331 models

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Number of fermion families

The SM has three generations of fermions

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad \begin{pmatrix} t' \\ b' \end{pmatrix}_L \quad \cdots$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \begin{pmatrix} \nu_4 \\ l_4 \end{pmatrix}_L \quad \cdots$$

SM does not offer any explanation why there is only three generations. In principle there could be more.

$\Rightarrow$ 331-models offer explanation to the number of fermion generations.

331-gauge extension

331-models replace the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with $SU(3)_c \times SU(3)_L \times U(1)_X$

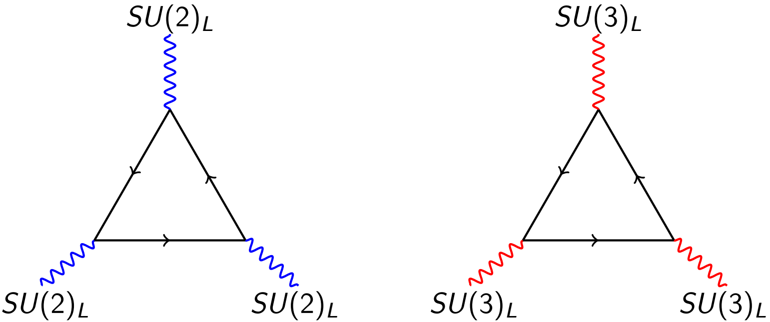
$\Rightarrow$ 5 more generators = 5 more gauge bosons
$\Rightarrow$ Scalar sector has to be extended in order to give masses to the new gauge bosons

Fundamental representation is now a $SU(3)_L$-triplet for left-handed fermions. $\Rightarrow$ Fermion sector needs to be extended (U, D_1, D_2, ν'_i)

$$Q_{L,i} = \begin{pmatrix} u_i \\ d_i \\ q_{exotic} \end{pmatrix}_L , \quad L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ l_{exotic} \end{pmatrix}_L .$$

Anomaly cancellation in 331-model

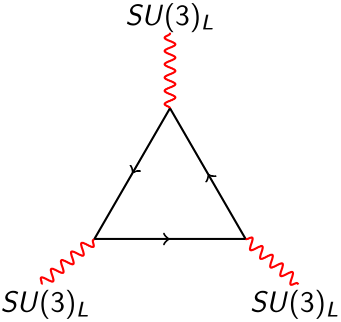
$$\text{Standard model} = SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$331-\text{model} = SU(3)_C \times SU(3)_L \times U(1)_X$$



$SU(2)_L$: $\left\{\frac{\sigma^a}{2}, \frac{\sigma^b}{2}\right\} = \frac{\delta^{ab}}{2} \Rightarrow$ Anomaly cancels automatically

$SU(3)_L$: $\left\{\frac{T^a}{2}, \frac{T^b}{2}\right\} \neq \delta^{ab} \Rightarrow$ Anomaly doesn't cancel automatically!

In 331 number of families = 3



Pure $SU(3)_L$-anomaly cancels only if the number of families is 3

Family discrimination

Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. How to assign them?

6 triplets (quarks have 3 colours):

$$L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \nu_i' \end{pmatrix}_L , \quad Q_{L,1} = \begin{pmatrix} u_1 \\ d_1 \\ U \end{pmatrix}_L$$

6 antitriplets (quarks have 3 colours):

$$Q_{L,2} = \begin{pmatrix} d_2 \\ -u_2 \\ D_1 \end{pmatrix}_L , \quad Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ D_2 \end{pmatrix}_L$$

One of the quarks is in different representation!

Family discrimination

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$$L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \nu'_i \end{pmatrix}_L , \quad Q_{L,2} = \begin{pmatrix} u_2 \\ d_2 \\ U \end{pmatrix}_L$$

6 antitriplets (quarks have 3 colours):

$$Q_{L,1} = \begin{pmatrix} d_1 \\ -u_1 \\ D_1 \end{pmatrix}_L , \quad Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ D_2 \end{pmatrix}_L$$

But which generation to pick?

