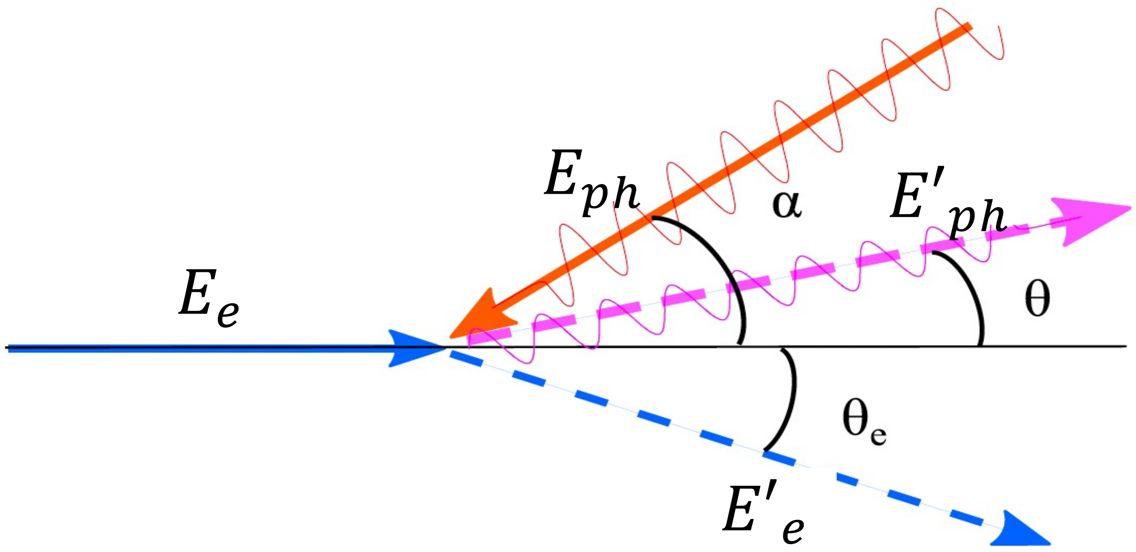
General Compton Scattering geometry between an incident eletron E_e and a photon E_{ph} at a collision angle α, photon E'_{ph} scattering angle θ and electron E'_e scattering angle θ_e



Second Series *May, 1923* *Vol. 21, No. 5*

beginning of the story – the photon, quantum of energy

THE

PHYSICAL REVIEW

A QUANTUM THEORY OF THE SCATTERING OF X-RAYS BY LIGHT ELEMENTS

BY ARTHUR H. COMPTON

ABSTRACT

A quantum theory of the scattering of X-rays and γ-rays by light elements. —The hypothesis is suggested that when an X-ray quantum is scattered it spends all of its energy and momentum upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in

The change in

$$\lambda' - \lambda = \lambda_c (1 + \cos \Delta\vartheta)$$

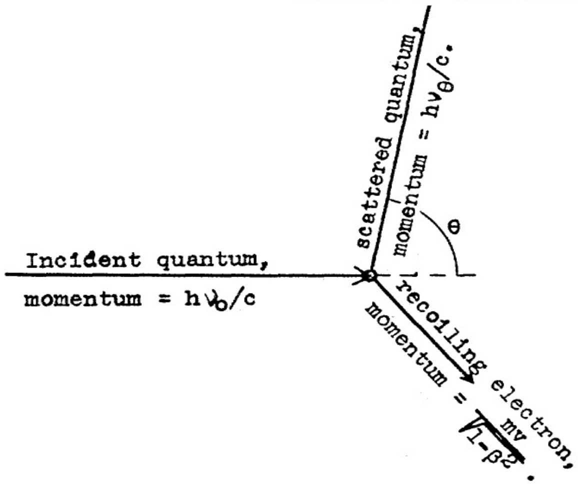


Fig. 1 A

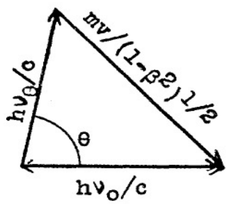
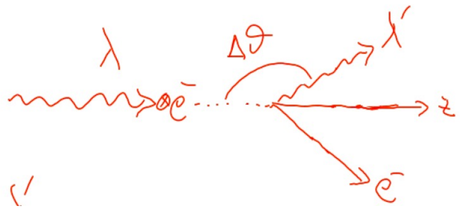


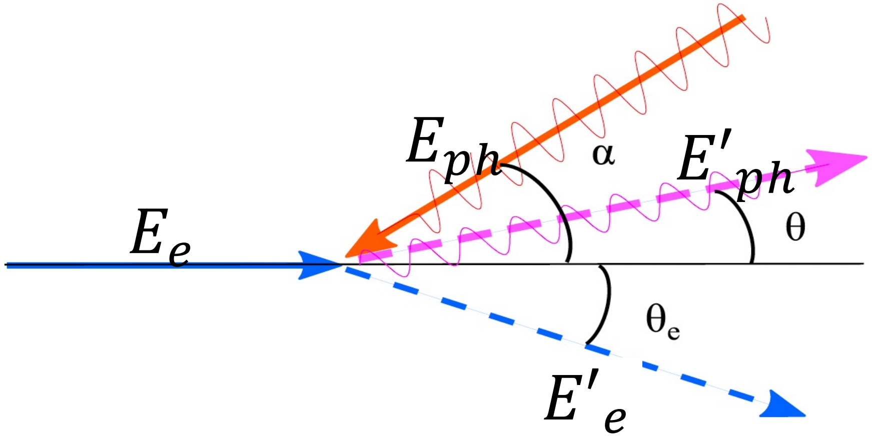
Fig. 1 B



$$E'_{ph} = \frac{E_{ph}}{1 + \frac{E_{ph}}{mc^2}(1 + \cos\vartheta)}$$

that an X-ray quantum of frequency ν_0 is scattered by an electron of mass m. The momentum of the incident ray will be $h\nu_0/c$, where c is

$$E'_{ph} = \frac{\gamma^2(1+\beta)}{\gamma^2(1-\beta\cos\theta) + \frac{X}{4}(1+\cos\theta)} E_{ph}$$



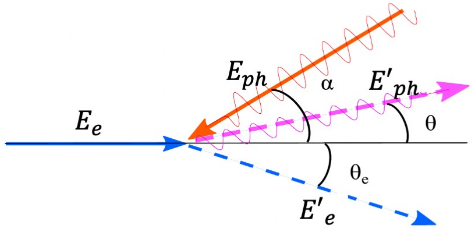
$X \equiv 4\gamma^2 E_{ph} / E_e$

$A \equiv \beta\gamma^2 - X/4 = \gamma^2(\beta - E_{ph}/E_e)$

$$E'_{ph} = \frac{4(\gamma^2 + A) + X}{4(\gamma^2 - A\cos\theta) + X} E_{ph}$$

$A=0$, *i.e.* Symmetric Compton Scattering cancels the $\gamma^2\theta^2$ correlation

N. Ranjan *et al.*, **PRAB 21**, 030701 (2018)



Considering the Compton interaction between photon pulses and counter-propagating electrons, we can derive the well-known equation for the photon energy ($E'_{\mathrm{ph}} = \hbar\omega'$, with ω' being the photon angular frequency and $\hbar$ the reduced Planck constant) scattered at an angle θ. Following the notation of Eq. 3 in Ref. [18], we can write:

α=0

$$E'_{\mathrm{ph}}(\theta) = \frac{(1+\beta)\gamma^2}{\gamma^2(1-\beta\cos\theta) + \frac{X}{4}(1+\cos\theta)} E_{\mathrm{ph}}, \quad (1)$$

where the incident photon energy is $E_{\mathrm{ph}} = \hbar\omega$, $\beta = v_e/c$ is the dimensionless electron velocity v_e (c being the speed of light), $\gamma = 1/\sqrt{1-\beta^2}$ is electron Lorentz factor and X is the electron recoil factor that introduces an important contribution at high energy of both incident photons and electrons. X has been defined in [17] (eq. 4) as:

$$X = \frac{4E_e E_{\mathrm{ph}}}{(m_0c^2)^2} = \frac{4\gamma E_{\mathrm{ph}}}{m_0c^2} = 4\gamma^2 \frac{E_{\mathrm{ph}}}{E_e}, \quad (2)$$

with m_0 the electron rest mass and $E_e = \gamma m_0 c^2$. Eq. (1) can be cast in a more schematic form as a function of the incident particle energies.

$$E'_{\mathrm{ph}} = \frac{(1+\beta)\, E_{\mathrm{ph}} E_e}{(1-\beta\cos\theta)\, E_e + (1+\cos\theta)\, E_{\mathrm{ph}}}$$

An invariant view at Compton effect - 1

(any inertial ref. frame)

Simulation of inverse Compton scattering and its implications on the scattered linewidth

Phys. Rev. AB (2018) **21**, 030701

N. Ranjan[1], B. Terzić[2,*] G. A. Krafft[2,3], V. Petrillo[4,5], I. Drebot[4], and L. Serafini[4]
[1] *Princess Anne High School, Virginia Beach, Virginia 23462, USA*
[2] *Department of Physics, Center for Accelerator Science, Old Dominion University, Norfolk, Virginia 23529, USA*
[3] *Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[4] *INFN-Milan, via Celoria 16, 20133 Milano, Italy*
[5] *Universita degli Studi di Milano, via Celoria 16, 20133 Milano, Italy*

$$X = \frac{4\gamma\hbar\omega}{mc^2}$$

$$\omega'(\theta) = \frac{\omega(1+\beta)^2\gamma^2}{\gamma^2(1-\beta\cos\theta)(1+\beta) + \frac{X}{4}(1+\cos\theta)(1+\beta)}. \quad (3)$$

A<0 photon looses energy

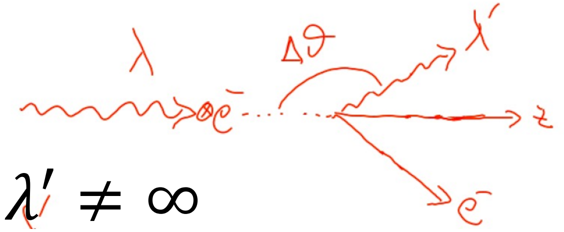
A=0

A>0 electron looses energy

Arthur Compton, Nobel Prize 1927

$\beta = 0$

$\lambda' - \lambda = \lambda_c (1+\cos\Delta\vartheta)$



$\lambda' \neq \infty$

isolated e⁻ cannot absorb a photon

S.C.S.

$$\beta\gamma^2 = \frac{X}{4}$$

$$E'_{ph} = \beta E_e = \beta\gamma mc^2$$

$$E'_{ph} \neq f(\gamma^2\vartheta^2)$$

$$\gamma \gg 1 \;;\; \gamma^2 \gg \frac{X}{4}$$

I.C.S.

$$E'_{ph} = \frac{4\gamma^2 E_{ph}}{1 + X + \gamma^2\vartheta^2}$$

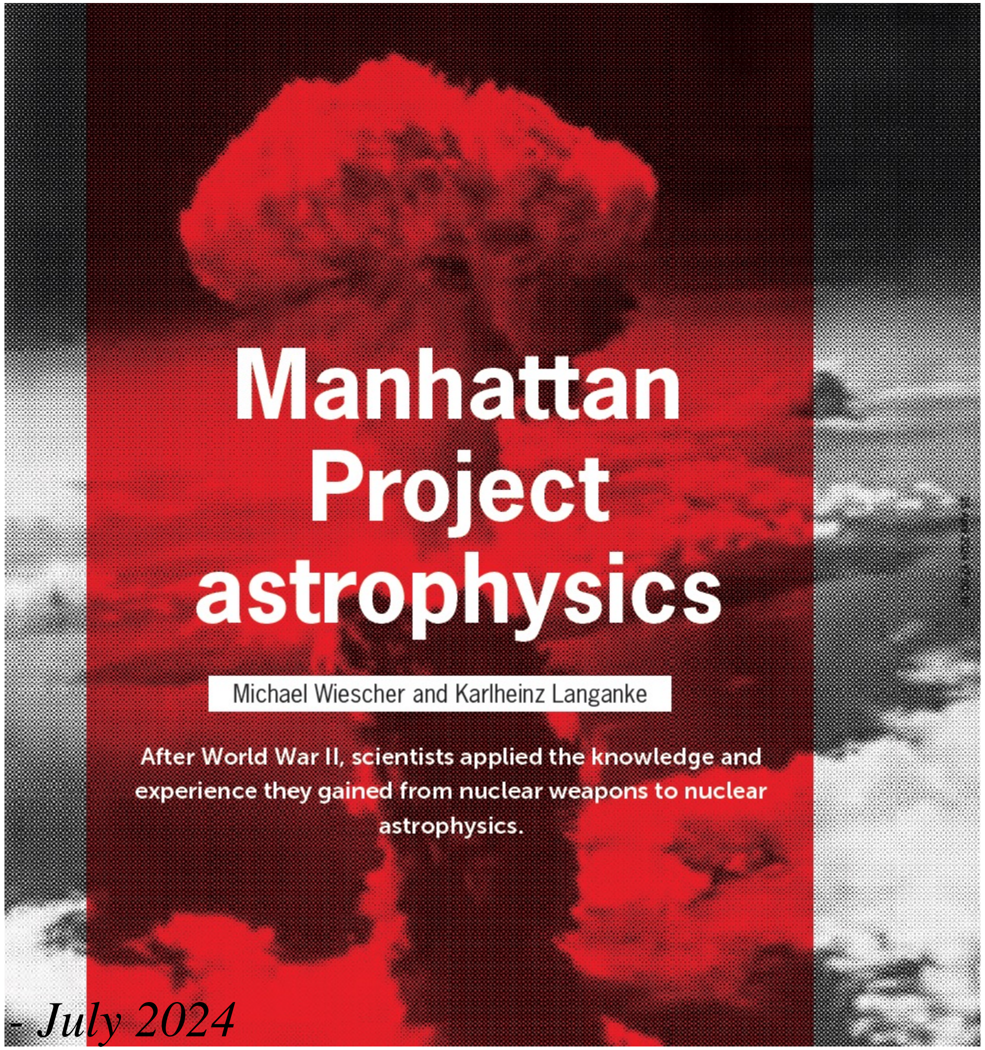
$$\lim_{X\to\infty} E'_{ph} = E_e$$

A. Compton 1923 - Direct Compton effect

First consideration and study of Inverse Compton Scattering….

During the development of the nuclear bomb!
The Manhattan Project

Will the back-scattered photons, by hot electrons of the plasma created in the initial stage of the nuclear bomb explosion, release energy from the fire-ball decreasing its temperature???



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AIP

PHYSICS TODAY

Manhattan Project astrophysics

After World War II, scientists applied the knowledge and experience they gained from nuclear weapons to nuclear astrophysics.

Michael Wiescher; Karlheinz Langanke

Check for updates

Physics Today **77** (3), 34–41 (2024);
https://doi.org/10.1063/pt.jksg.hage

of initiating nuclear fusion of the whole atmosphere!!!

an ignition could not be deemed impossible. The Trinity test took place in July 1945, and the atomic bombs were dropped on Hiroshima and Nagasaki shortly thereafter. Despite the bombs' tremendous damage, they did not set the atmosphere on fire.

Theory mitigates fear

The year after the test, Teller, his graduate student Emil Konopinski, and local technician Cloyd Marvin Jr wrote a classified Los Alamos National Laboratory report in which they summarized theoretical considerations on the possible ignition of the atmosphere by an atomic explosion.[1] The paper, declassified in 1979, argues that propagation of nuclear burning in the atmosphere is possible only if the energy gained from nuclear reactions is greater than the energy loss through the emitted gamma and beta radiation.

Konopinski, Teller, and Marvin considered the fusion of two ^{14}N nuclei as the most important energy-producing reaction, because ^{14}N is the dominant component in Earth's atmosphere. On the other hand, when compared to the stable oxygen-16 isotope, ^{14}N nuclei can easily be broken up. Therefore, the fusion of two ^{14}N atoms should lead mainly to a rearrangement of the nucleons by the nuclear force and produce a light fragment and a heavy fragment. Energetically, the most favorable result would be their breakup into alpha particles and a magnesium-24 nucleus.

Up to 17.7 MeV of kinetic energy from the reaction can be

FIGURE 2. J. ROBERT OPPENHEIMER in typical postures—at the blackboard and with a cigarette. His goal as scientific director of the Manhattan Project was to develop a nuclear device that exploded from the fission of uranium-235 and plutonium-239. (Illustration by David McMacken.)

The electron gas cools by inelastic scattering and by emitting bremsstrahlung in the form of a continuous x-ray spectrum. Because the atmosphere is transparent to that radiation, it loses energy. Konopinski, Teller, and Marvin found that the rate of gen content. Of even more concern were the tests of 20-megaton thermonuclear weapons (so-called hydrogen bombs), and scientists even considered the possibility of the fusion of ^{16}O atoms in ocean water.[2] Their explosions would increase the sudden energy release by up to three orders of magnitude. The uncertainties in the initial crude energy release and cooling calculations required experimental verification.

Experiment confirms theory

To experimentally clarify the troubling situation, a dedicated accelerator was built at Oak Ridge National Laboratory in the early 1950s, which made it possible to measure fusion cross sections for $^{14}N + ^{14}N$, $^{16}O + ^{16}O$, and other reactions of medium-heavy nuclei.[3] Alexander Zucker, one of the young scientists who was to measure the effective cross sections and who would later be director of Oak Ridge, noted that for security reasons he and other experimentalists were not directly told why there was interest in those data.

After the detonation of the Soviet 50-megaton "Tsar Bomba" in 1961 above Novaya Zemlya—a

fire-ball becomes transparent to photons, that can take energy off the fire-ball, limiting the maximum temperature down to a safe level

Teller and his coworkers. Because the atmosphere is heated only to temperatures of a few million degrees, the energies of the fusing nuclei—a few hundred kiloeletron volts—are well below the Coulomb barrier, and the likelihood of fusion is low.

The Oak Ridge fusion tests were not confined to nitrogen and oxygen nuclei; they also included tests on light isotopes such as deuterium and tritium and were meant to inform Teller's plans and ideas for developing the "Super," his label for a thermonuclear weapon based on fusion. The idea for the fusion bomb based on the fusion of deuterium and tritium

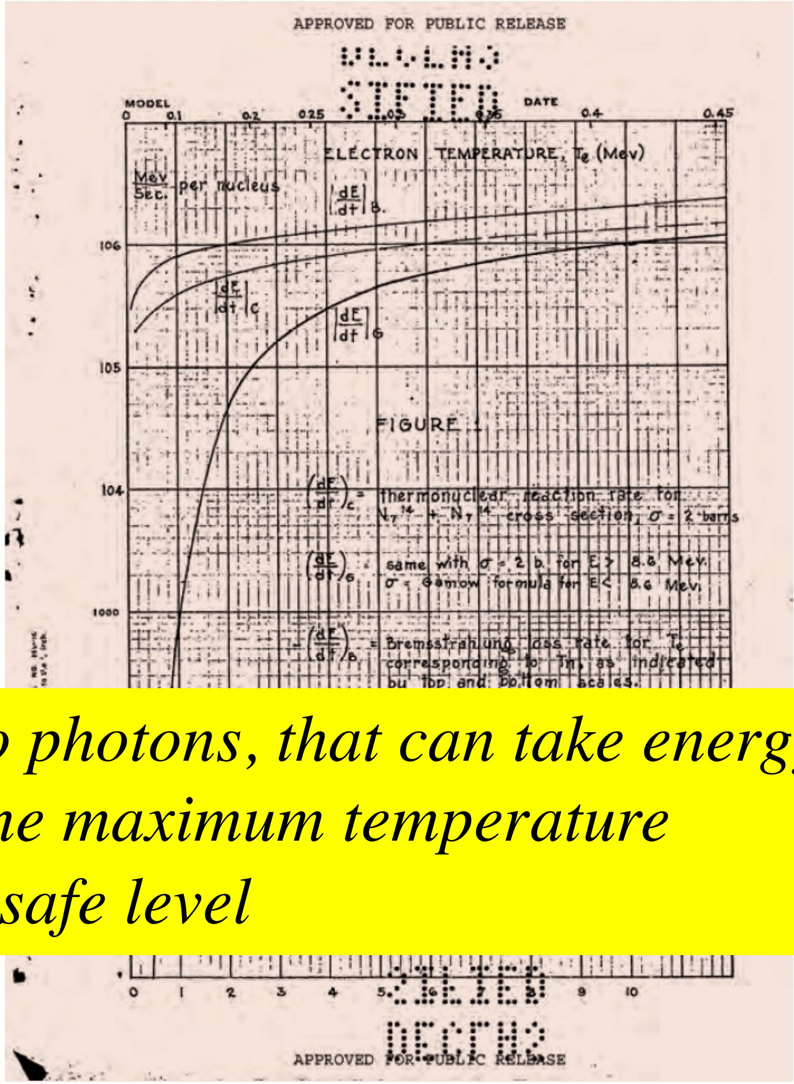

FIGURE 3. A CRITICAL PLOT of the rate of energy production as a function of temperature (in megaelectron volts), from the originally classified 1946 Los Alamos report *Ignition of the Atmosphere with Nuclear Bombs*.[1] Three curves characterize the energy-transport conditions for different temperatures in the nuclear fireball. The $(dE/dt)_C$ curve shows the reaction rate for the fusion of two nitrogen-14 nuclei when a constant cross section is assumed. The $(dE/dt)_G$ curve shows the $^{14}N + ^{14}N$ fusion reaction rate when the cross section is assumed to rapidly decrease at low energies, as predicted by George Gamow. And the $(dE/dt)_B$ curve shows the radiative energy loss through x-ray emission, as predicted by Arthur Compton. (From ref. 1.)

A. Compton 1923 - Direct Compton effect

J. Follin 1947 - Inverse Compton Scattering
*first published (non classified) study on ICS**

PROPAGATION OF COSMIC RAYS THROUGH
INTERSTELLAR SPACE

Thesis by
James Wightman Follin, Jr.

Second motivation to study ICS in the '40s was understanding why electrons are almost missing in cosmic rays bombarding the upper atmosphere

In Partial Fulfilment of the Requirements for the
Degree of Doctor of Philosophy

Both lines (nuclear bomb and astrophysics) were looking for a mechanism capable to transfer maximum energy from the electrons to the photons

California Institute of Technology
Pasadena, California
1947

* *but unknown and not credited in the whole literature on ICS*

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Publication referenced by everybody in ICS community as the first published paper on ICS

PHYSICAL REVIEW VOLUME 73, NUMBER 5 MARCH 1, 1948

Interaction of Cosmic-Ray Primaries with Sunlight and Starlight*

E. Feenberg and H. Primakoff
Washington University, St. Louis, Missouri
(Received November 20, 1947)

This paper discusses collision processes between cosmic-ray primaries (protons and electrons) and the thermal photons of sunlight and starlight. In particular, electron-positron pair production and Compton scattering in interplanetary, intragalactic, and intergalactic space are treated in detail. It is found that the number of collisions between primary particles and thermal photons in single traversals energetic scattered photons. The same statement holds for the primary protons even on an intergalactic scale. On the other hand, energetic primary electrons may experience a sufficient number of Compton collisions in intergalactic space (travel time of the order 2×10^9 years) to eliminate them effectively from the cosmic radiation reaching the neighborhood of the earth.

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* The research described in this paper was supported in part by contract N60RI-117, U.S. Navy Department.

[1] T. H. Johnson, Rev. Mod. Phys. **11**, 208 (1939); M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. **59**, 615 (1941); **59**, 930 (1941).

[2] Collisions between high energy photons, considered as cosmic-ray primaries, and thermal photons, with resultant electron-positron pair creation have been considered by G. Breit and J. A. Wheeler, Phys. Rev. **46**, 1087 (1934); 45, 134 (A) (1934). Extensive calculations similar to the present have been carried out by J. W. Follin, Bull. Am. Phys. Soc.July 11, 1947, Abstract D5. Through the courtesy of Dr. J. R. Oppenheimer, we have seen a manuscript copy of Dr. Follin's paper.

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First idea by Milburn: use electron accelerators to perform Inverse Compton Scattering in the laboratory vs. the cosmos

PHYSICAL REVIEW LETTERS

VOLUME 10 | 1 FEBRUARY 1963 | NUMBER 3

ELECTRON SCATTERING BY AN INTENSE POLARIZED PHOTON FIELD*

Richard H. Milburn
Department of Physics, Tufts University, Medford, Massachusetts

(Received 26 December 1962)

Compton scattering by starlight quanta has been postulated by Feenberg and Primakoff to be a mechanism for the energy degradation of high-energy electrons in interstellar space.[1] We shall discuss here the possibility of observing this phenomenon directly in the laboratory by scattering a multi-GeV electron beam against the intense flux of visible photons produced by a typical laser. It will be shown that using existing laser systems and electron accelerators, one may expect to obtain of the order of several thousand collimated high-energy scattered photons during each accelerator pulse, and that these quanta retain to a high degree the polarization of the original beam of optical photons.

The kinematic formulas for Compton scattering on moving electrons are given by Feenberg and Primakoff.[2] We shall consider the special case of an extreme-relativistic electron of energy $E = \gamma mc^2$, $\gamma = 1/(1-\beta^2)^{1/2} \gg 1$, incident head-on upon a beam of photons of energy $k_i = (1-3)$ eV propagating in the opposite direction. An observer moving with the incident electron will see a photon of energy $k_0 = 2\gamma k_i$. In Table I are listed for various laboratory electron energies, E, the corresponding values of k_f tabulated in terms

The approximation fails only near $x = 1$, for which $k_f = k_i$ is required. However, for large $\gamma = E/mc^2$ the bulk of the scattered photons is folded back and emerges in the laboratory in the direction of motion of the incident electron, making angles with that direction given by $\theta = 2\tan(\frac{1}{2}\theta) = (1/\gamma) \times \cot(\frac{1}{2}\theta_0)$. Thus for 1-GeV electrons, all photons having $23° < \theta_0 < 180°$ will end up within 0.0025 radian of the electron direction. We shall confine our discussion to these high-energy quanta. The

Table I. Energy, λ, polarization, and cross section for highest energy photons produced by ruby-laser photons scattered on electrons of energy E. The quantity $\sigma_{1/2}$ is the cross section for higher half of k_f spectrum.

E (GeV)	λ	$(k_f)_{max}$ (MeV)	P_{max}	$\sigma_{1/2}$ (mb)
1.02	0.014	28	1.00	320
2.92	0.040	216	1.00	310
4.16	0.057	426	0.99	300
4.60	0.063	515	0.99	290
5.11	0.070	628	0.99	290
5.48	0.075	715	0.99	290
5.84	0.080	806	0.99	280

1) R. Rabinowitz et al., Proc. Inst. Radio Engrs. (correspondence) 50 (1962) 2365.
2) A. Javan, E. A. Ballik and W. L. Bond, J. Opt. Soc. Am. 52 (1962) 96.
3) S. Jacobs and P. Rabinowitz, Proc. of the 3rd Quantum Electronics Conf., Paris, 1963 (to be published).
4) K. D. Froome and R. H. Bradsell, J. Sci. Instr. 38 (1961) 458.
5) J. Terrien, J. phys. radium 19 (1958) 390.
6) G. R. Hanes, Can. J. Phys. 37 (1959) 1283.
7) C. F. Bruce and R. M. Hill, Australian J. Phys. 14 (1961) 64; 15 (1962) 152.
8) R. M. Hill and C. F. Bruce, Australian J. Phys. 15 (1962) 194.

* * * * *

Almost at the same time as Arutyunian and co-workers

THE COMPTON EFFECT ON RELATIVISTIC ELECTRONS AND THE POSSIBILITY OF OBTAINING HIGH ENERGY BEAMS

F. R. ARUTYUNIAN and V. A. TUMANIAN
Physical Institute of the State Committee of the Council of Ministers of the USSR for the Use of Atomic Energy

Received 20 February 1963

A characteristic feature of the Compton effect on relativistic electrons is the appearance of photons with energies exceeding those of the primary photons. As a result, even when light photons are scattered on extremely relativistic electrons, the energies of the scattered photons will be of the same order of magnitude as those of the electrons. This feature may possibly be exploited for obtaining high energy γ-ray beams in electron accelerators. An important point to be mentioned is that the characteristics of such γ-beams will significantly differ from those obtained by bremsstahlung.

In the Compton effect involving moving electrons

Of course in order to obtain γ-beams by the method considered here high photon fluxes will be required. A high intensity photon source that should be feasible is the laser. At present ruby lasers seem to be the most reliable.

For ruby laser photons (λ = 6943 Å) scattered on 6 GeV electrons one gets $\omega_{2\ \mathrm{max.}}$ = 848 MeV. This effect rapidly grows with increase of the electron energy. Thus for the same ruby lasers and ϵ_1 = 40 and 500 GeV the maximal energy is correspondingly $\omega_{2\ \mathrm{max.}} \sim$ 21 and 497 GeV.

Of course if lasers emitting shorter wave lengths or other sources of high energy photons be employed, Istituto Nazionale di Fisica Nucleare

Volume 13, number 4 PHYSICS LETTERS 15 December 1964

First measured ICS – 500 MeV

COMPTON EFFECT ON MOVING ELECTRONS

O. F. KULIKOV, Y. Y. TELNOV, E. I. FILIPPOV and M. N. YAKIMENKO
Lebedev Physical Institute, Moscow University, Moscow, USSR

Received 3 November 1964

Until recent times only the Compton effect on electrons at rest has been investigated. The electron acceleration technique having being improved, there arises the possibility of investigating the scattering of photons by electrons moving with speeds near to the speed of light. New powerful sources of photon-lasers make possible the scattering of visible photons on electrons, moving in an orbit of a cyclic accelerator.

telescope tube (T) which was used while positioning the laser beam. A photomultiplier is installed beyond the telescope's ocular. The signals from the photomultiplier are proportional to the energy of the light. Gamma-quanta of scattered radiation, passing through the glass plate (G), the lens (L), the turning mirror (TM) and the collimator (C) (diameter 15 mm) cause scintillation in the crystal of NaI. This is registered by the photo-

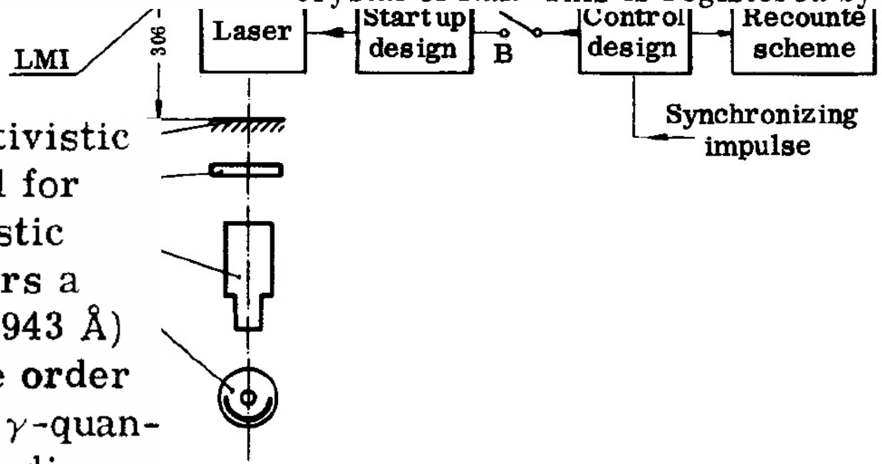


Fig. 1

The thoery of Compton's effect on relativistic electrons [1] has been considered in detail for interactions of laser photons with relativistic electrons [2 - 4]. According to these authors a head-on collision of laser radiation (γ = 6943 Å) with relativistic electrons of energy of the order of 500 MeV, will cause the appearance of γ-quanta of energy near 6.75 MeV, moving in the direction of motion of electrons.

Second measured ICS – 6 GeV

PHYSICAL REVIEW VOLUME 138, NUMBER 6B 21 JUNE 1965

High-Energy Photons from Compton Scattering of Light on 6.0-GeV Electrons*

CARLO BEMPORAD†, RICHARD H. MILBURN, AND NOBUYUKI TANAKA
Department of Physics, Tufts University, Medford, Massachusetts

AND

MIRCEA FOTINO
Cambridge Electron Accelerator, Harvard University, Cambridge, Massachusetts

(Received 28 January 1965; revised manuscript received 1 March 1965)

Compton scattering of optical photons on 6.0-GeV electrons has been observed at the Cambridge Electron Accelerator. A giant-pulsed ruby-laser burst of 0.2 J, impinging upon a 2-mA circulating electron current, was observed to yield about 8 scattered photons per pulse. These photons acquire, through a twofold Doppler shift, energies of hundred of MeV, and are expected to retain to a high degree the polarization of the laser beam. The observed yield is compatible with predictions based upon the theory of Compton scattering.

