NICPB
KBFI

CONNECTING SCIENCES

Inflation

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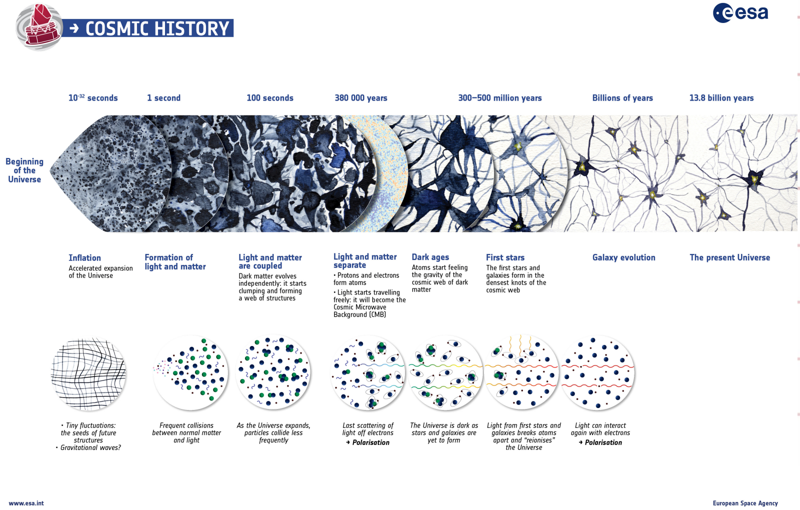
• History of the Universe •



→ COSMIC HISTORY

esa

10^-32 seconds

1 second

100 seconds

380 000 years

300–500 million years

Billions of years

13.8 billion years

Beginning of the Universe

Inflation
Accelerated expansion of the Universe

Formation of light and matter

Light and matter are coupled
Dark matter evolves independently: it starts clumping and forming a web of structures

Light and matter separate
- Protons and electrons form atoms
- Light starts travelling freely: it will become the Cosmic Microwave Background (CMB)

Dark ages
Atoms start feeling the gravity of the cosmic web of dark matter

First stars
The first stars and galaxies form in the densest knots of the cosmic web

Galaxy evolution

The present Universe

· Tiny fluctuations: the seeds of future structures
· Gravitational waves?

Frequent collisions between normal matter and light

As the Universe expands, particles collide less frequently

Last scattering of light off electrons
→ Polarisation

The Universe is dark as stars and galaxies are yet to form

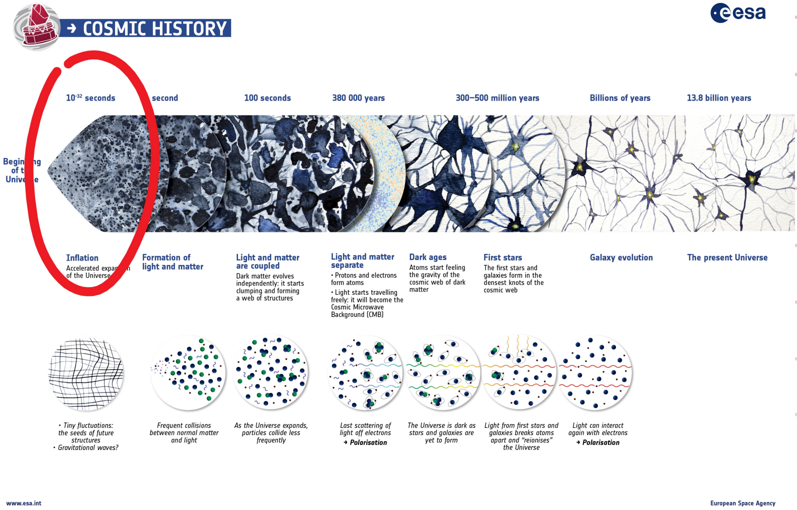
Light from first stars and galaxies breaks atoms apart and "reionises" the Universe