Family discrimination

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$$L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \nu'_i \end{pmatrix}_L , \quad Q_{L,3} = \begin{pmatrix} u_3 \\ d_3 \\ U \end{pmatrix}_L$$

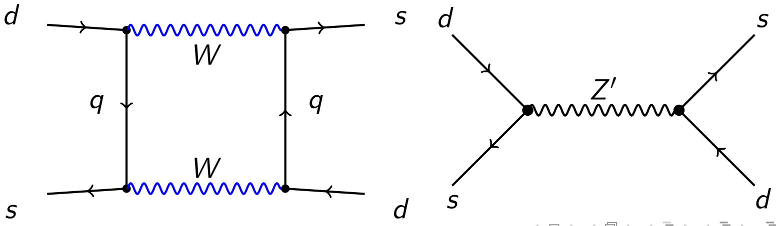
6 antitriplets (quarks have 3 colours):

$$Q_{L,1} = \begin{pmatrix} d_1 \\ -u_1 \\ D_1 \end{pmatrix}_L , \quad Q_{L,2} = \begin{pmatrix} d_2 \\ -u_2 \\ D_2 \end{pmatrix}_L$$

No one knows!

Flavour changing neutral currents

- In order to cancel the anomalies, one quark family has to be in a different representation than the other two. (And no one knows which!)
- $\Rightarrow$ Scalar mediated flavour changing neutral currents (FCNC) at tree-level!
- $\Rightarrow$ Neutral meson mixing at tree-level!
- Different quark generation assignments give different flavour violating couplings $\Rightarrow$ Can be tested $\Rightarrow$ New physics scale ~ 10 TeV



331-model

The $SU(3)_L$-gauge group has one additional diagonal generator compared to the $SU(2)_L$:
$\Rightarrow$ Freedom in electric charge definition: $Q = T_3 + \beta T_8 + X$ (compare to SM: $Q = T_3 + Y/2$).

Two types of models:

- $\beta = \pm\sqrt{3}$:
  F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
  P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

- $\beta = \pm\frac{1}{\sqrt{3}}$:
  M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

$\beta = (\pm)\frac{1}{\sqrt{3}}$

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Fermion representations:

$$\text{Triplets:} \quad L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$Antitriplets : Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \; \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

$\beta = (\pm)\frac{1}{\sqrt{3}}$, Gauge bosons

$$D_\mu = \partial_\mu - ig_3 \sum_{a=1}^{8} T_a W_{a\mu} - ig_x X B_\mu,$$

$$\sum_{a=1}^{8} T_a W_{a\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W'^{+}_{\mu} & X'^{0}_{\mu} \\ W'^{-}_{\mu} & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & V'^{-}_{\mu} \\ X'^{0*}_{\mu} & V'^{+}_{\mu} & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

The neutral mass eigenstates are: photon, Z_μ, Z'_μ, X^0_μ and X^{0*}_μ. The charged mass eigenstates are $W^\pm_\mu$ and $V^\pm_\mu$.

$\beta = (\pm)\sqrt{3}$

F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

$$\text{Triplets:}\quad L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ e_{L,i} \\ (e_{R,i})^c \end{pmatrix} \sim (1,3,0), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ q_1^{+5/3} \end{pmatrix} \sim (1,3,\frac{2}{3}),$$

$$\text{Antitriplets:}\quad Q_{L,2} = \begin{pmatrix} c \\ s \\ q_2^{-4/3} \end{pmatrix}, \quad Q_{L,3} = \begin{pmatrix} t \\ b \\ q_3^{-4/3} \end{pmatrix} \sim (1,3^*,-\frac{1}{3}).$$

$q_1^{+5/3}$, $q_2^{-4/3}$ and $q_3^{-4/3}$ are new quarks with exotic eletric charges!

$\beta = (\pm)\sqrt{3}$, scalar sector

$$\mathcal{L}_m = \epsilon_{\alpha\beta\gamma} G_{ij} \overline{L}^{\alpha}_{L,i} (L^{\beta}_{L,j})^c \langle \eta^* \rangle^{\gamma} = \left(\epsilon_{\alpha\beta\gamma} G_{ij} \langle \eta^{0*} \rangle \right) \overline{e}_{L,i} e_{R,j}$$

$\Rightarrow$ Antisymmetric charged lepton mass matrix! Need to add scalar sextet.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^+ \end{pmatrix} \sim (1,3,0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1,3,1),$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1,3,-1), \quad S = \begin{pmatrix} \sigma_1^0 & h_2^- & h_1^+ \\ h_2^- & H_1^{--} & \sigma_2^0 \\ h_1^+ & \sigma_2^0 & H_2^{++} \end{pmatrix} \sim (1,6,0).$$

$\beta = (\pm)\sqrt{3}$, Gauge bosons

$$D_\mu = \partial_\mu - ig_3 \sum_{a=1}^{8} T_a W_{a\mu} - ig_x X B_\mu,$$

$$\sum_{a=1}^{8} T_a W_{a\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W'^{+}_{\mu} & V'^{-}_{\mu} \\ W'^{-}_{\mu} & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & U^{--}_{\mu} \\ V'^{+}_{\mu} & U^{++}_{\mu} & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