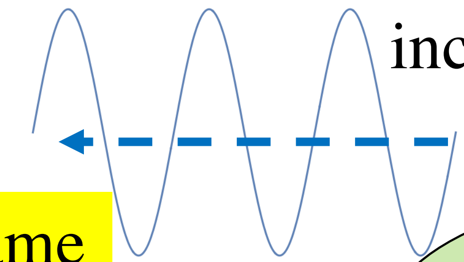
THE scattering of optical photons from a laser on extreme-relativistic electrons has been predicted[1–3] to yield a high-energy output photon beam which preserves to a high degree the polarization of the incident light beam. Photons of energy up to 0.85 GeV are expected from the interaction of 6943-Å quanta from

mercial laser cavity, a cylindrical reflector, together with and parallel to a single flash lamp.[5] The optical pumping energy was normally between 750 and 850 J. Total measured output energies were typically about 0.2 J appearing in two or three giant pulses, each about 30 nsec wide and 200–300 nsec apart. Electrical pulses de-

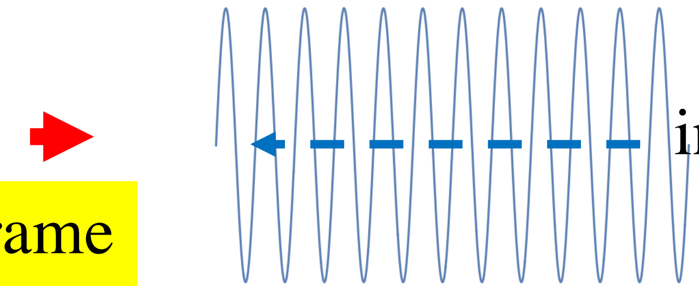
Kinematics of Inverse Compton Scattering a Cartoon

incident electron
$E_e = \gamma mc^2$

incident photon E_{ph}



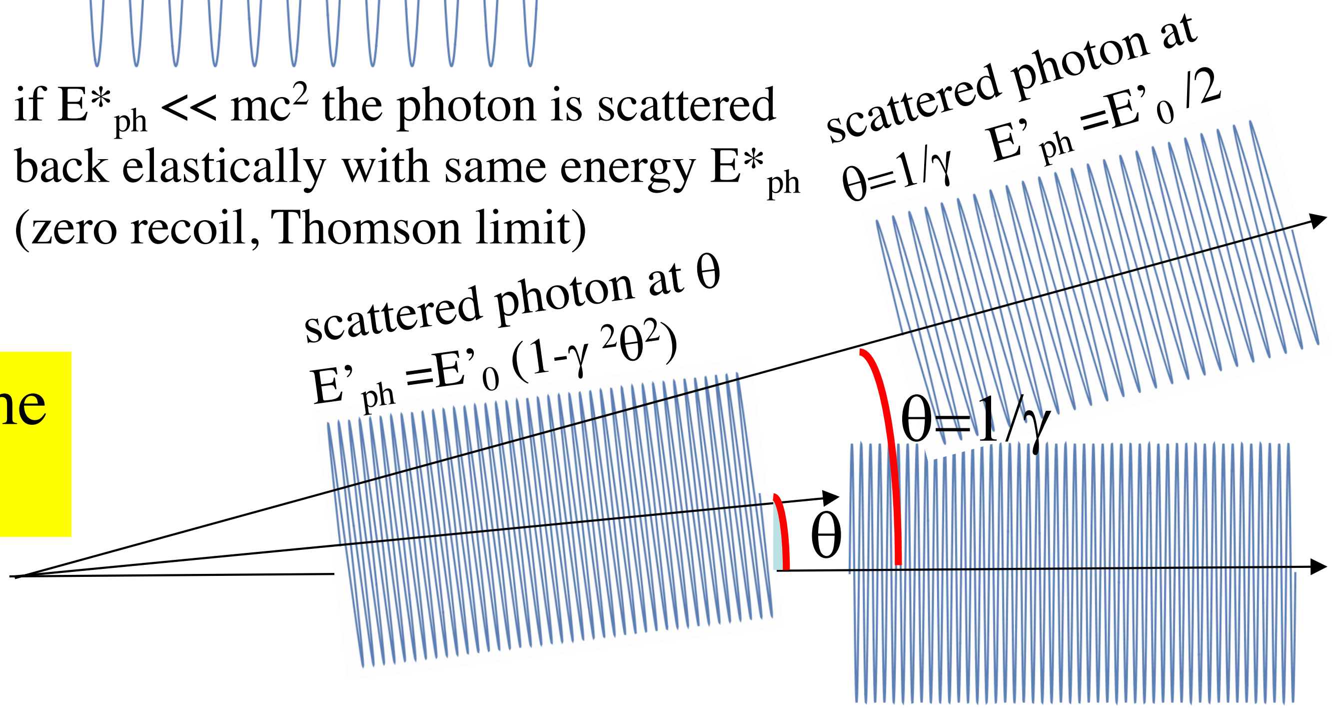
Lab Reference Frame



incident photon $E^*_{ph} = 2\gamma\, E_{ph}$

Electron rest frame
$E^*_e = mc^2$

if $E^*_{ph} << mc^2$ the photon is scattered back elastically with same energy E^*_{ph} (zero recoil, Thomson limit)



Lab Reference Frame after Scattering

scattered electron
$E'_e = \gamma mc^2 + E_{ph} - E'_{ph}$

energy of scattered photon at $\theta=0$ $E'_0 = 2\gamma\, E^*_{ph} = 4\gamma^2\, E_{ph}$

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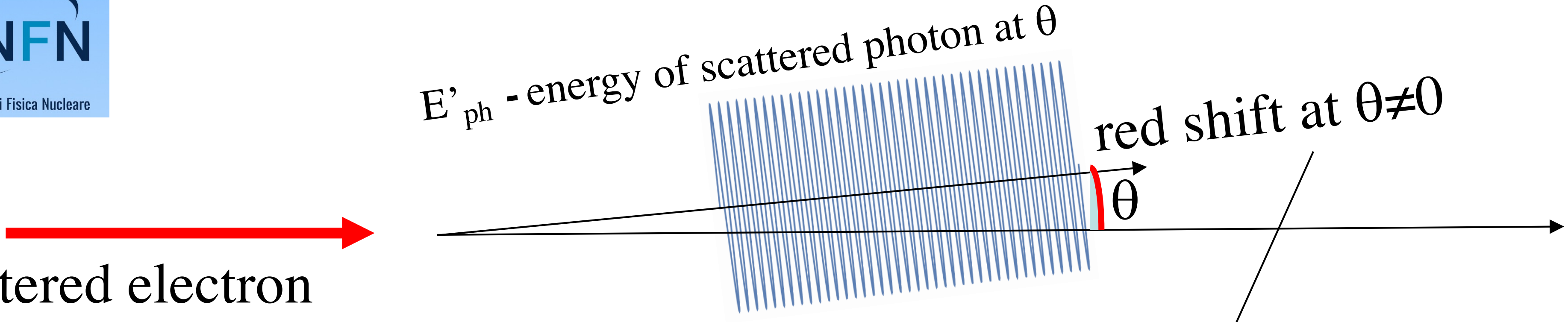


scattered electron

$E'_e = \gamma mc^2 + E_{ph} - E'_{ph}$

$$E'_{ph}(\theta) = \frac{4E_{ph}\gamma^2}{1 + X + \gamma^2\theta^2}$$

$$X = \frac{4E_e E_{ph}}{\ldots} = \frac{4\gamma E_{ph}}{\ldots} = 4\gamma^2 \frac{E_{ph}}{\ldots}$$

All I.C.S. X/γ ray Sources work at X<1 and A » 1

STAR (350 keV) $X_{STAR} < 2.6 \cdot 10^{-3}$ $A_{STAR} > 10^4$

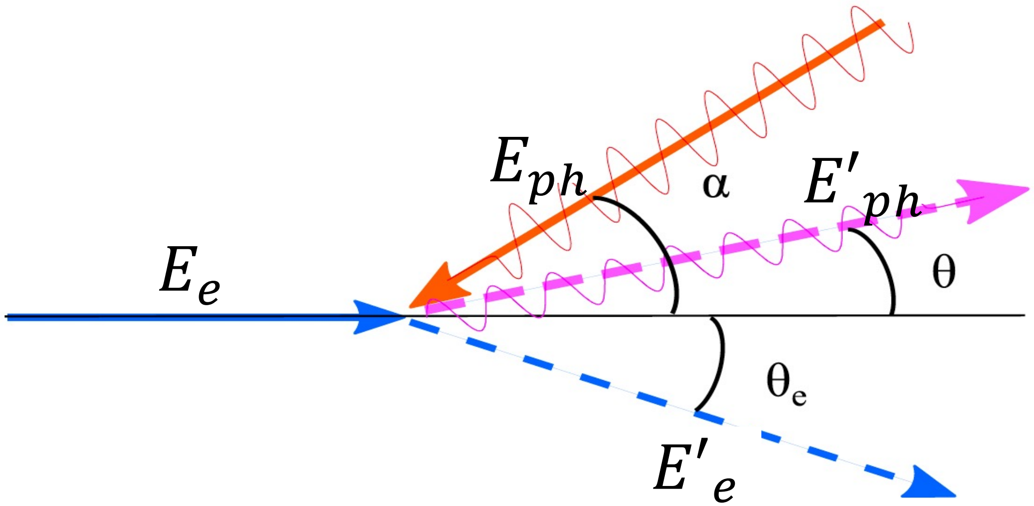
ELI-NP (20 MeV) $X_{ELI\text{-}NP} < 0.026$ $A_{ELI\text{-}NP} > 2.4 \cdot 10^5$

X = recoil … n seen

by the ele … d to mc²

The $\gamma^2\theta^2$ issue/disease

All radiation originated by a Lorentz Boost associated to relativistic emitting particles (electrons, heavy ions) is intrinsically poli-chromatic because of $\gamma\theta$ correlation (energy boost of scattered photons depends on scattering angle, at $\theta=1/\gamma$ photon energy is 50% of max photon energy at $\theta=0$) of single electron spectrum (on top of inhomogeneous effects)



$$E_e = \gamma mc^2$$

$$E_{ph} = \frac{4\gamma^2 E_l}{1 + X + \gamma^2\vartheta^2}$$

$$X \equiv \frac{4\gamma E_{ph}}{mc^2} = \frac{2E_{ph}^{ERF}}{mc^2}$$

True for all kinds of Undulatory and Collisional radiation (bremsstrahlung, wiggler/betatron, synchrotron, RRS, ICS), while resonant or amplified radiation (undulators, FELs), that are diffraction limited thanks to their beam quality, are not (or only partially) affected

How do we derive fundamental I.C.S. formula?

$$E'_{ph} = \frac{\gamma^2(1+\beta)}{\gamma^2(1-\beta\cos\theta)+\frac{X}{4}(1+\cos\theta)}E_{ph}$$

that is valid for head-on collision, where electron and photon counter-propagate along z-axis

$$\begin{cases} E_e + E_{ph} = E'_e + E'_{ph} \\ cp_e - E_{ph} = cp'_{ze} + E'_{ph}\cos\vartheta \\ 0 = cp'_{xe} + E'_{ph}\sin\vartheta \end{cases} ;$$

conservation of total momentum and total energy

$$\begin{cases} \gamma mc^2 + E_{ph} = \sqrt{c^2p'^2_{xe} + c^2p'^2_{ze} + m^2c^4} + E'_{ph} \\ mc^2\beta\gamma - E_{ph} = cp'_{ze} + E'_{ph}\cos\vartheta \\ 0 = cp'_{xe} + E'_{ph}\sin\vartheta \end{cases} ;$$

$$
\begin{cases}
\gamma mc^2 + E_{ph} = \sqrt{c^2 p'^2_{xe} + c^2 p'^2_{ze} + m^2 c^4} + E'_{ph} \\
mc^2 \beta\gamma - E_{ph} = cp'_{ze} + E'_{ph} \cos\vartheta \\
0 = cp'_{xe} + E'_{ph} \sin\vartheta
\end{cases}
\quad \vdots
$$

$$
c^2 p'^2_{xe} = E'^2_{ph} \sin^2\vartheta
$$

$$
c^2 p'^2_{ze} = \left(\beta\gamma mc^2 - E_{ph} - E'_{ph} \cos\vartheta\right)^2
$$

$$
\left(\gamma mc^2 + E_{ph} - E'_{ph}\right)^2 =
$$

$$
E'^2_{ph} \sin^2\vartheta + \left(\beta\gamma mc^2 - E_{ph} - E'_{ph} \cos\vartheta\right)^2 + m^2 c^4
$$

$$
\gamma mc^2 \left(E_{ph} - E'_{ph}\right) - E_{ph} E'_{ph} =
$$

$$
E_{ph} E'_{ph} \cos\vartheta - \beta\gamma mc^2 \left(E_{ph} + E'_{ph} \cos\vartheta\right)
$$

$$\gamma mc^2\left(E_{ph} - E'_{ph}\right) - E_{ph}E'_{ph} \; = \; E_{ph}E'_{ph}\cos\vartheta - \beta\gamma mc^2\left(E_{ph} + E'_{ph}\cos\vartheta\right)$$

$$E'_{ph}\left[\gamma mc^2(\beta\cos\vartheta - 1) - E_{ph}\,(1+\cos\vartheta)\right] = -E_{ph}\,\gamma mc^2(1+\beta)$$

$$E'_{ph} \; = \frac{\gamma(1+\beta)}{\gamma(1-\beta\cos\vartheta) + \dfrac{E_{ph}}{mc^2}(1+\cos\vartheta)} E_{ph}$$

$$X \equiv \frac{4\gamma E_{ph}}{mc^2} = \frac{4E_{ph}E_e}{(mc^2)^2}$$

valid for any value of β, γ, X, θ, E_{ph}

$$E'_{ph} \; = \frac{(1+\beta)\gamma^2}{\gamma^2(1-\beta\cos\vartheta) + \dfrac{X}{4}(1+\cos\vartheta)} E_{ph}$$

$$E'_{ph} = \frac{(1+\beta)\gamma^2}{\gamma^2(1-\beta\cos\vartheta) + \frac{X}{4}(1+\cos\vartheta)} E_{ph}$$

N.B. if $\theta = \pi$ $E'_{ph} = E_{ph}$ for any γ

$$E'_e = E_e + E_{ph} - E'_{ph}$$

Compton Scattering of X-rays on atomic electrons

$$\beta = 0\ ;\ \gamma = 1 \quad \Rightarrow \quad E'_{ph} = \frac{E_{ph}}{1 + \frac{X}{4}(1+\cos\vartheta)}$$

Inverse Compton Scattering of photons on relativistic electrons

$$\gamma \gg 1\ ;\ \beta \approx 1 - \frac{1}{2\gamma^2} \quad \Rightarrow$$

$$E'_{ph} = \frac{2\gamma^2 E_{ph}}{\gamma^2\left[1 - \cos\vartheta\left(1 - \frac{1}{2\gamma^2}\right)\right] + \frac{X}{4}(1+\cos\vartheta)}$$

$$E'_{ph} = \frac{2\gamma^2 E_{ph}}{\gamma^2\left[1 - \cos\vartheta\left(1 - \frac{1}{2\gamma^2}\right)\right] + \frac{X}{4}(1 + \cos\vartheta)}$$

$$\vartheta \ll 1 \;\; ; \quad \cos\vartheta \approx 1 - \frac{\vartheta^2}{2} \quad \Rightarrow$$

$$E'_{ph} = \frac{2\gamma^2 E_{ph}}{\gamma^2\left[\frac{\vartheta^2}{2} + \frac{1}{2\gamma^2}\right] + \frac{X}{2}}$$

$$\Downarrow$$

$$E'_{ph} = \frac{4\gamma^2 E_{ph}}{1 + \gamma^2\vartheta^2 + X}$$

A look at 4-vectors (head-on collision)

Inverse Compton

$$E_\omega \ll \gamma mc^2 \,;\; \gamma \gg 1$$

$$\overline{P_e} = \{\gamma mc, 0,0, \beta\gamma mc\}$$

$$\overline{P_\omega} = \{E_\omega/c\,, 0,0, -E_\omega/c\}$$

$$\overline{P_{tot}} \cong \left\{\gamma mc + \frac{E_\omega}{c}, 0,0, \beta\gamma mc - \frac{E_\omega}{c}\right\}$$

Symmetric Compton

$$E_\omega = p_e c = \beta\gamma mc^2$$

$$\overline{P_e} = \{\gamma mc, 0,0, \beta\gamma mc\}$$

$$\overline{P_\omega} = \{\beta\gamma mc, 0,0, -\beta\gamma mc\}$$

$$\overline{P_{tot}} = \{(1+\beta)\gamma mc, 0,0,0\}$$

$$E_{cm} \equiv c\sqrt{\overline{P_{tot}} \otimes \overline{P_{tot}}}$$

fixed target

$$E_{cm} = mc^2\sqrt{1 + \frac{(1+\beta)X}{2}} \approx$$

$$\approx mc^2\sqrt{1 + 4\gamma E_\omega/mc^2}$$

$$\gamma_{cm} \equiv \frac{E_{lab}}{E_{cm}}$$

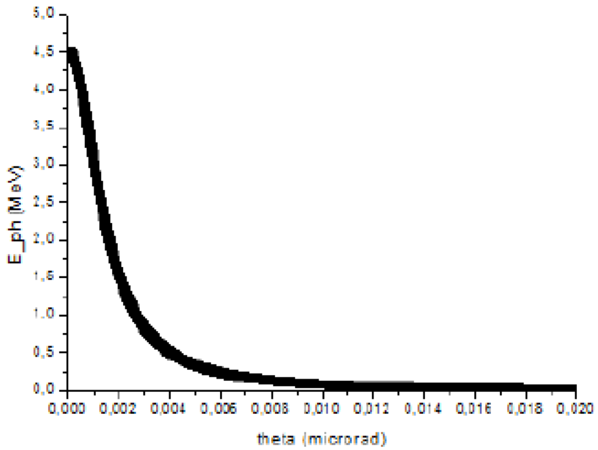
collider

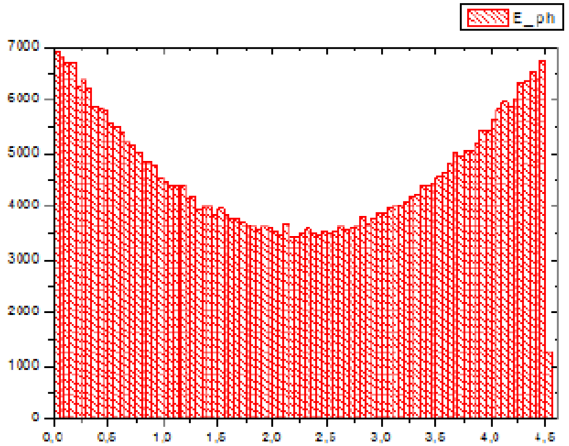
$$E_{cm} = (1+\beta)\gamma mc^2$$

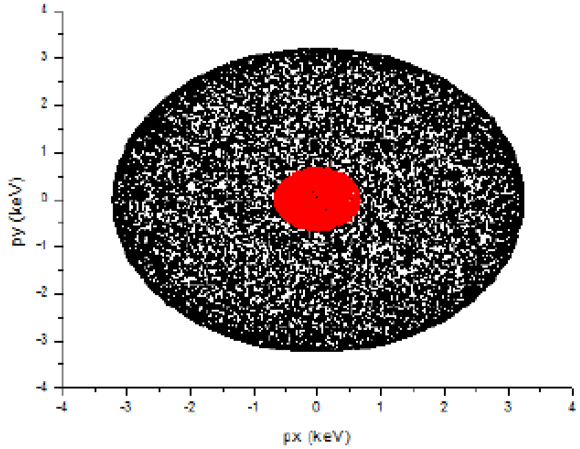
$$\gamma_{cm} \cong \gamma/\sqrt{1+X} \quad ; \quad \gamma_{cm} \xrightarrow[\gamma\to\infty]{} 1$$

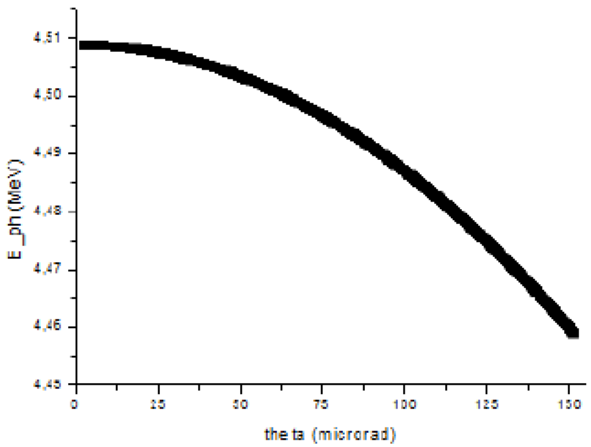
$$\gamma_{cm} = 1$$

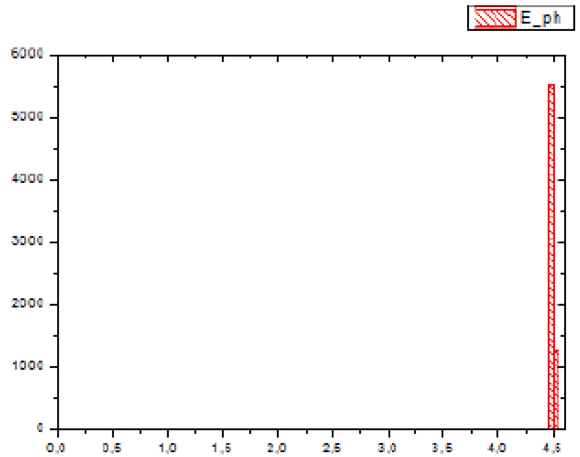
Single electron-photon spectra









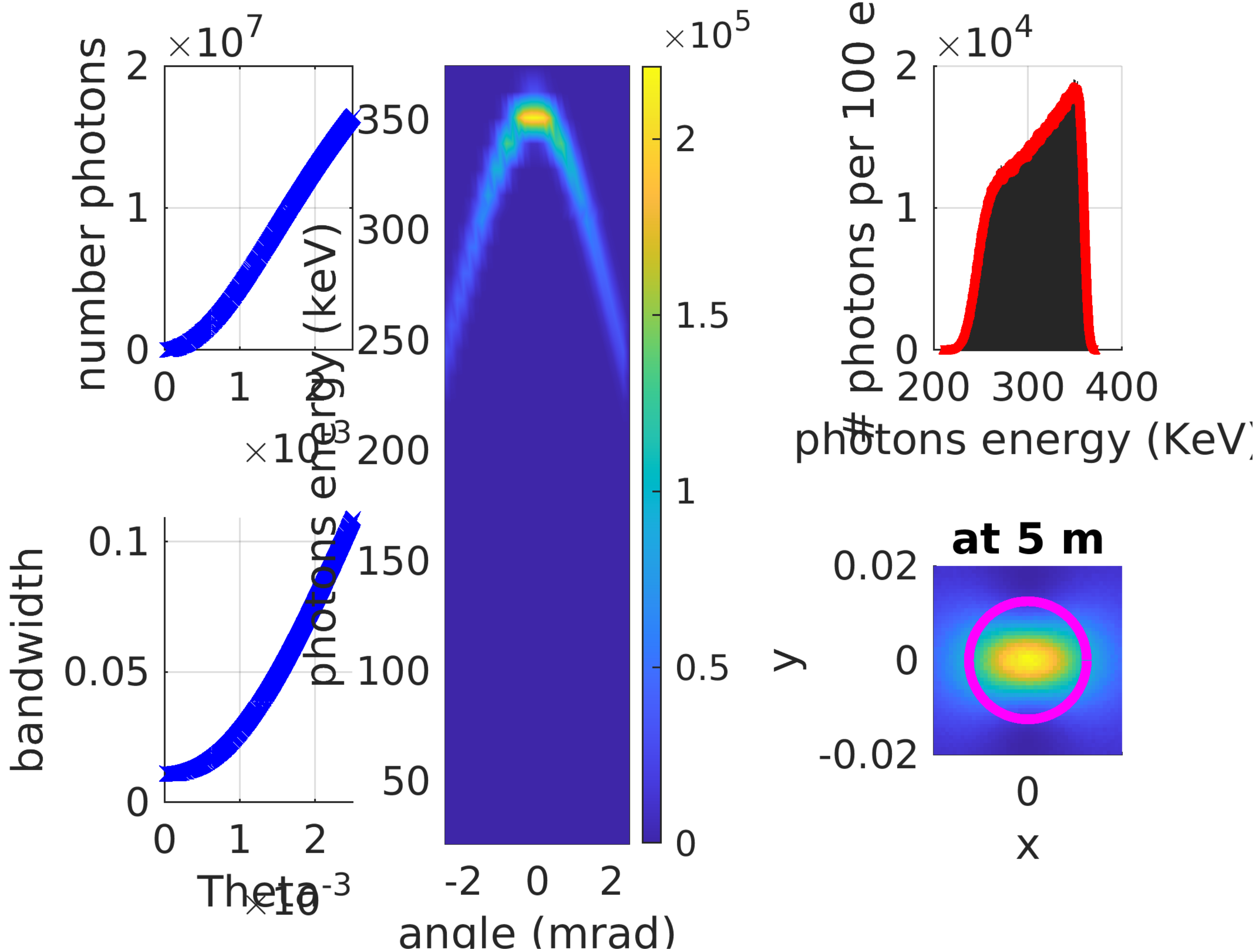


Energia elettroni 360 MeV
Energia fotoni 2.3 eV
Angolo alpha 0 gradi
Theta collimazione 151 microrad
Banda relativa 0.003

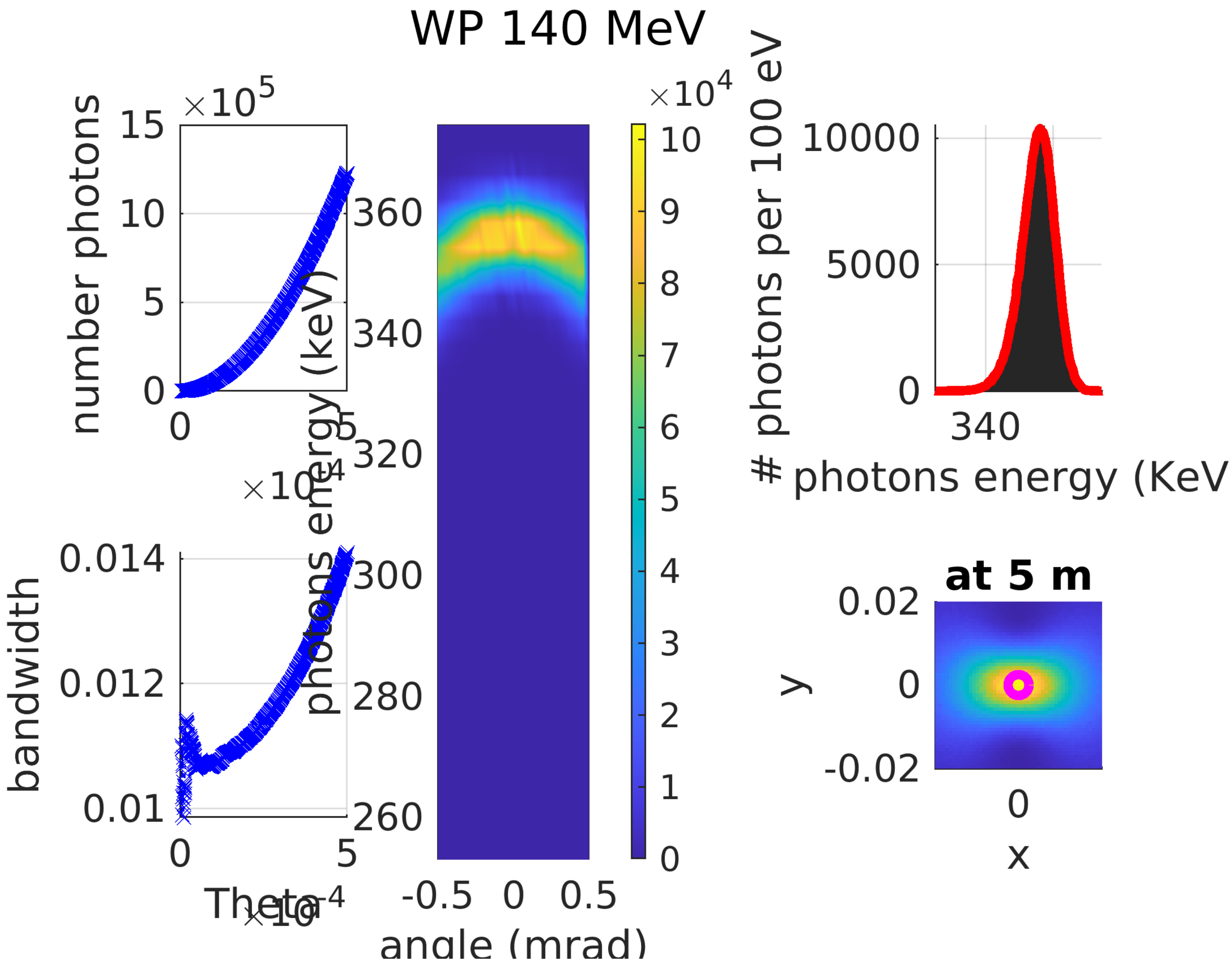
$$\gamma\vartheta = 0.11$$

What happens when we scatter beams of electron against beams of photons?

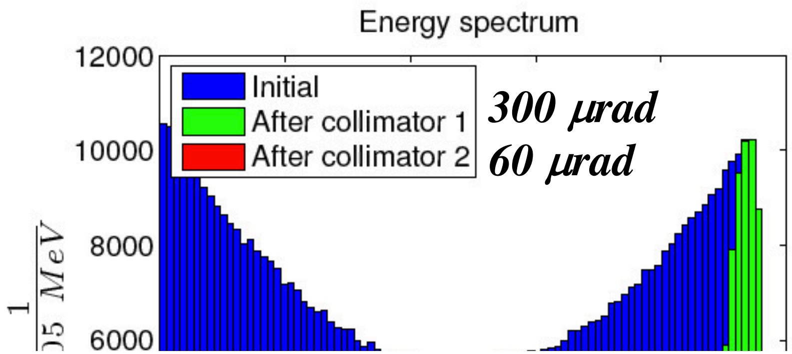
WP 140 MeV

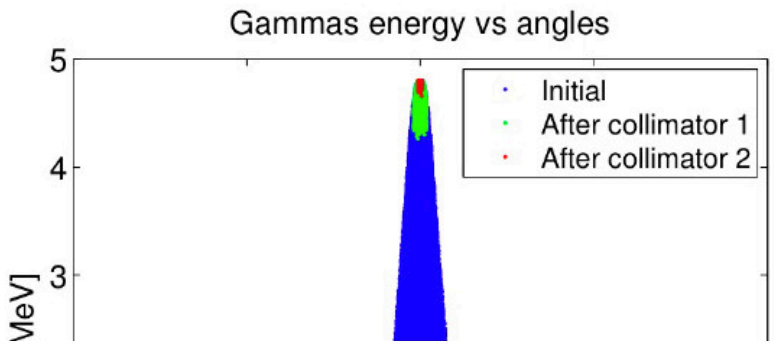


Courtesy I. Drebot



Courtesy I. Drebot





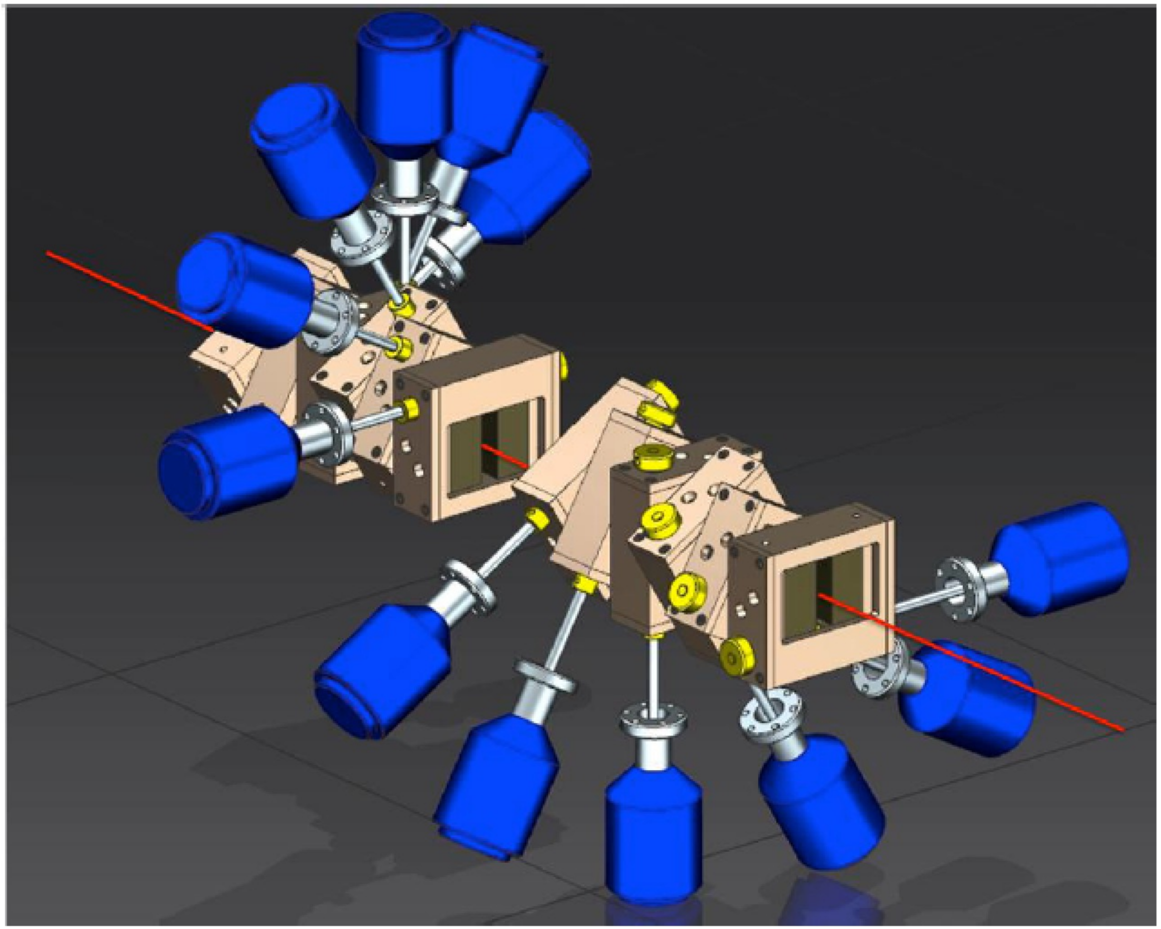
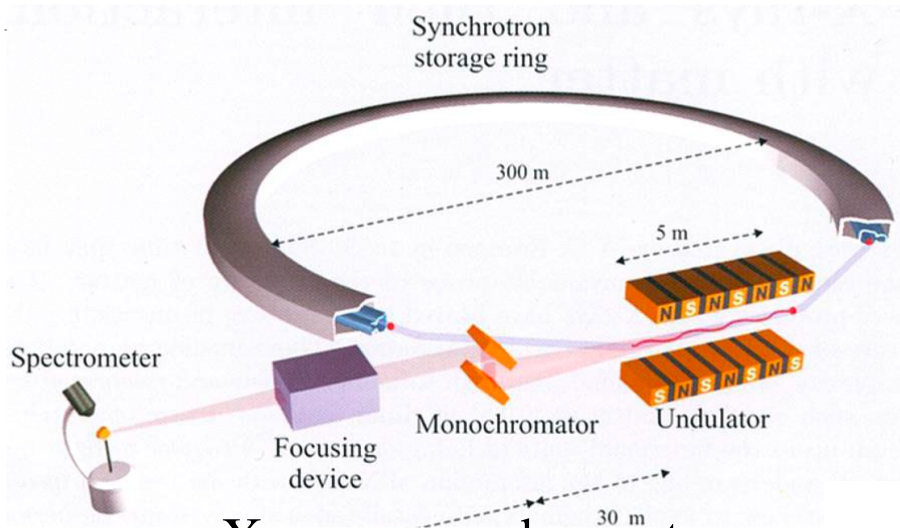


