Type II seesaw mechanism

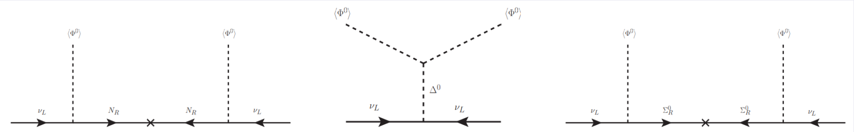
Aleksei Kubarski

National Institute of Chemical Physics and Biophysics

Talk based on work in progress by K. Kannike, A. K. and L. Marzola

Neutrino masses

Seesaw mechanism



Type I
$m_\nu \sim \frac{1}{M_R}$

Type II
$m_\nu \sim \langle \Delta^0 \rangle \sim \frac{1}{M_\Delta}$

Type III
$m_\nu \sim \frac{1}{M_\Sigma}$

Type II seesaw neutrino mass term

$$\mathcal{L}_Y \supset -(Y_\nu)_{\alpha\beta} \overline{L^c}_\alpha \epsilon \Delta L_\beta + \text{h.c.},$$

$$(m_\nu)_{\alpha\beta} = \sqrt{2} (Y_\nu)_{\alpha\beta} v_\Delta$$

Type II seesaw model

Scalar fields

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}$$

Potential

$$V = \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \lambda_\Delta [\mathrm{tr}\left(\Delta^\dagger \Delta\right)]^2 + \lambda'_\Delta \,\mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right)$$
$$+ \mu_\Delta^2 \,\mathrm{tr}\left(\Delta^\dagger \Delta\right) + \frac{1}{2}\mu_{H\Delta}[H^T \epsilon \Delta^\dagger H + \text{h.c.}]$$
$$+ \lambda_{H\Delta} H^\dagger H \,\mathrm{tr}\left(\Delta^\dagger \Delta\right) + \lambda'_{H\Delta} H^\dagger \Delta \Delta^\dagger H$$

Type II seesaw model

Our neutral extremum $N_{H\Delta}$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v_H}{\sqrt{2}} \end{pmatrix}, \qquad \langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ \frac{v_\Delta}{\sqrt{2}} & 0 \end{pmatrix}$$

Z and W gauge boson masses

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)(v_H^2 + 4v_\Delta^2), \qquad M_W^2 = \frac{1}{4}g^2(v_H^2 + 2v_\Delta^2),$$

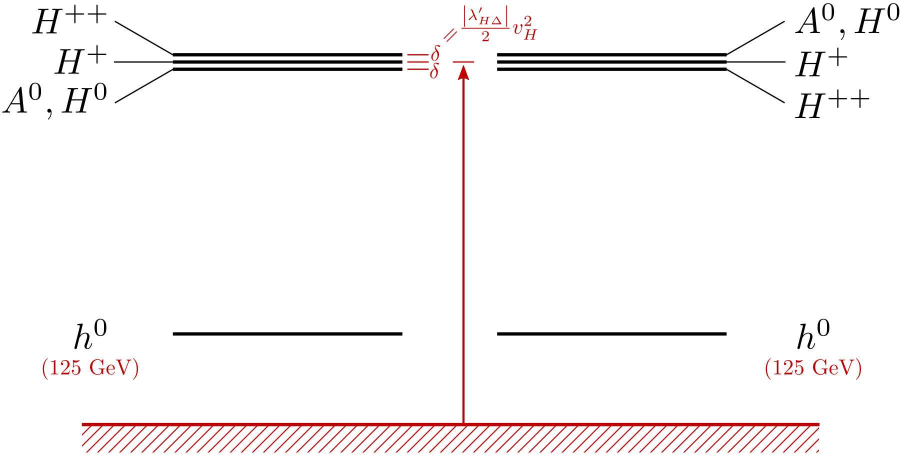
where $0 \leq v_\Delta \leq 2.58$ GeV and $v_H^2 + 2v_\Delta^2 = 246.22$ GeV

