# Gauge fixing in coset spaces and supergravities


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Mons, meeting Gauge invariance: quantization and geometry

**A workshop in memory of Igor Batalin**

mostly in collaboration with Renata Kallosh

# Igor Batalin

- April 1987, on a bus in Trieste: discussion with Renata Kallosh, who then organizes my participation to a “Quantum Gravity seminar” in Moscow, end of May 1987.

- At Renata’s place: Igor Batalin for a further discussion on ``Symmetries of superparticle and superstring action“

- (we had no smartphones yet: no picture)

# What I learned from Igor Batalin

- The famous formalism to look at symmetries and quantization.
- We continued in the following years with this formalism.
  We had chosen Lagrangian Batalin-Vilkovisky formulation
  In Brussels: Hamiltonian Batalin-Vilkovisky formulation

- Further work on that with
  Renata Kallosh, Walter Troost, Peter van Nieuwenhuizen, ….

- What I learned in person from him that evening:
  "*In order to be able to drink a lot, you have to eat a lot*".

# Quantum theory

$$Z(J,\Phi^*) = \int \mathcal{D}\Phi \ \exp \tfrac{\mathrm{i}}{\hbar}\left[W(\Phi,\Phi^*) + J(\Phi)\right]$$ with $$W = S + \hbar M_1 + \hbar^2 M_2 + \dots\ ,$$

- Which is the measure ? related to regularization procedure.
- changing measure ↔ changing regularization ↔ changing $M_i$
- one method: Pauli-Villars (PV): field and PV partner of opposite statistics enter simultaneously in the measure, which is then invariant for symmetries apart from the mass terms of PV fields.
  The choice of regularization is the choice of the mass term of PV fields.

# Anomalies

$$\langle \mathcal{A} \rangle = \frac{1}{Z(J,\Phi^*)} \int \mathcal{D}\Phi \ \mathcal{A}(\Phi,\Phi^*) \ \exp \frac{\mathrm{i}}{\hbar}(W + J\Phi)$$

$$\begin{aligned} \mathcal{A}(\Phi,\Phi^*) &= \Delta W + \frac{\mathrm{i}}{2\hbar}(W,W) \\ &= \frac{\mathrm{i}}{2\hbar}(S,S) + \left[\Delta S + \mathrm{i}(S,M_1)\right] + \dots\ . \end{aligned}$$

$$\Delta = \frac{\partial}{\partial \Phi^A} \frac{\partial}{\partial \Phi^*_A}$$

W. Troost, P. van Nieuwenhuizen and AVP, 1990

# This talk further

1. Finiteness calculations and counterterms of maximal supergravity
2. Repetition on first construction of N=8 in D=4 from D=11
3. Symmetries of N=8 and a remark of E. Cremmer and B. Julia
4. Cosets with different gauge fixings
5. Duality: as proper symmetry and enhanced duality
6. Various supergravities in D=4 and D=6

Conclusions

# 1. Finiteness calculations and counterterms of maximal sugra

$$S_{cr} = \kappa^{2\,(L_{cr}-1)} \int d^{4\mathcal{N}} d^D x \det E\, \mathcal{L}(x,\theta)$$

G/H coset space supergravities

- $L_{cr}$ critical loop order where the first counterterm exist that is global G and local H invariant
- (for lower loop, e.g. L=7 in N=8, we have to integrate over subspaces, breaking the H symmetry)
- e.g. D=4: $L_{cr} = \mathcal{N}$ ; D=5 $L_{\rm cr} = 6$
- But divergences are found below this order: e.g. L=5 at D=5

Z. Bern, J.J.M. Carrasco, M. Chiodaroli, H. Johansson, R. Roiban, 2304.07392

All D>4 UV divergences are at loop order below critical! ⟹ Local H-symmetry and G-symmetry must have anomalies!

D=4 $\mathcal{N}$ **> 4** no UV divergences so far

Why not ? Enhanced dualities ?

R. Kallosh, 2304.13926; 2312.06794; 2402.03453; 2405.20275.

# 2. D=11 to D=4, N=8

- Step by step dimensional reduction: i= 1,…7 times: $x^M = \{x^\mu, y\}$

$$G_{MN} = \mathrm{e}^{2\varphi}\begin{pmatrix} g_{\mu\nu} + \mathcal{A}_\mu \mathcal{A}_\nu & \mathcal{A}_\mu \\ \mathcal{A}_\nu & 1 \end{pmatrix}$$

7 times: defining for $i = 1, \ldots, 7$: $\varphi_i$ and 2-forms $\mathcal{F}^{(2)i} = \mathrm{d}\mathcal{A}^{(1)i}$