Light can interact again with electrons
→ Polarisation

www.esa.int

European Space Agency

History of the Universe



COSMIC HISTORY

esa

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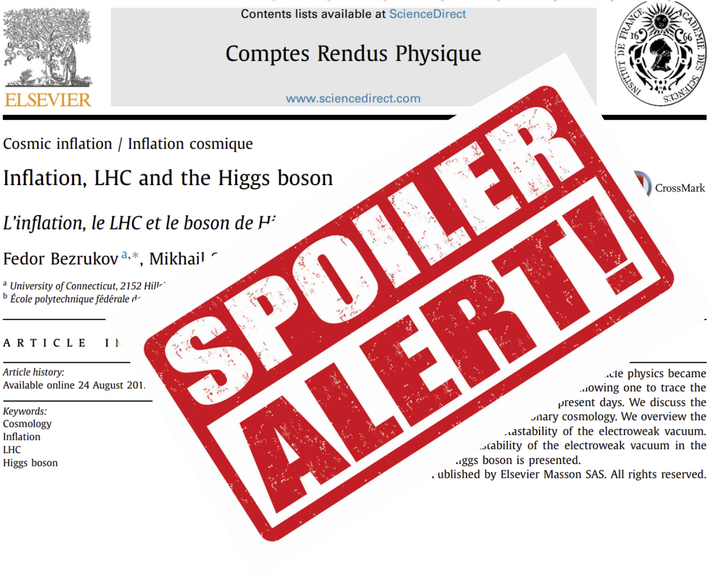
• Higgs Inflation •



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ELSEVIER

Cosmic inflation / Inflation cosmique

Inflation, LHC and the Higgs boson

L'inflation, le LHC et le boson de H

CrossMark

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A R T I C L E I

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Keywords:
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Inflation
LHC
Higgs boson

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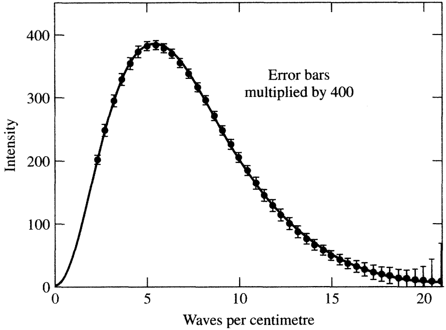
CMB



The cosmic microwave background (CMB) is a radiation bathing the Earth from all directions, with the form of a black-body radiation with $T \simeq 2.725K$. Furthermore, the T coming from different parts of the sky is astonishingly uniform.

• CMB anisotropies •

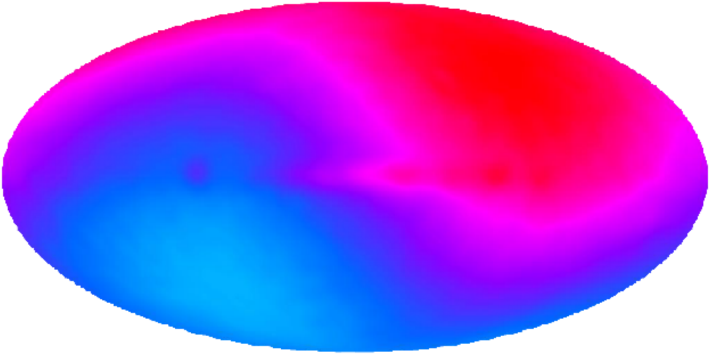
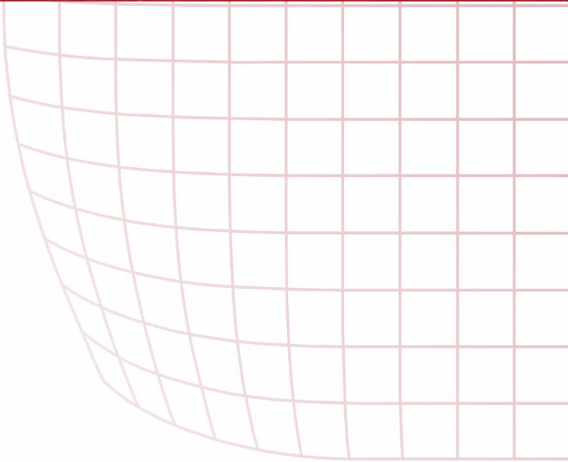


Figure: The COBE (1989-1993) measurement of the CMB anisotropy. The oval is a map of the sky showing the dipole anisotropy $\Delta T/T \sim 10^{-3}$.

