High Brightness Electron Beams for Radiation Sources

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On behalf of BD & ICS Milano Group

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Outline

- Basic concepts in Accelerator Physic & Beam Dynamics (BD) in the frame of HBB machines:
  - Relativity – beam Rigidity
  - Main beam optics: Dipoles, Quadrupoles, Sextupoles, Solenoids
  - The transverse phase-space, emittance & brightness. The longitudinal phase-space
  - Accelerators cavities basic concepts
  - Sketch of a High Brightness Linear Accelerator (LINAC) for Thomson/Compton sources
  - STAR-II upgrade, a real LINAC; some BD and machine images
- Useful codes for space charge dominated beam simulations
  - The Astra code & examples of its use
  - Examples of the Astra Use
  - Cain, a Montecarlo quantum code to simulate electron-photon interactions & examples
  - Simulation of Three different scattering cases, what will be done this coffee-break in the laboratory

Relativity basic concepts

Important:
When we speak of beam particle energy in an accelerator,
we refer to Kinetic Energy! (unless specified)

Mostly we use the I.S., with few exceptions:

- The Beam energy: eV (keV, MeV, …)------[1eV =1.6 x 10^{-19} J]
- Mass: eV/c^2 ----------------------------------[Proton= 1.67×10^{-27} kg → 938 MeV/c^2]
  ----------------------------------[Electron = 9.11×10^{-31} kg →0.511 MeV/c^2]
- Momentum: eV/c ----------------------------[Proton @ β=0.9 → 1.94 GeV/c]

From an ATP Tennis player like Matteo Berrettini its service is about 90 J i.e. (5.6×10^{20} eV)

Extreme-energy cosmic ray (EECR) are typically protons with energy ≥ 10^{18} (0.8 J)
(LHC beams: 7 x 10^{12})

$$\beta \equiv \frac{v}{c} = \frac{pc}{E}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \rightarrow \boldsymbol{\beta\gamma = \sqrt{\gamma^2 - 1}}$$

$$\text{momentum } p = \gamma m v$$

$$\text{total energy } E = \gamma m c^2$$

$$\text{kinetic energy } \boldsymbol{K} = E - mc^2 = \boldsymbol{mc^2(\gamma - 1)}$$

$$E = \sqrt{(mc^2)^2 + (pc)^2}$$

Beam Rigidity

It's a relation between:
radius - magnetic field - momentum - charge
HOW HARD (or EASY) is it to defect PARTICLES?

Centrifugal force

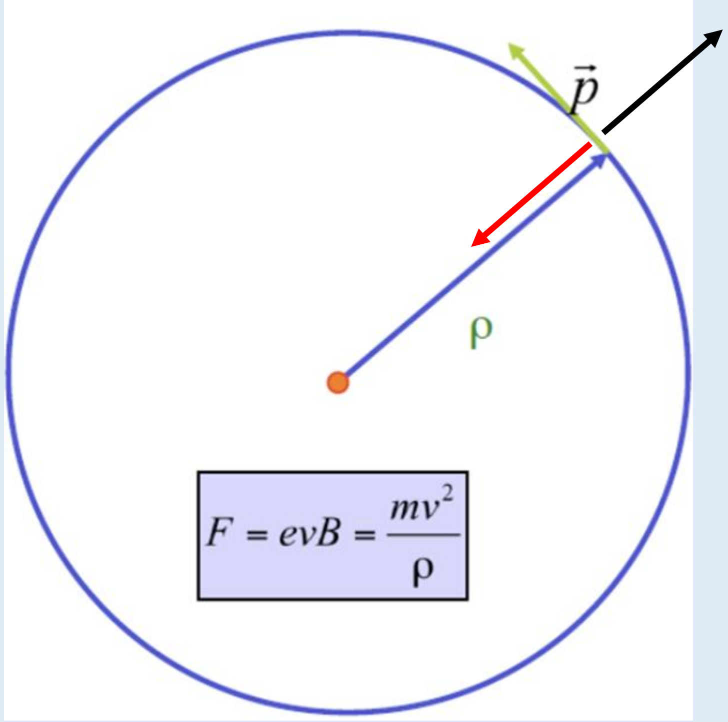
$$\boldsymbol{F}_{CF} = -m\boldsymbol{a}_{CF} = -m[\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})] = -m\omega(\omega^2 r)\hat{\boldsymbol{r}} = -m\frac{v_t^2}{r}\hat{r}$$

Centripetal force (or generally Lorentz Force)

$$\boldsymbol{F}_{CP} = q(v\times B)\hat{r}$$

$$B\rho = \frac{p}{q}$$

$$B\rho[Tm] \approx 3.33\frac{p\left[\frac{GeV}{c}\right]}{q[C]}$$



	LEP	LHC
ρ [m]	3096.175	2803.95
p_0 [GeV/c]	104	7000
B [T]	0.11	8.33

Room-temperature coils

Superconducting coils

$8\,[T] \simeq 0.11[T]\cdot\frac{P_0\,[\text{LHC}]}{P_0\,[\text{LEP}]}$ Linear scaling from LEP

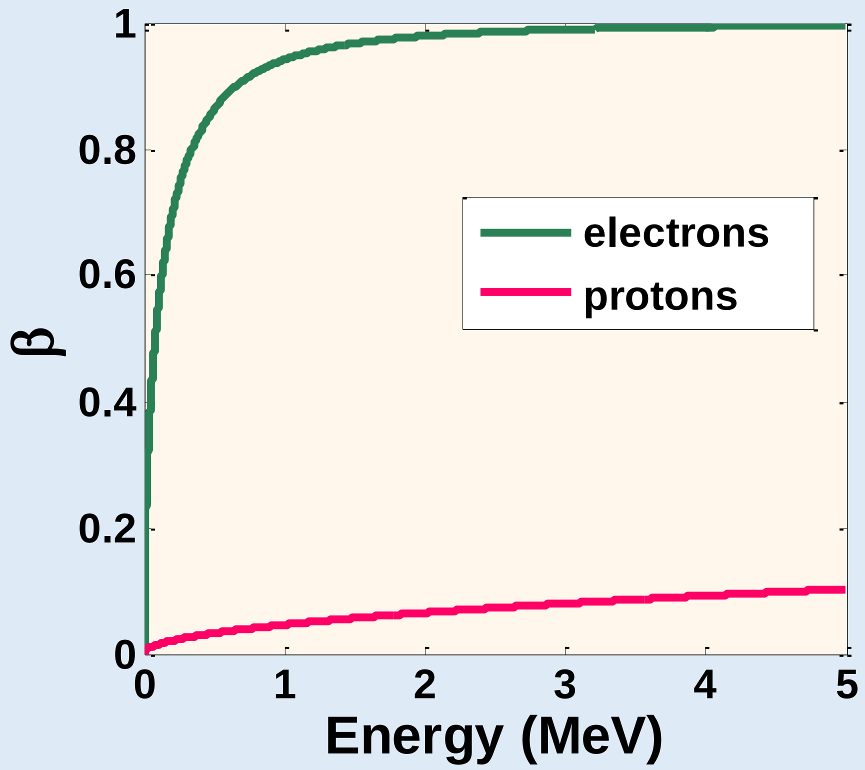
Particle velocity as function of kinetic energy 1/2

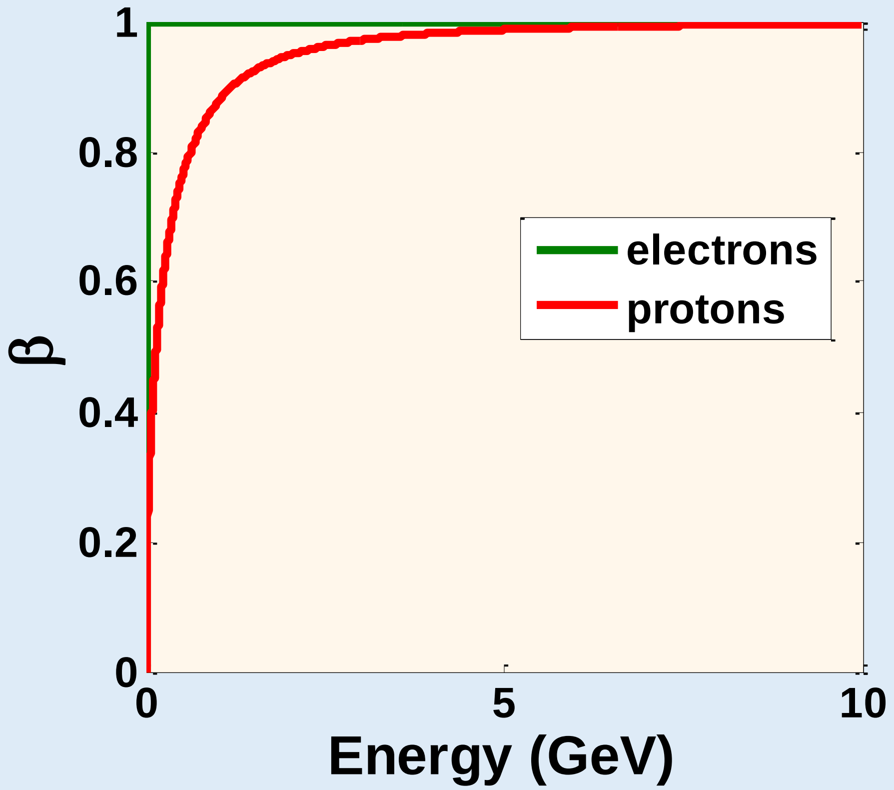
$$\beta\gamma = \sqrt{\gamma^2 - 1}$$

$$\boldsymbol{\beta} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\gamma = 1 + \frac{\boldsymbol{k}}{\boldsymbol{mc^2}}$$

$\beta = \dfrac{v}{c}$ $\beta = 1$ Particle at light velocity *c*





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Electrons are ultra-relativistic @ few MeV
Protons @ few GeV (mass ~2000 times electron mass)
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Particle velocity as function of kinetic energy 2/2

$$E_o = 0.511\ MeV \quad or \quad 938.27\ MeV$$

$$E_{tot} = E_{kin} + E_o$$

$$\gamma = \frac{E_{tot}}{E_o},\quad \beta = \sqrt{1-\frac{1}{\gamma^2}},\quad p = \beta E_{tot}$$

$$B\rho = 3.33\ 10^{-3} p$$

Energy lost per turn

$$\Delta U = \frac{e^2}{3\epsilon_0}\frac{\gamma^4}{\rho}$$

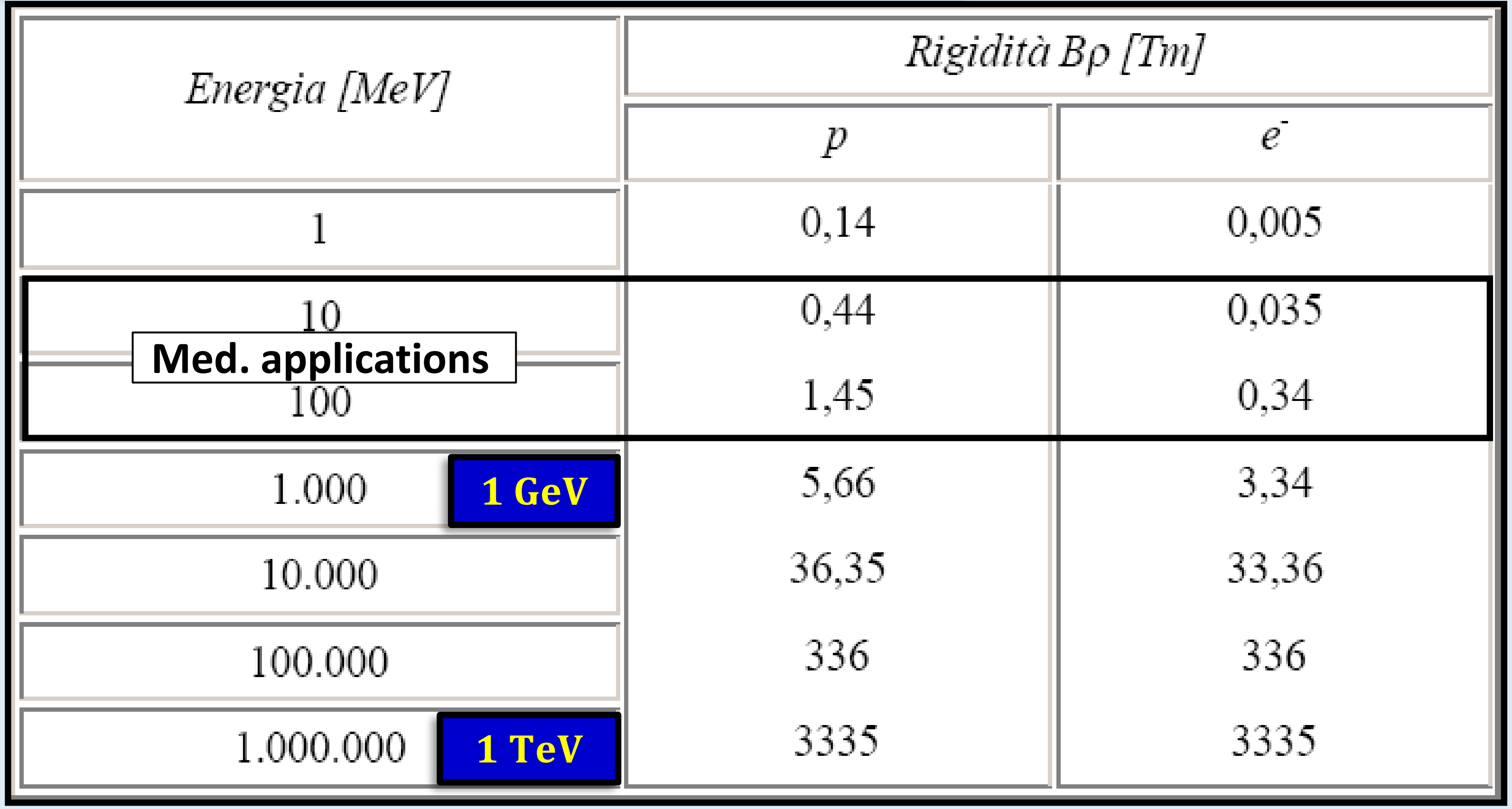
For Proton:

$$\Delta U_p(keV) = 6.03\frac{E(TeV)^4}{\rho(m)}$$

For e-beam:

$$\Delta U_e(keV) = 88.46\frac{E(GeV)^4}{\rho(m)}$$

@ Ultra High energy m_0 becomes negligible



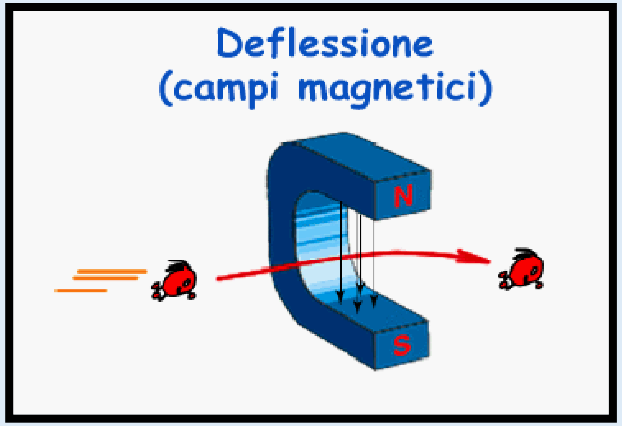
Energia [MeV]	Rigidità Bρ [Tm]	
	p	e⁻
1	0,14	0,005
10	0,44	0,035
Med. applications		
100	1,45	0,34
1.000 **1 GeV**	5,66	3,34
10.000	36,35	33,36
100.000	336	336
1.000.000 **1 TeV**	3335	3335

LHC= 7 TeV
ρ=5km,ΔU=0.7 keV

FCC= 50 TeV
ρ=10km,ΔU=5 MeV

Basic optics for accelerators – 1/3

Dipole fields to change beam direction

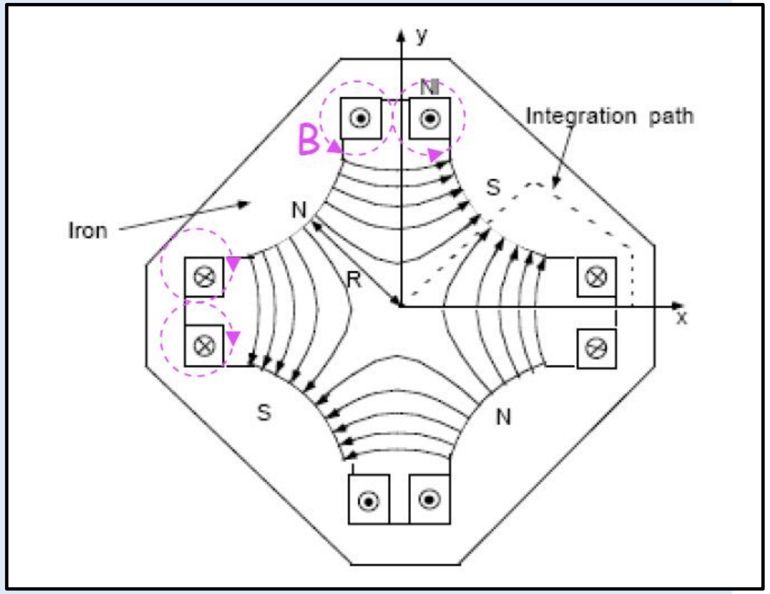


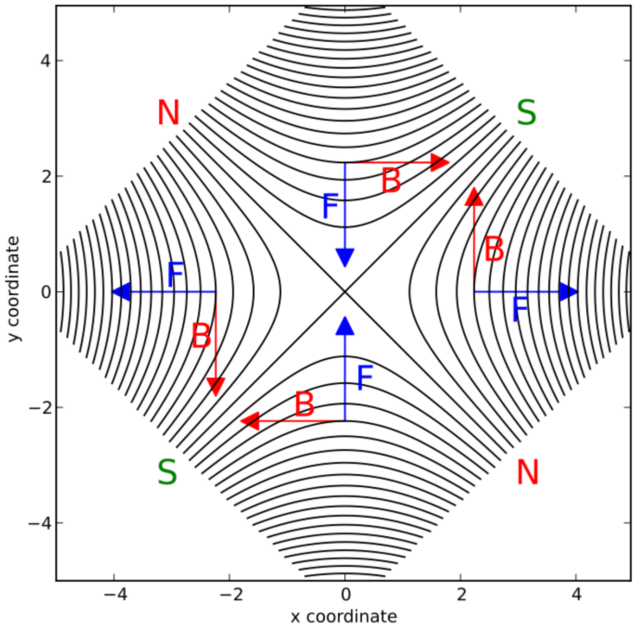
Bending Strength of a dipole

From ridIdity $B\rho[Tm] \approx 3.33 \frac{p[\frac{GeV}{c}]}{q[C]}$

$$\frac{1}{\rho}\left[\frac{1}{m}\right] = \frac{0.2998 \cdot B\ [T]}{p[\frac{GeV}{c}]}$$

Quadrupole fields to focus or defocus the beam





The gradient is:

$$G\left[\frac{T}{m}\right] = \frac{dB_x}{dx} = \frac{dB_y}{dy}$$

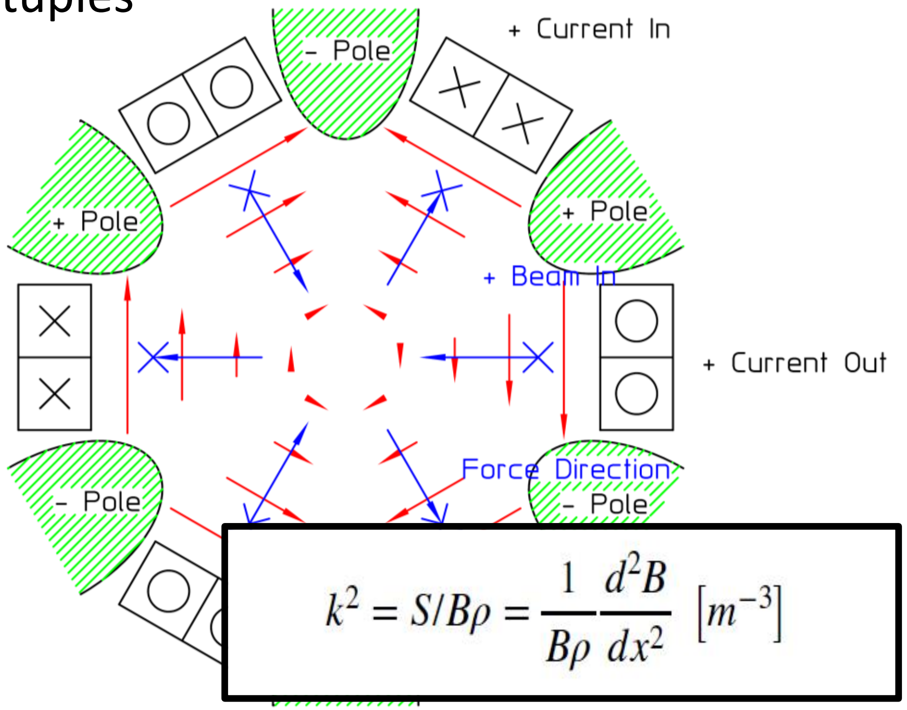
$$G\left[\frac{T}{m}\right] = \frac{B_{tip}}{R}$$

Linear Force with displacement (x):

$$F_x = qvGx\ , F_y = qvGy$$

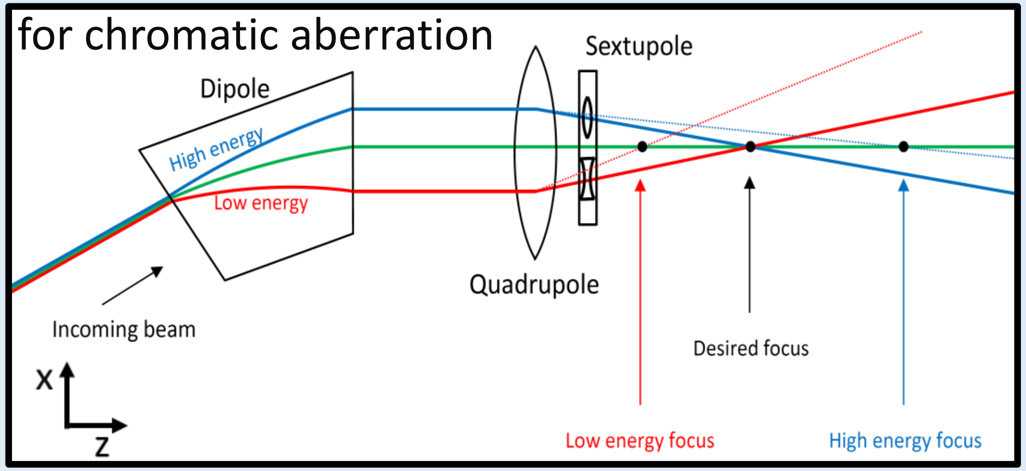
Basic optics for accelerators – 2/3

Sextuples

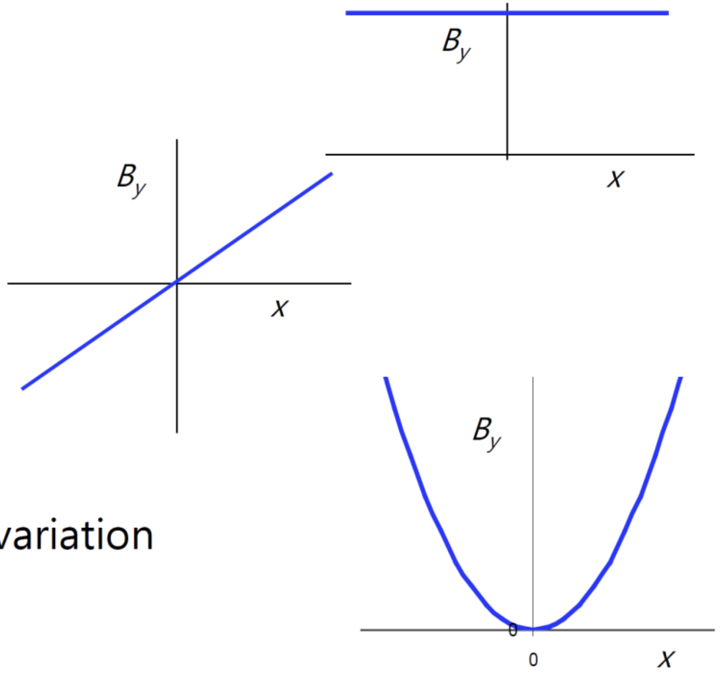


$$k^2 = S/B\rho = \frac{1}{B\rho}\frac{d^2B}{dx^2} \quad [m^{-3}]$$

for chromatic aberration



Fast recap: fields behaviour

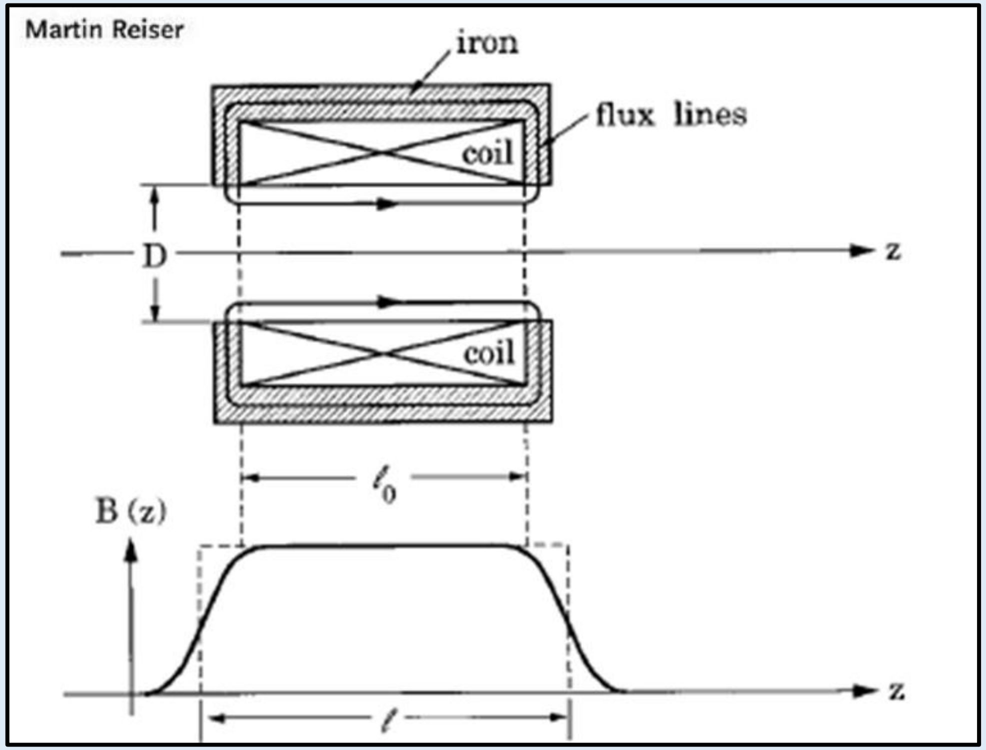


Dipole - constant field

Quad - linear variation

Sextupole - quadratic variation

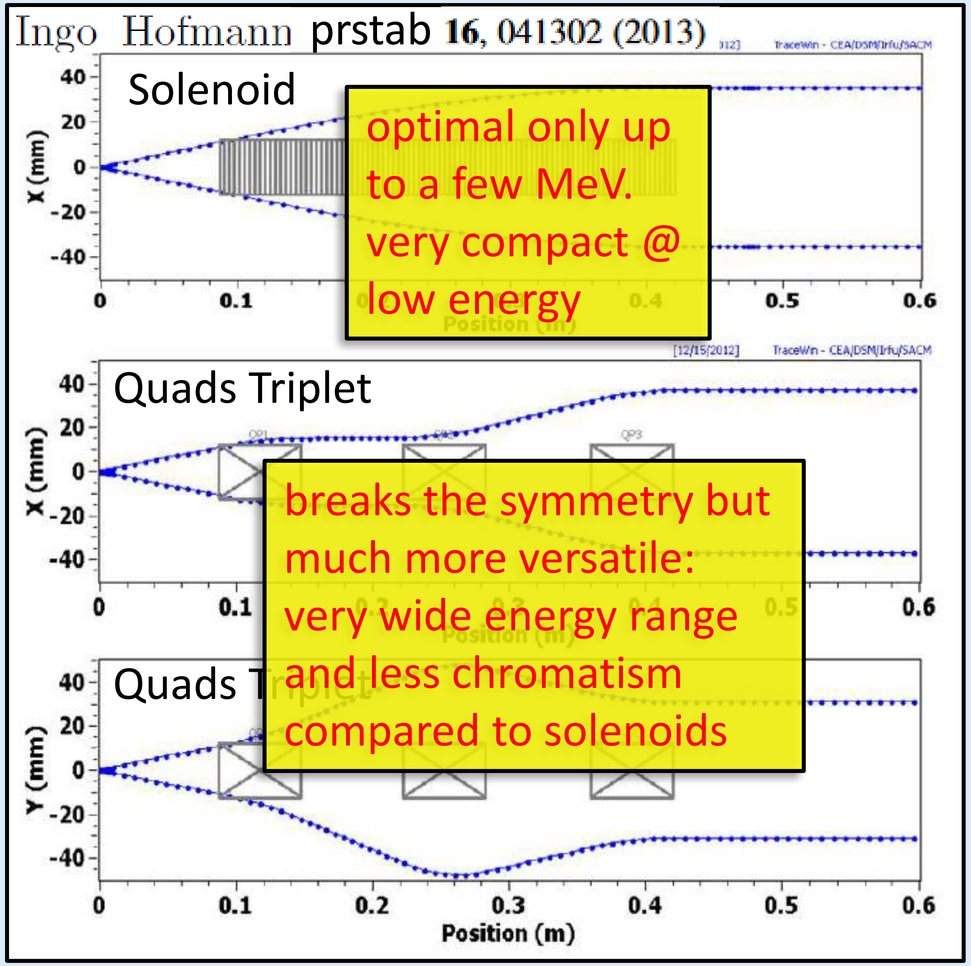
Basic optics for accelerators – 3/3



Martin Reiser

$$\frac{1}{f} = -\frac{r'}{r} = \left(\frac{q}{2mc\beta\gamma}\right)^2 \int_{z_1}^{z_2} B^2\, dz$$

Solenoid VS Quads, Same solution



Ingo Hofmann prstab 16, 041302 (2013)

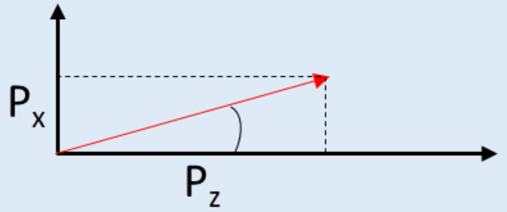
optimal only up to a few MeV. very compact @ low energy

breaks the symmetry but much more versatile: very wide energy range and less chromatism compared to solenoids

Important differences to remember:

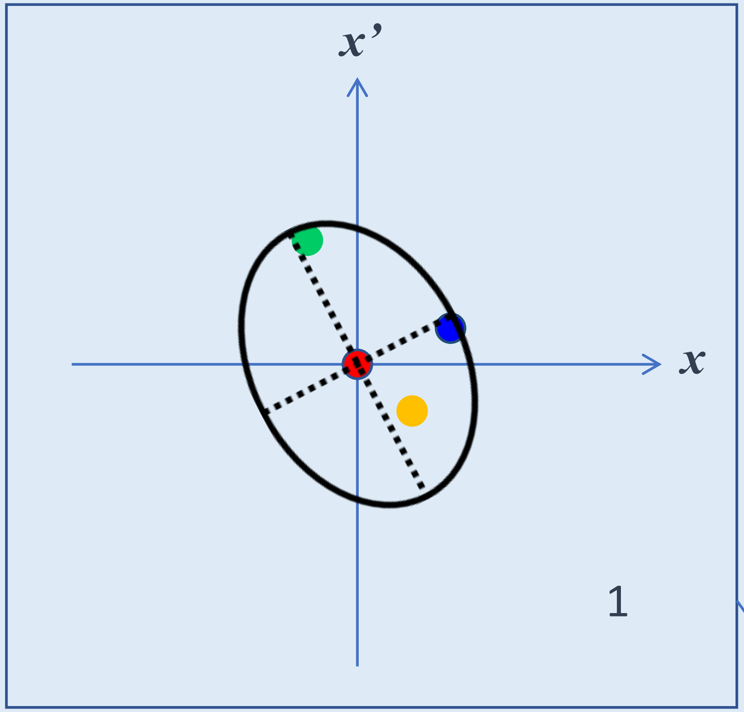
- solenoids work well only at low beam energy: few MeV for electrons.
- solenoids give larger beam quality degradation if the beam is large (sig_x) into the optic (chromatism effects)

Bunch transverse Phase-space & emittance – 1/5



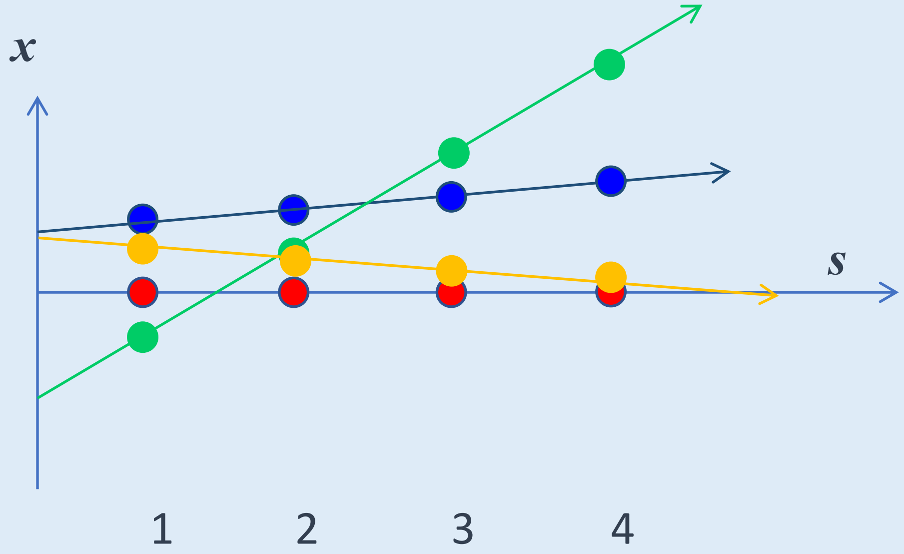
Paraxial approximation

$$x' = \frac{P_x}{P_z} \ll 1 \;\rightarrow \frac{P_x}{P_z} = Tan(\theta) \cong \theta$$



