European Union
European Regional
Development Fund

Investing
in your future

Keemilise ja
Bioloogilise Füüsika Instituut
National Institute of Chemical Physics and Biophysics

Quantum information at collider experiments

Luca Marzola
luca.marzola@cern.ch

Based on:

- "*Quantum entanglement and Bell inequality violation at colliders*", A. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — *Prog.Part.Nucl.Phys. 139 (2024)*

-"*Bell inequality is violated in $B^0 \to J/\psi\, K^*(892)^0$ decays*", M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — *Phys.Rev.D 109 (2024)*,

-"*Bell inequality is violated in charmonium decays*", M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — *Phys.Rev.D 110 (2024)*

4th CERN Baltic Conference, 15-17/10/2024, Tallinn.

What are you even talking about?

What are you even talking about?

Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

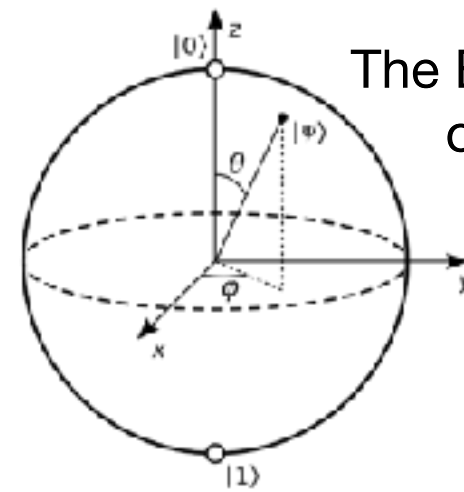
What are you even talking about?

Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

Focus on a qubit (two-level system):

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,; \qquad |\alpha|^2 + |\beta|^2 = 1$$

Born rule



The Block sphere of a qubit

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\varphi}\sin\frac{\theta}{2}$$

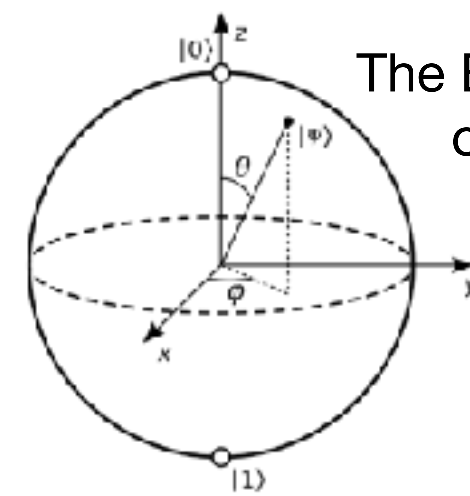
What are you even talking about?

Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

Focus on a qubit (two-level system):

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,; \qquad |\alpha|^2 + |\beta|^2 = 1$$

Born rule



The Block sphere of a qubit

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\varphi}\sin\frac{\theta}{2}$$

???

What are you even talking about?

Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

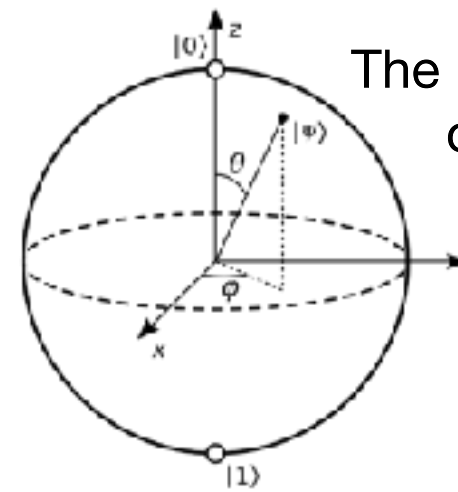
Focus on a qubit (two-level system):

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,; \qquad |\alpha|^2 + |\beta|^2 = 1$$

$|0\rangle \equiv$ spin down; $|1\rangle \equiv$ spin up

$|\alpha|^2 + |\beta|^2 = 1$: Born rule

electrons ARE qubits!



The Block sphere of a qubit

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\varphi}\sin\frac{\theta}{2}$$

Particles are inherently quantum systems and properties such as *spin* or flavor can be used to encode *information!*

What are you even talking about?

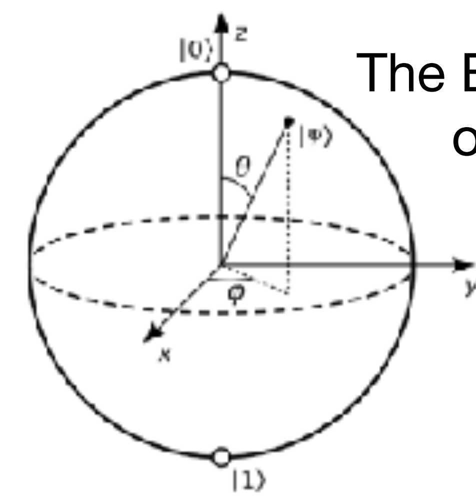
Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

Focus on a qubit (two-level system):

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,; \qquad |\alpha|^2 + |\beta|^2 = 1$$

$|0\rangle \equiv$ spin down; $|1\rangle \equiv$ spin up ← *electrons ARE qubits!*

Born rule



The Block sphere of a qubit

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\varphi}\sin\frac{\theta}{2}$$

Particles are inherently quantum systems and properties such as *spin* or flavor can be used to encode *information!*

"*What information?!?*" you say? At the moment it doesn't really matter as the identification "electron = qubit" allows us to:

- *use QIT methods to explore particle physics*
- *use particle physics to explore QIT (quantum mechanics)*

What are you even talking about?

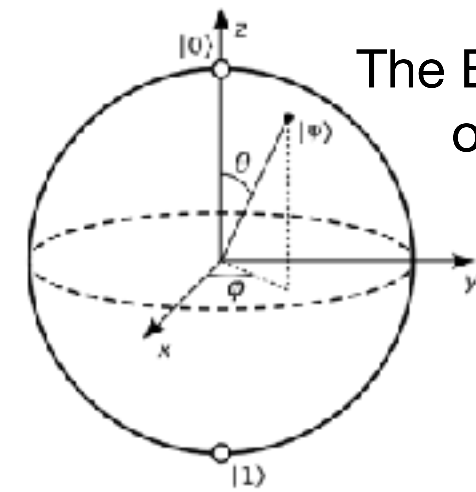
Quantum Information Theory (QIT) describes how *information* can be *encoded* in quantum systems, *manipulated*, *transferred* and *decoded*.

Focus on a qubit (two-level system):

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,; \qquad |\alpha|^2 + |\beta|^2 = 1$$

$|0\rangle \equiv$ spin down; $|1\rangle \equiv$ spin up — *electrons ARE qubits!*

Born rule: $|\alpha|^2 + |\beta|^2 = 1$



The Block sphere of a qubit

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\varphi}\sin\frac{\theta}{2}$$

Particles are inherently quantum systems and properties such as *spin* or flavor can be used to encode *information!*

"*What information?!?*" you say? At the moment it doesn't really matter as the identification "electron = qubit" allows us to:

- *use QIT methods to explore particle physics*
- *use particle physics to explore QIT (quantum mechanics)*

In fact, it is quite remarkable that the LHC detectors —built to measure cross sections— can also be used to investigate notions that are central to QIT: *entanglement* and *Bell inequality violation*.

Entanglement and Bell Inequality violation

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

- iv postulate: $\mathscr{H}_{A \cup B} = \mathscr{H}_A \otimes \mathscr{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle$ can describe $(A \cup B)$

  $|a_i\rangle \in \mathscr{H}_A, \, |b_i\rangle \in \mathscr{H}_B$

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

- iv postulate: $\mathscr{H}_{A\cup B} = \mathscr{H}_A \otimes \mathscr{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle$ can describe $(A \cup B)$
  $|a_i\rangle \in \mathscr{H}_A,\ |b_i\rangle \in \mathscr{H}_B$
- superposition: $|\psi\rangle = \sum_i c_i |n_i\rangle$ can also describe $(A \cup B)$

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

- iv postulate: $\mathscr{H}_{A\cup B} = \mathscr{H}_A \otimes \mathscr{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle$ can describe $(A \cup B)$

  $|a_i\rangle \in \mathscr{H}_A,\ |b_i\rangle \in \mathscr{H}_B$
- superposition: $|\psi\rangle = \sum_i c_i |n_i\rangle$ can also describe $(A \cup B)$

The subsystems A and B are *entangled if the (pure) state* $|\psi\rangle$ *of the system:*

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \quad \forall\, |\psi_A\rangle \in \mathcal{H}_A,\ |\psi_B\rangle \in \mathcal{H}_B$$

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

- iv postulate: $\mathscr{H}_{A\cup B} = \mathscr{H}_A \otimes \mathscr{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle$ can describe $(A \cup B)$
  $|a_i\rangle \in \mathscr{H}_A,\ |b_i\rangle \in \mathscr{H}_B$
- superposition: $|\psi\rangle = \sum_i c_i |n_i\rangle$ can also describe $(A \cup B)$

The subsystems A and B are *entangled if the (pure) state $|\psi\rangle$ of the system:*

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \quad \forall\, |\psi_A\rangle \in \mathcal{H}_A,\ |\psi_B\rangle \in \mathcal{H}_B$$

For a *mixed state,* described by a *density matrix ρ*, this generalizes to

$$\rho \neq \sum_{ij} p_{ij}\, \rho_i^{(A)} \otimes \rho_j^{(B)}\ , \quad with \quad p_{ij} > 0 \quad and \quad \sum_{ij} p_{ij} = 1$$

Entanglement and Bell Inequality violation

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

- iv postulate: $\mathscr{H}_{A\cup B} = \mathscr{H}_A \otimes \mathscr{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle$ can describe $(A \cup B)$
  $|a_i\rangle \in \mathscr{H}_A,\ |b_i\rangle \in \mathscr{H}_B$
- superposition: $|\psi\rangle = \sum_i c_i\, |n_i\rangle$ can also describe $(A \cup B)$

The subsystems A and B are *entangled if the (pure) state $|\psi\rangle$ of the system:*

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \quad \forall\, |\psi_A\rangle \in \mathcal{H}_A,\ |\psi_B\rangle \in \mathcal{H}_B$$

For a *mixed state,* described by a *density matrix ρ*, this generalizes to

$$\rho \neq \sum_{ij} p_{ij}\, \rho_i^{(A)} \otimes \rho_j^{(B)}\ , \quad with \quad p_{ij} > 0 \quad and \quad \sum_{ij} p_{ij} = 1$$

Physically, *entanglement is the hallmark of quantum mechanics* as classical configurations are described by product states.

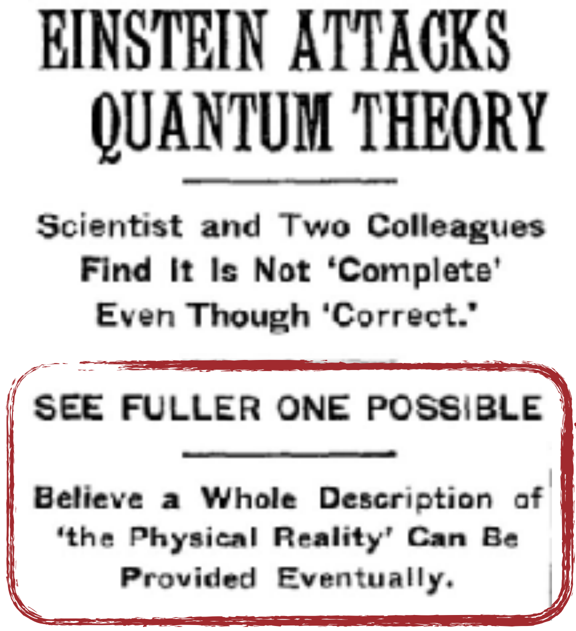
EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

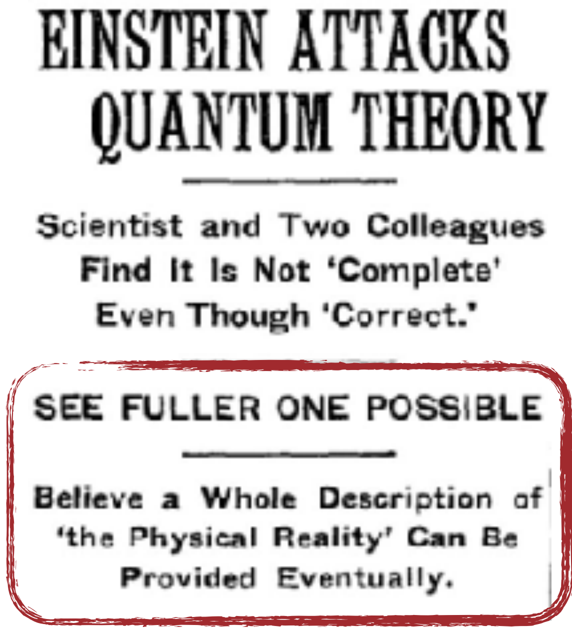
SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

