NUCLEAR STRUCTURE AND REACTIONS

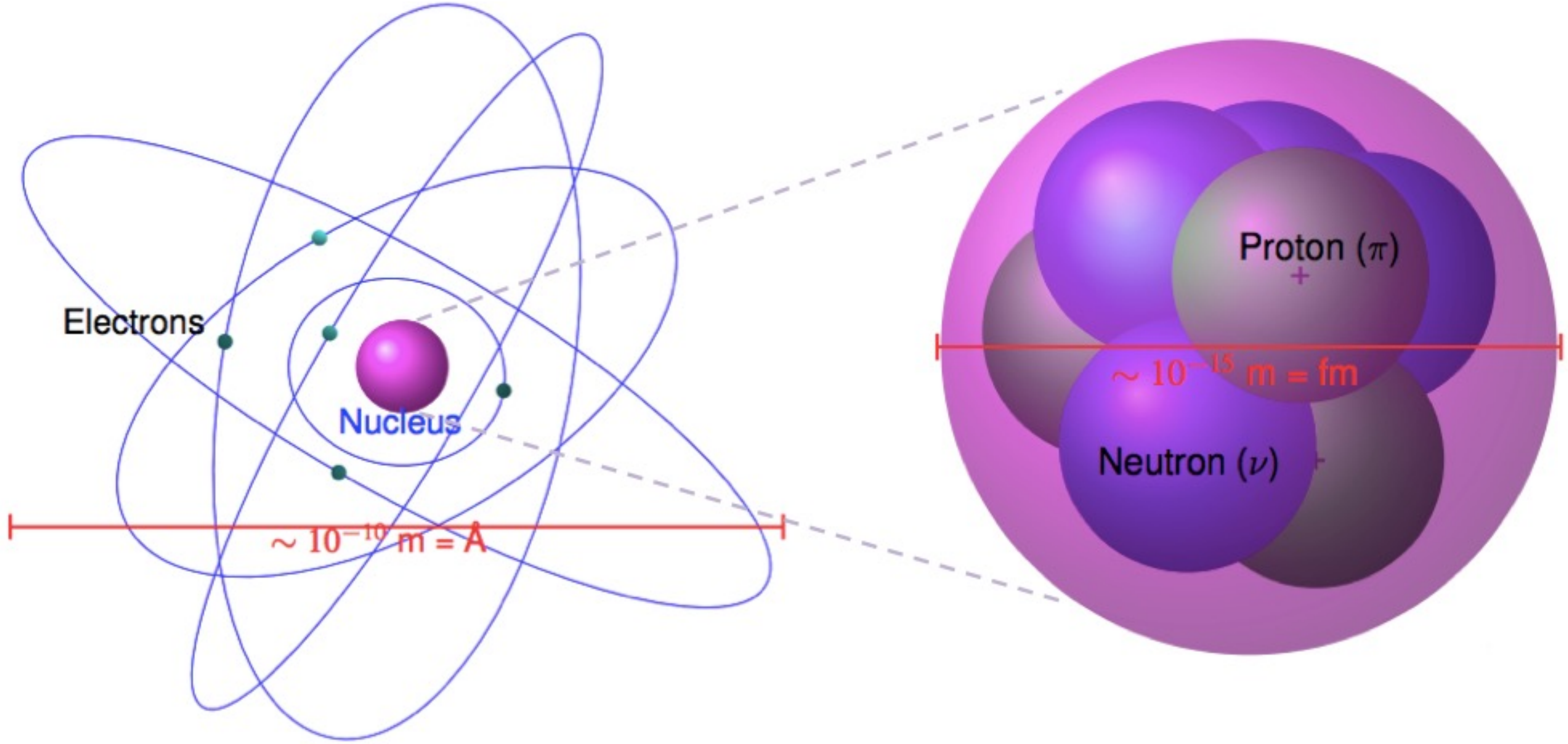
Luna Pellegri – University of the Witwatersrand and iThemba LABS

OUTLINE

- Nuclear properties
  - The Nucleus
  - The Nuclear Chart
  - Interactions in the nucleus
  - Mass and Binding Energy
  - Properties of nuclear states
- Nuclear Models
  - The Semi-empirical mass formula
  - Nuclear Shell Model
- Nuclear Reactions
  - General characteristics
  - Conservation Laws
  - Q-value and Cross Section
  - Direct reactions
  - Compound nuclear reactions
- Radioactive Decay
  - Alpha decay
  - Spontaneous fission
  - Beta decay
  - Gamma decay

NUCLEAR PROPERTIES

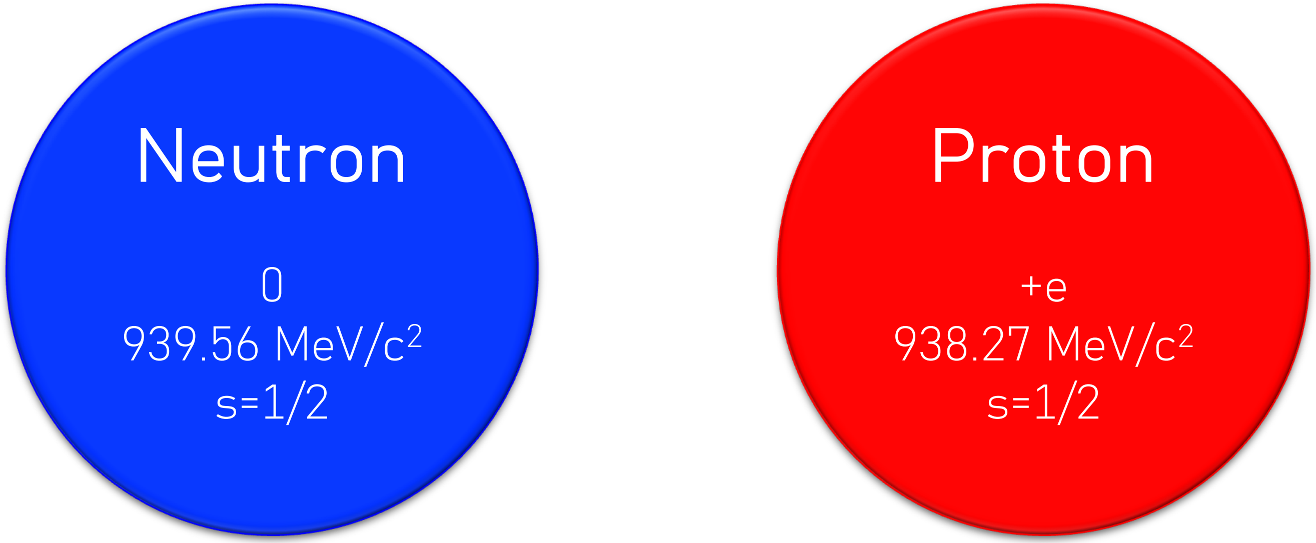
THE ATOM AND THE NUCLEUS



- Atom is a neutral system;
- Atom dimension: $\sim 10^{-10}$ m;
- Atomic excitations: $\sim$ 1-10^5 eV, caused by transitions between electronic states

- Nucleus is a positively charged;
- Nucleus dimension: $\sim 10^{-15}$ m;
- Nuclear excitations: $\sim 10^5$–10^8 eV, caused by transitions between nuclear states

THE NUCLEUS



Nucleons are quantum mechanical objects:

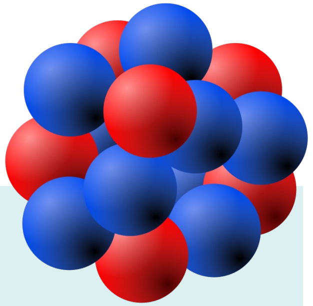
Fermions → their intrinsic spin is half-integer (specifically, $s = 1/2$, $m_s = \pm 1/2$);

Radius: $r \sim 1 \times 10^{-15}$ m, or 1 fm (femtometer or fermi);

Charge: p → + e and n → 0

Mass: p → 938.27 MeV/c^2 and n → 939.56 MeV/c^2

SOME BASICS…



- *Nucleons*, a.k.a. neutrons and protons
- Convention for referring to a particular configuration, a.k.a. *nuclide* $^{A}_{Z}\boldsymbol{Element}_{N}$

  A = # of nucleons, Z = # of protons, N = # of neutrons

- The chemical symbol is from the periodic table that corresponds to Z (i.e. He, Zn, Pb …)
- Since A=Z+N and Z is indicated by the chemical symbol, more common notation is: $^{A}\boldsymbol{Element}$
- Nuclides with the **same Z** but different N are ***isotopes*** *(this term is often used in lieu of nuclides)*

  ^{207}Pb, ^{208}Pb, ^{209}Pb ...

- Nuclides with the **same N** but different Z are ***isotones:*** ^{142}Nd, ^{141}Pr, ^{140}Ce, ^{139}La, ^{138}Ba, …
- Nuclides with the **same A** are ***isobars:*** ^{36}Ar, ^{36}Cl, ^{36}S, …
- Nuclides with special names: ^{1}H = proton, ^{2}H = deuteron (d), ^{3}H = triton (t), ^{4}He = α, ^{3}He = helion *(rarely used)*

SOME BASICS…

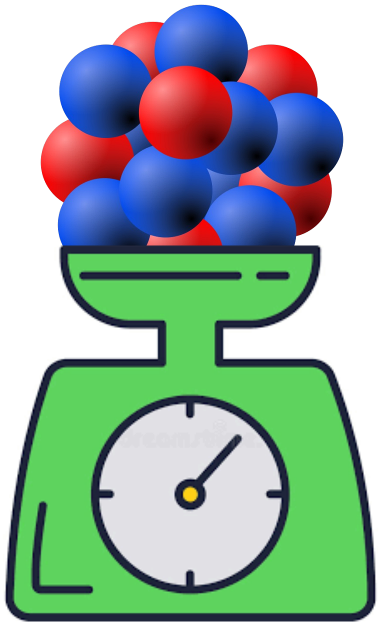
- A nucleus's mass is roughly: M(Z,A) = A*amu

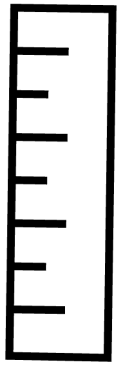
  1 amu = atomic mass unit = u = 931.494 MeV/c^2 ≈ 1.66 x 10^{-24} g

- The amu is defined such that M(^{12}C) ≡ 12 u
- A nucleus's (charge) radius is roughly: R(Z,A) = (1.2 fm)*$A^{1/3}$

  fm = femtometer (a.k.a. fermi) = 10^{-15}m

- The radius of a nucleon is often referred to as r_0=1.2 fm





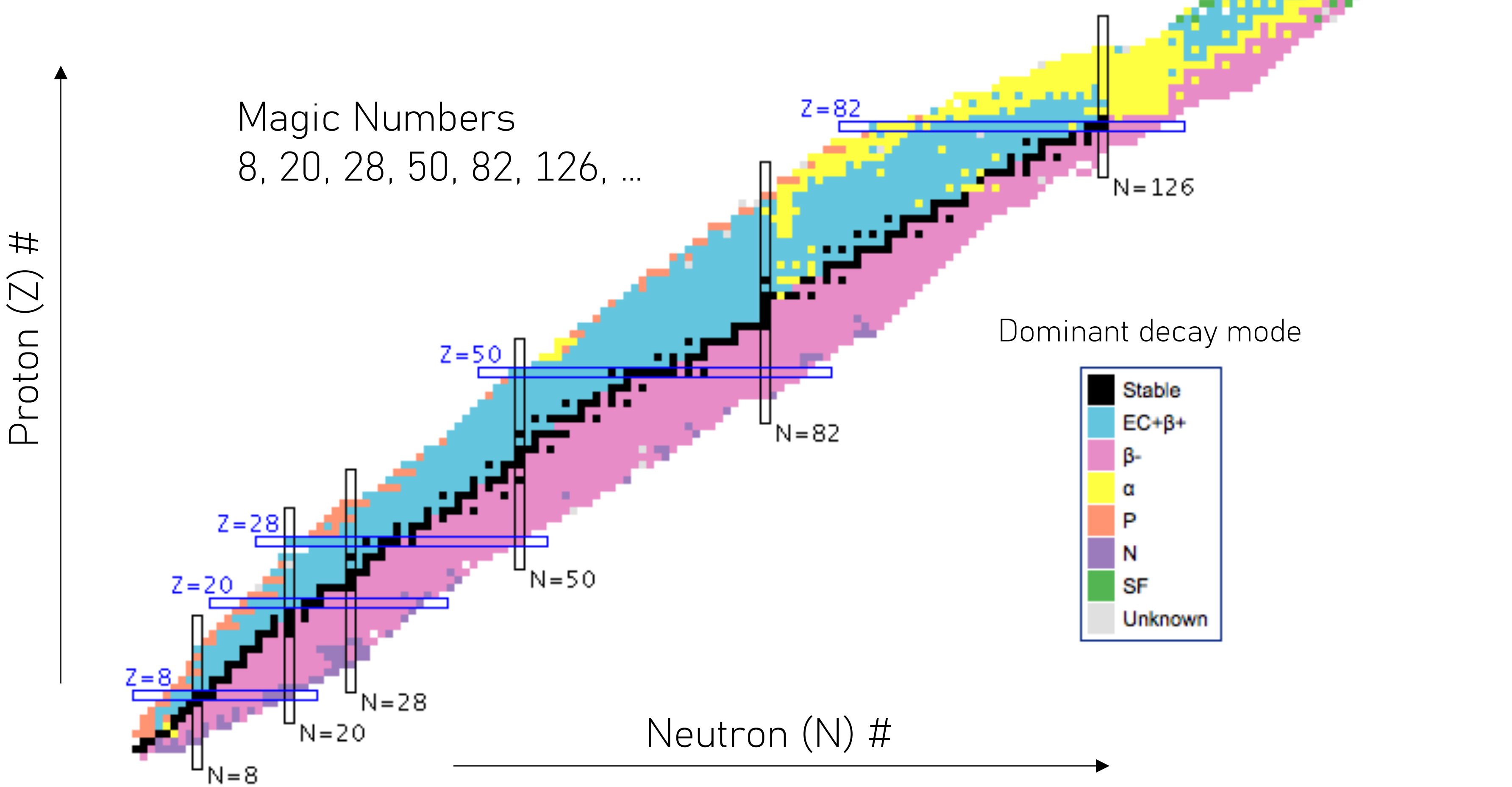
The atomic mass unit (AMU or amu) of an element is a measure of its atomic mass. Also known as the dalton (Da) or unified atomic mass unit (u), the AMU expresses both atomic masses and molecular masses. AMU is defined as one-twelfth the mass of an atom of carbon-12 (12C).

THE NUCLEAR CHART

Nuclear charts are used to classify the nuclei and they can reveal some interesting effects based on what it has been observed over several decades.

There are roughly **3000** known (**experimentally observed**) nuclei in the universe (**~ 250 stable**).
Recent predictions suggest that there may be more than **7000 bound nuclear systems** that are able to exist [J. Erler et al., Nature 486, 509512 (2012)].

Most radioactive nuclei are not found in nature yet many of them are critical ingredients in the creation of the stable isotopes/elements in the cosmos.



For information on nuclei

National Nuclear Data Center

https://www.nndc.bnl.gov

INTERACTIONS IN THE NUCLEUS

Force	Theory	Mediator	Relative Strength	Range (m)
Strong	QCD	gluons(g), pions (π) - nucleons	1	$\sim 10^{-15}$
Electromagnetic	QED	photon (γ)	$\alpha = \frac{1}{137} \approx 10^{-2}$	∞
Weak	Electroweak	$W^{\pm}$ and Z bosons	$\sim 10^{-5}$	$\sim 10^{-18}$
Gravitational	*Gravity*	*unknown*	$\sim 10^{-38}$	∞

Strong interaction: attractive at internucleon distance $\simeq$ 1 fm = 10^{-15} m. Very short range, negligible when the nucleons are only few femtometers apart.