● Reference particle

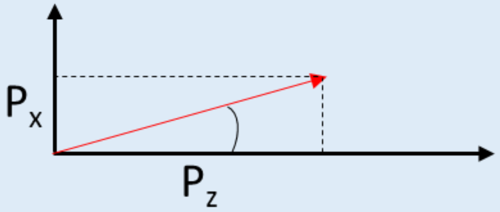
● ● ● Other particles



Phase space

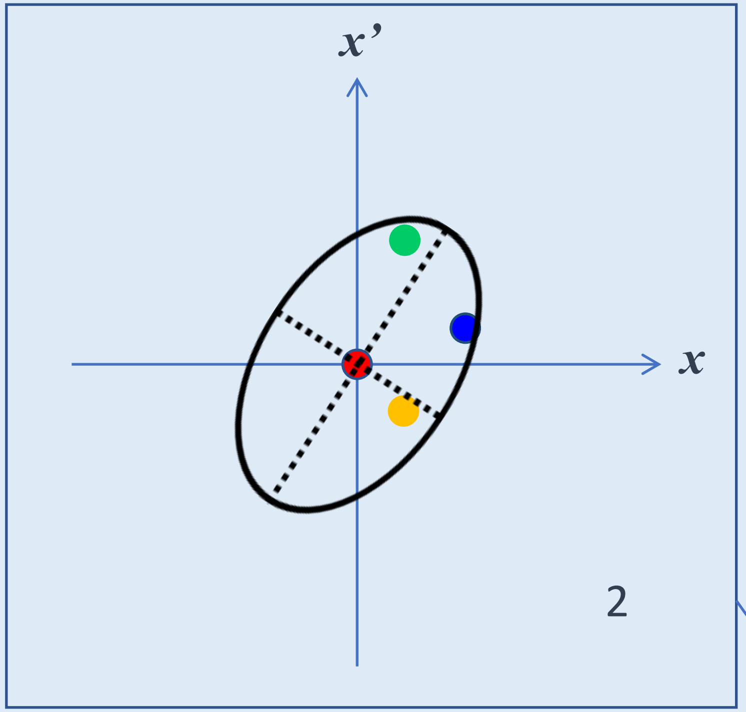
No external fields – Drift propagation

Bunch transverse Phase-space & emittance – 2/5



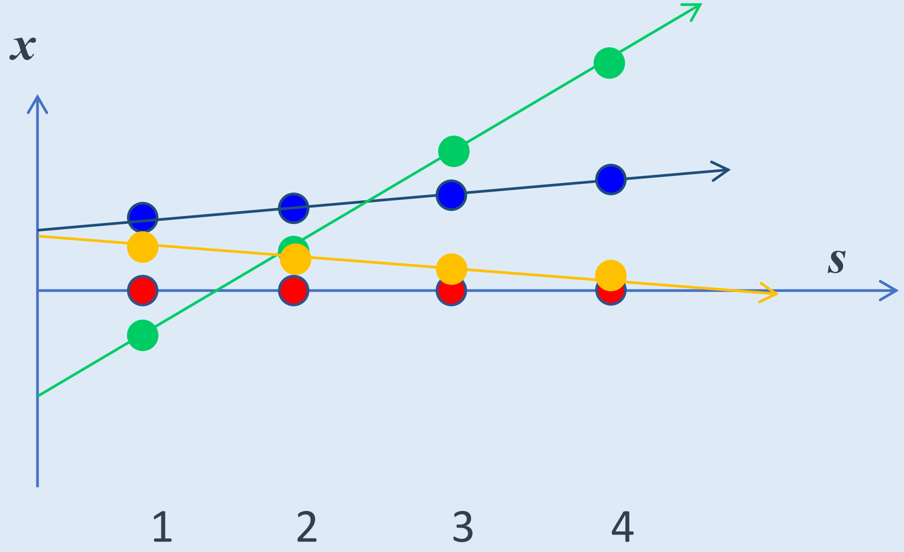
Paraxial approximation

$$x' = \frac{P_x}{P_z} \ll 1 \;\rightarrow \frac{P_x}{P_z} = Tan(\theta) \cong \theta$$



● Reference particle

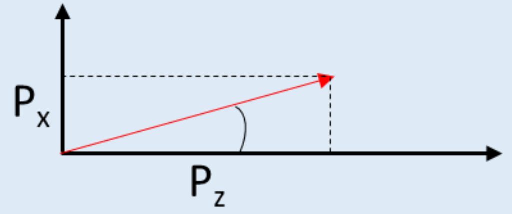
● ● ● Other particles



Phase space

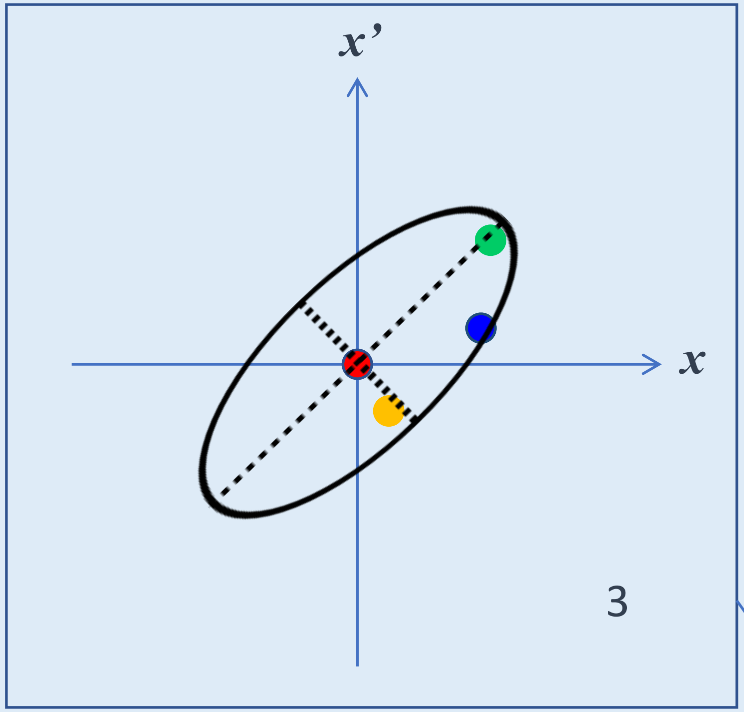
No external fields – Drift propagation

Bunch transverse Phase-space & emittance – 3/5



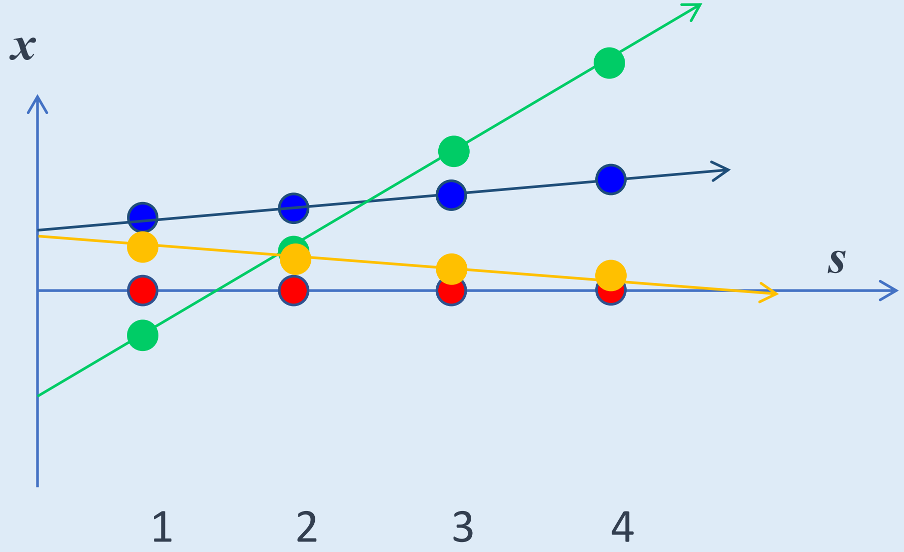
Paraxial approximation

$$x' = \frac{P_x}{P_z} \ll 1 \;\rightarrow \frac{P_x}{P_z} = Tan(\theta) \cong \theta$$



● Reference particle

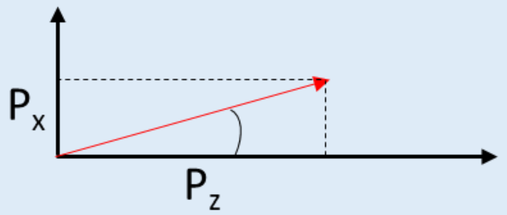
● ● ● Other particles



Phase space

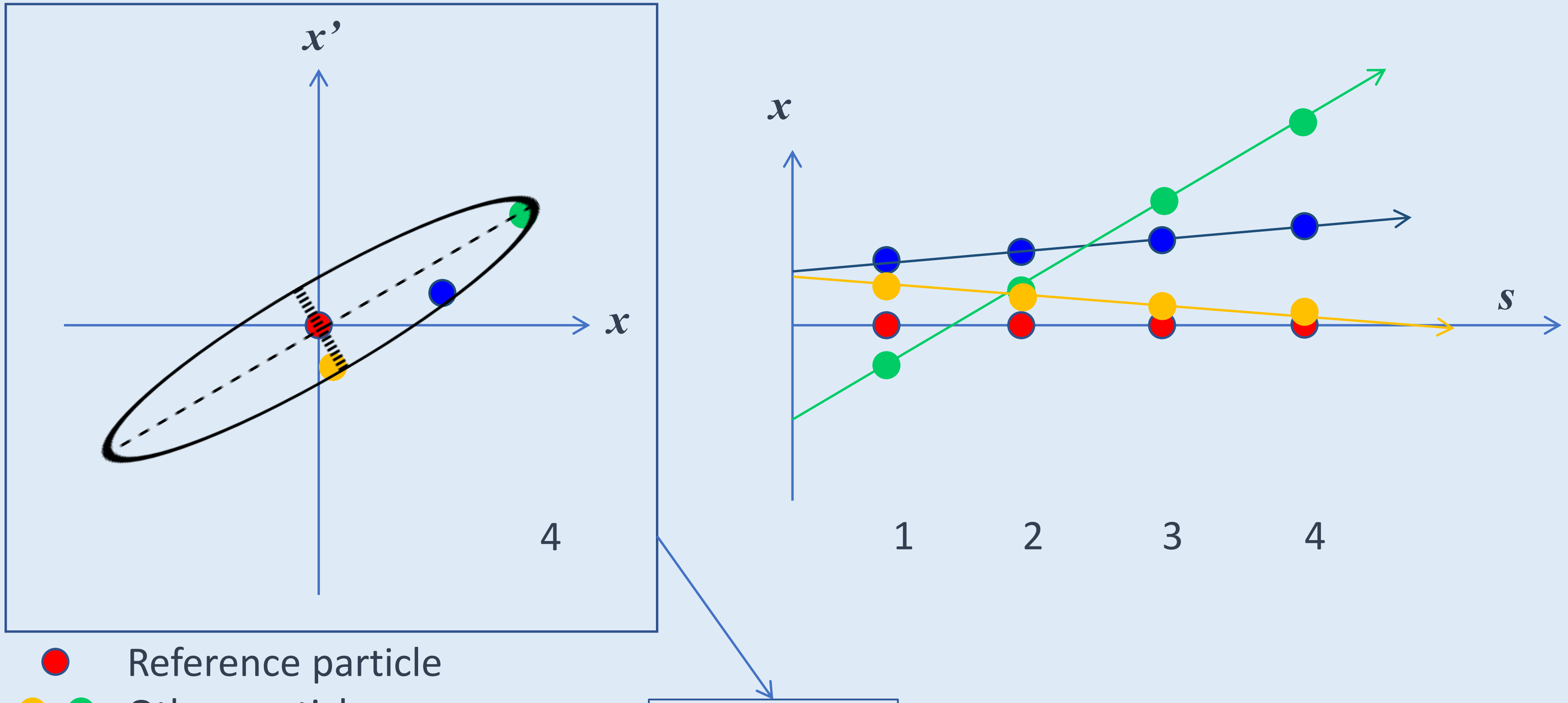
No external fields – Drift propagation

Bunch transverse Phase-space & emittance – 4/5



Paraxial approximation

$$x' = \frac{P_x}{P_z} \ll 1 \rightarrow \frac{P_x}{P_z} = Tan(\theta) \cong \theta$$



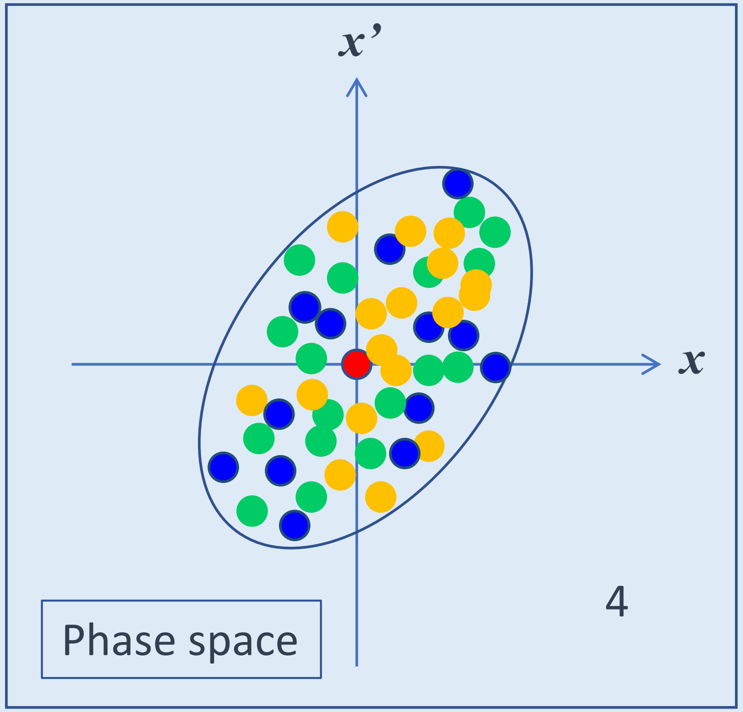
● Reference particle

● ● ● Other particles

Phase space

No external fields – Drift propagation

Bunch transverse Phase-space & emittance – 5/6



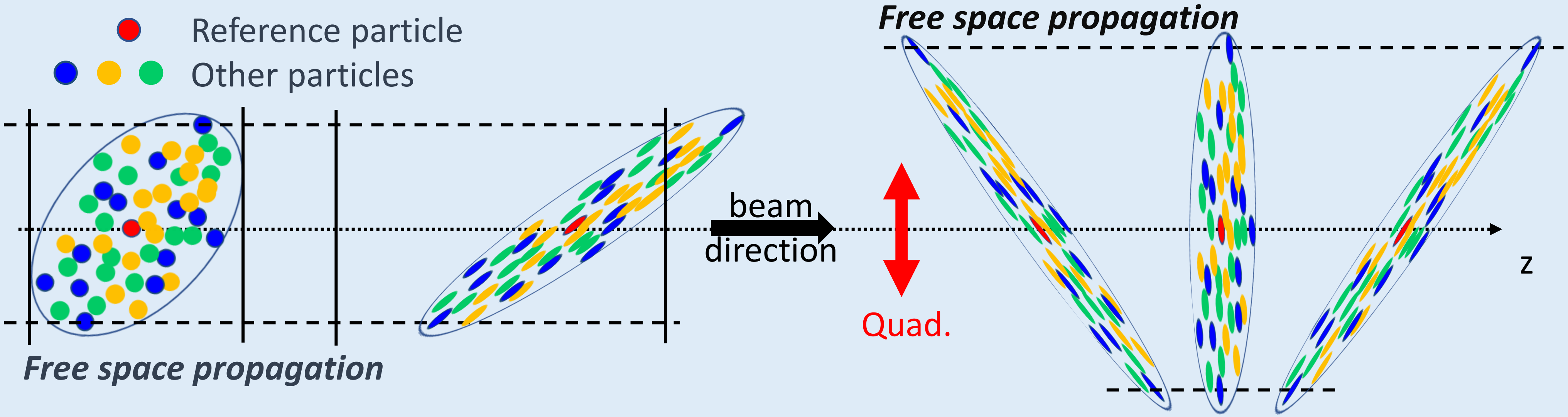
● Reference particle

● ● ● Other particles

- Emittance = Area of phase space

- Horizontal phase space (x, x’)
- Vertical phase space (y, y’)
- Longitudinal phase space (Time-Energy)

For linear fields (↕) the emittance will be constant



Bunch transverse Phase-space & emittance – 6/6

rms-geometrical emittance

$$\varepsilon_{n,x} = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x\cdot x'\rangle^2}$$

can be normalized

Ellipse equation (The Area is επ)

$$x'_j = \frac{p_{x,j}}{p_{z,j}},\ y'_j = \frac{p_{y,j}}{p_{y,j}}$$

Twiss parameters

$$\beta_x = \frac{\langle x^2\rangle}{\varepsilon_x}$$

$$\gamma_x = \frac{\langle x'^2\rangle}{\varepsilon_x}$$

$$\alpha_x = -\frac{\langle xx'\rangle}{\varepsilon_x}$$

$$\beta_x\gamma_x - \alpha_x^2 = 1$$

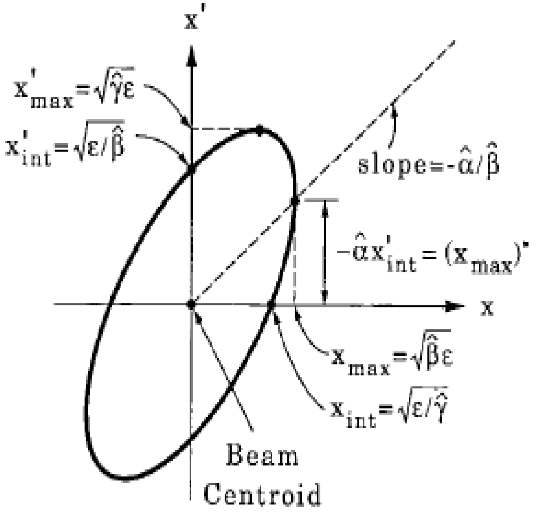


Fig. 3.26 Trace-space ellipse described by equation $\hat{\gamma}x^2 + 2\hat{\alpha}xx' + \hat{\beta}x'^2$ and relations for several important points on the circumference of the ellipse.

Martin Reiser – Theory and Design of Charged Particle Beams

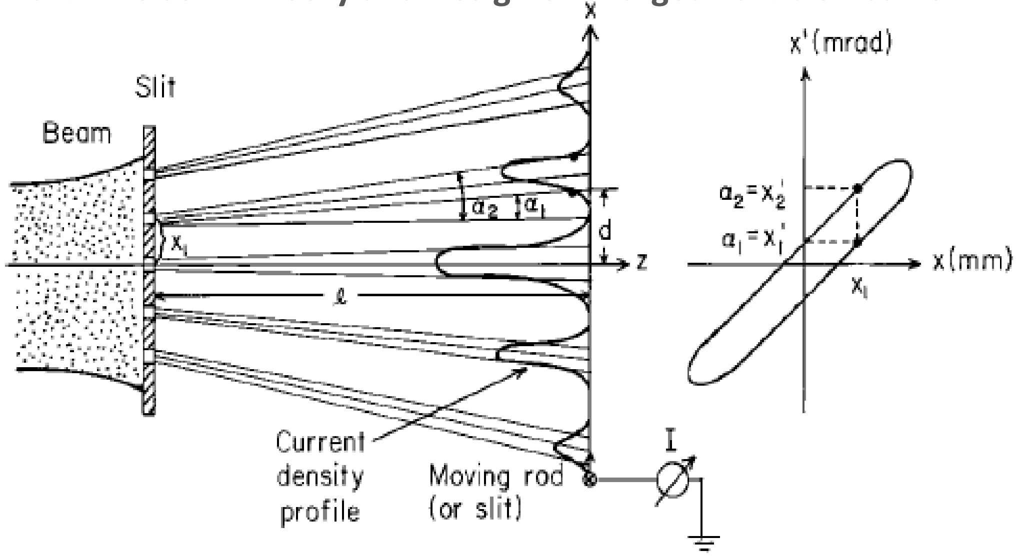


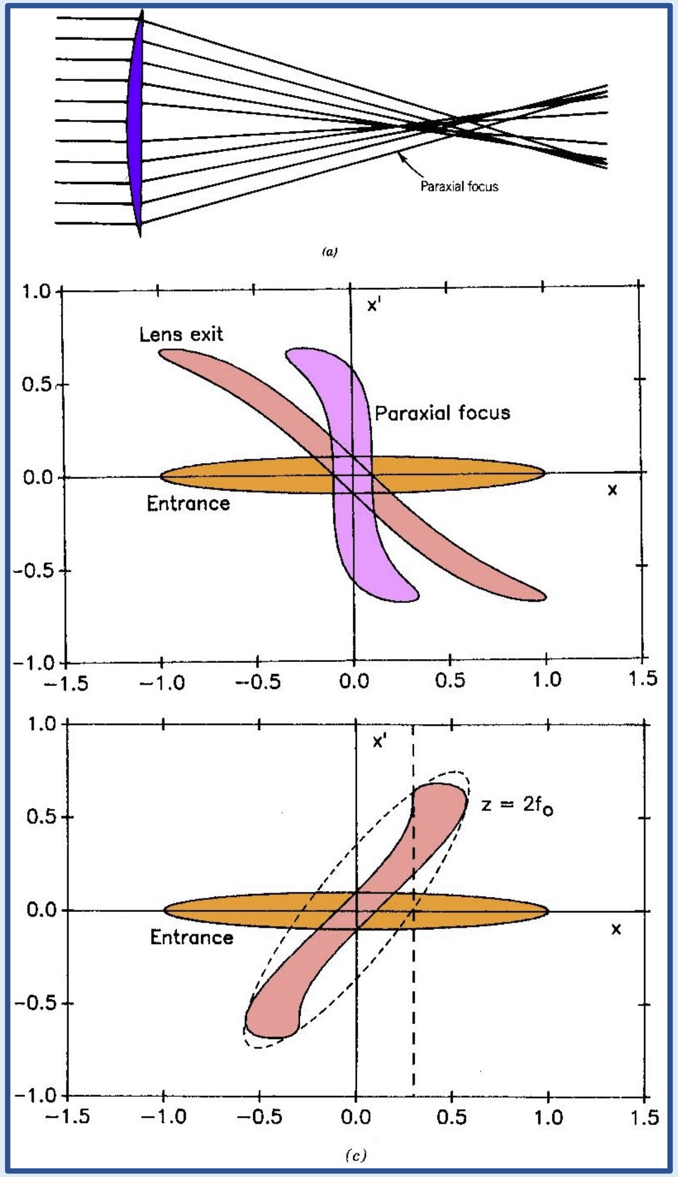
Fig. 3.1 Method of measuring the trace-space distribution of a beam.

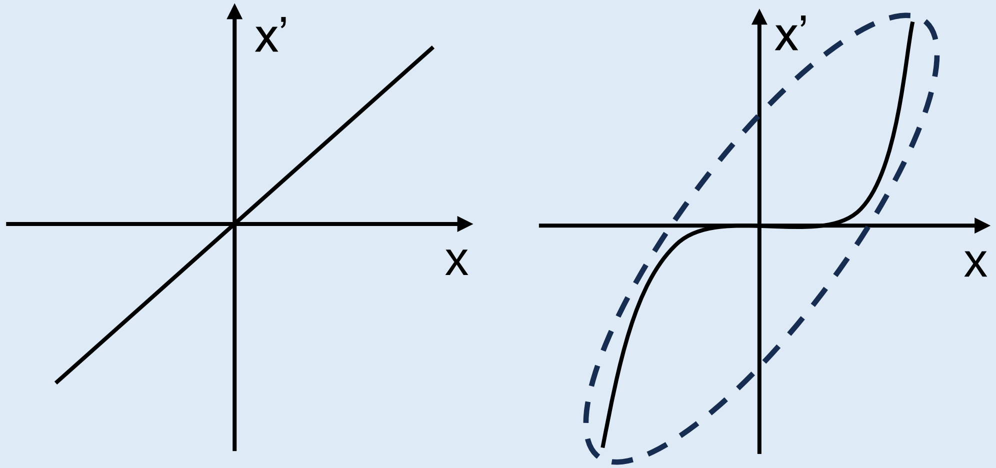
- Beam brightness definition: Current density x unit solid angle

$$B = \frac{J}{d\Omega} = \frac{dI}{dS\,d\Omega} \qquad \langle B_n\rangle = \frac{I}{\iint dS\,d\Omega} = \frac{2I}{\pi^2\varepsilon_{n,x}\varepsilon_{n,y}}\left[\frac{A}{m^2}\right]$$

Brightness or Emittance Degradation

A focusing channel





Considering that for any position x the divergece of the particle is $x' = Cx^n$

$$\varepsilon_x^2 = \overline{x^2x'^2} - \overline{xx'}^2 = C^2\left(\overline{x^2x^{2n}} - \overline{xx^{n+1}}^2\right)$$

with n=1 the staight line gives rms emittance equal 0. For n≠1 the emittance is not 0, also if the two distribution area are 0

❖ The Brightness

Brightness

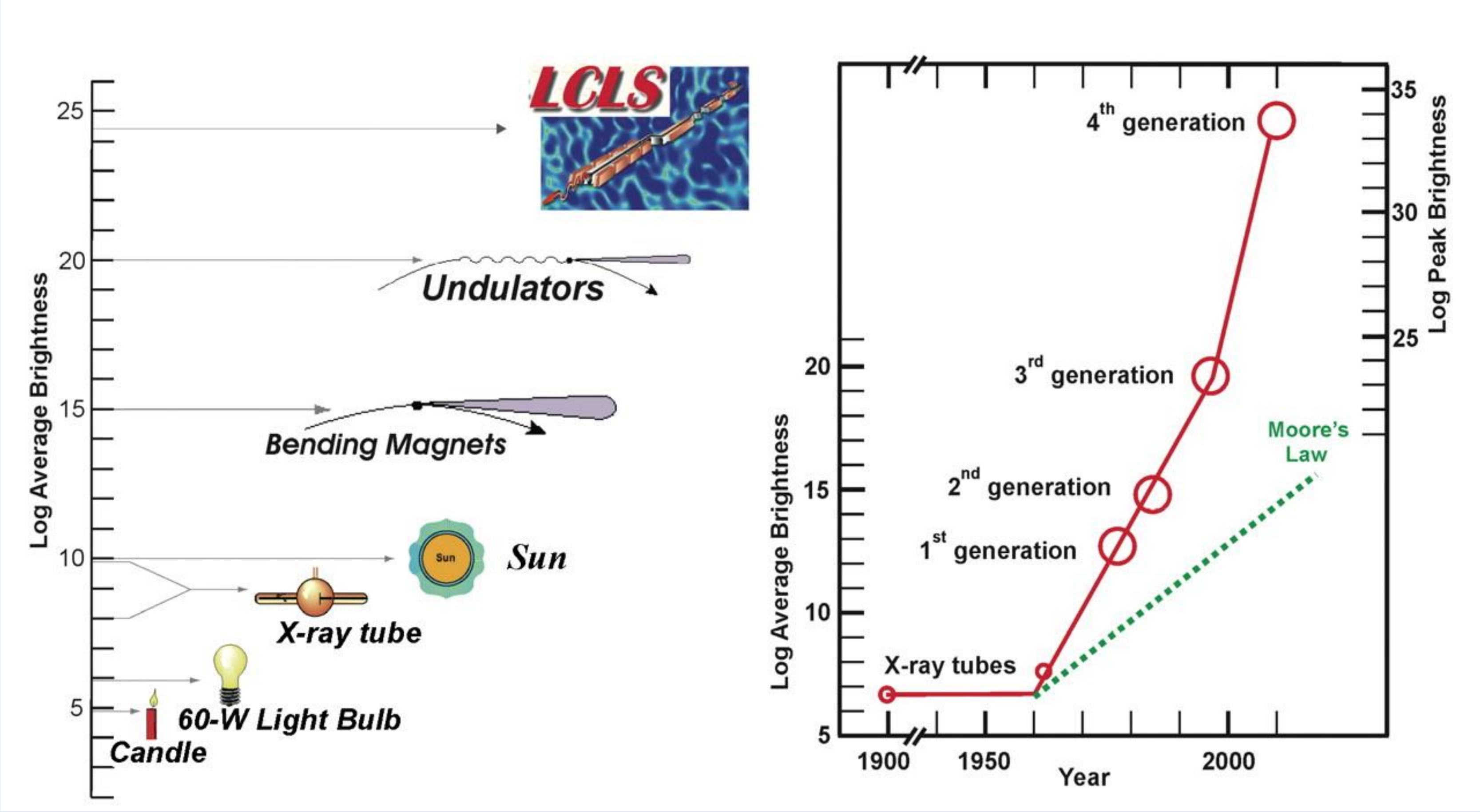
$$B = \frac{2I}{\pi^2 \varepsilon_{x,n} \varepsilon_{y,n}}$$

Current

$$I[A] = \frac{Q[C]c[\frac{m}{s}]}{\sigma_z[m]\sqrt{12}}$$

Emittance

$$\varepsilon_{n,x} = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x \cdot x'\rangle^2}$$

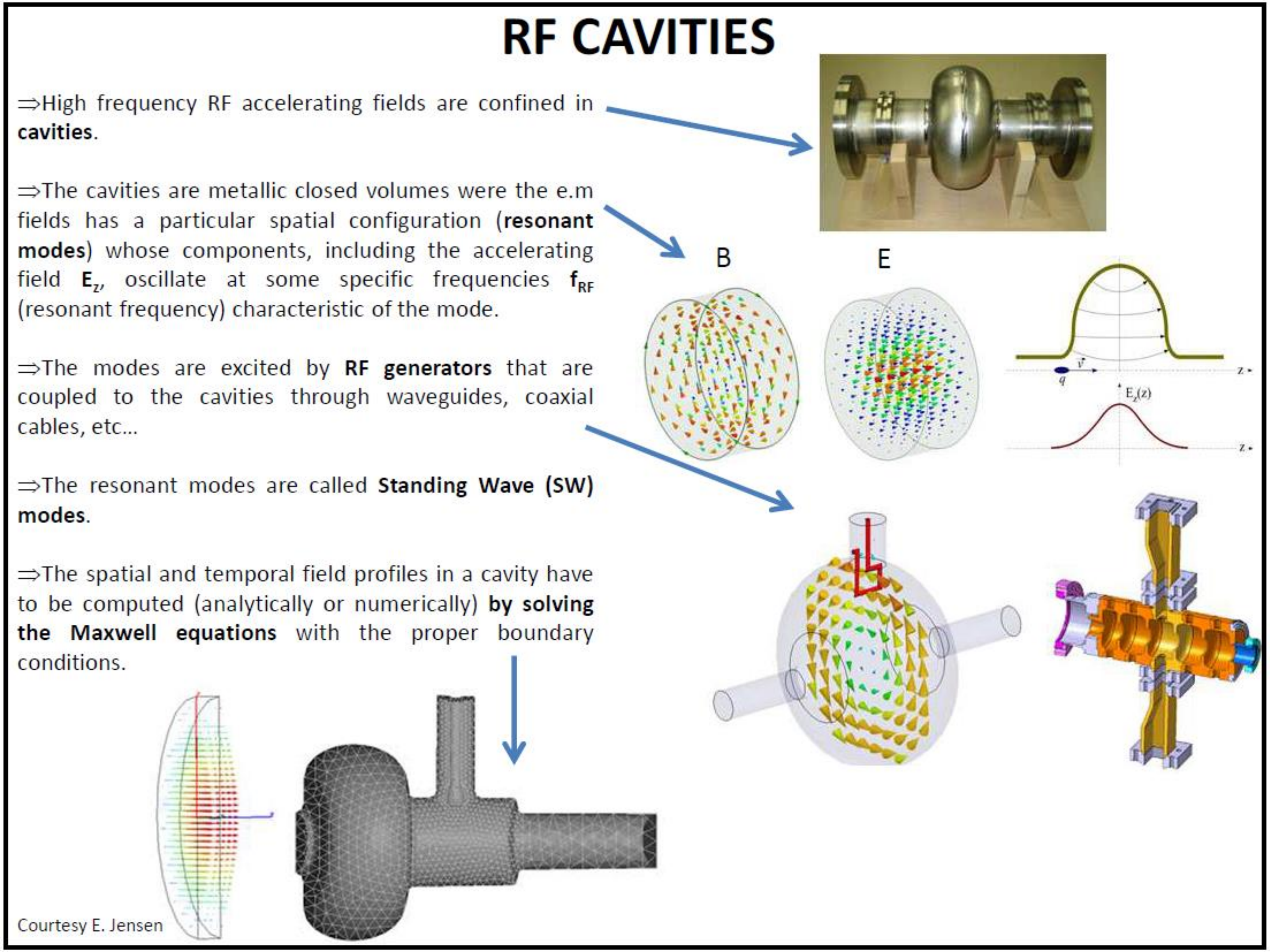


Accelerators cavities basic concepts:

Resonant Modes, Phase velocity, Standing & Traveling, low β cavities

Accelerat

The Accelera



RF CAVITIES

⇒High frequency RF accelerating fields are confined in **cavities**.

⇒The cavities are metallic closed volumes were the e.m fields has a particular spatial configuration (**resonant modes**) whose components, including the accelerating field **E_z**, oscillate at some specific frequencies **f_{RF}** (resonant frequency) characteristic of the mode.