Hidden-variable theories, built on two pillars

- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

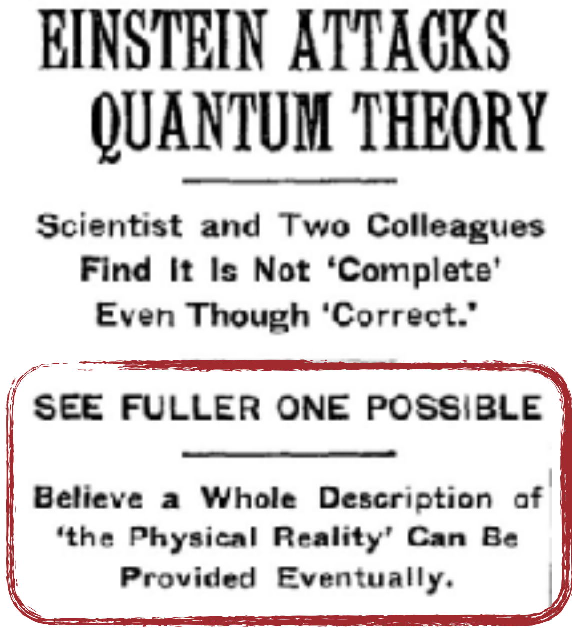
SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

Hidden-variable theories, built on two pillars

- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*
- *Locality*: for independent measurements it has to hold $P(A \cap B) = P(A)P(B)$ — *no action at a distance!*



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

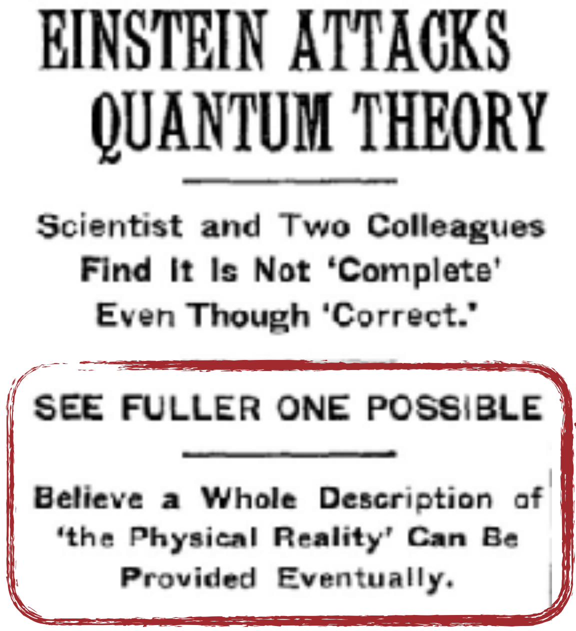
Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

Hidden-variable theories, built on two pillars

- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*
- *Locality*: for independent measurements it has to hold $P(A \cap B) = P(A)P(B)$ – *no action at a distance!*

So, is *quantum mechanics incomplete*?

This was the question until 1964, when J. Bell identified an objective way to distinguish between the two frameworks.


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

Hidden-variable theories, built on two pillars

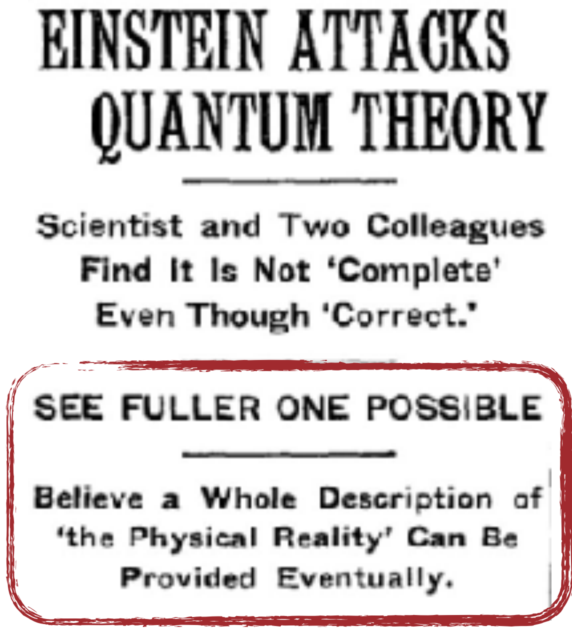
- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*
- *Locality*: for independent measurements it has to hold $P(A \cap B) = P(A)P(B)$ — *no action at a distance!*

So, is *quantum mechanics incomplete*?

This was the question until 1964, when J. Bell identified an objective way to distinguish between the two frameworks.

Two *independent observers (A, B)* have, each, *two observables* at their disposal ($\hat{A}_1, \hat{A}_2$ and $\hat{B}_1, \hat{B}_2$) all with possible outcomes 0 or 1. They test a *bipartite system* and look at the combination of expectation values (i.e. combination of average probabilities) given by (CHSH version)

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle$$


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

Hidden-variable theories, built on two pillars

- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*
- *Locality*: for independent measurements it has to hold $P(A \cap B) = P(A)P(B)$ – *no action at a distance!*

So, is *quantum mechanics incomplete*?

This was the question until 1964, when J. Bell identified an objective way to distinguish between the two frameworks.

Two *independent observers (A, B)* have, each, *two observables* at their disposal ($\hat{A}_1, \hat{A}_2$ and $\hat{B}_1, \hat{B}_2$) all with possible outcomes 0 or 1. They test a *bipartite system* and look at the combination of expectation values (i.e. combination of average probabilities) given by (CHSH version)

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle$$

Theorem (*Bell*): *if locality and realism hold, then* $\mathcal{I}_2 \leq 2$.

- When we compute the same quantity with the rules of *quantum mechanics we obtain* $\mathcal{I}_2 \leq 2\sqrt{2}$, hence *measuring* $2 < \mathcal{I}_2 \leq 2\sqrt{2}$ *would strongly favor quantum mechanics over hidden-variable theories.*

KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

4 October 2022

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2022 to

Alain Aspect
Institut d'Optique Graduate School – Université Paris-Saclay and École Polytechnique, Palaiseau, France

John F. Clauser
J.F. Clauser & Assoc., Walnut Creek, CA, USA

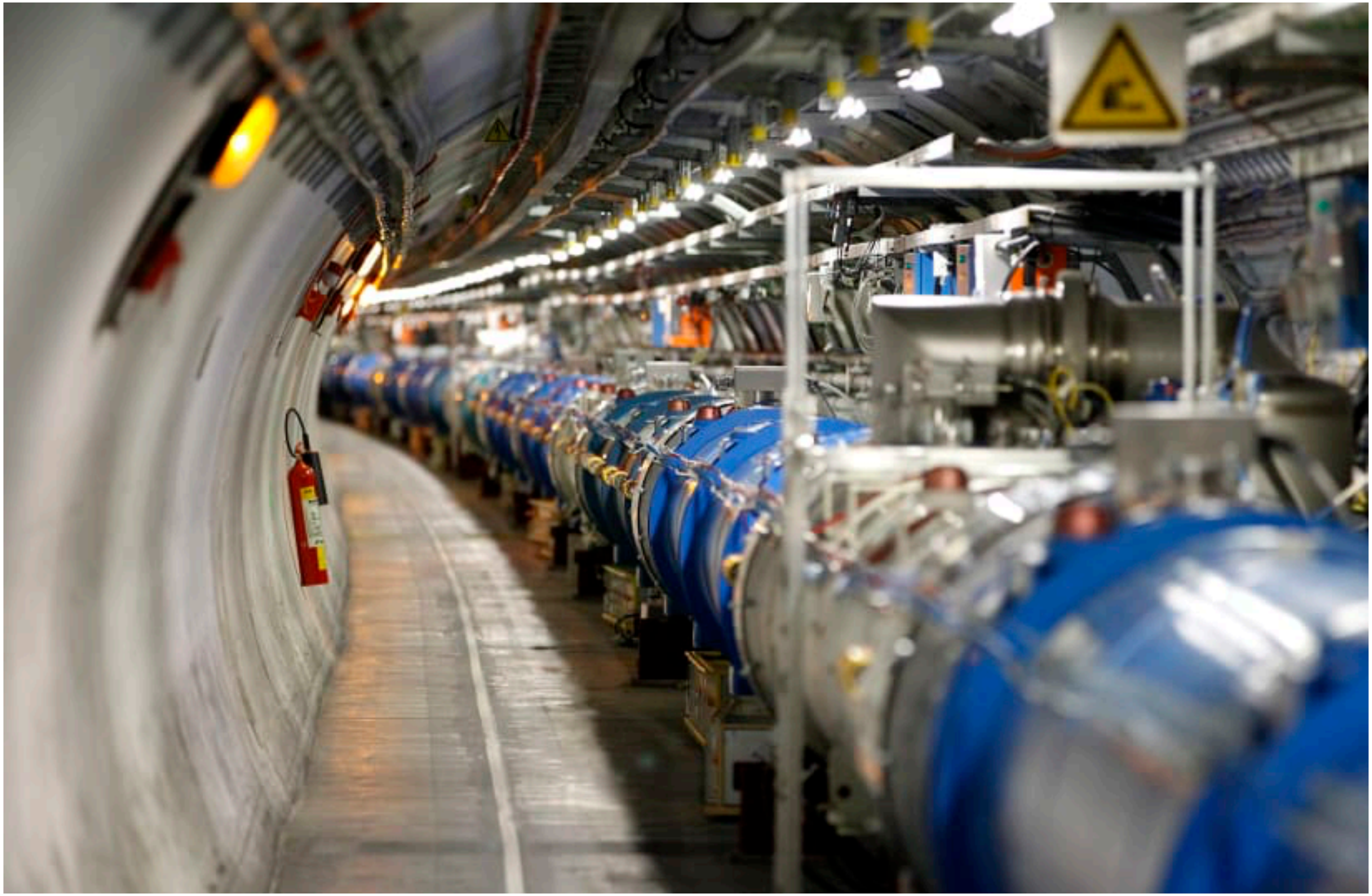
Anton Zeilinger
University of Vienna, Austria

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

↓

Quantum mechanics is NOT incomplete!

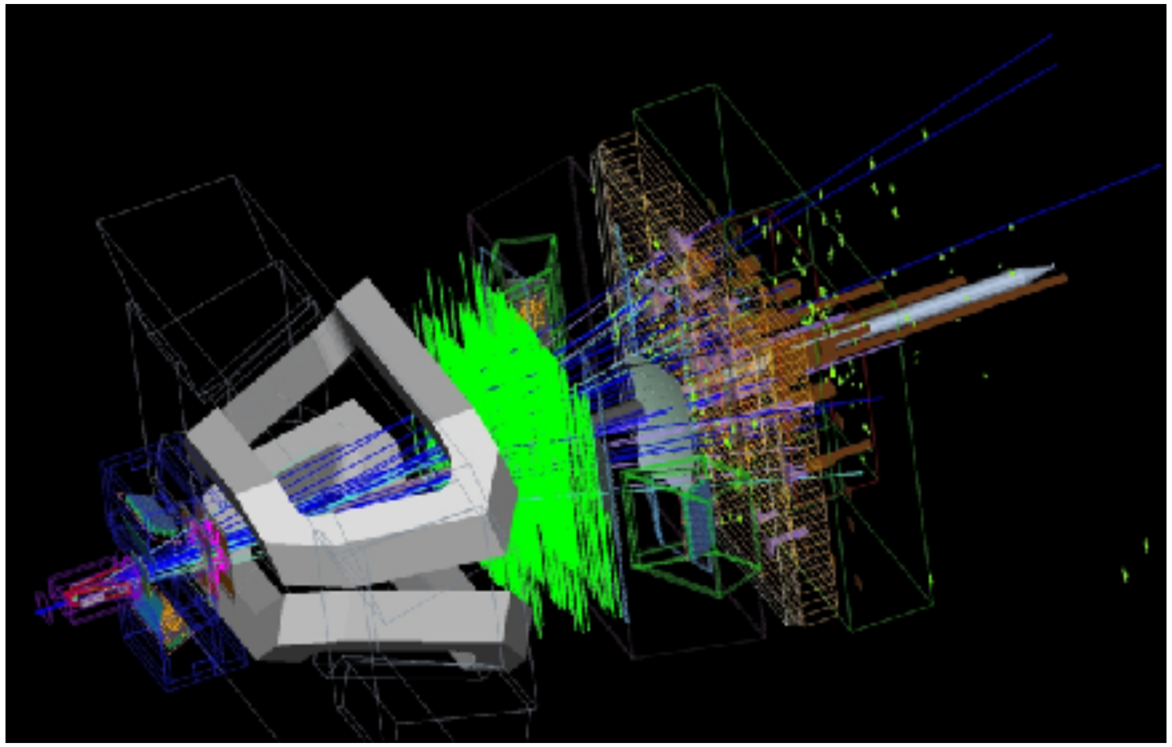
Can we test this stuff at colliders?



