\delta\tilde{J} = d^\dagger\tilde{\rho}$ where $\tilde{\rho}$ is a 3-form current localised on $\mathscr{E}$

Generalised Symmetries in Maxwell Theory

Gaiotto, Kapustin, Seiberg, Willett

$$dF = 0 \qquad\qquad d*F = 0$$

$F, *F$ conserved 2-form currents

$$F = dA$$

0-form gauge symmetry $\delta A = d\sigma$

1-form symmetry $\delta A = \lambda$ $\qquad d\lambda = 0$

Constrained parameter: regard as global symmetry

1-form symmetries modulo 0-form symmetries: Cohomology class $[\lambda]$

Gauging 1-form Symmetry

Seek theory with symmetry for *unconstrained* λ

Introduce 2-form gauge field B

$$\delta B = d\lambda, \quad \delta A = \lambda$$

$$F = dA - B$$ invariant

$$S = \int \frac{1}{2} F \wedge *F$$

Dual Symmetry

$$* F = d\tilde{A}$$

$$\delta\tilde{A} = \tilde{\lambda}, \quad \delta\tilde{B} = d\tilde{\lambda}$$

$$* F = d\tilde{A} - \tilde{B}$$

Dual Gauge Symmetry in Original Theory

$$F = dA$$

Couple $\tilde{B}$ to Noether current F

$$S = \frac{1}{2}\int F \wedge *F + \tilde{B} \wedge F$$

Invariant under $\delta\tilde{B} = d\tilde{\lambda}$

Anomaly

Obstruction to gauging both symmetries

$$S = \int \frac{1}{2} F \wedge {*F} + \tilde{B} \wedge dA \qquad F = dA - B$$

Invariant under $\delta \tilde{B} = d\tilde{\lambda}$

Under $\delta B = d\lambda, \quad \delta A = \lambda$ $\qquad \delta S = \int d\lambda \wedge \tilde{B}$

Alternative action $\qquad S = \int \frac{1}{2} F \wedge {*F} + \tilde{B} \wedge F$

Invariant under $\delta B = d\lambda, \quad \delta A = \lambda$, but $\qquad \delta S = -\int d\tilde{\lambda} \wedge B$

Obstruction to simultaneous gauge of $\lambda, \tilde{\lambda}$ 1-form symmetries:

“Mixed Gauge Anomaly”

Ungauged theory has “mixed ’t Hooft anomaly” in $\lambda, \tilde{\lambda}$ 1-form symmetries

Gaiotto, Kapustin, Seiberg, Willett

BV BRST Structure

For gauged theory

$$QB_2 = dC_1$$

$$QC_1 = dC_0$$

$$QA_1 = C_1 + dc_0$$

$$Qc_0 = C_0$$

For ungauged theory

$$B_2 = 0$$

Global symmetry, so constrained ghosts

$$dC_1 = 0 \qquad QC_1 = dC_0 \quad \Longrightarrow \quad C_1 \sim C_1 + d\alpha$$

Cohomology class

$$QA_1 = C_1 + dc_0$$

Dirac Action

Write $\rho = *\tilde{\lambda}, \;\; \tilde{\rho} = *\lambda$ $\qquad B = -*\tilde{J}, \;\; \tilde{B} = -*J$

$$F = *\tilde{J} + dA \qquad \rightarrow \qquad F = dA - B$$

Dirac action becomes gauged Maxwell theory, with Dirac string currents $J, \tilde{J}$ reinterpreted as gauge fields $\tilde{B}, B$. 1-form symmetries with 1-form parameters $\lambda, \tilde{\lambda}$

The anomaly and the veto

Under gauge transformation

$$\delta S = \int_{\mathscr{M}} \lambda \wedge *j = \sum_i q_i \int_{\mathscr{M}} \lambda \wedge *\delta_{\mathscr{C}_i}$$

For a λ arising from change of $\tilde{J} = \sum_i p_i \delta_{\mathscr{D}_i}$ that results from moving the Dirac strings

$$\lambda = \sum_i p_i * \delta_{\mathscr{E}_i},$$

$$\delta S = \sum_{i,j} q_i p_j \int_{\mathscr{M}} \delta_{\mathscr{E}_j} \wedge \delta_{\mathscr{C}_i}$$

$$\delta S = \sum_{i,j} q_i p_j \int_{\mathscr{M}} \delta_{\mathscr{E}_j} \wedge \delta_{\mathscr{C}_i}$$

Family of Dirac string world-sheets $\mathscr{D}_j(\xi)$ parameterised by ξ sweeps out a 3-dimensional surface $\mathscr{E}_j$

Veto: None of these must cross any of the electric worldlines $\mathscr{C}_i$

For $i \neq j$ Dirac veto $\Longrightarrow \delta_{\mathscr{E}_j} \wedge \delta_{\mathscr{C}_i} = 0$

For $i = j$, $\mathscr{C}_i$ is tangent to $\mathscr{E}_i \Longrightarrow \delta_{\mathscr{E}_j} \wedge \delta_{\mathscr{C}_i} = 0$

Dirac Veto $\Longrightarrow \delta S = 0$

Dirac Veto: restricts to configurations of background gauge fields $B, \tilde{B}$ for which ano
vanishes

Conclusion

- Dirac's action has generalised 1-form symmetries
- Veto implies anomaly vanishes
- Treats case in which electric and magnetic particles both light and dynamical
- Generalises to p-form gauge fields coupling to branes: higher form symmetries
- Field theory for electric and magnetic particles? [Zwanziger, Schwinger]