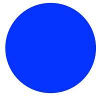
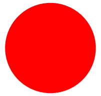
Coulomb interaction: repulsive in the nucleus due to the presence of protons. Long range interaction $\left(\propto \frac{1}{r}\right)$
It will dominate for large enough mass numbers ⇒ number of nucleons will be limited.

Weak interaction: short range interaction but it is orders of magnitude weaker than the strong interaction ⇒ the contribution to the nuclear binding energy is negligible. Governs particle decay.

Another key factor ⇒ **Pauli principle**: the probability to find two identical particles of half-integral spin in the same quantum state must always vanish. This determines why atomic subshells fill in a certain way. (Nuclear shell model)

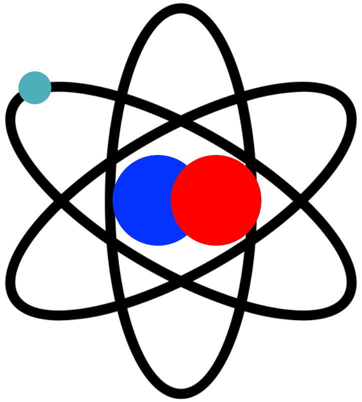
MASS OF NUCLEI AND BINDING ENERGY (BE)

^{2}H mass ?

 +  +  = 1878.348 MeV/c^2

939.565 MeV/c^2 938.272 MeV/c^2 0.511 MeV/c^2

^{2}H



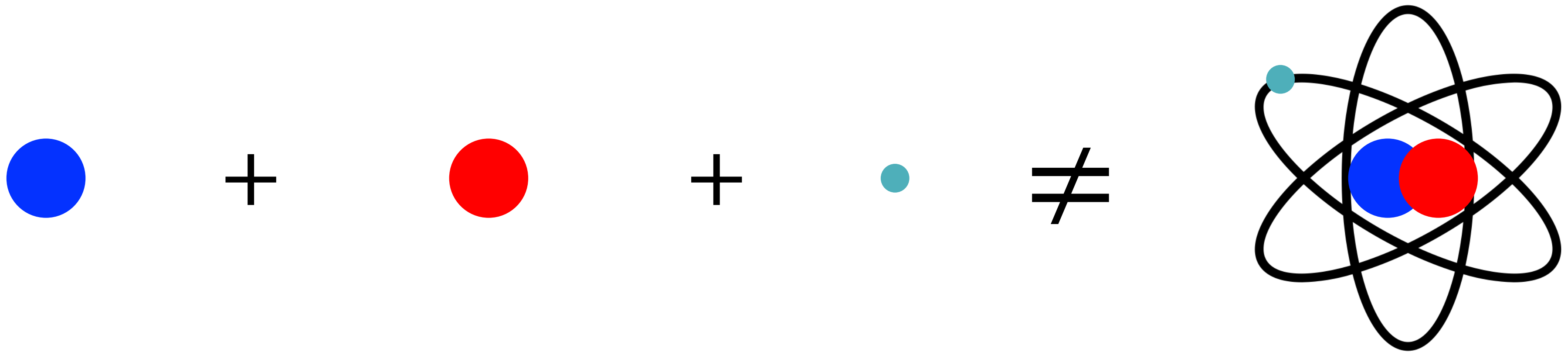
Atomic Mass Data Center – https://www-nds.iaea.org/amdc/

2.014102 amu → 1876.1240 MeV/c^2

Atomic Mass ≠ sum of the mass of constituent

Mass difference – 2.224 MeV

MASS OF NUCLEI AND BINDING ENERGY (BE)



- This difference in mass is known as the **mass defect**

  The nuclear mass is slightly less then the sum of the individual nucleon masses.

- Nucleons are bound together, and this binding requires energy which is provided by a reduction in mass.

- This is a result of Einstein's postulate $E = mc^2$

  The **binding energy (BE) is paid-for via a mass reduction**

- This lower-energy state of a nucleus is the only reason why nucleons cluster together forming nuclei.

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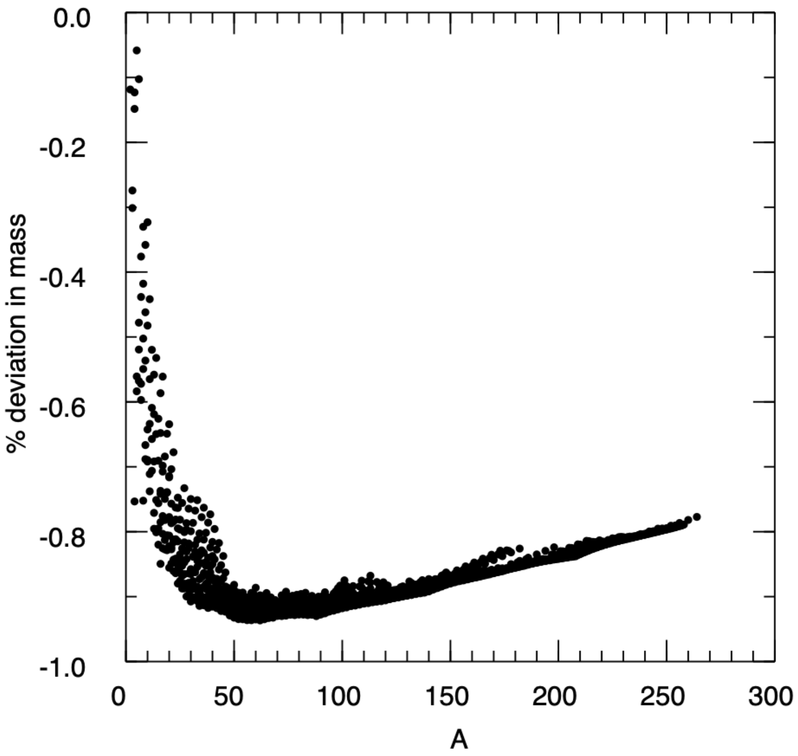
Deviation of the atomic masses from that expected from the sum of its constituents.



Lecture Notes in Nuclear Structure Physics
B. Alex Brown

MASS OF NUCLEI AND BINDING ENERGY (BE)

Binding energy - BE(N,Z): Energy released in assembling a given nucleus from its constituents' nucleons or the energy required to separate a nucleus into its constituents.

$$BE(N,Z) = \{ZM(\,^{1}H) + NM_{n} - M(N,Z)\}c^{2}$$

With the masses generally given in atomic mass units, it is convenient to include the unit conversion factor in c^2, thus: c^2 = 931.494 MeV/u.

^{2}H: $BE(1,1) = \{1(1.007825\ u) + 1(1.008665\ u) - 2.014102\ u\}\, 931.494\ MeV/u = 2.224\, MeV$

Commonly used instead of binding energy is the “mass excess”

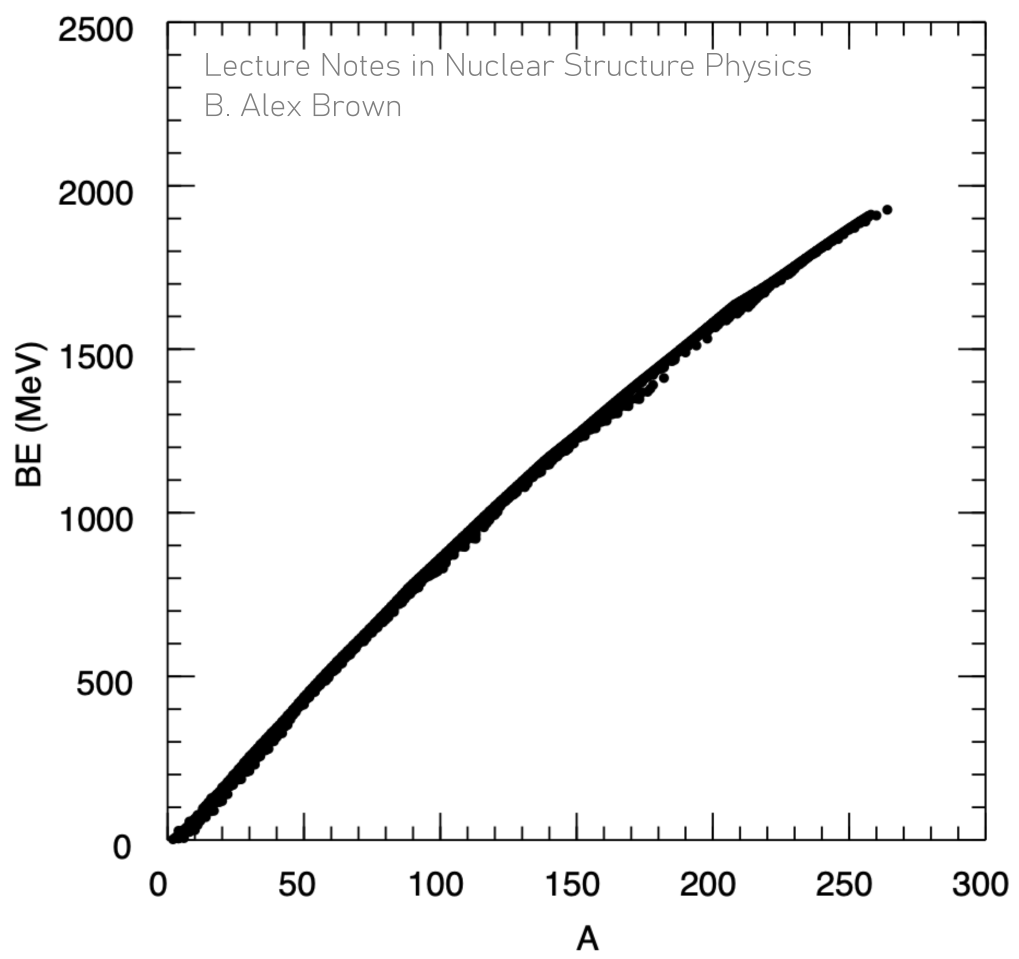
$$ME(N,Z) = \{M(N,Z) - (N+Z)\}c^{2}$$

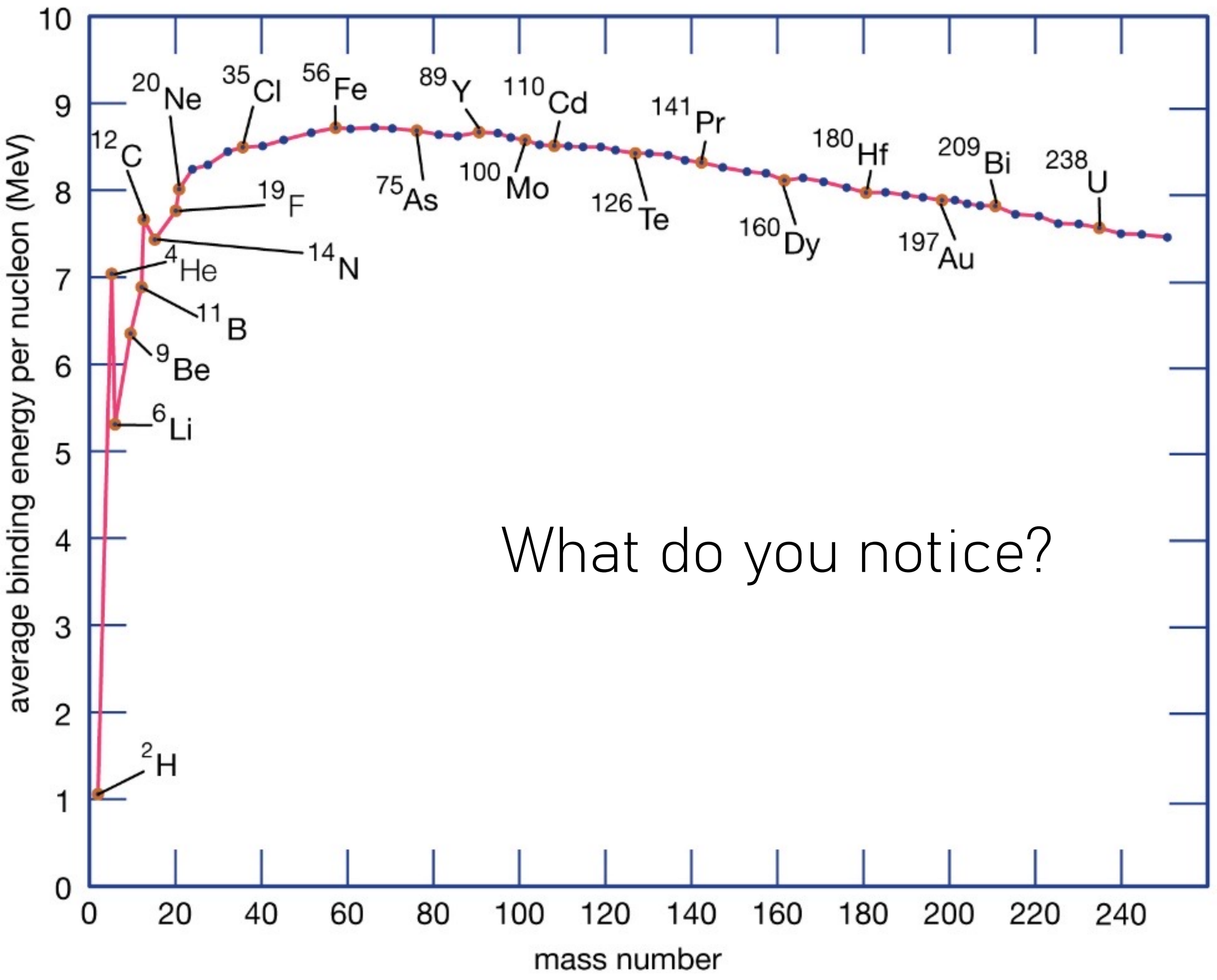
Difference between its actual mass and its mass number in atomic mass unit

Mass defect is mass excess with a minus sign

^{2}H: $ME(1,1) = \{2.014102\ u\ - 2\}\, 931.494\ MeV/u = 13.1357\, MeV$

BINDING ENERGY (BE)