• Comoving coordinates •



CMB uniformity

CMB anisotropy

• Comoving coordinates •

CMB uniformity → **Cosmological Principle**

“The Universe, at large scales, is homogeneous and isotropic”

CMB anisotropy

• Comoving coordinates •

CMB uniformity → **Cosmological Principle**

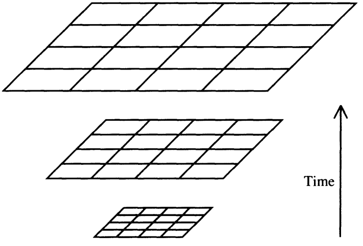
> “The Universe, at large scales, is homogeneous and isotropic”

CMB anisotropy → Universe expands

$\Downarrow$

comoving coordinates: $\boxed{\vec{r} = a(t)\vec{x}}$

- physical distance: $\vec{r}$
- comoving distance: $\vec{x}$
- scale factor of the Universe: $a(t)$



Think about a grid which expands with time: galaxies are fixed in the comoving system, but they move in the physical one.

Friedmann equations

The evolution of the Universe is described by the Friedmann eqs.

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \qquad \boxed{M_P = (8\pi G)^{-1/2},\ c = 1}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- k describes the geometry of the Universe $\begin{cases} k < 0 & \text{open} \\ k = 0 & \text{flat} \\ k > 0 & \text{closed} \end{cases}$
- ρ: energy density of the Universe
- p: pressure of the Universe
- **critical density**: $\boxed{\rho_c = \frac{3(\dot a/a)^2}{8\pi G}} \Rightarrow k = 0 \qquad \boxed{\rho_c(t)!!!!}$

Now is $\rho_c(t_0) = 1.88\,h^2 \times 10^{-26}$ kg m^{-3}, $h = 0.674 \pm 0.005$

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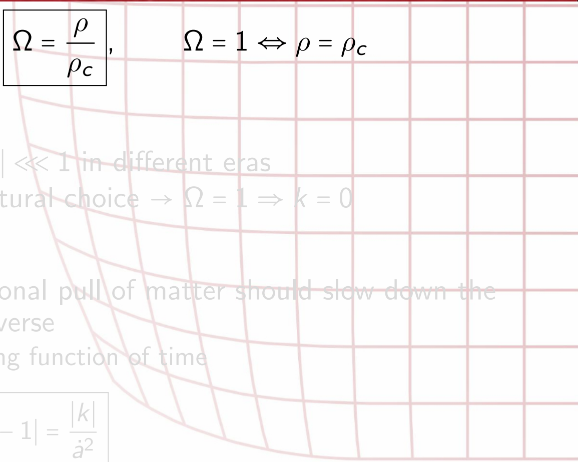
Flatness Problem



- **density parameter:** $\boxed{\Omega = \frac{\rho}{\rho_c}}$, $\quad \Omega = 1 \Leftrightarrow \rho = \rho_c$

- **observations**: $|\Omega - 1| \lll 1$ in different eras
  $\Rightarrow$ simplest but unnatural choice $\to \Omega = 1 \Rightarrow k = 0$

- **theory:** the gravitational pull of matter should slow down the expansion of the Universe
  $\Rightarrow \dot{a}(t)$ is a decreasing function of time

$$\text{Friedmann eq.} \quad \Rightarrow \boxed{|\Omega - 1| = \frac{|k|}{\dot{a}^2}}$$

$\Rightarrow |\Omega - 1|$ is an increasing function of time
$\Rightarrow \Omega$ will inevitably depart from 1

$\Rightarrow$ **flatness problem**

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• Inflation •

Alan Guth proposed a solution (PRD 23(1981) 347-356):

inflation: $\boxed{\ddot{a} > 0}$

Looking at the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

we see immediately that inflation implies $\rho + 3p < 0$.
Since we always assume $\rho > 0$, this requires

$$p < -\frac{\rho}{3}$$

How can we get a $p < 0$? 1st idea: **Cosmological Constant (CC)**

• Cosmological constant •

CC is a fluid with a constant energy density: $\boxed{\rho_\Lambda = \frac{\Lambda}{8\pi G}}$