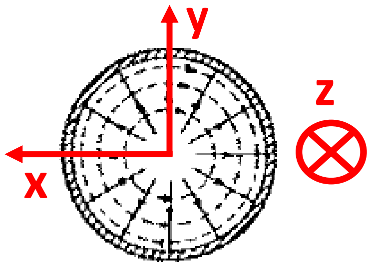
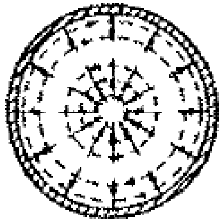
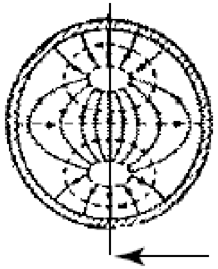
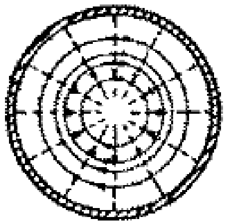
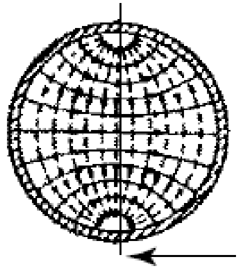
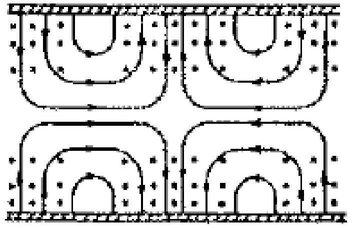
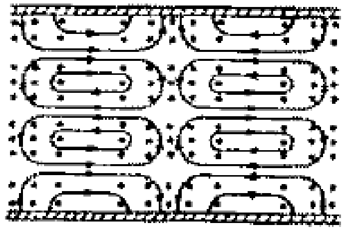
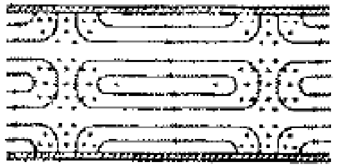
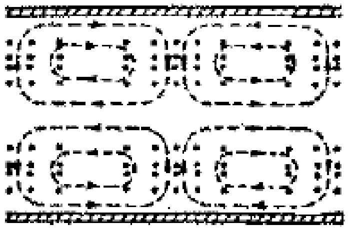
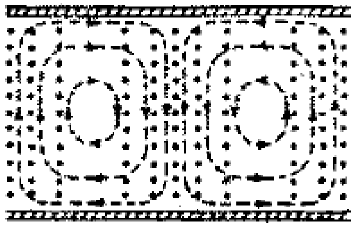
⇒The modes are excited by **RF generators** that are coupled to the cavities through waveguides, coaxial cables, etc…

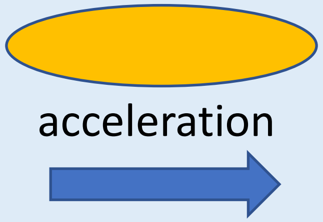
⇒The resonant modes are called **Standing Wave (SW) modes**.

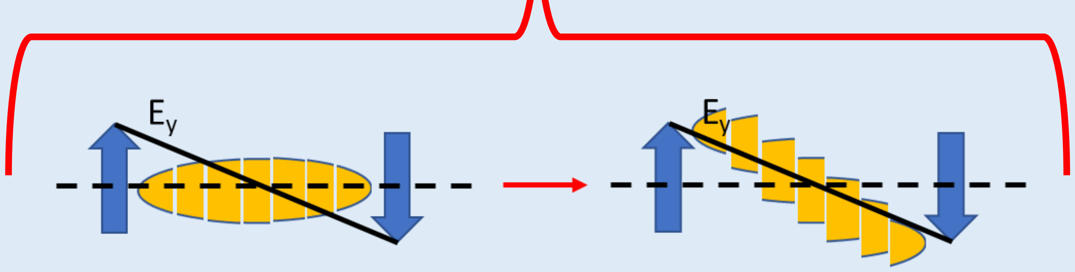
⇒The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.

Courtesy E. Jensen

Ramo, John R. Whinnery - Fields and Waves in Communication Electronics – Circular pipe guides

Wave Type	TM_{01}	TM_{02}	TM_{11}	TE_{01}	TE_{11}
Field distributions in cross-sectional plane, at plane of maximum transverse fields			 Distributions below along this plane		 Distributions below along this plane
Field distributions along guide					
Field components present	E_z, E_r, H_ϕ	E_z, E_r, H_ϕ	$E_z, E_r, E_\phi, H_r, H_\phi$	H_z, H_r, E_ϕ	$H_z, H_r, H_\phi, E_r, E_\phi$


acceleration



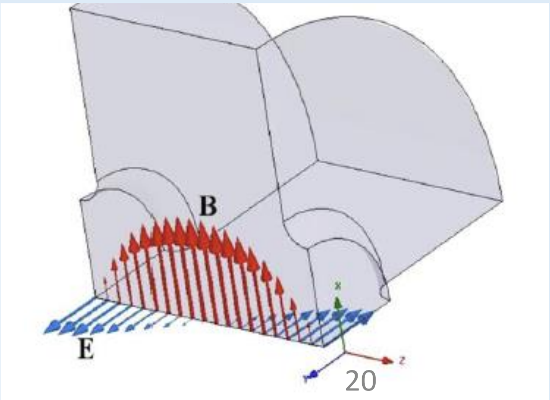


Fig. 13. Field profiles of the deflecting TM_{110}-like mode.

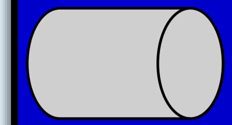
Dispersion curve and Phase Velocity v_p to accelerate $v_p = c\beta$

$$E_z = E_0 J_0(k_c r)\exp[j(\omega t - \beta z)]$$
$$E_r = j\eta E_0[1-(\omega_c/\omega)^2]^{\frac{1}{2}} J_1(k_c r)\exp[j(\omega t - \beta z)]$$
$$H_\theta = jE_0 J_1(k_c r)\exp[j(\omega t - \beta z)]$$

TM_{01}

where J_0 and J_1 are Bessel functions, ω_c is the cut-off frequency, $k_c = \omega_c/c$, $\beta = \omega/v_p$, $\beta^2 = k^2 - k_c^2$, $k = \omega/c$ and η is the intrinsic impedance of the medium. For these solutions to exist, the propagation constant β must be real; this condition is fulfilled only if $v_p > c$.

Into a cylindrical guide:



$$\omega^2 = (k_c \cdot c)^2 + (\beta \cdot c)^2$$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-(k_c \cdot c/\omega)^2}} > c$$

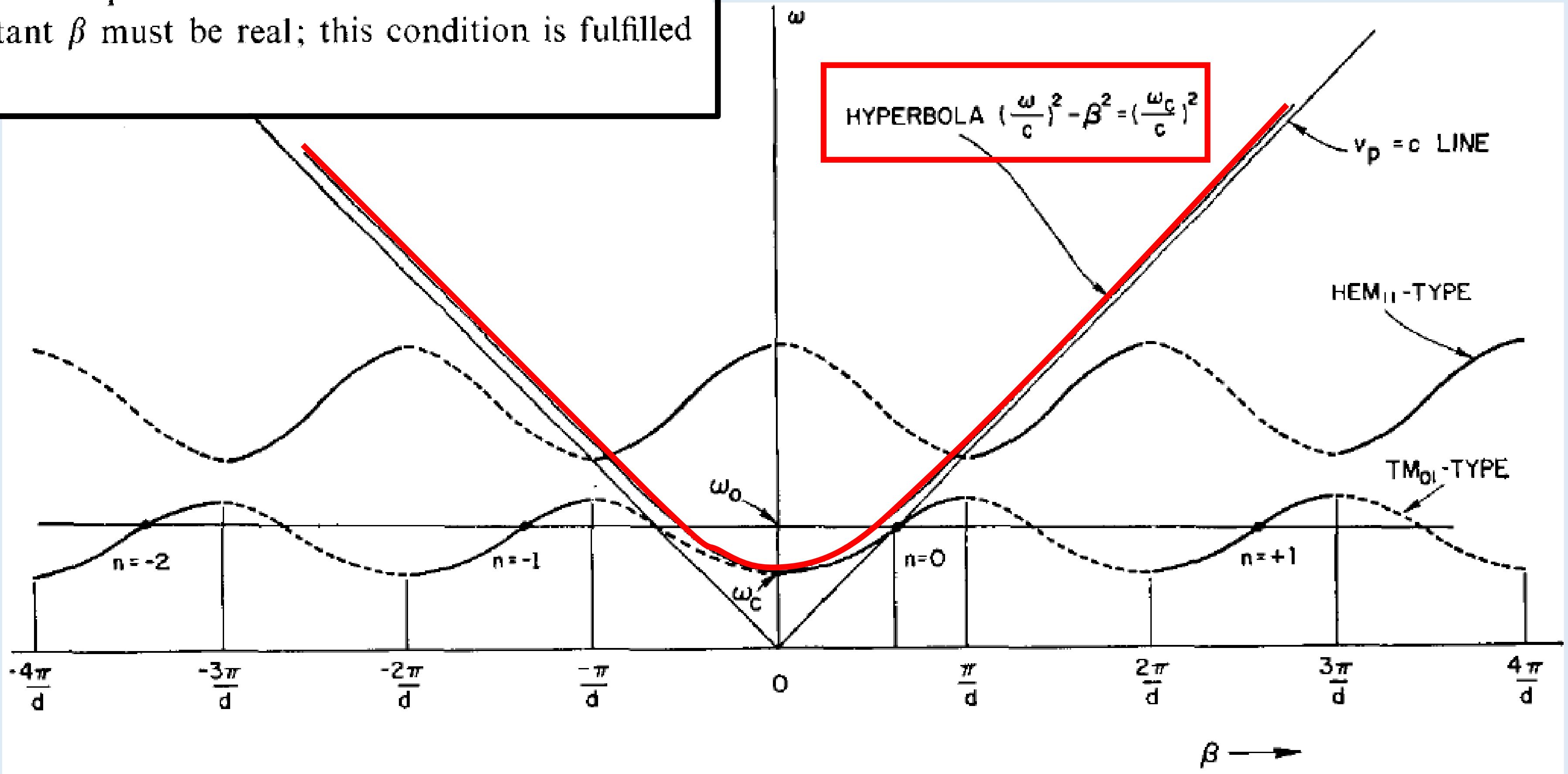


Fig. 2. Brillouin (or ω-β) diagram showing propagation characteristics for uniform and periodically loaded structures.

From Lapostolle (1970) – Linear Accelerators

Disk loaded cavities

SLAC-PUB-2884

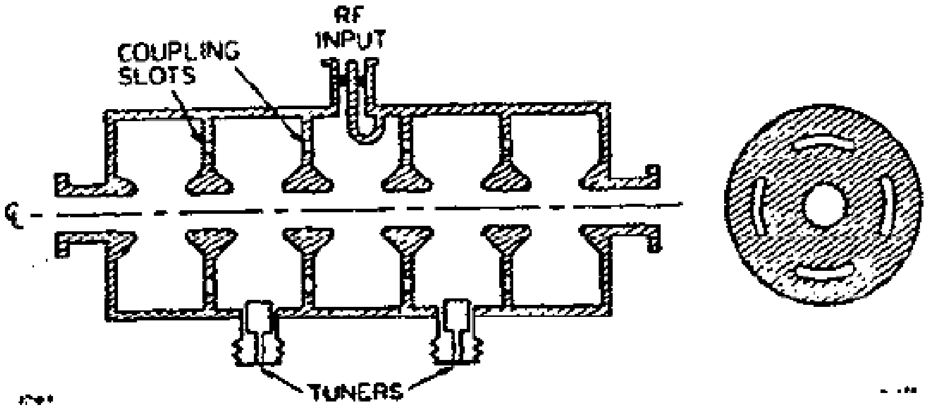


Fig. 3.3. Diagram showing the important features of a five-cell π-mode structure with magnetic field coupling.

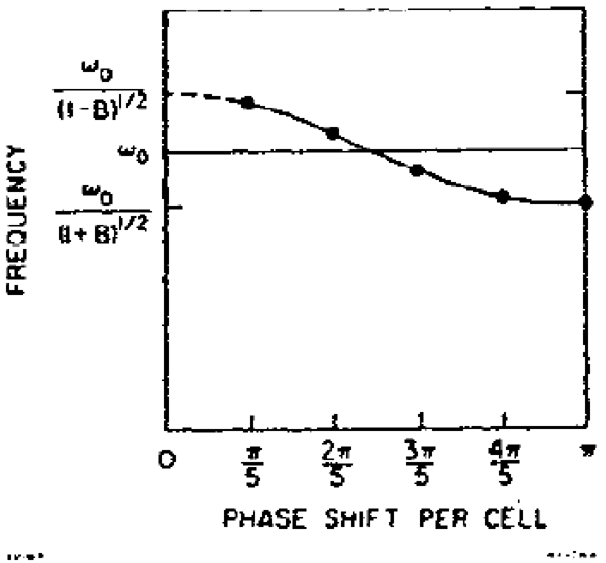
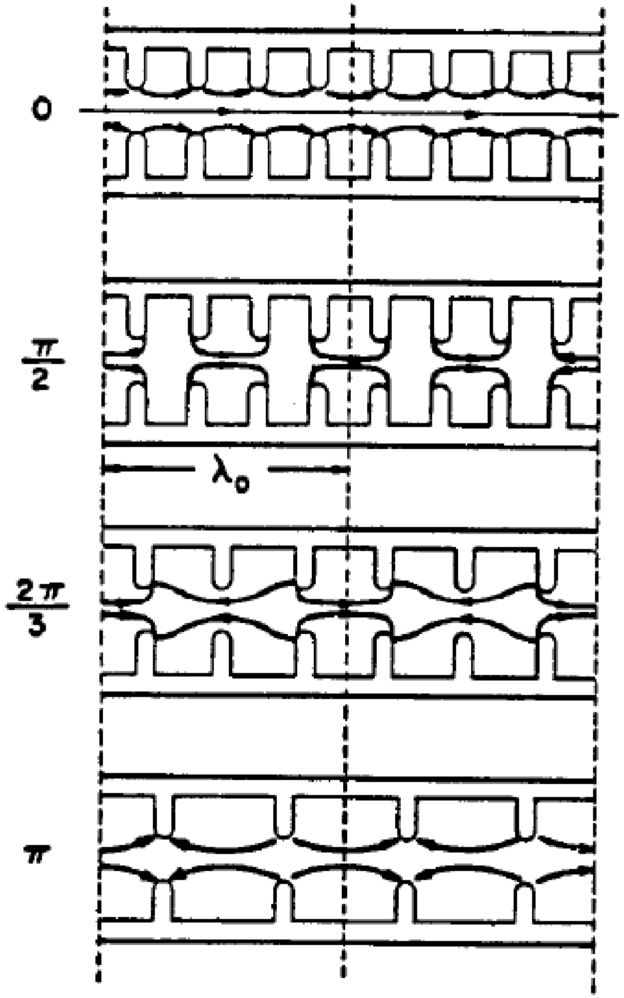
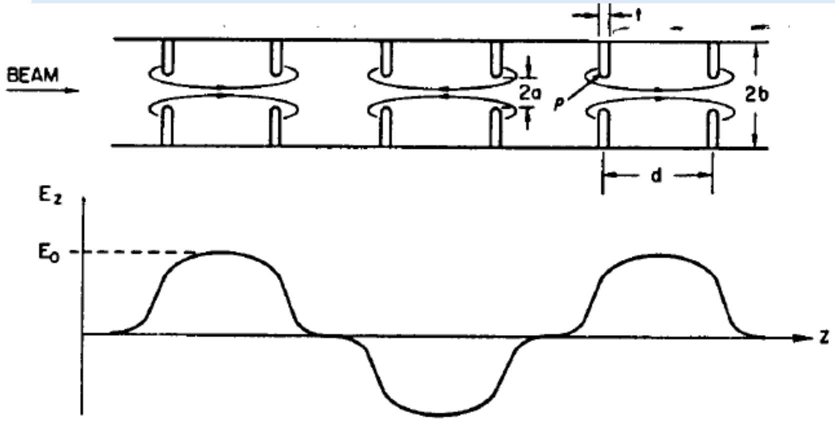


Fig. 3.4. Dispersion diagram for a five-cell structure with "flat" π-mode.



SLAC-PUB-3221



Low Beta cavities $\beta < 1$

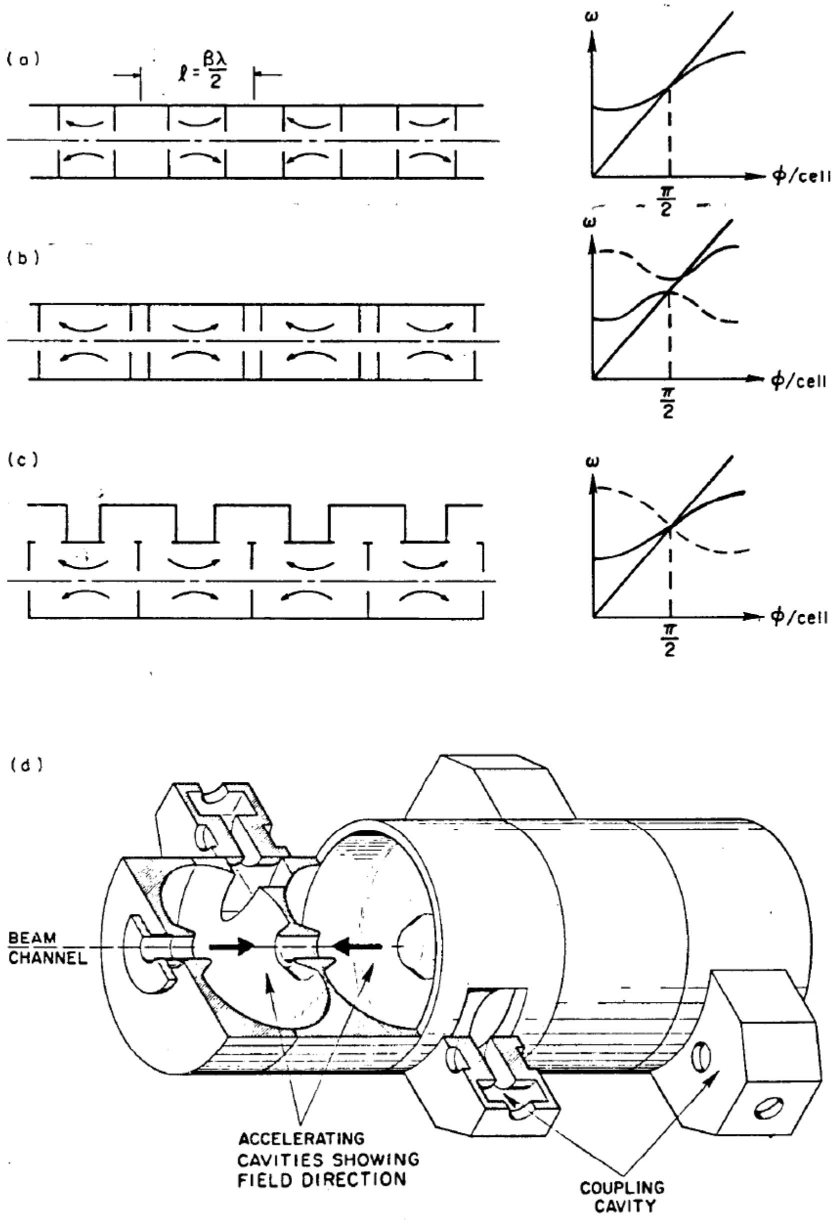
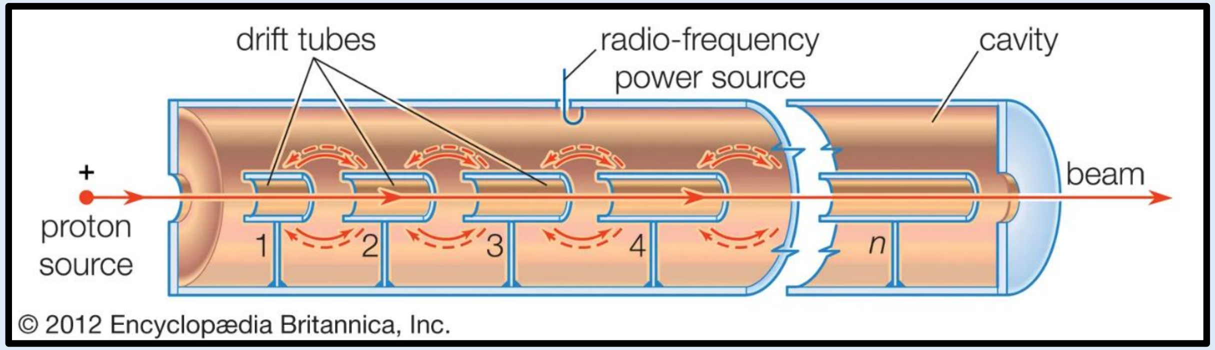


Fig. 31. Evolution of $\pi/2$-mode iris-loaded structure to π-mode side-coupled structure.

DTL – Drift Tube Linear Accelerator



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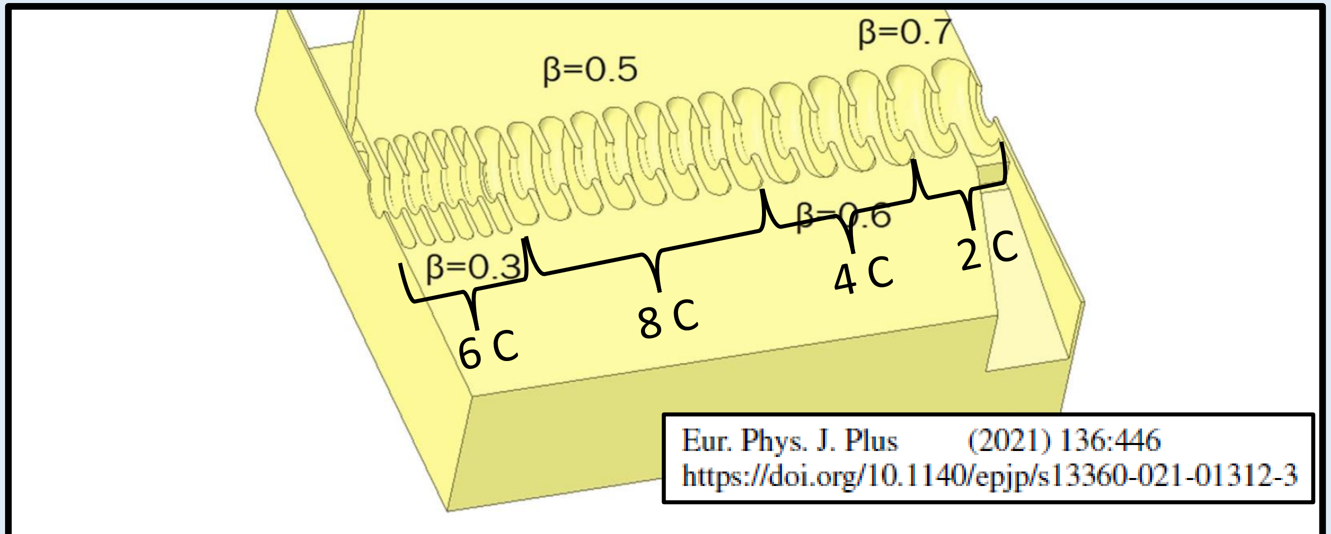
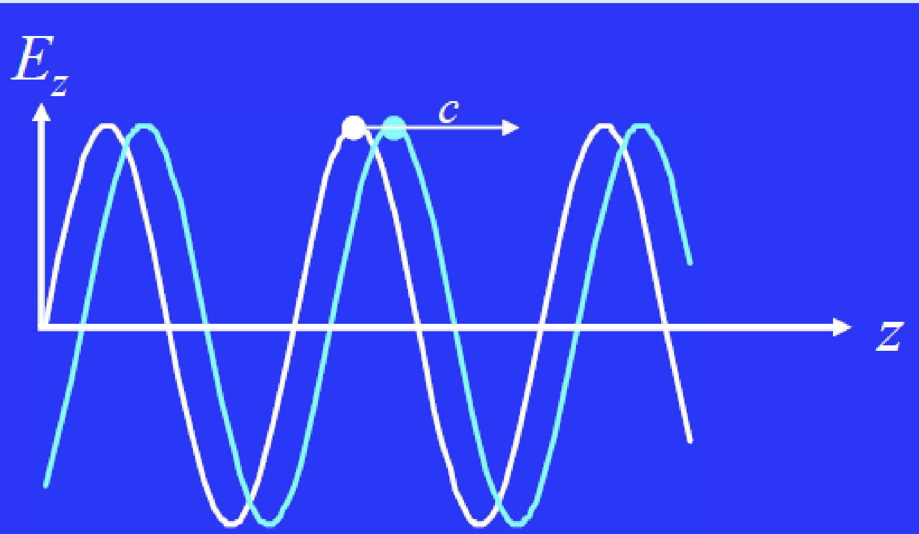


Eur. Phys. J. Plus (2021) 136:446
https://doi.org/10.1140/epjp/s13360-021-01312-3

Fig. 31 Example of a buncher for a 40-kW 16.4 GHz linac, consisting of four β_{ph} = const sections: six cells with $\beta_{ph} = 0.3$, eight cells with $\beta_{ph} = 0.5$, four cells with $\beta_{ph} = 0.6$, and two cells with $\beta_{ph} = 0.7$. The available power restricts the length due to low field amplitude A

Sergey V. Kutsaev (RadiaBeam Tech) – Electron bunchers for industrial RF linear accelerators: Theory and design guide

TW & SW cavities – 1/4



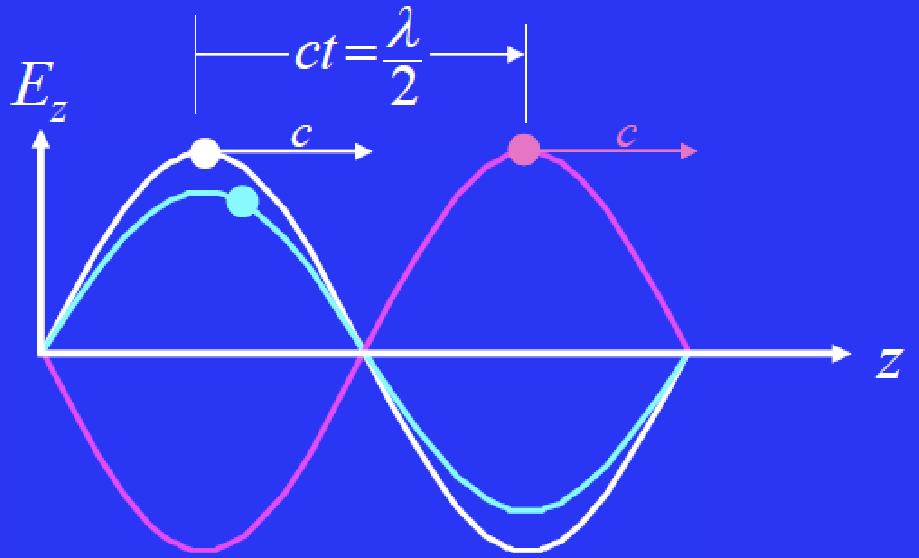
travelling wave structure:

need *phase velocity* = c

(*disk-loaded structure*)

bunch sees constant field:

$$E_z = E_0 \cos(\phi)$$



standing wave cavity:

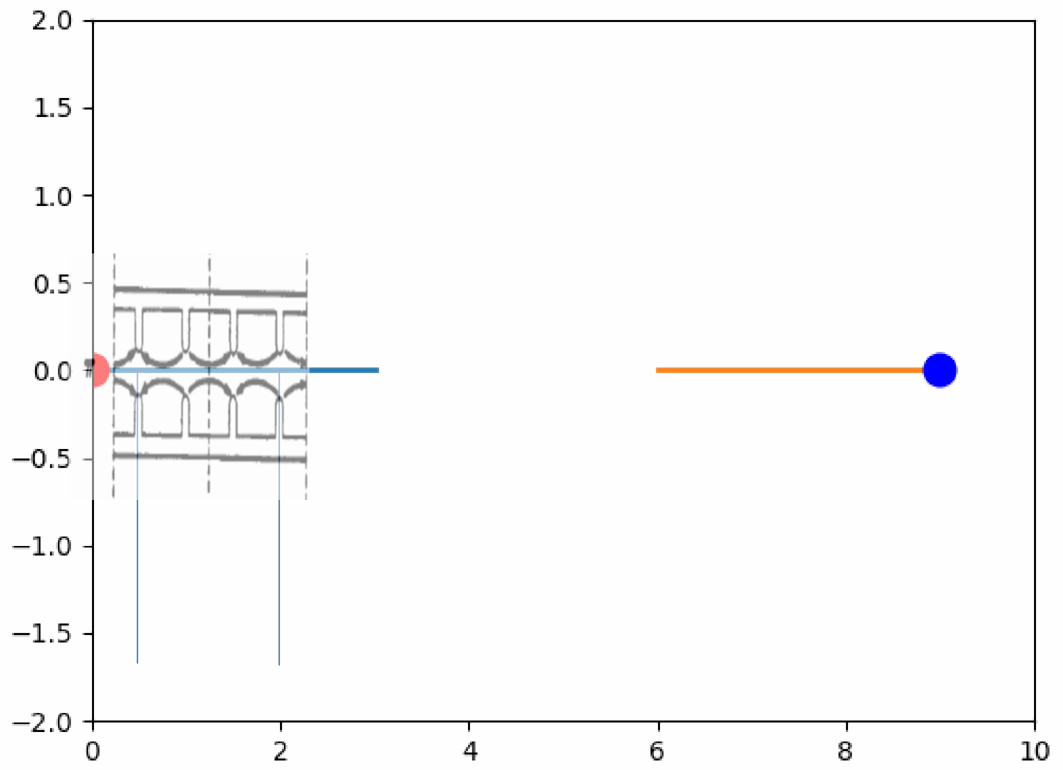
bunch sees field:

$$E_z = E_0 \sin(\omega t + \phi)\sin(kz)$$

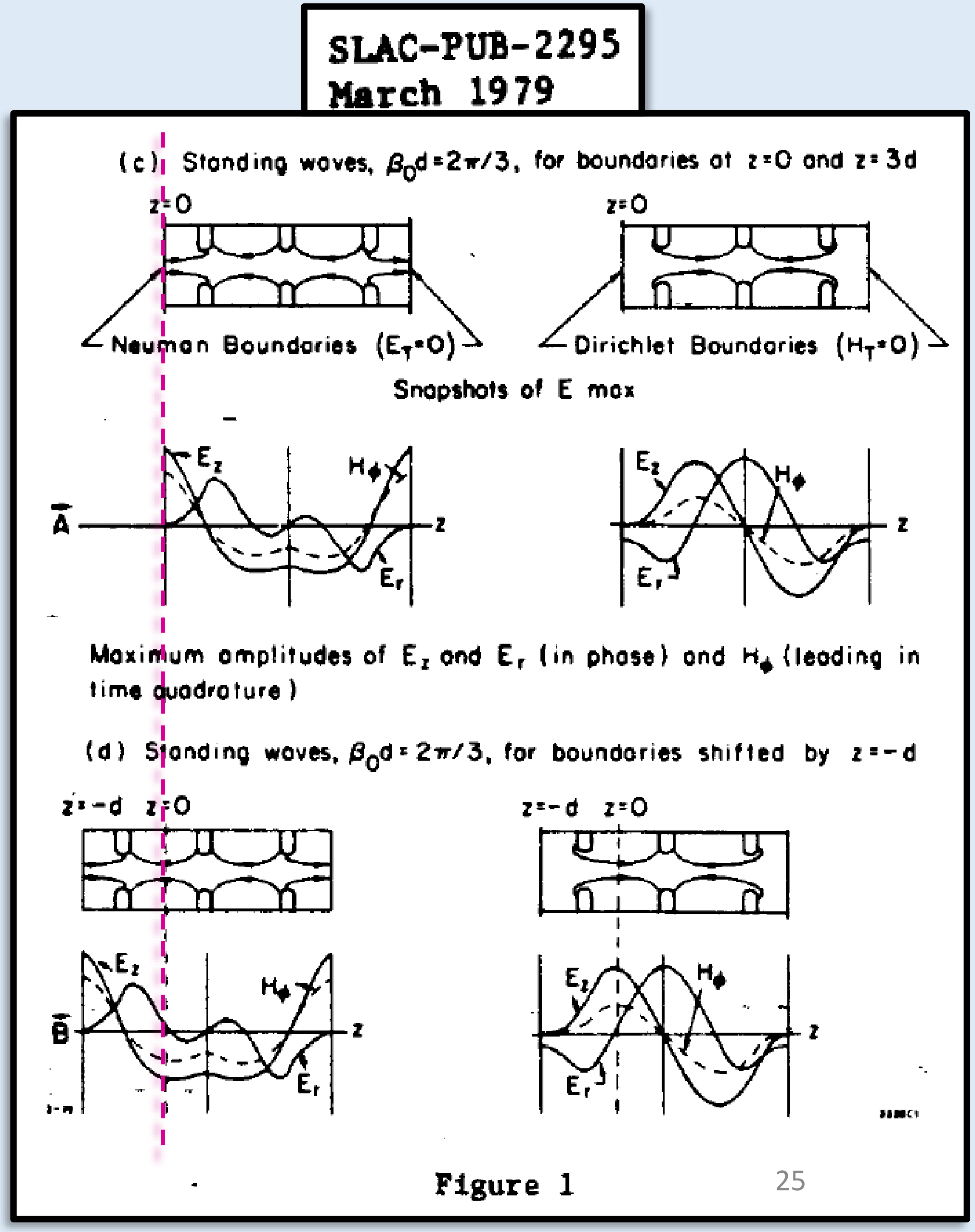
$$= E_0 \sin(kz + \phi)\sin(kz)$$

TW & SW cavities – 2/4

Standing Wave case; e.g. of an
Energy **R**ecovery **L**inac (**ERL**) case



TW = SW1 + SW2



SLAC-PUB-2295
March 1979

(c) Standing waves, $\beta_0 d = 2\pi/3$, for boundaries at $z = 0$ and $z = 3d$

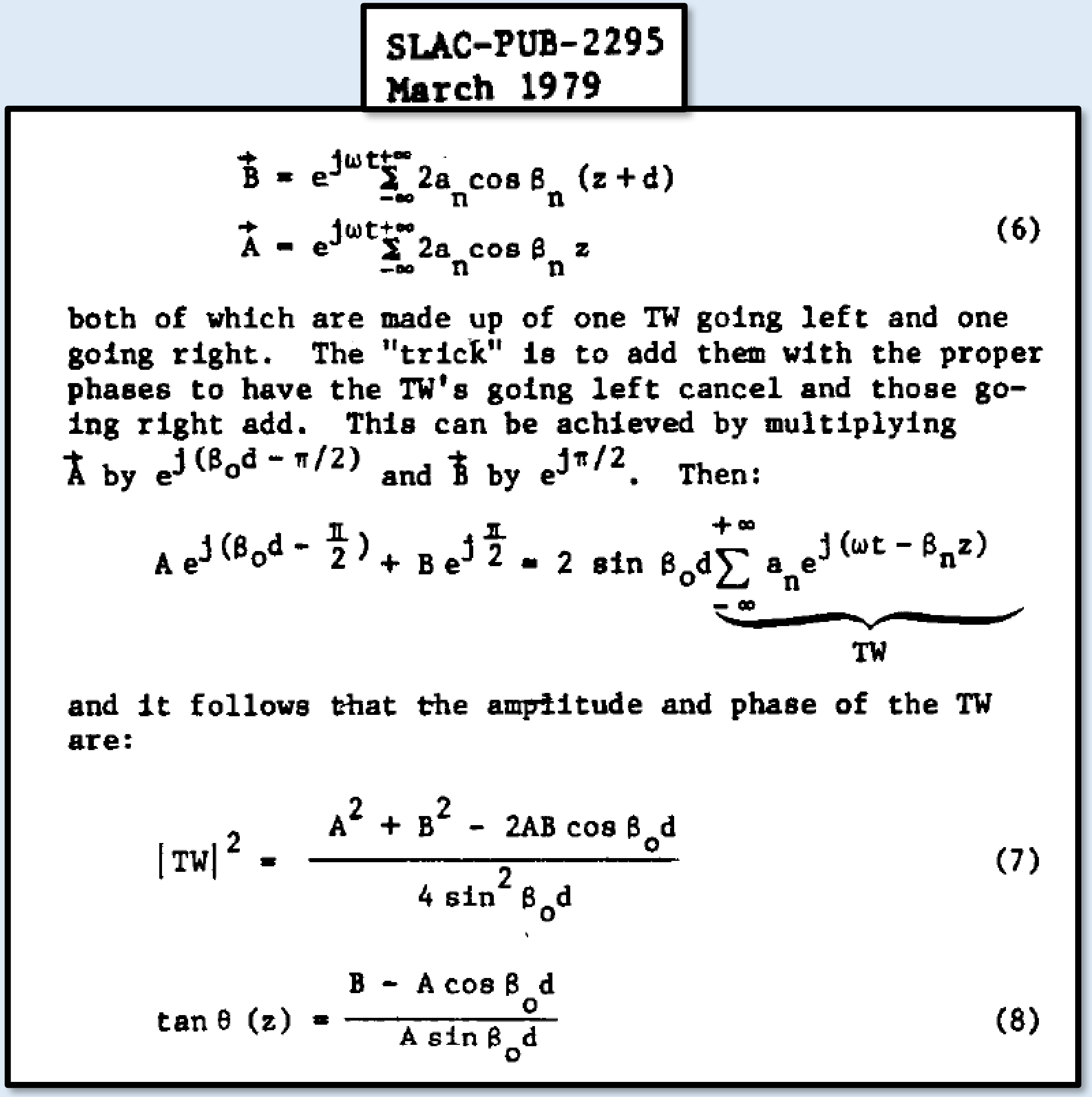
Snapshots of E max

Maximum amplitudes of E_z and E_r (in phase) and H_ϕ (leading in time quadrature)