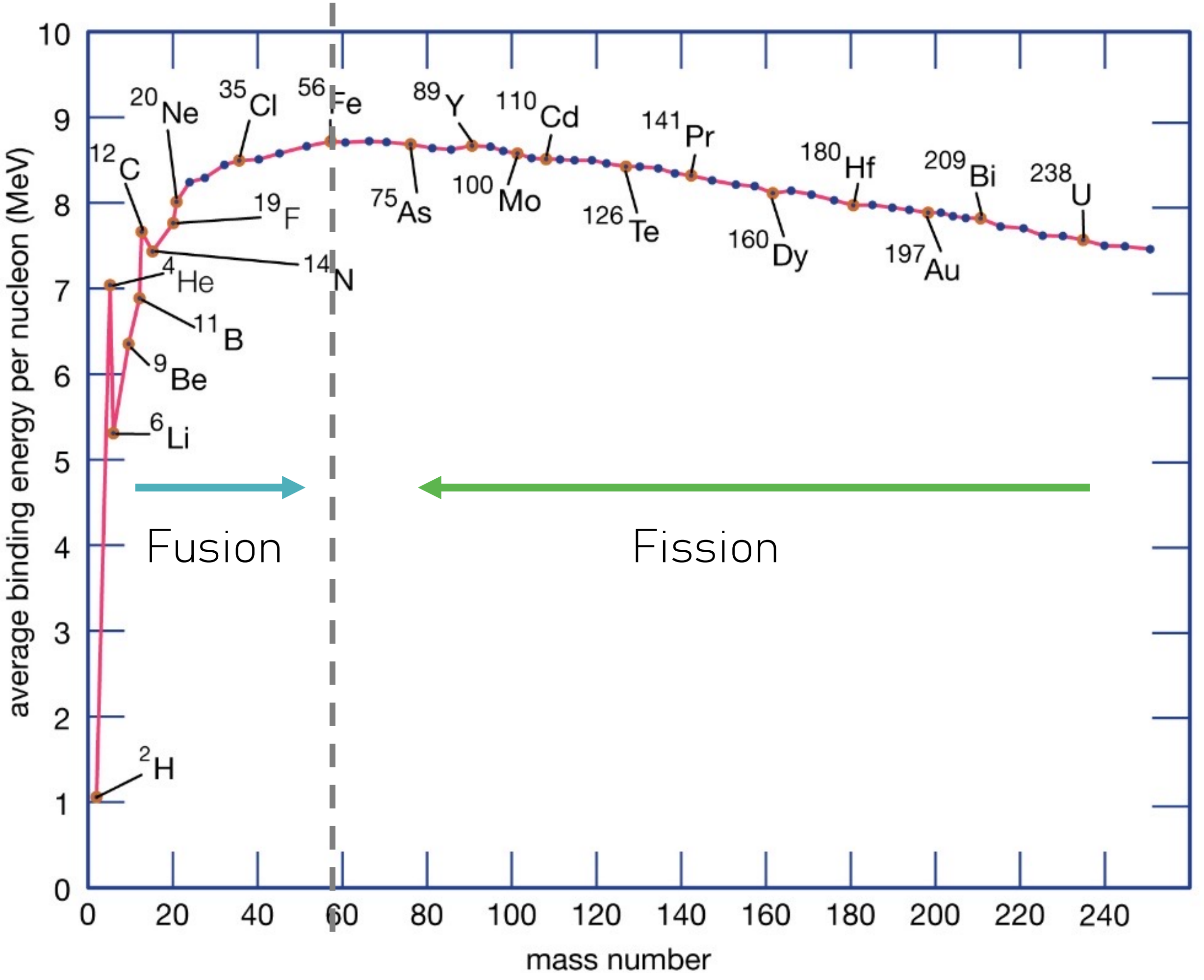
The binding energy increases nearly linearly with energy
More nucleons would lead to more binding since they attract each other via the strong force

BINDING ENERGY (BE)

- **BE/A ~ 8 MeV/A**, for most nuclei

  ⇒ **nuclear force saturates** such that only each nucleon can interact with a few of its neighbors. (nuclear force – short distance range ~ 1 fm).

- The **most bound nuclei** are in the region of **A ~ 56-62.**
- **Some structure in this curve** also exists (particularly for ^{4}He) that **results from quantum effects of the nucleus**. (Shell effects and magic numbers).
- Nuclei on the left of the peak can release energy by joining together (**Nuclear Fusion**)
- Nuclei on the right of the peak can release energy by breaking apart (**Nuclear Fission**)



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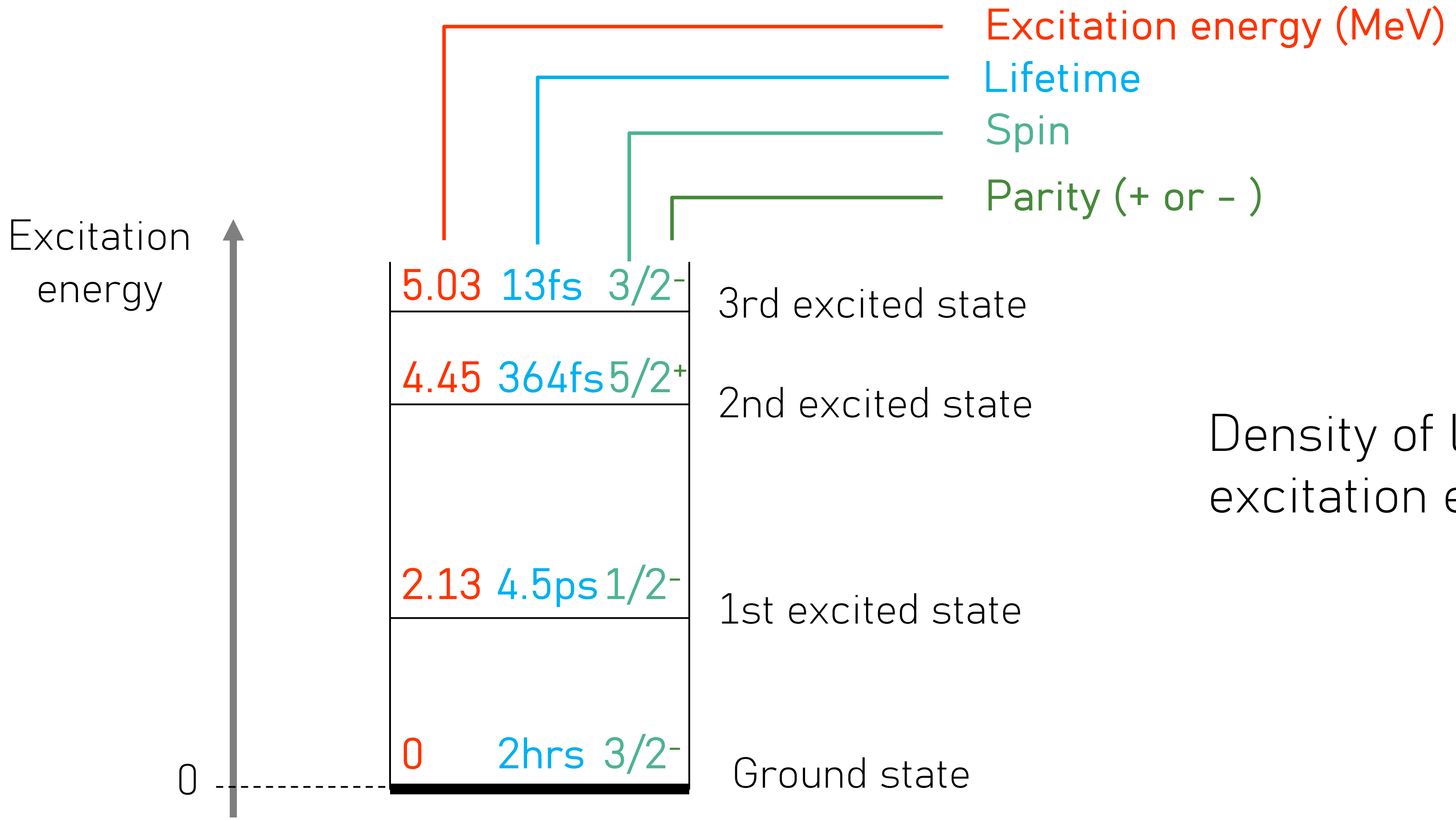
PROPERTIES OF NUCLEAR STATES

Nucleons in the nucleus can only have discrete energies. Therefore, the nucleus as a whole can be excited into discrete energy levels (excited states)



Density of levels increases with excitation energy exponentially.

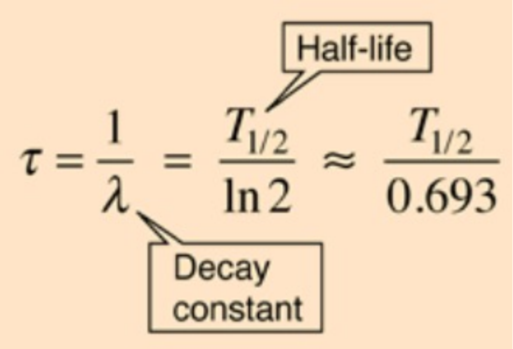
WIDTH/LIFETIME OF NUCLEAR STATES

Each state is characterized by:

- energy (mass)
- spin
- parity
- lifetimes against γ, p, n, and α emission

The lifetime is usually given as a width as it corresponds to a width in the excitation energy of the state according to Heisenberg:

$$\Delta E \cdot \Delta t = \hbar$$



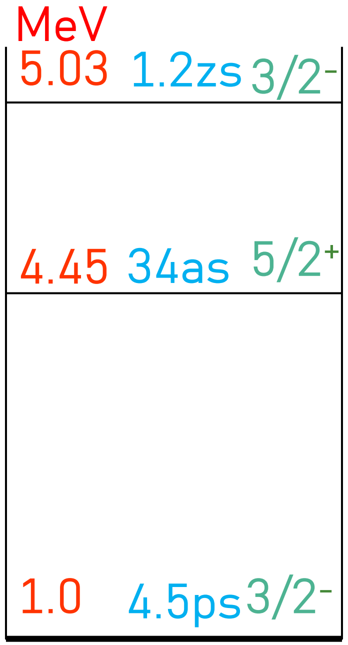
therefore, a lifetime **τ** corresponds to a width Γ:

$$\Gamma = \frac{\hbar}{\tau}$$

the lifetime against the individual "channels" for γ, p, n, and α emission are given as partial widths

$$\Gamma_\gamma, \Gamma_p, \Gamma_n, \Gamma_\alpha, \ldots \quad \text{with} \quad \Gamma = \sum \Gamma_i$$

WIDTH/LIFETIME OF NUCLEAR STATES



Calculate the widths of the three states.

z – zepto (10^{-21})
a – atto (10^{-18})
p – pico (10^{-12})

Quantity	Symbol	Value (eV units)
Planck's constant	h	4.1357×10^{-15} eV s
reduced Planck's constant	$\hbar = h/2\pi$	6.5821×10^{-16} eV s

$$\Gamma = \frac{\hbar}{\tau}$$

h-bar (eVs)	lifetime (s)	width (eV)	width (keV)
6.58E-16	1.20E-21	5.49E+05	5.49E+02
	3.40E-17	1.94E+01	1.94E-02
	4.50E-12	1.46E-04	1.46E-07

NUCLEAR SPIN

Orbital Angular Momentum ⇒ Nucleons move in an average central potential of the nucleus, which gives each a constant orbital angular momentum $\vec{\ell}$

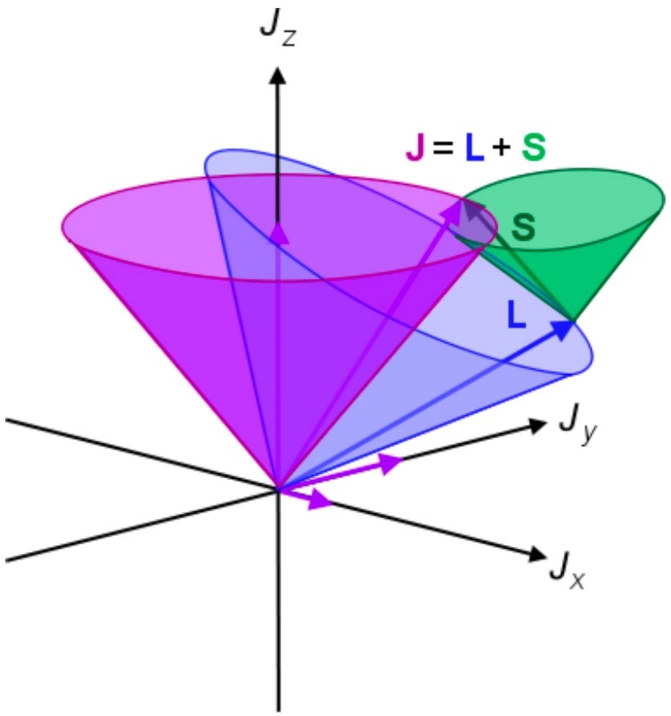
Intrinsic Angular Momentum – Spin ⇒ Nucleons are Fermions ⇒ intrinsic spin is $\vec{s} = \frac{\vec{1}}{2}$.

Total Angular Momentum – The orbital angular momentum ($\vec{\ell}$) and the intrinsic spin ($\vec{s}$) sum to give the total angular momentum ($\vec{j}$) of a nucleon.

Nuclear Angular Momentum (Nuclear Spin) ⇒ the total angular momentum for a given nucleus that contains A nucleons can be written as the vector sum of the angular momenta for the constituent nucleons. This is referred to as the Nuclear Spin and is typically denoted by $\vec{I}$ or $\vec{J}$.

$$\vec{J}^2 = \hbar^2 J(J+1), \text{ and } \vec{J}_z = \hbar m_J \quad (m_J = -J \; \ldots\ldots +J)$$

The nucleus behaves as if it was a single entity with an intrinsic angular momentum of $\vec{J}$ (e.g. Zeeman effect ⇒ $2J + 1$ substates splits). No fields of sufficient strength that can break the coupling of the nucleons can be produced (no break between $\vec{\ell}$ and $\vec{s}$)



NUCLEAR SPIN

The measured values for Nuclear Spin J can tell us a great deal about the nuclear structure

- The values of J are Integer if A is even, and Half-integer if A is odd
- All **even Z-even N** nuclei have $J = 0$ ground state

  (which indicates that identical nucleons tend to pair their angular momenta in the opposite directions – pairing effect)

- The ground state spin of **odd-A nuclei** must be equal to the $\vec{j}$ of the odd proton or neutron which is $\frac{1}{2}$.

  Nuclear spin coincides with the last unpaired nucleon ($\vec{J} = \vec{j}_{odd}$)

- For **odd-odd nuclei**, the ground states is determined by $\vec{J} = \vec{j}_p + \vec{j}_n$

PARITY

Parity is the inversion of sign for all spatial coordinates about some reflection point (mirror image).