Performing all the computations of the case:

- negative pressure! $\boxed{p_\Lambda = -\rho_\Lambda}$
- $a(t)$ increases exponentially with time

$$\boxed{a(t) = a_{t=0} \exp\left(\sqrt{\frac{\Lambda}{3}}t\right)} \Rightarrow |\Omega - 1| \propto \exp\left(-\sqrt{\frac{4\Lambda}{3}}t\right)$$

$\Omega \to 1$ very fast. We want to get so close that what happens after inflation cannot move it away again.

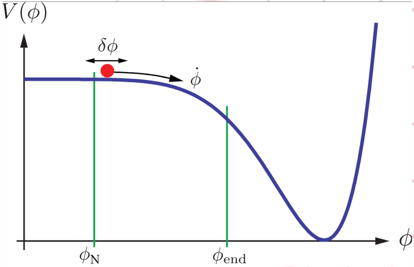
How much inflation is needed? The result is expressed in N_e

$$\frac{a(t_{\text{final}})}{a(t_{\text{initial}})} = \exp[N_e]$$

More accurate model building will provide $N_e \in [50, 60]$

• Slow-roll inflation •

- Inflation via a CC never stops → PROBLEM
- solution → a *temporary* CC
- a scalar particle (ϕ, **inflaton**): $\mathcal{L}_m = T - V$



  - kinetic energy density : $T = \frac{1}{2}\dot\phi^2$
  - potential energy density : $V(\phi)$
  - If $T \ll V$, then $\rho \simeq V \simeq -p \approx$ CC behaviour

Like a ball slowly rolling down the slope from the top of a hill

• PSRP •

Enough to know $V(\phi)$ → Potential Slow Roll Parameters (PSRP)

$$\boxed{\epsilon_V(\phi) = \frac{M_P^2}{2}\left(\frac{V_{,\phi}(\phi)}{V(\phi)}\right)^2}$$

$$\boxed{\eta_V(\phi) = M_P^2\,\frac{V_{,\phi\phi}(\phi)}{V(\phi)}}$$

Potential Slow Roll Approximation (PSRA): $\epsilon_V, \eta_V \ll 1$

- end of inflation: $\epsilon_V(\phi_{\text{end}}) = 1$
- number of e-folds: $N_e \simeq \frac{1}{M_P^2}\int_{\phi_{\text{end}}}^{\phi_N} \frac{V}{V_{,\phi}}\,\mathrm{d}\phi$

where $\begin{cases} \phi_N \text{ is taken at the } \textit{beginning} \text{ of} \\ \phi_{\text{end}} \text{ is taken at the end of} \end{cases}$ inflation

• CMB anisotropies •

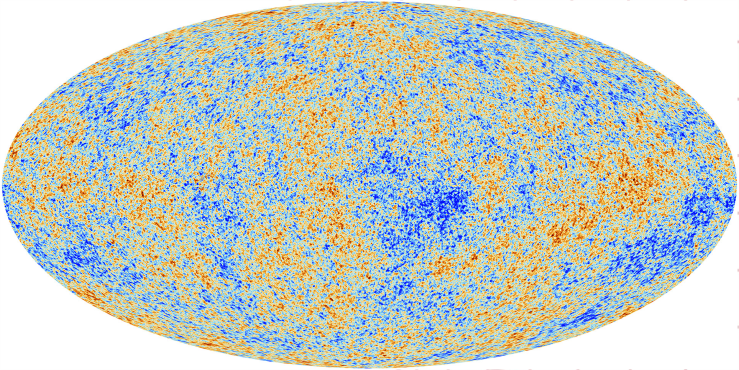


Figure: Planck (2009-2013): CMB anisotropy (dipole removed).

tiny fluctuations in the CMB ⇐ pertubations via quantum effects

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x}) \,, \qquad g_{\mu\nu}(t,\mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\mathbf{x})$$