- Further any *p*-form in the higher dimension

$$\hat{F}^{(p+1)} = \mathrm{d}\hat{A}^{(p)} = F^{(p+1)} + F^{(p)} \wedge (\mathrm{d}y + \mathcal{A}^{(1)})$$

$$F^{(p+1)} = \mathrm{d}A^{(p)} - \mathrm{d}A^{(p-1)} \wedge \mathcal{A}^{(1)}\,, \qquad F^{(p)} = \mathrm{d}A^{(p-1)}$$

# Scalar sector in D=4

$$\begin{array}{c|cccccccccc} D=11 & g_{\mu\nu}, & & & & & & A^{(3)}, & & & \\ D=10 & g_{\mu\nu}, & & & \varphi_1, & \mathcal{A}^{(1)1}, & & A^{(3)}, & & A_1^{(2)}, & \\ D=9 & g_{\mu\nu}, & \varphi_2, & \mathcal{A}^{(1)2}, & \varphi_1, & \mathcal{A}^{(1)1}, & \mathcal{A}^{(0)1}{}_2, & A^{(3)}, & A_2^{(2)}, & A_1^{(2)}, & A_{12}^{(1)} \end{array}$$

*D=4* scalars : $\varphi_i$ or $\vec{\varphi}$, $\mathcal{A}^{(0)i}{}_j$ , $A^{(0)}_{ijk}$ $\quad i > j > k$

7 + 7*6/2 + 7*6*5/6 = 63

*D=4* 2-forms :

$A_i^{(2)}$ which in $D = 4$ dualized to 7 scalars: $\chi^i$

$$-2\mathcal{L}_0 = *\mathrm{d}\vec{\varphi}\cdot\wedge\mathrm{d}\vec{\varphi} + \sum_{i<j<k} \mathrm{e}^{\vec{a}_{ijk}\cdot\vec{\varphi}} *F^{(1)}_{ijk}\wedge F^{(1)}_{ijk} + \sum_{i<j} \mathrm{e}^{\vec{b}^i{}_j\cdot\vec{\varphi}} *\mathcal{F}^{(1)i}{}_j\wedge\mathcal{F}^{(1)i}{}_j + \sum_i \mathrm{e}^{-\vec{a}_i\cdot\vec{\varphi}} *G^{(1)}_i\wedge G^{(1)}_i$$

$$F^{(1)}_{ijk} = \mathrm{d}A^{(0)}_{ijk}\,, \qquad \mathcal{F}^{(1)i}{}_j = \mathrm{d}\mathcal{A}^{(0)i}{}_j\,, \qquad G^{(1)}_i = \mathrm{d}\chi_i$$

# 3. Symmetries of D=4, N=8

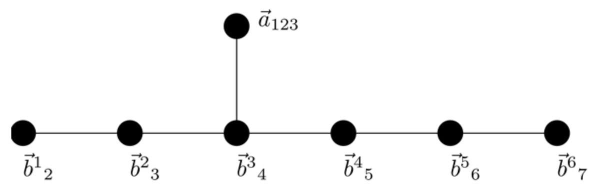
$$-2\mathcal{L}_0 = *\mathrm{d}\vec{\varphi}\cdot\wedge\mathrm{d}\vec{\varphi} + \sum_{i<j<k} \mathrm{e}^{\vec{a}_{ijk}\cdot\vec{\varphi}} *F^{(1)}_{ijk}\wedge F^{(1)}_{ijk} + \sum_{i<j} \mathrm{e}^{\vec{b}^i{}_j\cdot\vec{\varphi}} *\mathcal{F}^{(1)i}{}_j\wedge\mathcal{F}^{(1)i}{}_j + \sum_i \mathrm{e}^{-\vec{a}_i\cdot\vec{\varphi}} *G^{(1)}_i\wedge G^{(1)}_i$$

$$F^{(1)}_{ijk} = \mathrm{d}A^{(0)}_{ijk}\,, \qquad \mathcal{F}^{(1)i}{}_j = \mathrm{d}\mathcal{A}^{(0)i}{}_j\,, \qquad G^{(1)}_i = \mathrm{d}\chi_i$$

Shift symmetries of $\vec{\varphi}$ form a 7d CSA $\vec{H}$
shifts of other 63 scalars are roots $E^{ijk}$, ${E_j}^i$, $E_i$ with weight vectors :

$$\vec{a}_{ijk},\ \vec{b}^i{}_j,\ \vec{a}_i$$

$\vec{b}^i{}_{i+1}$ and $\vec{a}_{123}$ are the 7 simple roots with inner products represented as