Fig. 184. Drawing of the configuration of low energy collimator made up of 12 tungsten adjustable slits with a relative 30° rotation each

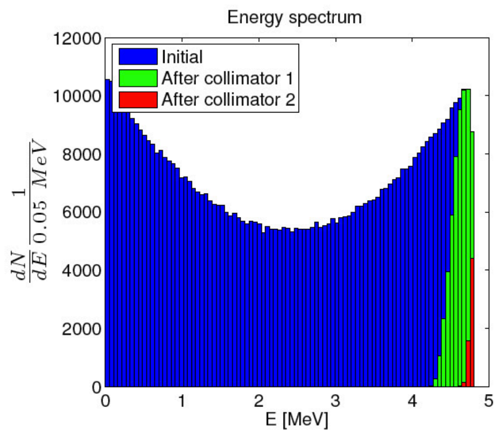
Courtesy M. Gambaccini

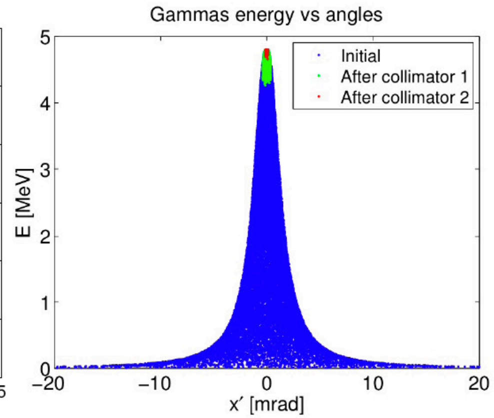
Poli-chromaticity implies using mono-chromators of different kinds (bragg-reflectors, collimators) to select a narrow bandwidth line from a broad-band spectrum



X-ray monochromator

ELI-NP-GBS γ–beam collimator (2-19 MeV)





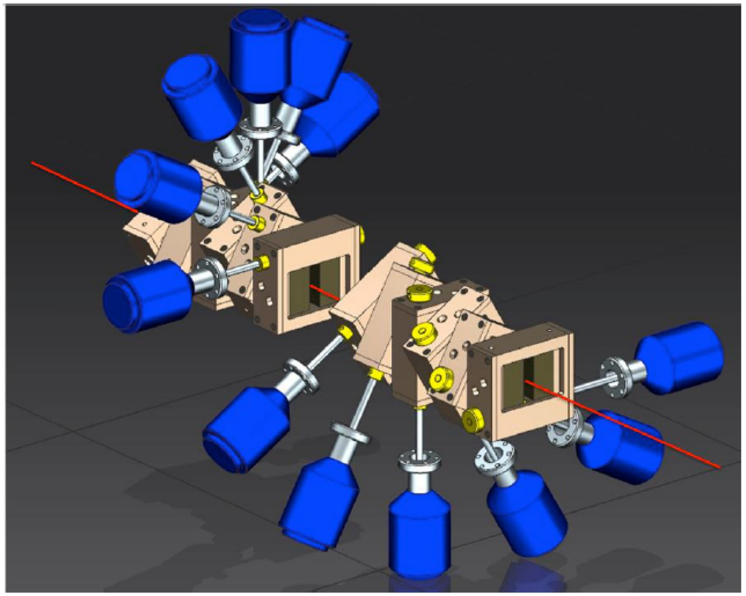


Fig. 184. **Drawing of the configuration of low energy collimator made up of 12 tungsten adjustable slits with a relative 30° rotation each**

Recalling Compton differential cross-section

$$\frac{d\sigma}{d\theta' d\phi'} = r_e^2 \left(\frac{2}{2+\Delta(1-cos\theta')}\right)^2 \left(\frac{1+cos^2\theta'}{2}\right). \quad (2.11)$$

$$\left(1+\frac{\Delta^2(1-cos\theta')^2}{2(1+cos^2\theta')(2+\Delta(1-cos\theta'))}\right) sin\theta'$$

total cross-section can be obtained from eq. (2.11) by integrating over θ' and ϕ'

$$\sigma_{tot} = 2\pi r_e^2 \frac{1}{\Delta}\left[\left(1-\frac{4}{\Delta}-\frac{8}{\Delta^2}\right)\log(1+\Delta)+\frac{1}{2}+\frac{8}{\Delta}-\frac{1}{2(1+\Delta)^2}\right] \quad (2.14)$$

and

$$\begin{cases} \lim_{\Delta\to 0} \sigma_{tot} = \frac{8\pi r_e^2}{3}(1-\Delta) = \sigma_T(1-\Delta) & \text{non-relativistic case} \\ \lim_{\Delta\to\infty} \sigma_{tot} = \frac{2\pi r_e^2}{\Delta}\left(\log\Delta+\frac{1}{2}\right) & \text{ultra-relativistic case.} \end{cases} \quad (2.15)$$

$\sigma_T = 670\ mbarn$

For example, the recoil parameter Δ associated with the head-on scattering of an electron at $E_e = 400$ MeV and a photon with $h\nu_0 = 2.4047$ eV (these energies are in LAB) is given by

$$\Delta = \frac{2h\nu_0'}{mc^2} = \frac{4\gamma_i h\nu_0}{mc^2} = 7.37\cdot 10^{-3}$$

$$E_{cm} = m_e c^2\sqrt{1+\Delta}$$

$$\Delta = \left(E_{cm}/m_e c^2\right)^2 - 1$$

We ... **ollider,**
tha ... ***nosity*,**

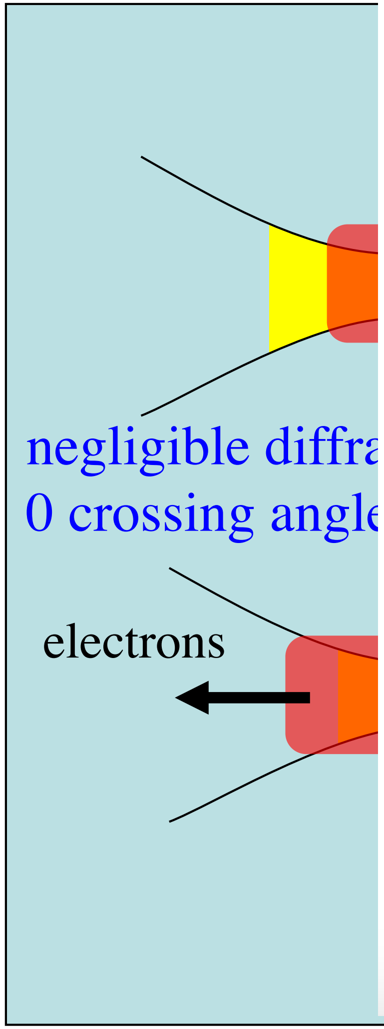
KEK-76-3

GENERAL FORMULAE OF LUMINOSITY FOR VARIOUS TYPES OF COLLIDING BEAM MACHINES

Toshio SUZUKI

JULY 1976

NATIONAL LABORATORY FOR
HIGH ENERGY PHYSICS
OHO-MACHI, TSUKUBA-GUN
IBARAKI, JAPAN



$...7\cdot 10^{-24}\,cm^2 = 0.67\ \ barn$

$= \mathbf{L}\sigma_T$ $\sigma_T = \frac{8\pi}{3} r_e^2$

...HEP collisions

...ectrons

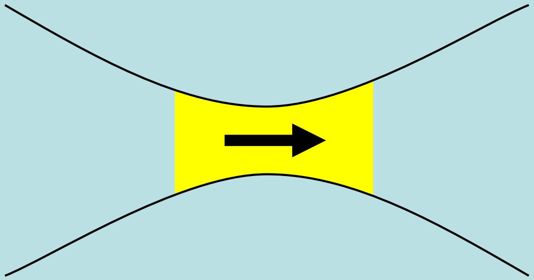
$= \frac{N_L N_{e-}}{4\pi\sigma_x^2} f$

$L_S \equiv \frac{L}{\Delta\nu_\gamma}$

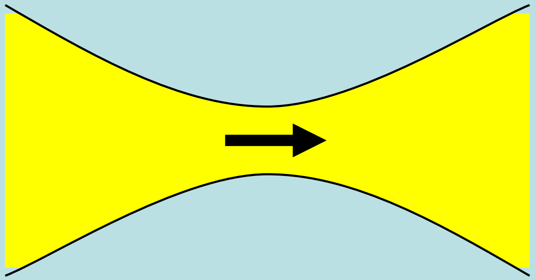
$...^{1}) = 2.5\cdot 10^{35}\,cm^{-2}s^{-1}$

cf. LHC 10..., Hi-Lumi LHC 10^{35}

Matching Laser Pulse Length and Focus Size



Laser pulse length matched to focus size (Rayleigh length).



Laser pulse too long for small focus size.

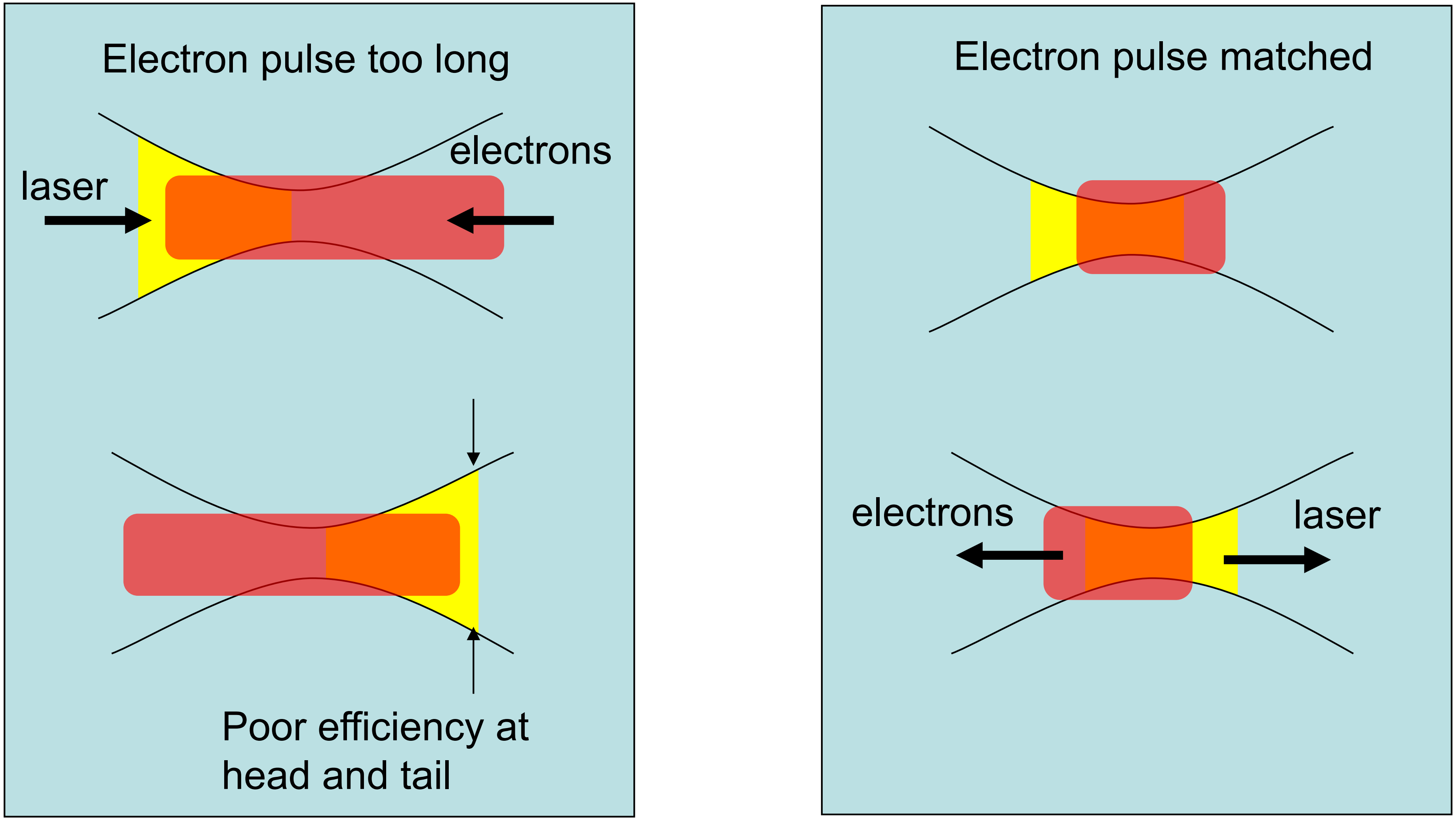
Laser pulse must be short compared to Rayleigh length so that whole pulse is focused simultaneously.

Laser may be shorter than Rayleigh length, but less than 0.5 ps is not practical, and could lead to non-linear effects not included in our spectral model.

courtesy of D. Moncton

$$L = \frac{N_{el} N_{las}}{2\pi\left(\sigma_0^2 + w_0^2/4\right)} f$$

Electron Bunch Length Matched to Rayleigh Length



courtesy of D. Moncton

Bandwidth due to collection angle, laser and electron beam phase space distribution

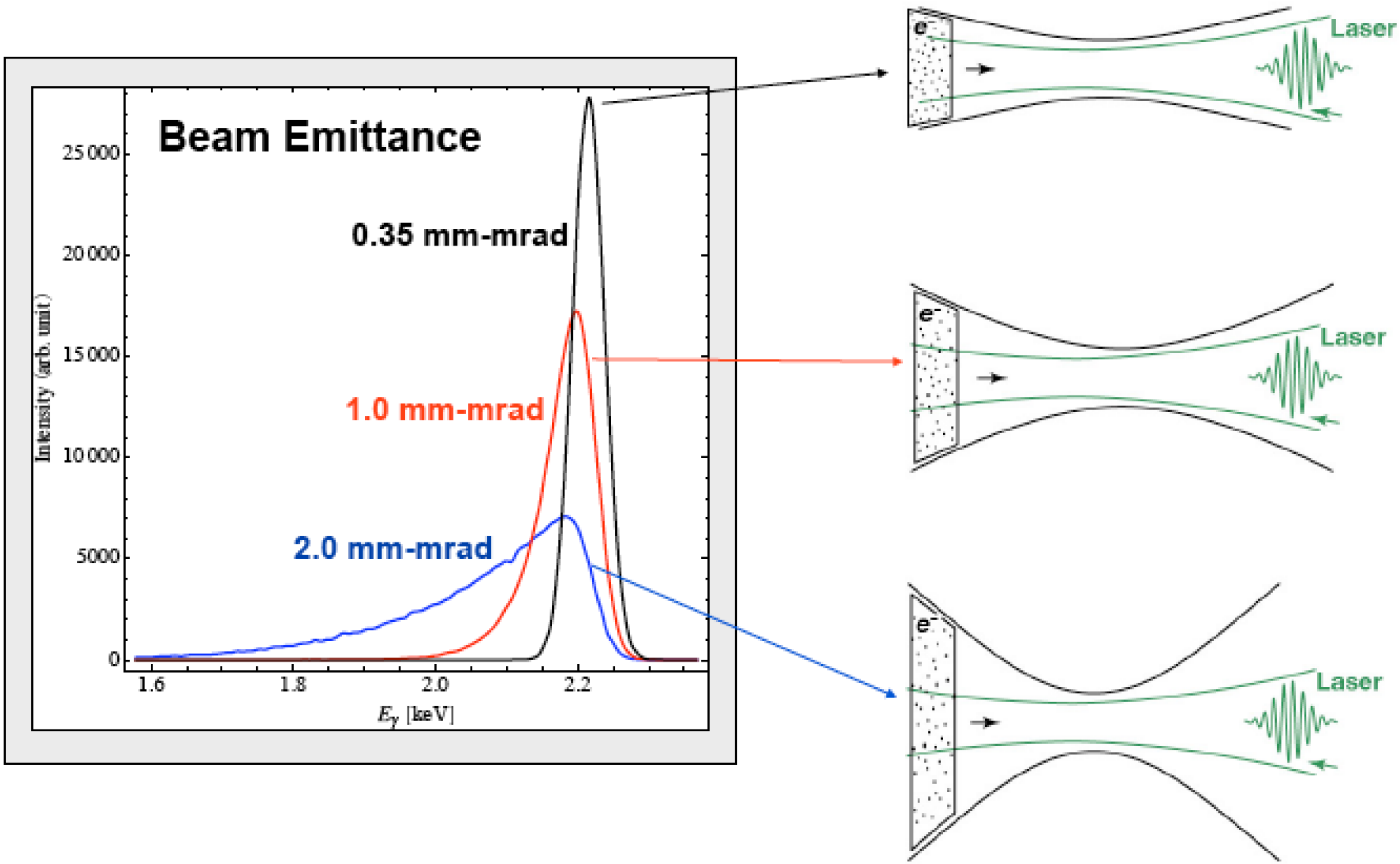
$$\nu_X = \frac{4\gamma^2\nu_L}{1+\Delta}\left(1-\frac{\gamma^2\vartheta^2}{1+\Delta}-\frac{a_0^2}{2}\right) \qquad \Delta = 4\gamma h\nu/mc^2$$

$$\left.\frac{\delta\nu_X}{\nu_X}\right|_{\nu_L} = \frac{\partial\nu_X}{\partial\nu_L}\frac{\nu_L}{\nu_X}\frac{\delta\nu_L}{\nu_L} \;;\; \left.\frac{\delta\nu_X}{\nu_X}\right|_{\gamma} = \frac{\partial\nu_X}{\partial\gamma}\frac{\gamma}{\nu_X}\frac{\delta\gamma}{\gamma} \;;\; \left.\frac{\delta\nu_X}{\nu_X}\right|_{\vartheta} = \frac{1}{2}\frac{\partial^2\nu_X}{\partial\vartheta^2}\frac{\delta\vartheta^2}{\nu_X} \quad etc$$

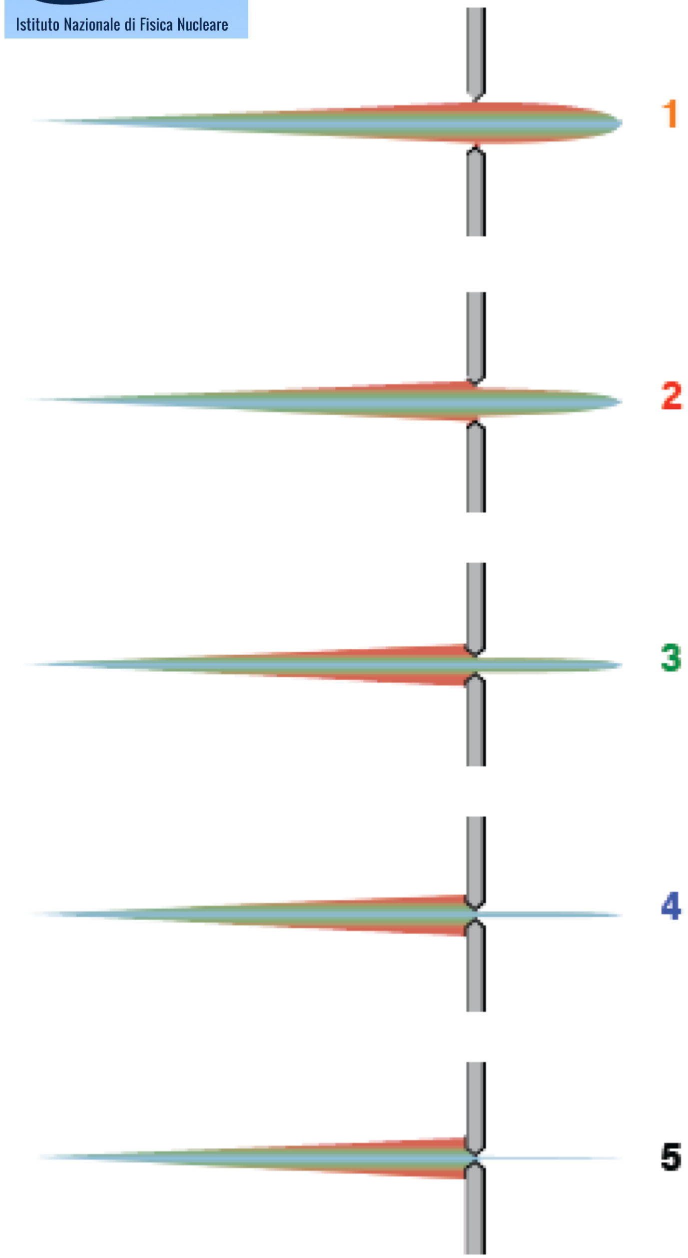
angular spread due to scattering angle and angular spread due to single electron incoming angle (emittance) are treated symmetrically

$$\left\langle\gamma^2\theta^2\right\rangle \cong \left\langle\gamma^2\vartheta^2\right\rangle + \left\langle\gamma^2\vartheta_e^2\right\rangle \cong \gamma^2\vartheta_{rms}^2 + (\sigma_{p\perp}/mc)^2 \cong \gamma^2\vartheta_{rms}^2 + 2(\varepsilon_n/\sigma_x)^2$$

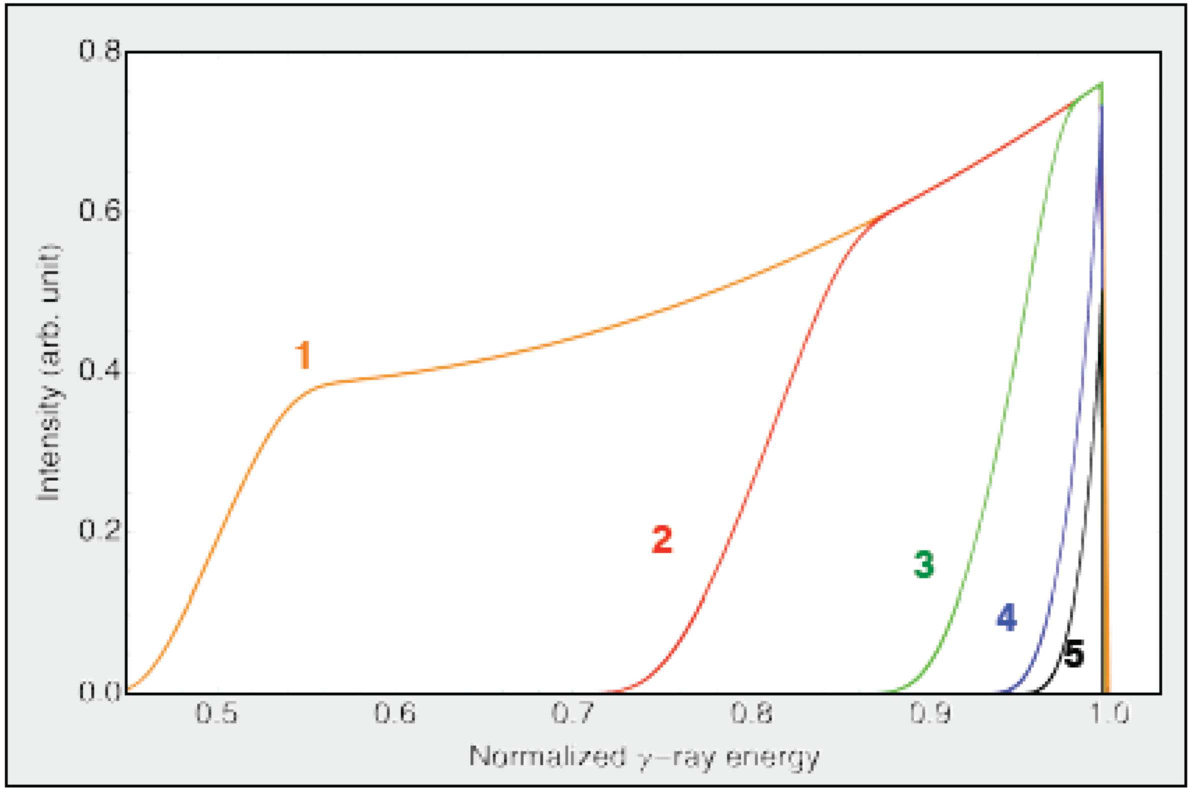
$$\frac{\delta\nu_X}{\nu_X} = \sqrt{\left(\left.\frac{\delta\nu_X}{\nu_X}\right|_{\nu_L}\right)^2 + \left(\left.\frac{\delta\nu_X}{\nu_X}\right|_{\gamma}\right)^2 + \left(\left.\frac{\delta\nu_X}{\nu_X}\right|_{\vartheta}\right)^2 + \ldots\ldots}$$



Courtesy C. Barty - LLNL



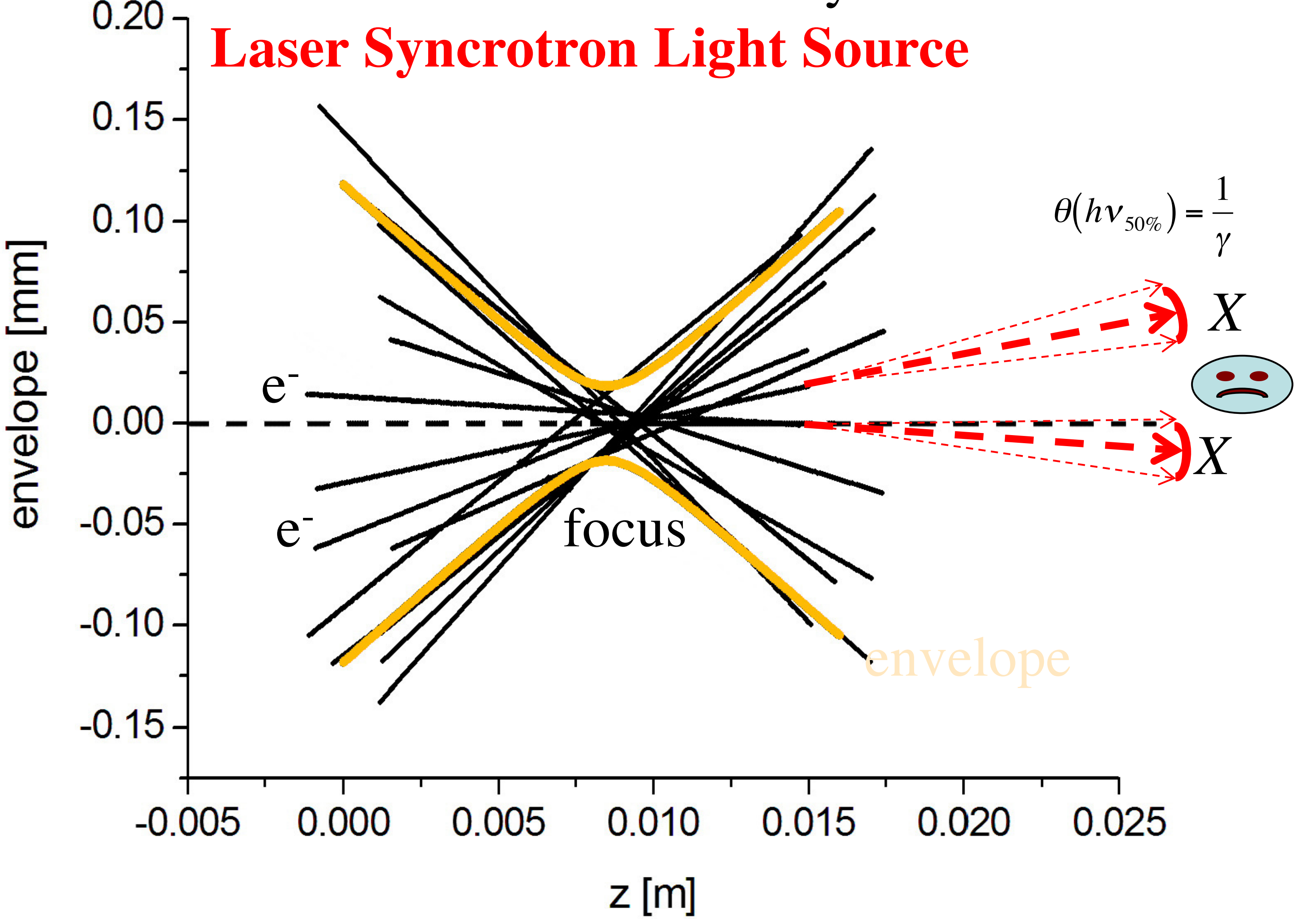
$$\Delta\theta \approx \frac{1}{\gamma} \ ; \ \Delta\nu_\gamma \approx 50\%$$



$$\Delta\theta \approx \sigma_{x'} = \frac{\varepsilon_n}{\gamma\sigma_x} \ ; \ \Delta\nu_\gamma \approx \left(\frac{\varepsilon_n^2}{\sigma_x^2}\right)$$

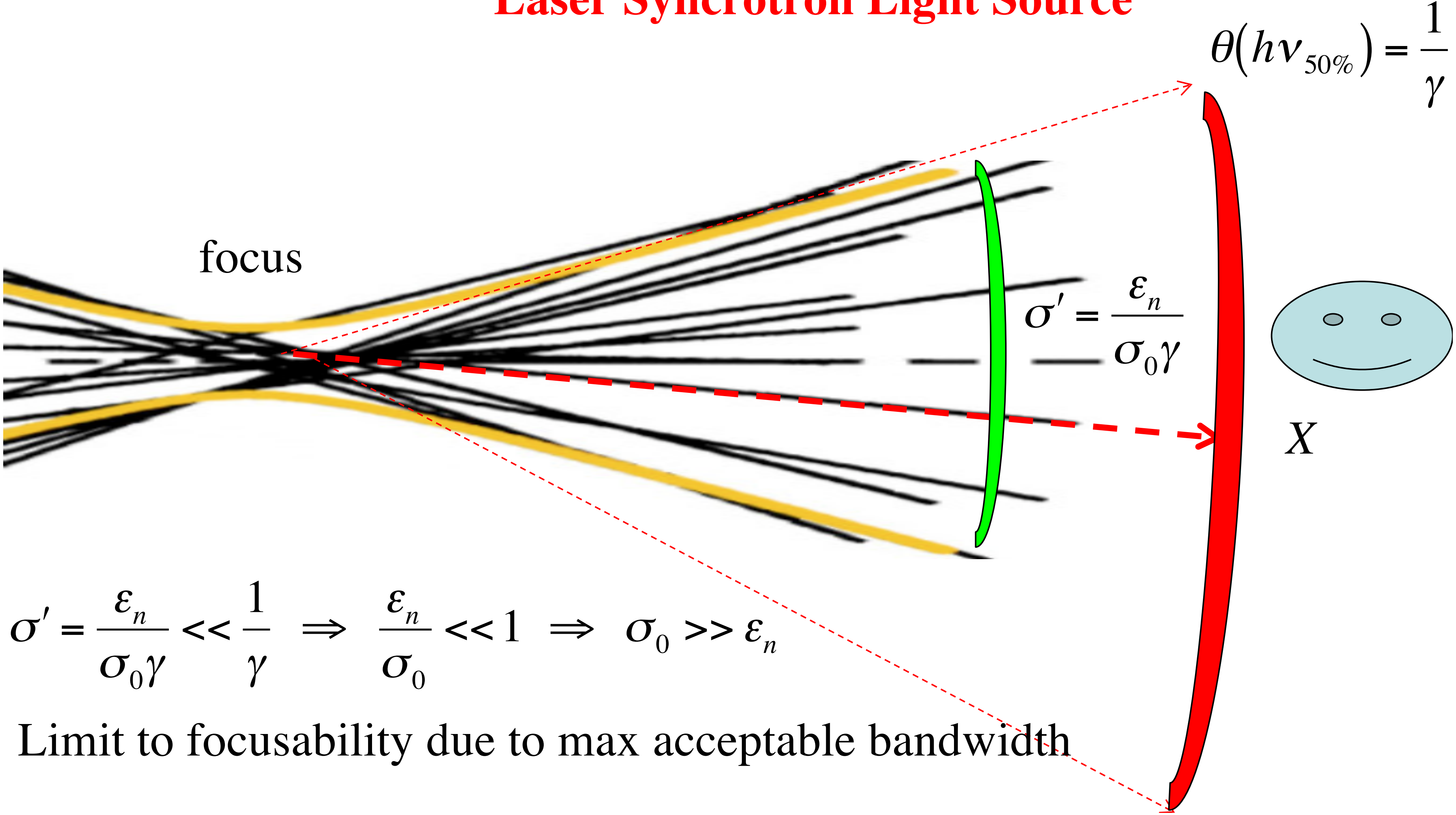
Courtesy C. Barty - LLNL

Spectral broadening due to ultra-focused beams:
Thomson Source classically described as a
Laser Syncrotron Light Source