The case of a B meson that walks into a bar…

The case of a B meson that walks into a bar…

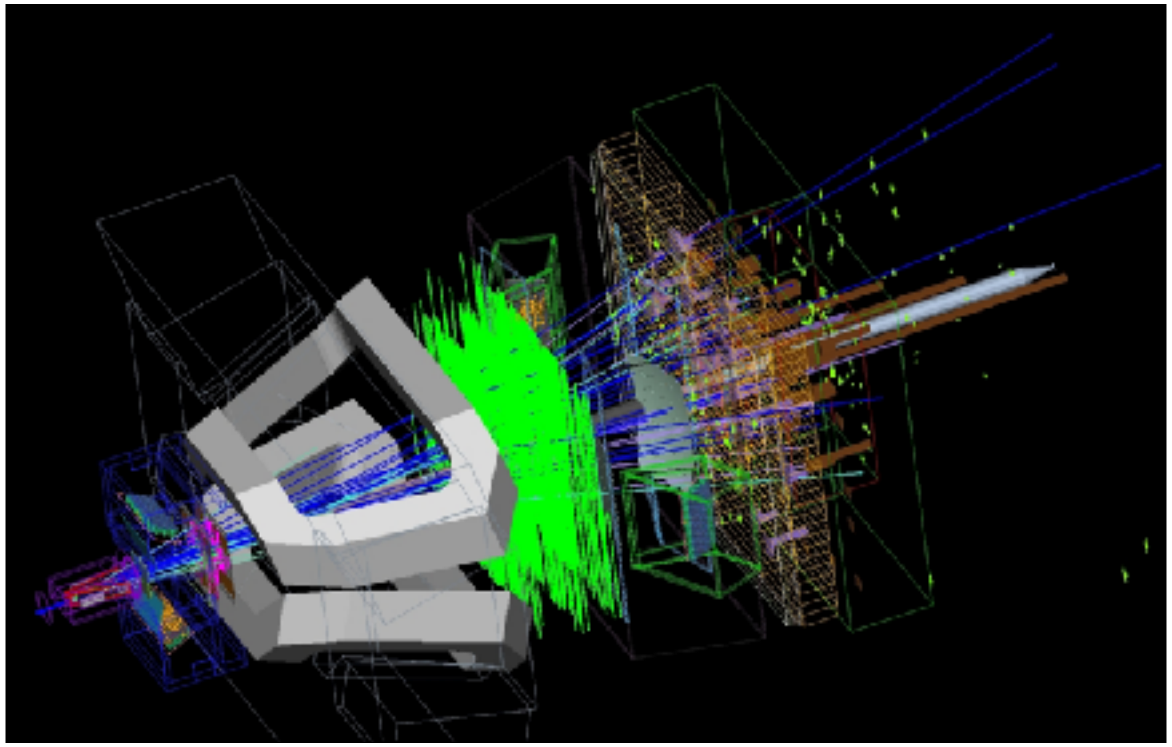
… and decays into two vector mesons. It happens plenty of times at the LHCb(ar).



LHCb Monte Carlo: $B_s \to J/\psi\ \phi$ — stolen from the LHCb website

The case of a B meson that walks into a bar…

… and decays into two vector mesons. It happens plenty of times at the LHCb(ar).

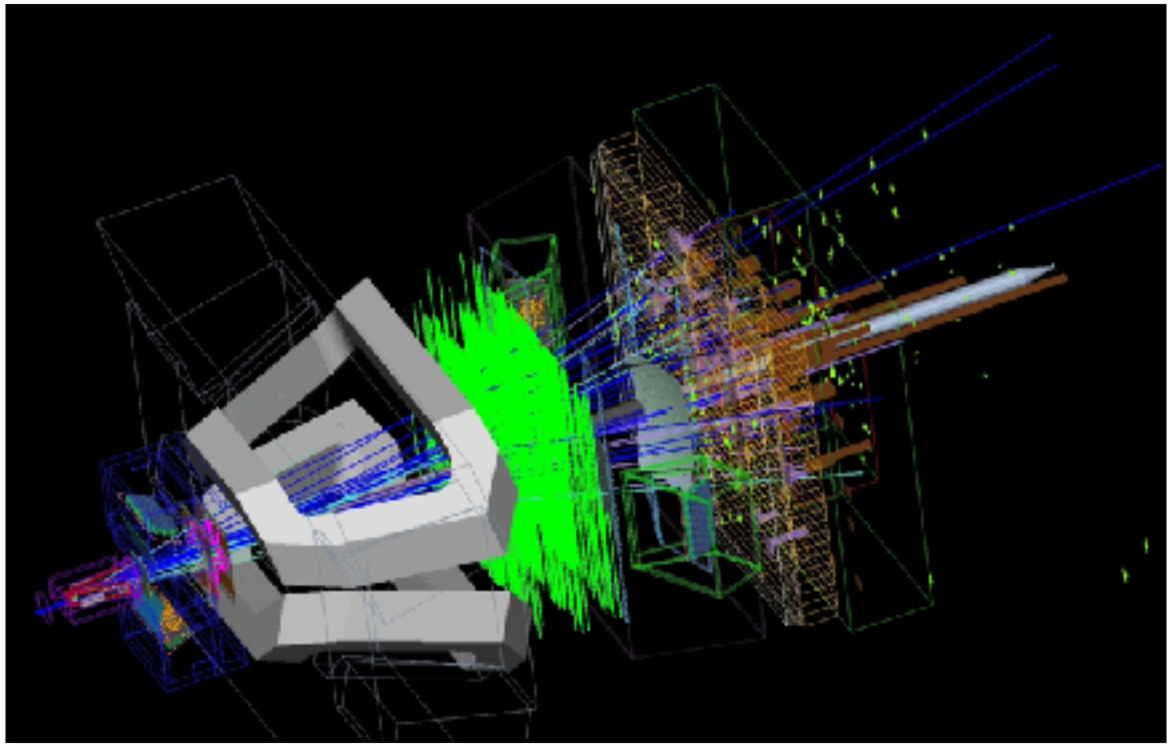


LHCb Monte Carlo: $B_s \rightarrow J/\psi\ \phi$ — stolen from the LHCb website

Particle	*Mass/ GeV*	*Quark content*	J^P
B^0	5.279	$d\bar{b}$	0^-
B_s	5.366	$s\bar{b}$	0^-

The case of a B meson that walks into a bar…

… and decays into two vector mesons. It happens plenty of times at the LHCb(ar).



LHCb Monte Carlo: $B_s \to J/\psi\, \phi$ — stolen from the LHCb website

Particle	*Mass/ GeV*	*Quark content*	J^P
B^0	5.279	$d\bar{b}$	0^-
B_s	5.366	$s\bar{b}$	0^-
J/ψ	3.097	$c\bar{c}$	1^-
ϕ	1.019	$s\bar{s}$	1^-
ρ^0	0.770	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	1^-
$K^*(0.892)^0$	0.892	$d\bar{s}$	1^-

We focus on these decays:

$B^0 \to J/\psi\, K^*(892)^0$ — *R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)*

$B^0 \to \phi\, K^*(892)^0$ — *K. F. Chen et al. [Belle], Phys. Rev. Lett. 94, 221804 (2005)*

$B^0 \to \rho\, K^*(892)^0$ — *R. Aaij et al. [LHCb], JHEP 05, 026 (2019)*

$B_s \to \phi\, \phi$ — *R. Aaij et al. [LHCb], [arXiv:2304.06198 [hep-ex]].*

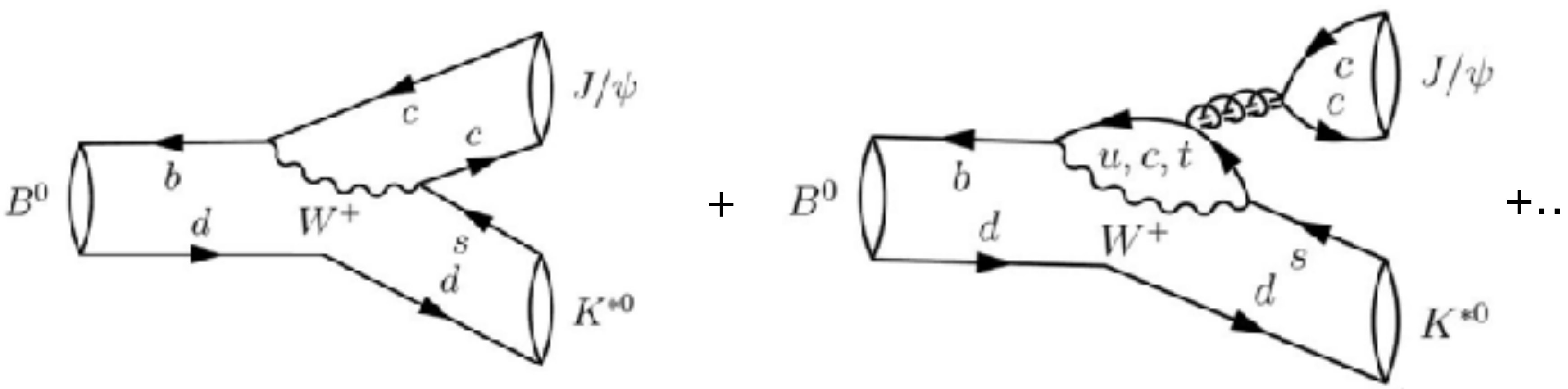
$B_s \to J/\psi\, \phi$ — *G. Aad et al. [ATLAS], Eur. Phys. J. C 81, no.4, 342 (2021)*

The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*

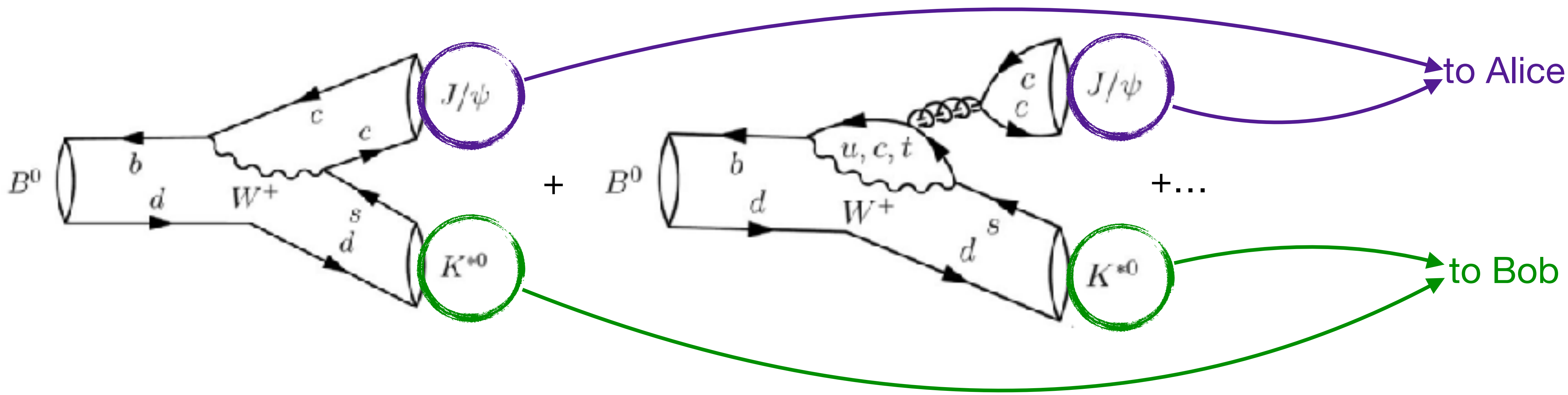
The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*