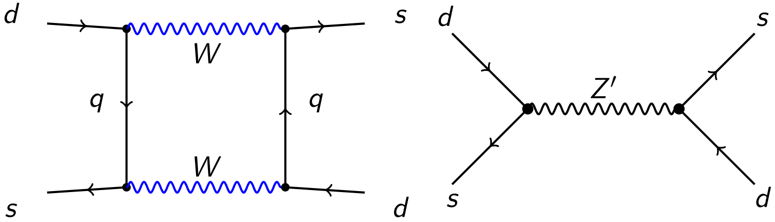
The neutral mass eigenstates are: photon, Z_μ and Z'_μ. The charged mass eigenstates are $W^{\pm}_\mu$, $V^{\pm}_\mu$ and $U^{\pm\pm}$.

Summary

- In 331 models anomaly cancellation fixes number of generations to be 3
- The new physics scale is $\gtrsim 10$ TeV
- Probable at the LHC

Neutral meson mixing

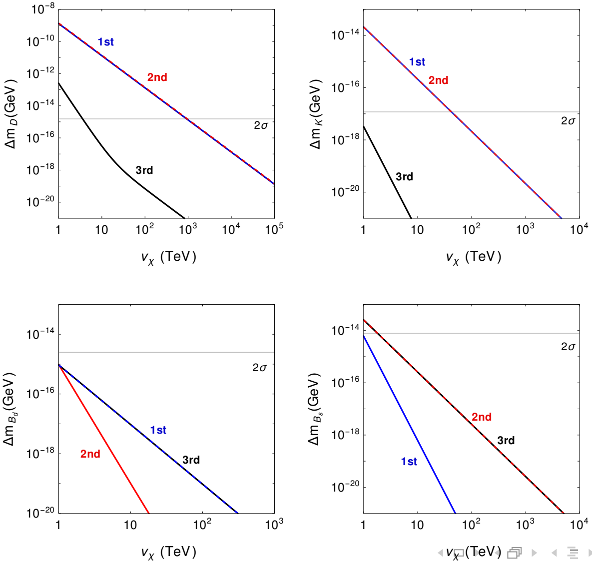
Different quark generation assignments give different flavour violating couplings ⇒ can be tested



Neutral mesons:

$$
\begin{aligned}
D^0 &= c\bar{u} & \Delta m_D &= (6.56 \pm 0.763) \times 10^{-15}\ \mathrm{GeV} \\
K^0 &= d\bar{s} & \Delta m_K &= (3.48 \pm 0.00592) \times 10^{-15}\ \mathrm{GeV} \\
B_d^0 &= d\bar{b} & \Delta m_{B_d} &= (3.33 \pm 0.0125) \times 10^{-13}\ \mathrm{GeV} \\
B_s^0 &= s\bar{b} & \Delta m_{B_s} &= (1.17 \pm 0.000395) \times 10^{-11}\ \mathrm{GeV}
\end{aligned}
$$

Plot



Benchmarks for LHC

	BP1	**BP2**	**BP3**
m_U/TeV	1.362	2.368	1.829
m_{D1}/TeV	2.150	6.931	9.076
m_{D2}/TeV	16.007	36.369	26.922

How to make predictions?

- The different quark generation assignments lead to different flavour violating structure ⇒ Can be used to test which generation is discriminated
- One needs to make assumptions about the structure of quark mass matrices
- We assume that CKM, $V_{\rm CKM} = U_L^u U_L^{d\dagger}$, is generated without accidental cancellations between elements of U_L^u and $U_L^{d\dagger}$. This is achieved when ($\epsilon = 0.23$, Cabibbo angle)

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \quad U_L^u,\ U_L^d \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

$$m_u \sim \frac{v}{\sqrt{2}} \begin{pmatrix} \epsilon^{L_1+R_1} & \epsilon^{L_1+R_2} & \epsilon^{L_1+R_3} \\ \epsilon^{L_2+R_1} & \epsilon^{L_2+R_2} & \epsilon^{L_2+R_3} \\ \epsilon^{L_3+R_1} & \epsilon^{L_3+R_2} & \epsilon^{L_3+R_3} \end{pmatrix}, \quad \begin{matrix} L_1 = 3 \\ L_2 = 2 \\ L_3 = 0 \end{matrix} \Rightarrow \text{CKM}$$