Scattering angle in Thomson limit (no recoil) is small, i.e. < $1/\gamma$

Spectral broadening due to ultra-focused beams:
Thomson Source classically described as a
Laser Syncrotron Light Source



$$\sigma' = \frac{\varepsilon_n}{\sigma_0 \gamma} << \frac{1}{\gamma} \Rightarrow \frac{\varepsilon_n}{\sigma_0} << 1 \Rightarrow \sigma_0 >> \varepsilon_n$$

Limit to focusability due to max acceptable bandwidth

Petrillo-Serafini Formula* for ICS photon beam bandwidth

collimation

electron beam phase space

$$\frac{\delta\nu_\gamma}{\nu_\gamma} \cong \sqrt{\left[\left(\frac{\gamma^2\vartheta^2/\sqrt{12}}{1+\gamma^2\vartheta^2/2} + \frac{2\varepsilon_n^2/\sigma_x^2}{1+\sqrt{12}\,\varepsilon_n^2/\sigma_x^2}\right)\frac{1}{1+\Delta}\right]^2 + \left(\frac{2+\Delta}{1+\Delta}\frac{\delta\gamma}{\gamma}\right)^2 + \left(\frac{1}{1+\Delta}\frac{\delta\nu}{\nu}\right)^2 + \left(\frac{M^2\lambda_L}{2\pi w_0}\right)^4 + \left(\frac{a_0^2/3}{1+a_0^2/2}\right)^2}$$

laser beam phase space

laser collective effects

PHYSICAL REVIEW ACCELERATORS AND BEAMS **20,** 080701 (2017)

Analytical description of photon beam phase spaces in inverse Compton scattering sources

C. Curatolo,[1,*] I. Drebot,[1] V. Petrillo,[1,2] and L. Serafini[1]
[1]*INFN-Milan, via Celoria 16, 20133 Milano, Italy*
[2]*Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy*

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with the hope that an ICS compact hard X-ray source will start being developed somewhere in Africa by the next African School of Physics, in the context of the African Light Source pan-african initiative

INFN
Istituto Nazionale di Fisica Nucleare

Grazie per l'attenzione
Merci beaucoup
Shukrân - شكرا
ⵜⴰⵏⵎⵎⵉⵔⵜ

An invariant view at Compton effect - 1

(any inertial ref. frame)

Simulation of inverse Compton scattering and its implications on the scattered linewidth

Phys. Rev. AB (2018) **21**, 030701

N. Ranjan[1], B. Terzić[2,*] G. A. Krafft[2,3], V. Petrillo[4,5], I. Drebot[4], and L. Serafini[4]
[1] *Princess Anne High School, Virginia Beach, Virginia 23462, USA*
[2] *Department of Physics, Center for Accelerator Science, Old Dominion University, Norfolk, Virginia 23529, USA*
[3] *Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[4] *INFN-Milan, via Celoria 16, 20133 Milano, Italy*
[5] *Universita degli Studi di Milano, via Celoria 16, 20133 Milano, Italy*

$$X = \frac{4\gamma\hbar\omega}{mc^2}$$

$$\omega'(\theta) = \frac{\omega(1+\beta)^2\gamma^2}{\gamma^2(1-\beta\cos\theta)(1+\beta) + \frac{X}{4}(1+\cos\theta)(1+\beta)}. \quad (3)$$

Arthur Compton, Nobel Prize 1927

$\beta = 0$

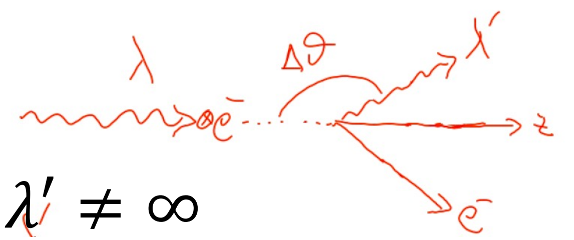
$A<0$ photon looses energy

$A=0$ → **S.C.S.**

$A>0$ electron looses energy → **I.C.S.**

$\gamma \gg 1\ ;\ \gamma^2 \gg \frac{X}{4}$

$\lambda' - \lambda = \lambda_c(1+\cos\Delta\vartheta)$



$\lambda' \neq \infty$

isolated e⁻ cannot absorb a photon

S.C.S.

$$\beta\gamma^2 = \frac{X}{4}$$

$$E'_{ph} = \beta E_e = \beta\gamma mc^2$$

$$E'_{ph} \neq f(\gamma^2\vartheta^2)$$

I.C.S.

$$E'_{ph} = \frac{4\gamma^2 E_{ph}}{1 + X + \gamma^2\vartheta^2}$$

$$\lim_{X\to\infty} E'_{ph} = E_e$$

INFN Istituto Nazionale di Fisica Nucleare

An invariant view at Compton effect - 2

(any inertial ref. frame)

$$X = \frac{4\gamma\hbar\omega}{mc^2}$$

$$\omega'(\theta) = \frac{\omega(1+\beta)^2\gamma^2}{\gamma^2(1-\beta\cos\theta)(1+\beta) + \frac{X}{4}(1+\cos\theta)(1+\beta)}. \quad (3)$$

A<0 — photon looses energy

A=0

A>0 — electron looses energy

Direct Compton

$$\beta = 0\,;\ \gamma = 1$$

$$\nu' = \frac{\nu}{1 + \frac{X_{DC}}{4}(1+\cos\vartheta)}$$

$$X_{DC} = \frac{4\hbar\omega}{mc^2} = \frac{4\lambda_C}{\lambda}$$

S.C.S.

$$\overrightarrow{p_e} = -\overrightarrow{p_{ph}}$$

$$X = 4\beta\,\gamma^2$$

$$E_{ph} = p_e c = \beta\gamma mc^2$$

$$E'_{ph} = E_{ph}\ \forall\vartheta$$

$$E'_e = E_e = \gamma mc^2\ \forall\vartheta$$

$$\gamma_{cm} = 1$$

CM ref. frame = Lab ref. frame

I.C.S.

$$E'_{ph} = \frac{4\gamma^2 E_{ph}}{1 + X + \gamma^2\vartheta^2}$$

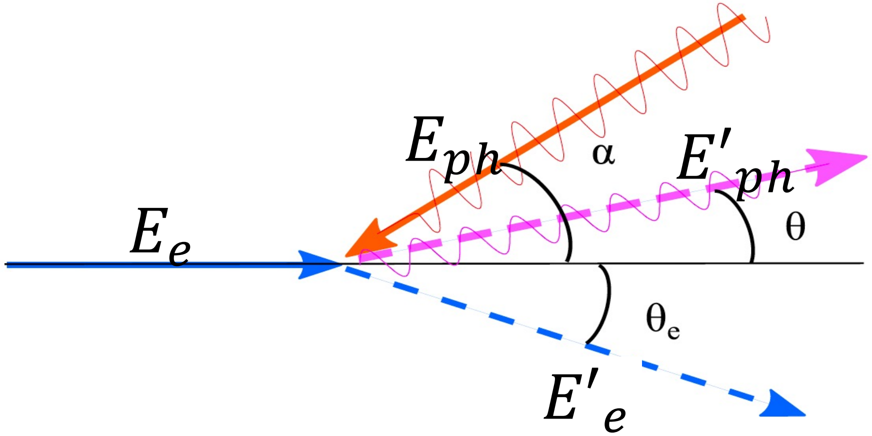
$$\gamma \to \infty$$

$$X \to \infty$$

$$E'_{ph} \to E_e$$

$$E_{e'} \to mc^2$$

Let's consider the condition of maximum energy/momentum transfer between electron and photon, $i.e.\ \theta = 0$



$$E'_{ph} = \frac{\gamma^2(1+\beta)}{\gamma^2(1-\beta\cos\theta) + \frac{X}{4}(1+\cos\theta)} E_{ph}$$

$\theta = 0$

$$E'_{ph} = \frac{2\gamma^2}{\gamma^2(1-\beta) + \frac{X}{2}} E_{ph}$$

$\theta = 0$ corresponds to:
maximum energy of back-scattered photon E'_{ph-max}
and
minimum energy of electron after scattering E'_{e-min}

$$E'_{ph-max} = \frac{4\gamma^2 E_{ph}}{1+X}$$

Thomson limit: X << 1

$$E'_{ph-max} = 4\gamma^2 E_{ph}$$

Deep recoil Compton: X >> 1

$$E'_{ph-max} \sim \left(1 - \frac{1}{X}\right) E_e$$

$$E_{TOT} = E_e + E_{ph} = E'_{e-min} + E'_{ph-max}$$

$$E'_{e-min} = E_e + E_{ph} - E'_{ph-max} = E_e + E_{ph} - \frac{4\gamma^2 E_{ph}}{1+X}$$

All quantities normalized to the total energy $E_{tot} = E_e + E_{ph}$

E_e = 100 MeV

$$\frac{E'_{ph}}{E_{tot}} = \frac{X}{(1+X)(1+\frac{X}{4\gamma^2})}$$

$$\frac{E'_e}{E_{tot}} = 1 - \frac{X}{(1+X)(1+\frac{X}{4\gamma^2})}$$

Energy Budget towards γ-rays with high spectral density

- **25 GeV** electrons would be needed to generate
  2 MeV photons via ***synchrotron radiation***
  (highest spectral density S 10^{12} $s^{-1}eV^{-1}$, very small bdw 10^{-4})

- **850 MeV** electrons were used to ***Channeling Radiate***
  2 MeV γ-rays (high S 10^5-10^6 $s^{-1}eV^{-1}$, broad bdw 10 %)

- **350 MeV** e^-s are needed to ***Inverse Compton Scatter***
  2 MeV γ-rays (good S 10^4-10^5 $s^{-1}eV^{-1}$, small bdw 10^{-3})

- **3.5 MeV** electrons to ***bremsstrahlung***
  2 MeV γ-rays (poor S 1 $s^{-1}eV^{-1}$, very broad bandwidth)

- **2 MeV** e^-s to ***Symmetric Compton Scatter*** a photon target
  2 MeV γ-ray photons (S 10^4 $s^{-1}eV^{-1}$) **spectral purification!**

when recoil X is large electron swaps with photon,
maximum energy loss by the electron in favour to the photon

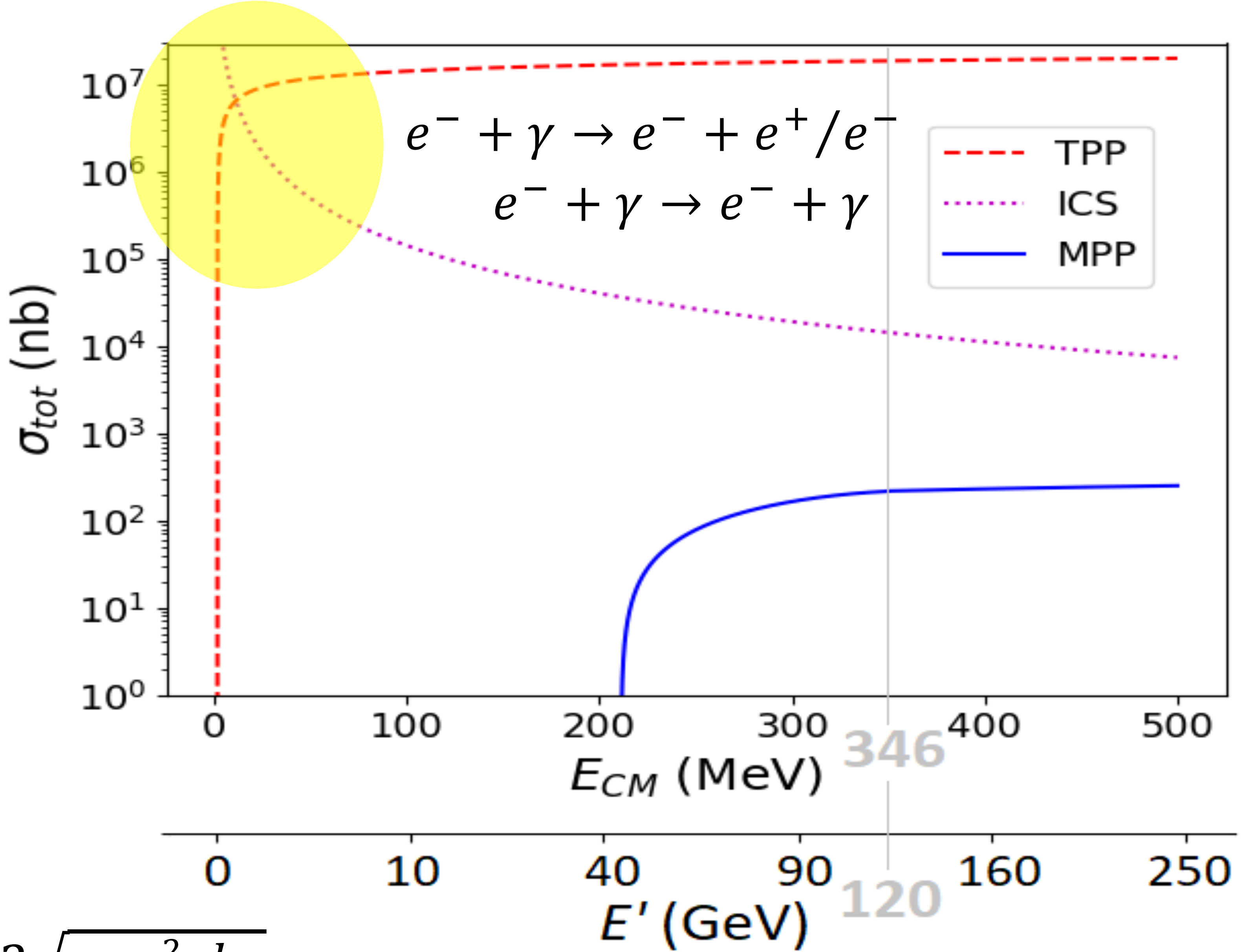
$$E'_{ph-max} = \frac{4E_{ph}{E_e}^2/(mc^2)^2}{1+4E_{ph}E_e/(mc^2)^2}$$

$$if\ \gamma \gg 1 \qquad E'_{e-min} \approx E_e \frac{1+(1+X)\,E_{ph}/E_e}{1+X}$$

X<<1: $E'_{e-min} \approx E_e$

X>>1: $E'_{e-min} \approx E_{ph}$

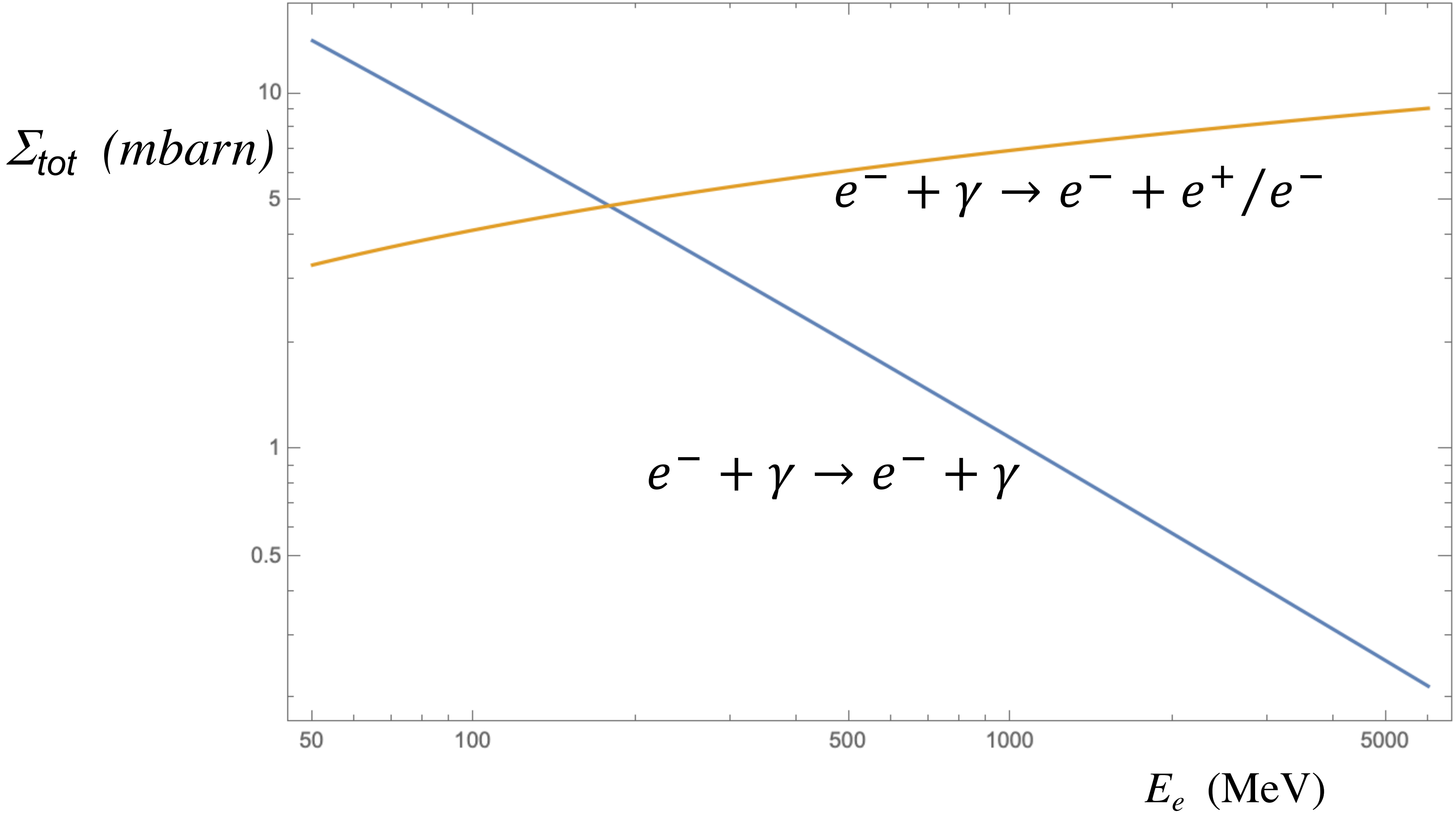
Total cross-section for QED (e,γ) reactions



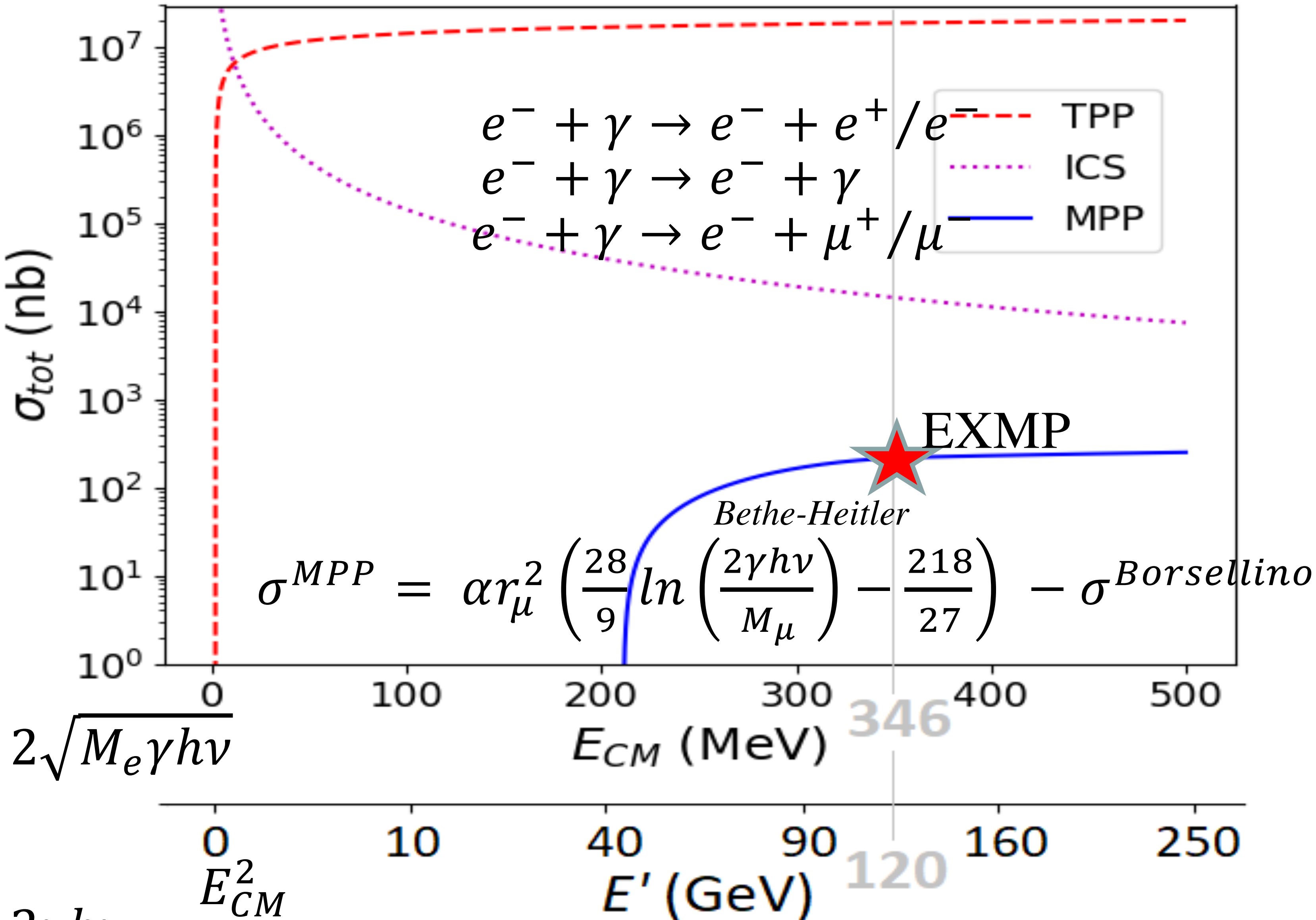
$$E_{CM} = 2\sqrt{m_e c^2 \gamma h\nu}$$

Total cross-sections for Compton and Bethe-Heitler

$E_{ph} = 255.5\ keV$ (E_e from 50 MeV to 5 GeV)



Total cross-section for MPP (muon pair production), Bethe-Heitler: fraction of a μbarn at photon energies > 100 GeV onto electrons at rest



$$E_{CM} = 2\sqrt{M_e \gamma h\nu}$$

$$E' = 2\gamma h\nu = \frac{E_{CM}^2}{2M_e}$$

Inverse Compton Sources rivaling/overcoming Synchrotron Light Sources at photon energies above 80-100 keV

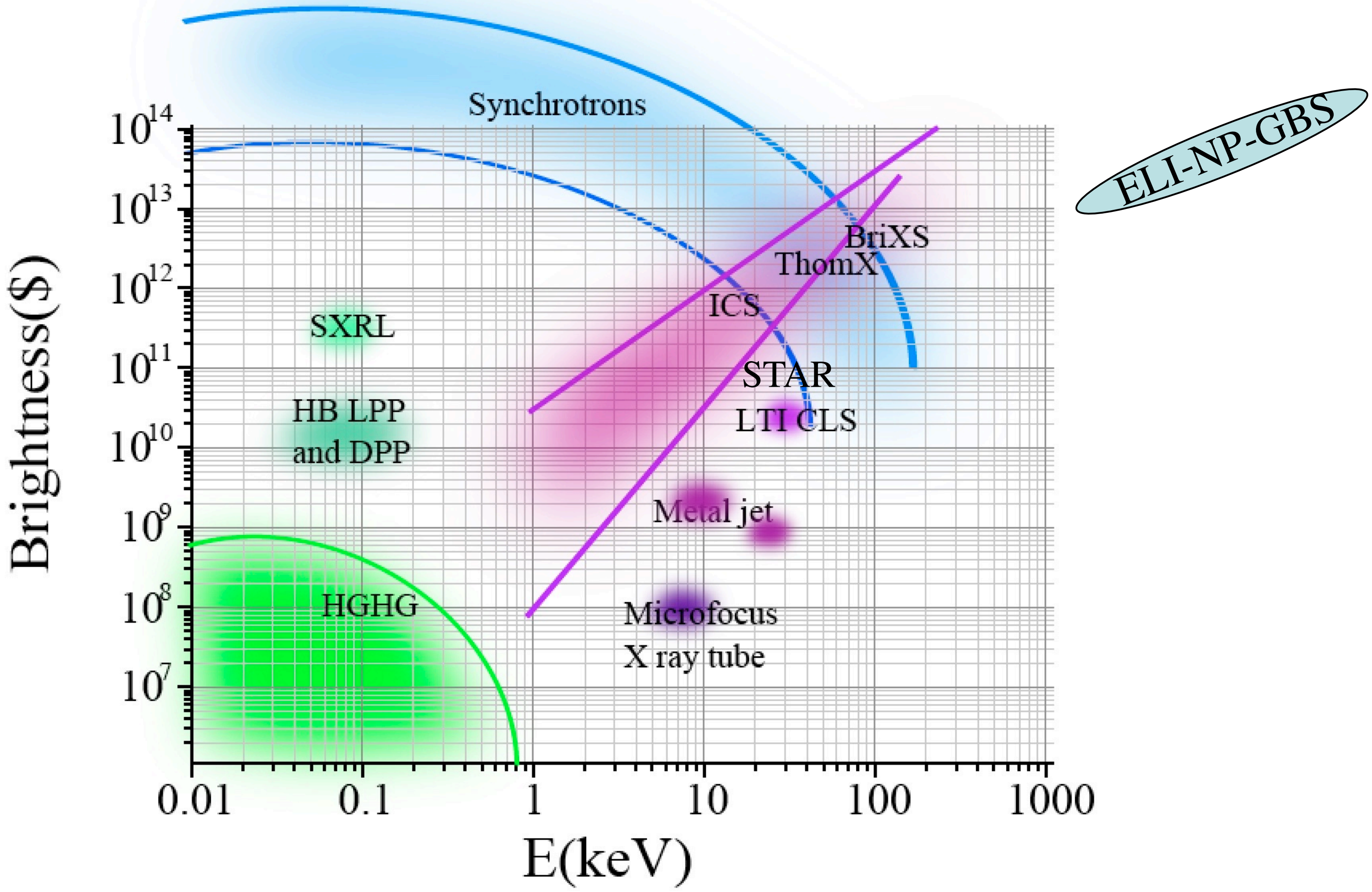


Figure 1: Brightness of several radiation sources as a function of the photon energy. \$: Photon $number/s/mm^2/mrad^2/(0.1\%$.
I.C.S. Sources (LTI-CLS,ThomX, STAR, UH-FLUX and BriXS) are compared to Synchrotron Light Sources and the most performing X-ray tube so far (Metal Jet).

3^{rd}-4^{th} Generation Light Sources

- Synchrotron light sources: < 50 keV, > 50 ps (100 m, 300 M$)
- X-ray FEL (LCLS): energy ≤25 (50?) keV, 1-100 fs (1 km, 1 G$)

- **New approach: inverse Compton scattering (ICS) 20-200 keV , sub-ps, (10 m , 10 M$) – sometimes called Laser Synchrotron since a laser pulse substitutes the magnetic undulators**

Brilliance of Lasers and X-ray sources

$$N_{ph} = 10^{19} - 10^{20}$$

$$\sigma_t = 10 - 20 \quad fs$$

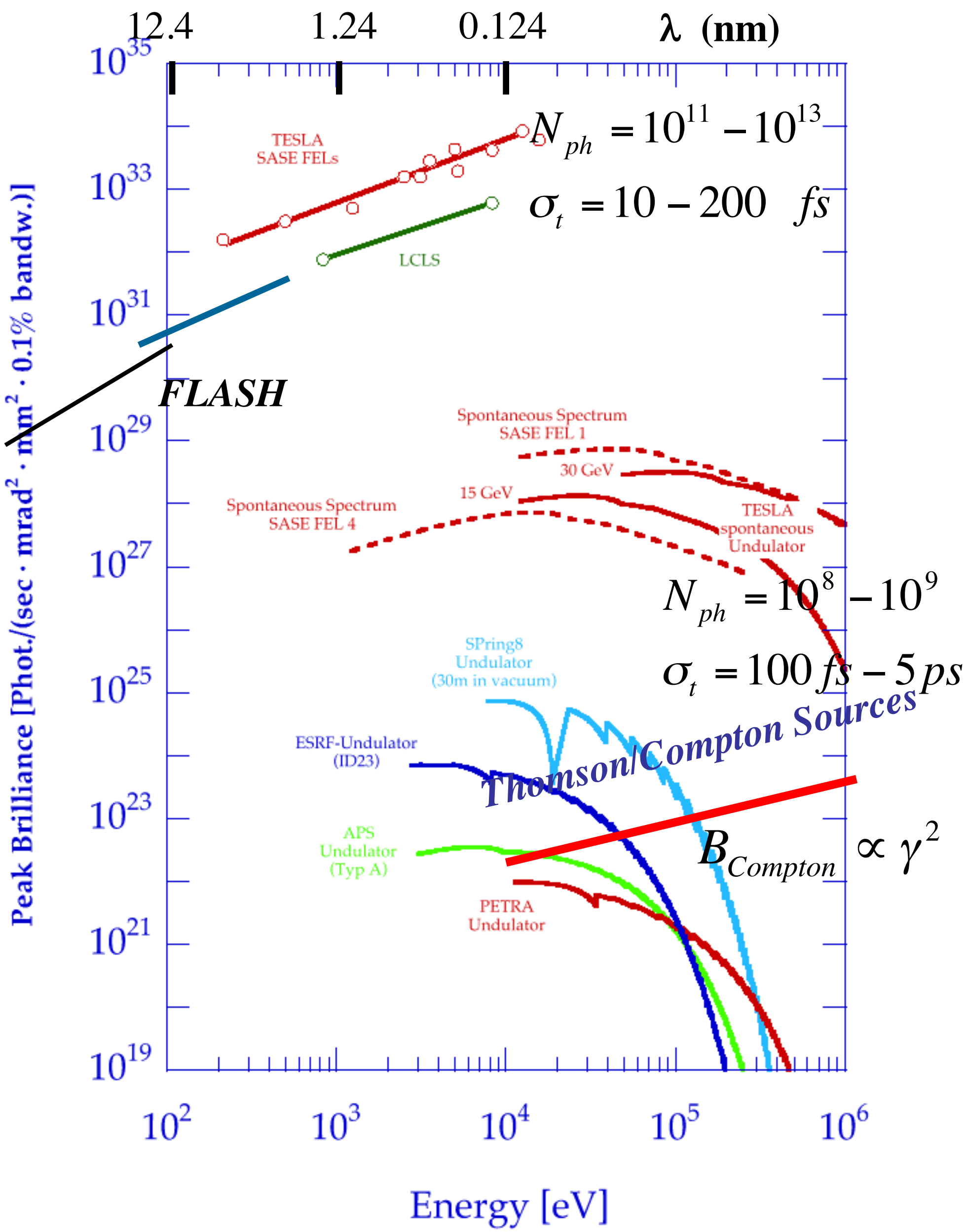
ELI

BELLA

$$B = \frac{N_{ph}}{\sqrt{2\pi}\sigma_t \left(M^2\lambda\right)^2 \frac{\Delta\lambda}{\lambda}}$$

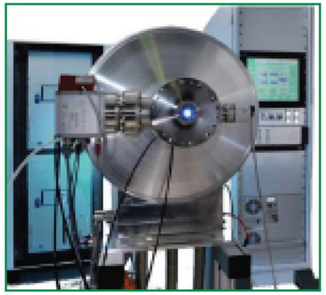
$$B_{peak} = \frac{N_{ph}}{\sqrt{2\pi}\sigma_t \varepsilon_x^2 \frac{\Delta E_X}{E_X}}$$

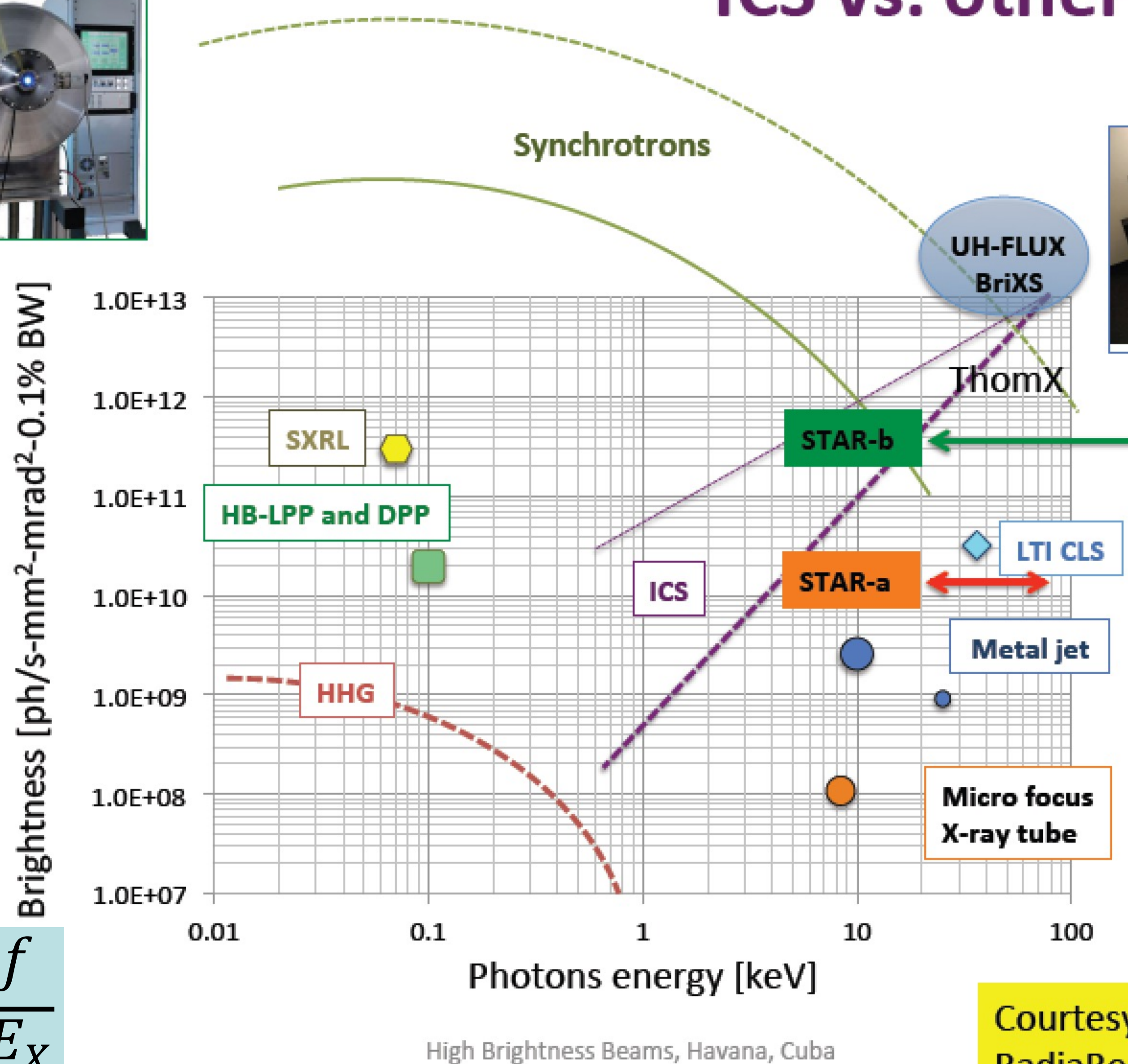
$$B_{av} = \frac{N_{ph} f}{\varepsilon_x^2 \frac{\Delta E_X}{E_X}}$$