Nuclear states are described by a wave function that can be either even or odd:

Even wave-functions are symmetric about the origin → when spatial coordinate and spin are flipped

$\psi(r, s) = \psi(-r, -s)$ - This is known as **positive parity**

Odd wave-functions are antisymmetric about the origin → when spatial coordinate and spin are flipped

$\psi(r, s) = -\psi(-r, -s)$ - This is known as **negative parity**

For a spherical symmetric potential (i.e. $V(r, \theta, \varphi) = V(r)$), the parity of a particle is given by its orbital angular momentum $\pi = (-1)^{\ell}$

For a state of several nucleons, $\pi = \prod_i \pi_i$ (note that as for the spin, it is the unpaired nucleons that matter)

Parity is conserved for strong and electromagnetic interactions

The parity is denoted by a + or – superscript to the nuclear spin J^{π}. Examples are 0^{+}, 2^{-}, $1/2^{-}$, $3/2^{+}$

NUCLEAR MODELS

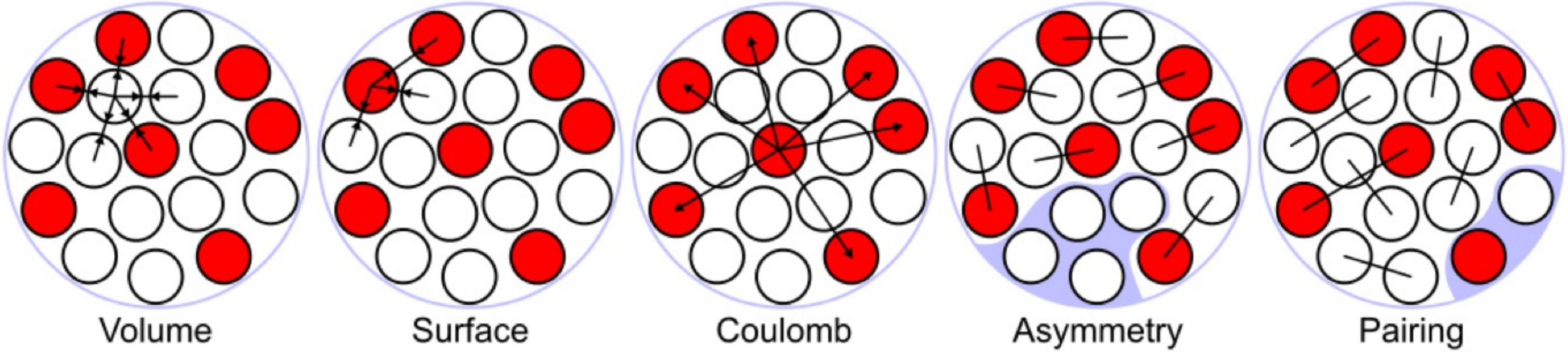
THE SEMI-EMPIRICAL MASS FORMULA

- The **Liquid drop model** was historically the first model to describe the nuclear properties.
- First proposed by George Gamow and developed by Niels Bohr and John Archibald Wheele
- The fluid is made of nucleons (protons and neutrons), which are held together by the strong nuclear force.
- The Liquid drop model is a model in nuclear physics which treats the **nucleus as a drop of incompressible nuclear fluid**
- This is a crude model that does not explain all the properties of the nucleus but does explain the spherical shape of most nuclei. It also **helps to predict the binding energy of the nucleus**.
- The idea came primarily from the observation that nuclear forces exhibit saturation properties:

  Non-saturated $BE/A \propto (A-1)/2$ *while the nuclear binding energy shows a constant behaviour* $BE/A \propto const \Rightarrow$ *each nucleon attract only its closest neighbours*
- Nucleus present a low compressibility and so a well-defined nuclear surface

THE SEMI-EMPIRICAL MASS FORMULA

- Let's consider all the interaction within the nucleus:



$$BE(N,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \begin{cases} +\delta & if\ even-even \\ 0 & if\ odd-even \\ -\delta & if\ odd-odd \end{cases}$$

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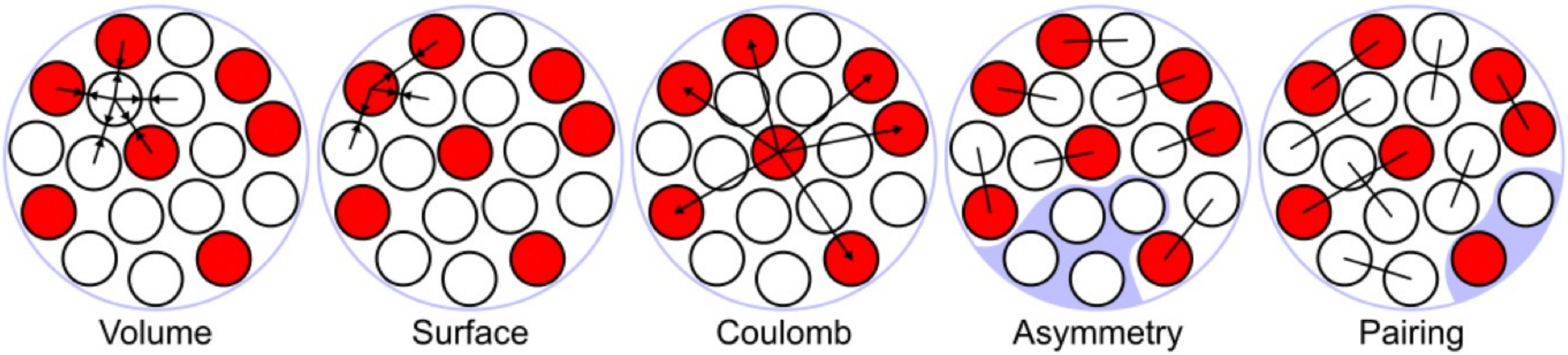

$$BE(N,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \begin{cases} +\delta & if\ even - even \\ 0 & if\ odd - even \\ -\delta & if\ odd - odd \end{cases}$$

VOLUME – expresses the fact that the nuclear force is saturated. The strong force has a very limited range, and a given nucleon may only interact strongly with its nearest neighbors and next nearest neighbors. Therefore, the number of pairs of particles that interact is roughly proportional to A and not to A(A-1)/2

THE SEMI-EMPIRICAL MASS FORMULA

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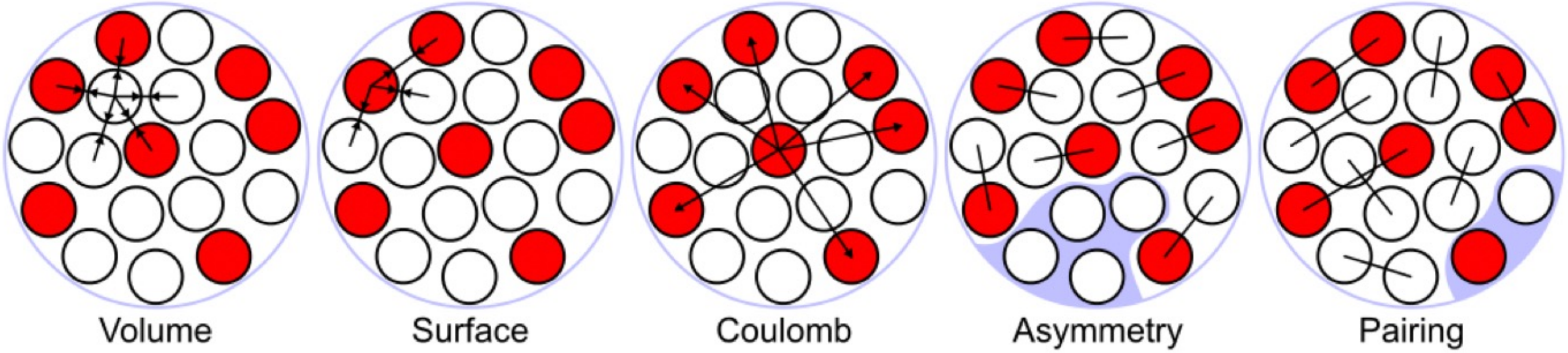

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SURFACE – considers the fact that the nucleons at the nuclear surface will have a reduced binding energy since only partially surrounded by nucleons. Correction proportional to the nuclear surface area $4\pi R^2$ with $R \propto A^{2/3}$

THE SEMI-EMPIRICAL MASS FORMULA

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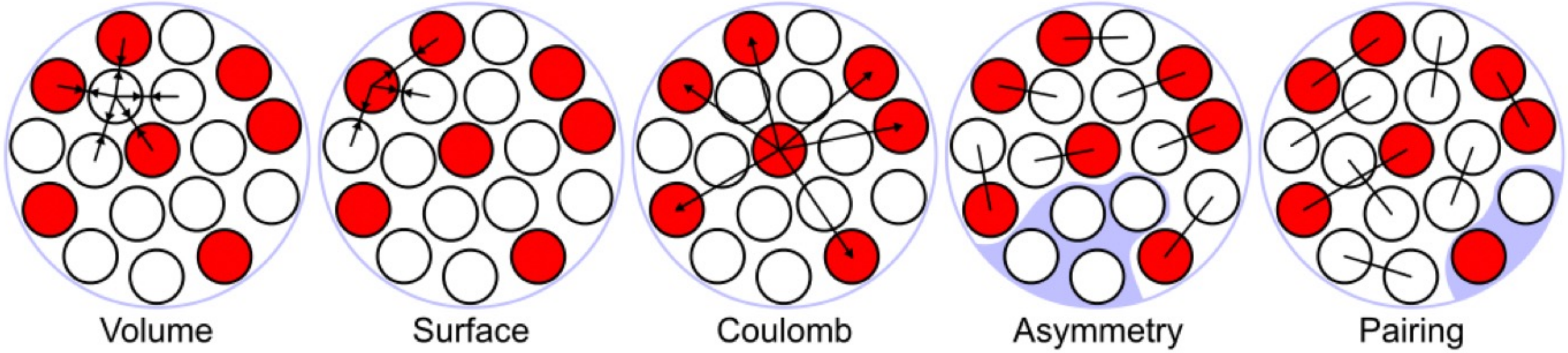

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COULUMB – considers the Ze charge of the nucleus. The electric repulsion between each pair of protons in a nucleus contributes toward decreasing its binding energy.

THE SEMI-EMPIRICAL MASS FORMULA

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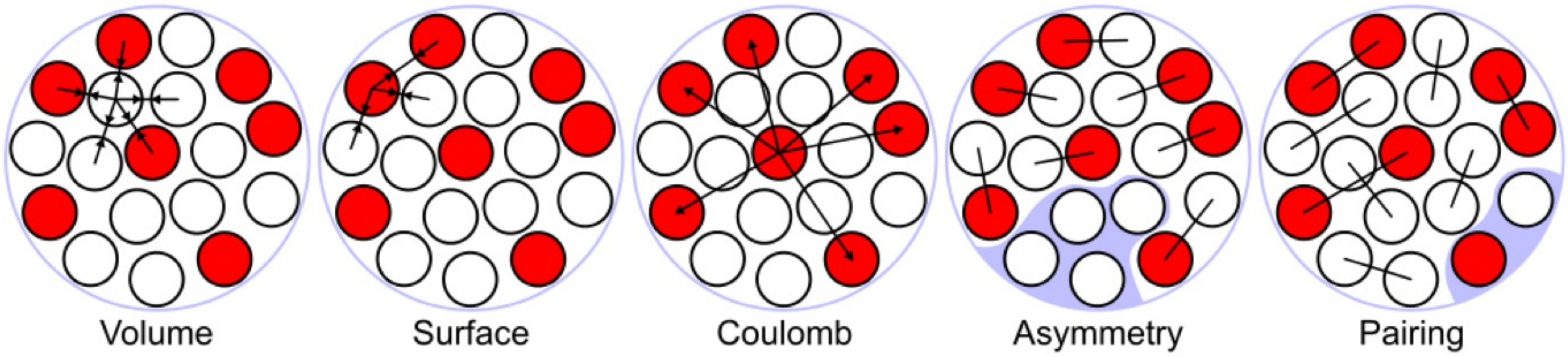
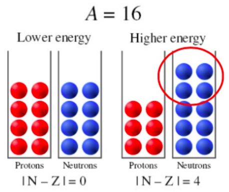


If there are significantly more neutrons than protons in a nucleus, some of the neutrons will be higher in energy than the available states in the proton pool.



$$BE(N,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \begin{cases} +\delta & if\ even-even \\ 0 & if\ odd-even \\ -\delta & if\ odd-odd \end{cases}$$

ASYMMETRY - considers that p-n attraction is stronger than p-p and n-n. For Pauli principle: more nucleons are added to the nuclei, these particles must occupy higher energy levels, increasing the total energy of the nucleus (and decreasing the binding energy)

THE SEMI-EMPIRICAL MASS FORMULA

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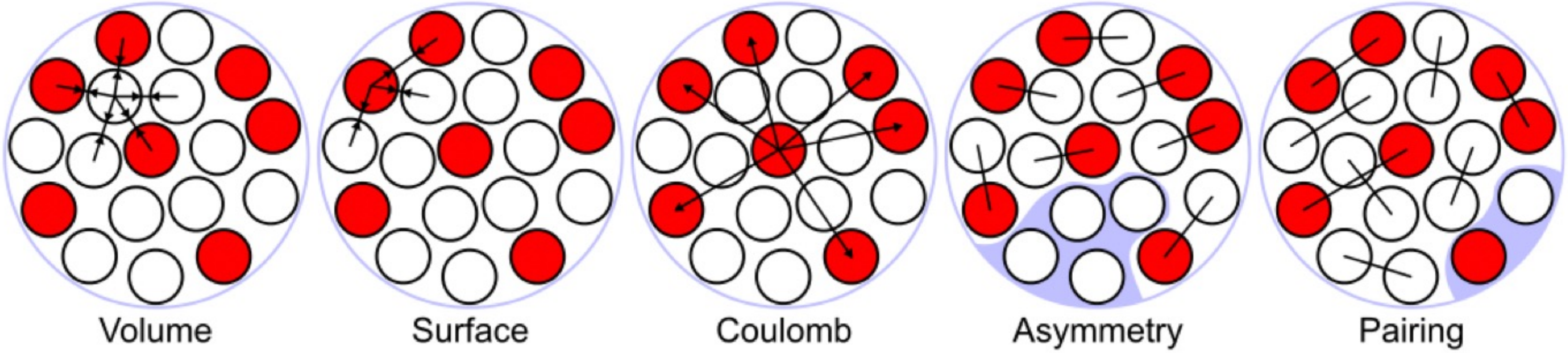

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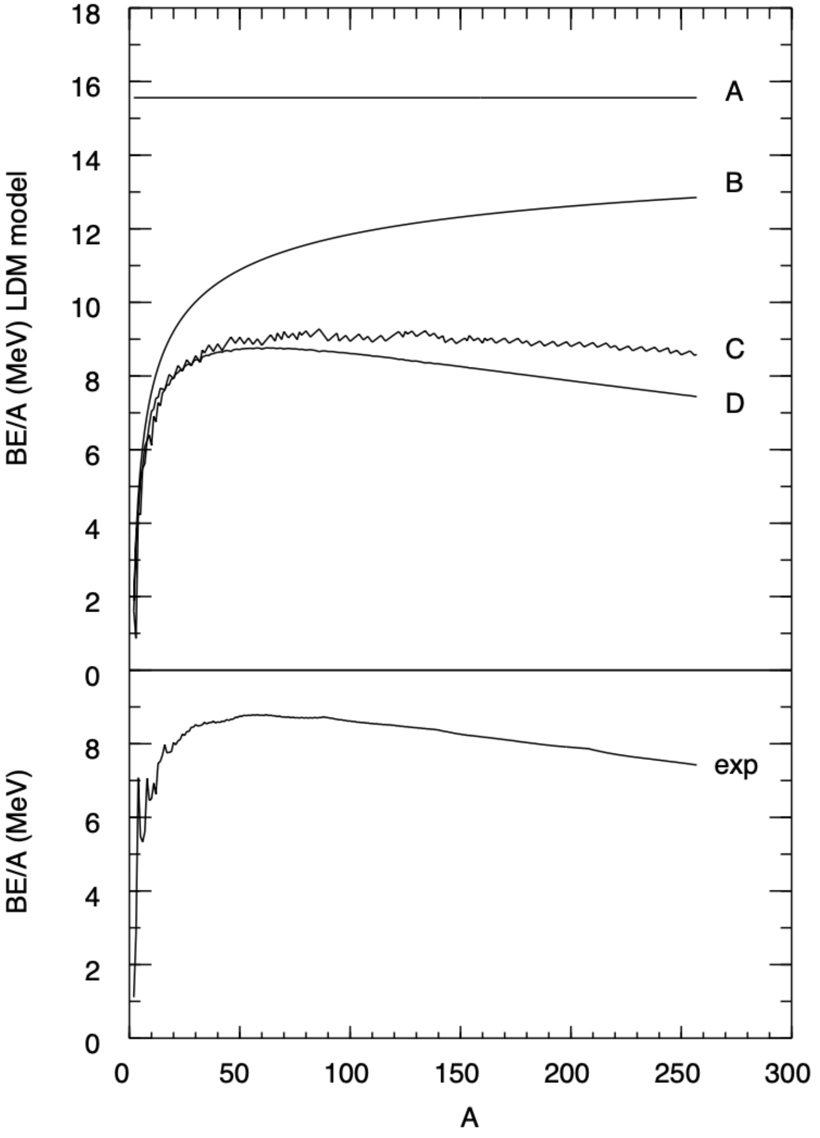
PARING – considers that nucleons preferentially form pairs (proton pairs, neutron pairs). Due to the Pauli exclusion principle the nucleus would have a lower energy if the number of protons with spin up will be equal to the number of protons with spin down. This is also true for neutrons. Only if both Z and N are even, both protons and neutrons can have equal numbers of spin up and spin down particles.