(d) Standing waves, $\beta_0 d = 2\pi/3$, for boundaries shifted by $z = -d$

Figure 1

TW & SW cavities – 3/4



SLAC-PUB-2295
March 1979

$$\vec{B} = e^{j\omega t}\sum_{-\infty}^{+\infty} 2a_n \cos\beta_n (z+d)$$

$$\vec{A} = e^{j\omega t}\sum_{-\infty}^{+\infty} 2a_n \cos\beta_n z \qquad (6)$$

both of which are made up of one TW going left and one going right. The "trick" is to add them with the proper phases to have the TW's going left cancel and those going right add. This can be achieved by multiplying $\vec{A}$ by $e^{j(\beta_o d - \pi/2)}$ and $\vec{B}$ by $e^{j\pi/2}$. Then:

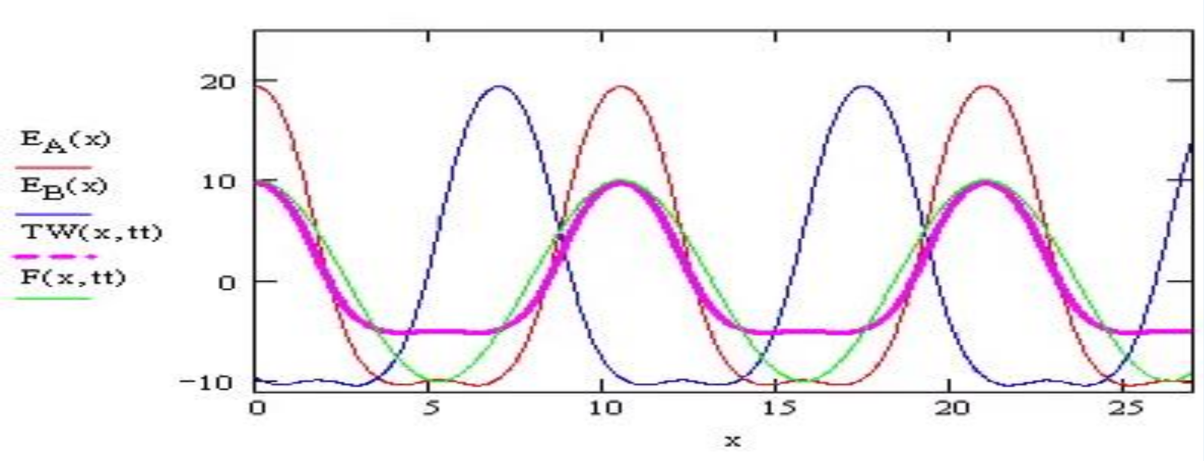
$$A\,e^{j(\beta_o d - \frac{\pi}{2})} + B\,e^{j\frac{\pi}{2}} = 2\sin\beta_o d\underbrace{\sum_{-\infty}^{+\infty} a_n e^{j(\omega t - \beta_n z)}}_{TW}$$

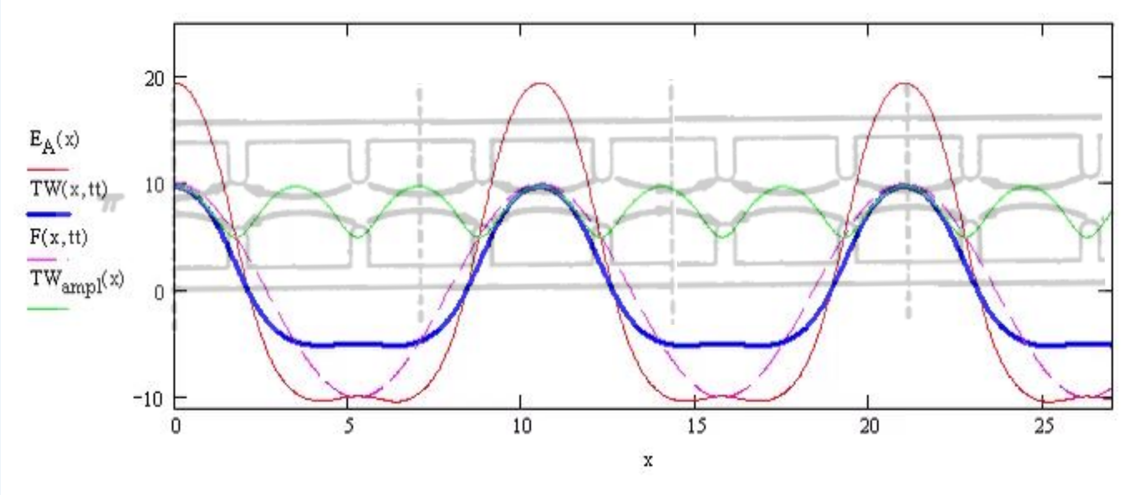
and it follows that the amplitude and phase of the TW are:

$$|TW|^2 = \frac{A^2 + B^2 - 2AB\cos\beta_o d}{4\sin^2\beta_o d} \qquad (7)$$

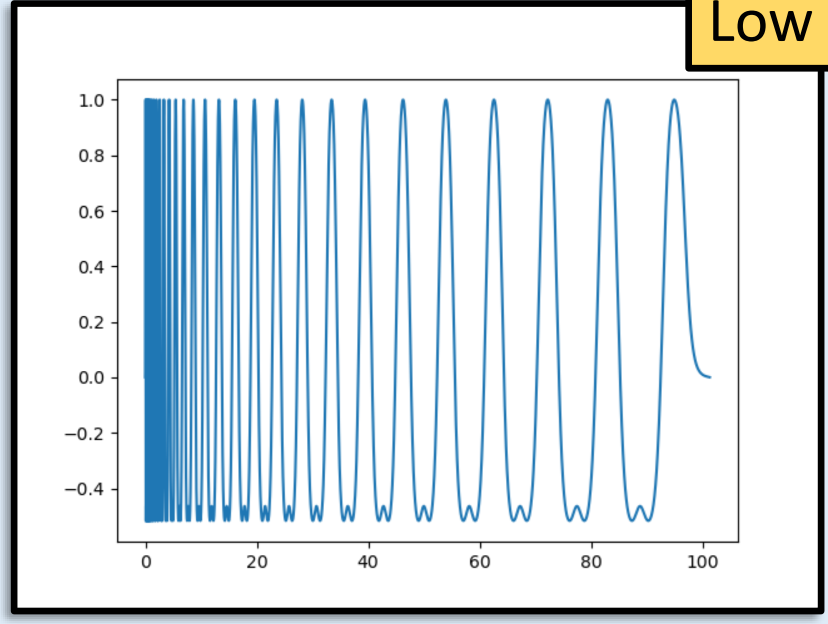
$$\tan\theta(z) = \frac{B - A\cos\beta_o d}{A\sin\beta_o d} \qquad (8)$$

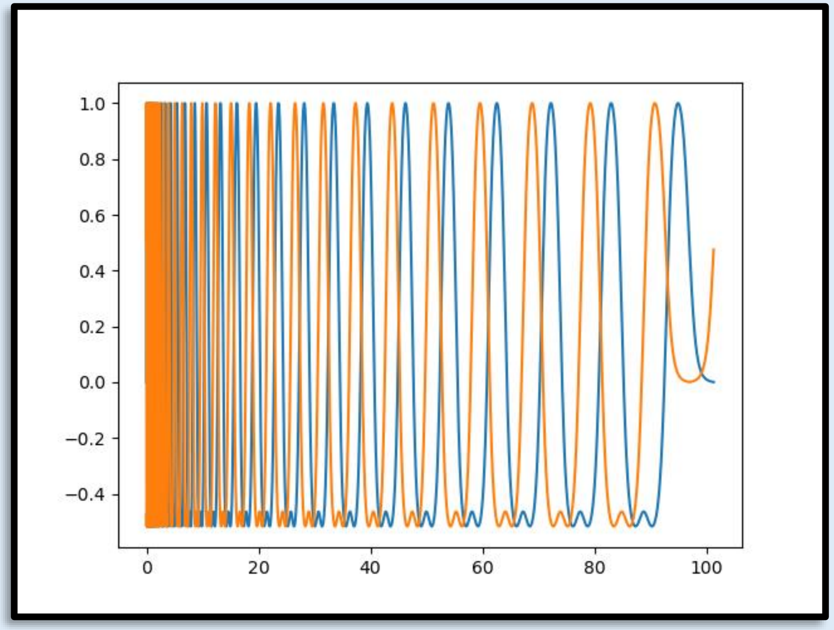
Traveling Wave case = two SW's dephased



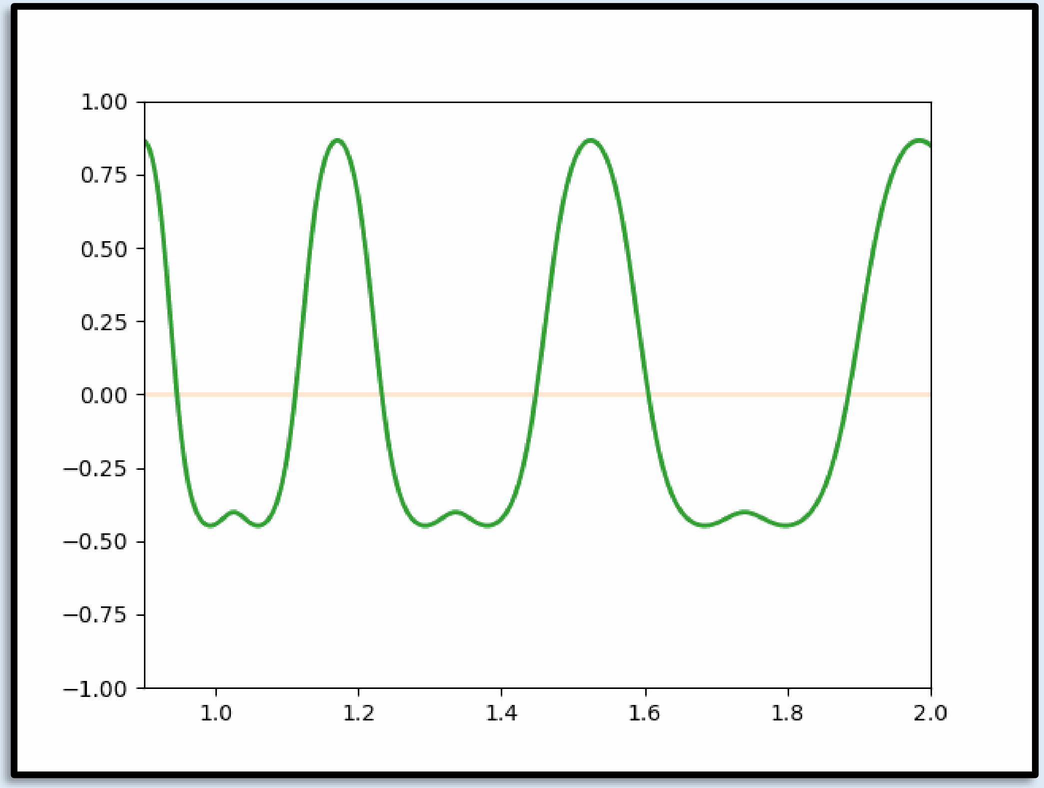


TW & SW cavities – 4/4

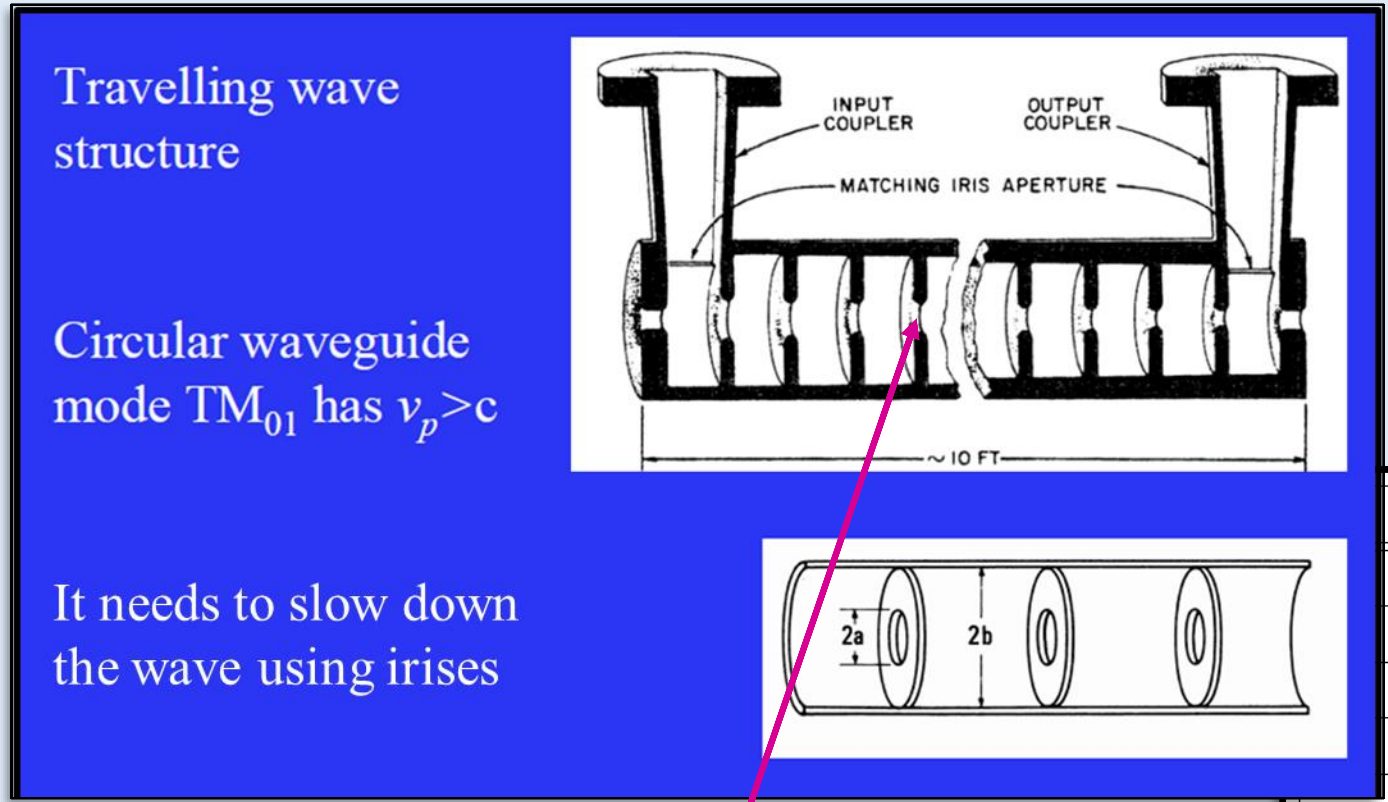




Low Beta cavities $\beta < 1$



e.g.: 3 meters long **SLAC type** S-band module



Travelling wave structure

Circular waveguide mode TM_{01} has v_p>c

It needs to slow down the wave using irises

$$G\left[\frac{\mathrm{MV}}{\mathrm{m}}\right] = \sqrt{r_l\left[\frac{M\Omega}{m}\right] \cdot \frac{P[MW]}{l_{cav}}}$$

$$r_l \propto \sqrt{f}$$

$$Grad(3\ GHz) \sim 35\ MeV/m$$

$$Grad(100\ GHz) \sim 200\ MeV/m$$

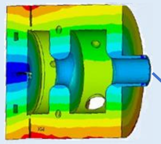
$$Grad(30\ THz) \sim 3.5\ GeV/m$$

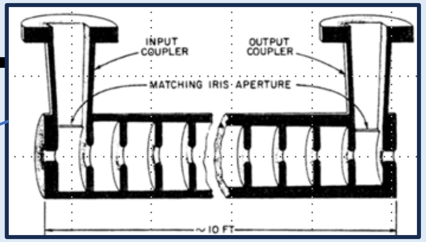
Parameter	symbol	Units	Value
Frequency	$\omega/2\pi$	Hz	2856 MHz
Length	L	m	3.048
Cell radius	b	cm	4.17–4.09
Iris radius	a	cm	1.31–0.96
Cell length	d	cm	3.50
Phase shift per cell	ψ	-	$2\pi/3$
Disc thickness	h	cm	0.584
Quality Factor	Q	-	13,000
Shunt impedance per meter	r_l	MΩ/m	52–60
Filling Time	t_f	nsec	830
Group Velocity	v_{gr}	%c	2.0–0.65
Attenuation	τ	"nepers"	0.57

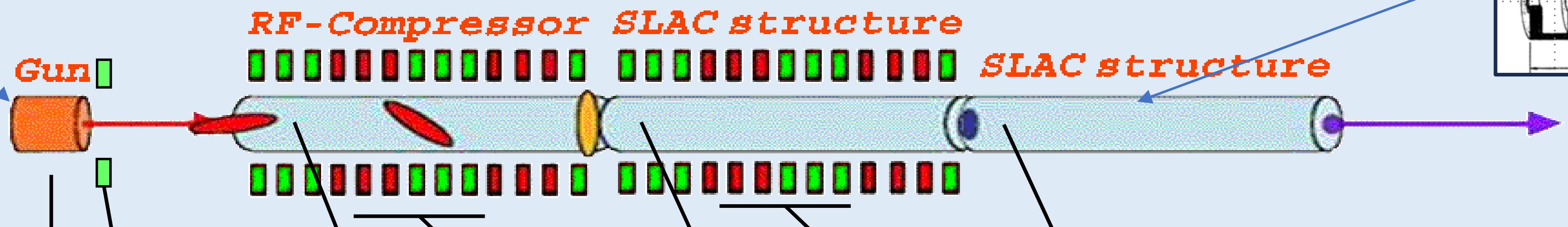
Parameters of the SLAC **constant-gradient** Travelling-Wave (TW) structure.
From G. A. Loew, R. B. Neal, "Accelerating Structures in Linear Accelerators" (1970)

Sketch of a typical High Brigthness electron LINAC

A typical ideal High Brigthness electron BeamLine







First Solen. array
-Position
-Intesity

Second Solen. array
-Position
-Intesity

Gun Solenoid:
-Position
-Intensity

First Acc. Cavity
-Position
-Inj.phase
-RF E_{peak}

Second Acc. Cavity
-Position
-Inj.phase
-RF E_{peak}

Third Acc. Cavity
-Position
-Inj.phase
-RF E_{peak}

The Gun:
-Laser Longitudinal profile
-Laser transverse dimension
(relative uniformity)
-Inj. phase in RF Acc.Field
-RF E_{peak}

19 parameters, some strongly coupled (non linearly) to each other
In-blue harder parameters to be set (12/19)
In-black easier parameters to be set (7/19)

GIOTTO – Genetic Interface for OpTimising Tracking with Optics

From 2007 up to day, the code is grew in power and versatility

What makes the difference:

Nowadays "**quasi-classic**" optimization techniques >> elitism; advanced mutation operators; hill climbing; regeneration from best solutions; parallelization *(Open-MPI, MS-mpi)*

fitness function freely defined by the user, by using all the tracking code's outputs (Astra) or by a dedicated **post processor** for the Lcomb configuration:
En, Den, SigZ, Xemit, sigX, Yemit, SigY, emitY ….

Constraints freely defined by the user

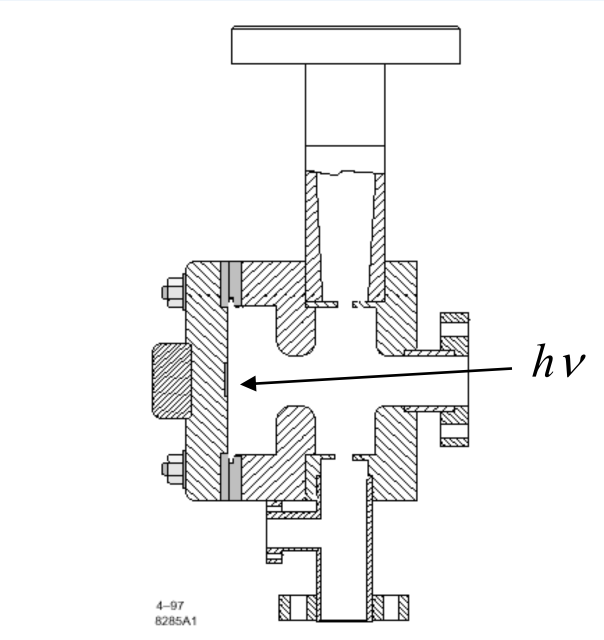
NameList (nml) can be imported into a DB and each nml variables can be used as a Giotto variable to be optimized (genes) (ex. Phi(1)…Phi(50),maxe(1),maxb(1), sig_x,sig_clock --- No limit in the number)

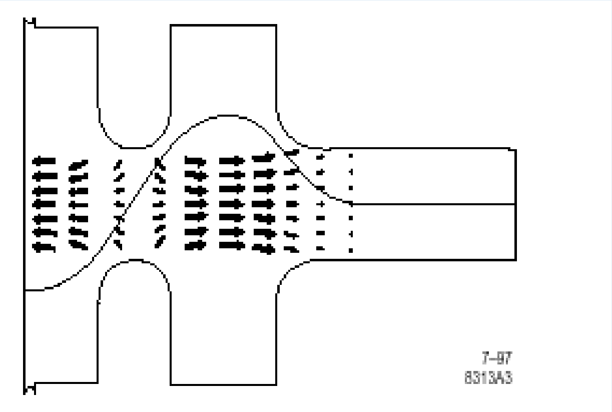
switches from Genetic Optimizations to Statistical Analysis. Each variable can be analyzed. The **sampling interval** can be sampled in **uniform** or **Gaussion** way – very fast stat. analysis.

GIOTTO soon on the ASTRA reposity -- https://www.desy.de/~mpyflo/
or write to alberto.bacci@mi.infn.it or marcello.rosseti@mi.infn.it

Radio-Frequency Photo-Injectors

UCLA/SLAC/BNL
S-band next gen. RF Gun





$$Q_{eff} = N_{electrons} / N_{laser-photons}$$

$$Q_{eff}\left(Cu \quad photo-cathode\right) \cong 5 \cdot 10^{-5}$$

$$W_{Cu} = 4.2eV, \; h\nu = 4.6eV$$

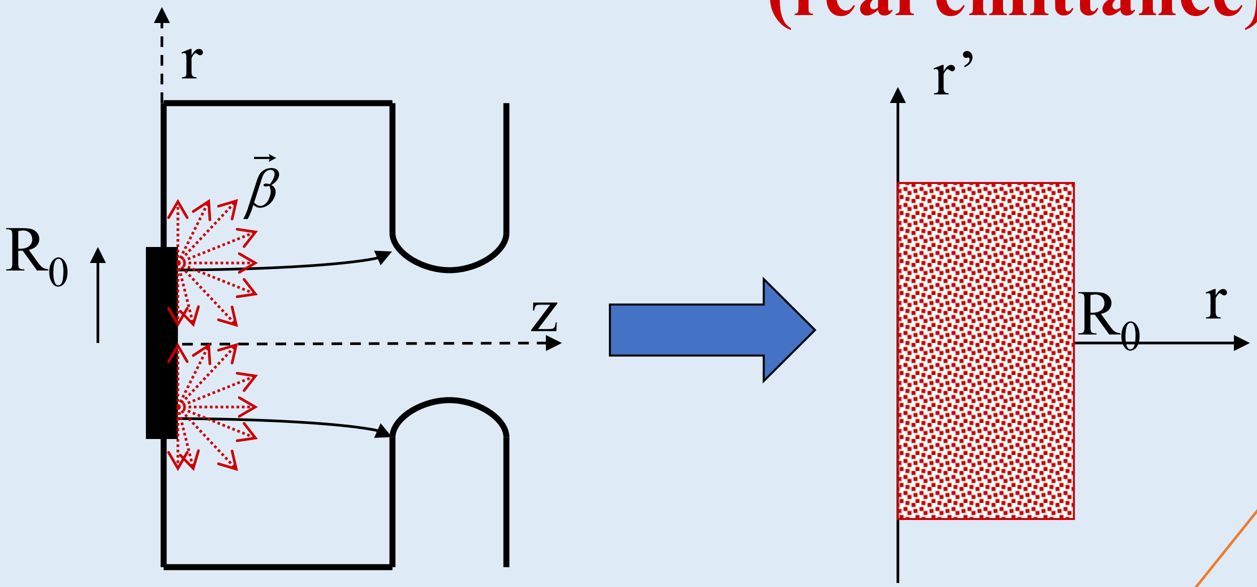
$$Q = 1 \;\; nC \;\; needs \;\; U_{las} = \frac{h\nu \cdot Q_{bunch}}{Q_{eff}} = 92 \;\; \mu J$$

Photo-Cathode Emissivity **J < 10 kA/cm²**
(t)Prompt emission on a ps time scale

Thermoionic Injectors
Cathode Emissivity J < 20 A/cm2

Beam Dinamycs in Photo-Injectors

themperature emittance @ photo-cathode (real emittance)



$$\varepsilon_n^{th} = \frac{\langle\beta\gamma\rangle}{2}\sqrt{\langle r^2\rangle\langle r'^2\rangle - \langle rr'\rangle^2}$$

$$\sigma_x = \frac{R_0}{2}$$

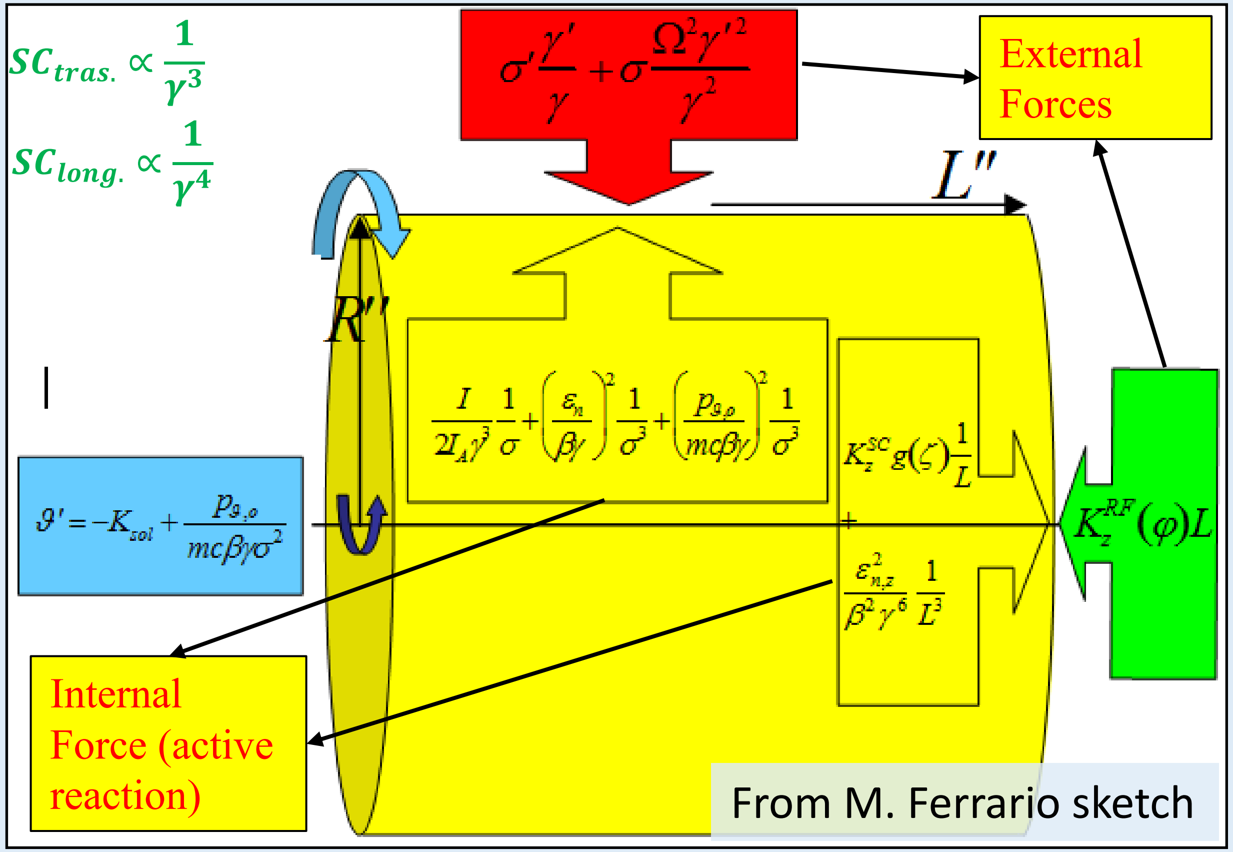
T_e (or r' is relative to the difference between [Photo Energy - Photo-Electric working function]

$$\langle\beta\gamma\rangle \cong \langle\beta\rangle \cong \sqrt{2T_e/m_e c^2} = 2\cdot 10^{-3}\sqrt{T_e[eV]} \Rightarrow \varepsilon_n^{th} = \frac{\pi\langle\beta\rangle R_0}{4\sqrt{6}}$$

$$\varepsilon_n^{th}\ [mm\cdot mrad] = 0.64\ R_0[mm]\ \sqrt{T_e[eV]}$$

Optimize High Brigthness BeamLines is challenging – 1

The electron beam is an ‘reactive’ distribution:



$SC_{tras.} \propto \frac{1}{\gamma^3}$

$SC_{long.} \propto \frac{1}{\gamma^4}$

$\sigma'\frac{\gamma'}{\gamma} + \sigma\frac{\Omega^2\gamma'^2}{\gamma^2}$

External Forces

L''

R''

$\frac{I}{2I_A\gamma^3}\frac{1}{\sigma} + \left(\frac{\varepsilon_n}{\beta\gamma}\right)^2\frac{1}{\sigma^3} + \left(\frac{p_{\vartheta,o}}{mc\beta\gamma}\right)^2\frac{1}{\sigma^3}$

$K_z^{sc}g(\zeta)\frac{1}{L}$

$\frac{\varepsilon_{n,z}^2}{\beta^2\gamma^6}\frac{1}{L^3}$

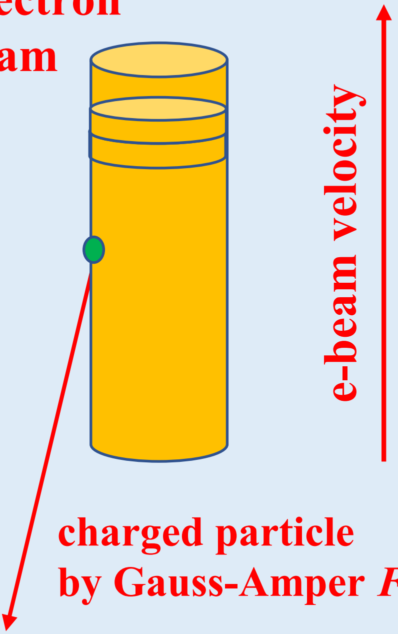
$K_z^{RF}(\varphi)L$

$\vartheta' = -K_{sol} + \frac{p_{\vartheta,o}}{mc\beta\gamma\sigma^2}$

Internal Force (active reaction)

From M. Ferrario sketch

A Traveling Electron beam



charged particle by Gauss-Amper *Fs*

$$\frac{Coulomb\ rep.\ F}{Mag.\ attrac.\ F} = \left(\frac{c}{v}\right)^2 = \beta^2$$

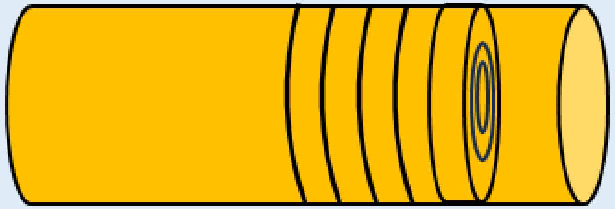
Space-Charge dominated bunches are hard to be tamed

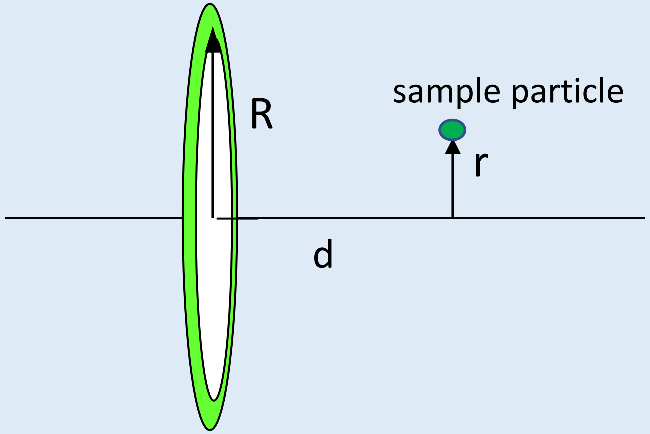
A. Bacci, A.R. Rossi / Nuclear Instruments and Methods in Physics Research A 740 (2014) 42–47 *on sc and laminar beam*

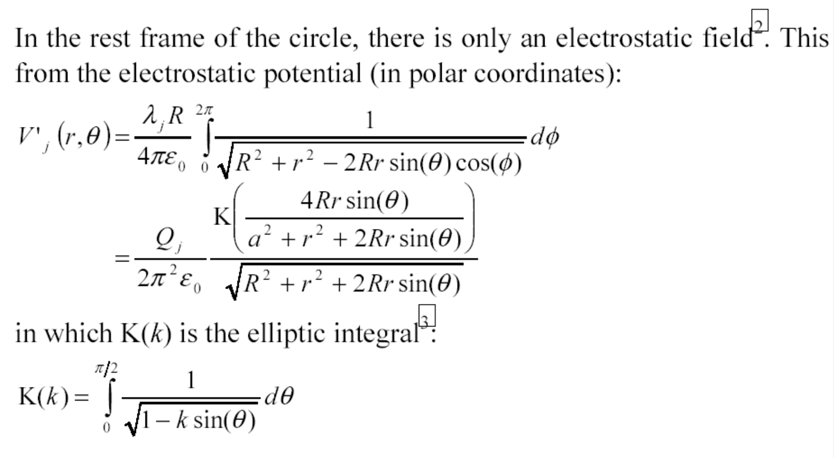
CODES Model (2D cylindrical symmetry)

Space charge computation recipe:
Lorentz-transforming the particles position and field maps into the **average rest frame of the beam**.