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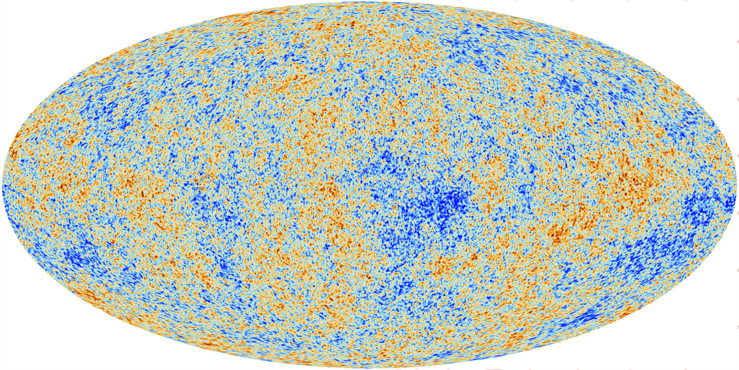

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• SR & relevant parameters •

- All observables can expressed in terms of $V(\phi)$, ϵ_{V} and η_{V}

  amplitude of P_S: $\boxed{A_s = \frac{1}{24\pi^2}\frac{V^*}{M_P^4}\frac{1}{\epsilon_{\mathrm{V}}^*}}$ → scale of inflation

  scalar spectral index: $\boxed{n_{\mathrm{s}} = 1 + 2\eta_{\mathrm{V}}^\star - 6\epsilon_{\mathrm{V}}^\star}$ → tilt of $V(\phi)$

  tensor-to-scalar ratio: $\boxed{r = \frac{A_T}{A_s} = 16\epsilon_{\mathrm{V}}^\star}$ → quantum gravity

  where * stands for evaluated at $\phi = \phi_N$

- we can take any $V(\phi)$ and start computing

- From the Planck 2018 constraints (arXiv:1807.06211)

$$\ln\left(10^{10} A_s^{\exp}\right) = 3.044 \pm 0.014 \quad \Rightarrow \quad A_s^{\exp} \simeq 2.1 \times 10^{-9}$$

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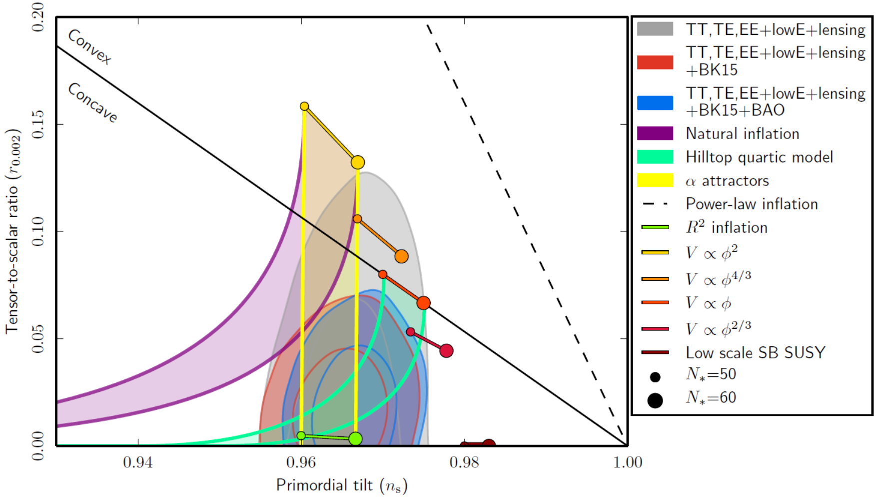

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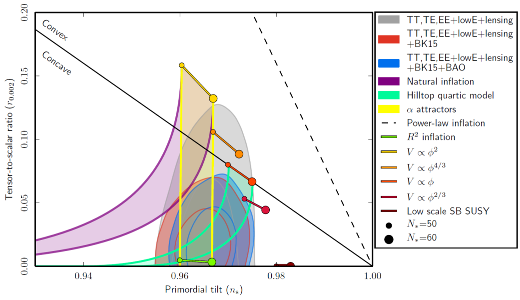