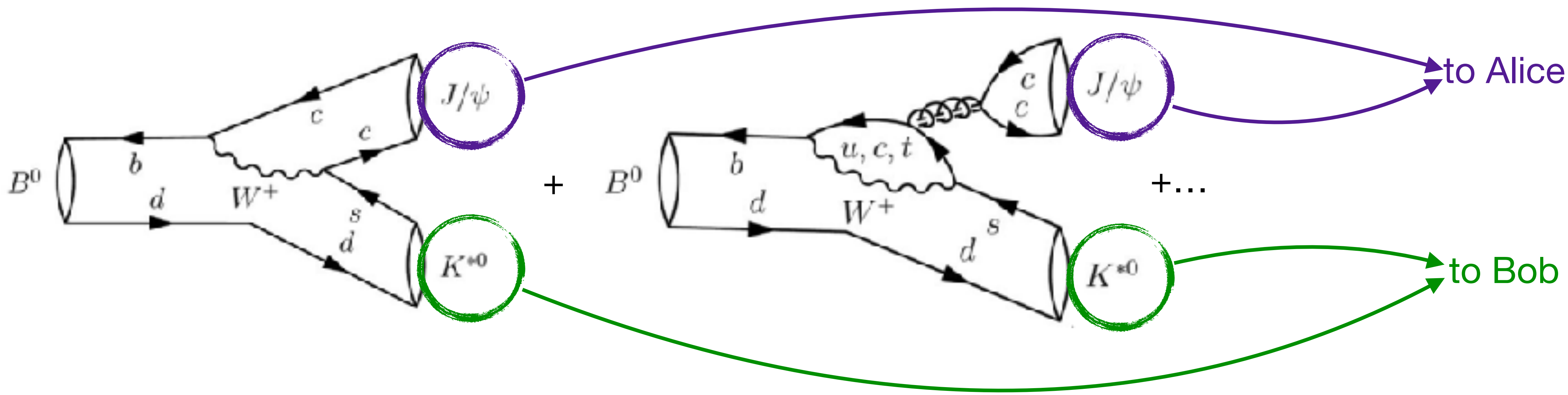
The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*



The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*



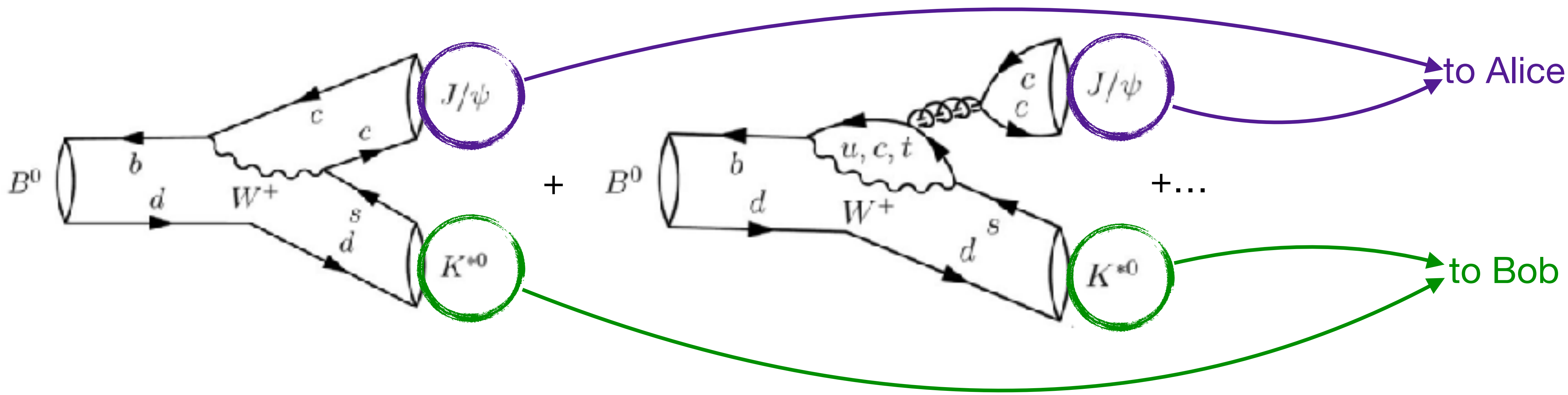
Polarizations and spin correlations can be reconstructed experimentally, from the decays of the spin-1 particles. This yields the *density matrix*

Gell-Mann matrices

$$\rho_{1\otimes 1} = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a\left[T^a\otimes\mathbb{1}\right] + \sum_a g_a\left[\mathbb{1}\otimes T^a\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]$$

The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*



Polarizations and spin correlations can be reconstructed experimentally, from the decays of the spin-1 particles. This yields the *density matrix*

Gell-Mann matrices

$$\rho_{1\otimes 1} = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a \left[T^a \otimes \mathbb{1}\right] + \sum_a g_a \left[\mathbb{1}\otimes T^a\right] + \sum_{ab} h_{ab} \left[T^a \otimes T^b\right]$$

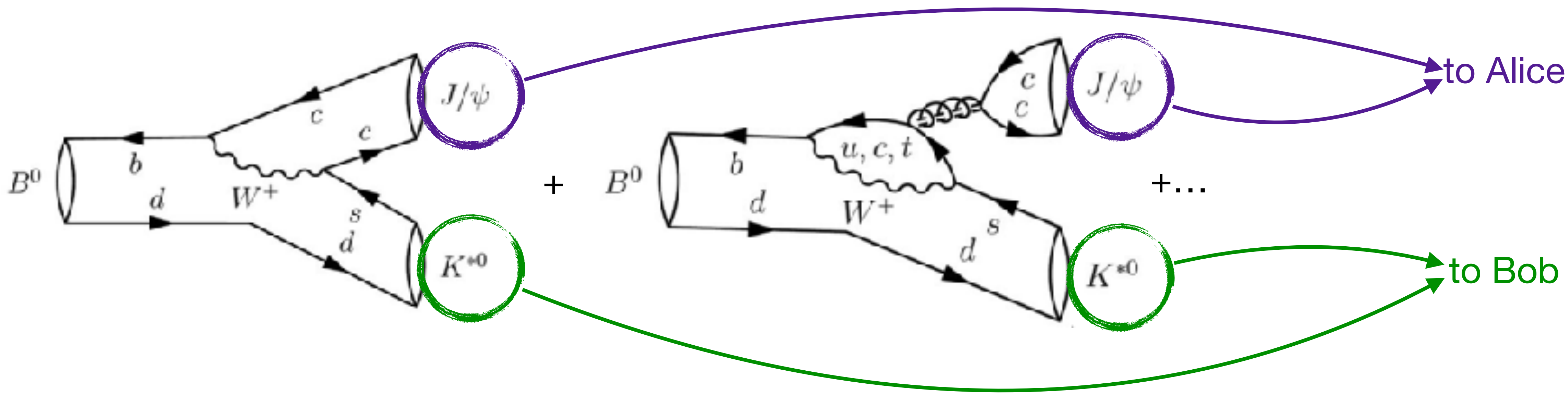
Information about vector and tensor polarizations

spin correlations

Entanglement?

The quantum info side of B meson decays

Spin-one objects are qutrits, hence the B meson decays produce *bipartite qutrit systems*



Polarizations and spin correlations can be reconstructed experimentally, from the decays of the spin-1 particles. This yields the *density matrix*

Gell-Mann matrices

$$\rho_{1\otimes 1} = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a \left[T^a \otimes \mathbb{1}\right] + \sum_a g_a \left[\mathbb{1}\otimes T^a\right] + \sum_{ab} h_{ab}\left[T^a \otimes T^b\right]$$

Information about vector and tensor polarizations   *spin correlations*

Entanglement? Pure state needs: $\rho_{1\otimes 1} \neq \rho_1 \otimes \rho_1 \iff h_{ab} \neq f_a \otimes g_b$

A pure bipartite qutrit system

Because the B meson is a (pseudo)scalar, the spin state of the vector bosons V_1 and V_2 emitted in its decays is uniquely “prepared” — it is *pure*:

A pure bipartite qutrit system

Because the B meson is a (pseudo)scalar, the spin state of the vector bosons V_1 and V_2 emitted in its decays is uniquely "prepared" — it is *pure*:

$$|\Psi\rangle \; = \; \frac{1}{\sqrt{|H|^2}}\Big[h_+\,|V_1(+)V_2(-)\rangle \, + \; h_0\,|V_1(0)V_2(0)\rangle + h_-\,|V_1(-)V_2(+)\rangle\Big]$$

J.A. Aguilar-Saavedra, A. Bernal, J.A. Casas and J.M. Moreno, Phys.Rev.D 107 (2023)
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Eur.Phys.J.C (2023) 83:823
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, JHEP 09 (2023) 195

A pure bipartite qutrit system

Because the B meson is a (pseudo)scalar, the spin state of the vector bosons V_1 and V_2 emitted in its decays is uniquely "prepared" — it is *pure*:

$$|\Psi\rangle \; = \; \frac{1}{\sqrt{|H|^2}}\Big[h_+\,|V_1(+)V_2(-)\rangle \; + \; h_0\,|V_1(0)V_2(0)\rangle + h_-\,|V_1(-)V_2(+)\rangle\Big]$$

J.A. Aguilar-Saavedra, A. Bernal, J.A. Casas and J.M. Moreno, Phys.Rev.D 107 (2023)
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Eur.Phys.J.C (2023) 83:823
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, JHEP 09 (2023) 195

The weights of the components are the *helicity amplitudes*

$$h_\lambda = \langle V_1(\lambda)V_2(-\lambda)|\mathcal{H}|B\rangle \qquad\qquad |H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

with $\mathcal{H}$ being the interaction Hamiltonian and $\lambda \in \{+,0,-\}$ denoting the spin state with respect to the quantization axis of one of the produced vector boson in its rest frame.

A pure bipartite qutrit system

Because the B meson is a (pseudo)scalar, the spin state of the vector bosons V_1 and V_2 emitted in its decays is uniquely "prepared" — it is *pure*:

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\Big[h_+\,|V_1(+)V_2(-)\rangle + h_0\,|V_1(0)V_2(0)\rangle + h_-\,|V_1(-)V_2(+)\rangle\Big]$$

J.A. Aguilar-Saavedra, A. Bernal, J.A. Casas and J.M. Moreno, Phys.Rev.D 107 (2023)
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Eur.Phys.J.C (2023) 83:823
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, JHEP 09 (2023) 195

The weights of the components are the *helicity amplitudes*

$$h_\lambda = \langle V_1(\lambda)V_2(-\lambda)|\mathcal{H}|B\rangle \qquad |H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

with $\mathcal{H}$ being the interaction Hamiltonian and $\lambda \in \{+,0,-\}$ denoting the spin state with respect to the quantization axis of one of the produced vector boson in its rest frame.

Experimentalists measure the *polarization amplitudes A_0, $A_\parallel$, $A_\perp$*

$$\frac{h_0}{|H|} = A_0\,, \quad \frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}}\,, \quad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}$$

and so we can easily reconstruct the density matrix

Quantum tomography @ LHCb:

Quantum tomography @ LHCb:

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum tomography @ LHCb:

$$B^0 \to J/\psi\, K^*(892)^0$$

LHCb ↓

$$|A_\parallel|^2 = 0.227 \pm 0.004\ (\text{stat}) \pm 0.011\ (\text{sys})$$
$$|A_\perp|^2 = 0.201 \pm 0.004\ (\text{stat}) \pm 0.008\ (\text{sys})$$
$$\delta_\parallel\ [\text{rad}] = -2.94 \pm 0.02\ (\text{stat}) \pm 0.03\ (\text{sys})$$
$$\delta_\perp\ [\text{rad}] = 2.94 \pm 0.02\ (\text{stat}) \pm 0.02\ (\text{sys})$$

R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum tomography @ LHCb:

$$B^0 \to J/\psi\, K^*(892)^0$$

LHCb

$$|A_\parallel|^2 = 0.227 \pm 0.004\ (\text{stat}) \pm 0.011\ (\text{sys})$$
$$|A_\perp|^2 = 0.201 \pm 0.004\ (\text{stat}) \pm 0.008\ (\text{sys})$$
$$\delta_\parallel\ [\text{rad}] = -2.94 \pm 0.02\ (\text{stat}) \pm 0.03\ (\text{sys})$$
$$\delta_\perp\ [\text{rad}] = 2.94 \pm 0.02\ (\text{stat}) \pm 0.02\ (\text{sys})$$

R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}} \qquad \frac{h_0}{|H|} = A_0 \qquad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Quantum tomography @ LHCb:

$$B^0 \to J/\psi\, K^*(892)^0$$

LHCb

$$|A_{\parallel}|^2 = 0.227 \pm 0.004\ (\text{stat}) \pm 0.011\ (\text{sys})$$
$$|A_{\perp}|^2 = 0.201 \pm 0.004\ (\text{stat}) \pm 0.008\ (\text{sys})$$
$$\delta_{\parallel}\ [\text{rad}] = -2.94 \pm 0.02\ (\text{stat}) \pm 0.03\ (\text{sys})$$
$$\delta_{\perp}\ [\text{rad}] = 2.94 \pm 0.02\ (\text{stat}) \pm 0.02\ (\text{sys})$$

R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_+}{|H|} = \frac{A_{\parallel} + A_{\perp}}{\sqrt{2}} \qquad \frac{h_0}{|H|} = A_0 \qquad \frac{h_-}{|H|} = \frac{A_{\parallel} - A_{\perp}}{\sqrt{2}}$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Once ρ is reconstructed we can *probe entanglement*. Choose your favorite *monotone/measure*.