It then applies static forces to the various rings of the cylindrical map assuming a constant charge density inside a ring.
This algorithm requires to have some particles in each of the cell of the cylindrical grid.







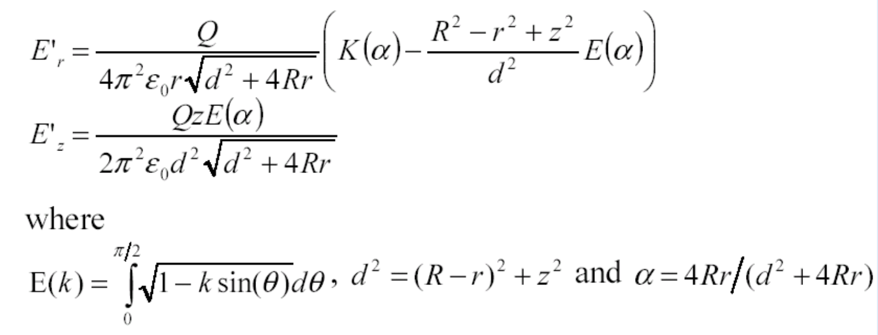
In the rest frame of the circle, there is only an electrostatic field[2]. This from the electrostatic potential (in polar coordinates):

$$V'_j(r,\theta) = \frac{\lambda_j R}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{1}{\sqrt{R^2 + r^2 - 2Rr\sin(\theta)\cos(\phi)}} d\phi$$

$$= \frac{Q_j}{2\pi^2\varepsilon_0} \frac{\mathrm{K}\left(\frac{4Rr\sin(\theta)}{a^2 + r^2 + 2Rr\sin(\theta)}\right)}{\sqrt{R^2 + r^2 + 2Rr\sin(\theta)}}$$

in which $\mathrm{K}(k)$ is the elliptic integral[3]:

$$\mathrm{K}(k) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - k\sin(\theta)}} d\theta$$



$$E'_r = \frac{Q}{4\pi^2\varepsilon_0 r\sqrt{d^2 + 4Rr}} \left(K(\alpha) - \frac{R^2 - r^2 + z^2}{d^2} E(\alpha) \right)$$

$$E'_z = \frac{QzE(\alpha)}{2\pi^2\varepsilon_0 d^2 \sqrt{d^2 + 4Rr}}$$

where

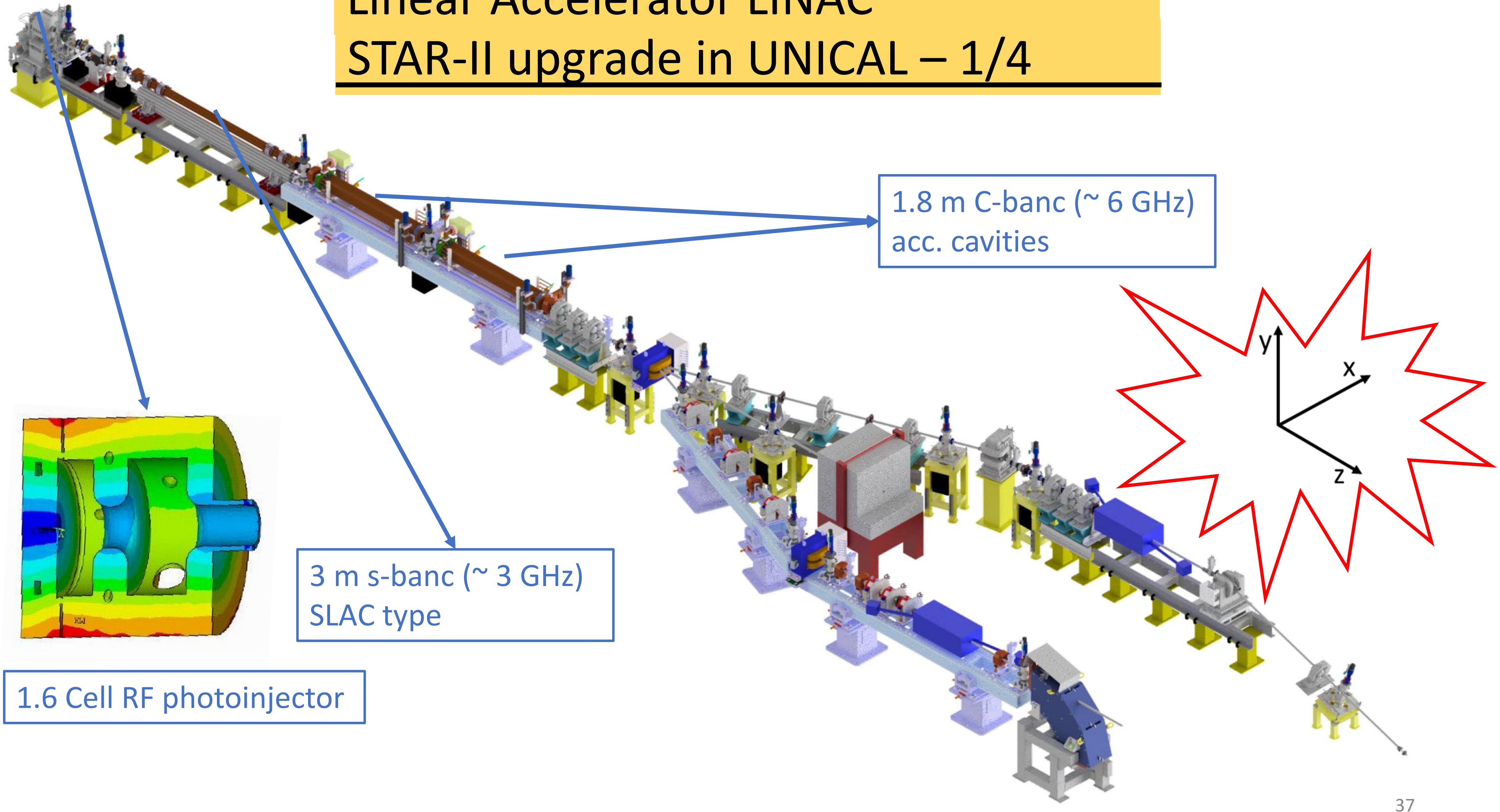
$$\mathrm{E}(k) = \int_0^{\pi/2} \sqrt{1 - k\sin(\theta)}\, d\theta,\quad d^2 = (R - r)^2 + z^2 \text{ and } \alpha = 4Rr/(d^2 + 4Rr)$$

After seeing:
- relativity and acceleration
- optics in accelerators
- Cavities basic comcepts

Let’s see a real accelerator

- Then come back to bem parameters

Linear Accelerator LINAC
STAR-II upgrade in UNICAL – 1/4

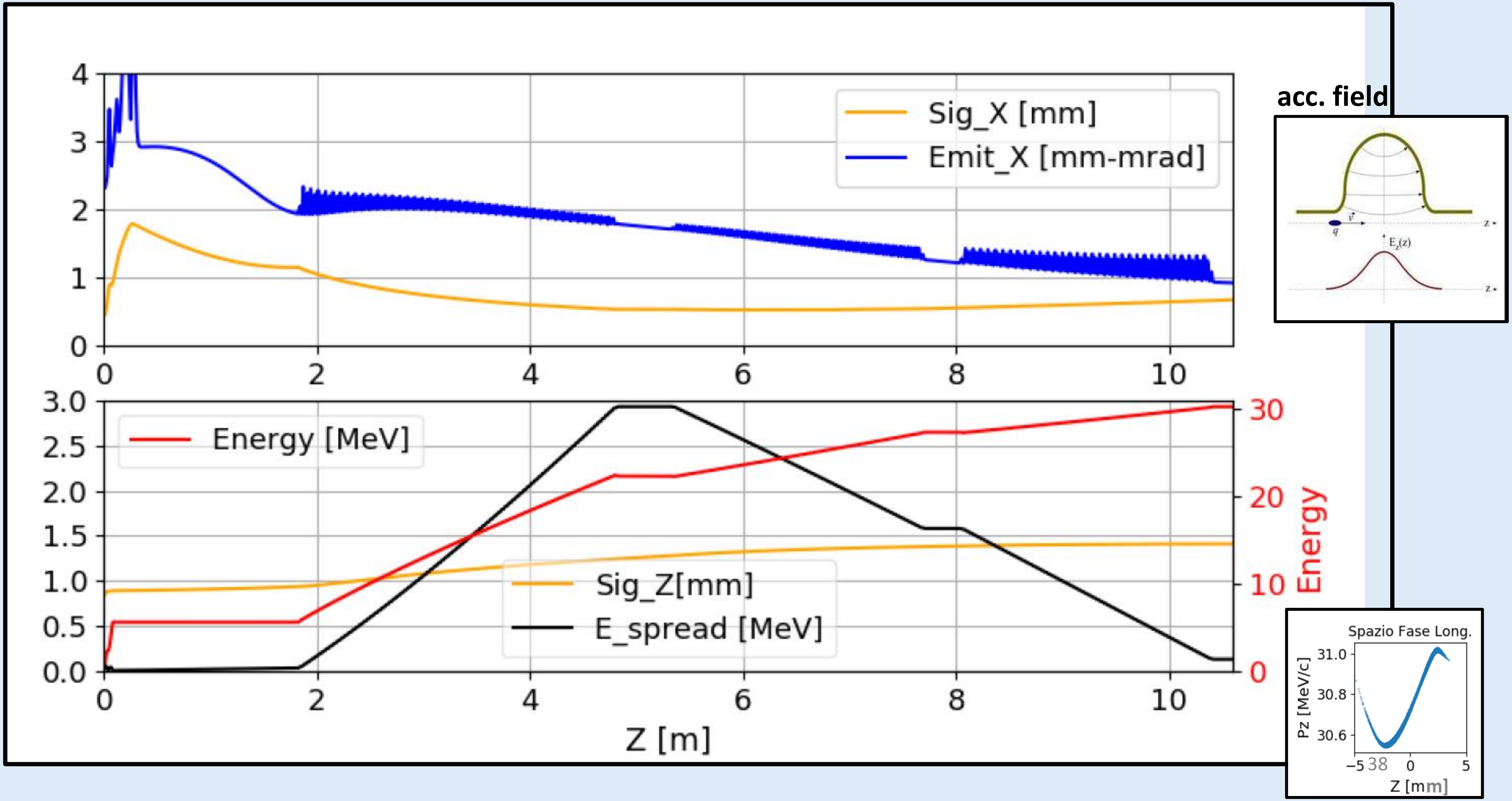


1.8 m C-banc (~ 6 GHz)
acc. cavities

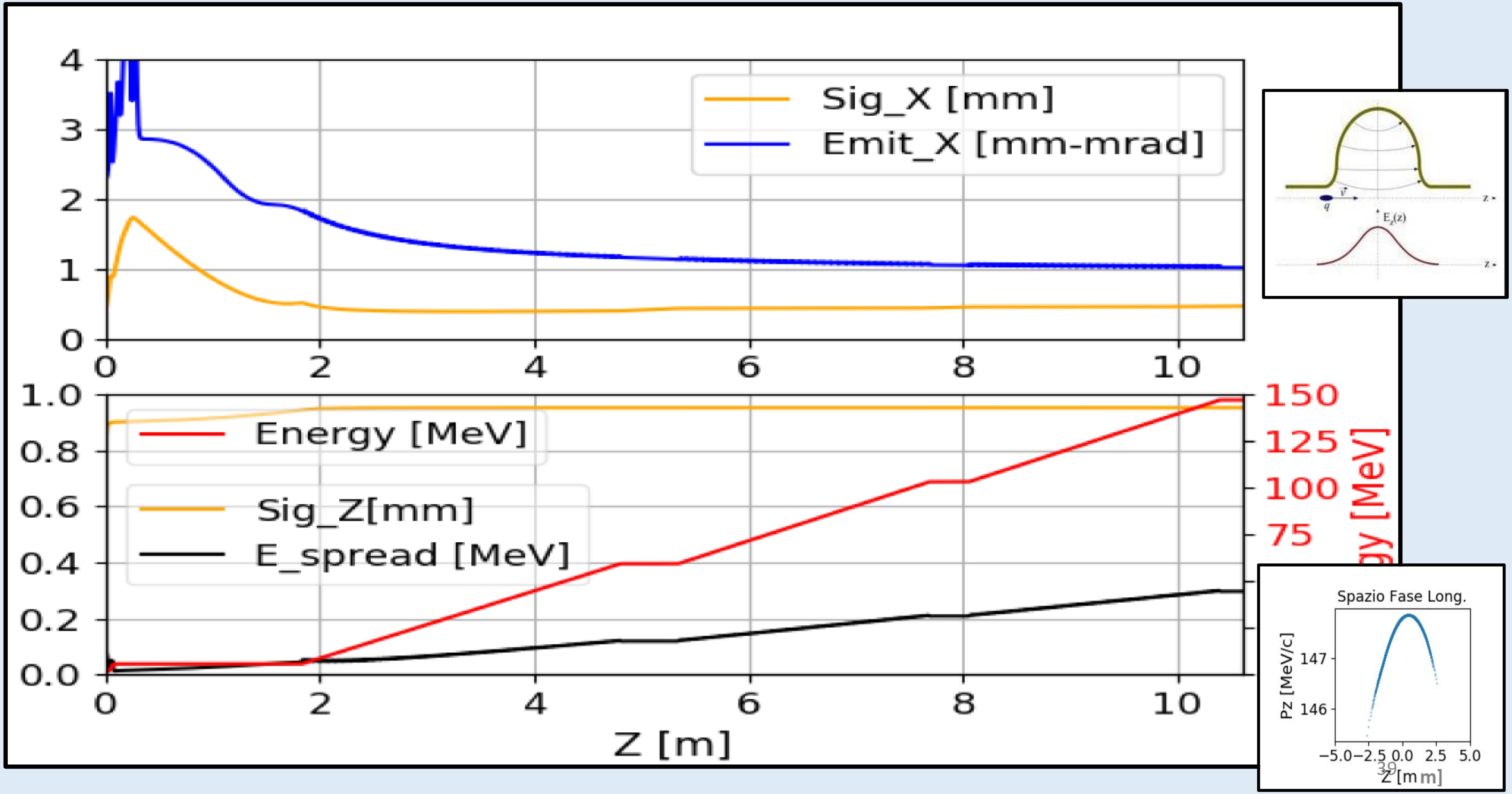
3 m s-banc (~ 3 GHz)
SLAC type

1.6 Cell RF photoinjector

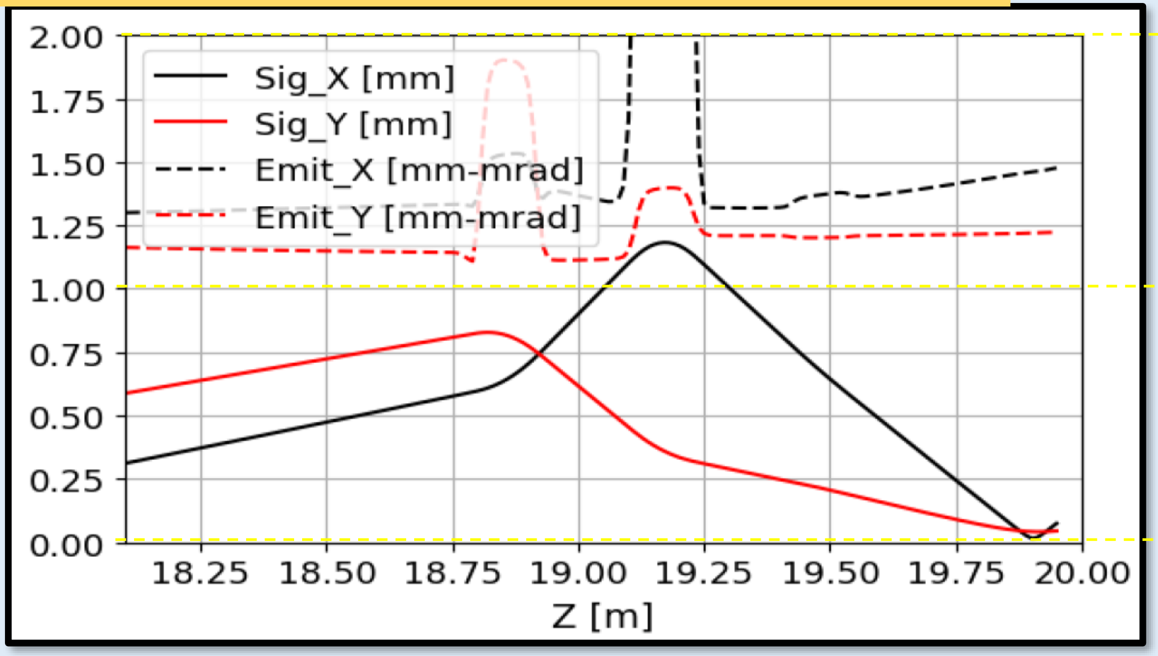
Typical beam parameters for a LINAC (30 MeV – STAR case) – 2/4

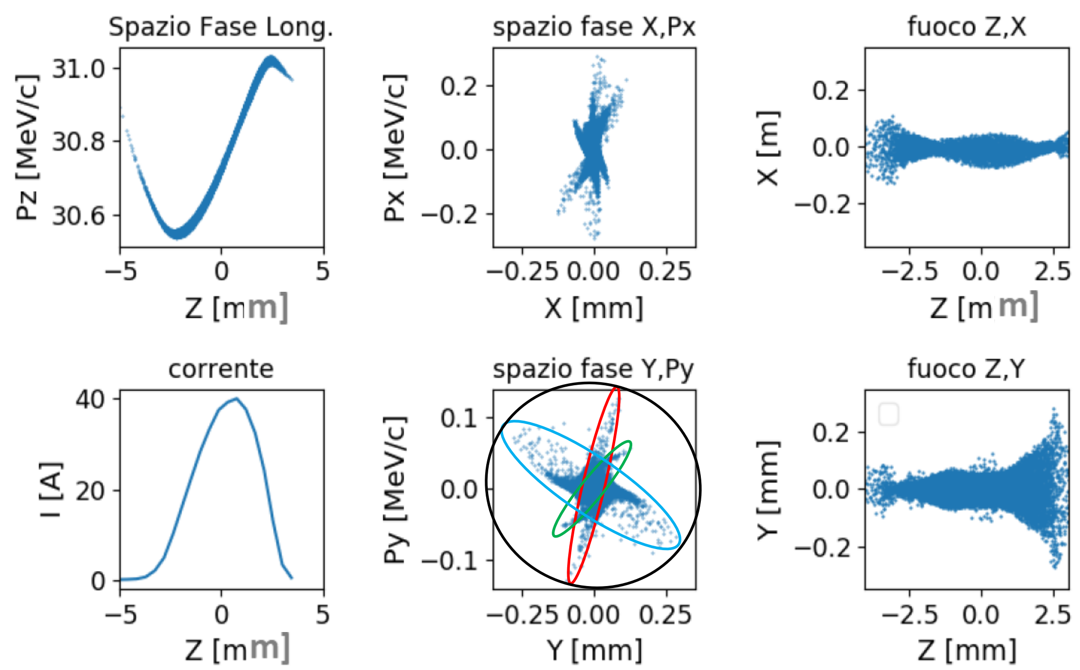


Typical beam parameters for a LINAC (150 MeV – STAR case) – 3/4

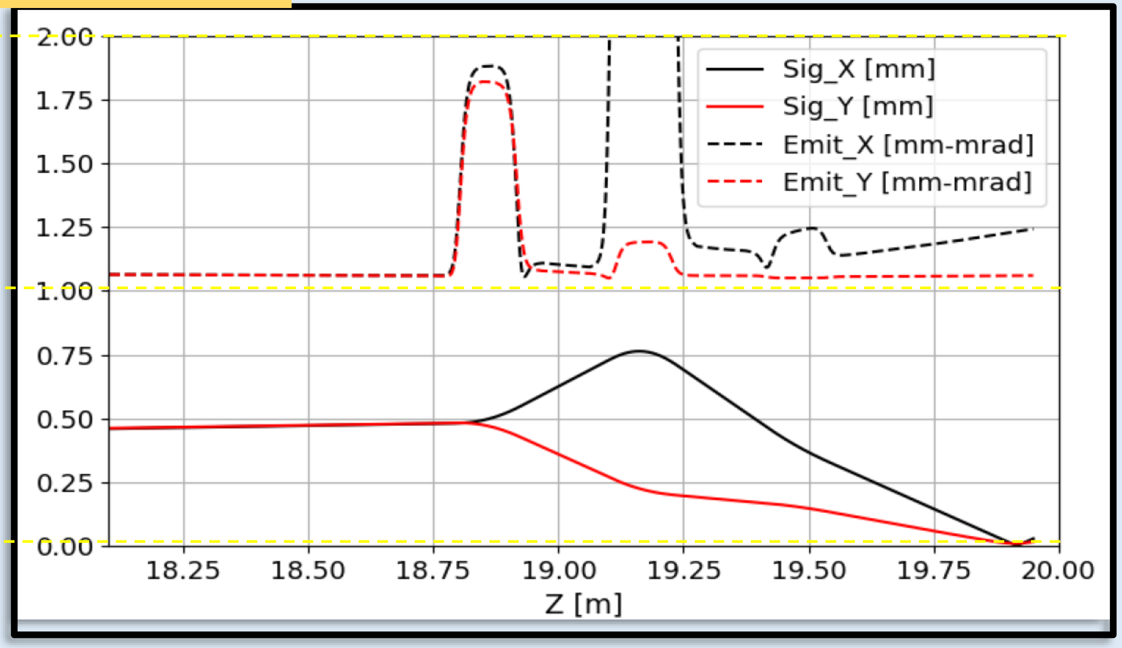


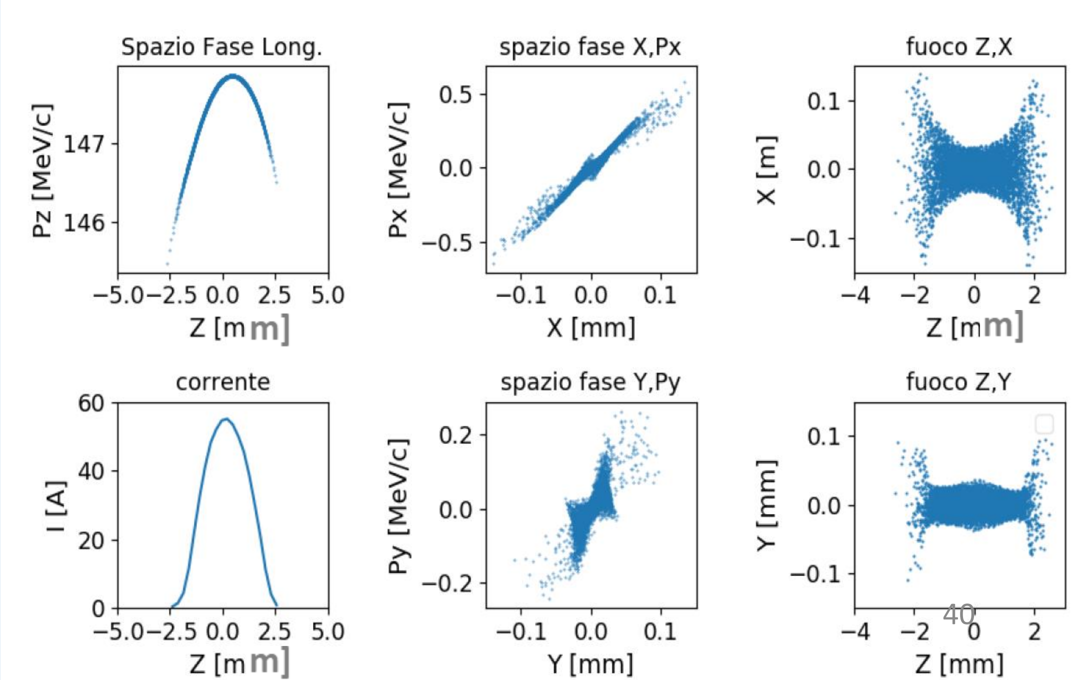
STAR – focusing channel – 30 MeV case





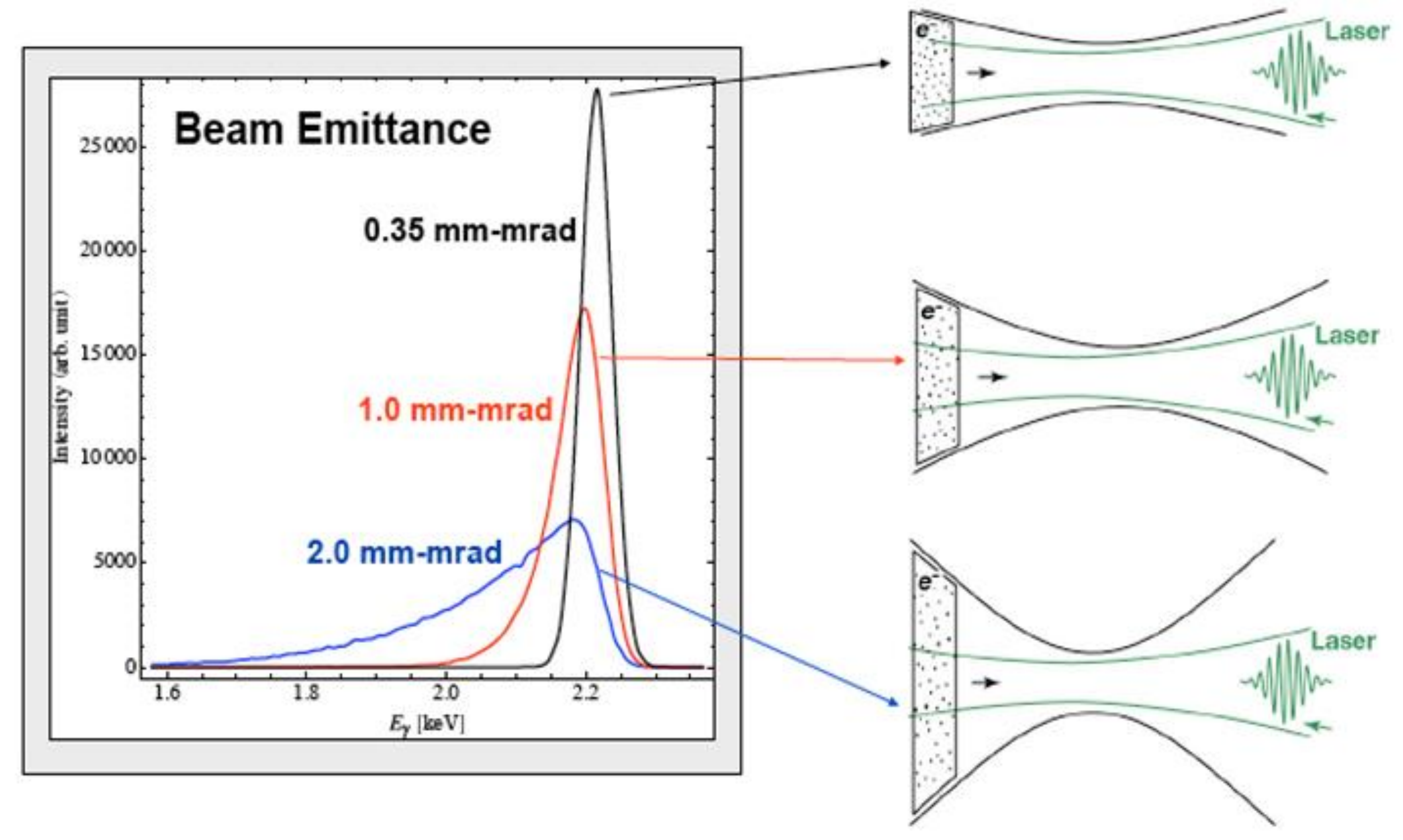
150 MeV case





Figurative affect of High Brightness in ICS

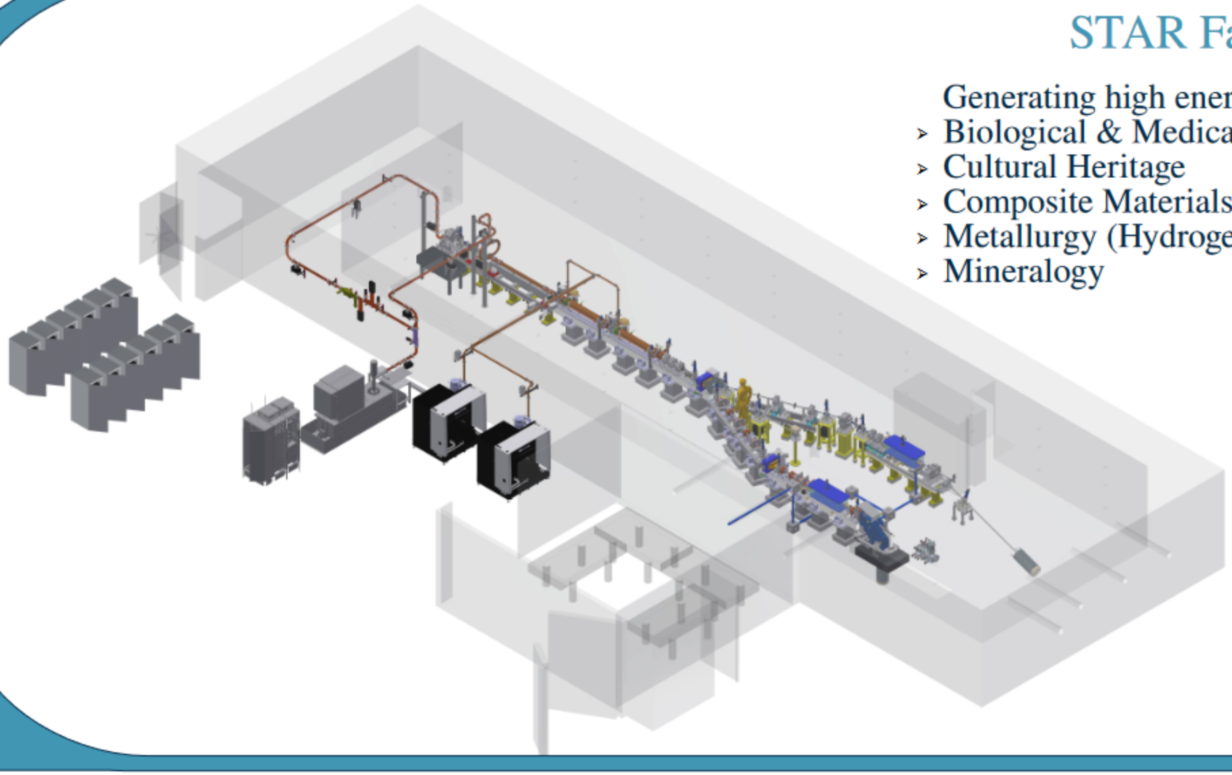
In Thomson/compton source – To drammatically improve the Spectral Density



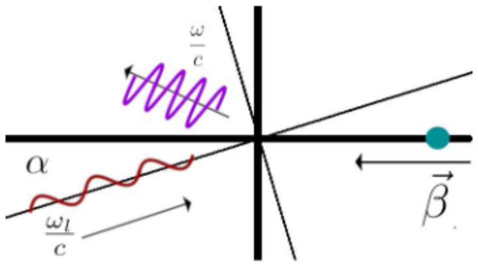
Istituto Nazionale di Fisica Nucleare

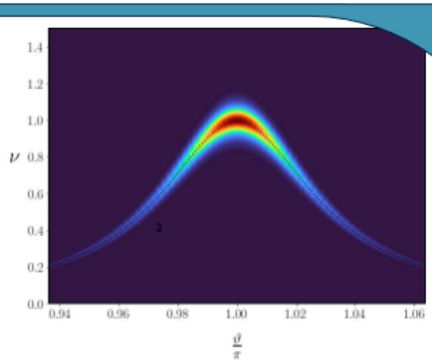
STAR HE-Linac

STAR Facility



Generating high energy radiation for
- Biological & Medical Imaging
- Cultural Heritage
- Composite Materials
- Metallurgy (Hydrogen embrittlement)
- Mineralogy

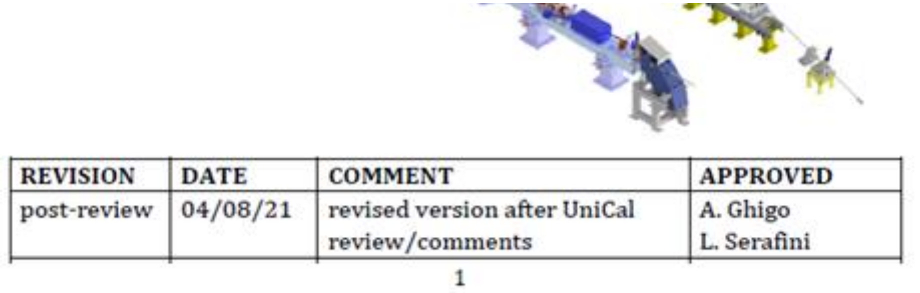




	Electron [Mev]	Photon [keV]
LE – line	23 -65	40 - 150
HE – line	40-150	25 - 350

Electrons
- Emittance : 1 [mm mrad]
- Charge : 100 – 500 [pC]
- Bunch length : < 0.7 [mm]
- Energy spread : 0.1 %, 0.05%

(CPA) Laser
- Energy : > 0.5 [Joule]
- Wavelength : 1030 [nm]
- Bandwidth : 1 [nm]

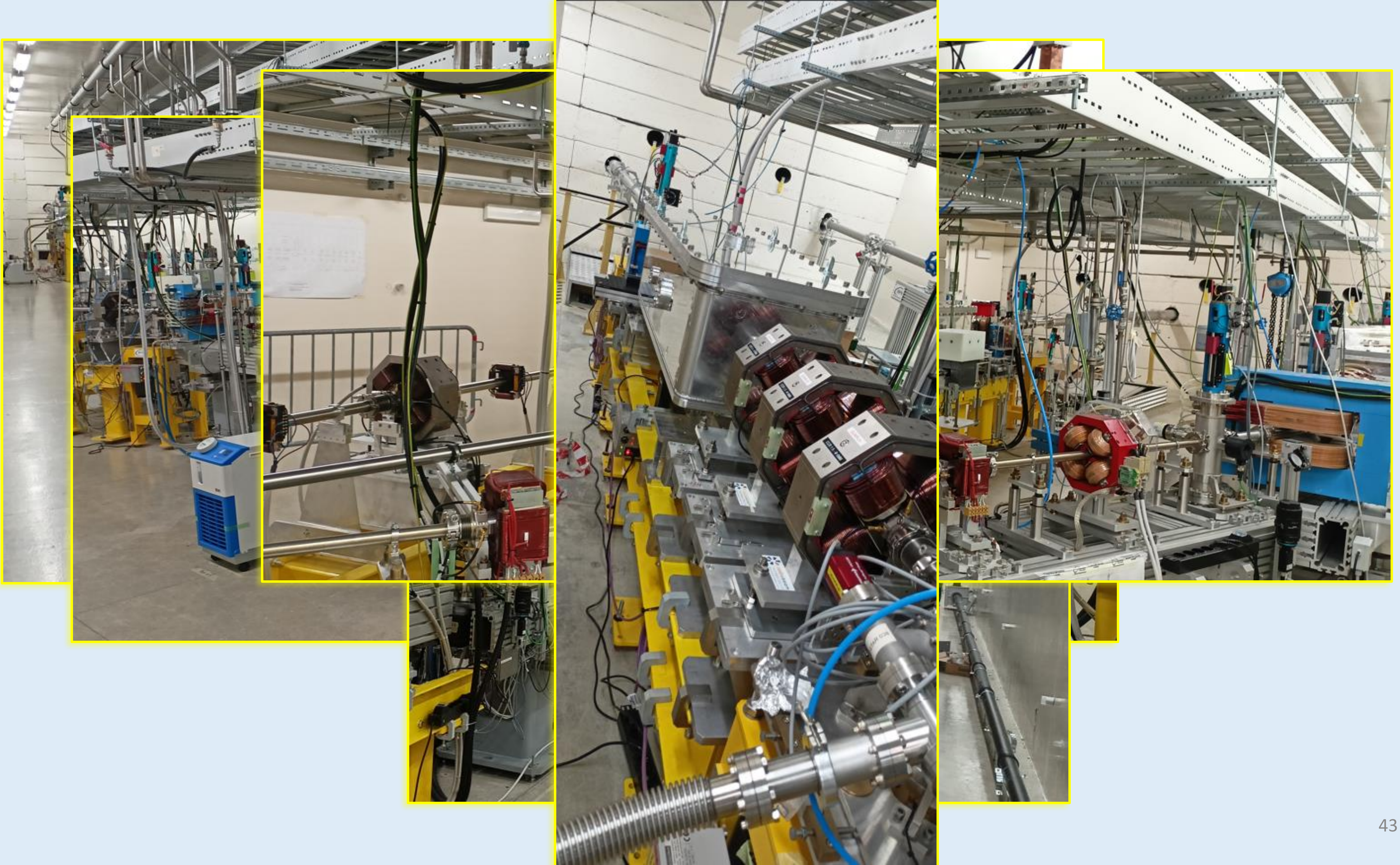


REVISION	DATE	COMMENT	APPROVED
post-review	04/08/21	revised version after UniCal review/comments	A. Ghigo L. Serafini

1

1. - Pendaglio a coppia antropomorfa da Francavilla Marittima tipo A (lato 1 e 2).