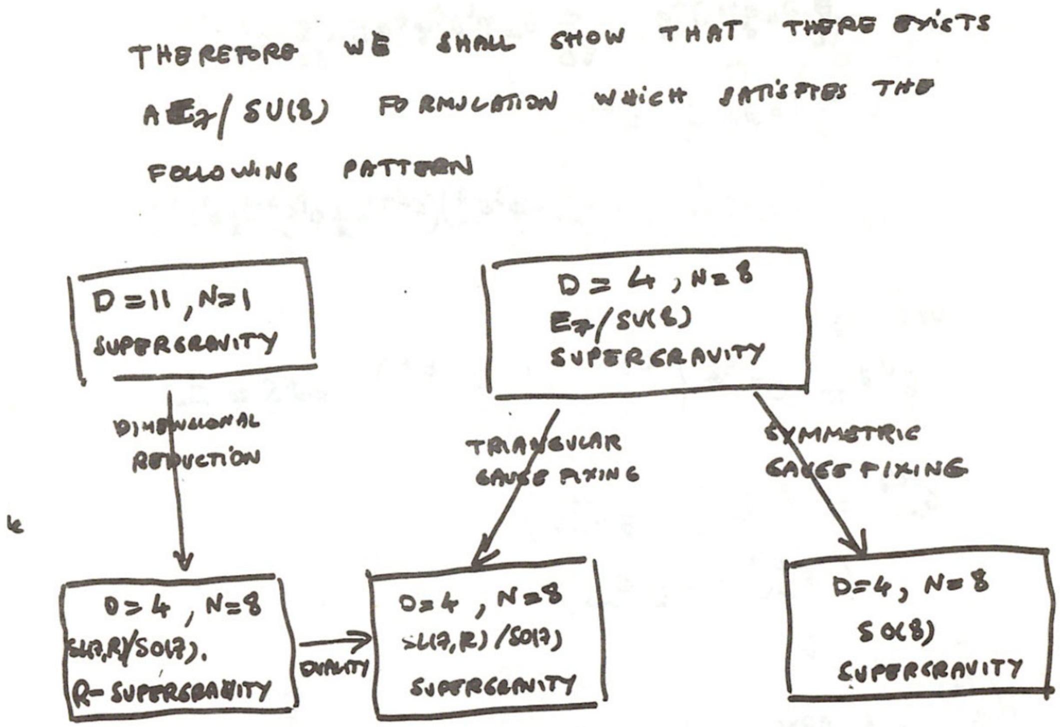

$\{\vec{H},\ E^{ijk},\ {E_j}^i,\ E_i\}$ generate the Borel subalgebra (Cartan subalgebra + positive roots) of $E_7$,

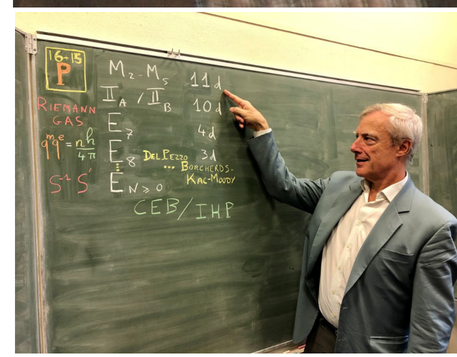
70 scalars described by representative of $E_7$

$$\mathcal{V}_{\mathrm{Iwa}} = \exp\left(\mathcal{A}^j{}_i {E_j}^i + A_{ijk}E^{ijk} + \chi^i E_i\right)\exp\left(\tfrac{1}{2}\vec{\varphi}\cdot\vec{H}\right)$$

Cremmer-Julia, 1979 :“we have a candidate for the bosonic part of the N = 8 supersymmetric action .. but in a dissymetrical form. This is to be contrasted with the lowest order results of [de Wit, Freedman, 1977] where a global S0(8) internal symmetry is manifest. In the next chapter we shall restore this symmetry”. Then they go to symmetric gauge …

# Slide of Eugène Cremmer, Trieste 1981




Cremmer and Julia, 1979: “The key operation in what follows, is the exchange of the order of elimination of two fields and is allowed classically ; the quantum theory however depends on the choice of dynamical fields and requires a careful computation of functional determinants arising when one integrates over Gaussian fields. So let us discuss only the classical transformation.”

# 4. Coset gauge fixings

$$\mathbb{G} = \mathbb{H} \oplus \mathbb{K}$$

$$\{T_A\} = \{M_i, K_a\}$$

Supergravities (higher N) have symmetric algebras

$$[\mathbb{H},\, \mathbb{H}] \subset \mathbb{H}$$
$$[\mathbb{H},\, \mathbb{K}] \subset \mathbb{K}$$
$$[\mathbb{K},\, \mathbb{K}] \subset \mathbb{H}\,, \quad \mathbb{H} \text{ is max. compact part}$$

$\mathcal{V}(x) \to \mathbf{g}\,\mathcal{V}(x)h^{-1}(x)$ representative in $\mathbb{G}$.