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- concave potentials are strongly FAVORED!!!
- $V(\phi_N) = \frac{3\pi^2 A_s}{2}\, r\, M_P^4 \lesssim (1.6 \times 10^{16}\,\mathrm{GeV})^4$
  → inflation a GUT scale phenomenon or just accident?
- non-minimal Higgs inflation still in the game
  → C. Dioguardi's talk tomorrow

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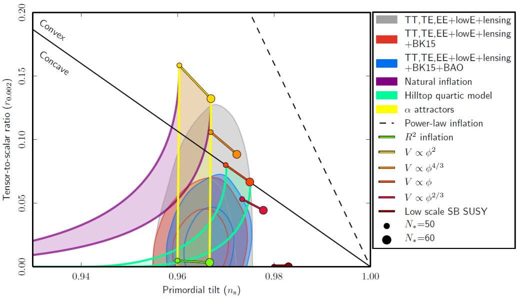

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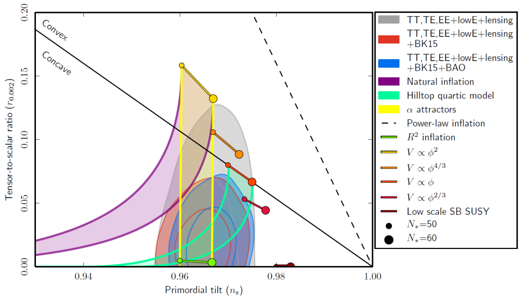

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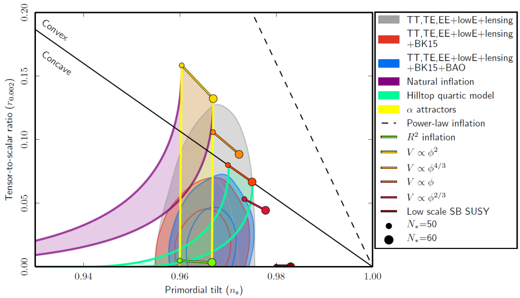

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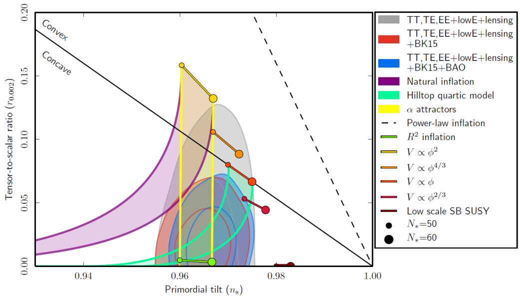

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• Summary & Conclusions •

- inflation is a stage of accelerated expansion of the Universe
- it solves the flatness problem of the Universe
- a scalar particle (inflaton) can drive inflation
- the Higgs boson (in non-minimal setups) can be the inflaton

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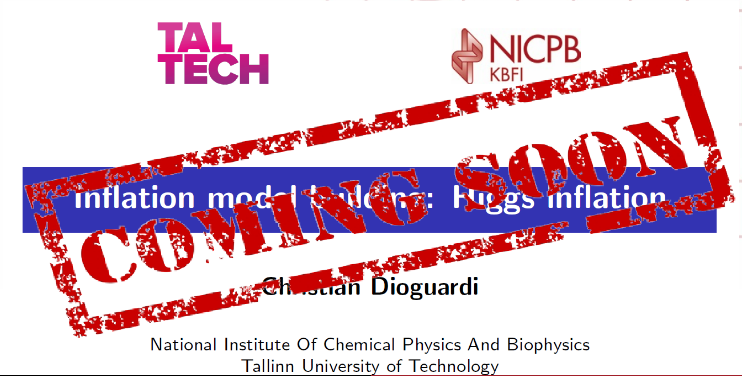


TALTECH

NICPB KBFI

COMING SOON

inflation model building: Higgs inflation

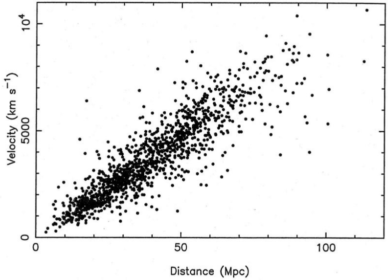
Christian Dioguardi

National Institute Of Chemical Physics And Biophysics
Tallinn University of Technology