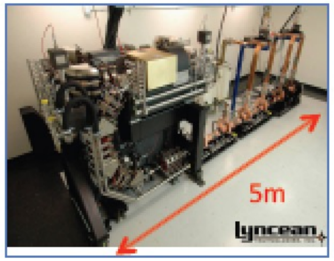


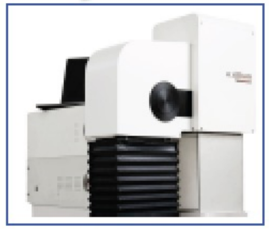
Rivaling with Synchr. Light Sources for energies above 50 keV

ICS vs. other sources









$$B_{av} = \frac{N_{ph} f}{\varepsilon_x^2 \frac{\Delta E_X}{E_X}}$$

Courtesy of A. Murokh
RadiaBeamTechnology

INFN Istituto Nazionale di Fisica Nucleare

Large Recoil in ICS damps the effect of large bandwidth incident photon beams onto the bandwidth of scattered photons

PHYSICAL REVIEW ACCELERATORS AND BEAMS **20,** 080701 (2017)

Analytical description of photon beam phase spaces in inverse Compton scattering sources

C. Curatolo,[1,*] I. Drebot,[1] V. Petrillo,[1,2] and L. Serafini[1]
[1]*INFN-Milan, via Celoria 16, 20133 Milano, Italy*
[2]*Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy*
(Received 9 March 2017; published 3 August 2017)

equivalent to FELs Kim-Pellegrini crit. on 3D inhomogeneous effects on photon bandwidth

$$\frac{\Delta E_{ph}}{E_{ph}} \simeq \sqrt{\left[\frac{\Psi^2/\sqrt{12}}{1+\Psi^2} + \frac{\bar{P}^2}{1+\sqrt{12}\bar{P}^2}\right]^2 + \left[\left(\frac{2+X}{1+X}\right)\frac{\Delta\gamma}{\gamma}\right]^2 + \left(\frac{1}{1+X}\frac{\Delta E_L}{E_L}\right)^2 + \left(\frac{M^2\lambda_0}{2\pi w_0}\right)^4 + \left(\frac{a_0^2/3}{1+a_0^2/2}\right)^2}$$

collimation angle

beam emittance

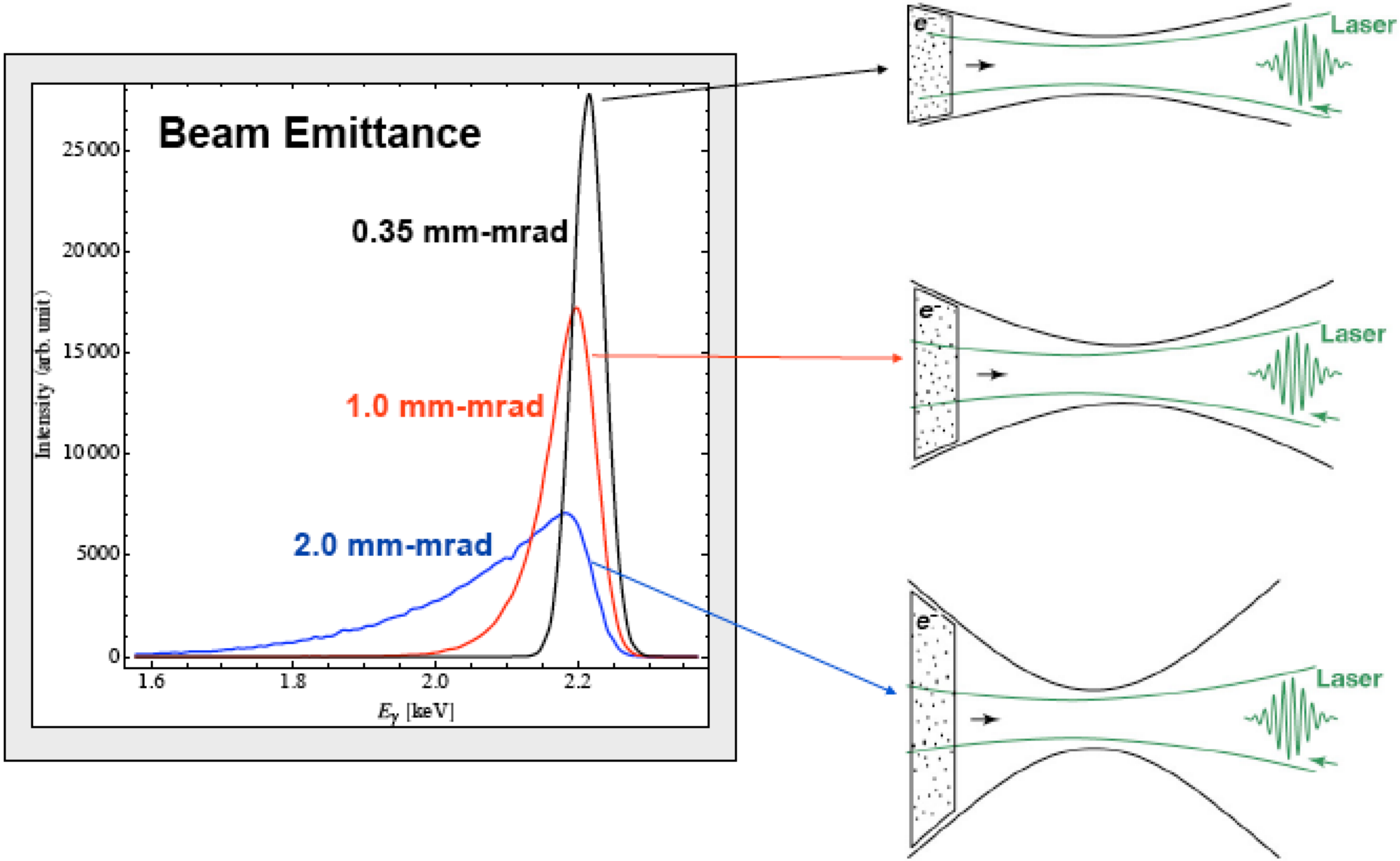
beam en. spread

incident photons en. spread

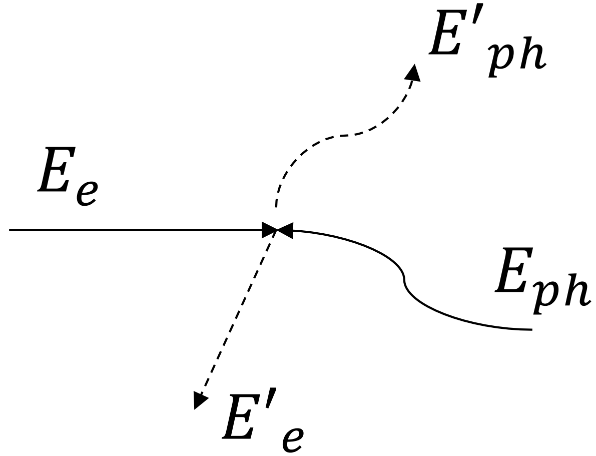
diffraction

non linearity

Lorentz boosted radiation (synchrotron, ICS, FEL, etc) is strongly affected by the emittacne of the electron beam



Courtesy C. Barty - LLNL



$$X \equiv 4\gamma^2 E_{ph} / E_e$$
$$A \equiv \beta\gamma^2 - X/4 = \gamma^2(\beta - E_{ph}/E_e)$$

$$E'_{ph} = \frac{4(\gamma^2+A)+X}{4(\gamma^2 - A\cos\theta)+X} E_{ph}$$

I.C.S. low recoil $X<<1$

$A \sim \beta\gamma^2 \sim \gamma^2 - 1/2$

$$E'_{ph-max} \sim 4\gamma^2 E_{ph}$$

I.C.S. deep recoil $X>>1$

$A \sim \beta\gamma^2 - X/4 \sim \gamma^2 - 1/2 - X/4$

$$E'_{ph-max} \sim \frac{4\gamma^2}{1+X} E_{ph} \sim \left(1 - \frac{1}{X}\right) E_e$$

S.C.S. (A = 0) or
quasi-SCS (|A| <<1)

$$E'_{ph-max} \sim E_{ph}\left(1 + \frac{2A}{(1+\beta)\gamma^2}\right)$$
$$E'_{e-min} \sim E_e - E_{ph}\frac{2A}{(1+\beta)\gamma^2}$$

D.C. $\gamma=1$, $\beta=0$, $A = -X/4$

$$E'_{ph-min} = \frac{1}{1+X/2} E_{ph} = \frac{1}{1-2A} E_{ph}$$

Direct Compton $\gamma=1$, $\beta=0$, $X = 4E_{ph}/mc^2$

$$E'_{ph-min} = \frac{E_{ph}}{1 + 2E_{ph}/mc^2} \qquad if \quad E_{ph} \gg mc^2 \quad E'_{ph-min} = \frac{mc^2}{2}$$

Very energetic photons are scattered back at 255 keV
and electrons pushed to $E_{ph} + 0.5mc^2$

$$E'_{e-max} = mc^2 + E_{ph} - E'_{ph-min} \;, if \; E_{ph} \gg mc^2 \quad E'_{e-max} = E_{ph} + \frac{mc^2}{2}$$

General Formula expressed in terms of energies of primary colliding particles, valid for any γ, A, X, θ

$$E'_{ph} = \frac{(1+\beta)\, E_{ph} E_e}{(1-\beta\cos\theta)\, E_e + (1+\cos\theta)\, E_{ph}}$$

$$E'_{ph-max} = \frac{(1+\beta)E_{ph}E_e}{(1-\beta)E_e + 2E_{ph}}$$

Generalization to any Collision Angle α , $\alpha = \pi$ *head-on*

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **17,** 020706 (2014)

Dual color x rays from Thomson or Compton sources

V. Petrillo,[1,2] A. Bacci,[1] C. Curatolo,[1,2] M. Ferrario,[3] G. Gatti,[3] C. Maroli,[2] J. V. Rau,[4] C. Ronsivalle,[5] L. Serafini,[1] C. Vaccarezza,[3] and M. Venturelli[2]
[1]*INFN Milano, Via Celoria, 16 20133 Milano, Italy*
[2]*Università degli Studi di Milano, Via Celoria, 16 20133 Milano, Italy*
[3]*LNF, INFN Via E. Fermi, 40 Frascati (Roma), Italy*
[4]*ISM-CNR Via del Fosso del Cavaliere, 100 00133 Roma, Italy*
[5]*ENEA Via E. Fermi, 45 Frascati (Roma), Italy*
(Received 12 September 2013; published 28 February 2014)

Each electron, characterized by normalized velocity $\underline{\beta}_i$ forming an angle θ_i with the z axis, scatters photons with frequency ν_p given by

$$\nu_p = \nu_0 \frac{1 - \underline{e}_k \cdot \underline{\beta}_i}{1 - \underline{n} \cdot \underline{\beta}_i + \frac{h\nu_0}{mc^2\gamma_i}(1 - \underline{e}_k \cdot \underline{n})}, \qquad (1)$$

where ν_0 is the frequency of the incident laser photon, $\underline{e}_k$ the unit vector of its direction, $\underline{n}$ is the direction of the scattered photon, h the Planck constant and γ_i the electron Lorentz factor before the scattering. The last term in the

$$E'_{ph} = \frac{4\gamma^2(1 - \beta\cos\alpha)}{4\gamma^2(1 - \beta\cos\theta) + X(1 - \cos\alpha\cos\theta + \sin\alpha\sin\theta)} E_{ph}$$

$\alpha = \pi$, *head-on* $\Rightarrow$

$$E'_{ph} = \frac{\gamma^2(1 + \beta)}{\gamma^2(1 - \beta\cos\theta) + \frac{X}{4}(1 + \cos\theta)} E_{ph}$$

in agreement with Eq.3 in *N. Ranjan et al.*, **PRAB 21**, 030701 (2018)

$$E'_{ph} = \frac{4\gamma^2(1-\beta\cos\alpha)}{4\gamma^2(1-\beta\cos\theta) + X(1-\cos\alpha\cos\theta + \sin\alpha\sin\theta)} E_{ph}$$

$$if\ \gamma \gg 1\ \ and\ \ \beta \approx 1 - \frac{1}{2\gamma^2}\ and\ \theta \ll 1$$

$$E'_{ph} = \frac{4\gamma^2\left(\frac{1-\cos\alpha}{2}\right)}{1+\gamma^2\theta^2 + X\left(\frac{1-\cos\alpha}{2}\right)} E_{ph}$$

$$if\ \theta = 0 \quad E'_{ph} = E'_{ph-max} \qquad E'_{ph-max} = \frac{4\gamma^2\left(\frac{1-\cos\alpha}{2}\right)}{1 + X\left(\frac{1-\cos\alpha}{2}\right)} E_{ph}$$

in agreement with Eq.1 in *I. Drebot et al.*, ***EPL*** **120**, 14002 (2017)

$$E'_{ph-max} = \frac{4\gamma^2\left(\frac{1-\cos\alpha}{2}\right)}{1+X\left(\frac{1-\cos\alpha}{2}\right)} E_{ph}$$

$X << 1 \Rightarrow$ $$E'_{ph-max} = 4\gamma^2\left(\frac{1-\cos\alpha}{2}\right) E_{ph}$$

$\alpha = \pi$, *head-on* $\Rightarrow$ $$E'_{ph-max} = \frac{4\gamma^2}{1+X} E_{ph}$$

$\alpha = \pi/2$, $X << 1 \Rightarrow$ $$E'_{ph-max} = 2\gamma^2 E_{ph}$$

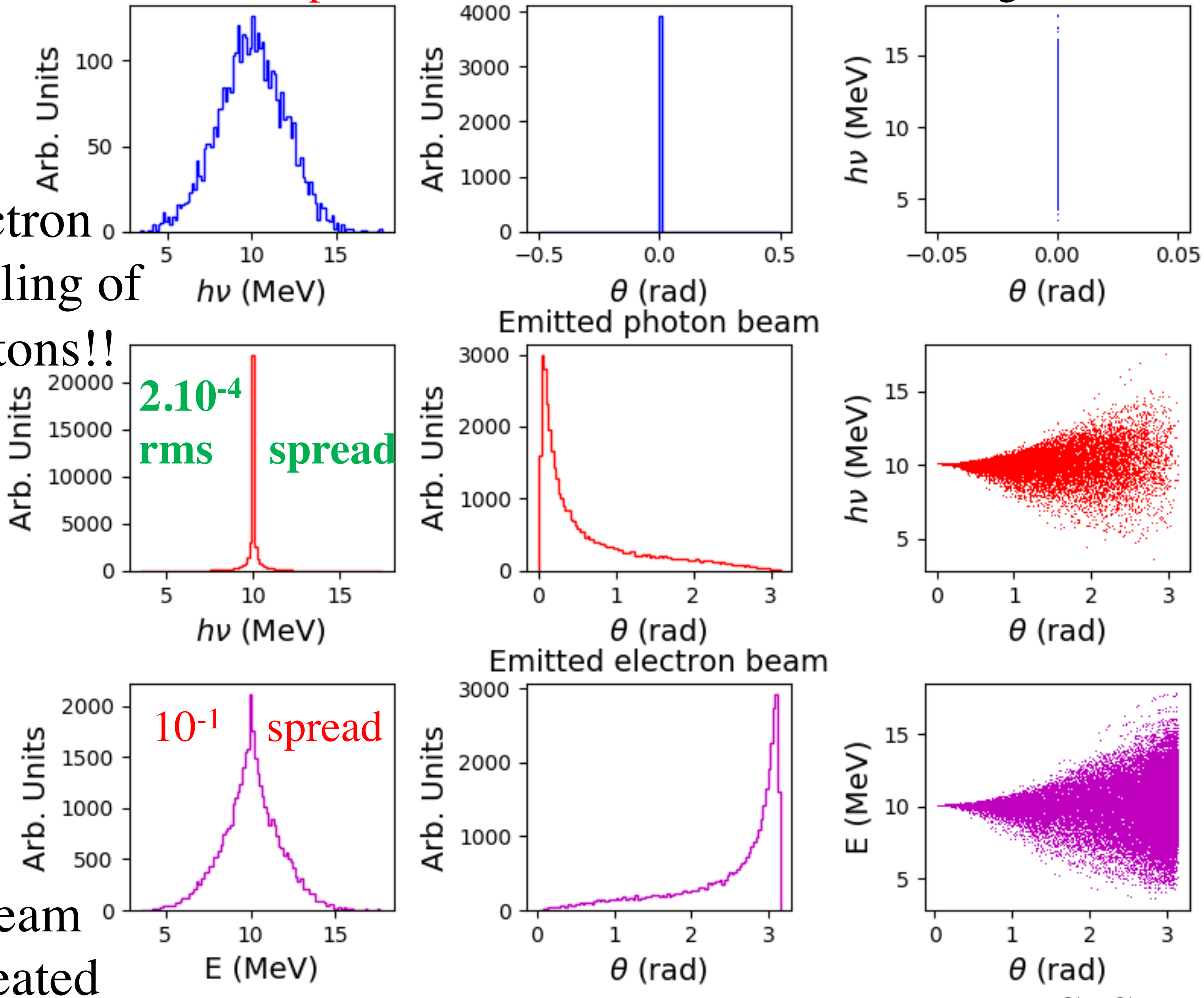
$X >> 1 \Rightarrow$ $$E'_{ph-max} = \frac{4\gamma^2}{X} E_{ph} = E_e \quad \forall\alpha \text{ !!}$$

γ-ray in-vacuum mono-chromatization, SCS at large Recoil

20% rms spread

Incoming photon beam

Incoming e- beam 10^{-4} rms spread



Electron Cooling of photons!!

no energy-angular correlation

e- beam is heated

C. Curatolo - Whizard

Symmetric Compton Scattering suppresses the $\gamma^2\theta^2$ correlation

Photons are scattered at same energy at any angle

Lorentz Boost is damped

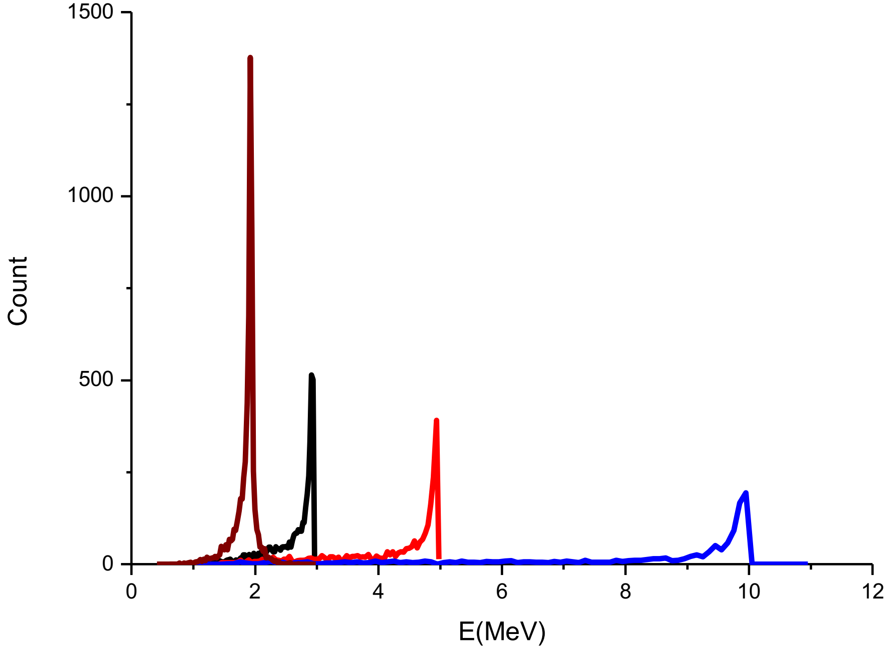
Radiation emission is intrinsically mono-chromatic

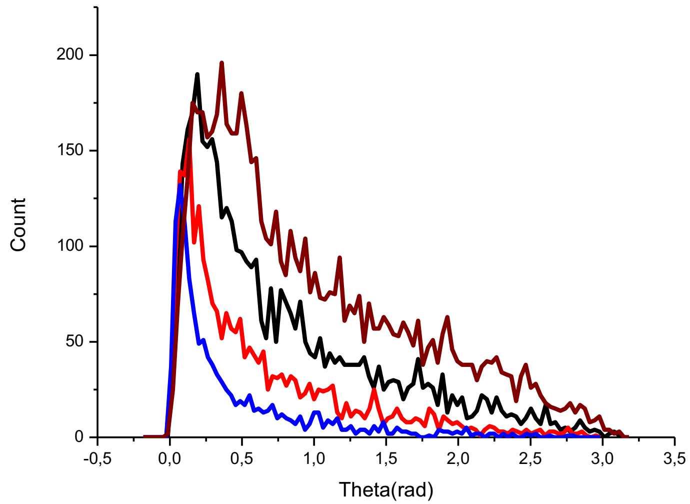
Poli-chromaticity of incident photon beam is transferred to the scattered electron beam and viceversa (photon cooling, electron heating)

SCS - What Matters?

- **SCS may allow to design a laser-less γ-ray source for nuclear photonics, aka ELI-NP-GBS, using a compact low energy Linac (20-30 MeV versus 750 MeV)**
- **It can be used to extend the photon energy range of Light Sources and Free Electron Lasers up to MeV's photon beams (LCLS 12 keV, XFEL 19 keV, ESRF 100 keV ⇒ 1-10 MeV)**
- **Follow-ups in Astrophysics: Synchro-Compton catastrophe (see *Malcolm Longair, High Energy Astro-Physics*)**
- **Applications to Plasma Physics: additive trapping of electrons (positrons?) in magnetic bottles**

Colliding a gaussian distributed (20% rms spread) broad-band radiation beam, representing the first peak of channeling spectrum at 2 MeV, with a low energy (variable) electron beam (2,3,5,10 MeV)





Mono-chromatization, Tunability

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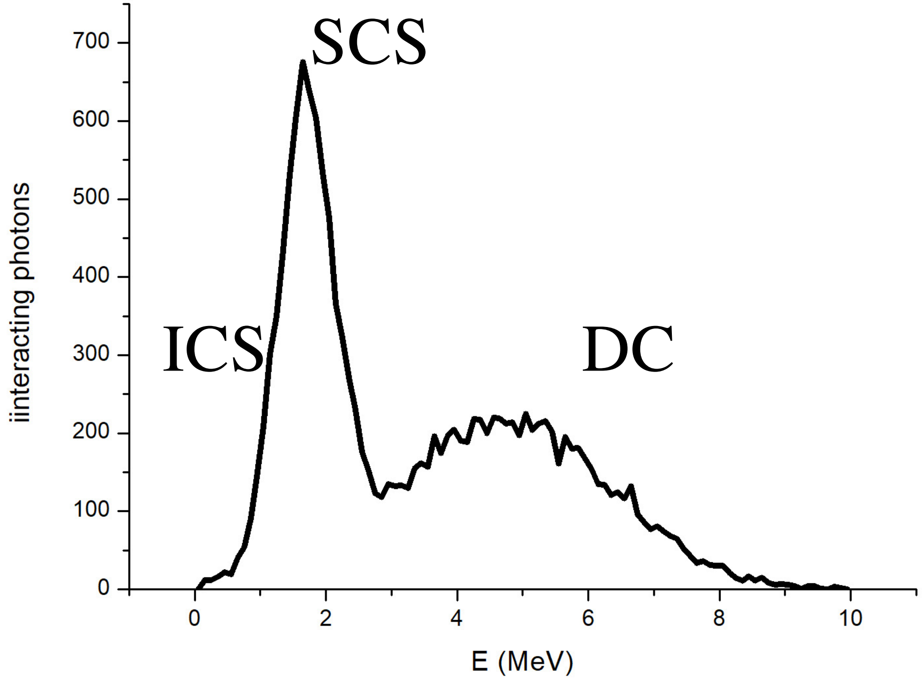
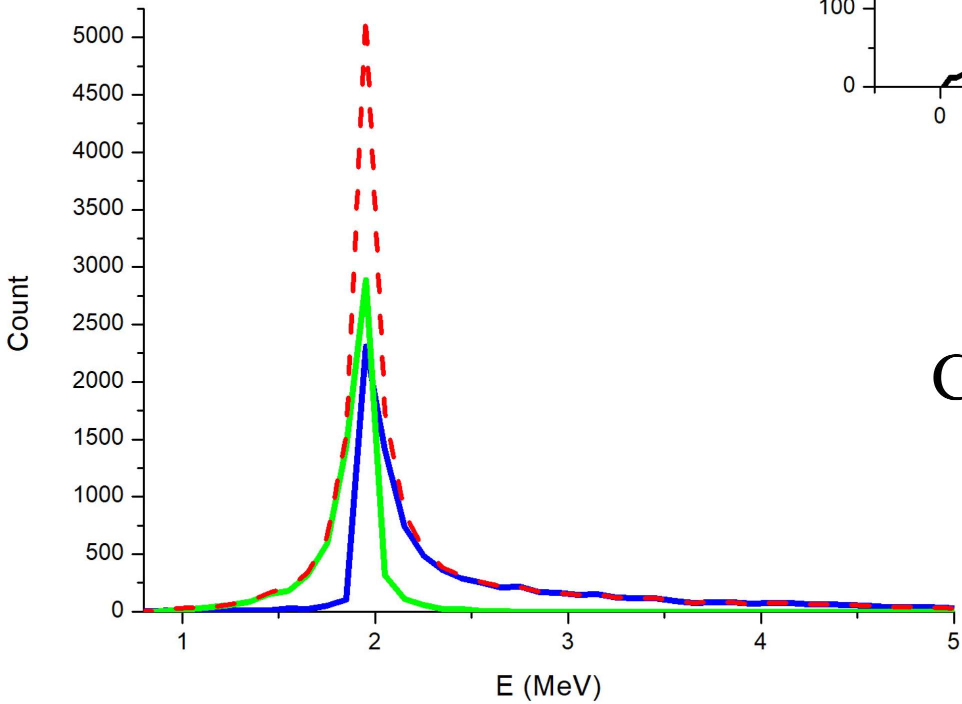
Colliding the full spectrum





Spectral purification
Compton Scattering across SCS

ELSEVIER
Optics Communications
Volume 50, Issue 6, 15 July 1984, Pages 373-378

Collective instabilities and high-gain regime in a free electron laser

R. Bonifacio *, C. Pellegrini, L.M. Narducci

Nuclear Inst. and Methods in Physics Research, A 909 (2018) 309–313

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima

A muon source based on plasma accelerators

L. Serafini[a], I. Drebot[a,*], A. Bacci[a], F. Broggi[a], C. Curatolo[a], V. Petrillo[a,c], A.R. Rossi[a], M. Rossetti Conti[a,c]

[a] INFN - Sezione di Milano, via Celoria 16, 20133 Milano, Italy
[b] INFN - Laboratori Nazionali di Frascati, Via Enrico Fermi 40, 00044 Frascati, Italy
[c] Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy

Spectral purification towards tunable mono-chromatic γ-rays with laser plasma deep recoil ICS-SCS?

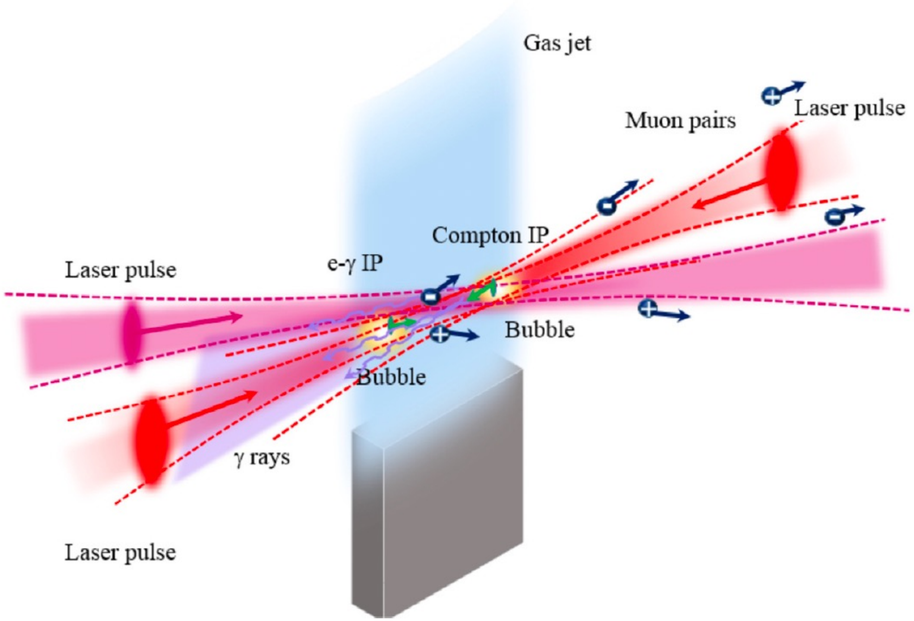
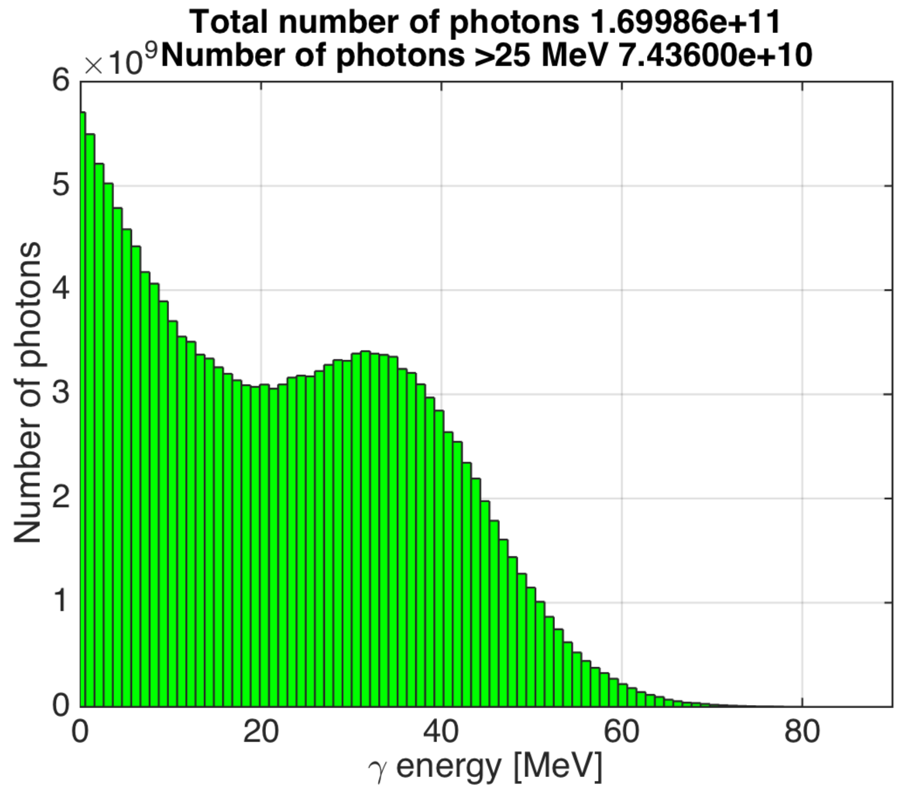
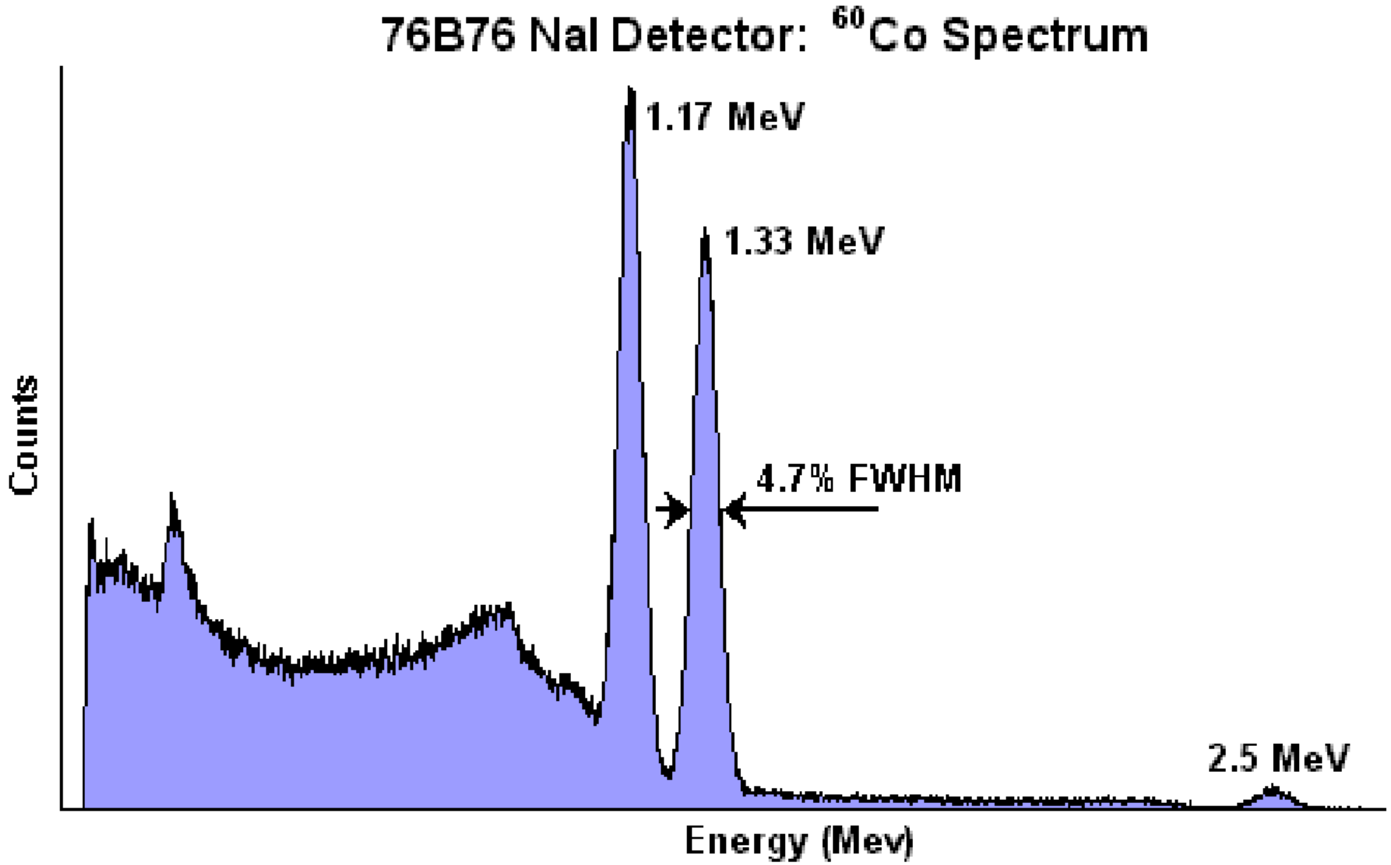