- Symmetric gauge: preserves global H-symmetry

$$\mathcal{V} = \mathrm{e}^{\phi^a K_a}\mathrm{e}^{\theta^i M_i}$$

$\phi^a$: scalars in the manifold; $\theta^i$ gauge fixed by local H to 0.

polar decomposition : matrix = symmetric × orthogonal

$$\mathcal{V}_{sym}(\phi^a) = \mathrm{e}^{\phi^a K_a} \in \exp(\mathbb{K}) \qquad a = 1, \ldots, n_{sc}$$

- Iwasawa gauge (or: triangular gauge): does not preserve H-symmetry

$$\mathbb{G} = \mathtt{C} \oplus \mathtt{N} \quad \oplus \tilde{\mathtt{N}}$$ CSA + positive + negative

$$= \quad \mathcal{S} \quad \oplus \tilde{\mathtt{N}}$$ Borel subalgebra $\{ t_a \}$+ negative

$$\mathcal{V}_{\rm Iwa} = \mathrm{e}^{\phi^a t_a} \in \exp \mathcal{S}$$

(before gauge fixing: Iwasawa decomposition G= K A N )

- Are they really equivalent, or anomalies to go from one to the other ?

They are all unitary gauges: gauge fixing function depends on fields, not on derivatives → no propating FP ghosts

# Comparison Sl(2,R)/U(1)

- Cartan subalgebra and positive root

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

- Coset representative in Iwasawa gauge

$$\mathcal{V}_{\rm Iwa} = {\rm e}^{-\chi\, E}{\rm e}^{-\frac{1}{2}\varphi\, H} = \begin{pmatrix} {\rm e}^{-\varphi/2} & -\chi{\rm e}^{\varphi/2} \\ 0 & {\rm e}^{\varphi/2} \end{pmatrix}$$

$$\Omega = -\mathcal{V}^{-1}{\rm d}\mathcal{V} = e^a K_a + \omega^1 M_1$$

$$\{K_a\} = \{H,\, E + E^T\}\,, \qquad M_1 = E - E^T$$

$$\{e^a\} = \tfrac{1}{2}\{{\rm d}\varphi, {\rm e}^{\varphi}{\rm d}\chi\}\,, \qquad \omega^1 = \tfrac{1}{2}{\rm e}^{\varphi}{\rm d}\chi\,.$$

$$e^1 \otimes e^1 + e^2 \otimes e^2 = ({\rm d}\varphi)^2 + {\rm e}^{2\varphi}({\rm d}\chi)^2$$

- compare N=8

70 scalars described by representative of $E_7$

$$\mathcal{V}_{\rm Iwa} = \exp\left(\mathcal{A}^j{}_i E_j{}^i + A_{ijk}E^{ijk} + \chi^i E_i\right)\exp\left(\tfrac{1}{2}\vec{\varphi}\cdot\vec{H}\right)$$

$$-2\mathcal{L}_0 = *{\rm d}\vec{\varphi}\cdot\wedge{\rm d}\vec{\varphi} + \sum_{i<j<k}{\rm e}^{\vec{a}_{ijk}\cdot\vec{\varphi}}*F^{(1)}_{ijk}\wedge F^{(1)}_{ijk} + \sum_{i<j}{\rm e}^{\vec{b}^i{}_j\cdot\vec{\varphi}}*\mathcal{F}^{(1)i}{}_j\wedge\mathcal{F}^{(1)i}{}_j + \sum_i {\rm e}^{-\vec{a}_i\cdot\vec{\varphi}}*G^{(1)}_i\wedge G^{(1)}_i$$

# Comparison Sl(2,R)/U(1)

- Cartan subalgebra and positive root $$H=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\,,\qquad E=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$
- Coset representative in Iwasawa gauge $$\mathcal{V}_{\rm Iwa}=\mathrm{e}^{-\chi\,E}\mathrm{e}^{-\frac{1}{2}\varphi\,H}=\begin{pmatrix}\mathrm{e}^{-\varphi/2} & -\chi\mathrm{e}^{\varphi/2}\\ 0 & \mathrm{e}^{\varphi/2}\end{pmatrix}$$

$$\Omega=-\mathcal{V}^{-1}\mathrm{d}\mathcal{V}=e^aK_a+\omega^1M_1$$

$$\{K_a\}=\{H,\,E+E^T\}\,,\qquad M_1=E-E^T$$

$$\{e^a\}=\tfrac{1}{2}\{\mathrm{d}\varphi,\mathrm{e}^{\varphi}\mathrm{d}\chi\}\,,\qquad \omega^1=\tfrac{1}{2}\mathrm{e}^{\varphi}\mathrm{d}\chi\,.$$

$$e^1\otimes e^1+e^2\otimes e^2=(\mathrm{d}\varphi)^2+\mathrm{e}^{2\varphi}(\mathrm{d}\chi)^2$$

- Symmetric gauge

$$\mathcal{V}_{\rm sym}=\mathrm{e}^{\phi^aK_a}=\mathrm{e}^{\phi^1H+\phi^2(E+E^T)}=\mathbb{1}_2\cosh r+\phi^aK_a\frac{\sinh r}{r}$$