Type II seesaw model

Scalar mass eigenstates

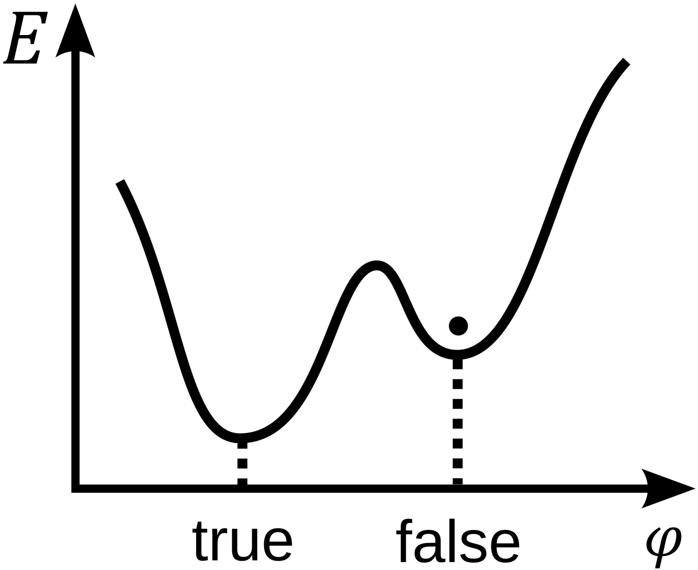
- Doubly charged field $H^{\pm\pm}$
- Singly charged field $H^{\pm}$
- Charged Goldstone boson $G^{\pm}$
- Neutral scalar fields H^0 & h^0 (SM-like Higgs boson)
- Neutral Goldstone boson G^0
- Neutral pseudoscalar field A^0

Type II seesaw model



Source: C. Bonilla *et al.*, Phys. Rev. D **92** (2015) no.7, 075028 [arXiv:1508.02323 [hep-ph]].

Vacuum stability



Orbit space

Rewritten potential

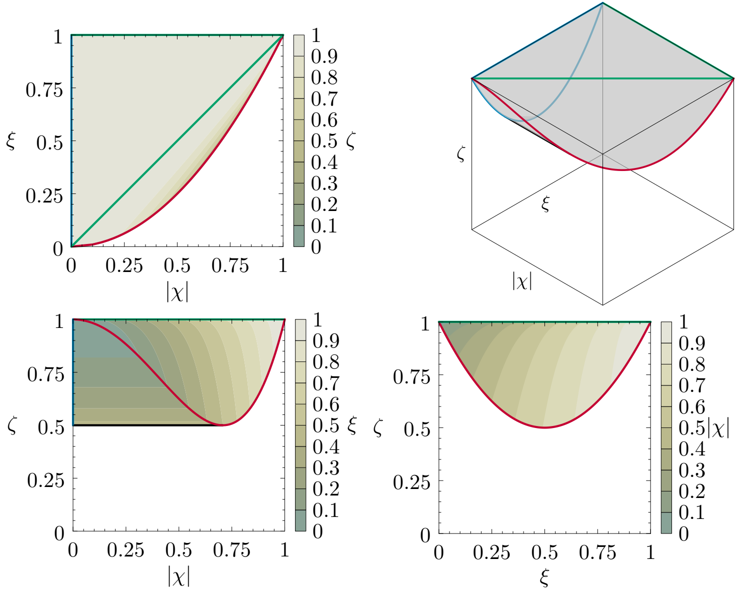
$$V = \frac{1}{2}\mu_H^2 h^2 + \frac{1}{2}\mu_\Delta^2 \delta^2 + \frac{1}{2\sqrt{2}}\mu_{H\Delta}\chi h^2\delta + \frac{1}{4}\lambda_H h^4 + \frac{1}{4}\lambda_\Delta \delta^4 + \frac{1}{4}\lambda'_\Delta \zeta \delta^4 + \frac{1}{4}\lambda_{H\Delta} h^2\delta^2 + \frac{1}{4}\lambda'_{H\Delta}\xi h^2\delta^2$$

Orbit space variables and norms

$$\xi = \frac{H^\dagger \Delta \Delta^\dagger H}{H^\dagger H \,\mathrm{tr}(\Delta^\dagger \Delta)}, \qquad \zeta = \frac{\mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right)}{(H^\dagger H)^2}, \qquad \chi = \frac{\frac{1}{2}[H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}]}{\sqrt{H^\dagger H}\,\mathrm{tr}(\Delta^\dagger \Delta)},$$

$$H^\dagger H = \frac{1}{2}h^2, \quad \mathrm{tr}\left(\Delta^\dagger \Delta\right) = \frac{1}{2}\delta^2$$