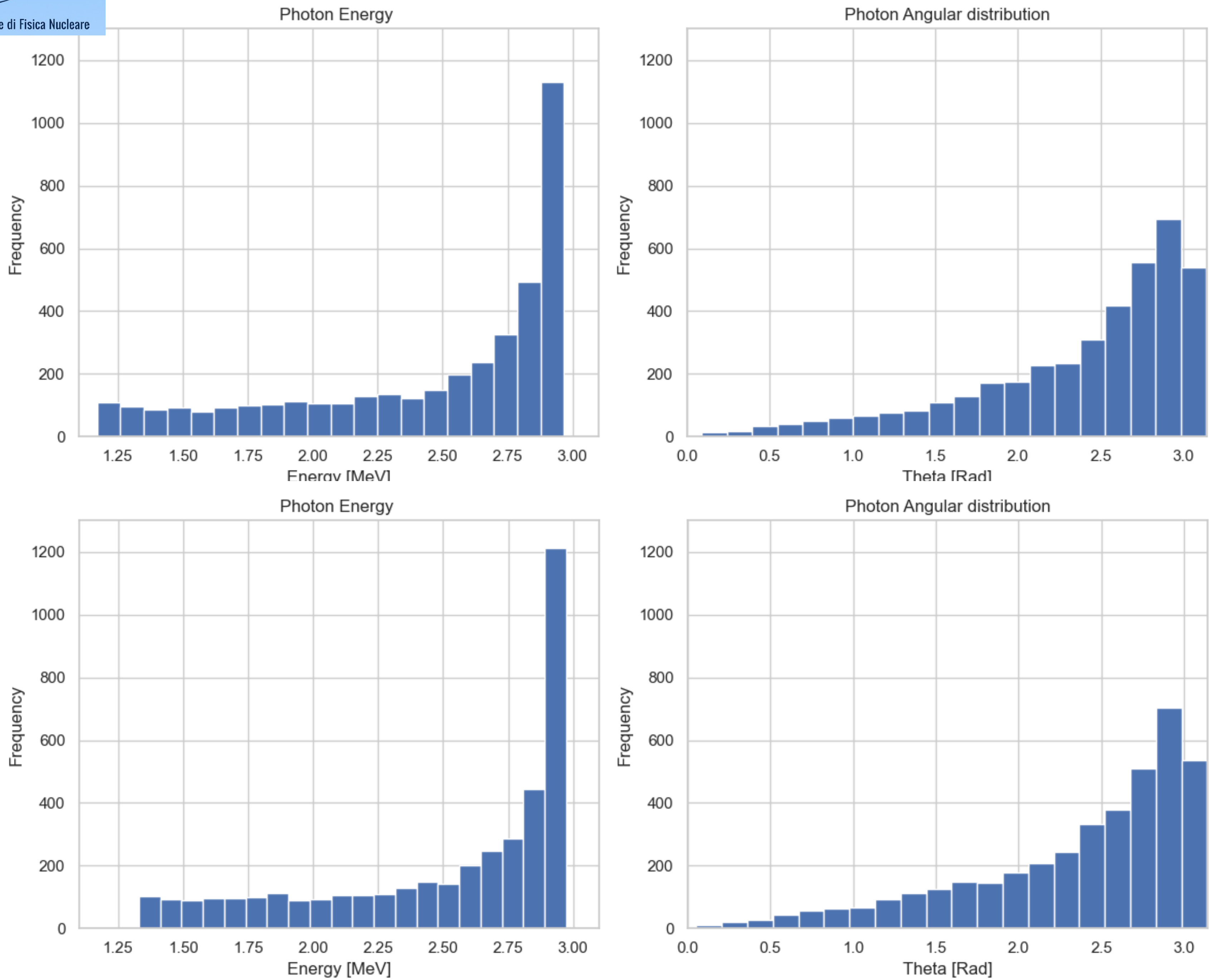
Fig. 2. Scheme of source.



Turning a radio-active Cobalt-60 fixed energy gamma-ray source into a tunable sorce of gamma-rays



2 spectral lines merged into a single tunable line



Trapping electrons (positrons) into a magnetic bottle by SCS at low recoil (72 keV photon beam heats up 5 keV e^- beam)

$$\frac{v_z}{v_r} < \sqrt{\frac{B_{\max}}{B_{\min}} - 1},$$

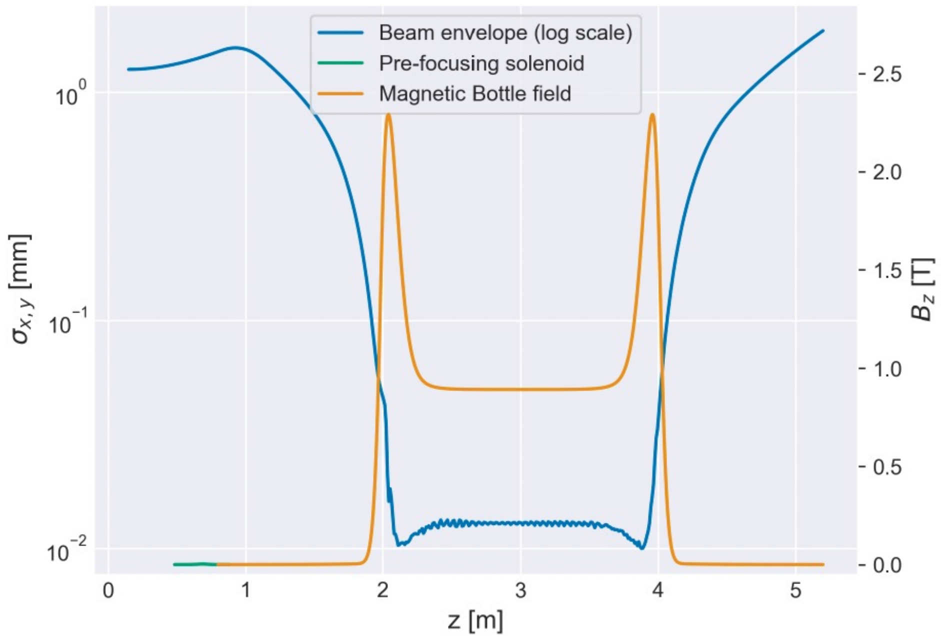
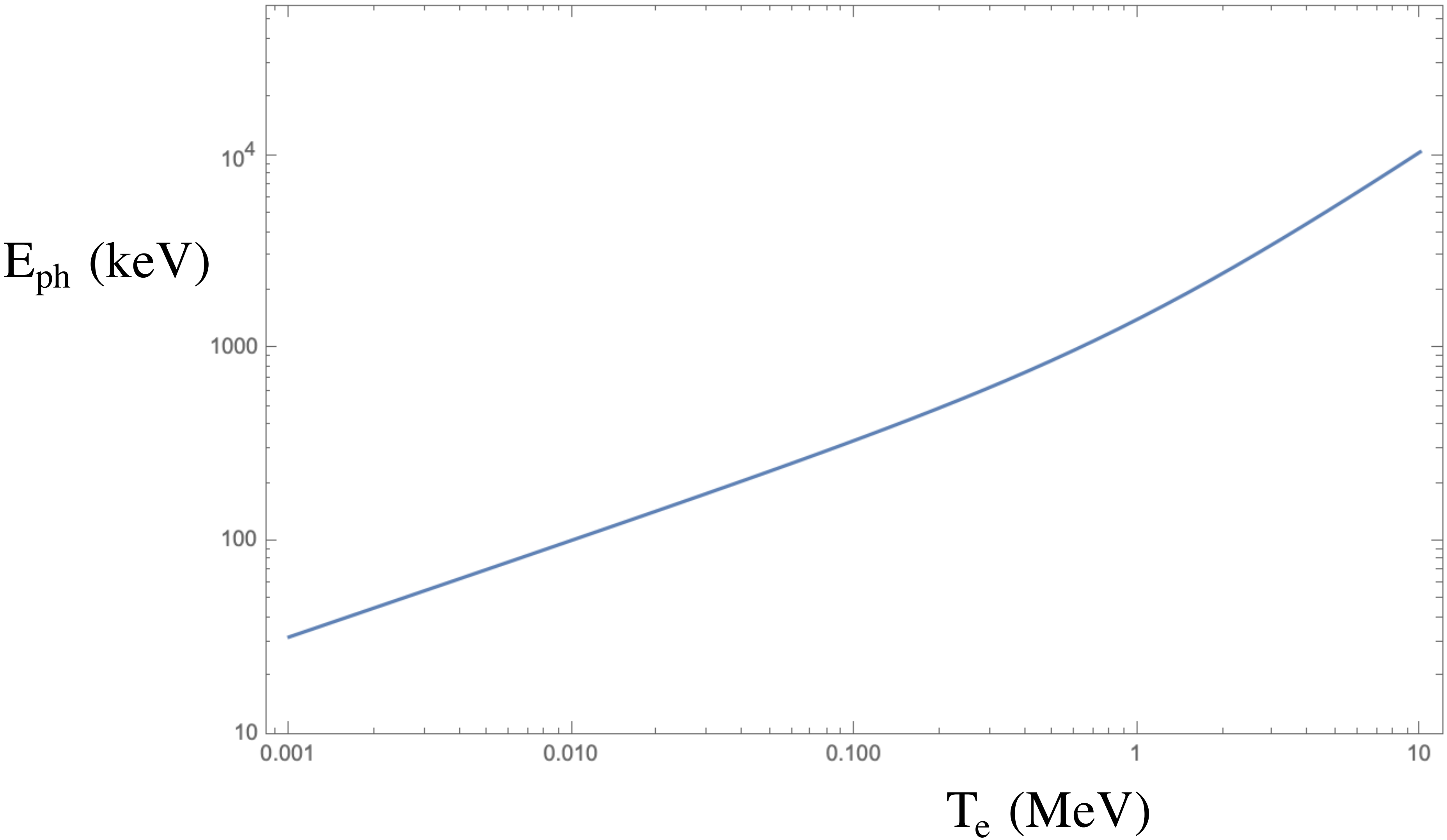


Figure 8: This image shows the transverse envelope of the primary electron beam (in blue) before, during, and after propagation in the MB field (in gold the Bz field distribution). Before the bottle, the weak field (in green) of a solenoid, peaking at 2.5 mT, is visible and is used for matching into the bottle.

S.C.S. – incident photon energy vs. incident electron kinetic energy



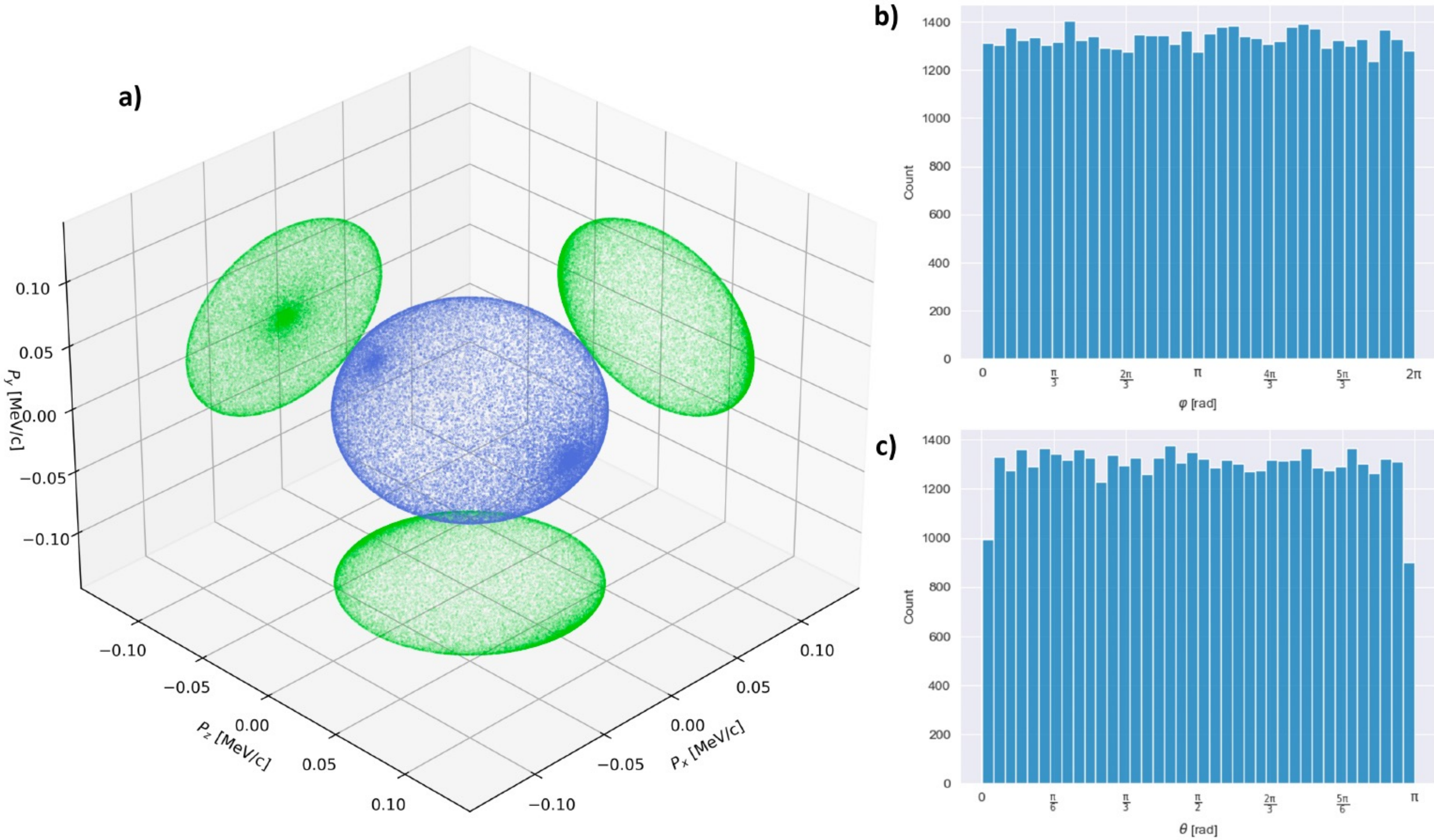


Figure 9: Representation of the momenta of the electrons that interacted with the photons in SCS regime. a) 3D representation of the momenta with their projections. b) Distribution of the momenta respect the φ angle around the z-axis. c) Distribution of the momenta respect the θ angle with the z-axis.

60% of scattered electrons are (additively) trapped into the magnetic bottle (w.o. any external field)

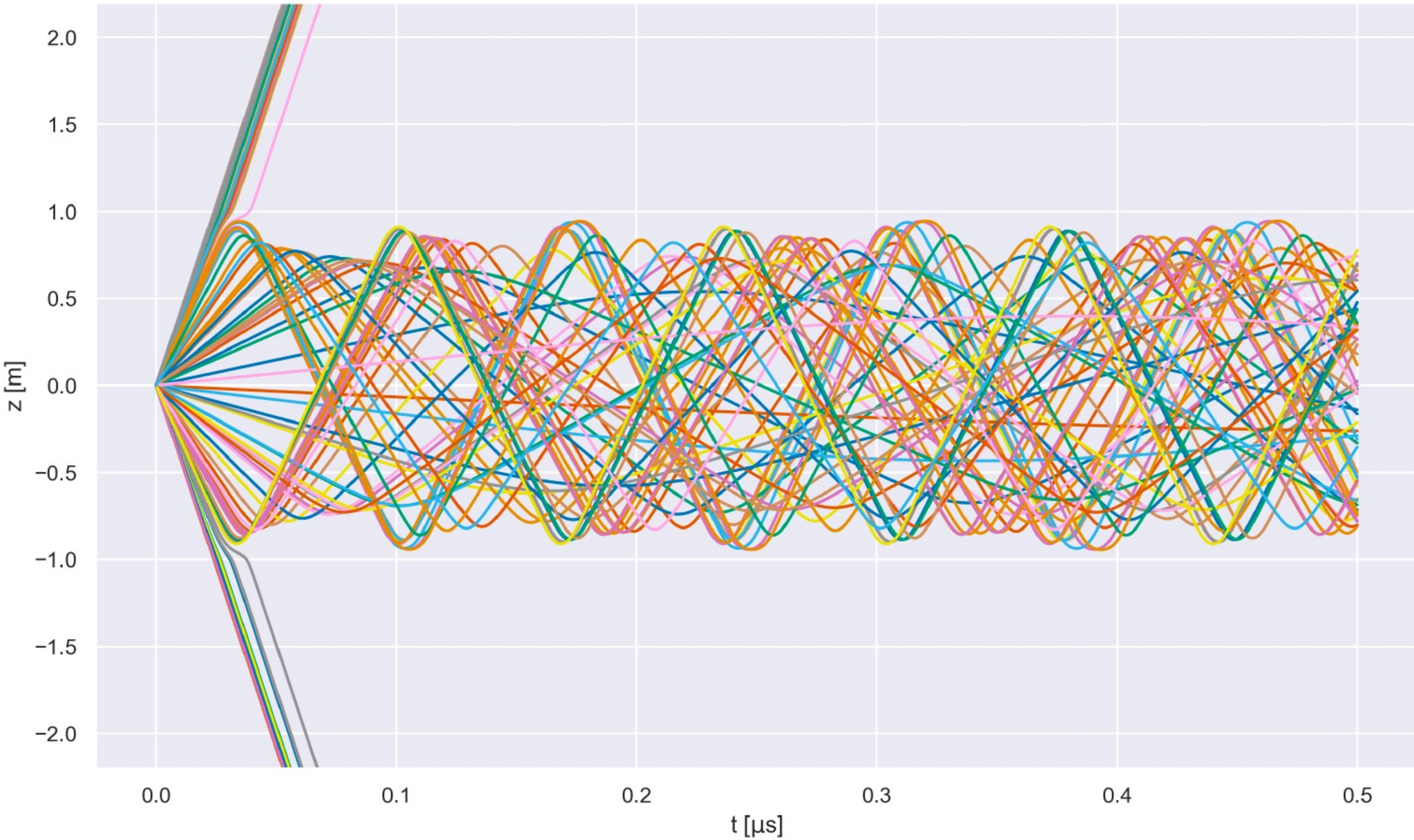


Figure 10: Evolution of the longitudinal position of 100 particles tracked in the MB, 60% where trapped.

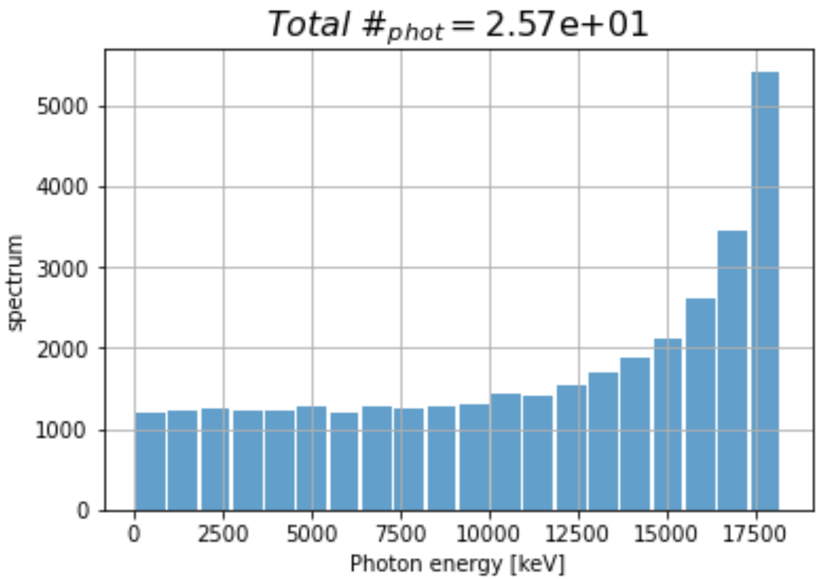
Conclusions

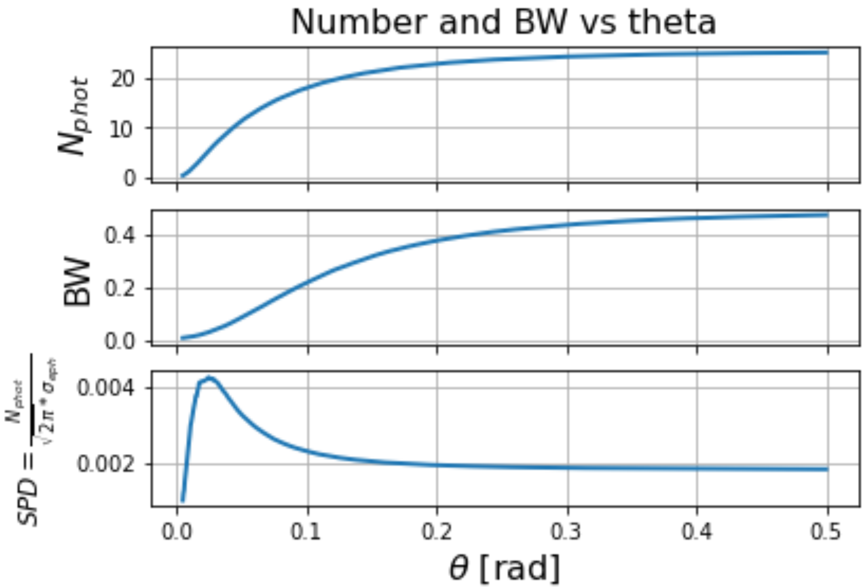
If S.C.S. will be the base of
Next Generation hard-X-Ray and gamma-rays,
we are not yet able to say,
but chances are quite promising.

If S.C.S. will play a role in plasma heating,
soon we'll be able to say.

Within Astro-physics context more studies
must be pursued to check about
Compton Catastrophe and related topics.

Undulator radiation (33 keV from Elettra) vs. 20 MeV e- beam from BriXSinO ERL about 10^9 photons/s in 1% bdw (100 MHz rep rate)





V. Petrillo, I. Drebot (INFN-Milan)
S. Dimitri (Elettra)

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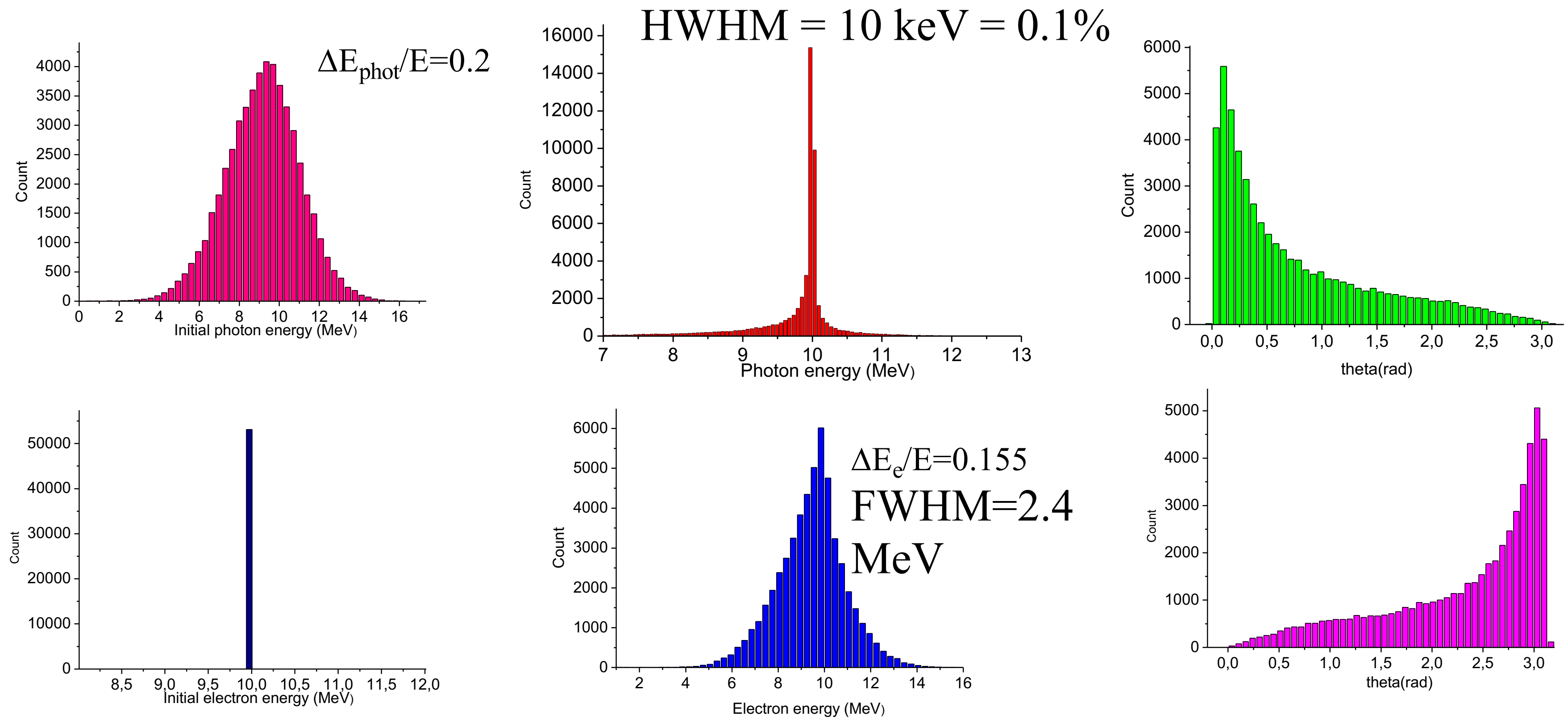
Symmetric Compton (CSC)
(10 MeV Linac vs. bremsstrahlung/betatron/ channeling/coherent bremsstrahlung beam)

Electron energy=10.013 MeV, **Photon energy=10 MeV,** $\Delta E_{phot}/E$=20% **Deep Recoil** X=1533

Q=1.e-9 C N_X=2.*10^8 σ_0=1 μm rep-rate=200 MHz



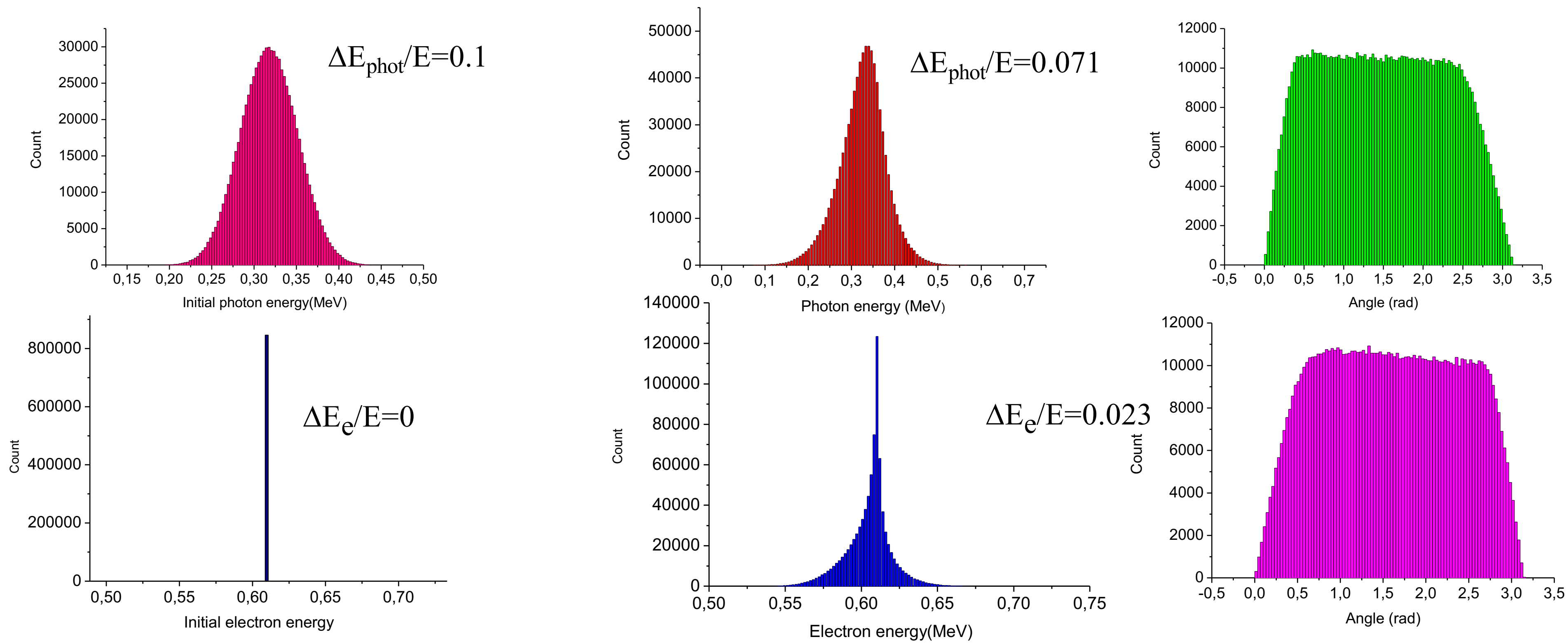
N_el=6.*10^9 N_X=2.*10^8 Σ=2.6*10^{-27} =2.6 mbarn N_X' (s^{-1})=5*10^6 S=500 $s^{-1}eV^{-1}$

$$\lim_{X\to 0}\sigma = \frac{8\pi r_e^2}{3}(1-X) = \sigma_T(1-X) \quad \lim_{X\to\infty}\sigma = \frac{2\pi r_e^2}{X}\left(\log X + \frac{1}{2}\right)$$

Vittoria Petrillo – Compton montecarlo

Symmetric Compton at moderate recoil

Initial Electron energy=611 keV, **Initial Photon energy=335 keV** , $\Delta E_{phot}/E=0.1$
Symmetric Compton ($\underline{p}_e=-\underline{p}_{phot}$), **moderate recoil** X=3.13

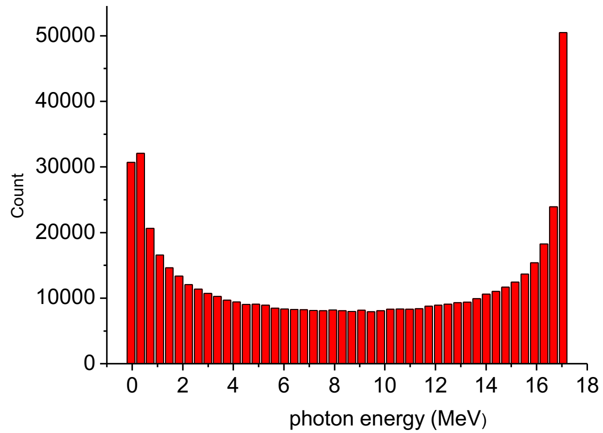


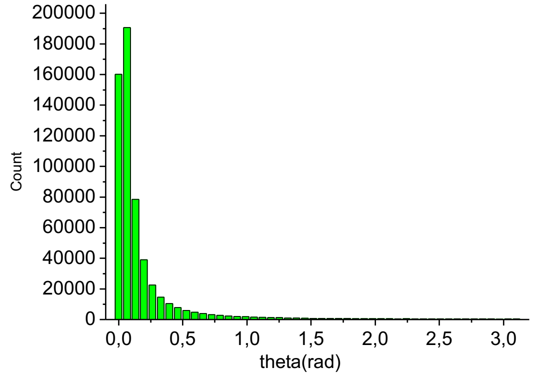
SCS and large recoil factors are both needed to mono-chromatize broad band incident photon beams