2. - Pendaglio a coppia antropomorfa da Francavilla Marittima tipo B (lato 1 e 2).



The hangar

The Laser

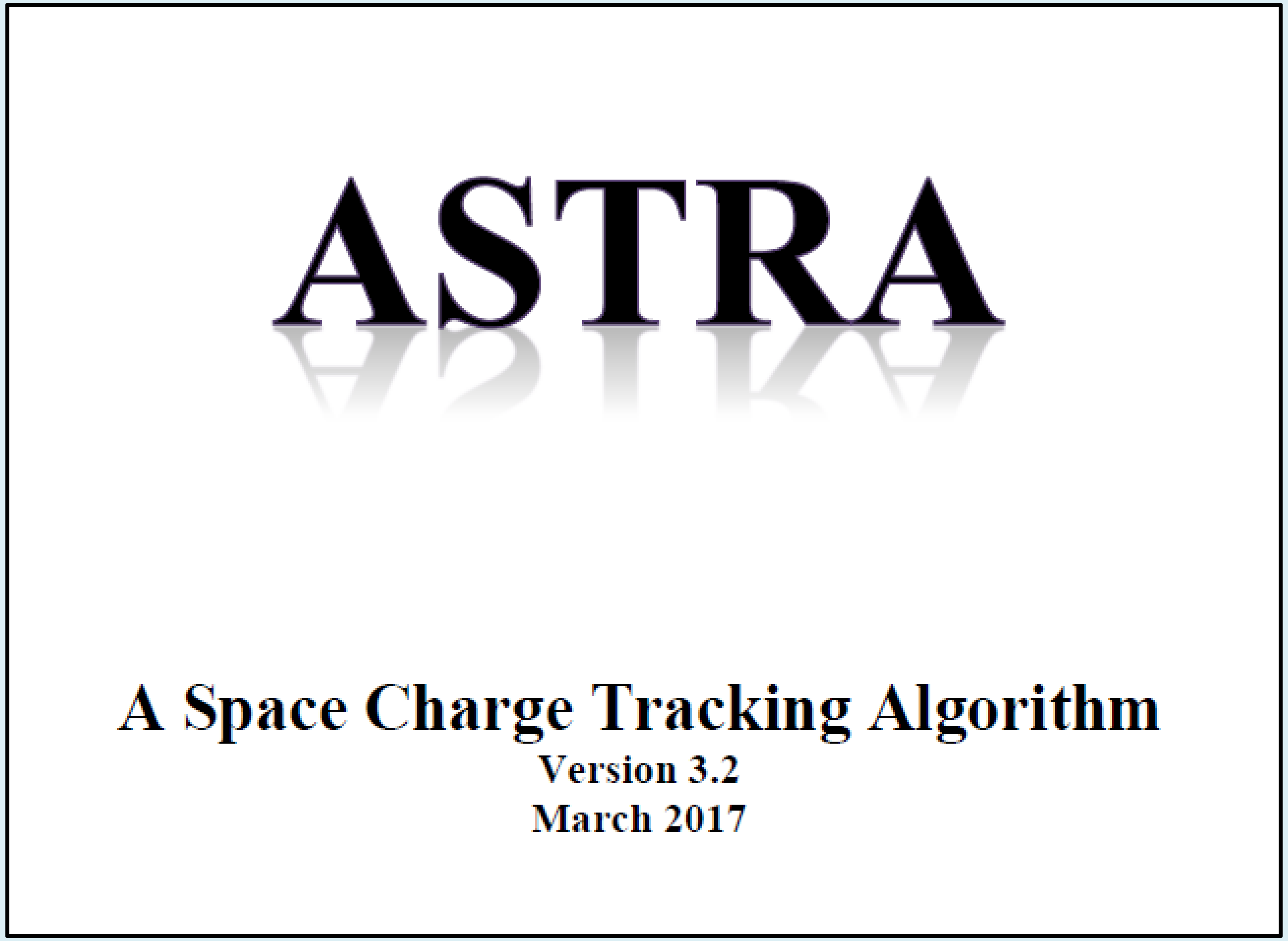
Some of the more used Codes for LINAC in space-charge regime

A consistent simulatin of the SC must be done by using PIC or P-P codes

Parmela	(Los Alamos National Lab. , L. Youg and J. Billen, PIC/P-P(?))
Tstep	(Parmela Son, from a Private Company, PIC)
Astra	(Desy, Klaus Floettmann, Free Code, PIC/P-P)
GPT	(Private company Pulsar Physics, Netherlands)
IMPACT-T and IMPACT-Z	(Berkeley Lab., Ji Qiang, Free Code)

Usually the Space-Charge is a main issue for Linac injectors up to 100 MeV. Typical applications: FEL, Thomson/Compton sources or ultra short bunches for Plasma Wave Accelerators)

Let's introduce:



ASTRA

A Space Charge Tracking Algorithm

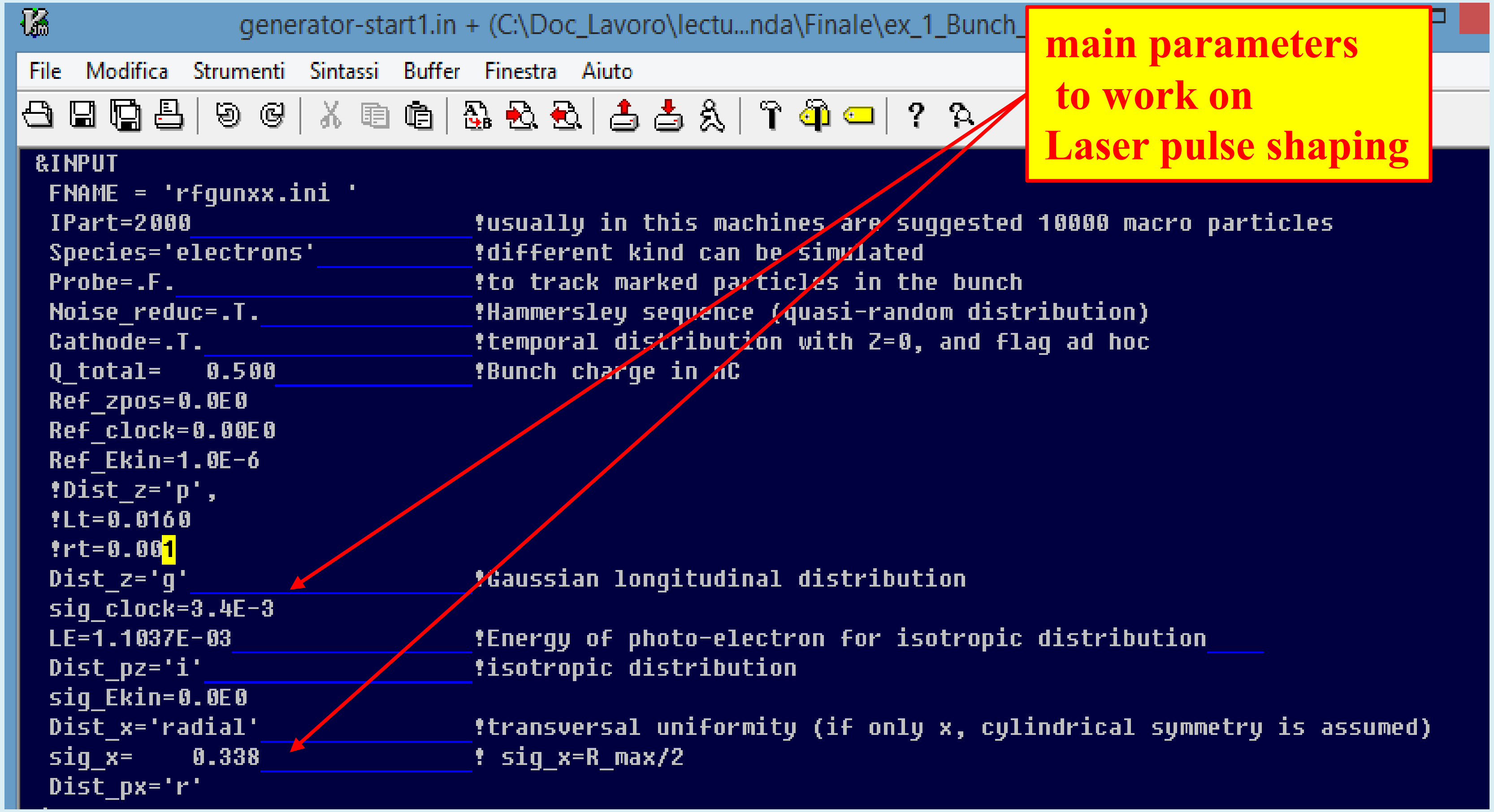
Version 3.2
March 2017

Astra input files and free parameters

Let see the STAR-project input files

All 3D (or 2D) pic tracking code have two main algorithms:
1) Bunch extraction from cathode or particles generation, 2) bunch tracker into the beam-line.

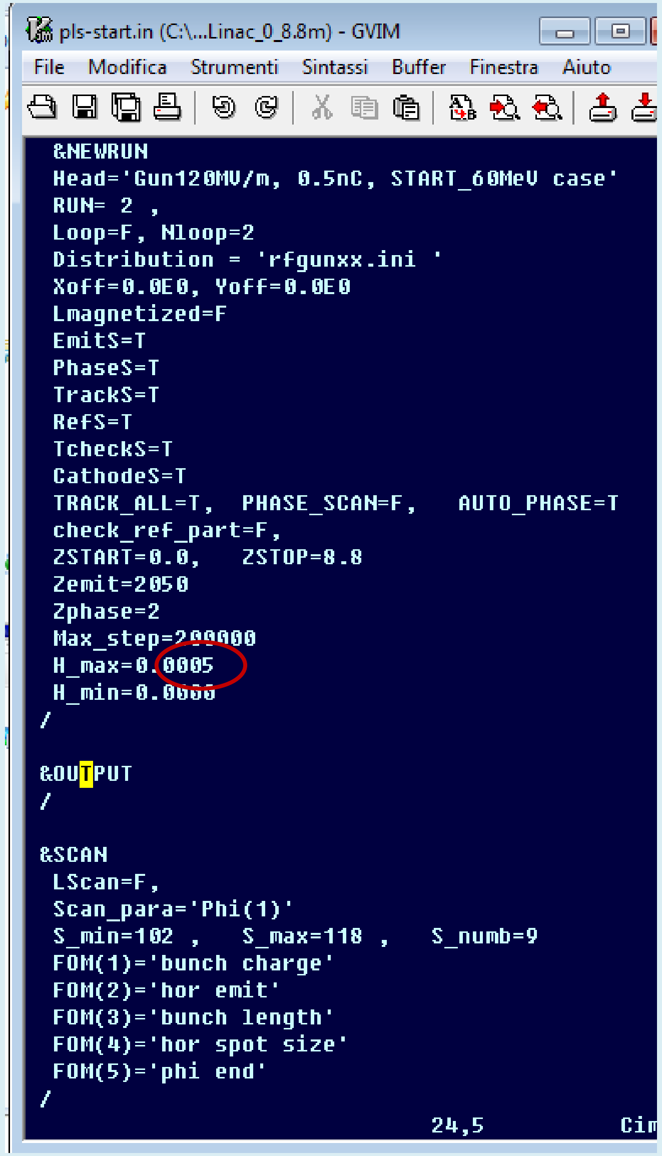
Input for e-bunch extraction:



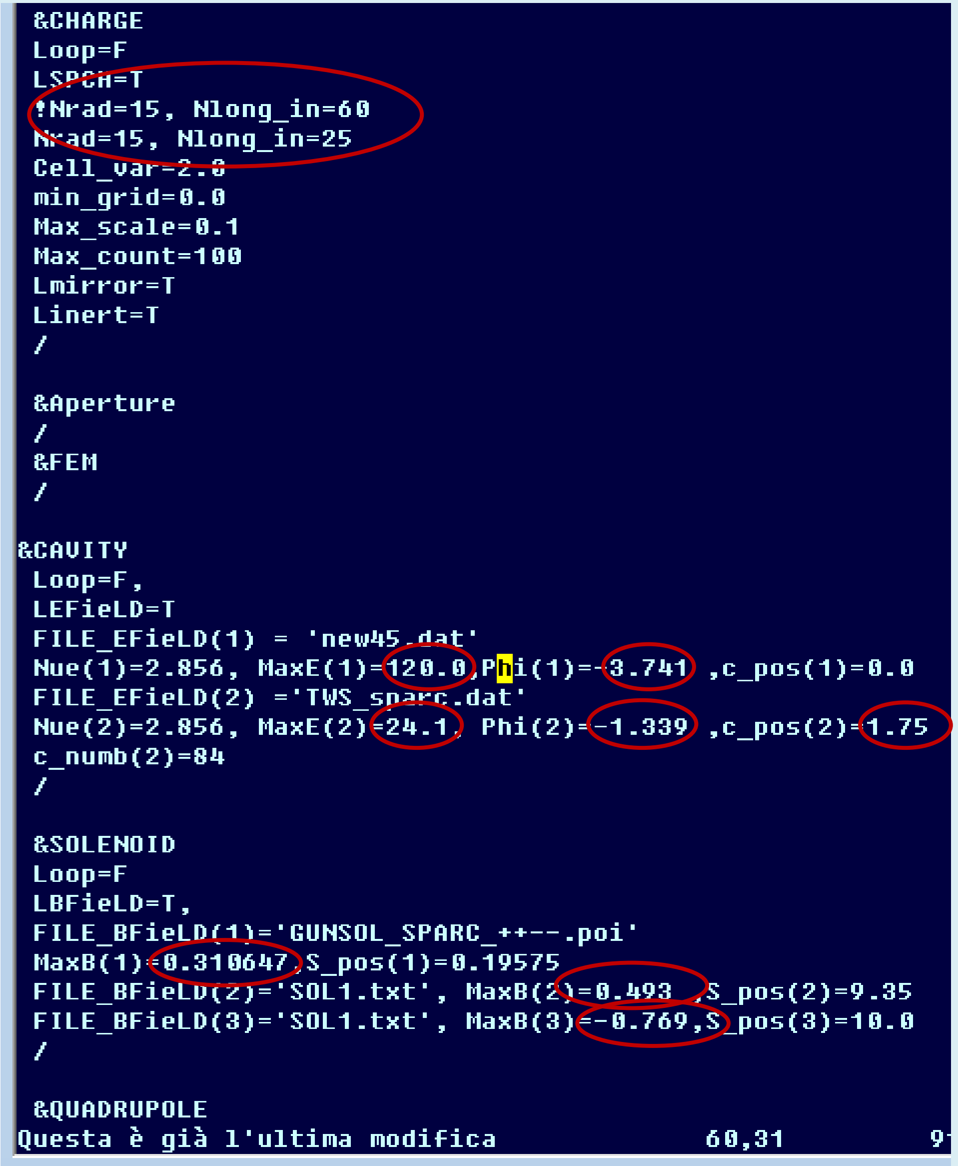
```
&INPUT
 FNAME = 'rfgunxx.ini '
 IPart=2000                        !usually in this machines are suggested 10000 macro particles
 Species='electrons'               !different kind can be simulated
 Probe=.F.                         !to track marked particles in the bunch
 Noise_reduc=.T.                   !Hammersley sequence (quasi-random distribution)
 Cathode=.T.                       !temporal distribution with Z=0, and flag ad hoc
 Q_total=   0.500                  !Bunch charge in nC
 Ref_zpos=0.0E0
 Ref_clock=0.00E0
 Ref_Ekin=1.0E-6
 !Dist_z='p',
 !Lt=0.0160
 !rt=0.001
 Dist_z='g'                        !Gaussian longitudinal distribution
 sig_clock=3.4E-3
 LE=1.1037E-03                     !Energy of photo-electron for isotropic distribution
 Dist_pz='i'                       !isotropic distribution
 sig_Ekin=0.0E0
 Dist_x='radial'                   !transversal uniformity (if only x, cylindrical symmetry is assumed)
 sig_x=    0.338                   ! sig_x=R_max/2
 Dist_px='r'
```

main parameters to work on Laser pulse shaping

Astra input files and free parameters

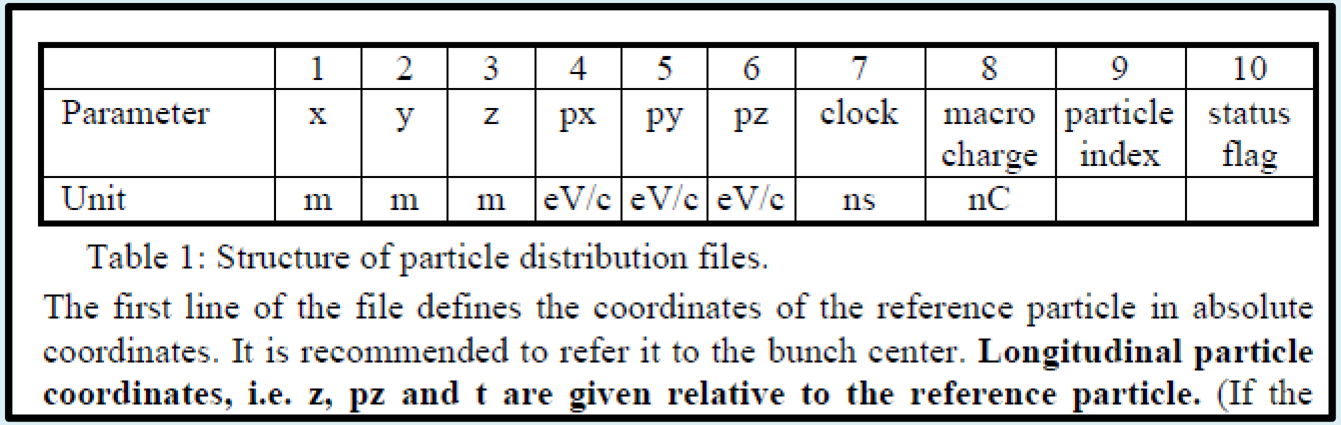


```
 &NEWRUN
 Head='Gun120MV/m, 0.5nC, START_60MeV case'
 RUN= 2 ,
 Loop=F, Nloop=2
 Distribution = 'rfgunxx.ini '
 Xoff=0.0E0, Yoff=0.0E0
 Lmagnetized=F
 EmitS=T
 PhaseS=T
 TrackS=T
 RefS=T
 TcheckS=T
 CathodeS=T
 TRACK_ALL=T,  PHASE_SCAN=F,   AUTO_PHASE=T
 check_ref_part=F,
 ZSTART=0.0,   ZSTOP=8.8
 Zemit=2050
 Zphase=2
 Max_step=200000
 H_max=0.0005
 H_min=0.0000
/

&OUTPUT
/

&SCAN
 LScan=F,
 Scan_para='Phi(1)'
 S_min=102 ,   S_max=118 ,   S_numb=9
 FOM(1)='bunch charge'
 FOM(2)='hor emit'
 FOM(3)='bunch length'
 FOM(4)='hor spot size'
 FOM(5)='phi end'
/
                            24,5         Cim
```



```
 &CHARGE
 Loop=F
 LSPCH=T
 !Nrad=15, Nlong_in=60
 Nrad=15, Nlong_in=25
 Cell_var=2.0
 min_grid=0.0
 Max_scale=0.1
 Max_count=100
 Lmirror=T
 Linert=T
 /

 &Aperture
 /
 &FEM
 /

&CAVITY
 Loop=F,
 LEFieLD=T
 FILE_EFieLD(1) = 'new45.dat'
 Nue(1)=2.856, MaxE(1)=120.0,Phi(1)=-3.741 ,c_pos(1)=0.0
 FILE_EFieLD(2) ='TWS_sparc.dat'
 Nue(2)=2.856, MaxE(2)=24.1, Phi(2)=-1.339 ,c_pos(2)=1.75
 c_numb(2)=84
 /

 &SOLENOID
 Loop=F
 LBFieLD=T,
 FILE_BFieLD(1)='GUNSOL_SPARC_++--.poi'
 MaxB(1)=0.310647,S_pos(1)=0.19575
 FILE_BFieLD(2)='SOL1.txt', MaxB(2)=0.493 ,S_pos(2)=9.35
 FILE_BFieLD(3)='SOL1.txt', MaxB(3)=-0.769,S_pos(3)=10.0
 /

 &QUADRUPOLE
Questa è già l'ultima modifica                  60,31         9
```

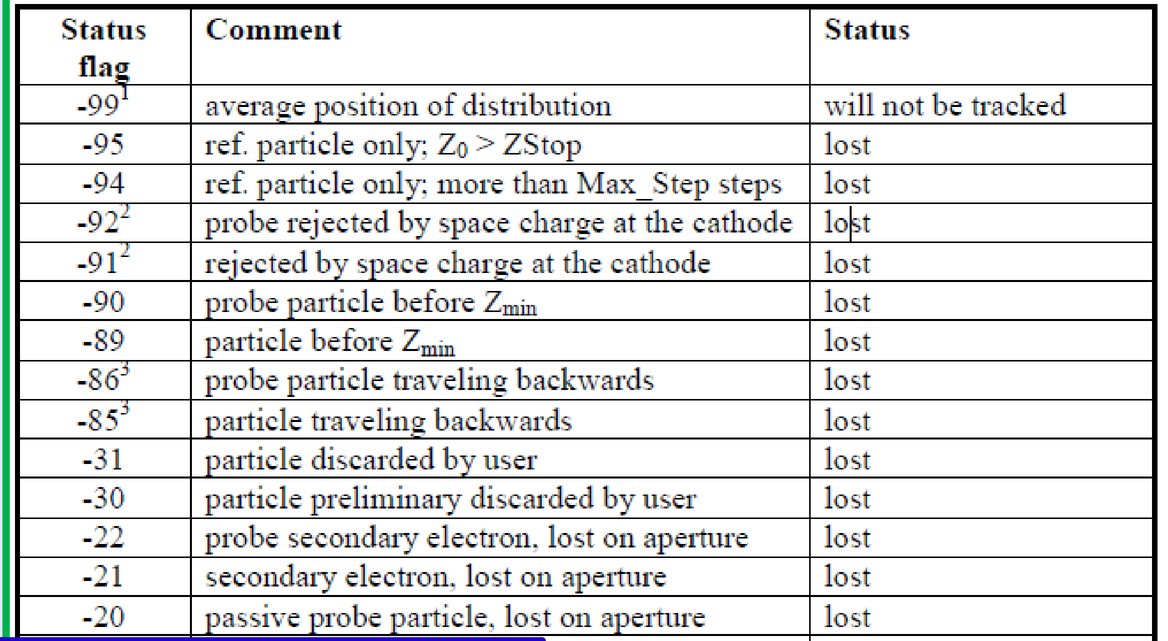
Astra main in/out files format



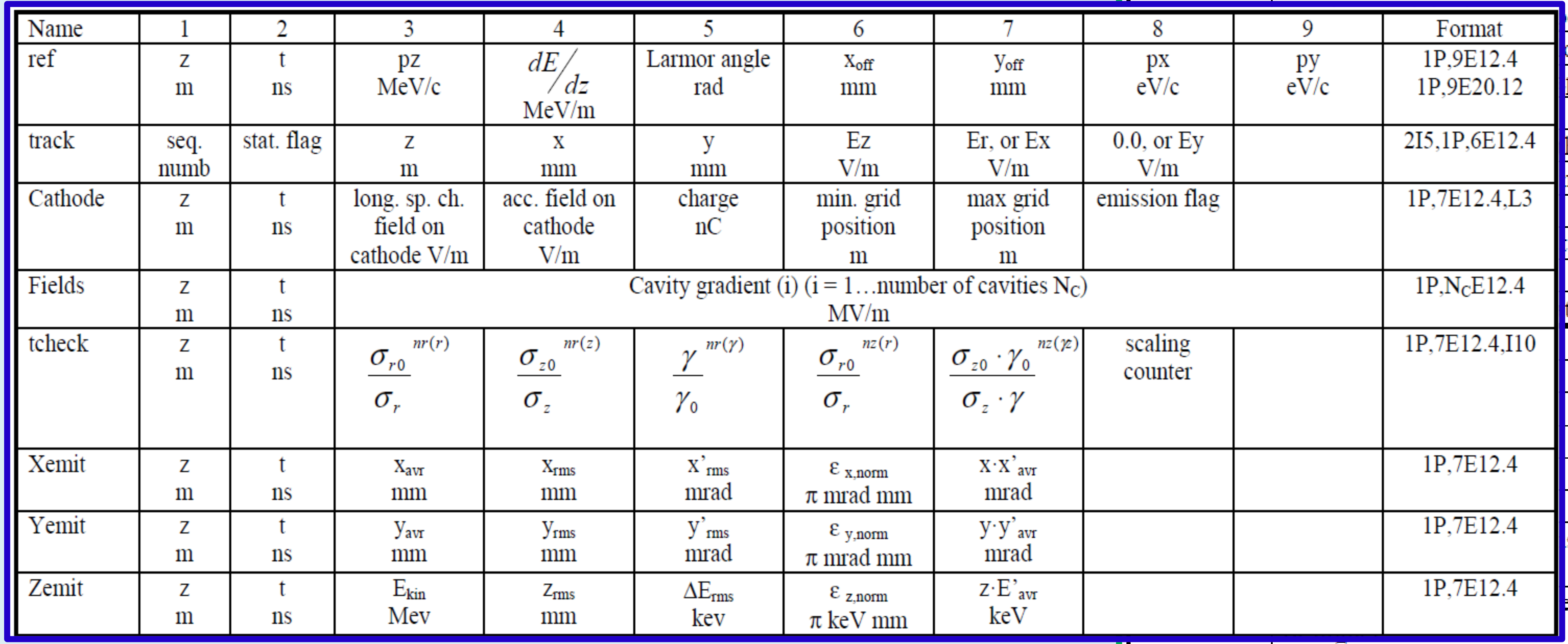
	1	2	3	4	5	6	7	8	9	10
Parameter	x	y	z	px	py	pz	clock	macro charge	particle index	status flag
Unit	m	m	m	eV/c	eV/c	eV/c	ns	nC		

Table 1: Structure of particle distribution files.