Orbit space



Vacuum structure and metastability

Other minima[1]

- panic vacua $N'_{H\Delta}$: other neutral vacuum solutions to minimisation equations
- Charged extrema $CB_{H\Delta}$: must be on curved edge (except special configurations)

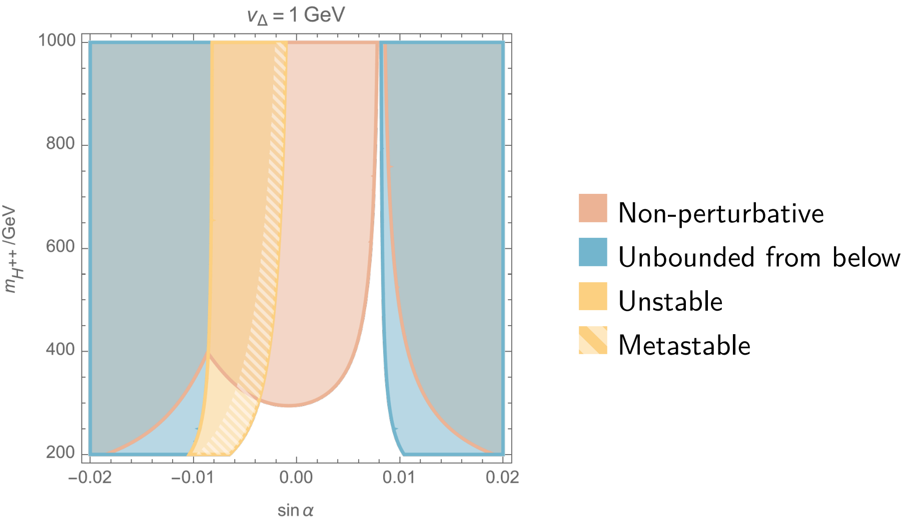
$$\xi = \chi^2, \quad \zeta = 1 - 2\xi + 2\xi^2, \quad -1 \leq \chi \leq 1$$

- Neutral extrema N_Δ or charged extrema CB_Δ ($\langle H \rangle = 0$):

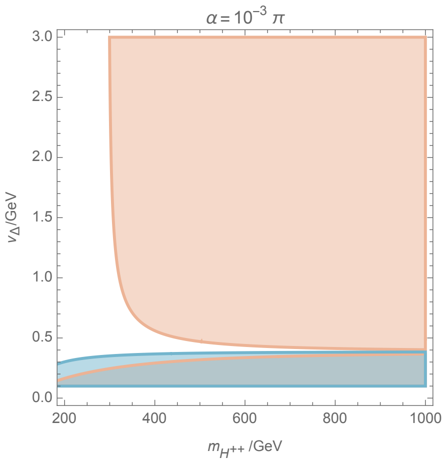
$$\delta^2 = -\frac{\mu_\Delta^2}{\lambda_\Delta + \zeta\lambda'_\Delta}, \qquad V_\Delta = -\frac{1}{4}\frac{\mu_\Delta^4}{\lambda_\Delta + \zeta\lambda'_\Delta}$$

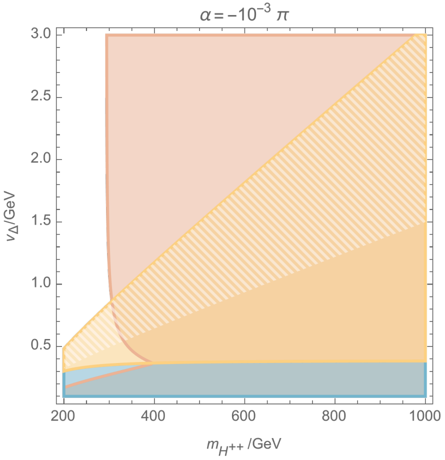
[1] P. M. Ferreira and B. L. Gonçalves, JHEP **02** (2020), 182 [arXiv:1911.09746 [hep-ph]].

Vacuum structure and metastability



Vacuum structure and metastability





Conclusions

- Type II seesaw offers a compelling framework for explaining the observed light neutrino masses.
- Utilizing the orbit space formalism has provided valuable insights into the vacuum structure of the model.
- While panic vacua are absent, regions of instability is present within the allowed parameter space.
- Stay tuned for phase transitions and gravitational waves.