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

• Expansion of the Universe •



Hubble's law: $\boxed{\vec{v} = H_0 \vec{r}}$

- velocity of recession proportional to the distance from us
- $H_0 = (67.4 \pm 0.5) km/s/Mpc$ is known as Hubble's constant
- not exact, but average behaviour
- everything is flying away from everything else
  → reverse time → initial singularity: **Big Bang**

• Geometry of the Universe •

Friedmann equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$

- k describes the geometry of the Universe:

$k = 0$ Euclidean (aka flat) geometry

$k > 0$ spherical (aka close) geometry

$k < 0$ hyperbolic (aka open) geometry

• Euclidean geometry •

Euclidean geometry is based on a set of simple axioms e.g.:

- a straight line is the shortest distance between 2 points
- parallel straight lines remain a fixed distance apart

which lead to the following conclusions:

- The angles of a triangle add up to 180°.
- The circumference of a circle of radius r is $2\pi r$

Such a geometry might well apply to our own Universe ⇒ the Universe must be infinite in extent, otherwise the edges would clearly violate homogeneity and isotropy. A Universe with this geometry is often called a **flat** Universe.

Spherical geometry

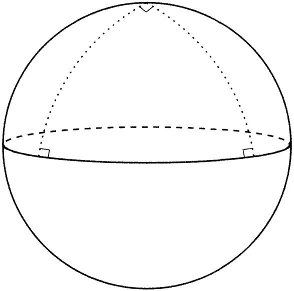


Figure A sketch of a spherical surface, representing positive *k*.

Main properties

- The angles of a triangle add up to more than 180°
- circumference of a circle is less than $2\pi r$

A Universe with $k > 0$ is also called **closed**, because of its finite size.

N.B. The picture shows a 2D spherical surface. Our Universe would eventually be a 3D spherical surface!

• Hyperbolic geometry •

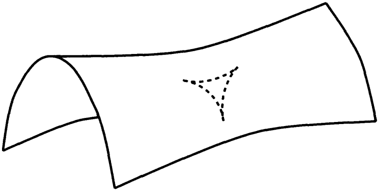


Figure 4.2 A sketch of a saddle surface, representing the hyperbolic geometry obtained when k is negative. A rather exaggerated triangle is shown with its sum of angles well below 180^o.

Main properties

- The angles of a triangle add up to less than 180°
- circumference of a circle is more than $2\pi r$

A Universe with $k < 0$ is also called **open**, because of its finite size.

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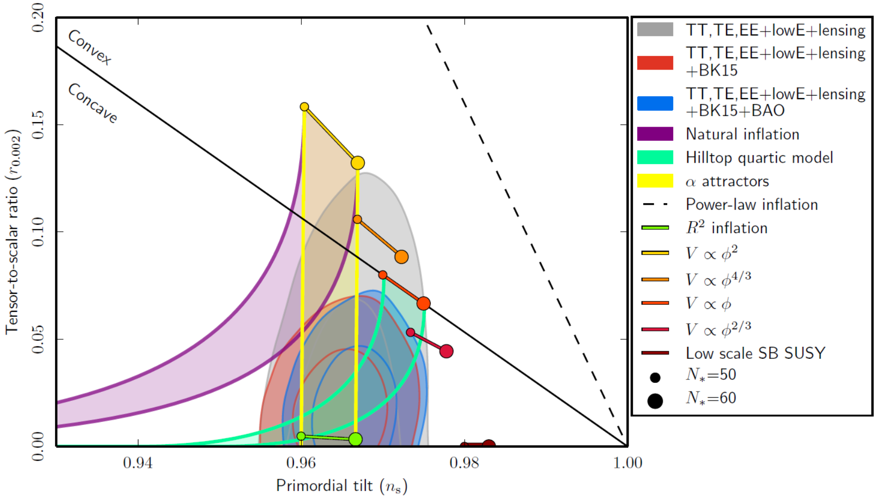
• Planck 2018 data •



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