THE SEMI-EMPIRICAL MASS FORMULA



Volume

Volume + Surface

Volume + Surface + Coulomb

Volume + Surface + Coulomb + Asymmetry

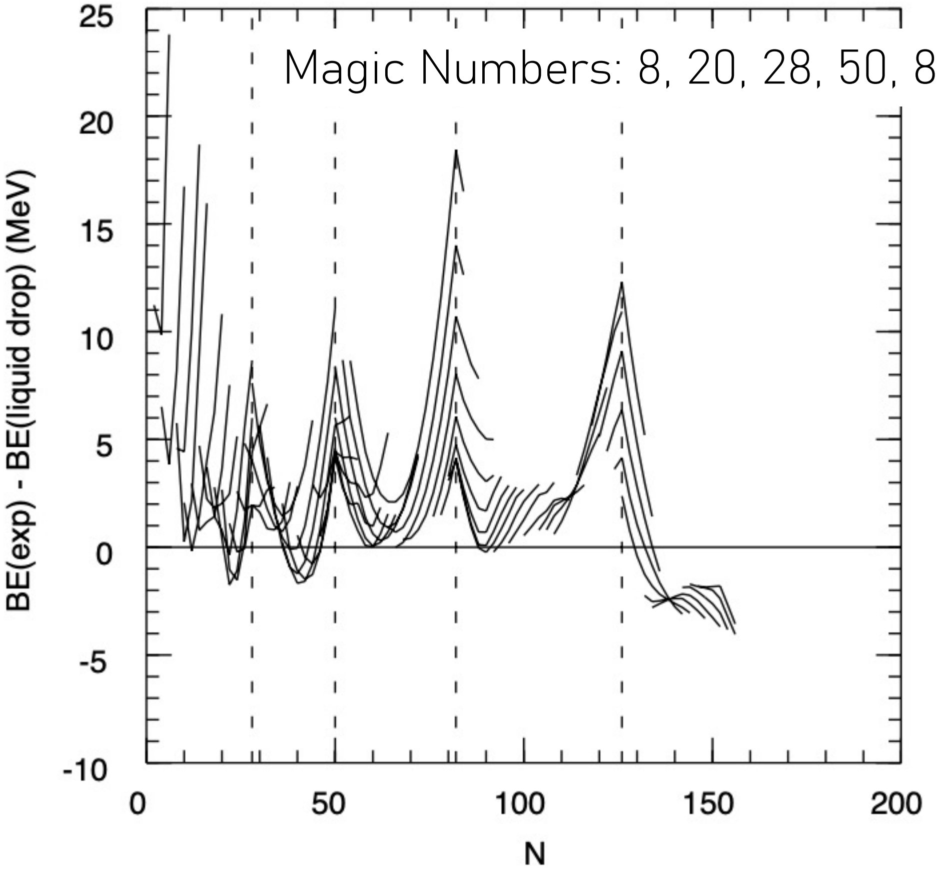
With this model – masses can be reproduced with good accuracy ~ 1 MeV deviation compared to ~ 8 MeV/A

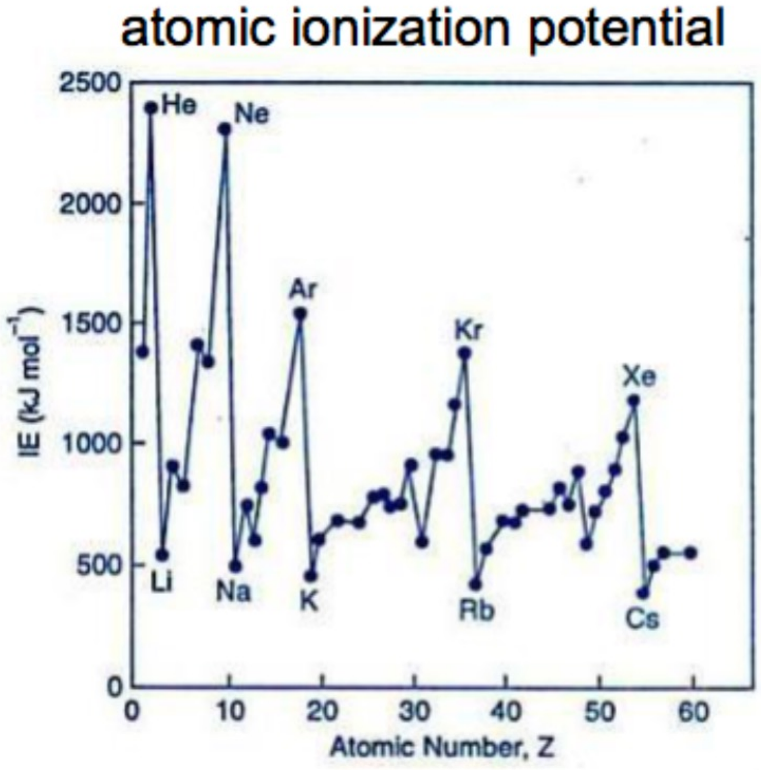
No explanation for the Magic Numbers

Lecture Notes in Nuclear Structure Physics - B. Alex Brown

NUCLEAR SHELL MODEL

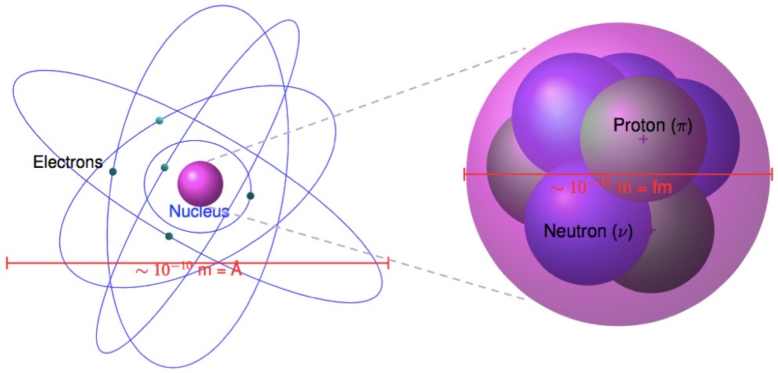
The semiempirical liquid drop model or binding-energy formula gives a good overall picture of the trends of nuclear binding energies. But does not account for finer details.





Analogy with Atomic Nucleus

NUCLEAR SHELL MODEL



In the analogy between **Atom** and **Nucleus** we need to notice:

<u>POTENTIAL</u>

In the **atomic case** the potential is supply by the **Coulomb field of the nucleus** while **in the nucleus** there is **no such an external agent** → the nucleons move in a potential that they themselves generate.

⇒ The motion of a single nucleon is governed by a potential caused by all the other nucleons.

<u>INTERACTION</u>

In the **atomic shell theory existence of spatial orbits (electrons are non-interacting)**.

Nucleons make many collision (strong interaction) → The energy transfer in a collision is not enough to allow the nucleus to move from a low-lying level to the valence band (all level are filled up - Pauli principle)

⇒ The collisions cannot occur, and the nucleons can indeed orbit as if they were transparent to one another.

<u>MAGIC NUBERS</u>

In the **atomic shell theory**, the **Schrödinger equation can be solved for the Coulomb potential** and the characteristic (energy) shells can be found at 2, 10, 18, 36, 54, 86

For the nucleus, the **Schrödinger equation can also be solved to obtain** the (energy) shell **but it must be different from the atomic** one since the shell are found at 8, 20, 28, 50, 82, 126

NUCLEAR SHELL MODEL

A non-relativistic quantum mechanics system is described by the wave function solutions to the **Schrodinger equation:**

$$-\frac{\hbar}{2m}\left(\frac{\partial^2\psi(\vec{r})}{\partial x^2}+\frac{\partial^2\psi(\vec{r})}{\partial y^2}+\frac{\partial^2\psi(\vec{r})}{\partial z^2}\right)+V(x,y,z)\psi(\vec{r}) = E\psi(\vec{r})$$

This equation will have **solutions only for certain values of the energy $\boldsymbol{E}$** which usually results from applying boundary conditions to $\psi(\vec{r})$, such as $\psi(\vec{r})$ and its derivative must be continuous across any boundary $\psi(\vec{r})$ must remain finite normalization condition.

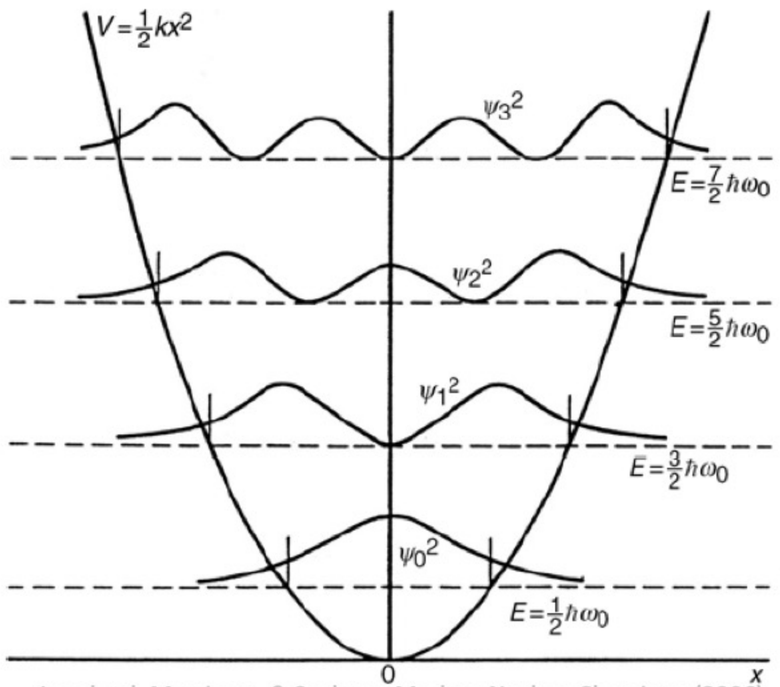
When $\psi(\vec{r})$ can be expressed in terms of spherical harmonics $\psi(\vec{r}) = R(r)Y_m^\ell(\theta,\varphi)$ we also can get the angular momentum for that eigenfunction and parity since the function is either even or odd.

NUCLEAR SHELL MODEL-POTENTIALS

To solve this problem, we need to assume a potential $V(x, y, z)$:

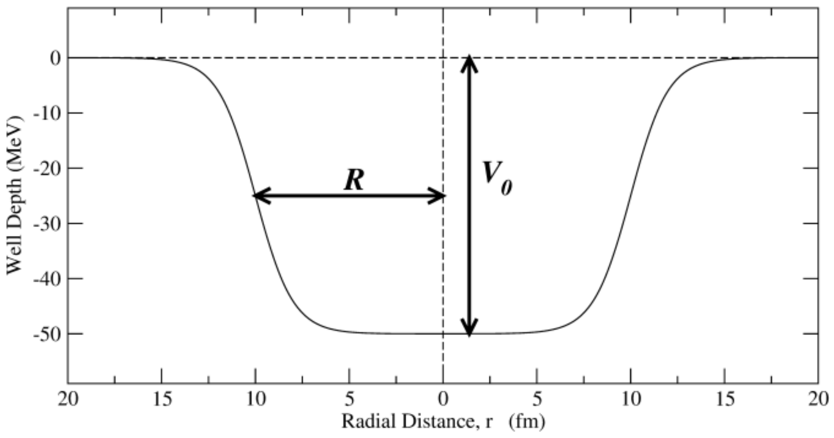
- Harmonic Oscillator Potential → produce evenly spaced energy levels → doesn't reproduced the magic numbers
- Wood-Saxon Potential → since the nuclear interaction is short-range, a natural improvement would be to adopt a central potential based on empirical density distribution (approximate with a square well with soft edges)

Harmonic Oscillator Potential



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

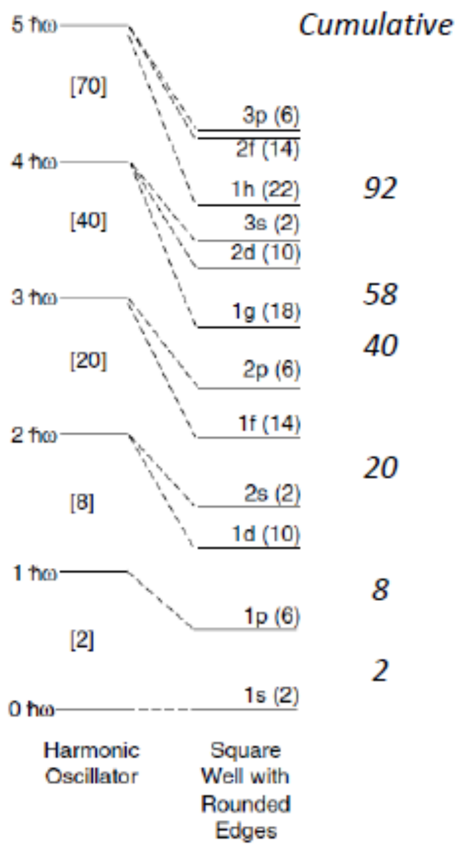
Wood-Saxon Potential



$$V(r) = \frac{-V_0}{1 + e^{(r-R)/d}}$$

From Leach lectures PHGN 422: Nuclear Physics

The degeneracy in ℓ is broken but magic numbers are not yet reproduced!



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

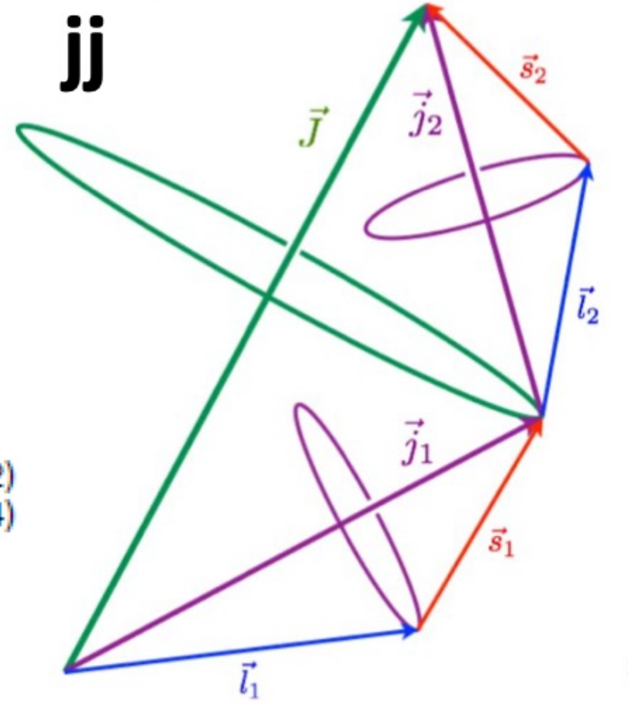
NUCLEAR SPIN-ORBIT INTERACTION

Interactions between the nucleons like the spin-orbit interacting in atoms.