Quantum tomography @ LHCb:

$$B^0 \to J/\psi\, K^*(892)^0$$

LHCb

$$|A_{\parallel}|^2 = 0.227 \pm 0.004\ (\text{stat}) \pm 0.011\ (\text{sys})$$
$$|A_{\perp}|^2 = 0.201 \pm 0.004\ (\text{stat}) \pm 0.008\ (\text{sys})$$
$$\delta_{\parallel}\ [\text{rad}] = -2.94 \pm 0.02\ (\text{stat}) \pm 0.03\ (\text{sys})$$
$$\delta_{\perp}\ [\text{rad}] = 2.94 \pm 0.02\ (\text{stat}) \pm 0.02\ (\text{sys})$$

R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_+}{|H|} = \frac{A_{\parallel} + A_{\perp}}{\sqrt{2}} \qquad \frac{h_0}{|H|} = A_0 \qquad \frac{h_-}{|H|} = \frac{A_{\parallel} - A_{\perp}}{\sqrt{2}}$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Once ρ is reconstructed we can *probe entanglement*. Choose your favorite *monotone/measure*.

For pure states we can use the *entropy of entanglement*, given by the *von Neumann entropy* of either of the composing subsystems A and B:

$$\mathscr{E} = -\,\text{Tr}[\rho_{V_1} \log \rho_{V_1}] = -\,\text{Tr}[\rho_{V_2} \log \rho_{V_2}] \qquad \rho_{V_{1(2)}} = \text{Tr}_{V_{2(1)}}\, \rho_{1\otimes 1}$$

Quantum tomography @ LHCb:

$$B^0 \to J/\psi\, K^*(892)^0$$

LHCb

$$|A_\parallel|^2 = 0.227 \pm 0.004\ (\text{stat}) \pm 0.011\ (\text{sys})$$
$$|A_\perp|^2 = 0.201 \pm 0.004\ (\text{stat}) \pm 0.008\ (\text{sys})$$
$$\delta_\parallel\ [\text{rad}] = -2.94 \pm 0.02\ (\text{stat}) \pm 0.03\ (\text{sys})$$
$$\delta_\perp\ [\text{rad}] = 2.94 \pm 0.02\ (\text{stat}) \pm 0.02\ (\text{sys})$$

R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}} \qquad \frac{h_0}{|H|} = A_0 \qquad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Once ρ is reconstructed we can *probe entanglement*. Choose your favorite *monotone/measure*.

For pure states we can use the *entropy of entanglement*, given by the *von Neumann entropy* of either of the composing subsystems A and B:

$$\mathscr{E} = -\,\text{Tr}[\rho_{V_1} \log \rho_{V_1}] = -\,\text{Tr}[\rho_{V_2} \log \rho_{V_2}] \qquad \rho_{V_{1(2)}} = \text{Tr}_{V_{2(1)}}\, \rho_{1\otimes 1}$$

The entropy of entanglement is a measure satisfying $0 \le \mathscr{E} \le \log 3$ and

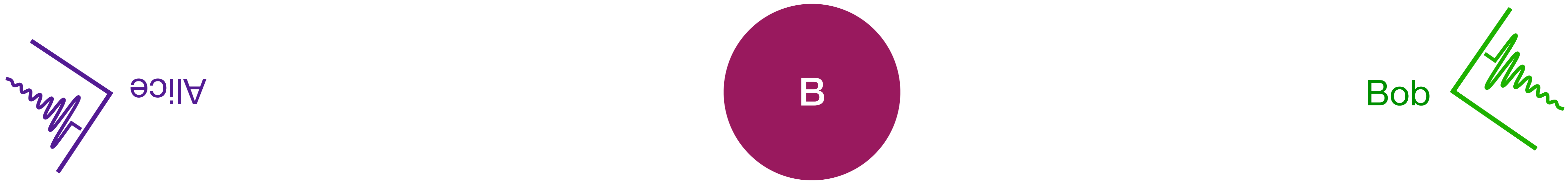
$$\mathscr{E} > 0 \Longleftrightarrow \textit{entangled state}$$

LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…

LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…



LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…

…doing cryptography and Bell tests since ’78. For a bipartite qutrit system, they usually rely on the *CGLMP inequality*. Alice and Bob do *two independent projective measurements* each, with possible outcome {0,1,2} (i.e. {+, 0, -}).

LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…

…doing cryptography and Bell tests since '78. For a bipartite qutrit system, they usually rely on the *CGLMP inequality*. Alice and Bob do *two independent projective measurements* each, with possible outcome {0,1,2} (i.e. {+, 0, -}).

Then we compute

$$\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$- P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$$

D. Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002).

LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…

…doing cryptography and Bell tests since '78. For a bipartite qutrit system, they usually rely on the *CGLMP inequality*. Alice and Bob do *two independent projective measurements* each, with possible outcome {0,1,2} (i.e. {+, 0, -}).

Then we compute

Probability that the second result obtained by Bob differs from the first one of Alice by a -1(mod 3)

$$\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$$

D. Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002).

LHCb(ell?)

Remember Alice and Bob from 3 slides ago? They have been busy…

…doing cryptography and Bell tests since '78. For a bipartite qutrit system, they usually rely on the *CGLMP inequality*. Alice and Bob do *two independent projective measurements* each, with possible outcome {0,1,2} (i.e. {+, 0, -}).

Then we compute

Probability that the second result obtained by Bob differs from the first one of Alice by a -1(mod 3)

$$\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$- P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$$

D. Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002).

Deterministic local models always satisfy $\mathcal{I}_3 \leq 2$ but *quantum mechanics may violate that bound!* To help seeing the effect we can maximize on Alice and Bob…

Alice and Bob know the density matrix formalism, so they compute

Alice and Bob know the density matrix formalism, so they compute

$$\mathcal{I}_3 = \mathrm{Tr}[\rho \mathcal{B}]$$

$$\mathcal{B} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\
0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\
0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

A. Acin, T. Durt, N. Gisin, and J.I. Latorre, Physical Review A, 65(5):052325, 2002.

Alice and Bob know the density matrix formalism, so they compute

$$\mathcal{I}_3 = \mathrm{Tr}[\rho\mathcal{B}]$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A. Acin, T. Durt, N. Gisin, and J.I. Latorre, Physical Review A, 65(5):052325, 2002.

The observable can still be *optimized* by means of *local unitary transformations*, which we apply to Alice and Bob (or to their reference frames).

Alice and Bob know the density matrix formalism, so they compute

$$\mathcal{I}_3 = \mathrm{Tr}[\rho \mathcal{B}]$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A. Acin, T. Durt, N. Gisin, and J.I. Latorre, Physical Review A, 65(5):052325, 2002.

The observable can still be *optimized* by means of *local unitary transformations*, which we apply to Alice and Bob (or to their reference frames).

After reconstructing ρ from the data, we then numerically maximize the observable by using

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

in the computation of I_3:

$$\mathcal{I}_3 = \mathrm{Tr}[\rho \mathcal{B}]$$

Plugging the numbers in

Plugging the numbers in

Process	$\mathscr{E}$ (>0)	$\mathcal{I}_3$ (>2)	#σ	Reference
$B^0 \to J/\psi\, K^*(892)^0$	0.756 ± 0.009	2.548 ± 0.015	≫5	R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)
$B^0 \to \phi\, K^*(892)^0$	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	~1.1	K. F. Chen et al. [Belle], Phys. Rev. Lett. 94, 221804 (2005)
$B^0 \to \rho\, K^*(892)^0$	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	~1.6	R. Aaij et al. [LHCb], JHEP 05, 026 (2019)
$B_s \to \phi\, \phi$	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	>5	R. Aaij et al. [LHCb], [arXiv:2304.06198 [hep-ex]].
$B_s \to J/\psi\, \phi$	0.731 ± 0.032	2.462 ± 0.080	>5	G. Aad et al. [ATLAS], Eur. Phys. J. C 81, no.4, 342 (2021)

*A * indicates that a conservative computation of the error has been employed (the error correlation matrix was not provided)*

Plugging the numbers in

Process	$\mathscr{E}$ (>0)	$\mathcal{I}_3$ (>2)	#σ	Reference
$B^0 \to J/\psi\, K^*(892)^0$	0.756 ± 0.009	2.548 ± 0.015	»5	R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)
$B^0 \to \phi\, K^*(892)^0$	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	~1.1	K. F. Chen et al. [Belle], Phys. Rev. Lett. 94, 221804 (2005)
$B^0 \to \rho\, K^*(892)^0$	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	~1.6	R. Aaij et al. [LHCb], JHEP 05, 026 (2019)
$B_s \to \phi\,\phi$	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	>5	R. Aaij et al. [LHCb], [arXiv:2304.06198 [hep-ex]].
$B_s \to J/\psi\,\phi$	0.731 ± 0.032	2.462 ± 0.080	>5	G. Aad et al. [ATLAS], Eur. Phys. J. C 81, no.4, 342 (2021)

*A * indicates that a conservative computation of the error has been employed (the error correlation matrix was not provided)*

Remarks:

i) the polarization amplitudes are generically complex (*final state interactions*)

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}} \qquad A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}} \qquad A_0 = |A_0| e^{i\delta_0}$$

Plugging the numbers in

Process	$\mathscr{E}$ (>0)	$\mathcal{I}_3$ (>2)	#σ	Reference
$B^0 \to J/\psi\, K^*(892)^0$	0.756 ± 0.009	2.548 ± 0.015	»5	R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)
$B^0 \to \phi\, K^*(892)^0$	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	~1.1	K. F. Chen et al. [Belle], Phys. Rev. Lett. 94, 221804 (2005)
$B^0 \to \rho\, K^*(892)^0$	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	~1.6	R. Aaij et al. [LHCb], JHEP 05, 026 (2019)
$B_s \to \phi\,\phi$	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	>5	R. Aaij et al. [LHCb], [arXiv:2304.06198 [hep-ex]].
$B_s \to J/\psi\,\phi$	0.731 ± 0.032	2.462 ± 0.080	>5	G. Aad et al. [ATLAS], Eur. Phys. J. C 81, no.4, 342 (2021)

*A * indicates that a conservative computation of the error has been employed (the error correlation matrix was not provided)*

Remarks:

i) the polarization amplitudes are generically complex (*final state interactions*)

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}} \qquad A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}} \qquad A_0 = |A_0| \cancel{e^{i\delta_0}}$$

We took the longitudinal phase as the overall phase and killed it.

Plugging the numbers in

Process	$\mathscr{E}$ (>0)	$\mathcal{I}_3$ (>2)	#σ	*Reference*
$B^0 \to J/\psi\, K^*(892)^0$	0.756 ± 0.009	2.548 ± 0.015	≫5	*R. Aaij et al. [LHCb], Phys. Rev. D 88, 052002 (2013)*
$B^0 \to \phi\, K^*(892)^0$	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	~1.1	*K. F. Chen et al. [Belle], Phys. Rev. Lett. 94, 221804 (2005)*
$B^0 \to \rho\, K^*(892)^0$	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	~1.6	*R. Aaij et al. [LHCb], JHEP 05, 026 (2019)*
$B_s \to \phi\, \phi$	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	>5	*R. Aaij et al. [LHCb], [arXiv:2304.06198 [hep-ex]].*
$B_s \to J/\psi\, \phi$	0.731 ± 0.032	2.462 ± 0.080	>5	*G. Aad et al. [ATLAS], Eur. Phys. J. C 81, no.4, 342 (2021)*

*A * indicates that a conservative computation of the error has been employed (the error correlation matrix was not provided)*

Remarks:

i) the polarization amplitudes are generically complex (*final state interactions*)

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}} \qquad A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}} \qquad A_0 = |A_0| \cancel{e^{i\delta_0}}$$

We took the longitudinal phase as the overall phase and killed it. 💀

ii) First proof that *Bell inequalities are violated in strong and weak interactions*

Further results

- *Pairs of top quarks*

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus 136 (2021) 907

Further results

- *Pairs of top quarks*

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus 136 (2021) 907

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\Big(1 - D\cos\phi\Big)$$

Φ is the angle between the emitted lepton as computed in the t and tbar rest frames

