Inflation model building: Higgs inflation

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Higgs Inflation

Total action is given by the **SM action** plus **Einstein-Hilbert** term plus a **non minimal coupling** between the Higgs boson and gravity (metric convention $-+++$)

$$\mathcal{S}_{tot} = \int d^4x\sqrt{-g}\left(\frac{M^2}{2}R + \xi H^{+}HR + \mathcal{L}_{SM}\right)$$

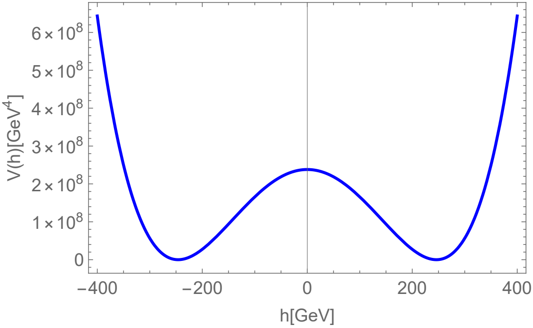
By going to the unitary gauge $H = \frac{h}{\sqrt{2}}$ and neglecting the gauge interactions we have:

$$\mathcal{S} = \int d^4x\left(\frac{M^2+\xi h^2}{2}R - \frac{1}{2}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}(h^2 - v^2)^2\right)$$

Higgs inflation - Minimal coupling

For $\xi = 0$ the potential $V(\phi) = \frac{\lambda}{4}(h^2 - v^2)^2$ with $v = 246\, GeV$

Expanding the potential we have the identification $m_H^2 = \lambda v^2$ which implies $\lambda = 0.26$ since $m_H = (125.25 \pm 0.17)\, GeV$ (PDG 2022)



Higgs inflation - Minimal coupling

Slow-roll approximation

$V(\phi) = \frac{\lambda}{4}(h^2 - v^2)^2$ tree-level Higgs potential

$\epsilon_V(\phi) = \frac{1}{2}\frac{V'(\phi)}{V(\phi)}$, $\epsilon_V < 1$ inflation

$\eta_V(\phi) = \frac{1}{2}\frac{V'(\phi)}{V(\phi)}$, $|\eta_V| < 1$ slow-roll condition

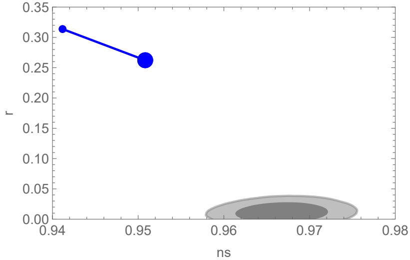
CMB observables

$r = 16\epsilon_V$ tensor-to-scalar ratio

$n_s = 1 - 2\eta_V + 6\epsilon_V$ spectral index

$A_s = \frac{V}{24\pi^2\epsilon_V}$ amplitude of the power spectrum

Higgs inflation - Minimal coupling



BICEP/Keck+Planck2018+BAO
arxiv.org/abs/2110.00483

Predicts $A_s \sim 4 \cdot 10^{-3}$, $r \sim 0.26$, $n_s \sim 0.95$ at 60 e-fold which are not in agreement with the observed values

Produces way too large matter fluctuations which **cannot seed the current large scale structure**

Higgs inflation with non-minimal coupling

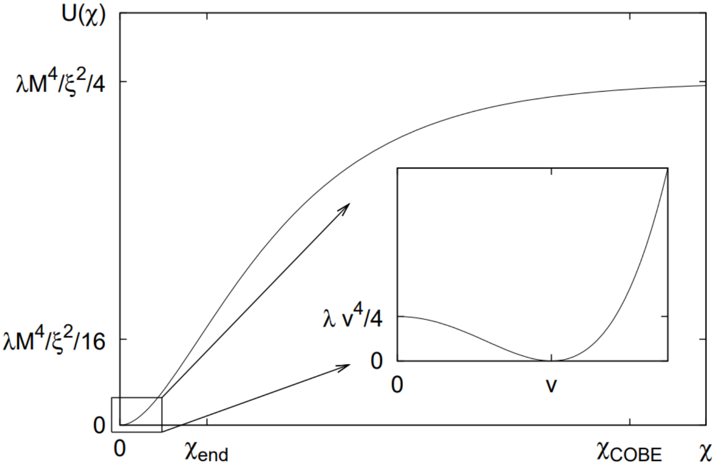
By a so called **conformal transformation** we can get rid of the non-minimal coupling