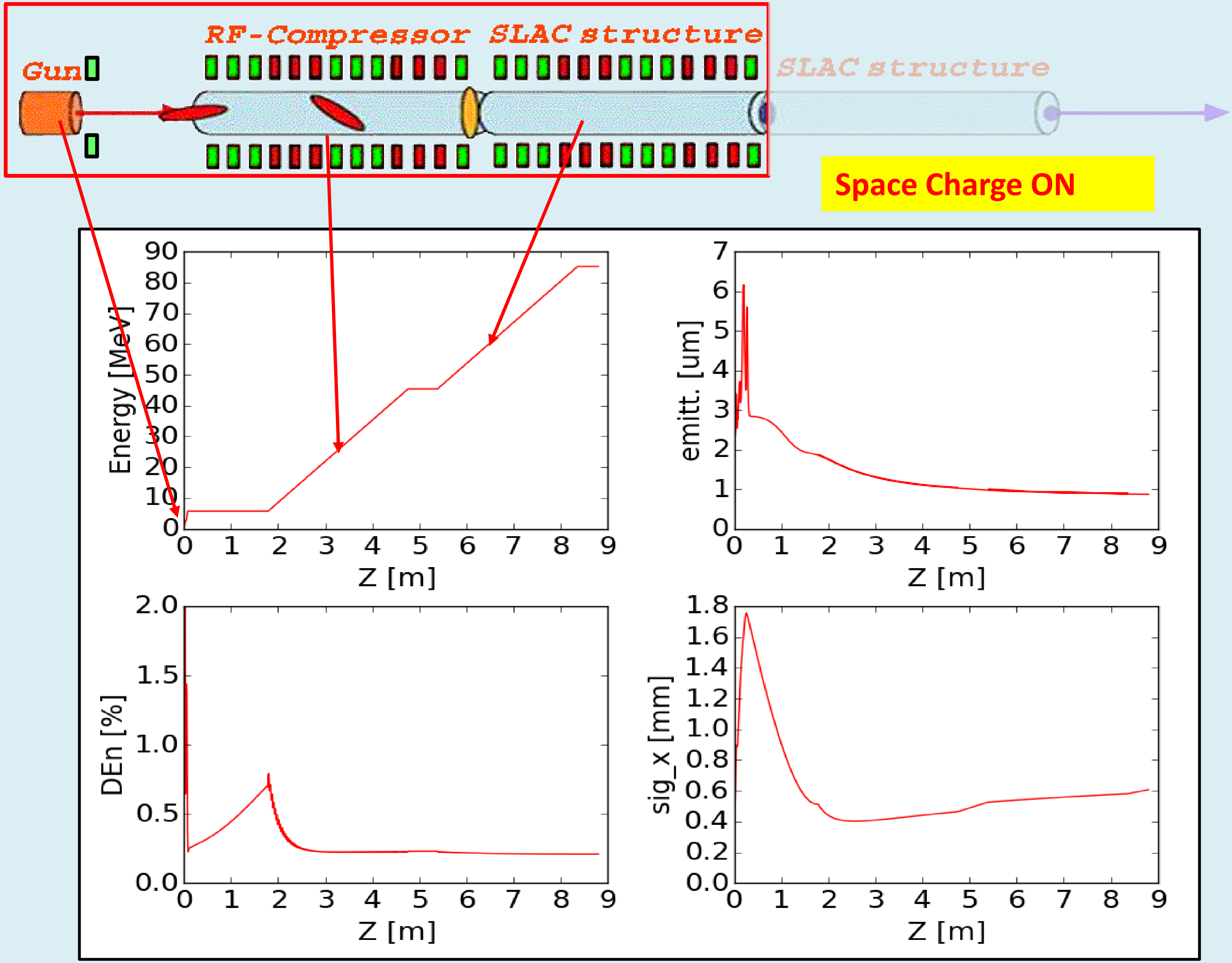
The first line of the file defines the coordinates of the reference particle in absolute coordinates. It is recommended to refer it to the bunch center. **Longitudinal particle coordinates, i.e. z, pz and t are given relative to the reference particle.** (If the



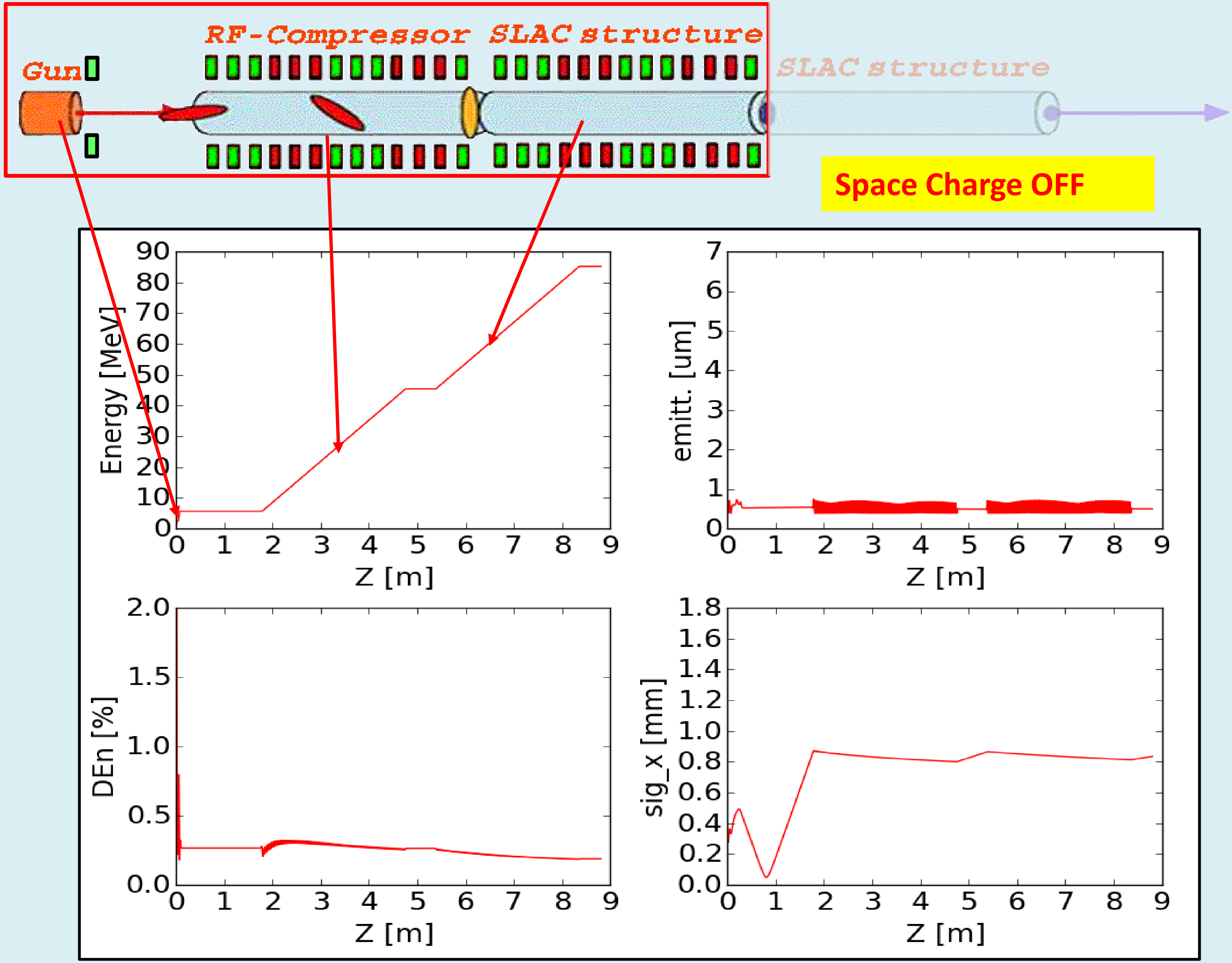
Status flag	Comment	Status
-99[1]	average position of distribution	will not be tracked
-95	ref. particle only; Z_0 > ZStop	lost
-94	ref. particle only; more than Max_Step steps	lost
-92[2]	probe rejected by space charge at the cathode	lost
-91[2]	rejected by space charge at the cathode	lost
-90	probe particle before Z_{min}	lost
-89	particle before Z_{min}	lost
-86[3]	probe particle traveling backwards	lost
-85[3]	particle traveling backwards	lost
-31	particle discarded by user	lost
-30	particle preliminary discarded by user	lost
-22	probe secondary electron, lost on aperture	lost
-21	secondary electron, lost on aperture	lost
-20	passive probe particle, lost on aperture	lost
	erture	lost
	ost on aperture	lost
	perture	lost
	he cathode	not yet started
	hode	not yet started
		not yet started
	the cathode	not yet started
	thode	not yet started
		tracking[4]
		tracking[4]
		tracking
		tracking
		tracking
	s of generation 1,	tracking
	eneration 1, 2…10	tracking



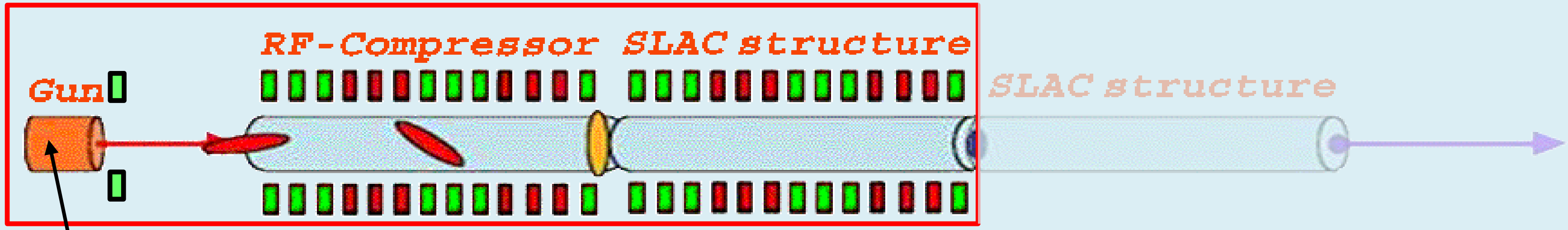
Name	1	2	3	4	5	6	7	8	9	Format
ref	z m	t ns	pz MeV/c	dE/dz MeV/m	Larmor angle rad	x_{off} mm	y_{off} mm	px eV/c	py eV/c	1P,9E12.4 1P,9E20.12
track	seq. numb	stat. flag	z m	x mm	y mm	Ez V/m	Er, or Ex V/m	0.0, or Ey V/m		2I5,1P,6E12.4
Cathode	z m	t ns	long. sp. ch. field on cathode V/m	acc. field on cathode V/m	charge nC	min. grid position m	max grid position m	emission flag		1P,7E12.4,L3
Fields	z m	t ns	Cavity gradient (i) (i = 1…number of cavities N_C) MV/m							1P,N_CE12.4
tcheck	z m	t ns	${\frac{\sigma_{r0}}{\sigma_r}}^{nr(r)}$	${\frac{\sigma_{z0}}{\sigma_z}}^{nr(z)}$	${\frac{\gamma}{\gamma_0}}^{nr(\gamma)}$	${\frac{\sigma_{r0}}{\sigma_r}}^{nz(r)}$	${\frac{\sigma_{z0}\cdot\gamma_0}{\sigma_z\cdot\gamma}}^{nz(\gamma)}$	scaling counter		1P,7E12.4,I10
Xemit	z m	t ns	x_{avr} mm	x_{rms} mm	x'_{rms} mrad	$\varepsilon_{x,norm}$ π mrad mm	$x\cdot x'_{avr}$ mrad			1P,7E12.4
Yemit	z m	t ns	y_{avr} mm	y_{rms} mm	y'_{rms} mrad	$\varepsilon_{y,norm}$ π mrad mm	$y\cdot y'_{avr}$ mrad			1P,7E12.4
Zemit	z m	t ns	E_{kin} Mev	z_{rms} mm	ΔE_{rms} kev	$\varepsilon_{z,norm}$ π keV mm	$z\cdot E'_{avr}$ keV			1P,7E12.4



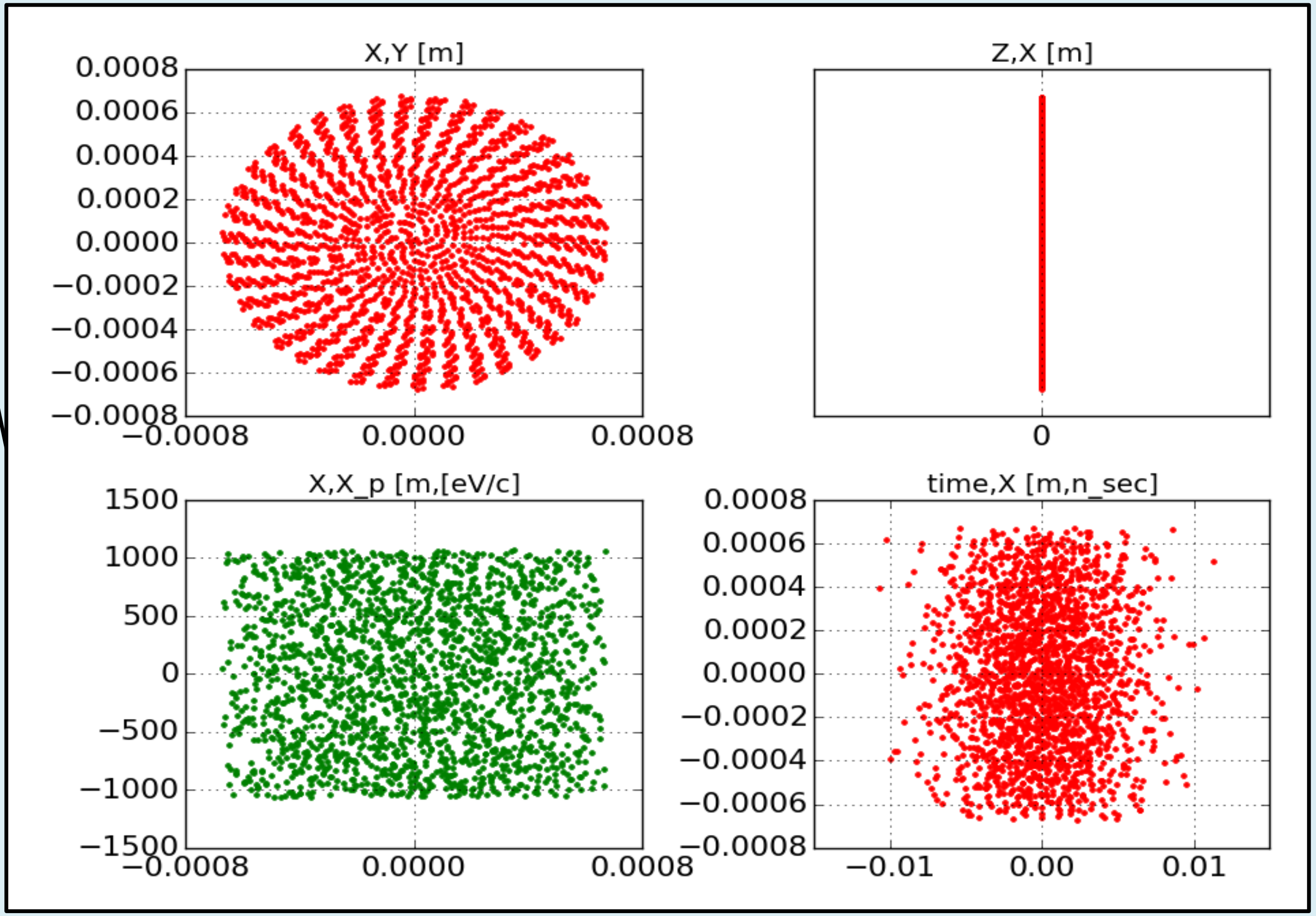
Space Charge ON

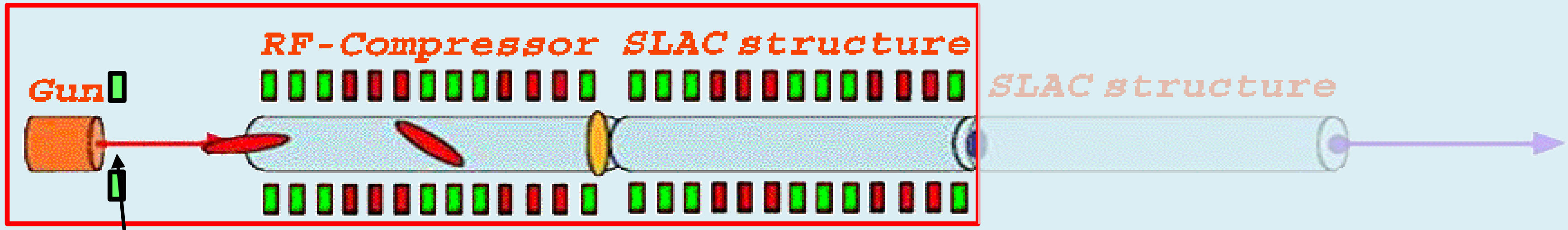


Space Charge OFF

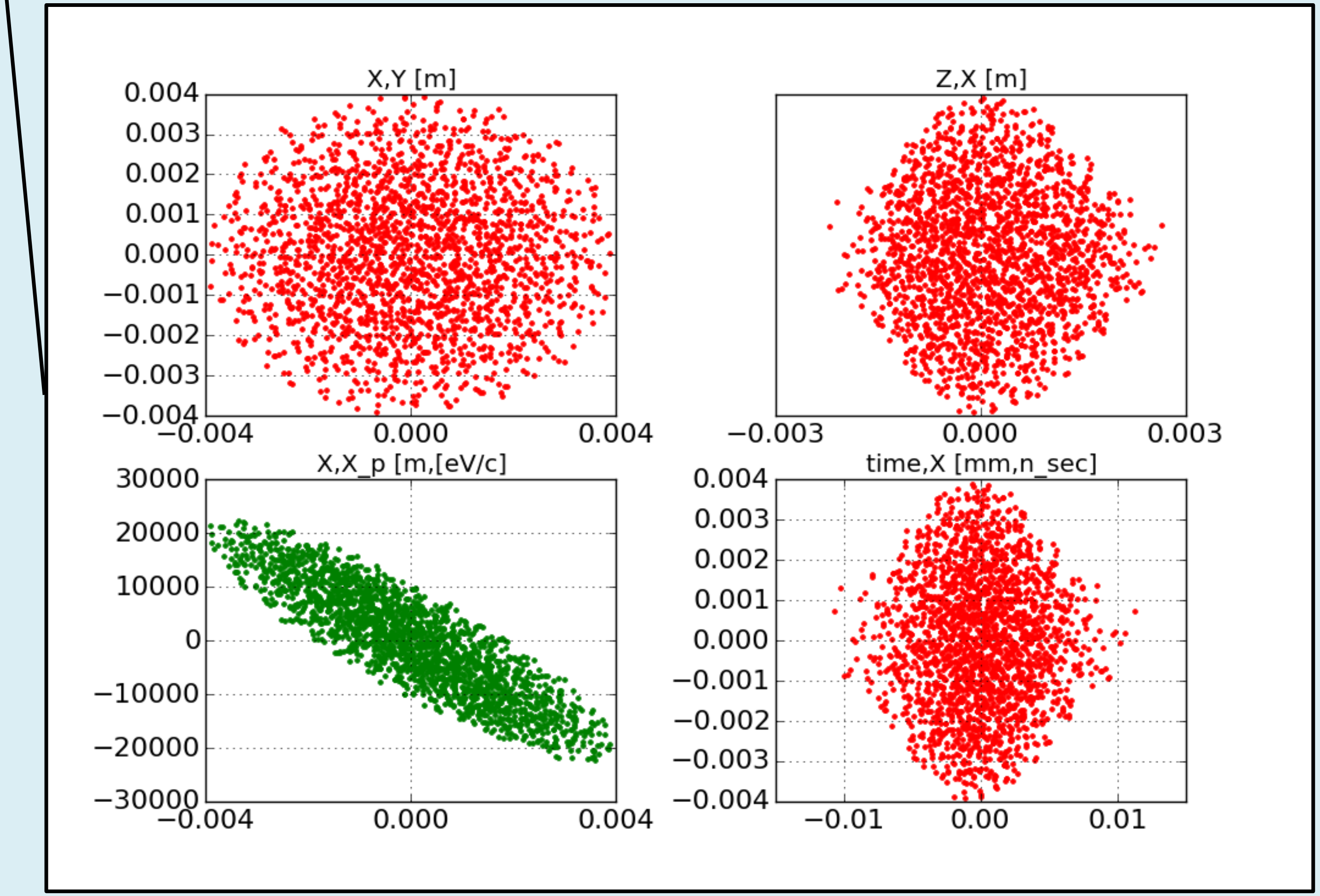


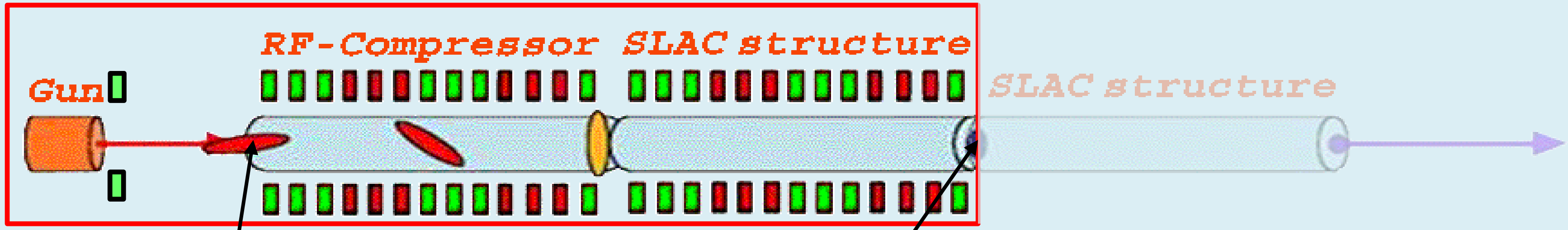
On the cathode



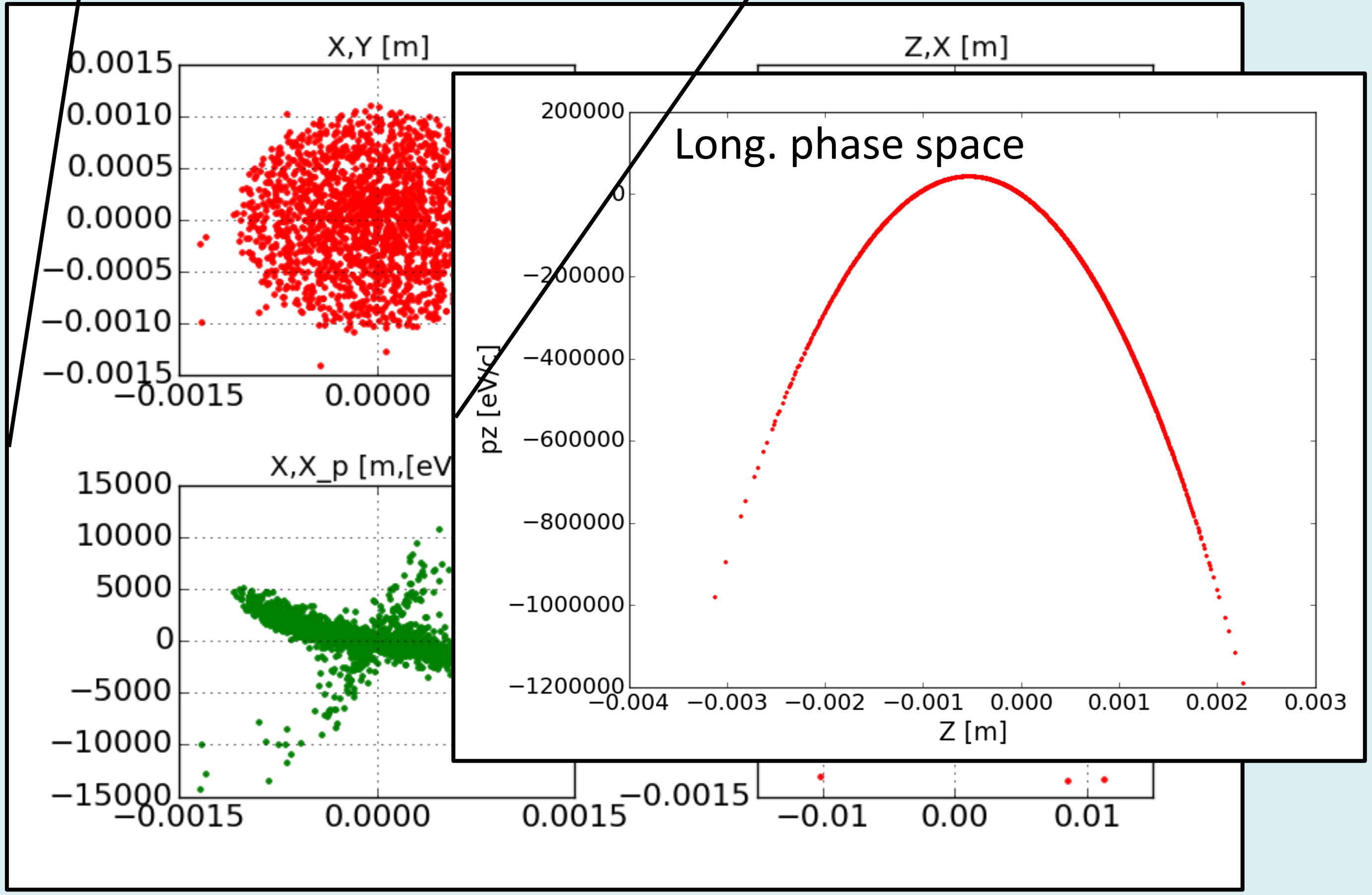

RF-Compressor
SLAC structure
Gun
SLAC structure

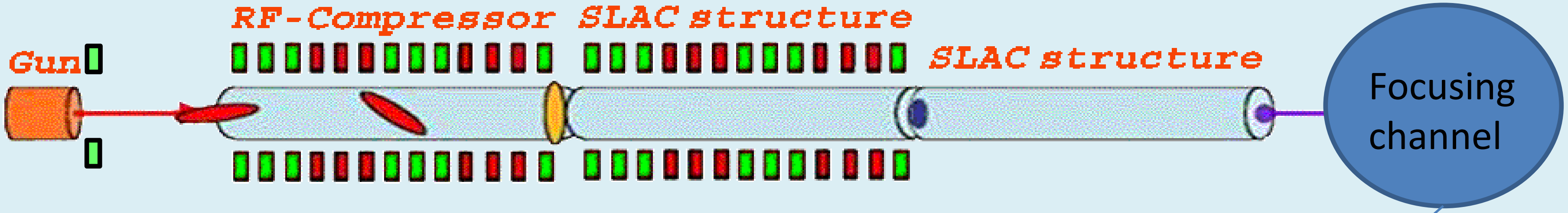
Gun Exit

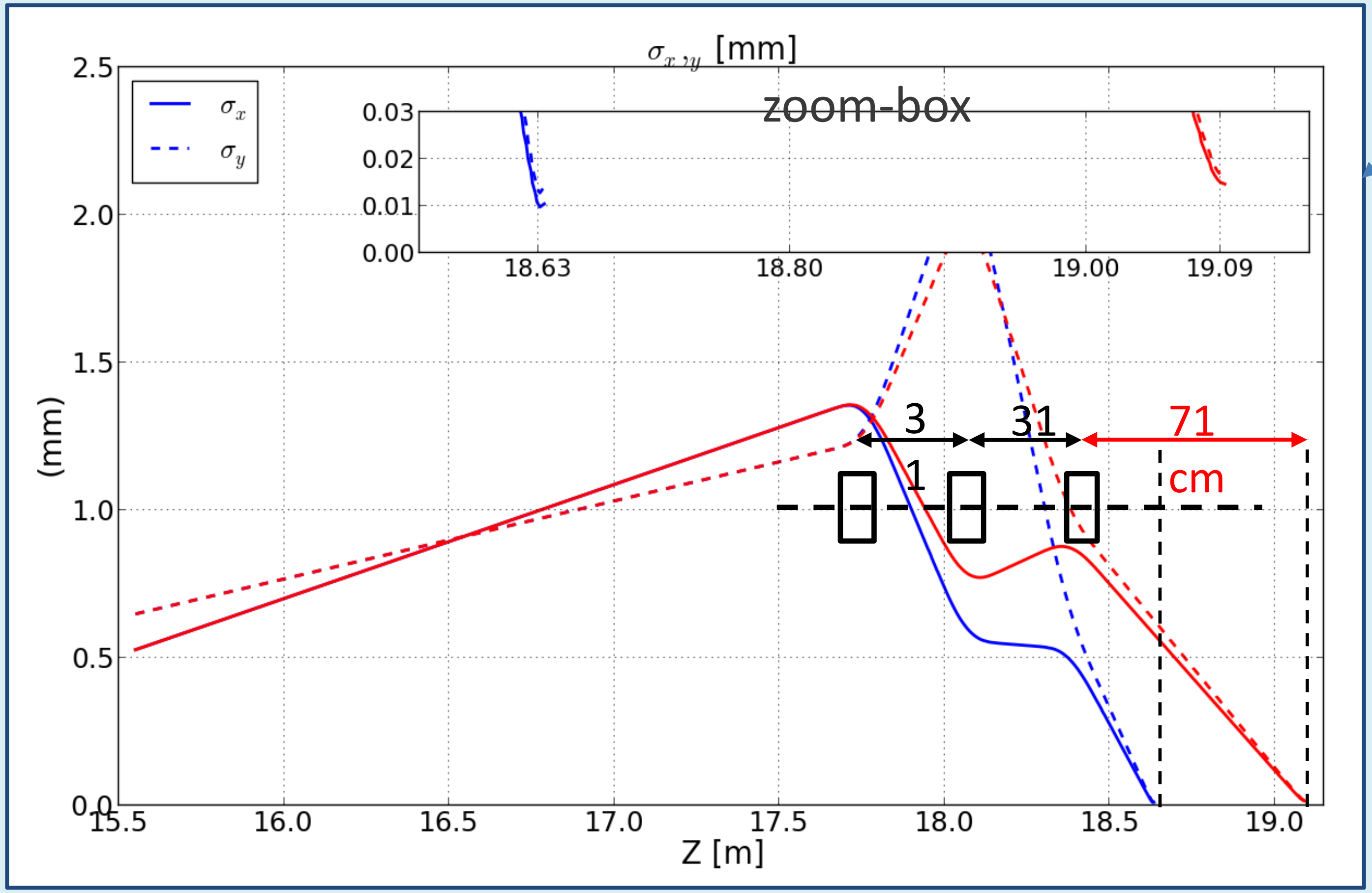

X,Y [m]
0.004
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
−0.004
−0.004
0.000
0.004
Z,X [m]
−0.003
0.000
0.003
X,X_p [m,[eV/c]
30000
20000
10000
0
−10000
−20000
−30000
−0.004
0.000
0.004
time,X [mm,n_sec]
0.004
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
−0.004
−0.01
0.00
0.01

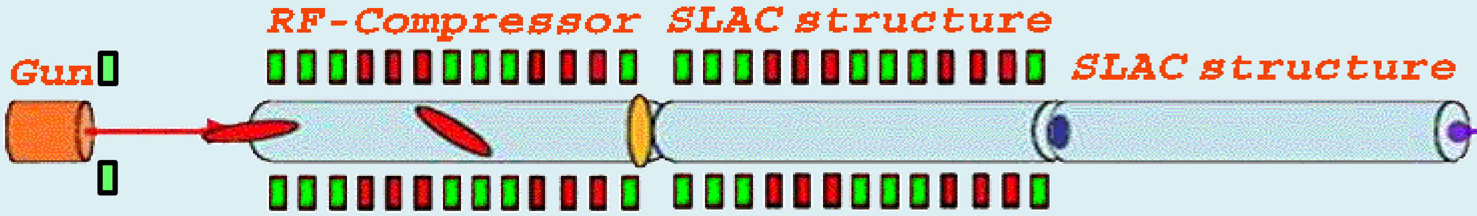


Linac entrance

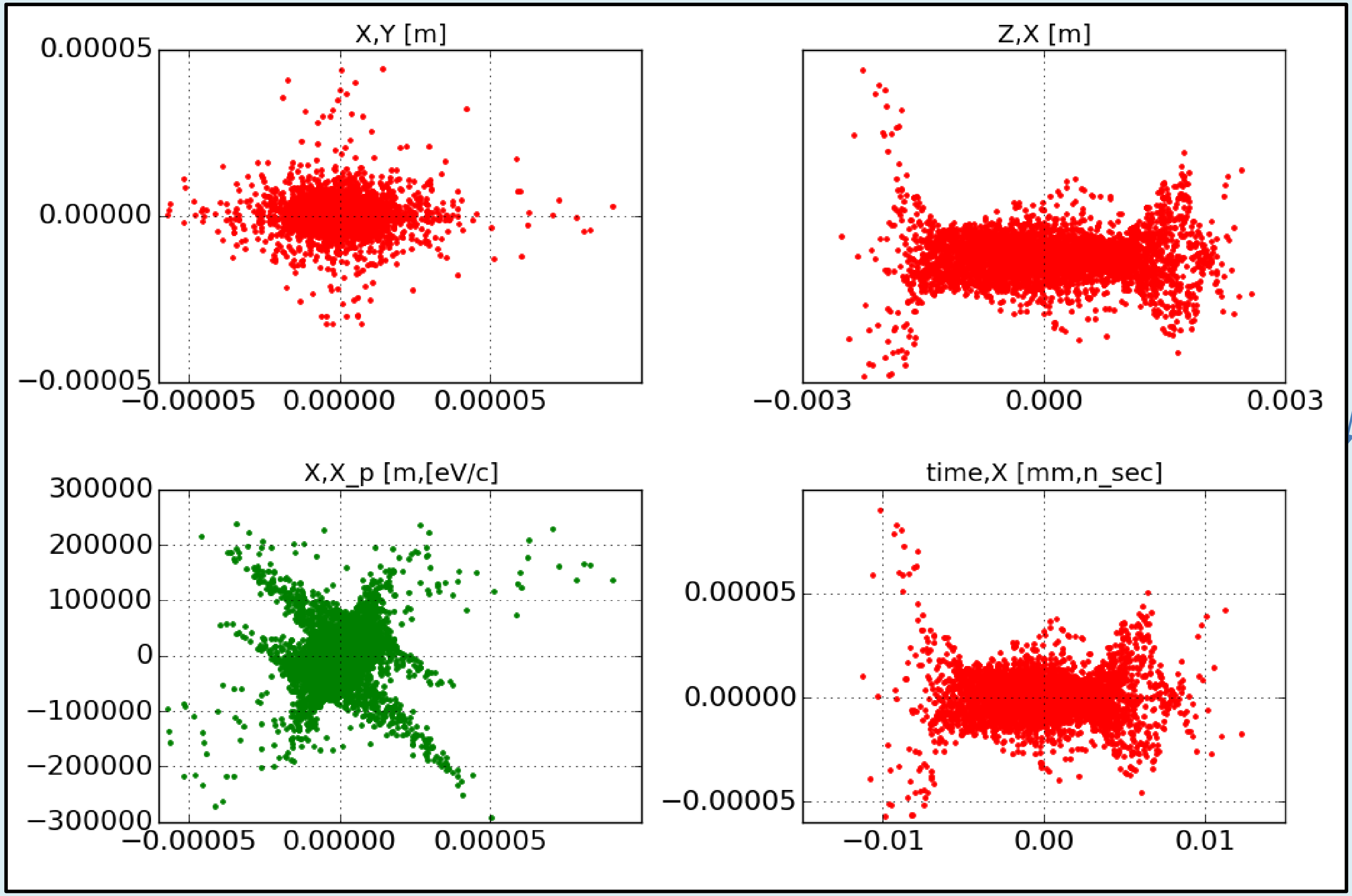



Gun
RF-Compressor
SLAC structure
SLAC structure
Focusing channel

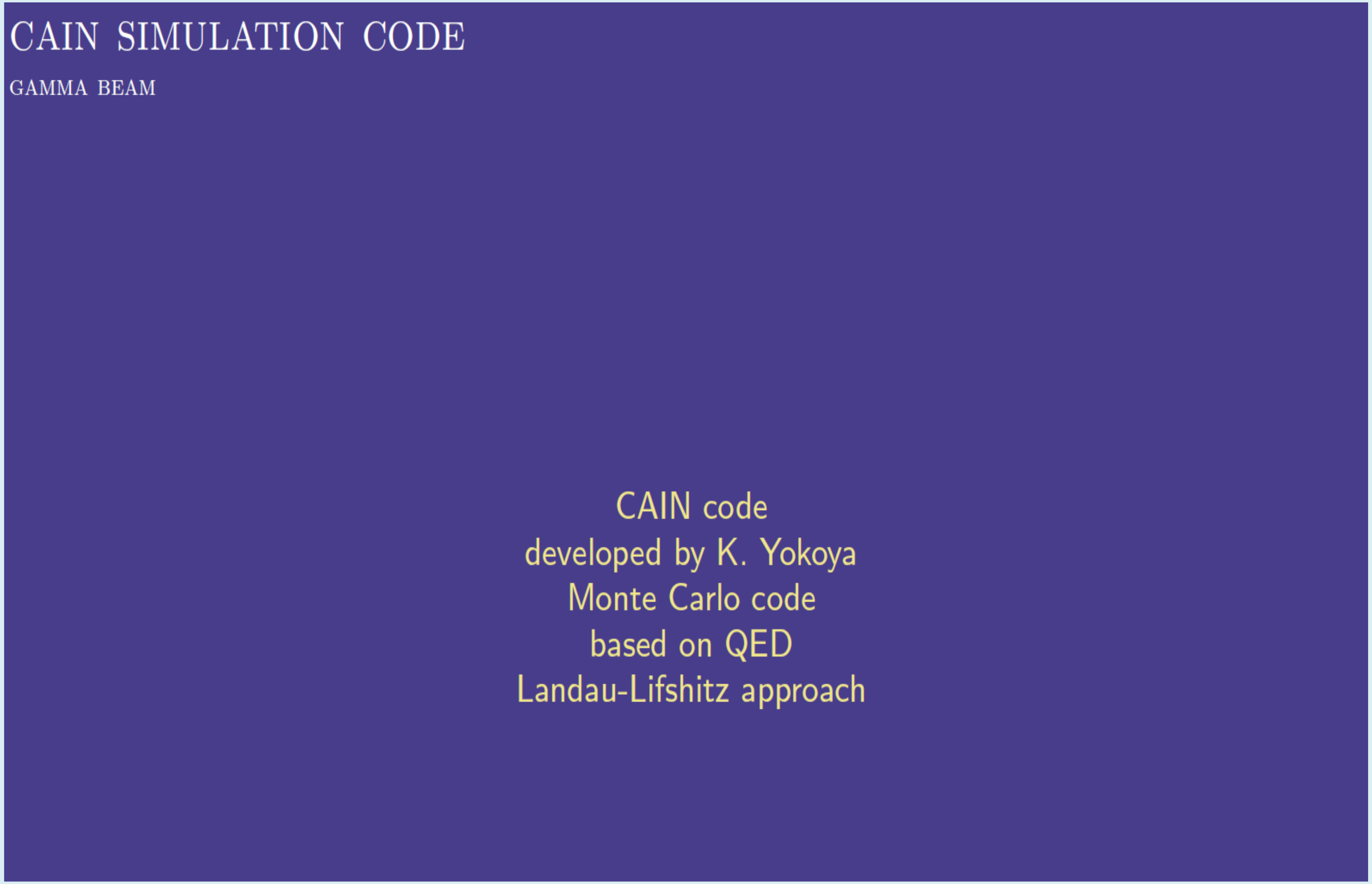

$\sigma_{x,y}$ [mm]
σ_x
σ_y
zoom-box
0.03
0.02
0.01
0.00
18.63
18.80
19.00
19.09
3
1
31
71
cm
2.5
2.0
1.5
1.0
0.5
0.0
(mm)
15.5
16.0
16.5
17.0
17.5
18.0
18.5
19.0
Z [m]