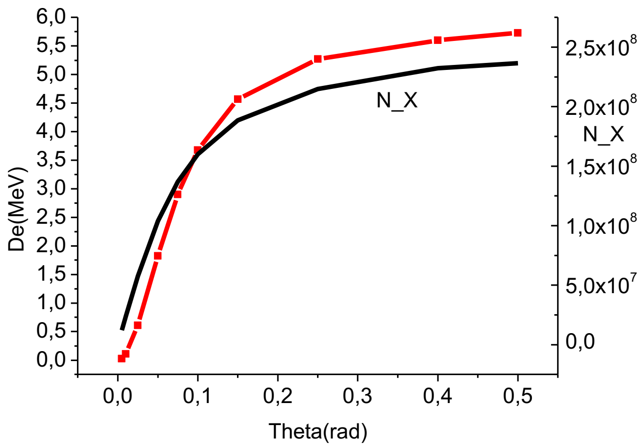
FEL beam vs. compact Linac

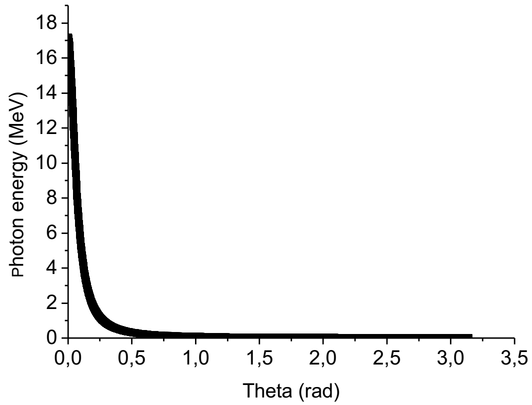
Initial Electron energy=20 MeV, **Initial Photon energy=20 keV ,** $\Delta E_{phot}/E=0.0001$

moderate recoil X=6.12





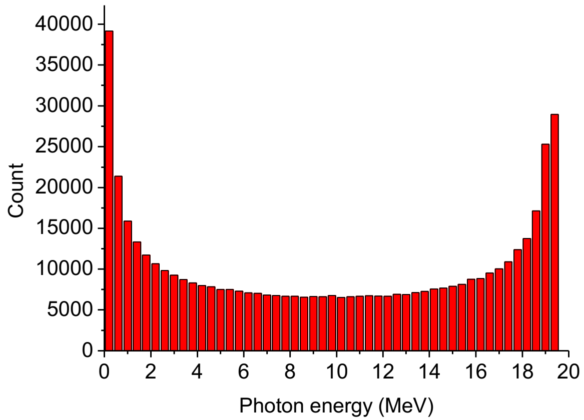


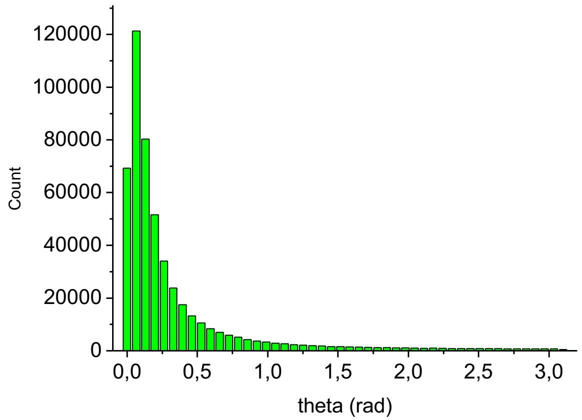


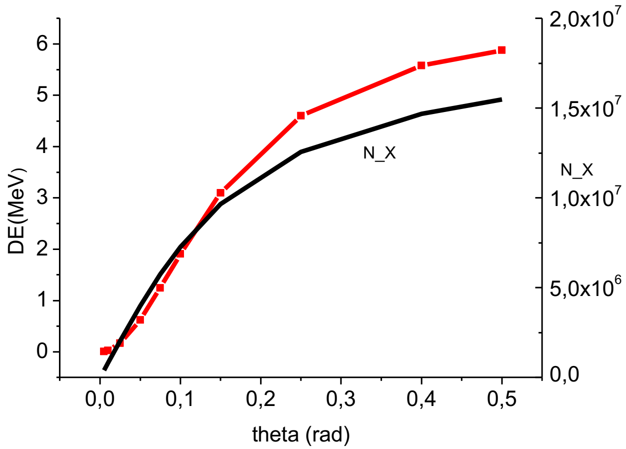
up to 10^8 photons/s at 17 MeV with 50 keV bdw: S 2000 $s^{-1}eV^{-1}$

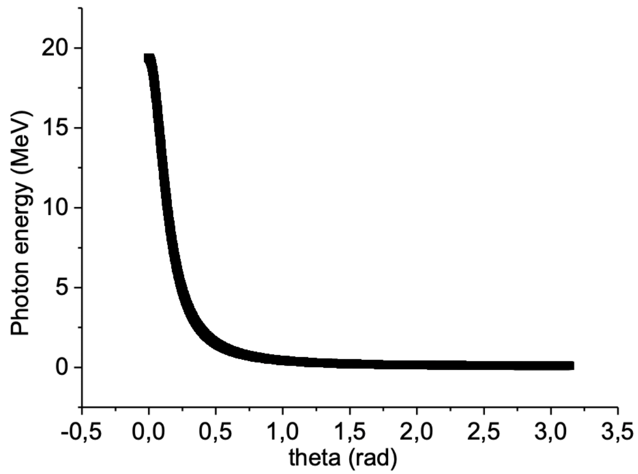
X-ray beam from Light Source vs. compact Linac

Electron energy=20 MeV, **Photon energy=100 keV ,** $\Delta E_{phot}/E=0.001$ **recoil** X=30.63









up to 10^7 photons/s at 17 MeV with 50 keV bdw

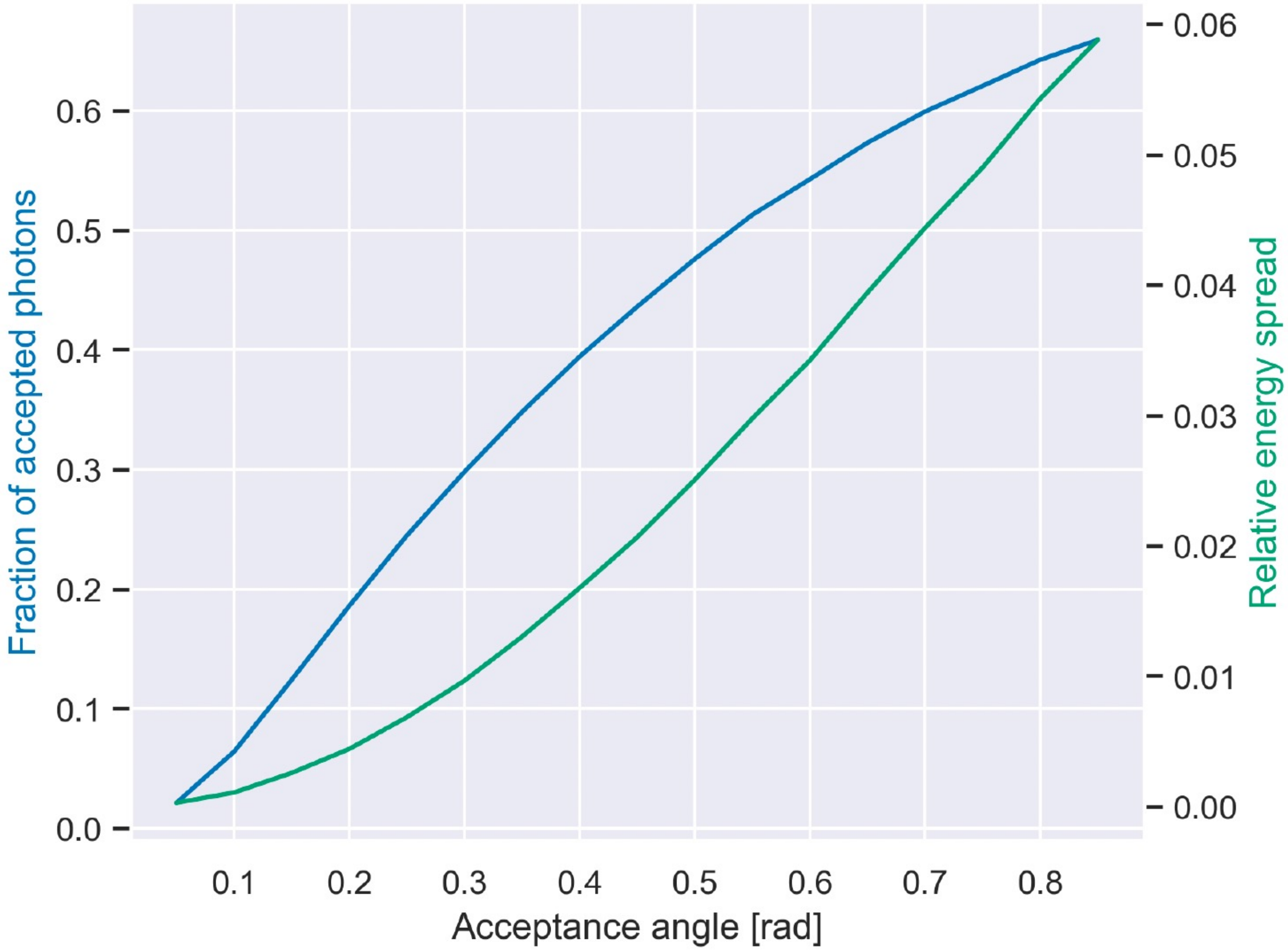
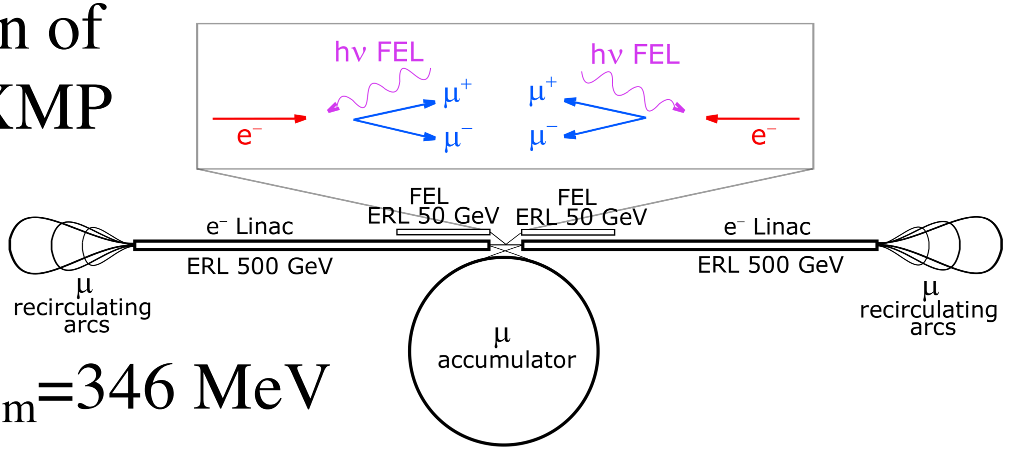


Figure 12: Relative fraction of scattered photons within the acceptance angle (blue curve) and relative bandwidth of the selected photon beam within the angular acceptance (green curve).

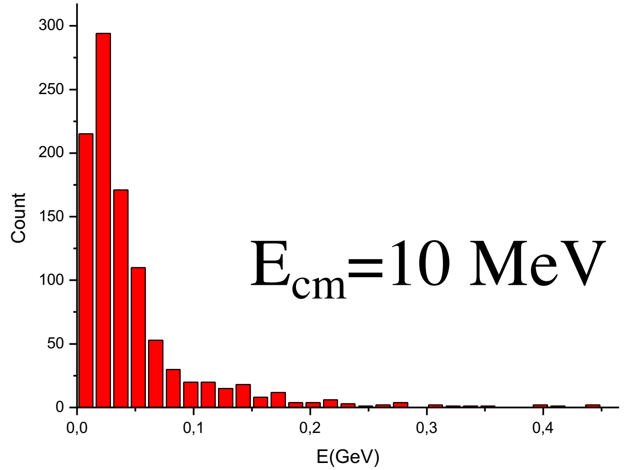
Ultra-low emittance positron beams from deep recoil electron-photon collider: 5 GeV ERL vs. 5 keV FEL, X=391

up to $10^{13\text{-}14}$ e^+/s at 50 MeV within 5% en. spread

Son of EXMP



E_{cm}=346 MeV

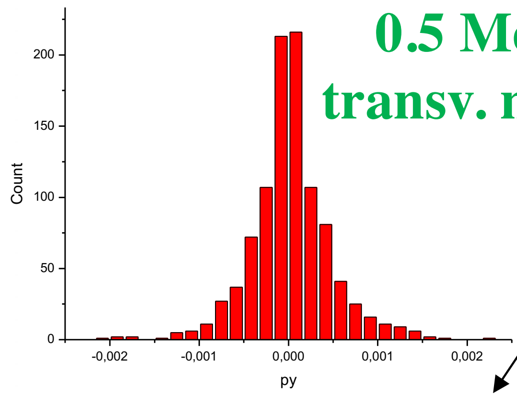


E_{cm}=10 MeV

Table 4: Future Positron Collider Projects [53, 59, 61–66].

Project	CLIC	ILC	LHeC (pulsed)	LEMMA	CEPC	FCC-ee
Final e^+ energy [GeV]	190	125	140	45	45	45.6
Primary e^- energy [GeV]	5	128** (3*)	10	–	4	6
Number of bunches per pulse	352	1312 (66*)	10^5	1000	1	2
Required charge [10^{10} e^+/bunch]	0.4	3	0.18	50	0.6	2.1
Horizontal emittance $\gamma\epsilon_x$ [μm]	0.9	5	100	–	16	24
Vertical emittance $\gamma\epsilon_y$ [μm]	0.03	0.035	100	–	0.14	0.09
Repetition rate [Hz]	50	5 (300*)	10	20	50	200
e^+ flux [10^{14} e^+/second]	1	2	18	10–100	0.003	0.06
Polarization	No/Yes***	Yes/(No*)	Yes	No	No	No

* The parameters are given for the electron-driven positron source being under consideration.
** Electron beam energy at the end of the main electron linac taking into account the looses in the undulator.
*** Polarization is considered as an upgrade option.



0.5 MeV/c rms transv. momentum

$0.5\text{-}1 \cdot 10^{-7}$ m·rad rms norm. transv. emittance with round beam (no-cooling)

V. Petrillo, A. Puppin – Whizard

Large Recoil in MPP damps the normalized emittance of the secondary generated muon beam

applied sciences

MDPI

Article

Electrons and X-rays to Muon Pairs (EXMP)

Camilla Curatolo * and Luca Serafini

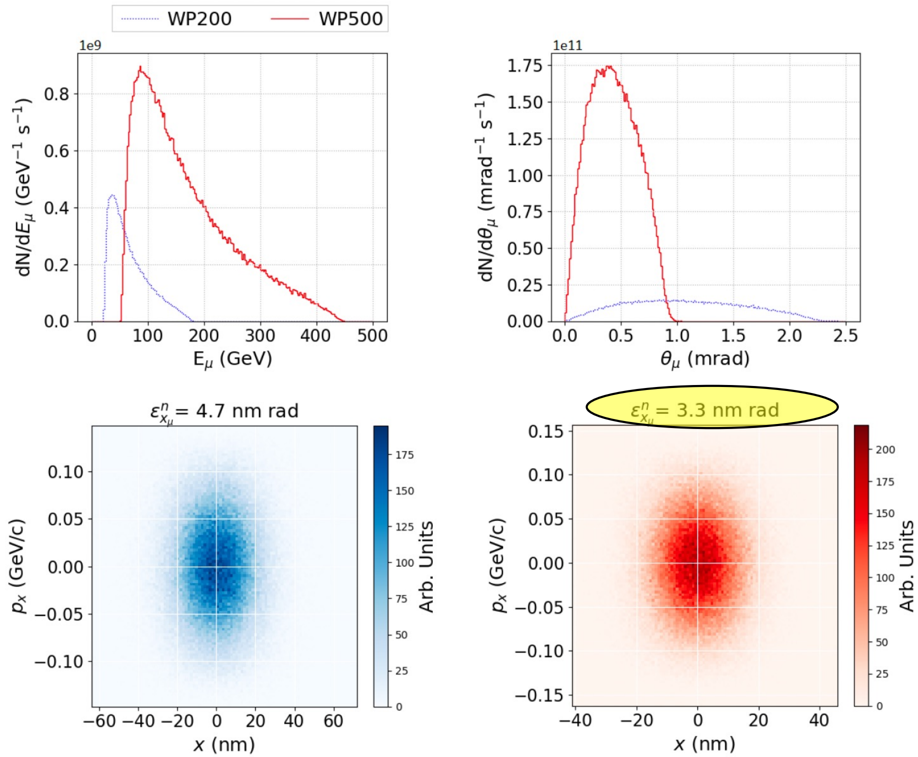
INFN Sezione di Milano, via Celoria 16, 20133 Milan, Italy; luca.serafini@mi.infn.it

* Correspondence: camilla.curatolo@mi.infn

muon beam norm. emittance

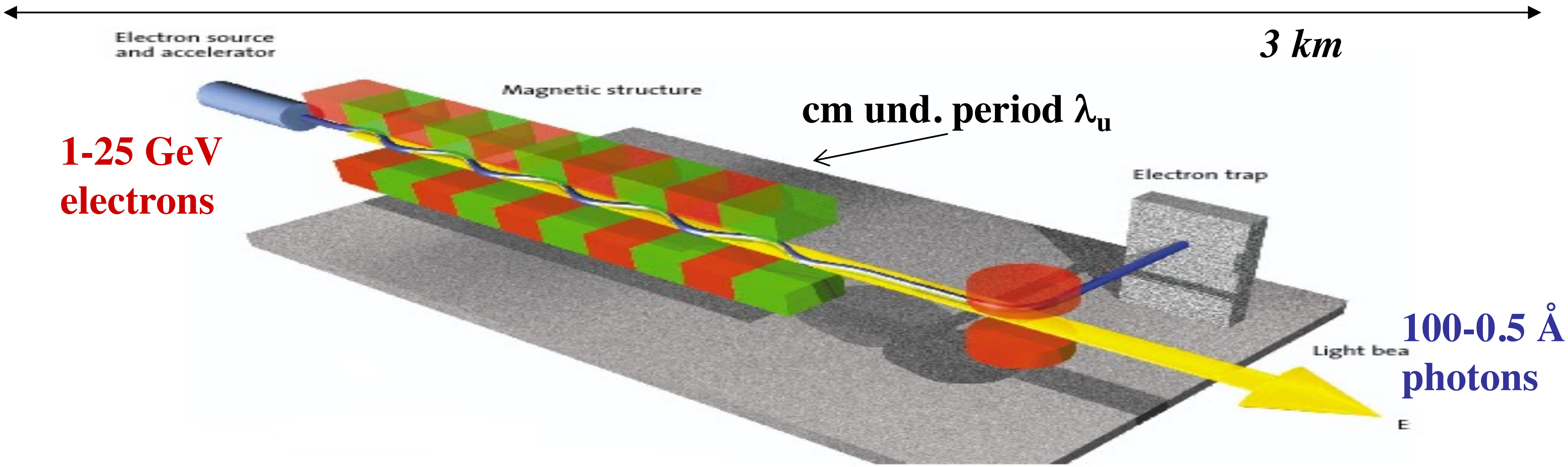
$$\epsilon_\mu^n \simeq \frac{2}{3}\sigma_0\left(\frac{M_e}{2M_\mu}\sqrt{X}-1\right)+\frac{\epsilon_e^n}{\sqrt{X}}$$

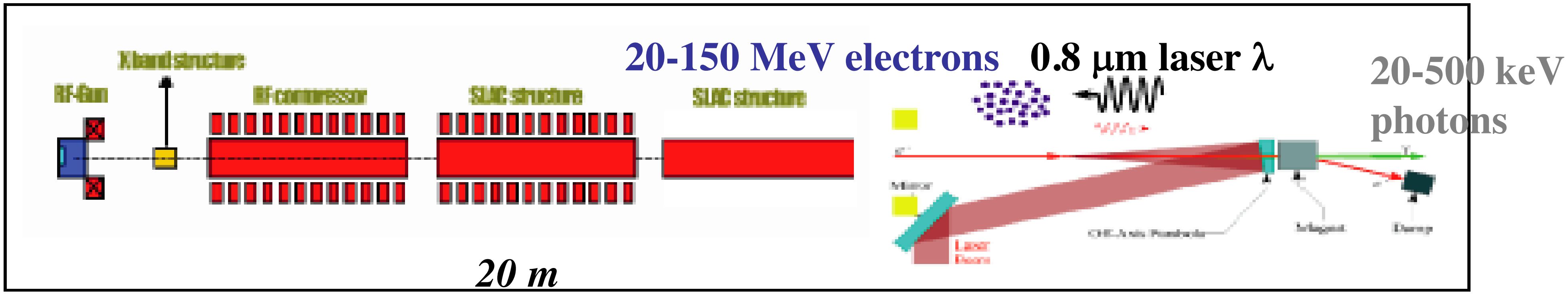
cmp. MAP norm. emitt. $\mathbf{2.5{\cdot}10^4}$ *nm·rad after ionization cooling*



The Classical E.M. view (Maxwell eq.): Thomson Sources as synchrotron radiation sources with electro-magnetic undulator

FEL's and Thomson/Compton Sources common mechanism:
collision between a relativistic electron and a (pseudo)electromagnetic wave





Spread of incidence angle
(resembles emittance)

cooling of photons still effective

as well as heating of electrons

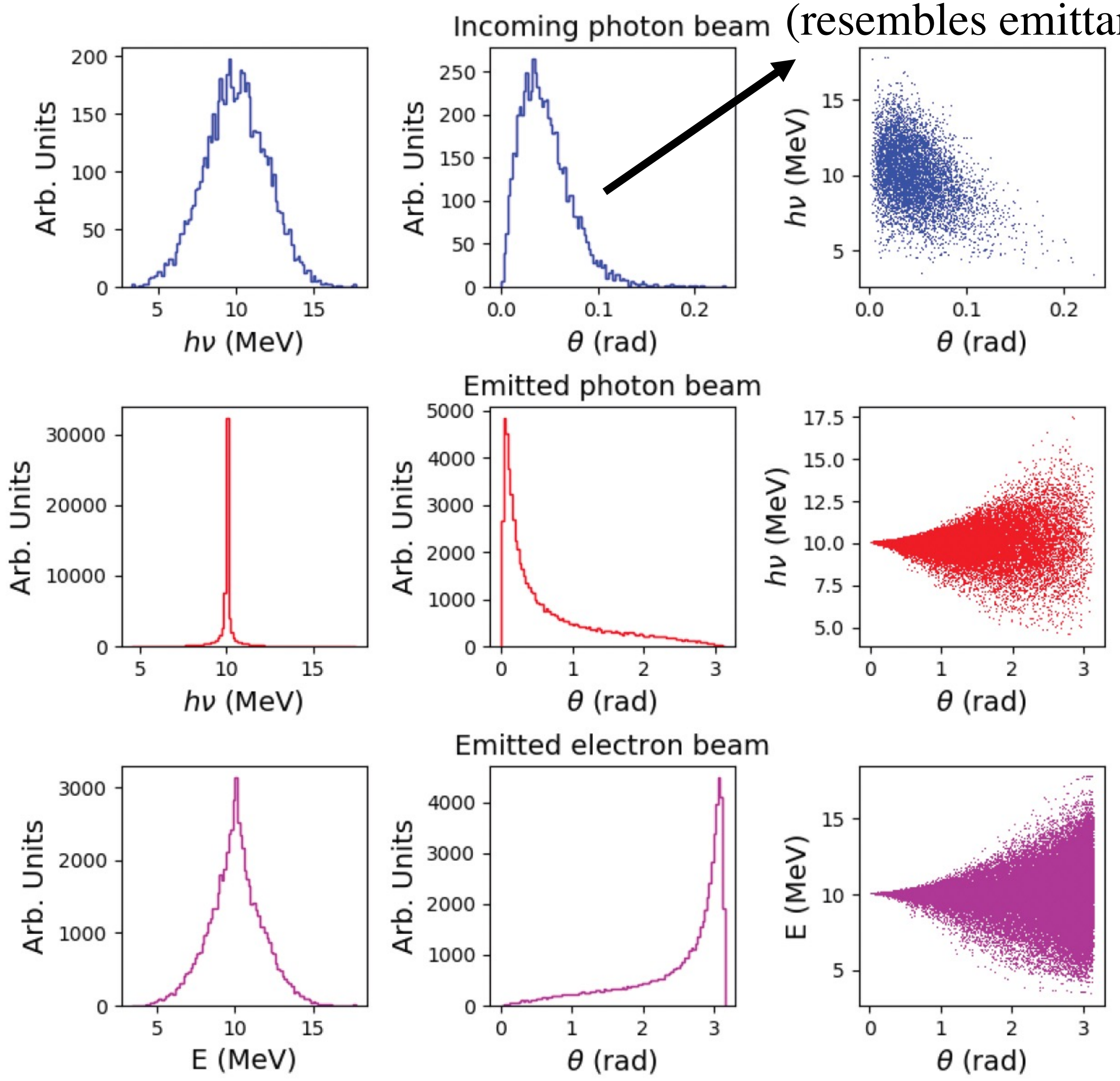


Figure 5: Simulations of SCS with an incoming photon beam displaying a correlation between angle of propagation and photon energy. The results are shown through 9 plots arranged in three rows as in fig. 4. The angular correlation of the incoming photon beam is removed in the interaction thanks to the high recoil factor ($X \sim 1500$).

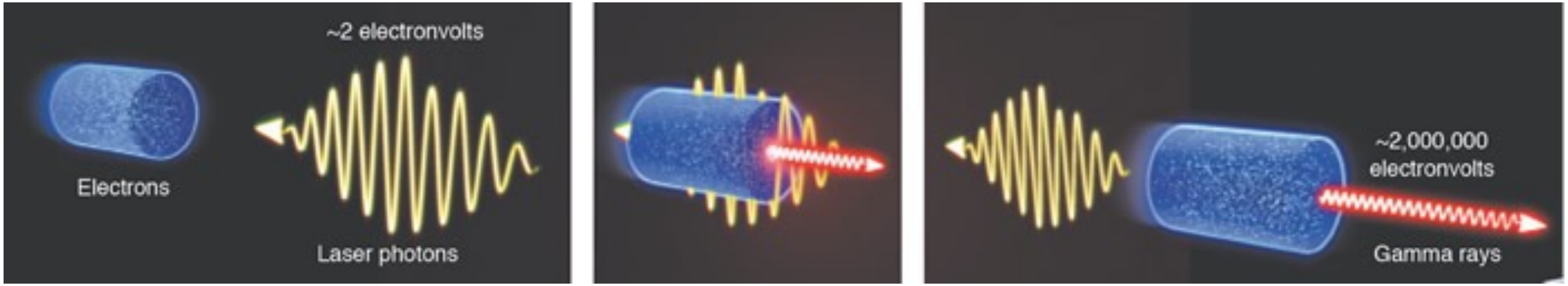
To transform to the Lab ref. system we need to compute γ_{cm}

$$\gamma_{cm} = \frac{E_{lab}}{E_{cm}} = \frac{E_e + h\nu_L}{m_e c^2 \sqrt{1+\Delta}} \cong \frac{\gamma}{\sqrt{1+\Delta}}$$

Then apply a Lorentz transformation

$$\begin{cases} E_{ph} = p^*_{ph}\gamma_{cm}\left(1 + \sqrt{1 - \frac{1}{\gamma^2_{cm}}}\cos\theta^*\right) \\ p_{phx} = p^*_{ph}\sin\theta^*\cos\phi^* \\ p_{phy} = p^*_{ph}\sin\theta^*\sin\phi^* \\ p_{phz} = p^*_{ph}\gamma_{cm}\left(\sqrt{1 - \frac{1}{\gamma^2_{cm}}} + \cos\theta^*\right) \end{cases}$$

I.C.S. : Inverse Compton Scattering



Inverse Compton Scattering: why Inverse?

(direct) Compton Scattering is performed by an energetic photon (X-rays) interacting with an atomic electron (eV)

Inverse Compton Scattering is performed by an energetic electron (MeV-GeV) onto a visible (eV) photon ("inverse" refers to the reaction kinematics, not the dynamics)

Deep Recoil I.C.S. helps attenuating the $\gamma^2\theta^2$ problem

strong focusing to maximize Peak Luminosity according to Petrillo-Serafini criterion

$$S_d \propto \frac{\langle I_e \rangle U_{las}}{\varepsilon_n^2 E_x}$$

30-150 MeV e⁻
20-350 keV X

$\Psi =$

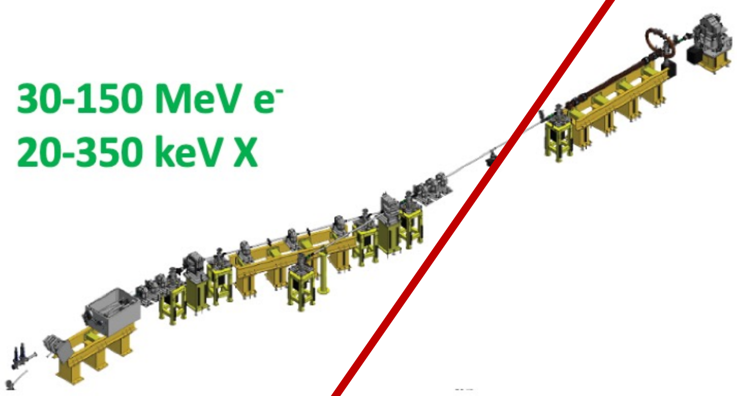


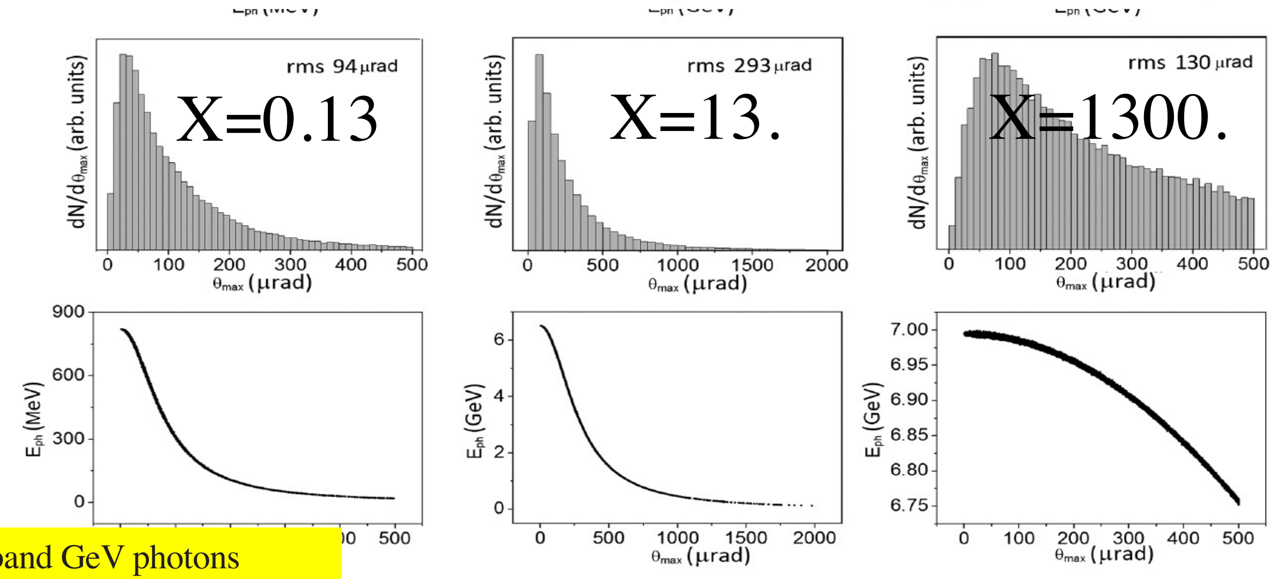
Fig.2 – STAR machine as an example of Paradigm A. Overall length about 12 m.

250-750 MeV e⁻
1-19.5 MeV γ

Fig. 197. Isometric 3D view of Building Layout of the Accelerator Hall & Experimental Areas

$$\bar{P} = \frac{\sqrt{2}\varepsilon_n}{\sigma_x \sqrt{1+X}}$$

$$\gamma_{cm} = \frac{\gamma}{\sqrt{1+X}}$$



…, CAIN simulations. First line spectrum, second line angular distribution, third line energy as a … ft column, case E middle column, case F right column.