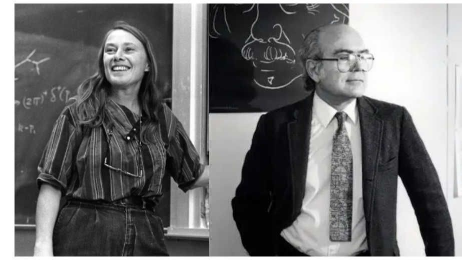
$$=\begin{pmatrix}\cosh r+\phi^1\frac{\sinh r}{r} & \phi^2\frac{\sinh r}{r}\\ \phi^2\frac{\sinh r}{r} & \cosh r-\phi^1\frac{\sinh r}{r}\end{pmatrix}\,,\qquad \phi^1=r\sin\theta,\ \phi^2=r\cos\theta$$

$$e^a=-\phi^a\frac{\mathrm{d}r}{r}+\varepsilon^{ab}\phi^b\frac{\sinh 2r}{2r}\mathrm{d}\theta\,,\qquad \omega^1=-(\sinh r)^2\mathrm{d}\theta$$

$$e^1\otimes e^1+e^2\otimes e^2=(\mathrm{d}r)^2+\frac{1}{4}(\sinh 2r)^2(\mathrm{d}\theta)^2$$

$\phi^1-\phi^2$ rigid symmetry remains.

# 5. Duality



$$S(F,\phi) = \frac{1}{4}\int \mathrm{d}^4x \left[e(\mathrm{Im}\,\mathcal{N}_{AB}(\phi))F^A_{\mu\nu}F^{B\mu\nu} - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\mathrm{Re}\,\mathcal{N}_{AB}(\phi))F^A_{\mu\nu}F^B_{\rho\sigma}\right]$$

$A, B = 1, \ldots n$

- M.K. Gaillard and B. Zumino (1981) in D=4:
  Transformations involving gauge field strengths and their duals, forming a symplectic group mixing field equations and Bianchi identities, in general not leaving the action invariant.
- Transforms the coupling $\mathcal{N}_{AB}(\phi)$ depending on fields and coupling constants.
- C.M. Hull and A.V.P. hep-th/ 9503022. ``Pseudo-Duality”: $D_{\mathrm{pseudo}} = \mathrm{Diff}(\mathcal{M}) \times \mathrm{Sp}(2N;\mathbb{R})$ ,
- For proper symmetry: must be an isometry of scalar manifold: $\mathrm{Iso}(\mathcal{M}) \subset \mathrm{Sp}(2n;\mathbb{R})$

$$D_{\mathrm{prop}} = \mathrm{Iso}(\mathcal{M}) \subset \mathrm{Iso}(\mathcal{M}) \times \mathrm{Iso}(\mathcal{M}) \subset \mathrm{Iso}(\mathcal{M}) \times \mathrm{Sp}(2n;\mathbb{R}) \subset D_{\mathrm{pseudo}}$$

- But there is the remainder (up to linear redefinitions of the vectors GL(*n*) ):
  “Enhanced duality group”

in earlier work: Ferrara, Scherk, Zumino, 1977; Cremmer, Scherk, Ferrara, 1978; de Wit, 1979; Cremmer and Julia, 1979
In Hamiltonian form: M. Henneaux, B. Julia, V. Lekeu and A. Ranjbar, 1709.06014

# Duality groups in 4 and higher (even) dimensions

- Relates $q$=D/2 forms $F^{(q=p+1)} = dA^p$ to their Hodge duals in D dimensions

$$D = 2q\,, \qquad F^{(q)} = \frac{1}{q!} F_{a_1\cdots a_q} e^{a_1} \wedge \cdots \wedge e^{a_q}$$

$$^*F^{(q)} = \frac{1}{q!} F_{a_1\cdots a_q} \frac{1}{q!} e^{b_1} \wedge \cdots \wedge e^{b_q} \varepsilon_{b_1\cdots b_q}{}^{a_1\cdots a_q}$$

$$^{**}F^{(q)} = \frac{1}{q!} F_{a_1\cdots a_q} \frac{1}{q!} \varepsilon_{b_1\cdots b_q}{}^{a_1\cdots a_q} \frac{1}{q!} e^{c_1} \wedge \cdots \wedge e^{c_q} \varepsilon_{c_1\cdots c_q}{}^{b_1\cdots b_q} = (-)^{q+1} F^{(q)}$$

for 1 time direction.

This implies a real structure for $q$ odd; largest duality group will be orthogonal if D=4m+2 and complex or antisymmetric for $q$ even; largest duality group will be symplectic if D=4m.

E.g. D=4: $q$=D/2 =2, $p$=1: vector fields give symplectic symmetry.