@ Interaction Point



A Quantum Code, To simulate the Electron-Phothon Bunches schattering

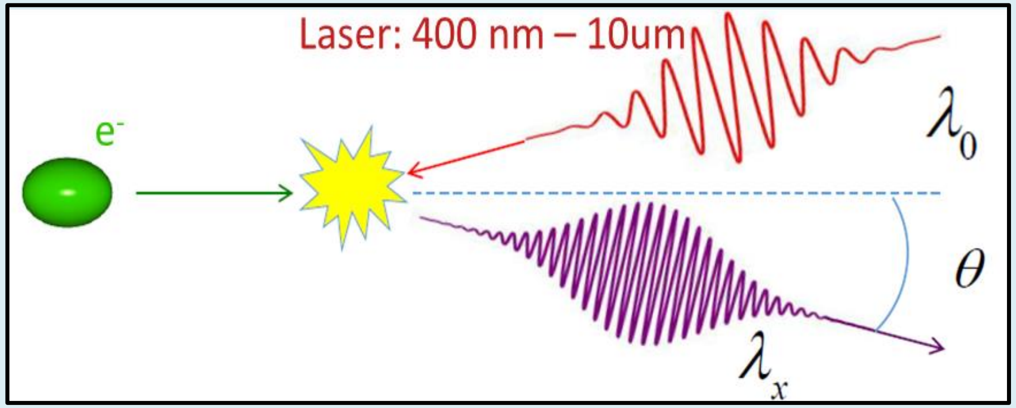


CAIN SIMULATION CODE

GAMMA BEAM

CAIN code
developed by K. Yokoya
Monte Carlo code
based on QED
Landau-Lifshitz approach

The Inverse Scattering Compton (ICS)

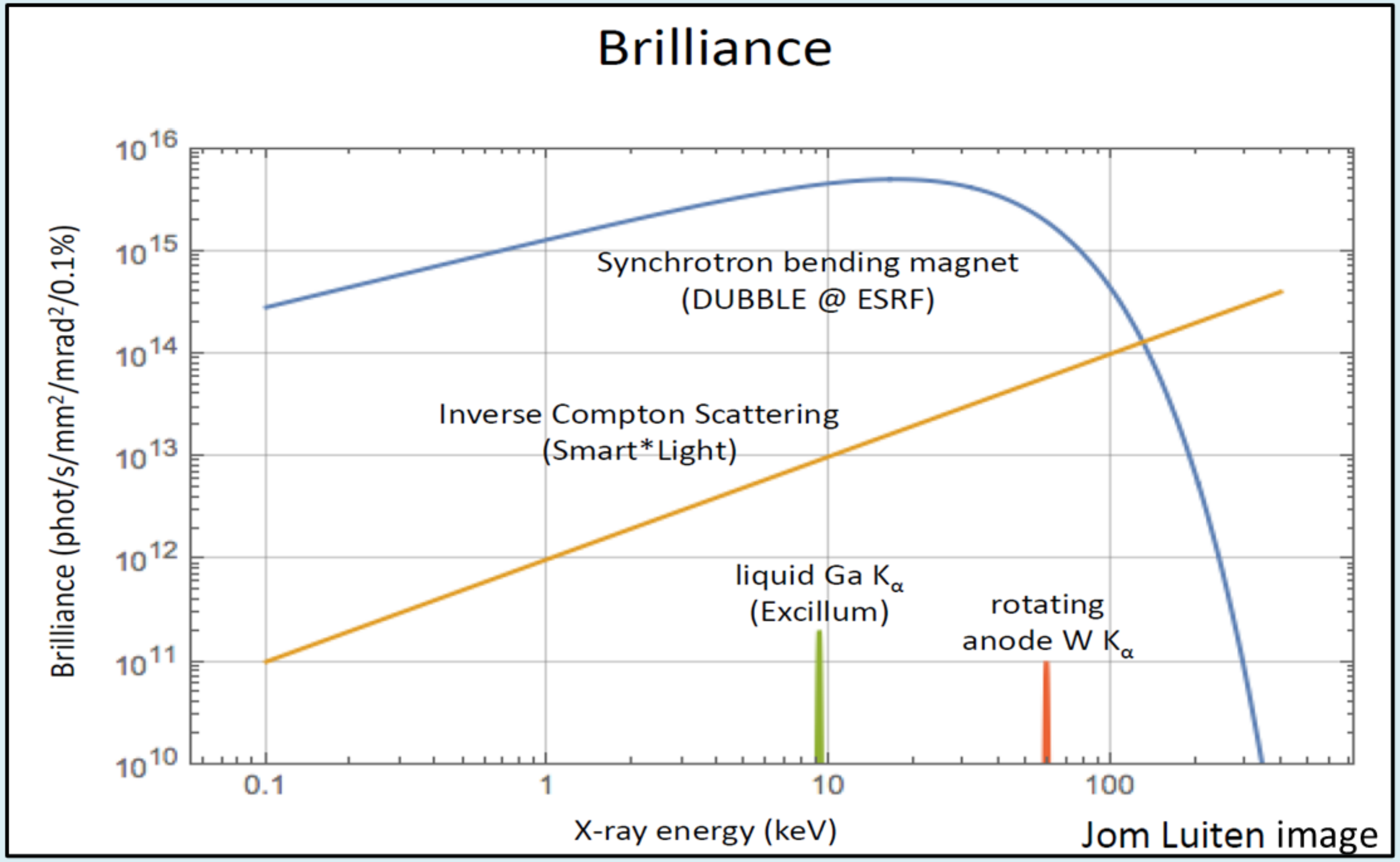


$$E'_{\mathrm{ph}}(\theta) = \frac{4E_{\mathrm{ph}}\gamma^2}{1+X+\gamma^2\theta^2}$$

$$X = \frac{4E_eE_{\mathrm{ph}}}{(m_0c^2)^2} = \frac{4\gamma E_{\mathrm{ph}}}{m_0c^2} = 4\gamma^2\frac{E_{\mathrm{ph}}}{E_e}$$

$$X << 1$$

$$\boxed{E'_{\mathrm{ph}}(\theta=0) = 4E_{\mathrm{ph}}\gamma^2}$$



Jom Luiten image

Number of x-ray photons

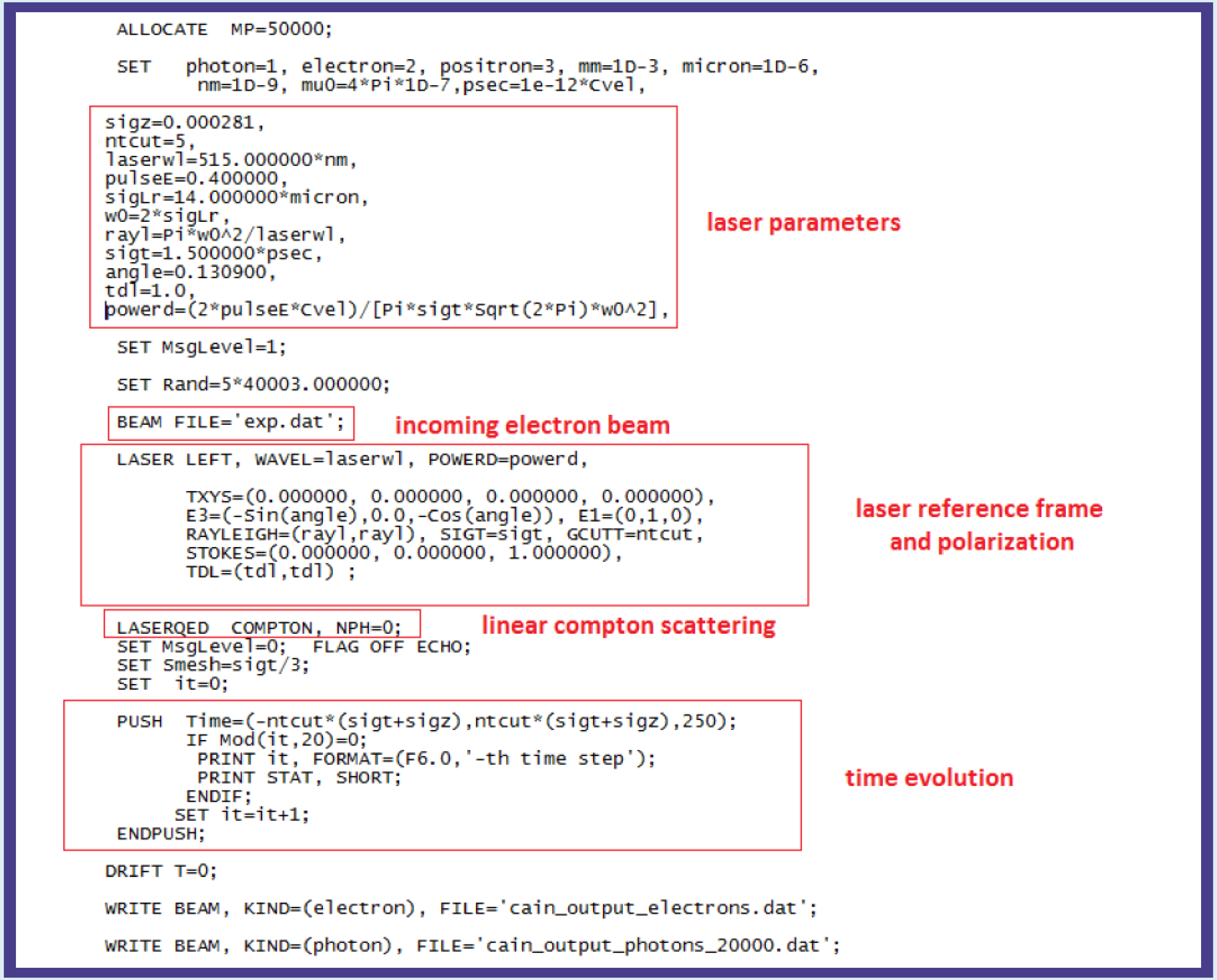
$$N_{x,total} = N_eN_0\frac{\sigma_T}{2\pi w_0^2}$$

Cross section

Lasere electron I-spot

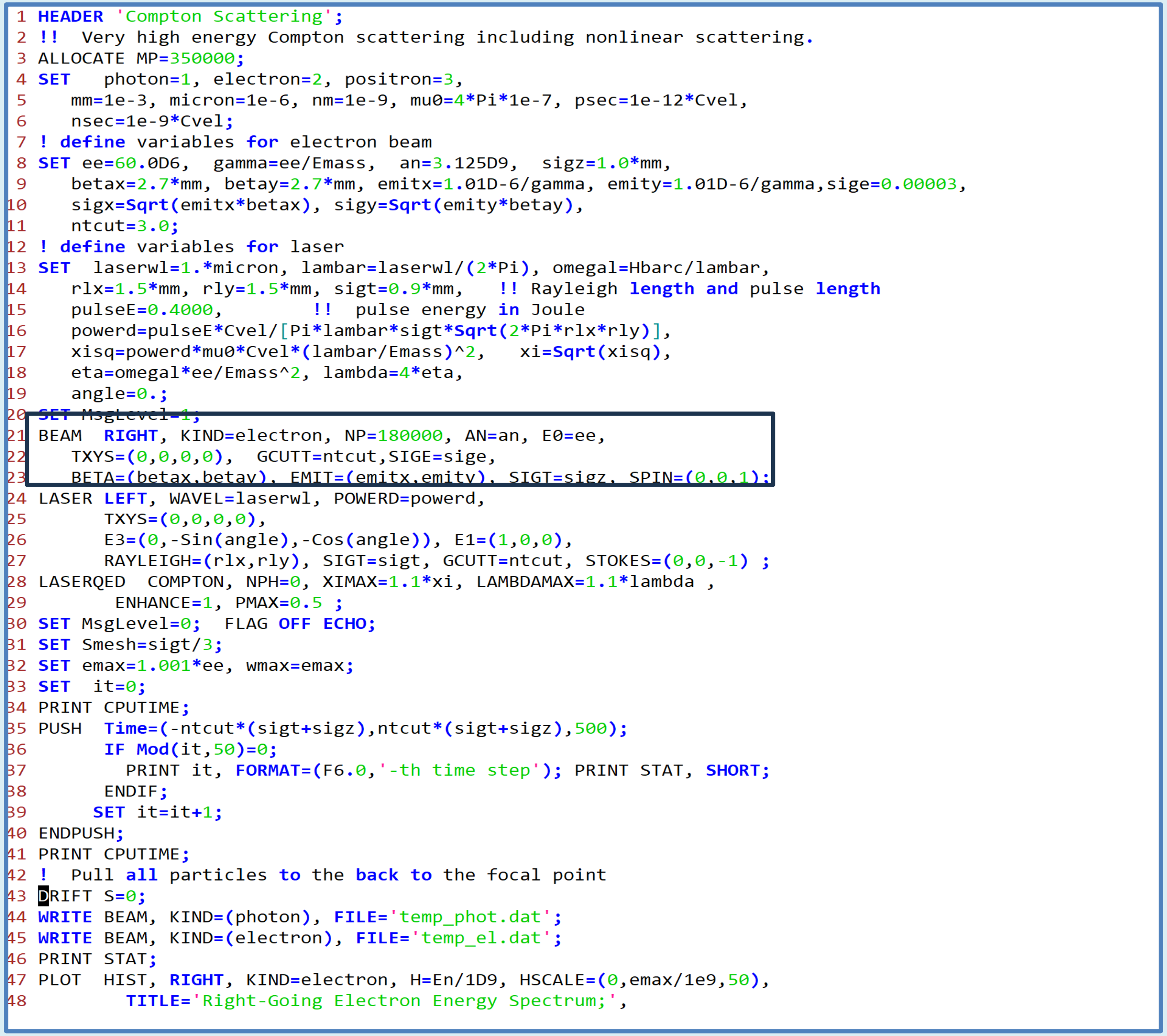
L. Serafini, A. Bacci, C. Curatolo et al. Fundamental Plasma Physics 7 (2023) 100026

Cain input file



```
 ALLOCATE  MP=50000;

 SET   photon=1, electron=2, positron=3, mm=1D-3, micron=1D-6,
        nm=1D-9, mu0=4*Pi*1D-7,psec=1e-12*Cvel,

sigz=0.000281,
ntcut=5,
laserwl=515.000000*nm,
pulseE=0.400000,
sigLr=14.000000*micron,
w0=2*sigLr,
rayl=Pi*w0^2/laserwl,
sigt=1.500000*psec,
angle=0.130900,
tdl=1.0,
powerd=(2*pulseE*Cvel)/[Pi*sigt*Sqrt(2*Pi)*w0^2],

 SET MsgLevel=1;

 SET Rand=5*40003.000000;

 BEAM FILE='exp.dat';

 LASER LEFT, WAVEL=laserwl, POWERD=powerd,

       TXYS=(0.000000, 0.000000, 0.000000, 0.000000),
       E3=(-Sin(angle),0.0,-Cos(angle)), E1=(0,1,0),
       RAYLEIGH=(rayl,rayl), SIGT=sigt, GCUTT=ntcut,
       STOKES=(0.000000, 0.000000, 1.000000),
       TDL=(tdl,tdl) ;

 LASERQED  COMPTON, NPH=0;
 SET MsgLevel=0;  FLAG OFF ECHO;
 SET Smesh=sigt/3;
 SET  it=0;

 PUSH  Time=(-ntcut*(sigt+sigz),ntcut*(sigt+sigz),250);
       IF Mod(it,20)=0;
        PRINT it, FORMAT=(F6.0,'-th time step');
        PRINT STAT, SHORT;
       ENDIF;
      SET it=it+1;
 ENDPUSH;

DRIFT T=0;

WRITE BEAM, KIND=(electron), FILE='cain_output_electrons.dat';

WRITE BEAM, KIND=(photon), FILE='cain_output_photons_20000.dat';
```

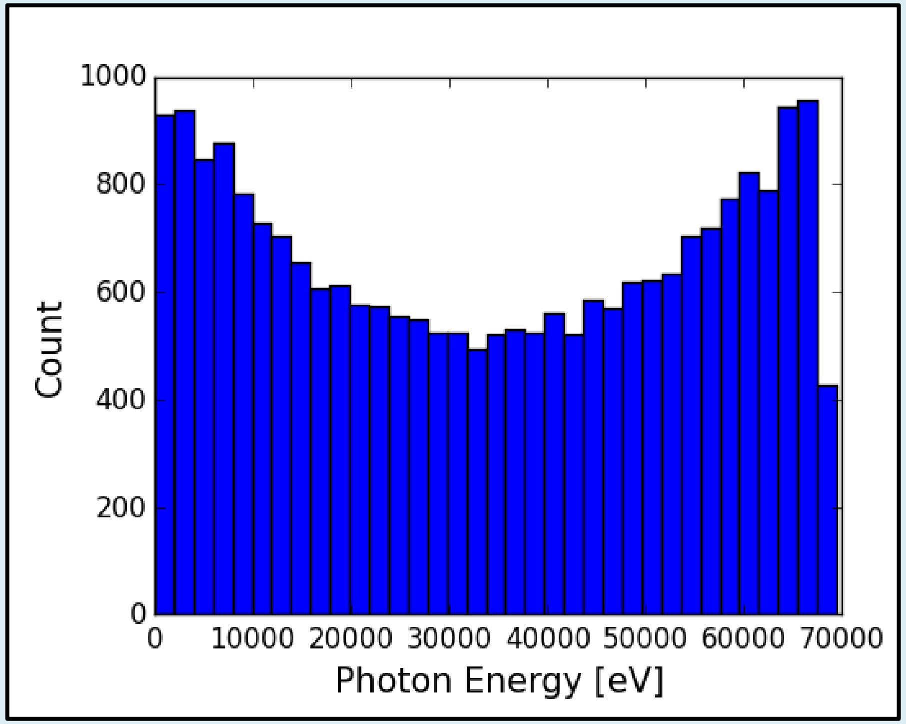
Cain input file: to define the bunch virtually

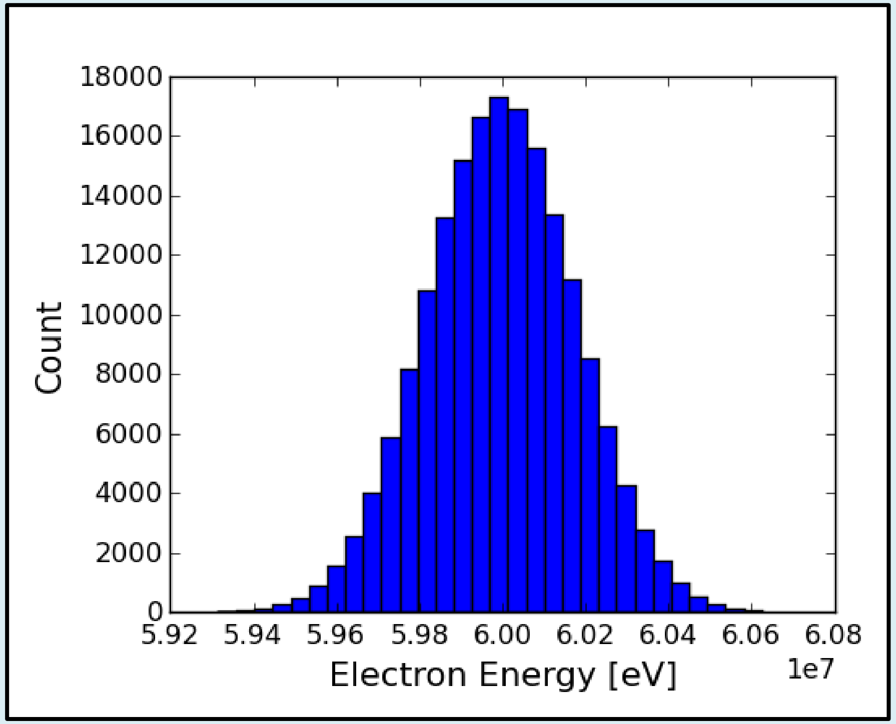


```
HEADER 'Compton Scattering';
!!  Very high energy Compton scattering including nonlinear scattering.
ALLOCATE MP=350000;
SET   photon=1, electron=2, positron=3,
   mm=1e-3, micron=1e-6, nm=1e-9, mu0=4*Pi*1e-7, psec=1e-12*Cvel,
   nsec=1e-9*Cvel;
! define variables for electron beam
SET ee=60.0D6,  gamma=ee/Emass,  an=3.125D9,  sigz=1.0*mm,
   betax=2.7*mm, betay=2.7*mm, emitx=1.01D-6/gamma, emity=1.01D-6/gamma,sige=0.00003,
   sigx=Sqrt(emitx*betax), sigy=Sqrt(emity*betay),
   ntcut=3.0;
! define variables for laser
SET  laserwl=1.*micron, lambar=laserwl/(2*Pi), omegal=Hbarc/lambar,
   rlx=1.5*mm, rly=1.5*mm, sigt=0.9*mm,   !! Rayleigh length and pulse length
   pulseE=0.4000,          !!  pulse energy in Joule
   powerd=pulseE*Cvel/[Pi*lambar*sigt*Sqrt(2*Pi*rlx*rly)],
   xisq=powerd*mu0*Cvel*(lambar/Emass)^2,   xi=Sqrt(xisq),
   eta=omegal*ee/Emass^2, lambda=4*eta,
   angle=0.;
SET MsgLevel=1;
BEAM  RIGHT, KIND=electron, NP=180000, AN=an, E0=ee,
   TXYS=(0,0,0,0),  GCUTT=ntcut,SIGE=sige,
   BETA=(betax,betay), EMIT=(emitx,emity), SIGT=sigz, SPIN=(0,0,1);
LASER LEFT, WAVEL=laserwl, POWERD=powerd,
      TXYS=(0,0,0,0),
      E3=(0,-Sin(angle),-Cos(angle)), E1=(1,0,0),
      RAYLEIGH=(rlx,rly), SIGT=sigt, GCUTT=ntcut, STOKES=(0,0,-1) ;
LASERQED  COMPTON, NPH=0, XIMAX=1.1*xi, LAMBDAMAX=1.1*lambda ,
       ENHANCE=1, PMAX=0.5 ;
SET MsgLevel=0;  FLAG OFF ECHO;
SET Smesh=sigt/3;
SET emax=1.001*ee, wmax=emax;
SET  it=0;
PRINT CPUTIME;
PUSH  Time=(-ntcut*(sigt+sigz),ntcut*(sigt+sigz),500);
      IF Mod(it,50)=0;
        PRINT it, FORMAT=(F6.0,'-th time step'); PRINT STAT, SHORT;
      ENDIF;
     SET it=it+1;
ENDPUSH;
PRINT CPUTIME;
!  Pull all particles to the back to the focal point
DRIFT S=0;
WRITE BEAM, KIND=(photon), FILE='temp_phot.dat';
WRITE BEAM, KIND=(electron), FILE='temp_el.dat';
PRINT STAT;
PLOT  HIST, RIGHT, KIND=electron, H=En/1D9, HSCALE=(0,emax/1e9,50),
        TITLE='Right-Going Electron Energy Spectrum;',
```

STAR Linac,
X-ray source @ 60 MeV – recoil << of the energy spread Δγ/γ

Let consider an electron-bunch with an Δγ/γ of 0.003 , λ_{LASER}=1um (0.4 J), W_0=20 um





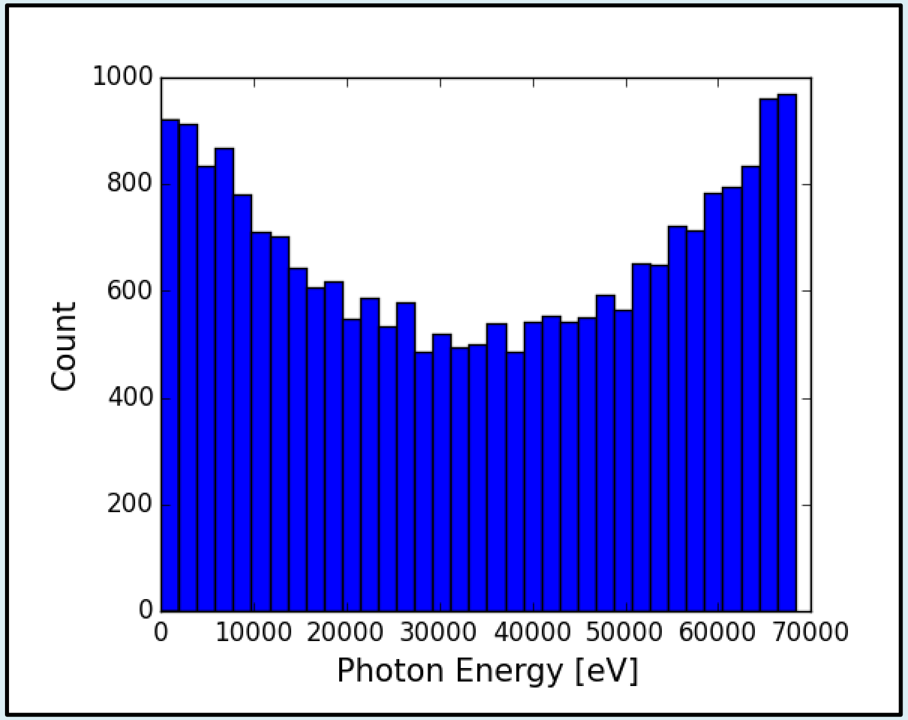
$$Emax = \frac{4\gamma^2 h\upsilon}{1+X}, \quad X = \frac{4\gamma h\upsilon}{511\ keV}$$

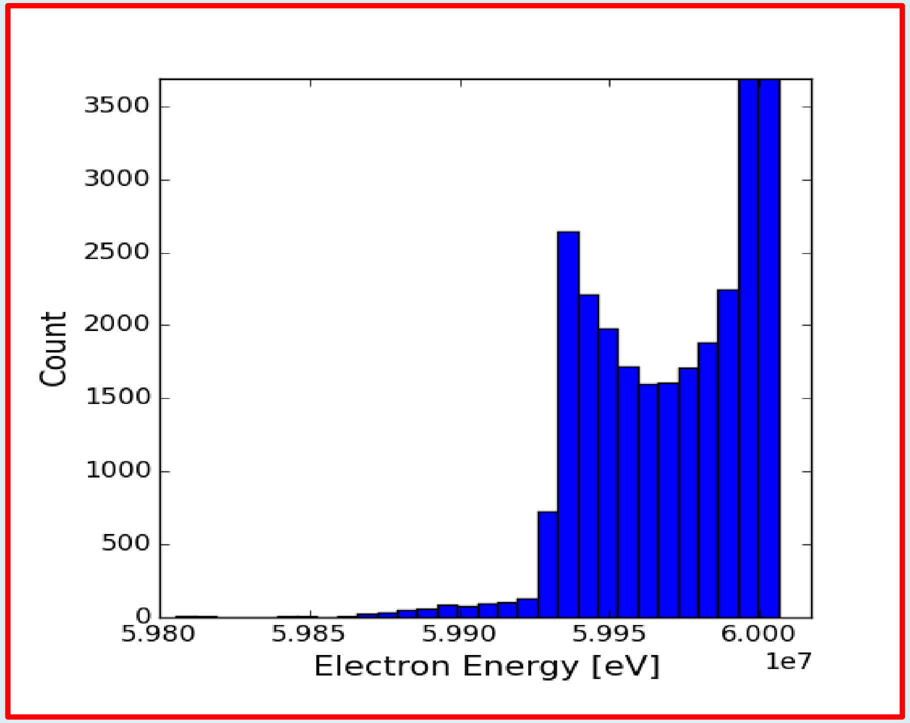
$Emax = 71.340 keV$, X=0.00116 (recoil)

$$E_{ph}\ (\lambda_{laser} = 1\ um) = 1.24\ eV$$

STAR Linac,
X-ray source @ 60 MeV – recoil as before but > $\Delta\gamma/\gamma$

Let consider an electron-bunch with ultra low $\Delta\gamma/\gamma$ of 0.00003 and same condition as before

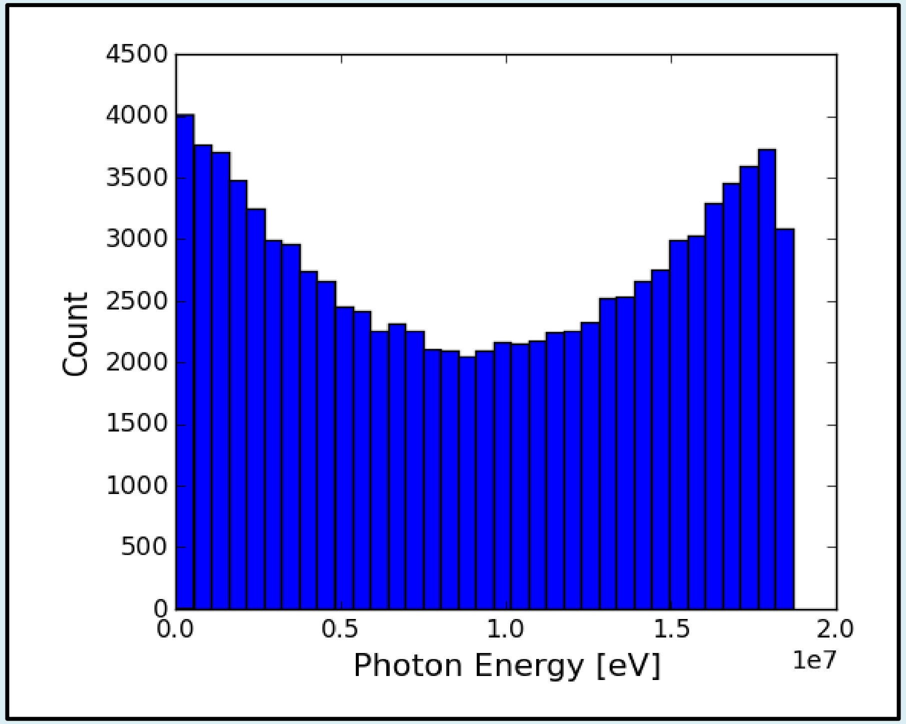


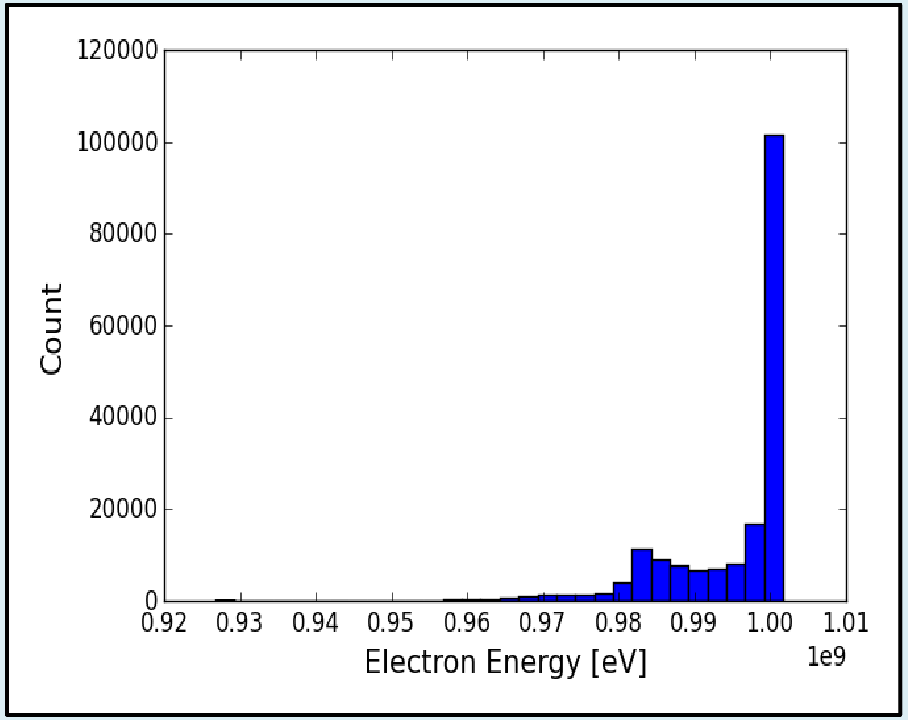


$Emax = 71.340keV, \quad X$=0.00116

STAR Linac,
X-ray source @ 1 GeV –not negligible recoil

Let consider an electron-bunch with an Δγ/γ of 0.0005, scattering λ=1um, 1 J, W0=20um laser





$Emax = \frac{4\gamma^2 h\nu}{1+X}$, $X = \frac{4\gamma h\nu}{511keV}$

$N_{phot} = \frac{8.4\cdot 10^8 \cdot Laser_{En}[J] Q_b[pC]}{Phot_{En}\,[eV]\cdot(\sigma_x^2\,[\mu m] + 0.25\cdot W_0[\mu m])}$

$Emax = 19.46MeV$, X=0.019

$E_{ph}\,(\lambda_{laser} = 1\ um) = 1.24\ eV$

$W_0 = \sqrt{R_L \cdot \lambda_L/\pi}$

$$Emax = \frac{4\gamma^2 h\nu}{1} \sim 20\ MeV$$

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