R. Hajima and M. Fujiwara, Narrow-band GeV photons generated from an x-ray free-electron laser oscillator, Phys. Rev. Accel. Beams 19, 020702 (2016). XFELO Project

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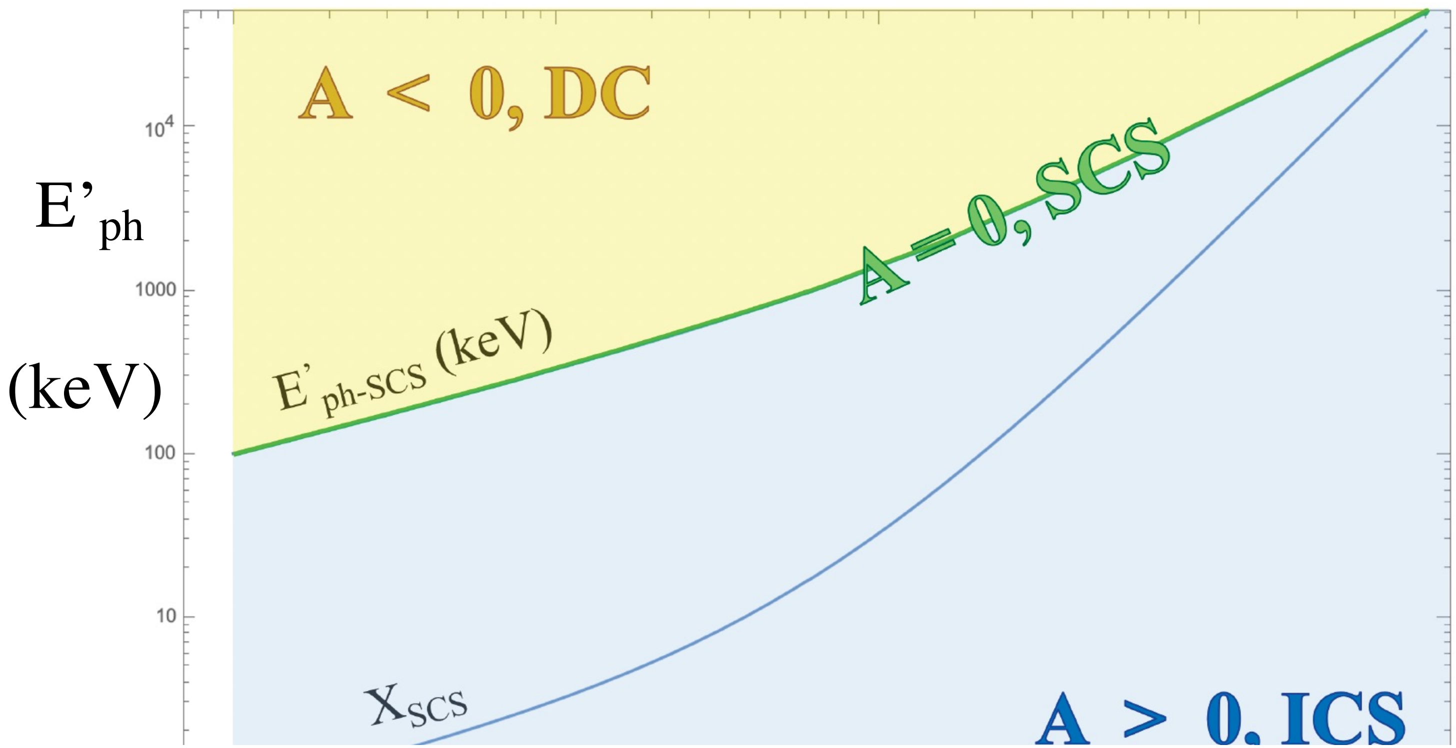

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Article

State of the Art of High-Flux Compton/Thomson X-rays Sources

Vittoria Petrillo [1,2,*,†], Illya Drebot [1,†], Marcel Ruijter [1,†], Sanae Samsam [1,†], Alberto Bacci [1], Camilla Curatolo [1], Michele Opromolla [1,2], Marcello Rossetti Conti [1], Andrea Renato Rossi [1] and Luca Serafini [1,†]

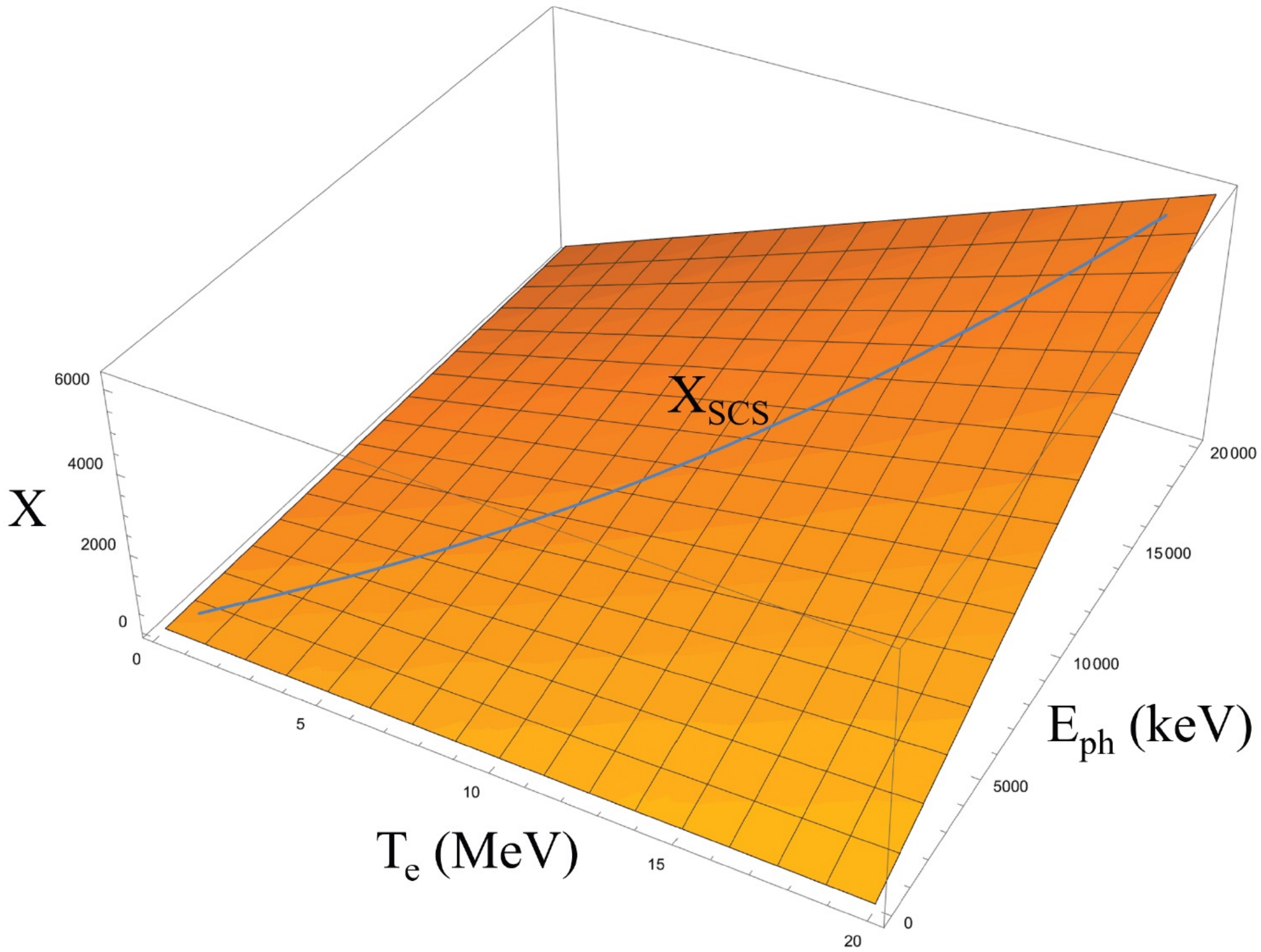
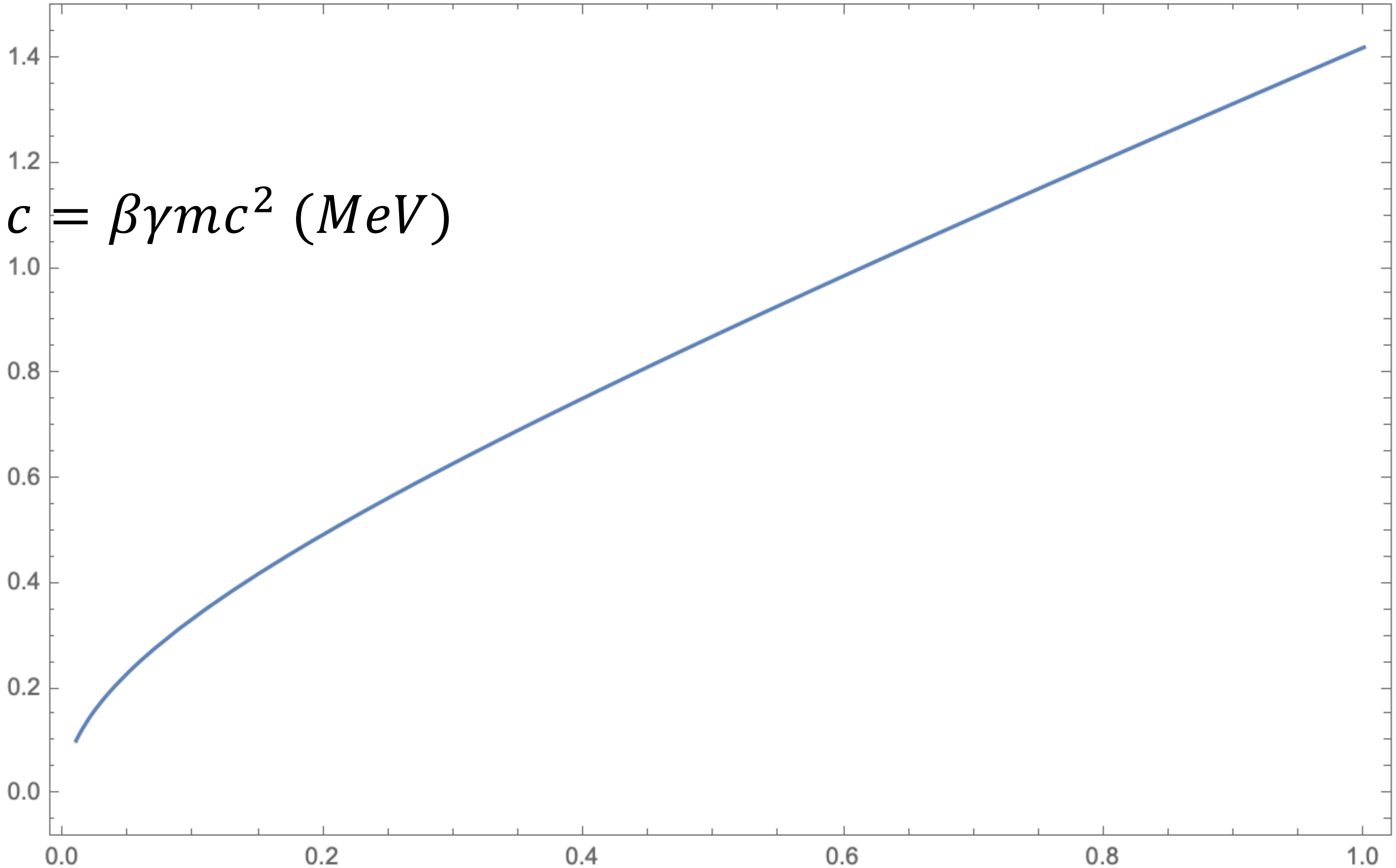


Figure 2: 3D representation of the value of the recoil factor X as a function of the interacting electron kinetic energy (T_e) and of the incident photon energy. The line shows the recoil value in SCS conditions

Symmetric Compton
Scaling of photon energy vs. electron kinetic energy

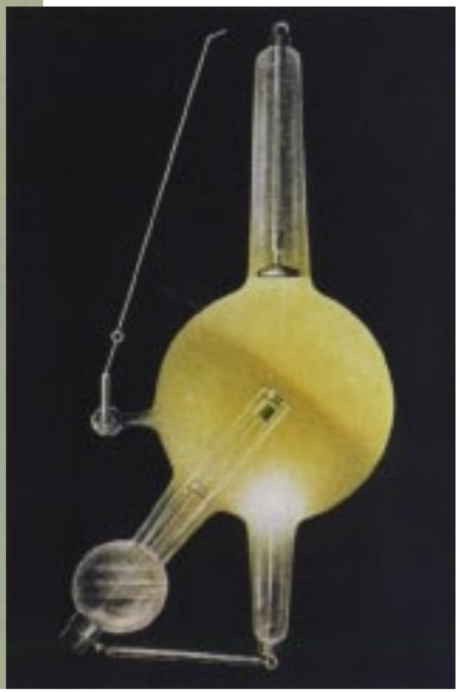
$$E_\omega = p_e c = \beta\gamma mc^2 \ (MeV)$$



$$T_e = E_e - mc^2 = (\gamma - 1)mc^2 \ (MeV)$$

But Arthur Compton fundamental experiments, leading to Compton scattering interpretation and the proof of light quanta existence (the photon) wouldn’t simply be possible without the discovery of X-rays by Roentgen (1895), who in turns couldn’t obtain his result without the vacuum tubes invented by William Crookes, who in turns exploited the glass-to-metal welding technique invented by Heinrich Geissler.

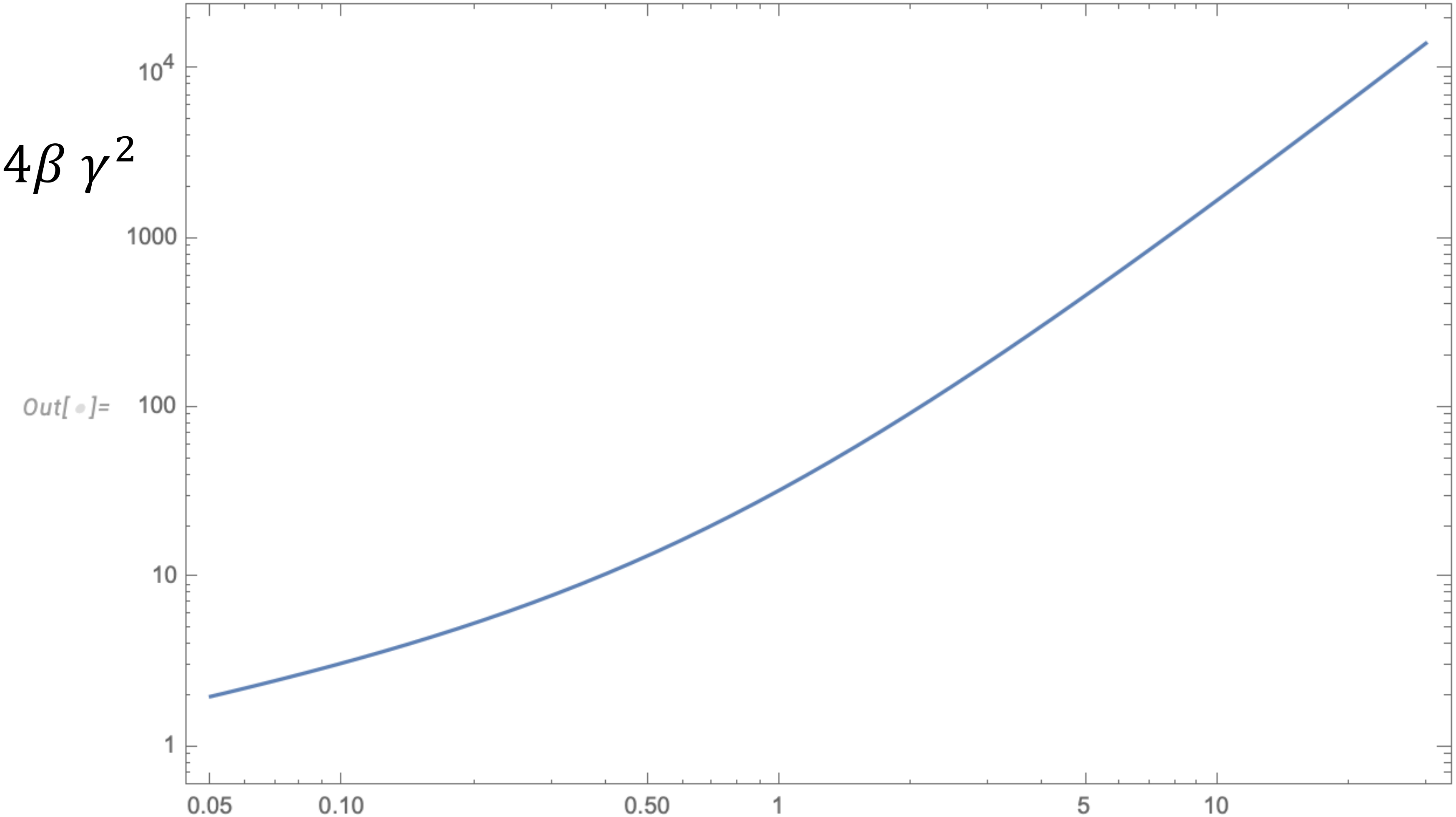
The Paradigm of Particle Accelerators!



Symmetric Compton
Scaling of X, recoil factor, vs. electron kinetic energy

$$E_\omega = p_e c = \beta\gamma mc^2$$

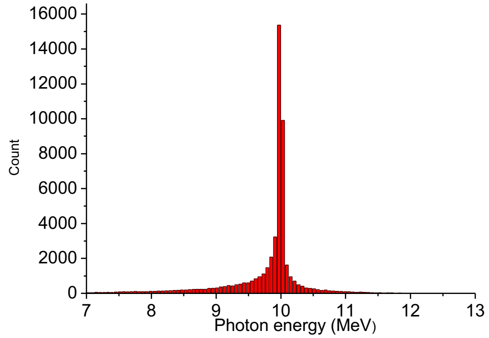
$$X = 4\beta\,\gamma^2$$

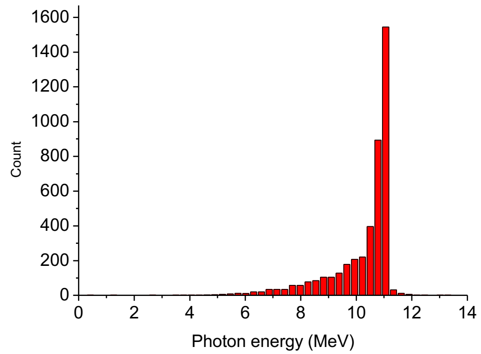


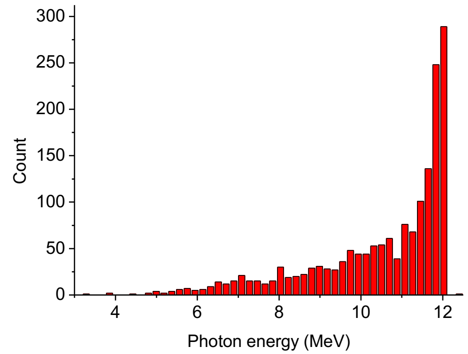
$$T_e = E_e - mc^2 = (\gamma - 1)mc^2 \text{ (MeV)}$$

Fixed recoil X=1531
Moving away from Symmetric Compton

incident $\Delta E_{phot}/E=20\%$



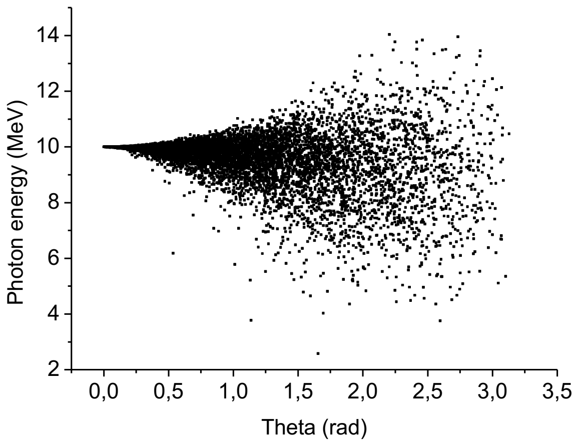


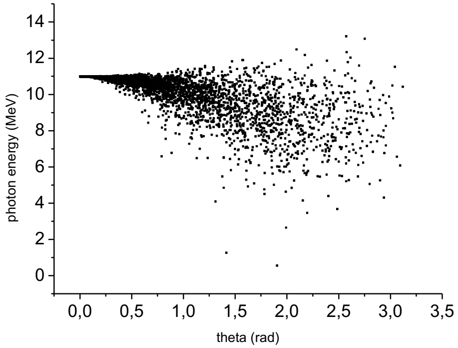


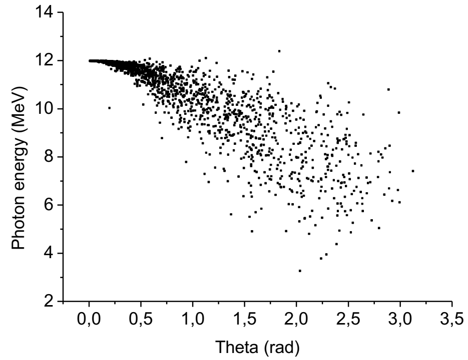
Initial Electron energy=10.013 MeV, **Initial Photon energy=10.0MeV**

Initial Electron energy=11.0 MeV, **Initial Photon energy=9.08MeV**

Initial Electron energy=12.0 MeV, **Initial Photon energy=8.33 MeV**



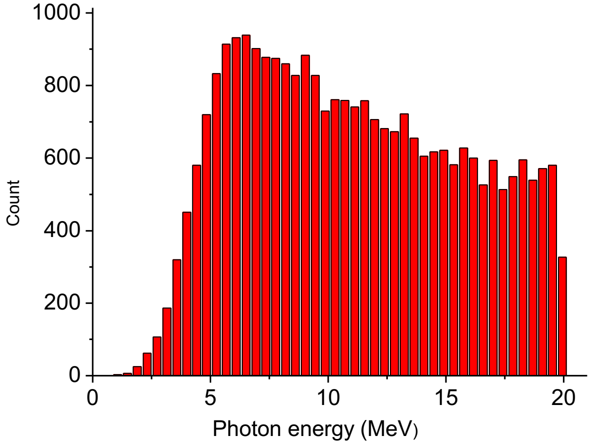


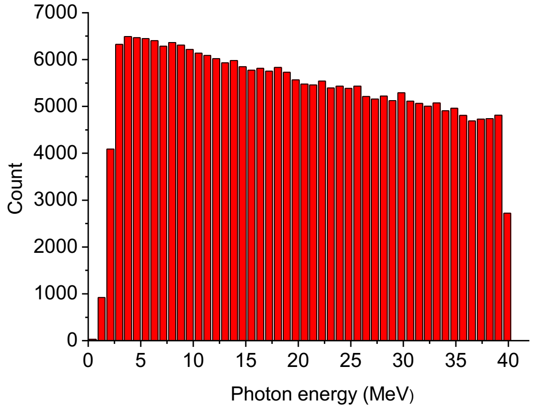


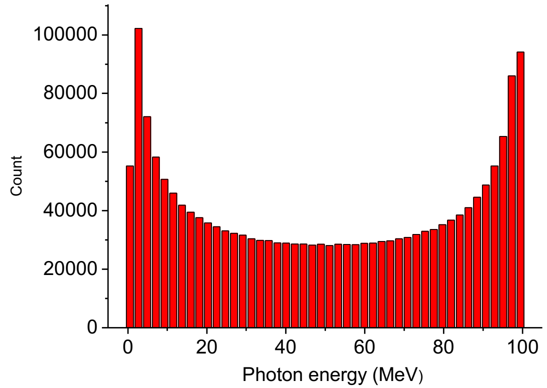
No $\gamma^2\theta^2$ disease

The onset of $\gamma^2\theta^2$ disease

Fixed recoil X=1531
going from SCS to ICS



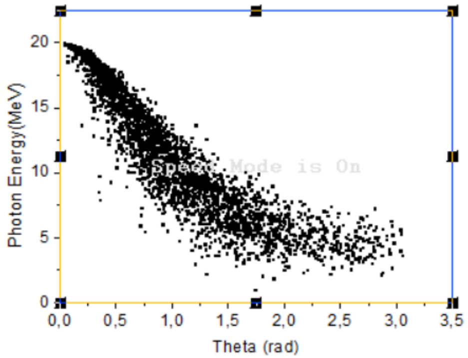


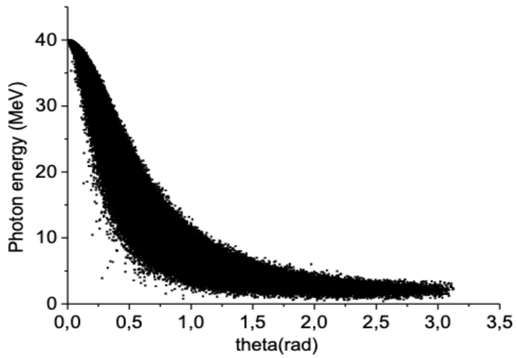


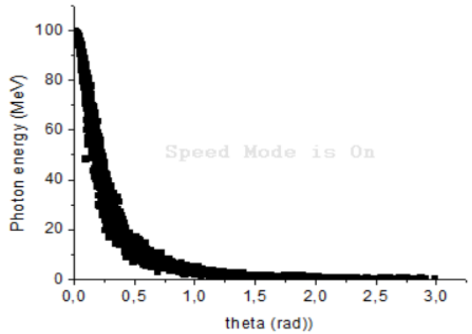
Initial Electron energy=20.0 MeV,
Initial Photon energy=5 MeV,
$\Delta E_{phot}/E=0.2$

Initial Electron energy=40.0 MeV,
Initial Photon energy=2.5 MeV,
$\Delta E_{phot}/E=0.2$

Initial Electron energy=100.0 MeV,
Initial Photon energy=1 MeV,
$\Delta E_{phot}/E=0.2$







full $\gamma^2\theta^2$ disease – the moustache pattern

The four-momentum of a particle is defined as $\mathbf{p} = \left(\frac{E}{c}, p_x, p_y, p_z\right)$, where E is the total energy of the particle, c is the speed of light in vacuum, and p_x, p_y, p_z are the components of the particle's momentum along the x, y, z axes respectively.

Let us consider the case of a head-on collision between a photon and a counter-propagating electron along the z-axis. Before the collision, the electron and the photon have the following four-momenta:

$$\begin{aligned} \mathbf{p_e} &= (\gamma m_0 c, 0, 0, \beta\gamma m_0 c), \\ \mathbf{p_{ph}} &= \left(\frac{E_{ph}}{c}, 0, 0, -\frac{E_{ph}}{c}\right), \end{aligned} \tag{25}$$

and the total four-momentum is:

$$\mathbf{p_{tot}} = \left(\gamma m_0 c + \frac{E_{ph}}{c}, 0, 0, \beta\gamma m_0 c - \frac{E_{ph}}{c}\right). \tag{26}$$

The energy available in the center of mass E_{cm}, in terms of the recoil factor introduced in Eq. (2), is:

$$E_{cm} = c\sqrt{\mathbf{p_{tot}} \cdot \mathbf{p_{tot}}} = m_0 c^2 \sqrt{(1+\beta)\frac{X}{2} + 1} = \\ = m_0 c^2 \sqrt{(1+\beta)\frac{2E_e E_{\text{ph}}}{(m_0 c^2)^2} + 1}. \tag{27}$$

The different regimes of Compton scattering can be analyzed in terms of their center of mass energy E_{cm}.

For the DC regime ($\beta = 0$, $\gamma = 1$):

$$E_{cm-DC} = m_0 c^2 \sqrt{\frac{2E_{\text{ph}}}{m_0 c^2} + 1}. \tag{28}$$

On the opposite side, in the ICS regime ($\beta \simeq 1$), we obtain:

$$E_{cm-ICS} = m_0c^2\sqrt{X+1} = m_0c^2\sqrt{\frac{4\gamma E_{\text{ph}}}{m_0c^2}+1}. \tag{29}$$

Finally, for the SCS regime ($E_{\text{ph}} = \beta E_e = \beta\gamma m_0c^2$):

$$E_{cm-SCS} = (1+\beta)\gamma m_0c^2. \tag{30}$$

In this peculiar situation, $E_{cm} \propto \gamma$ like in a collider. Being $\gamma_{cm} \equiv E_{lab}/E_{cm}$ the Lorentz boost factor associated to the center of mass reference frame. In SCS we have $\gamma_{cm} = 1$ (because $E_{lab-SCS} = E_{cm-SCS}$), meaning that the center of mass of the system is at rest in the laboratory system, and the radiation produced here has the same angular and spectral distribution seen by a detector at rest in the lab. On the other hand, DC and ICS exhibit a dependence of the available energy E_{cm} typical of a fixed target collision, where E_{cm} scales like $E_{cm} \propto \sqrt{T_p}$, where T_p is the projectile kinetic energy. ICS regime is characterized by $\gamma_{cm} \gg 1$ since the center of mass reference frame is almost traveling with the electron (as shown in Ref. [17] $\gamma_{cm} = \gamma/(1+X)$).

To obtain the momentum components of the emitted particles in the laboratory frame we have to apply the Lorentz transformations to the momenta values in CM:

$$\begin{cases} \nu = \gamma_{CM}(k^* + k_x^*\beta_x + k_y^*\beta_y + k_z^*\beta_z) \\ k_x = k^*\beta_x\gamma_{CM} + k_x^*\dfrac{1+\gamma_{CM}^2\beta_x^2}{1+\gamma_{CM}} + k_y^*\dfrac{\gamma_{CM}^2\beta_x\beta_y}{1+\gamma_{CM}} + k_z^*\dfrac{\gamma_{CM}^2\beta_x\beta_z}{1+\gamma_{CM}} \\ k_y = k^*\beta_y\gamma_{CM} + k_x^*\dfrac{\gamma_{CM}^2\beta_x\beta_y}{1+\gamma_{CM}} + k_y^*\dfrac{1+\gamma_{CM}^2\beta_y^2}{1+\gamma_{CM}} + k_z^*\dfrac{\gamma_{CM}^2\beta_y\beta_z}{1+\gamma_{CM}} \\ k_z = k^*\beta_z\gamma_{CM} + k_x^*\dfrac{\gamma_{CM}^2\beta_x\beta_z}{1+\gamma_{CM}} + k_y^*\dfrac{\gamma_{CM}^2\beta_y\beta_z}{1+\gamma_{CM}} + k_z^*\dfrac{1+\gamma_{CM}^2\beta_z^2}{1+\gamma_{CM}} \end{cases} \tag{C.0.6}$$

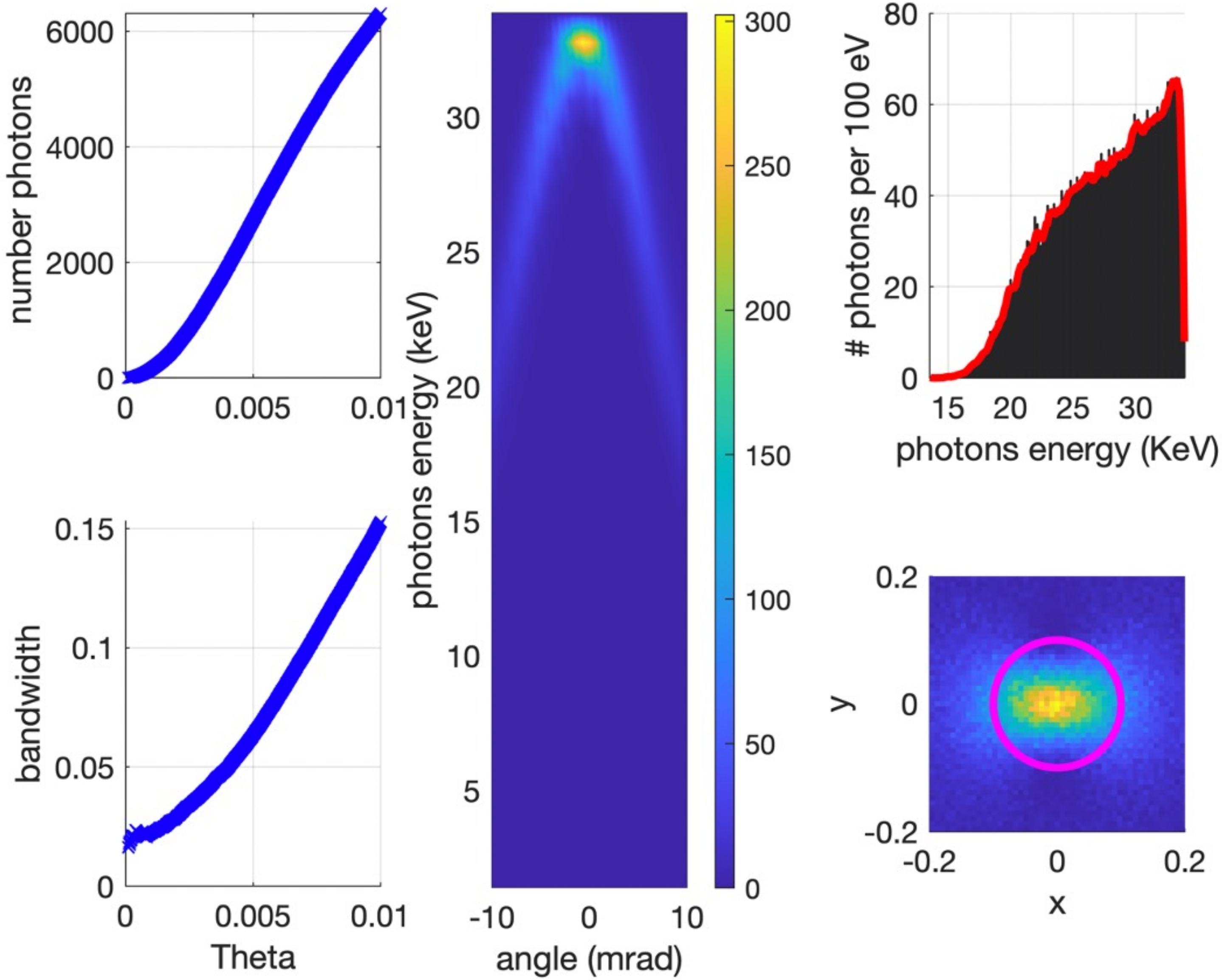
where $\underline{\beta}_{CM} = (\beta_x,\ \beta_y,\ \beta_z)$. If the scattering is head-on along the z axis, the above transformations simplify in

$$\begin{cases} \nu = k^*\gamma_{CM}\left(1+\beta_{CM}\cos\theta^*\right) \\ k_x = k^*\sin\theta^*\cos\phi^* \\ k_y = k^*\sin\theta^*\sin\phi^* \\ k_z = k^*\gamma_{CM}\left(\beta_{CM}+\cos\theta^*\right). \end{cases} \tag{C.0.7}$$

$$\frac{E'_{ph}}{E_{tot}} = \frac{X}{(1+X)(1+\frac{X}{4\gamma^2})}$$

$$\frac{E'_{e}}{E_{tot}} = 1 - \frac{X}{(1+X)(1+\frac{X}{4\gamma^2})}$$

BriXSinO's ICS source – Illya Drebot with CAIN – ICS Moustache



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