D=6: $q$=D/2=3, $p$=2 antisymmetric tensors give orthogonal symmetry

For D=4 with $n$ vectors Sp(2$n$;R); max. sugra : n=28 Sp(56;R)

D=6 with $n$ 2-tensors SO(*n,n;R);* max sugra: n=5 SO(5,5;R).

# Enhanced duality

In general: dualities define a class of actions.

- the Lagrangian is not uniquely defined (it can always be reparametrized via an electric-magnetic duality transformation) and neither is its invariance group.
- there exist different Lagrangians with different symmetry groups

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R})\backslash Sp(56,\mathbb{R})/GL(28,\mathbb{R})$$



Bernard de Wit, Henning Samtleben, Mario Trigiante
0212239 and 0705.2101

duality:
56*57/2= 1596

modulo 133
scalar reparam.

modulo $28^2 = 784$
vector reparam. (not mixing eom and Bianchi)

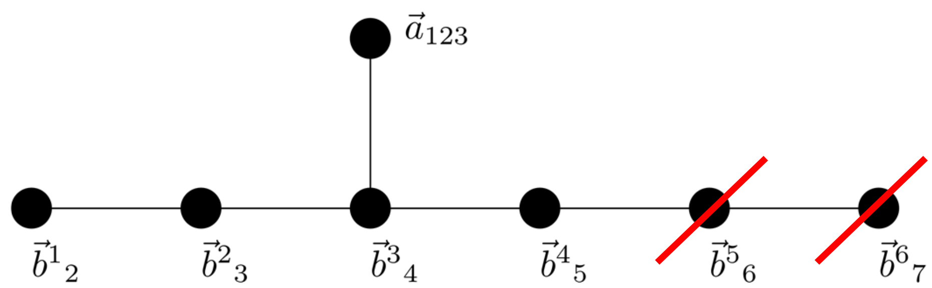
Double quotient
G\X/Y
where X is a group and
G,Y are subgroups of X

dimension of the
double quotient : 1596 – 133 – 784 = **679**

# no enhanced duality in D=6

- But not in D=6, (2,2) supergravity:
  pseudo-duality group from 5 two-forms
  (reduction of D=11  3-form over 11-6=5 circles): SO(5,5;R).

- That is also the duality group $E_{11-6,11-6} = E_{5,5} = SO(5,5;R)$.



# 6. Various supergravities

- Same amount of supersymmetry; different gauge fixings of H

I. Supergravity with G/H described in symmetric gauge: where $\phi$ is in the noncompact part of the algebra; local H symmetry = R-symmetry

4D: Cremmer Julia, 1979 de Wit, Nicolai 1982

6D Tanii,1984 Bergshoeff, Samtleben, Sezgin 2008

II. Dimensional reduced from higher dimension : coset in Iwasawa or partial Iwasawa gauge. Some scalars necessary have polynomial dependence; less symmetries (no global H remaining)

4D from 5D: Andrianopoli, D'Auria, Ferrara and Lledo, 2002
6D from 7D: Cowdall, 1998

classically the same by duality transformations

Cremmer and Julia, 1979: "The key operation in what follows, is the exchange of the order of elimination of two fields and is allowed classically ; the quantum theory however depends on the choice of dynamical fields and requires a careful computation of functional determinants arising when one integrates over Gaussian fields. So let us discuss only the classical transformation."

# D=4

$i$=1,…,8; $\phi_{[ijkl]}$: 35 complex or 70 real $\phi_a$
linear representation of SU(8)

$$\phi_{ijk\ell} = \pm\frac{1}{4!}\varepsilon_{ijk\ell pqmn}\bar{\phi}^{pqmn}$$

metric
28 vectors
70 scalars

$$\mathbb{G} = \mathbb{H} \oplus \mathbb{K}$$
$$133 = 63 + 70$$
$$E_7 \quad \mathrm{SU}(8) \quad \frac{E_{7(7)}}{SU(8)}$$

$$(\bar{\phi}\phi)^{ij}{}_{k\ell} = \frac{1}{2}\bar{\phi}^{ijmn}\phi_{mnk\ell}$$

considered as 28 × 28 matrix

Symmetric gauge

$$\mathcal{V}_{sym} = \mathcal{V}^{\dagger}_{sym} = \mathrm{e}^{\phi^a K_a} = \begin{pmatrix} \cosh\phi\bar{\phi} & \phi\frac{\sinh\bar{\phi}\phi}{\bar{\phi}\phi} \\ \bar{\phi}\frac{\sinh\phi\bar{\phi}}{\phi\bar{\phi}} & \cosh\bar{\phi}\phi \end{pmatrix}, \qquad \phi^a K_a = \begin{pmatrix} 0 & \phi_{ijkl} \\ \bar{\phi}^{mnpq} & 0 \end{pmatrix}$$