In Atomic Physics, the origin is magnetic, and the effect is a small correction. In the case of nuclear binding the effect is about 20 times larger, and it comes from a term in the nuclear potential itself which is proportional to $\vec{\ell} \cdot \vec{s}$

$$V_{spin-orbit} \propto -\frac{1}{r}\frac{\partial}{\partial r}V_{WS}(r)\,\vec{\ell} \cdot \vec{s}$$

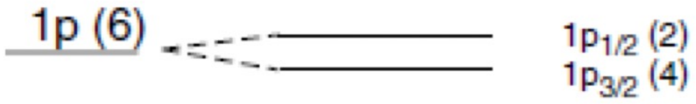
A. Kastberg, Lecture Notes Physique Atomique



The effect of the spin-orbit interaction is of coupling together the orbital angular momentum and the spin giving the total angular momentum: $\vec{j} = \vec{\ell} + \vec{s}$

→ the possible values of j are $|\ell - s| \leq j \leq |\ell + s|$

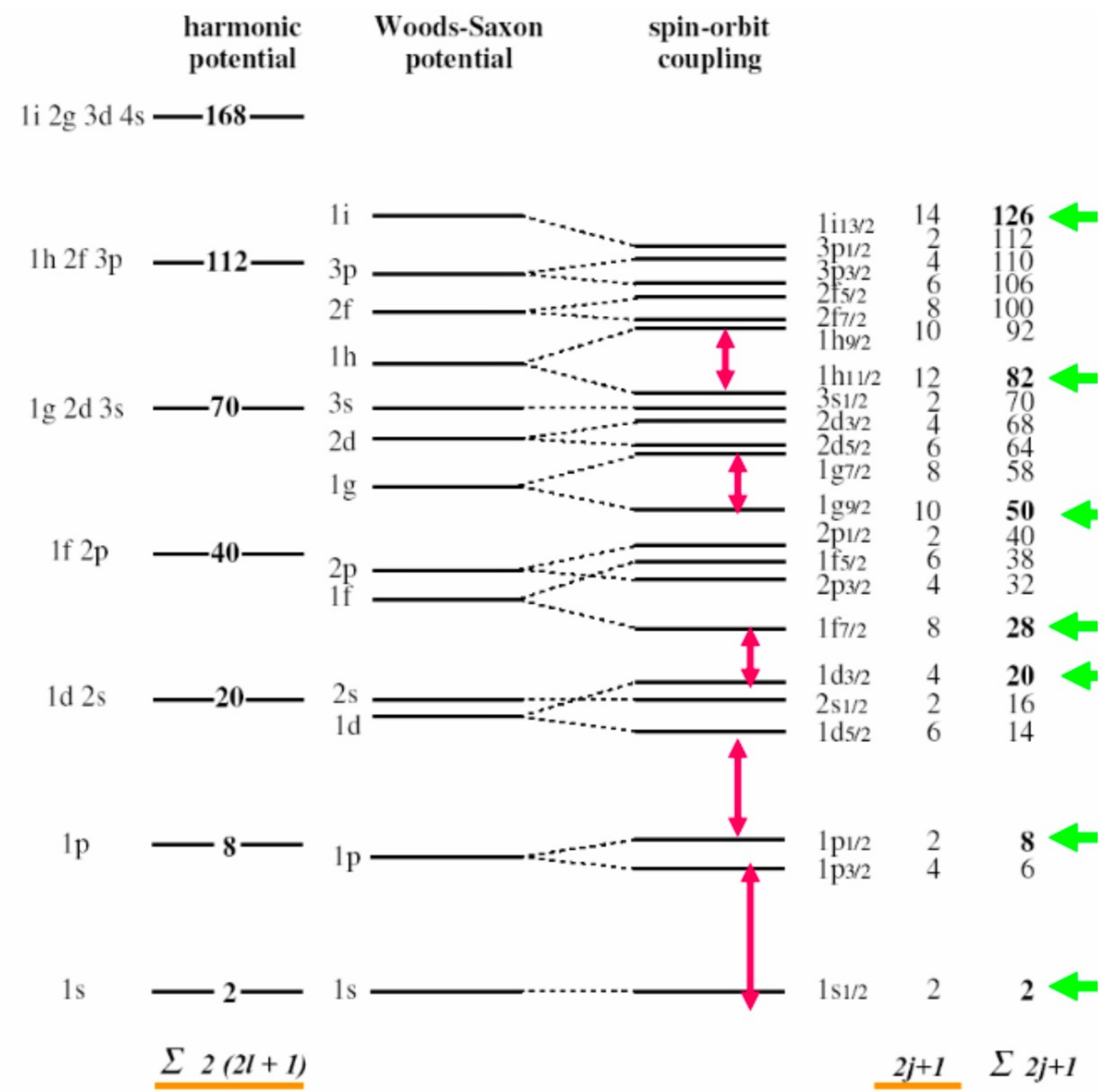
→ energy separation: $\frac{\hbar}{2}(2j+1)$



→ the energy splitting increases with increasing ℓ.

→ $V_{spin-orbit}$ is negative so the member of the pair with larger j is pushed downward.

NUCLEAR SPIN-ORBIT INTERACTION



Magic numbers:
Number of protons or neutrons in a full shell is called “magic” numbers, with numbers 2, 8, 20, 28, 50, 82, and 126.

Nuclei with magic neutron or proton numbers are characterized by a stronger binding, greater stability, and, therefore, are more abundant in nature.

Spectroscopic notation

$$\ell = 0, 1, 2, 3, 4, 5, \ldots \implies s, p, d, f, g, h, \ldots$$

NUCLEAR SPIN-ORBIT INTERACTION

Exercise:

Assume a **d-wave proton**.

- What angular momentum does the proton carry?
- What spin does the proton have?
- What is the total angular momentum?

Assume a **s-wave neutron**.

- What angular momentum does the neutron carry?
- What spin does the neutron have?
- What is the total angular momentum?

NUCLEAR SPIN-ORBIT INTERACTION

Assume a d-wave proton.

What angular momentum does the proton carry? $\ell = 2$

What spin does the proton have? $s = \frac{1}{2}$

What is the total angular momentum? $\mathrm{j} = \left|\ell - \frac{1}{2}\right| = \frac{3}{2}$ and $\mathrm{j} = \left|\ell + \frac{1}{2}\right| = \frac{5}{2}$

Assume a s-wave neutron.

What angular momentum does the neutron carry? $\ell = 0$

What spin does the neutron have? $s = \frac{1}{2}$

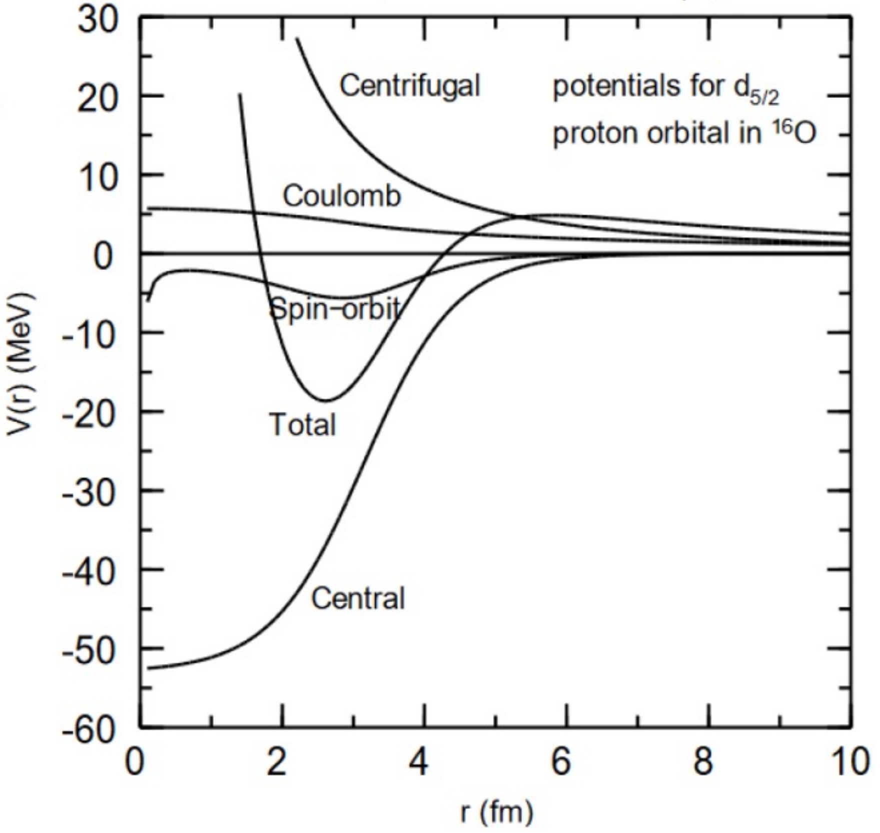
What is the total angular momentum? $\mathrm{j} = \left|\ell - \frac{1}{2}\right| = \left|\ell + \frac{1}{2}\right| = \frac{1}{2}$

NUCLEAR POTENTIAL

Nucleons within a nucleus can be treated as if they are:

- **Attracted** by a Woods-Saxon central potential
- **Repelled** by a Coulomb potential from a charged sphere (if proton)
- **Attracted or Repelled** if $\vec{\ell}$ and $\vec{s}$ are parallel or anti-parallel by the spin orbit force (Peaks at surface)
- **Repelled** by a centrifugal barrier (if the nucleon were to exit the nucleus, carrying away angular momentum $\vec{\ell} > 0$) – This is only relevant when a nucleon is removing/adding orbital angular momentum $\vec{\ell}$, so not to be included in calculating single particle levels

B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.



FILLING THE SHELL

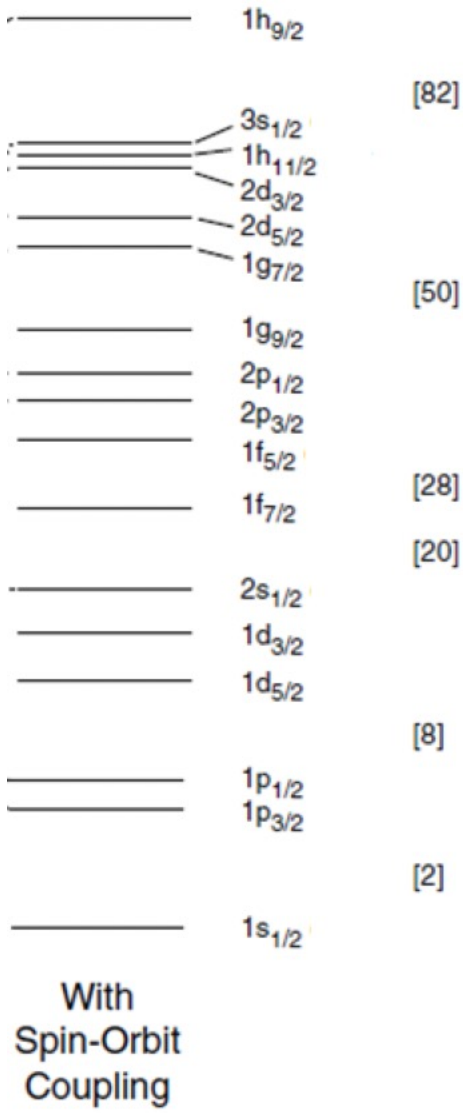
A nucleon will go in the lowest-energy level which isn't already filled, i.e.

- the largest angular momentum, j
- for the lowest orbital angular momentum, ℓ
- for the lowest oscillator shell, n

$2j + 1$ protons or neutrons are allowed per level

Each level is referred to by its $n\ell j$

- n by the # for the oscillator shell (convention either starts with 0 or 1)
- ℓ by spectroscopic notation (s=0,p=1,d=2,f=3,…)
- j by the half-integer corresponding to the spin



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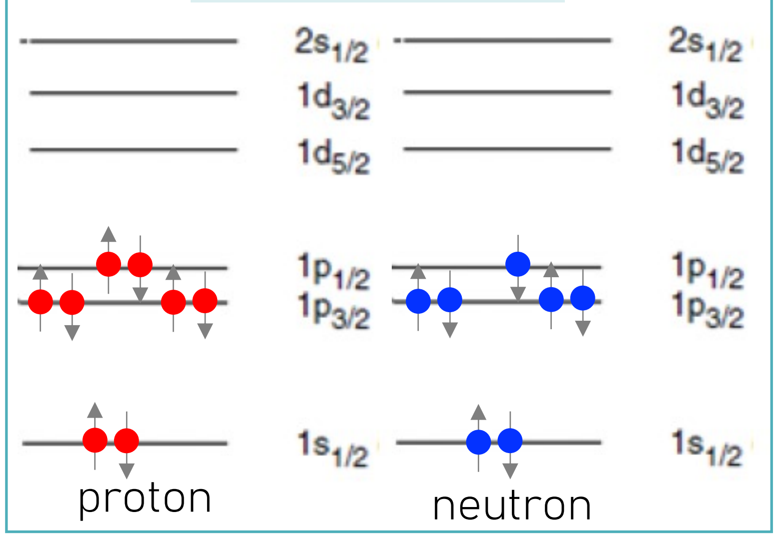
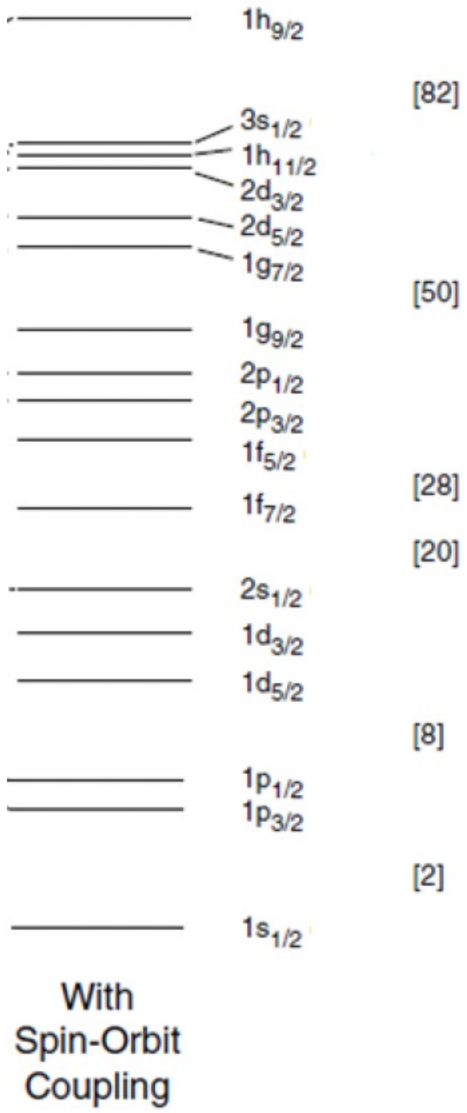
^{15}O – Z = 8 and N = 7