$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, $\Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$ the new transformed action reads:

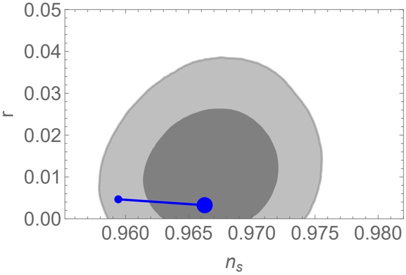
$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right)$$

with $U(\chi) = \frac{1}{\Omega^4(\chi)} \frac{\lambda}{4} (h^2(\chi) - v^2)^2$ and $\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}}$

Higgs inflation with non-minimal coupling



Higgs inflation with non-minimal coupling



We can use the **slow-roll** approximation to compute the **CMB observables**, this gives:

$$r = \frac{192}{(4N_e+3)^2} \sim 0.0031, n_s = 1 - \frac{8(4N_e+9)}{(4N_e+3)^2} \sim 0.967 \text{ at 60 efolds}$$

and by fixing $A_s = 2.1 \cdot 10^{-9}$ to the measured one we get $\xi = 1.76 \cdot 10^4$ and $E_{inf} \sim (\frac{r}{0.01})^{1/4} 10^{16} GeV \sim 10^{16} GeV$

Radiative corrections

At high energies the **Higgs potential** $V(h) = \frac{\lambda}{4}(h^2 - v^2)^2$ is **modified by quantum corrections**

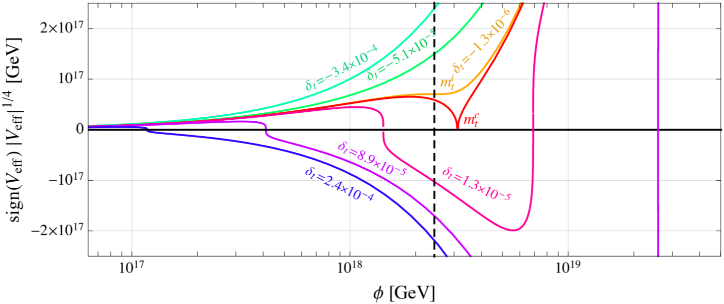
To extrapolate the form of the potential V_{eff} at high energies one can use $\bar{MS}$ scheme

The **potential** can be expressed as a **sum of tree level plus increasing loop contribution**:

$$V_{eff} = V^{(0)} + V^{(1)} + V^{(2)} + \ldots$$

From PDG 2022 world average:
$m_H^{exp} = (125.25 \pm 0.17) GeV$
$m_t^{exp} = (172.5 \pm 0.7) GeV$
$\alpha_s^{(5)exp} = 0.1179 \pm 0.0009$

Radiative corrections



Potential is gauge dependent (Landau gauge), but **stationary configurations are not** and only depend on the input parameters

$m_t = m_t^c(1+\delta_t)$ with $m_t^c = 171.0549 GeV$ being the **critical top mass** for which the potential is **degenerate** (after fixing $\alpha_s^{(5)}$, m_H to their central values)

Radiative corrections

Introducing a **non-minimal coupling** in the action can help **both** with **inflation** and the **stability** issue

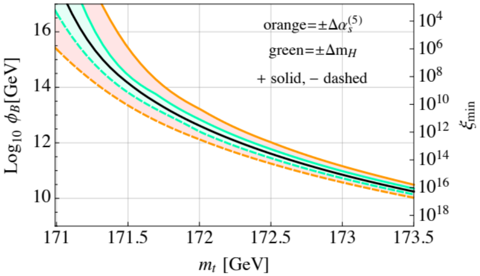
We recall that in this case the classical potential in the Einstein frame is given by:

$U(\chi) = \frac{1}{\Omega^4(\chi)} \frac{\lambda}{4} (h^2(\chi) - v^2)^2$

with $\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}}$ and $\Omega(\chi)^2 = 1 + \frac{\xi h(\chi)^2}{M_P^2}$

Again the high energy potential can be computed by **expanding at loop-order** $U_{eff} = U^0 + U^1 + U^2 + ...$

Radiative corrections



The non-minimal coupling can help stabilizing the potential at high energies

Value of ξ **required to stabilize the potential depends on** m_t

But what about inflation?