D < -1/3 witnesses the presence of entanglement

Further results

- *Pairs of top quarks*

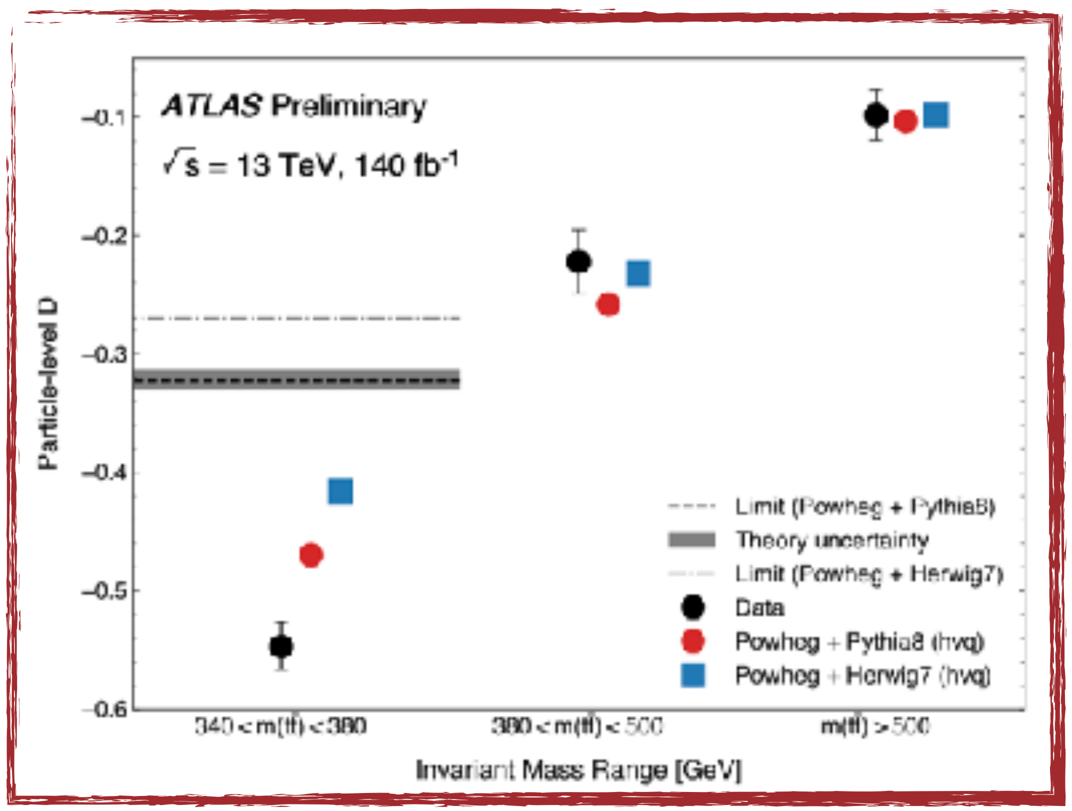
Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus 136 (2021) 907

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right)$$

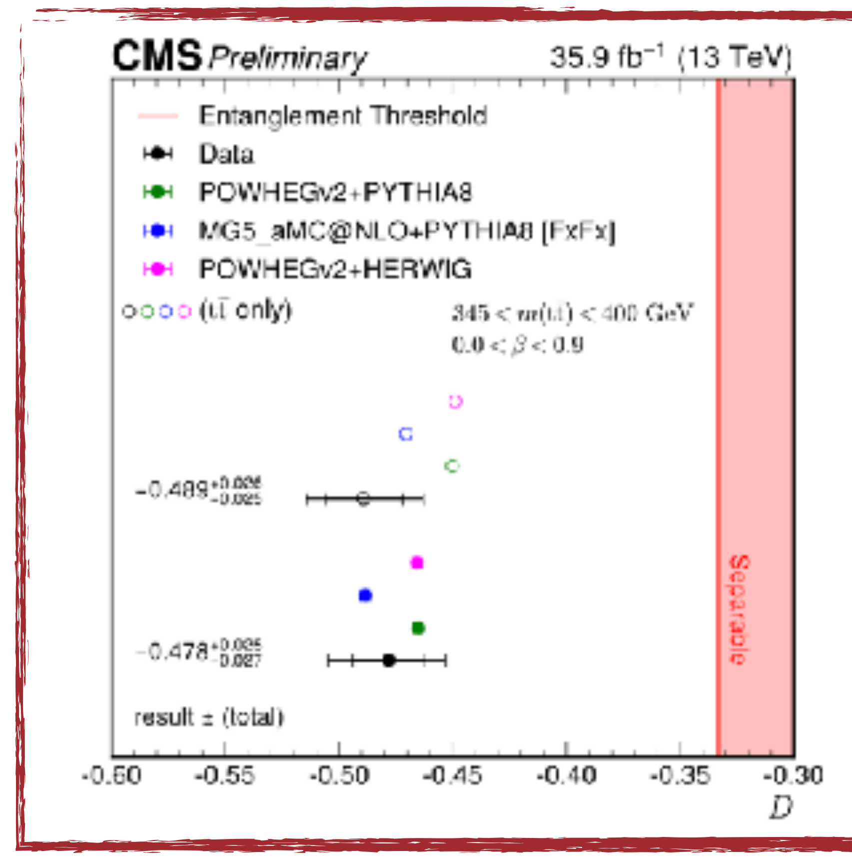
Φ is the angle between the emitted lepton as computed in the t and tbar rest frames

D < -1/3 witnesses the presence of entanglement



$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, Nature 633, 542–547 (2024)



$$D = -0.478^{+0.025}_{-0.027}$$

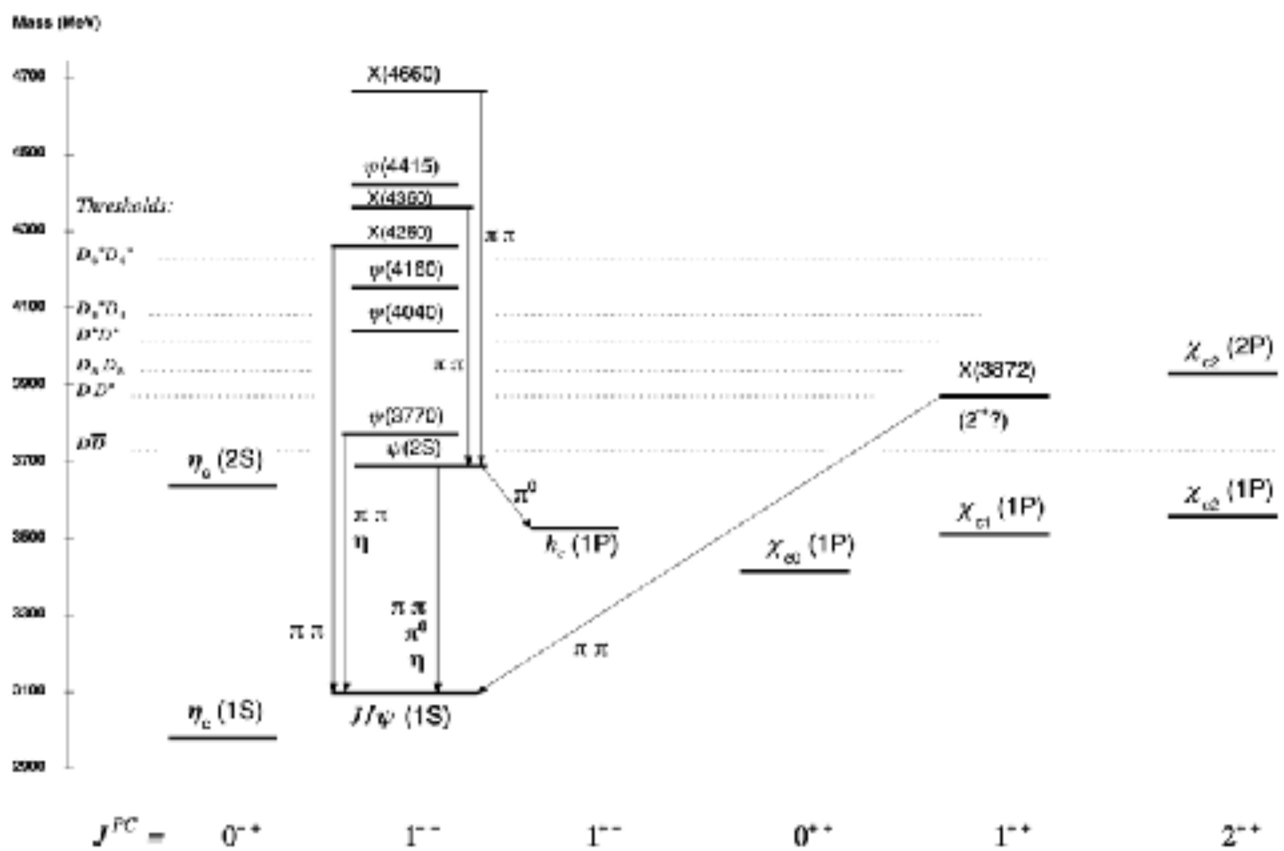
CMS Collaboration, CMS-TOP-23-001

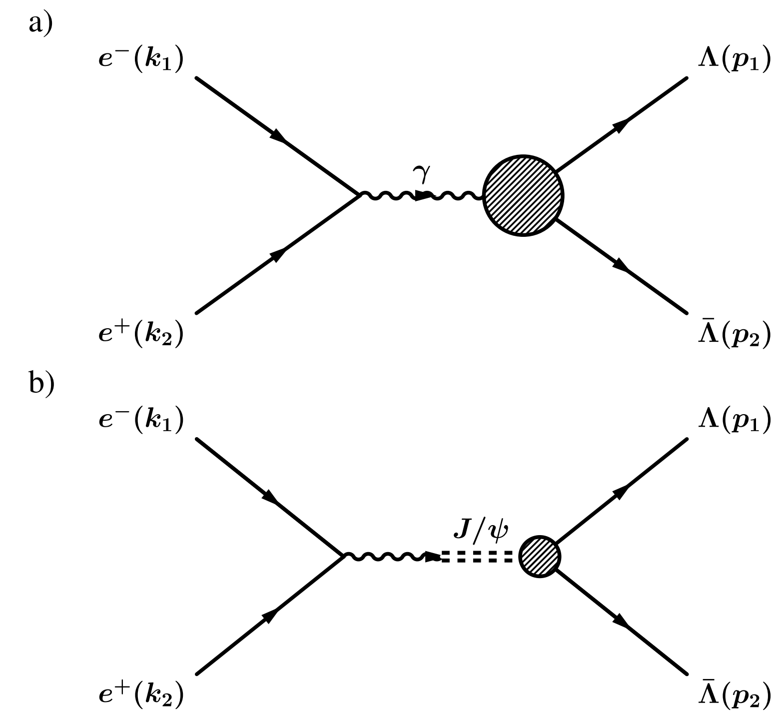
- *Charmonium decays*

M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012

• Charmonium decays

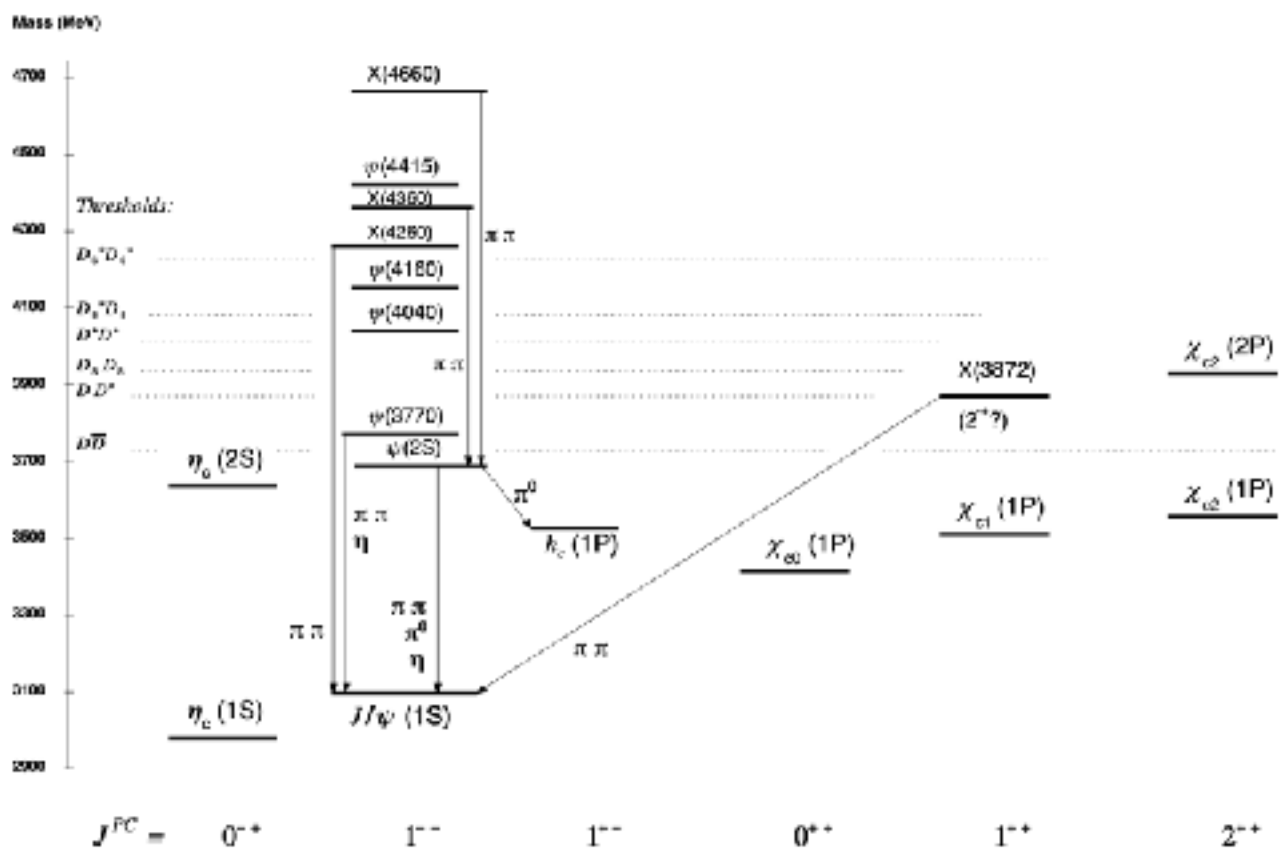
M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012