$$J_{gs} = j_{odd}\ and\ \pi_{gs} = (-1)^{\ell}$$

$$\Rightarrow J_{gs}^{\pi} = \frac{1}{2}^{-}$$



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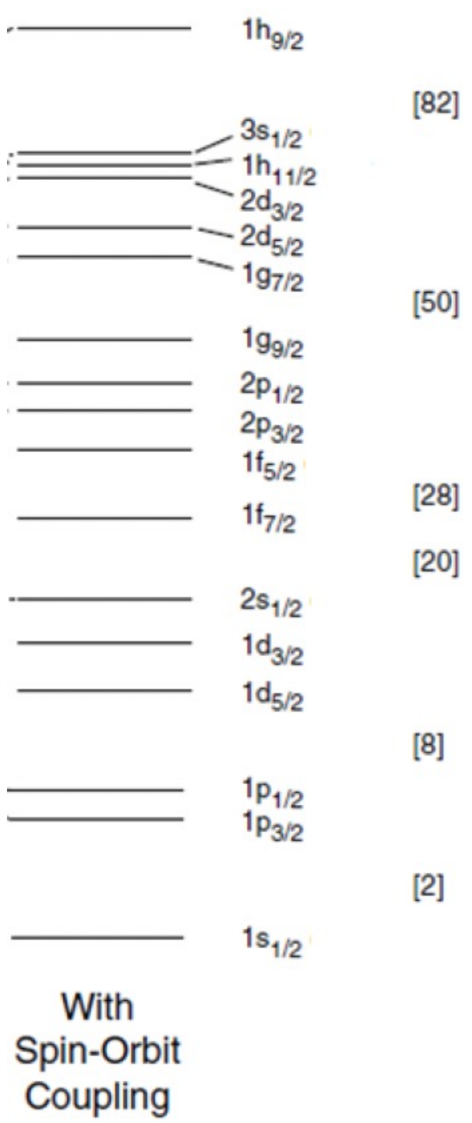
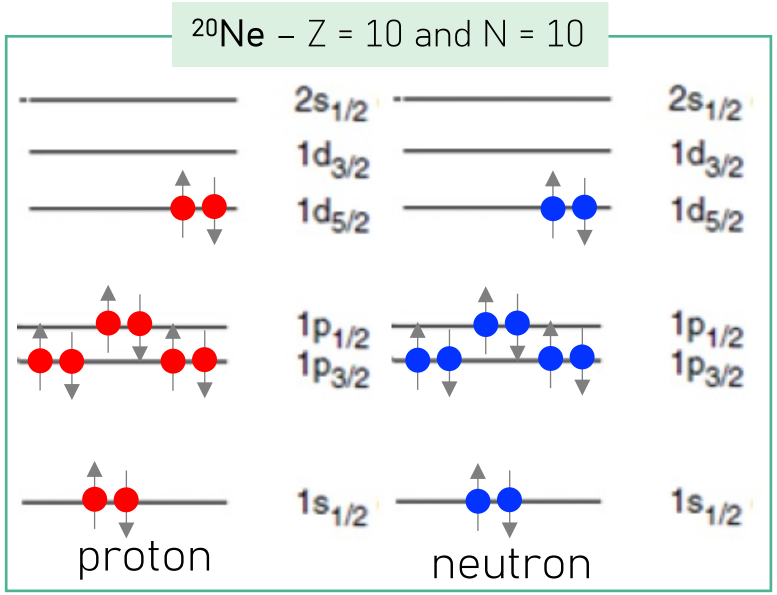


<u>Exercise</u>:

For ^{20}Ne, fill the shell and estimate the spin and parity of the ground state.

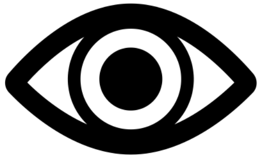
$$even - even\ nucleus$$
$$\Longrightarrow J^{\pi}_{gs} = 0^{+}$$



NUCLEAR REACTIONS

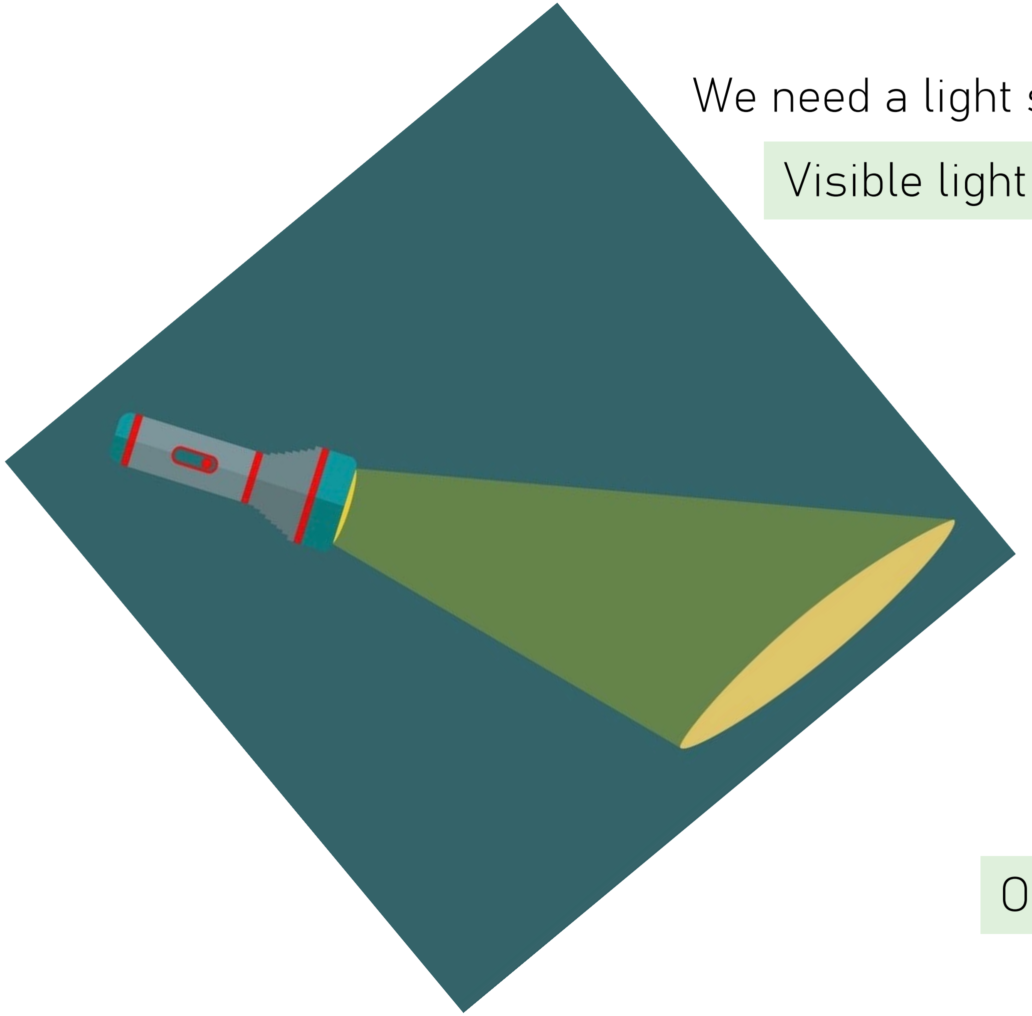
HOW CAN WE SEE AN OBJECT?

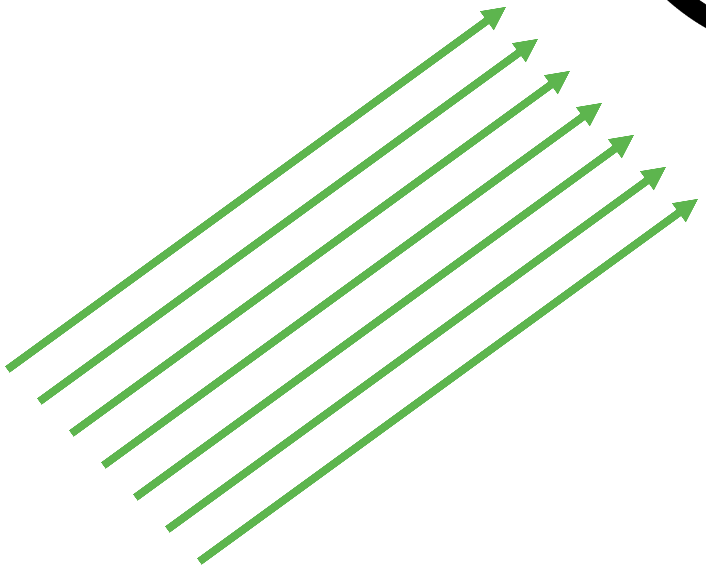
We need a detector

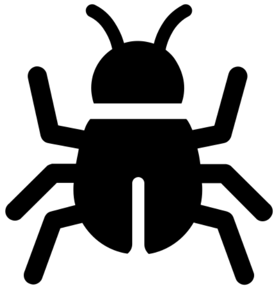


We need a light source

Visible light → $\lambda \sim 10^{-7} m$







We need a object

Object size → $l \sim 10^{-2} m$

Light source conditions:

the wavelength of the radiation must be smaller than the dimension of the object

HOW CAN WE “SEE” A NUCLEUS?

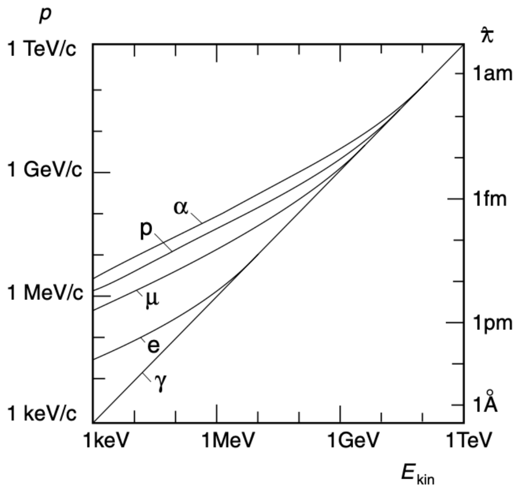
We need a radiation source

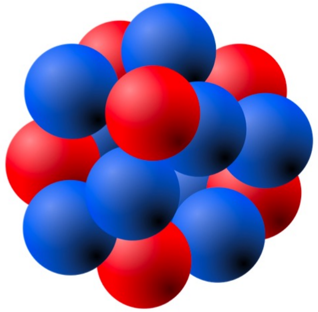
e⁻ e⁻
e⁻

de Broglie wavelength → $\lambda = \frac{h}{p}$

$$E(e^-) = 100\ \text{MeV} - 1\ \text{GeV}$$

$$\lambda = 2\ \text{fm} - 0.2\ \text{fm}$$





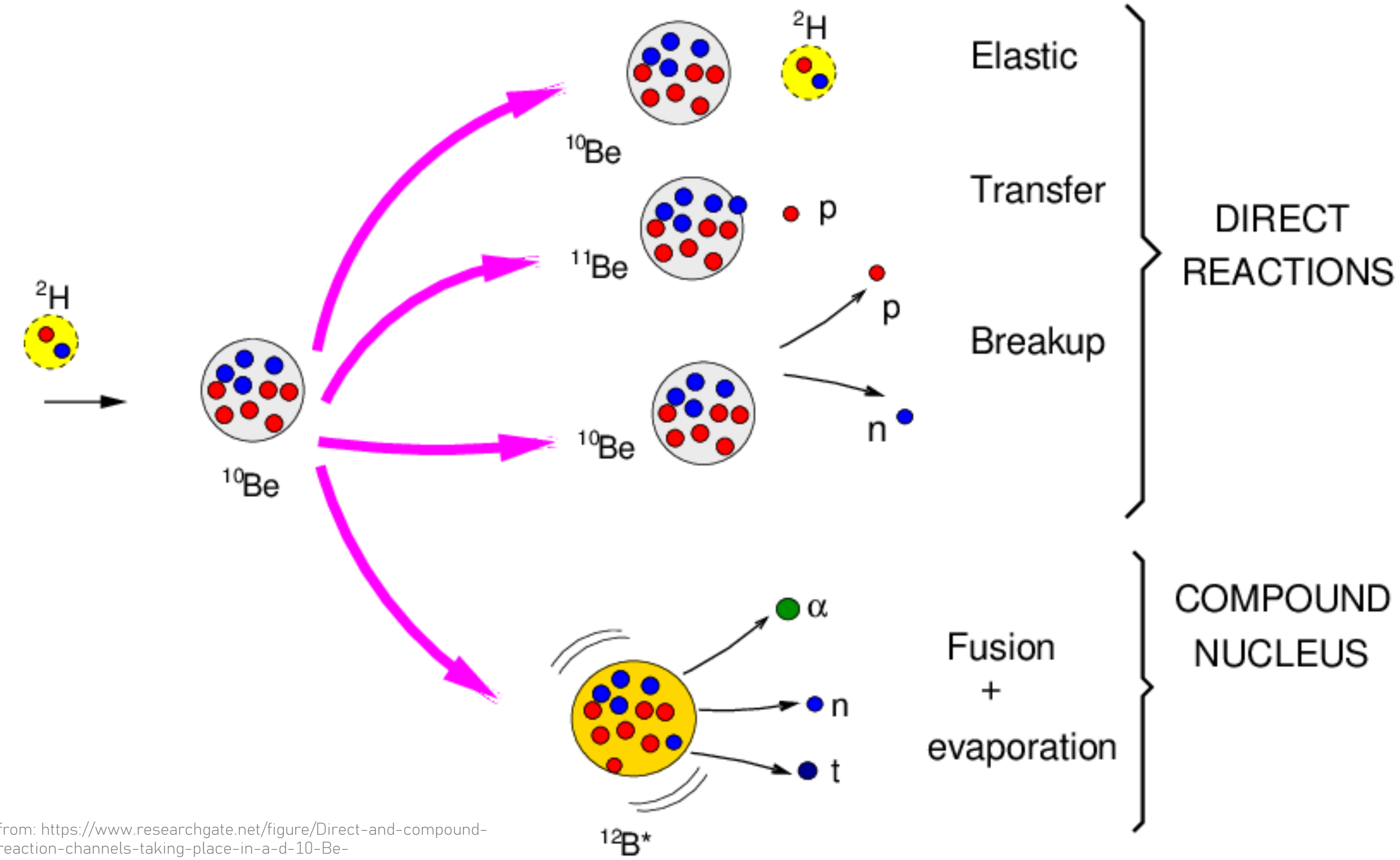
We need a nucleus

Nucleus diameter → $\mathrm{d} \sim 10^{-15}\ m$

We need a detector

3-Spectrometer facility
@ MAMI accelerator
Mainz University

GENERAL CHARACTERISTICS



only very few nucleons are involved

incoming and target merge, complete sharing of energy

Modified from: https://www.researchgate.net/figure/Direct-and-compound-nucleus-reaction-channels-taking-place-in-a-d-10-Be-reaction_fig8_326366718

CONSERVATION LAWS

- Conservation of total energy
- Conservation of linear momentum
- Conservation of charge

- **Conservation of proton and neutron number separately** (for low-energy reactions, no meson formation - production energy ~140 MeV - or quark rearrangement. The weak force is also very slow compared to be considered in the time of nuclear reactions) **or nucleon number** (for high-energy reactions)

- **Conservation of total angular momentum**: any change in the vector sum of the internal angular momenta (spins) of the nuclei must be compensated for by a corresponding change in the angular momentum of their relative motion.