Cremmer Julia, **1979** de Wit, Nicolai **1982**

# D=5

$$\mathbb{G} = \mathbb{H} \oplus \mathbb{K}$$
$$78 = 36 + 42$$
$$E_6 \quad \mathrm{USp}(8) \quad \frac{E_{6(6)}}{\mathrm{USp}(8)}$$

metric
27 vectors
42 scalars

In a symmetric gauge:
42 scalars $\phi_{abcd}$ (repr. of USp(8))

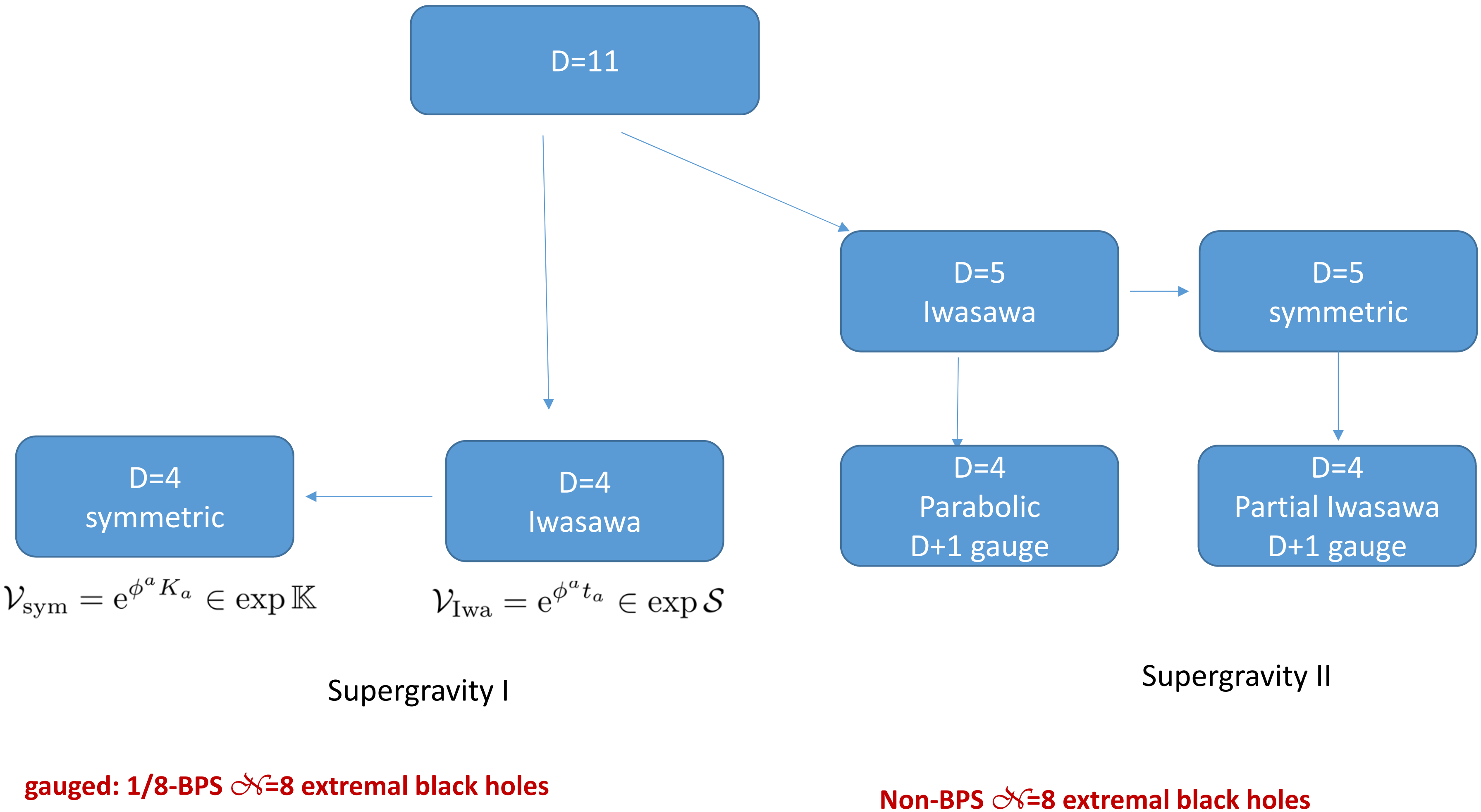
then dim.red. on circle to D=4
→ supergravity II

$$\mathcal{V}_{\mathrm{parab}} = \mathrm{e}^{a^\lambda t_\lambda}\,\mathrm{e}^{\hat{\phi}^r\hat{T}_r}\,\mathrm{e}^{\sigma D} \qquad \lambda = 1,\ldots,27;\ r = 1,\ldots,42$$
$$\mathcal{V}_{\mathrm{partial\ Iwa}} = \mathrm{e}^{a^\lambda t_\lambda}\,\mathrm{e}^{\phi^{abcd}K_{abcd}}\,\mathrm{e}^{\sigma D}$$