331 mass matrix

- In 331 exotic quarks mix with SM quarks
- We take it into account and do not ignore it!
- But mixing is suppressed by $SU(3)_L$-breaking scale u. $\delta = \frac{v_{\rm sm}}{u}$

$$m_u = \frac{v_{\rm sm}}{\sqrt{2}} \begin{pmatrix} y^u_{11} & y^u_{12} & y^u_{13} & y^u_{14} \\ y^u_{21} & y^u_{22} & y^u_{23} & y^u_{24} \\ y^u_{31} & y^u_{32} & y^u_{33} & y^u_{34} \\ \frac{u}{v_{\rm sm}} y^u_{41} & \frac{u}{v_{\rm sm}} y^u_{42} & \frac{u}{v_{\rm sm}} y^u_{43} & \frac{u}{v_{\rm sm}} y^u_{44} \end{pmatrix}$$

Write Yukawas is suggestive form:

$y_{ij} = \epsilon^{L_i + R_j} \Rightarrow (U_L)_{ij} \sim \epsilon^{|L_i - L_j|}, (U_R)_{ij} \sim \epsilon^{|R_i - R_j|}$

- $L_1 = 3, L_2 = 2, L_3 = 0$ gives CKM and $L_4 \equiv L_U$ affects mixing with exotic quarks
- R_i determine the right-handed rotion matrix (not relevant) $\Rightarrow$ R_i are chosen to produce correct SM masses

331 quark rotation matrices

$$U_L^u \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 & \delta\epsilon^{3-L_U} \\ \epsilon & 1 & \epsilon^2 & \delta\epsilon^{2-L_U} \\ \epsilon^3 & \epsilon^2 & 1 & \delta\epsilon^{-L_U} \\ \delta\epsilon^{3-L_U} & \delta\epsilon^{2-L_U} & \delta\epsilon^{-L_U} & 1 \end{pmatrix}, \quad \delta = \frac{v_{\rm sm}}{u}$$

$$U_L^d \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 & \delta\epsilon^{3-L_{D_1}} & \delta\epsilon^{3-L_{D_2}} \\ \epsilon & 1 & \epsilon^2 & \delta\epsilon^{2-L_{D_1}} & \delta\epsilon^{2-L_{D_2}} \\ \epsilon^3 & \epsilon^2 & 1 & \delta\epsilon^{-L_{D_1}} & \delta\epsilon^{-L_{D_2}} \\ \delta\epsilon^{3-L_{D_1}} & \delta\epsilon^{2-L_{D_1}} & \delta\epsilon^{-L_{D_1}} & 1 & \epsilon^{L_{D_1}-L_{D_2}} \\ \delta\epsilon^{3-L_{D_2}} & \delta\epsilon^{2-L_{D_2}} & \delta\epsilon^{-L_{D_2}} & \delta\epsilon^{L_{D_1}-L_{D_2}} & 1 \end{pmatrix}$$

$$V_{\rm CKM}^{331} = U_L^u \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U_L^{d\dagger} \quad (1)$$

- Quark rotation matrices determine the flavor violating couplings
- Each generation assignment gives different flavour violating couplings $\Rightarrow$ difference can be seen in neutral meson mixing!

Example: $\mathcal{L} = \bar{u}_L U^u_L(\lambda^u_{Z'}) U^{u\dagger}_L \gamma^\mu u_L Z'_\mu + \bar{d}_L U^d_L(\lambda^d_{Z'}) U^{d\dagger}_L \gamma^\mu d_L Z'_\mu$

$$\lambda'^u_{1st} = \begin{pmatrix} a+b & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a+c \end{pmatrix}, \lambda'^u_{2nd} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a+c \end{pmatrix}, \lambda'^u_{3rd} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a+b & 0 \\ 0 & 0 & 0 & a+c \end{pmatrix}$$

Neutral mesons: $K^0 = d\bar{s}$, $B^0_d = d\bar{b}, B^0_s = s\bar{b}$, $D^0 = c\bar{u}$

$$\begin{aligned} D^0 &= c\bar{u} & \Delta m_D &= (6.56 \pm 0.763) \times 10^{-15}\ \mathrm{GeV} \\ K^0 &= d\bar{s} & \Delta m_K &= (3.48 \pm 0.00592) \times 10^{-15}\ \mathrm{GeV} \\ B^0_d &= d\bar{b} & \Delta m_{B_d} &= (3.33 \pm 0.0125) \times 10^{-13}\ \mathrm{GeV} \\ B^0_s &= s\bar{b} & \Delta m_{B_s} &= (1.17 \pm 0.000395) \times 10^{-11}\ \mathrm{GeV} \end{aligned}$$