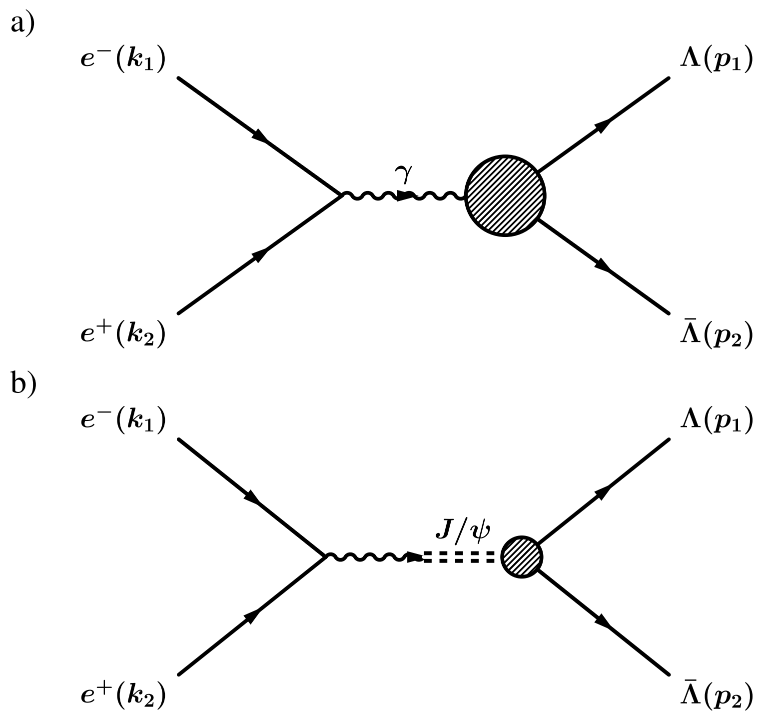




• Charmonium decays

M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012





Qubit final states

$$\rho = \frac{1}{4}\Big[\mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_{i=1}^{3} \mathrm{B}_i^+ (\sigma_i \otimes \mathbb{1}_2) + \sum_{i=1}^{3} \mathrm{B}_j^- (\mathbb{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} \mathrm{C}_{ij} (\sigma_i \otimes \sigma_j)\Big]$$

$$R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$$

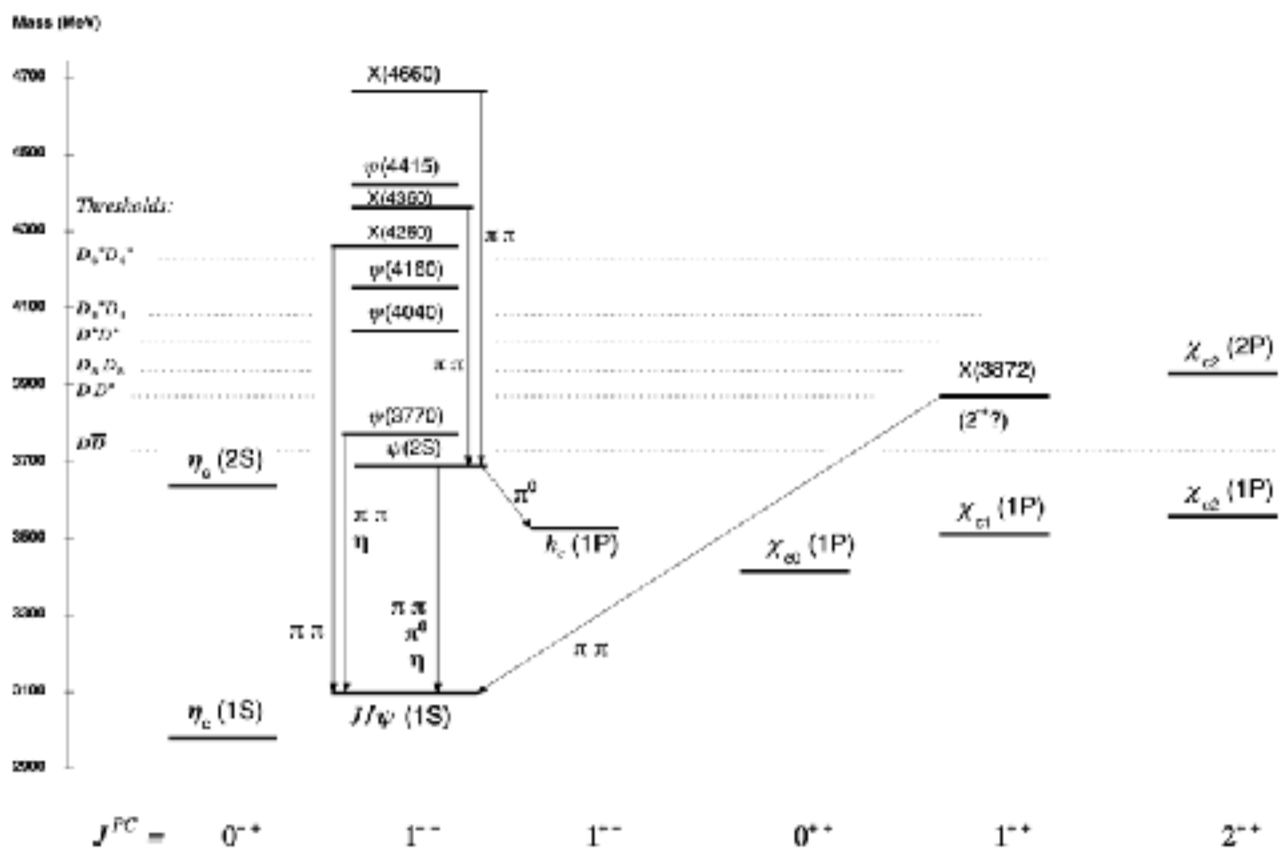
Concurrence $\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$

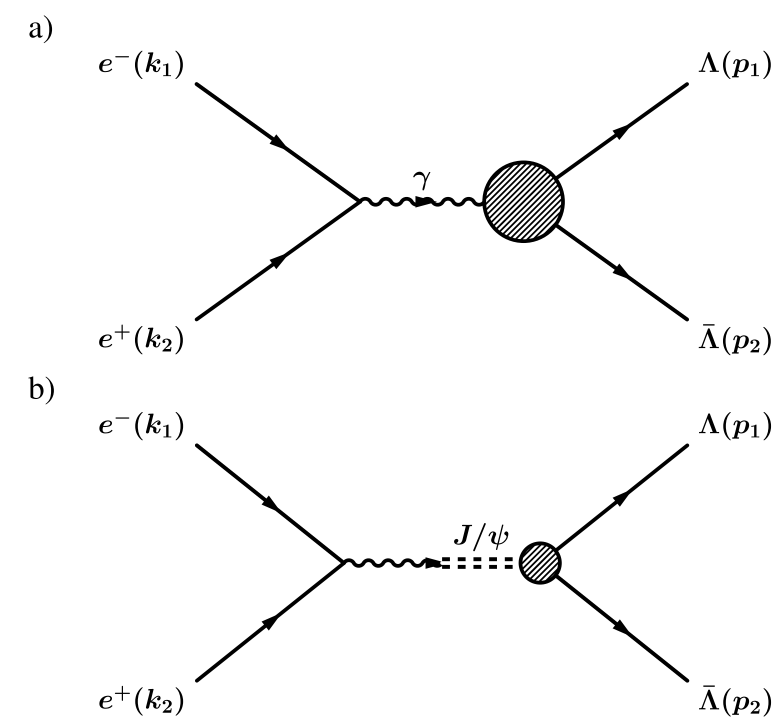
Horodecki condition $CC^T \;\; [m_1,\, m_2,\, m_3]$

$$\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$$

• Charmonium decays

M. Fabbrichesi, R. Floreanini, E. Gabrielli and LM, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012





Qubit final states

$$\rho = \frac{1}{4}\Big[\mathbb{1}_2\otimes\mathbb{1}_2 + \sum_{i=1}^{3} \mathrm{B}_i^{+}(\sigma_i\otimes\mathbb{1}_2) + \sum_{i=1}^{3} \mathrm{B}_j^{-}(\mathbb{1}_2\otimes\sigma_j) + \sum_{i,j=1}^{3} \mathrm{C}_{ij}(\sigma_i\otimes\sigma_j)\Big]$$

$$R = \rho\,(\sigma_y\otimes\sigma_y)\,\rho^*\,(\sigma_y\otimes\sigma_y)$$

Concurrence $\mathscr{C}[\rho] = \max\big(0, r_1 - r_2 - r_3 - r_4\big)$

Horodecki condition $CC^T \;\; [m_1,\, m_2,\, m_3]$

$$\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$$

Qutrits final states

$$\rho = \frac{1}{9}\,[\mathbb{1}_3\otimes\mathbb{1}_3] + \sum_{a=1}^{8} f_a\,[T^a\otimes\mathbb{1}_3] + \sum_{a=1}^{8} g_a\,[\mathbb{1}_3\otimes T^a] + \sum_{a,b=1}^{8} h_{ab}\,\big[T^a\otimes T^b\big]$$

$$\mathscr{C}_2 \;=\; 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,; \quad -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2,\; 0\Big]$$

Bell operator $\mathcal{I}_3 = \mathrm{Tr}\big[\rho\,\mathscr{B}_3\big]$

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\rm stat} \pm 0.019|_{\rm syst}\,.$$

BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$, JHEP* **05** (2023) 069, [arXiv:2301.12922].

$\mathscr{E}[\rho] = 0.531 \pm 0.0021$ (255σ)
(entanglement)

$\mathrm{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.2961 \pm 0.0165$ (18σ)
(Bell inequality violation)

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \pm 0.019|_{\text{syst}}\,.$$

BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$, JHEP* **05** (2023) 069, [arXiv:2301.12922].

$\mathscr{E}[\rho] = 0.531 \pm 0.0021 \quad (255\sigma)$
(entanglement)

$\mathrm{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.2961 \pm 0.0165 \quad (18\sigma)$
(Bell inequality violation)

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \pm 0.019|_{\text{syst}}\,.$$

BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$*, *JHEP* **05** (2023) 069, [arXiv:2301.12922].