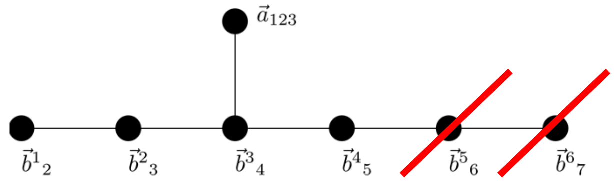
$$\frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)} \sim \frac{\mathrm{E}_{6(6)}}{\mathrm{USp}(8)} \times \mathrm{O}(1,1) \times \exp(\mathtt{N}^{[\mathbf{27}'_{+2}]})$$

: 70 = 42 + 1 + 27

Andrianopoli, D'Auria, Ferrara and Lledo, **2002**
Sezgin, Nieuwenhuizen, **1982**
Cremmer, Scherk, Schwarz, **1979**
**(Spontaneously broken N=8 supergravity)**


D=11
D=5
Iwasawa
D=5
symmetric
D=4
symmetric
D=4
Iwasawa
D=4
Parabolic
D+1 gauge
D=4
Partial Iwasawa
D+1 gauge
$\mathcal{V}_{\rm sym} = \mathrm{e}^{\phi^a K_a} \in \exp \mathbb{K}$
$\mathcal{V}_{\rm Iwa} = \mathrm{e}^{\phi^a t_a} \in \exp \mathcal{S}$
Supergravity I
Supergravity II
gauged: 1/8-BPS $\mathcal{N}$=8 extremal black holes
Non-BPS $\mathcal{N}$=8 extremal black holes

# D=6 analogue




- $E_{5,5} = SO(5,5)$ is $G$ isometry group of scalars,
  but also duality group since there are 11-6 = 5 two-forms (from $A_{\mu\nu i}$) with field strengths D/2- forms
- Symplectic spinors (2,2): R-symmetry group is USp(4) × USp(4)= SO(5) ×SO(5)
- Coset $\frac{SO(5,5)}{SO(5)\times SO(5)}$; described by 16 × 16 vielbein matrix
  $V_{\mu\dot{\mu}}{}^{\alpha\dot{\alpha}}$ with SO(5) spinor indices $\mu, \dot{\mu} = 1,2,3,4,\ \alpha, \dot{\alpha} = 1,2,3,4.$
  $a, \dot{a} = 1, \ldots 5\,, \qquad \underline{A} = \{a, \dot{a}\} = 1, \ldots, 10$
- Before gauge fixing $V = \mathrm{e}^{\frac{1}{4}\phi^{\underline{AB}}\Gamma_{\underline{AB}}}\,, \qquad \Gamma_{\underline{AB}} = \Gamma_{[\underline{A}}\Gamma_{\underline{B}]}$ with $10 \times 9/2 = 45$ independent entries $\phi^{\underline{AB}}$
- Symmetric gauge $V_{\mathrm{sym}} = \exp\left(\frac{1}{2}\phi^{a\dot{a}}\Gamma_{a\dot{a}}\right) \qquad \phi^{ab} = \phi^{\dot{a}\dot{b}} = 0 \qquad$ 25 scalars $\phi^{a\dot{a}}$

- That gives supergravity I: of Tanii-Bergshoeff-Samtleben-Sezgin

Gauge fixings: forthcoming paper R. Kallosh, H. Samtleben and AVP

# D=6 supergravity II

derived by Cowdall, 1998, from
7D supergravity of Pernici, Pilch, van Nieuwenhuizen, 1984,
and compactified on a circle, in the limit of vanishing gaugings

- D=7

metric
10 vectors
14 scalars

$(G/H)_{7D} = SL(5)/SO(5)$

$$\frac{\mathrm{SO}(5,5)}{\mathrm{SO}(5)\times\mathrm{SO}(5)} \sim \frac{\mathrm{SL}(5)}{\mathrm{SO}(5)} \times \mathrm{O}(1,1) \times \exp(\mathtt{N}^{[\mathbf{10}]})$$

in D=6 supergravity II

# Conclusions

- Finiteness calculations suggest anomalies of G/H symmetries in D>4 and not in D=4, $\mathcal{N} \geq 5.$

- Can enhanced dualities, which are present only in D=4, be a clue ?

- By different gauge choices of the coset spaces, some due to reductions from D+1, there are different supergravities.
  These are classically equivalent, but the quantum equivalence should be proven.

- All this: from analyses by R. Kallosh in papers in the previous year.
- In this investigation, we made technical progress for the 'symmetric' gauge fixing of D=6 supergravity,

R. Kallosh, H. Samtleben and AVP, in preparation