Radiative corrections

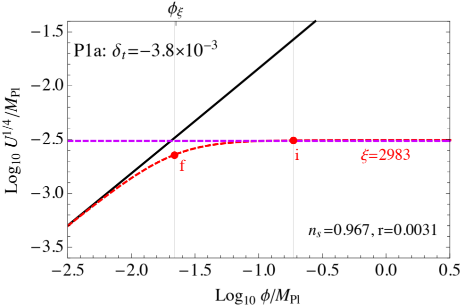
We will consider 4 cases trough examples:

- $m_t < m_t^i$ stable potential, no new minima appears
- $m_t = m_t^i$ potential with inflection point
- $m_t^i < m_t \leq m_t^c$ stable potential, but new minima appears at high energies
- $m_t > m_t^c$ metastable configuration, new minima appears with lower vacuum energy

Radiative corrections $m_t < m_t^i$

This case corresponds to stable configurations for which no new minima is developed at high energies ($m_t = 170.4 GeV$, 3σ lower value)

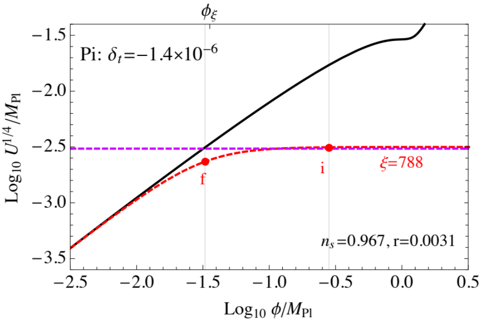
We flatten this potential with ξ that allows to predict the right value of $A_s = 2.1 \cdot 10^{-9}$ and get $\xi \sim 3 \cdot 10^3$



Radiative corrections $m_t = m_t^i$

This case corresponds to the stable configuration for which we have an inflection point at high energies ($m_t = 171.0547 \sim 2\sigma$ lower value)

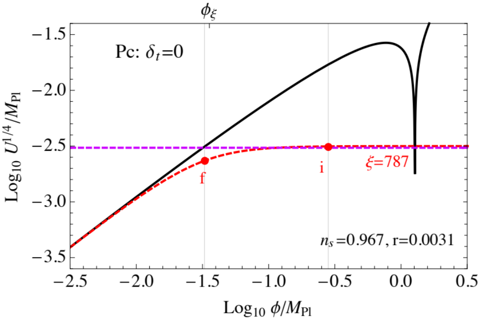
In this case we have $\xi \sim 8 \cdot 10^2$, the value is such that the inflection point disappears



Radiative corrections $m_t^i < m_t \leq m_t^c$

This case corresponds to the stable configuration for which we have a new minimum at high energies but at higher values of the EW vacuum ($m_t = 171.0549 \sim 2\sigma$ lower value)

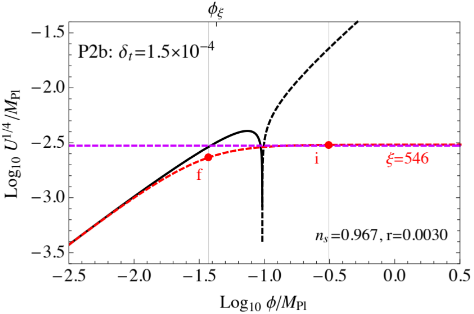
In this case we have $\xi \sim 8 \cdot 10^2$, the value is such that the second minimum disappears



Radiative corrections $m_t > m_t^c$

This case corresponds metastable configurations for which we have a new minimum at high energies but at higher values of the EW vacuum ($m_t = m_t^* = 171.08 \sim 2\sigma$ lower value)

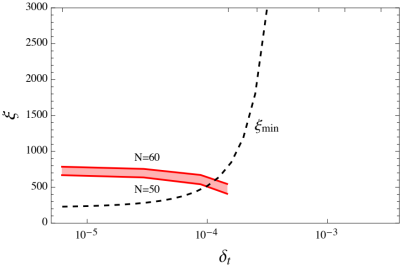
In this case we have $\xi \sim 5.5 \cdot 10^2$, the value is such that the second minimum disappears



Radiative corrections - Comments

Unfortunately **we cannot rescue** the potential **stability** and at the same time have **Higgs-inflation** for $m_t > m_t^*$

$\xi < \xi_{min}$ can't predict right A_s



The **predictions for** r, n_s **are not spoiled by quantum corrections** and essentially reproduce the tree-level predictions

Take-home message

We can have **viable metric Higgs inflation** but we need $m_t \leq 171.08 GeV$ ($\sim 2\sigma$ lower value) to solve metastibility issues

The end

Thank you for the attention!