$\mathscr{E}[\rho] = 0.531 \pm 0.0021 \;\; (255\sigma)$ (entanglement)

$\text{Tr}\, \rho_{\phi\phi}\, \mathscr{B} = 2.2961 \pm 0.0165 \;\; (18\sigma)$ (Bell inequality violation)

$J/\psi \to \Lambda + \bar{\Lambda}$ **and** $\psi(3686) \to \Lambda + \bar{\Lambda}$

$$|\psi_\uparrow\rangle \;\propto\; w_{\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\psi_\downarrow\rangle \;\propto\; w_{-\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

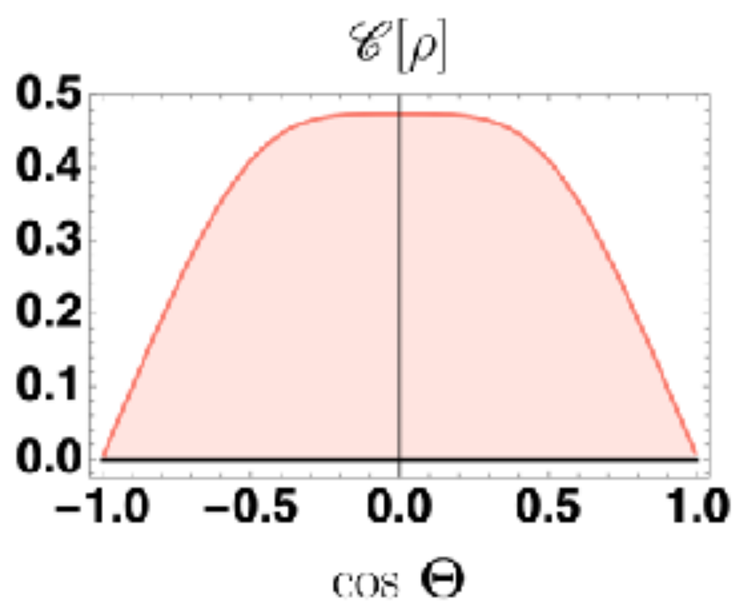
$$|\psi_0\rangle \;\propto\; w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

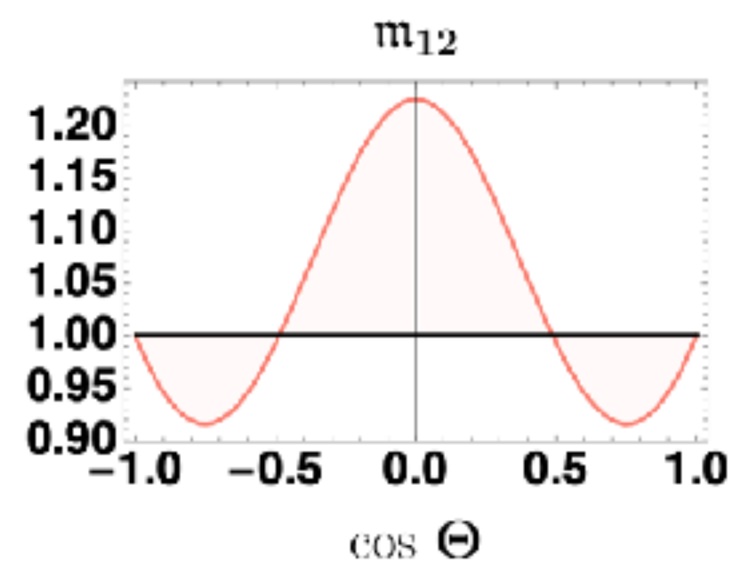
$$w_{\frac{1}{2}\,\frac{1}{2}} = w_{-\frac{1}{2}\,-\frac{1}{2}} = \frac{\sqrt{1-\alpha}}{\sqrt{2}}$$

$$w_{\frac{1}{2}\,-\frac{1}{2}} = w_{-\frac{1}{2}\,\frac{1}{2}} = \sqrt{1+\alpha}\,\exp[-i\Delta\Phi]\,.$$

$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}}$$

$$\Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}}\,.$$





$\mathscr{C} = 0.475 \pm 0.0039 \quad (122\sigma)$ (entanglement)

$\mathfrak{m}_{12} = 1.225 \pm 0.004 \quad (56\sigma)$ (Bell inequality violation)

BESIII Collaboration, M. Ablikim et al., *Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ-$\bar{\Lambda}$ Pairs*, *Phys. Rev. Lett.* **129** (2022), no. 13 131801, [arXiv:2204.11058].

Summary and outlook

Summary and outlook

- It is remarkable that detectors built, essentially, to measure cross section can be used for *quantum tomography* and fully reconstruct *spin correlations* in several collider processes.

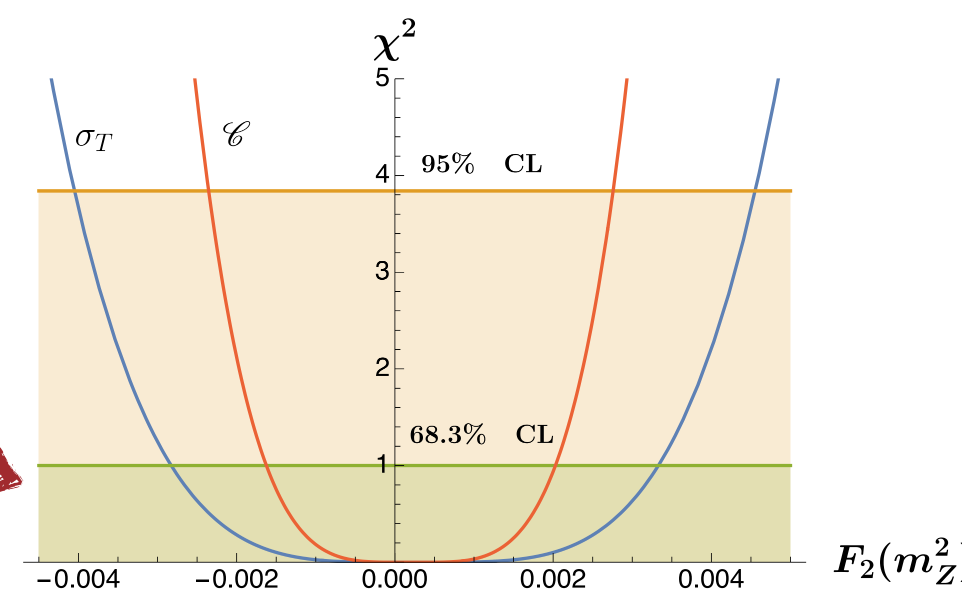
Summary and outlook

- It is remarkable that detectors built, essentially, to measure cross section can be used for *quantum tomography* and fully reconstruct *spin correlations* in several collider processes.
- At the *LHC this is already happening*, giving access to a wealth of *observables (entanglement, discord, magic, steering, Bell inequality violation…)* that can be used to *test* (and perhaps understand) *the Standard Model.*

Summary and outlook

- It is remarkable that detectors built, essentially, to measure cross section can be used for *quantum tomography* and fully reconstruct *spin correlations* in several collider processes.
- At the *LHC this is already happening*, giving access to a wealth of *observables (entanglement, discord, magic, steering, Bell inequality violation…)* that can be used to *test* (and perhaps understand) *the Standard Model.*
- Like cross sections, *these "quantum" observables can be used to constrain new physics* resulting, for example, in the tau lepton anomalous couplings:

$$i\,\frac{g}{2\cos\theta_W}\,\bar\tau\,\Gamma^\mu(q^2)\,\tau\,Z_\mu(q) =$$
$$i\,\frac{g}{2\cos\theta_W}\,\bar\tau\left[\gamma^\mu F_1^V(q^2) + \gamma^\mu\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau}F_3(q^2)\right]\tau\,Z_\mu(q)$$



Backup

Theoretical quantum tomography for qutrits

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon^{\mu *}_\lambda(p)$ be the *amplitude* for the production of a massive *V* boson, then:

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda) \varepsilon_\lambda^\nu$$ Quantum *state of the V boson*

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu} = -\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu|V_\mu\rangle}$$

Covariant density matrix; not good enough

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu} = -\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu|V_\mu\rangle}$$

Covariant density matrix; not good enough

S. Y. Choi, T. Lee, and H. S. Song, Phys. Rev. D, 40:2477–2480, Oct 1989

$$\mathcal{P}^{\mu\nu}_{\lambda\lambda'}(p) = \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2m_V}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_{i\,\beta}\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

Projector; S_i ($i \in \{1,2,3\}$) are the *spin matrices* and n_i are the *linear polarizations versors boosted by* $-p/m_V$

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$$

spin-2 guys

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu} = -\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu|V_\mu\rangle}$$

Covariant density matrix; not good enough

S. Y. Choi, T. Lee, and H. S. Song, Phys. Rev. D, 40:2477–2480, Oct 1989

$$\mathcal{P}^{\mu\nu}_{\lambda\lambda'}(p) = \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2m_V}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_{i\,\beta}\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

Projector; S_i ($i \in \{1,2,3\}$) are the *spin matrices* and n_i are the *linear polarizations versors boosted by -p/m$_V$*

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$$

spin-2 guys

$$\rho_{\lambda\lambda'} = \mathcal{P}^{\mu\nu}_{\lambda\lambda'}\,\rho_{\mu\nu}$$

projectors project

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda,p)=\mathcal{A}_\mu\varepsilon_\lambda^{\mu*}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle=\sum_\lambda\mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu}=-\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu|V_\mu\rangle}$$

Covariant density matrix; not good enough

S. Y. Choi, T. Lee, and H. S. Song, Phys. Rev. D, 40:2477–2480, Oct 1989

$$\mathcal{P}^{\mu\nu}_{\lambda\lambda'}(p)=\frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_V^2}\right)\delta_{\lambda\lambda'}-\frac{i}{2m_V}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_{i\,\beta}\,(S_i)_{\lambda\lambda'}-\frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

Projector; S_i (*i*∈*{1,2,3}*) are the *spin matrices* and n_i are the *linear polarizations versors boosted by -p/m_V*

$$S_{ij}=S_iS_j+S_jS_i-\frac{4}{3}\mathbb{1}\,\delta_{ij}$$

spin-2 guys

$$\rho_{\lambda\lambda'}=\mathcal{P}^{\mu\nu}_{\lambda\lambda'}\,\rho_{\mu\nu}$$

projectors project

$$\rho_{\lambda\lambda'}=\frac{\mathcal{M}(\lambda)\mathcal{M}^\dagger(\lambda')}{\sum_{\lambda''}\mathcal{M}^\dagger(\lambda'')\mathcal{M}(\lambda'')}=\frac{\mathcal{A}_\mu\mathcal{A}_\nu^\dagger\mathcal{P}^{\mu\nu}_{\lambda\lambda'}}{|\bar{\mathcal{M}}|^2}$$

after doing the math: *polarization/spin density matrix*

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu} = -\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu | V_\mu\rangle}$$

Covariant density matrix; not good enough

S. Y. Choi, T. Lee, and H. S. Song, Phys. Rev. D, 40:2477–2480, Oct 1989

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2m_V}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_{i\,\beta}\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

Projector; S_i (*i*∈*{1,2,3}*) are the *spin matrices* and n_i are the *linear polarizations versors boosted by -p/m$_V$*

$$S_{ij} = S_iS_j + S_jS_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$$

spin-2 guys

$$\rho_{\lambda\lambda'} = \mathcal{P}_{\lambda\lambda'}^{\mu\nu}\,\rho_{\mu\nu}$$

projectors project

$$\rho_{\lambda\lambda'} = \frac{\mathcal{M}(\lambda)\mathcal{M}^\dagger(\lambda')}{\sum_{\lambda''}\mathcal{M}^\dagger(\lambda'')\mathcal{M}(\lambda'')} = \frac{\mathcal{A}_\mu\mathcal{A}_\nu^\dagger\mathcal{P}_{\lambda\lambda'}^{\mu\nu}}{|\bar{\mathcal{M}}|^2}$$

after doing the math: *polarization/spin density matrix*

By writing S_i and S_{ij} in terms of Gell-Mann matrices (T^a, *a*∈*{1,…,8}*) and considering processes yielding two massive vector bosons:

$$\rho_{1\otimes 1} = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a\left[T^a\otimes\mathbb{1}\right] + \sum_a g_a\left[\mathbb{1}\otimes T^a\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]$$

Theoretical quantum tomography for qutrits

In theory, we can compute stuff. Let $\mathcal{M}(\lambda, p) = \mathcal{A}_\mu \varepsilon_\lambda^{\mu *}(p)$ be the *amplitude* for the production of a massive *V* boson, then:

$$|V^\nu\rangle = \sum_\lambda \mathcal{M}(\lambda)\varepsilon_\lambda^\nu$$

Quantum *state of the V boson*

$$\rho^{\mu\nu} = -\frac{|V^\mu\rangle\langle V^\nu|}{\langle V^\mu | V_\mu\rangle}$$

Covariant density matrix; not good enough

S. Y. Choi, T. Lee, and H. S. Song, Phys. Rev. D, 40:2477–2480, Oct 1989

$$\mathcal{P}^{\mu\nu}_{\lambda\lambda'}(p) = \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2m_V}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_{i\,\beta}\,(S_i)_{\lambda\lambda'} - \frac{1}{2}n_i^\mu n_j^\nu\,(S_{ij})_{\lambda\lambda'}$$

Projector; S_i (*i*∈*{1,2,3}*) are the *spin matrices* and n_i are the *linear polarizations versors boosted by -p/m*$_V$

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$$

spin-2 guys

$$\rho_{\lambda\lambda'} = \mathcal{P}^{\mu\nu}_{\lambda\lambda'}\,\rho_{\mu\nu}$$

projectors project

$$\rho_{\lambda\lambda'} = \frac{\mathcal{M}(\lambda)\mathcal{M}^\dagger(\lambda')}{\sum_{\lambda''}\mathcal{M}^\dagger(\lambda'')\mathcal{M}(\lambda'')} = \frac{\mathcal{A}_\mu\mathcal{A}_\nu^\dagger\mathcal{P}^{\mu\nu}_{\lambda\lambda'}}{|\bar{\mathcal{M}}|^2}$$

after doing the math: *polarization/spin density matrix*

By writing S_i and S_{ij} in terms of Gell-Mann matrices (T^a, *a*∈*{1,…,8}*) and considering processes yielding two massive vector bosons:

$$\rho_{1\otimes 1} = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a\left[T^a\otimes\mathbb{1}\right] + \sum_a g_a\left[\mathbb{1}\otimes T^a\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]$$

Information about vector and tensor polarizations

spin correlations

Bell inequality violation in Charmonium decays

decay	m_{12}	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2