Orbit space: introduction

Orbit space[2]

- Field vector transformation under G symmetry group: $\phi \to g \cdot \phi$
- Polynomial invariants transform under G symmetry group: $p_i \to p_i$
- $V(\phi)$ can be expressed as a function of finite set $p = (p_1(\phi), p_2(\phi), ..)$ of basic polynomial invariants.
- $\widehat{V}(p)$ has same range as $V(\phi)$, but is not affected by the same degeneracies.
- We can calculate the full orbit space by the means of the P-matrix formalism with

$$P_{ij} = \frac{\partial p_i}{\partial \Phi_a^\dagger} \frac{\partial p_j}{\partial \Phi^a}$$

[2] G. Sartori and G. Valente, Annals Phys. **319** (2005), 286-325

Orbit space: introduction

Invariants

$$p_1 = H^\dagger H$$

$$p_2 = \mathrm{tr}\left(\Delta^\dagger \Delta\right)$$

$$p_3 = \mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right)$$

$$p_4 = H^\dagger \Delta \Delta^\dagger H$$

$$p_5 = \frac{1}{2}[H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}]$$

$$\left(p_6 = \frac{1}{2}(\mathrm{tr}\, \Delta^{\dagger 2}\, H^T \epsilon \Delta H + \mathrm{tr}\, \Delta^2\, H^\dagger \Delta^\dagger \epsilon^\dagger H^*)\right)$$

$$\left(p_7 = H^\dagger \Delta^\dagger H H^\dagger \Delta H\right)$$

Orbit space: solutions

P-matrix

$$\mathbf{P} = \begin{pmatrix} 2p_1 & 0 & 0 & 2p_4 & 2p_5 \\ 0 & 2p_2 & 4p_3 & 2p_4 & p_5 \\ 0 & 4p_3 & 2p_2(6p_3 - 2p_2^2) & P_{34} & 2p_2 p_5 - p_6 \\ 2p_4 & 2p_4 & P_{43} & P_{44} & P_{45} \\ 2p_5 & p_5 & 2p_2 p_5 - p_6 & P_{54} & 2p_4 + \frac{1}{2}p_1^2 \end{pmatrix}$$

$$\begin{aligned} P_{34} &= P_{43} = 4p_2 p_4 + 2p_1(p_3 - p_2^2) \\ P_{44} &= p_1(p_3 - p_2^2) + 2p_2 p_4 + 2_{p_1} p_4 - p_7 \\ P_{45} &= P_{54} = p_1 p_5 + 2p_2 p_5 - p_6 \end{aligned}$$

Orbit space: solutions

Unitary gauge and real values for the triplet

$$\zeta = 1 - \frac{1}{2}\frac{[(\Delta^+)^2 + 2\Delta^0\Delta^{++}]^2}{[(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2]^2}$$

$$\xi = \frac{1}{2}\frac{2(\Delta^0)^2 + (\Delta^+)^2}{(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2}$$

$$\chi = -\frac{\Delta^0}{\sqrt{(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2}}$$

$\Delta^+ = 0$ solution

$$\xi = \chi^2, \quad \zeta = 1 - 2\xi + 2\xi^2, \quad -1 \le \chi \le 1$$

Couplings

$$\lambda_H = \frac{m_{h^0}^2 \cos\alpha + m_{H^0}^2 \sin\alpha}{2v_H^2},$$

$$\lambda_\Delta = \frac{1}{v_\Delta^2}\left(\frac{m_{h^0}^2 \sin^2\alpha + m_{H^0}^2 \cos^2\alpha}{2} + \frac{1}{2}\frac{v_H^2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - \frac{2v_H^2}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 + m_{H^{++}}^2\right),$$

$$\lambda'_\Delta = \frac{1}{v_\Delta^2}\left(\frac{2v_H^2}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 - \frac{v_H^2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - m_{H^{++}}^2\right),$$

$$\lambda_{H\Delta} = -\frac{2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 + \frac{4}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 + \frac{\sin\alpha\cos\alpha}{v_H v_\Delta}(m_{h^0}^2 - m_{h^0}^2),$$

$$\lambda'_{H\Delta} = \frac{4}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - \frac{4}{v_H^2 + 2v_\Delta^2} m_{H^+}^2, \quad \mu_{H\Delta} = \frac{2\sqrt{2}v_\Delta}{v_H^2 + 2v_\Delta^2} m_{A^0}^2$$