Example: ${}^{10}B + {}^{4}He \rightarrow {}^{1}H + {}^{13}C$

${}^{10}B$ has $\mathrm{J} = 3$ in the g.s., ${}^{4}He$ has $\mathrm{J} = 0$. Assuming that the ${}^{4}He$ particle is capture in an s-wave ($\ell_i = 0$), the intermediate compound nucleus will be in a state with spin $J_c = 3$. The final nuclei have both spins $\frac{1}{2} \Longrightarrow$ their sum will be 0 or 1 . For the conservation of the total angular momentum the relative angular momentum of the final products will be $\ell_f = 2, 3\ or\ 4$

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^{10}B (Z=5 and N=5) → odd-odd, therefore $\vec{J} = \vec{j}_n + \vec{j}_p \rightarrow \vec{j}_n = \vec{j}_p = \frac{\vec{3}}{2} \Longrightarrow \left|\frac{3}{2} - \frac{3}{2}\right| \le J \le \left|\frac{3}{2} + \frac{3}{2}\right| \Longrightarrow \mathrm{J} = 0, 1, 2, \mathbf{3}$

^{4}He (Z=2 and N=2) → even-even, therefore $\mathrm{J} = 0$

^{1}H (Z=1 and N=0) → even-even, therefore $\mathrm{J} = \frac{1}{2}$

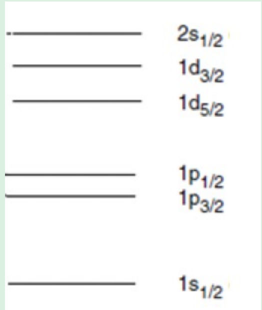
^{13}C (Z=6 and N=7) → even-odd, therefore $\vec{J} = \vec{j}_{odd} = \frac{\vec{1}}{2} \Longrightarrow \mathrm{J} = \frac{1}{2}$

$$\vec{J}(^{10}B) + \vec{J}(\ ^{4}He) + \vec{\ell}_i = 3 = \vec{J}_c$$

$$\vec{J}(\ ^{1}H) + \vec{J}(^{13}C) + \vec{\ell}_f = \vec{J}_c = 3$$

$$\Longrightarrow \vec{J}(\ ^{1}H) + \vec{J}(^{13}C) = 0, 1$$

$$\vec{\ell}_f = \mathbf{2, 3, 4}$$



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- **Conservation of parity**: any change in the product of their intrinsic parities must be reflected in a change in the parity of their relative motion. (remember $(-1)^{\ell}$ rule)

Q-VALUE

The Q-value is the energy released/consumed by a nuclear reaction. It is defined as the initial mass energy minus final mass energy

$$Q = \left(\sum m_{initial} - \sum m_{final}\right) c^2$$

When calculating the Q-value remember to balance the mass of the electrons.

Exothermic reaction → produces energy

If $\boldsymbol{Q}$-value $> \mathbf{0}$ (Exothermic), this means that the nuclear mass or binding energy is released as the kinetic energy of final products

Endothermic reaction → requires energy

If $\boldsymbol{Q}$ -value $< \mathbf{0}$ (Endothermic), this means that the initial kinetic energy is converted into nuclear mass or binding energy

If the reaction reaches excited states of residual nucleus, the Q-value equation should include the mass energy of the excited state

$$Q_{ex} = Q_0 - E_{ex}$$

Q-VALUE

Exercise: which is the Q-value of the $\alpha + \alpha \rightarrow {}^{7}Li + p$ that leaves ${}^{7}Li$ in the ground state?

$$Q_0 = 2\times m({}^{4}He) - m({}^{7}Li) - m({}^{1}H) = 2\times 4.001506 - 7.014357 - 1.007276 = -0.018621\,u$$

$$Q_0 = -0.018621\,u \times 931.494\,MeV/u = -17.35\,MeV$$

ENDOERGIC REACTION!

This is an endoergic reaction ⇒ you need energy above 17.35 MeV (in the CM frame) to initiate the reaction.

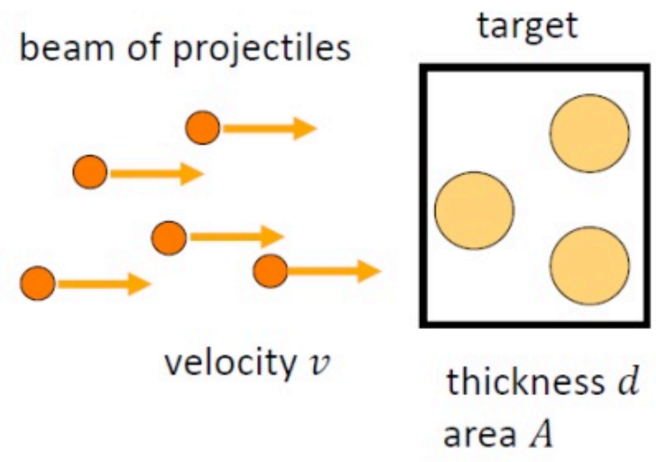
In the CM frame since the system is defined as to have the total momentum equal to zero all the kinetic energy is available for excitation while in the LAB frame some of the kinetic energy will be taken by the recoil nuclei.

Therefore, in this case the minimum energy in the LAB frame will be

$$E_\alpha(lab) = E_\alpha(CM)\frac{m_\alpha + m_\alpha}{m_\alpha} = 17.35\frac{4.001506 + 4.001506}{4.001506} = 17.35\times 2 = 34.7\,MeV$$

CROSS SECTION

The cross section describes the probability that a nuclear reaction will occur.
The concept of a nuclear cross section can be quantified physically in terms of "characteristic area" where a larger area means a larger probability of interaction.



$$\sigma = \frac{\#\,reactions/s}{\#\,projectiles/s\ \times \#\,targets/s}$$

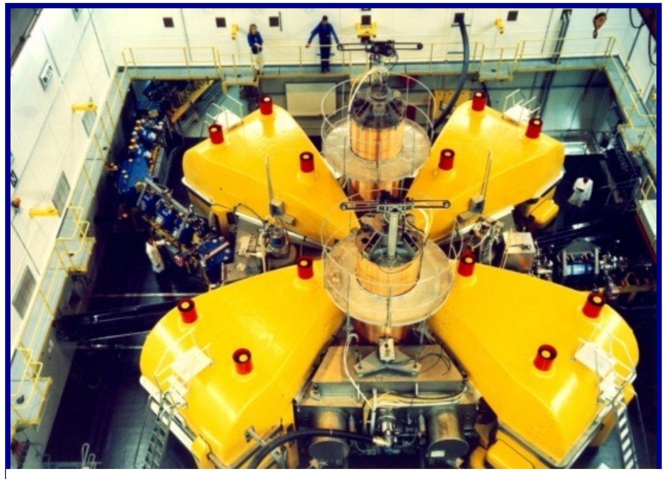
Unit for cross section: $1\ barn = 1\ b = 10^{-24} cm^2 = 100\ fm^2$

Reaction rate: $R[s^{-1}]$ $\quad \mathrm{R} = \sigma I \rho_A = \sigma j A n_t d = \sigma v n_p n_t V$

Reaction rate per volume: $r = \frac{R}{V} = \sigma v n_p n_t\ [s^{-1} cm^{-3}]$

Beam → intensity $I[s^{-1}]$
particle density $n_p[cm^{-3}]$
particle flux $j = \frac{I}{A} = n_p v\ [s^{-1} cm^{-2}]$

Target → particle density $n_t[cm^{-3}]$
areal density $\rho_A = n_t d[cm^{-2}]$

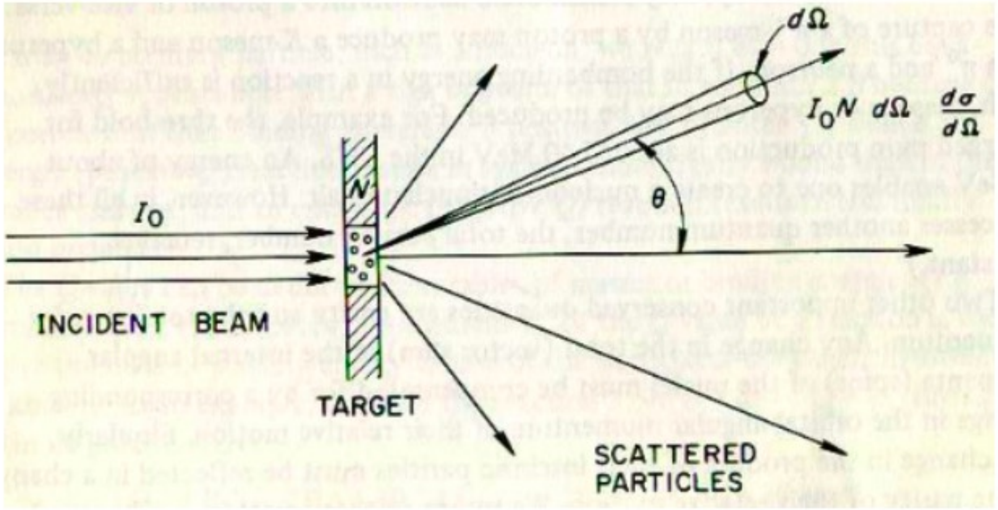


In laboratory environment one particle is typically stationary. In a star or LHC both particles are in motion.

CROSS SECTION

Notice that

- Detectors can only measure a small amount of σ because the occupy a small solid angle $d\Omega$.
- The outgoing particles will not be emitted uniformly but according to an angular distribution that will depend on θ and possibly on ϕ.



The detector occupies only a small solid angle $d\Omega$.

Therefore, only a small fraction dR of the outgoing particles can be observed

Hence, only a fraction of the cross section $d\sigma$ can be deduced

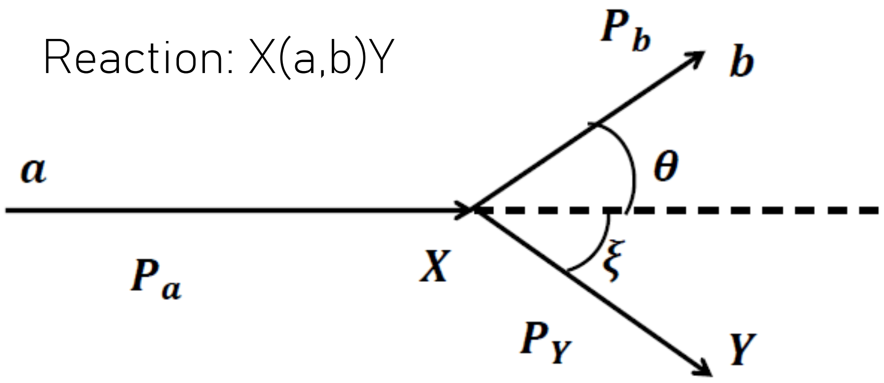
Differential cross section $\frac{d\sigma}{d\Omega} = \frac{r(\theta,\phi)}{(4\pi IN)}$

Units of barn/steradian [b/sr]

The reaction cross section σ can be determined by integrating $\frac{d\sigma}{d\Omega}$ over all angles, with $d\Omega = \sin\theta\, d\theta d\phi$

REACTIONS KINEMATICS

Reaction: X(a,b)Y



LAB frame: reaction plane → defined by the beam direction and one of the outgoing particles. Then for the conservation of the perpendicular component of the momentum, the second particle must lie in the plane as well.

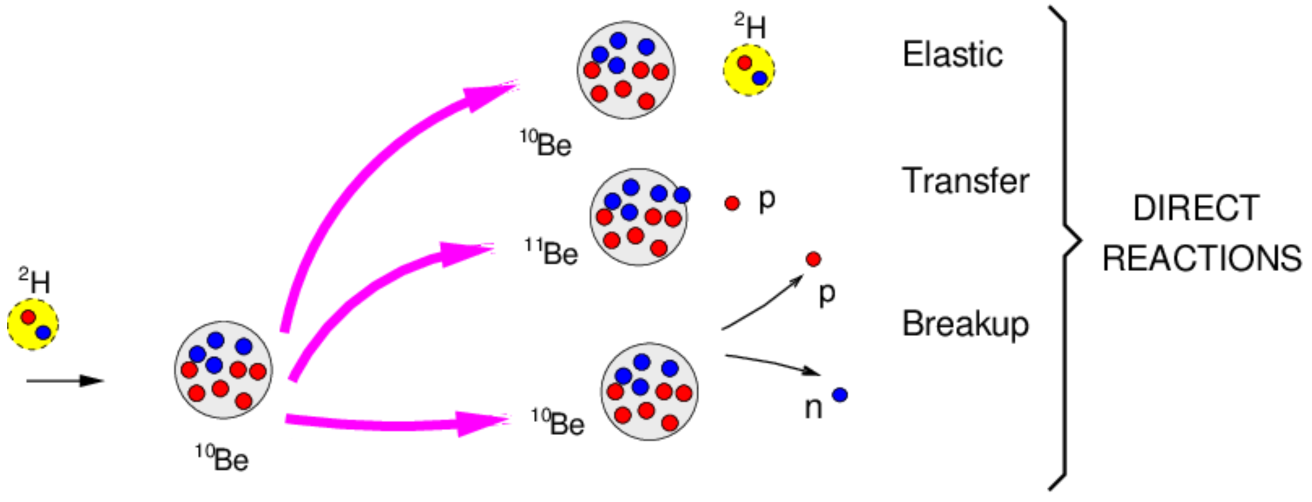
Using the conservation of the angular momentum, energy and definition of the Q-value it is possible to determine the kinetic energy of the outgoing scattering particle.

$$T_b^{1/2} = \frac{(m_a m_b T_a)^{1/2}\cos\theta \pm \{m_a m_b T_a \cos^2\theta + (m_Y + m_b)[m_Y Q + (m_Y - m_a)T_a]\}^{1/2}}{m_Y + m_b}$$

Threshold in the lab frame: $T_{th} = (-Q)\frac{m_Y + m_b}{m_Y + m_b - m_a}$

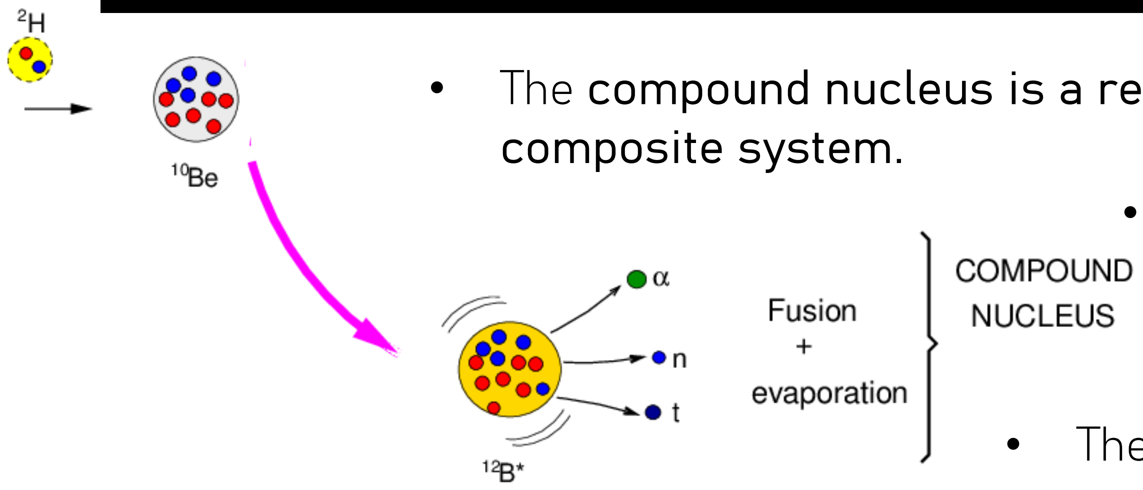
Kinematics calculators available:
Excel spreadsheet – Catkin by W. N. Catford: http://personal.ph.surrey.ac.uk/%7Ephs1wc/kinematics/
Window/Apple app – LISE++ kinematics calculator by O. Tarasov and D. Bazin: https://lise.nscl.msu.edu/lise.html
Online tool – SkiSickness by S.K.L. Sjue: http://skisickness.com/2010/04/21/

DIRECT REACTIONS



- Direct reactions are **fast** ($\sim 10^{-22}$ s)
- Direct reactions **happen on the surface** rather than in the volume of interacting nuclei.
- Direct reaction **products have highly anisotropic** – forward focused angular distributions in the centre of mass reference frame.
- **Angular distributions of direct reaction products are sensitive to the momentum transfer and parity change during the reactions.**
- Based on the selection rules from angular momentum and parity conservation the angular distribution measurements in direct reactions yield spin and parities of states populated in the exit channel.

COMPOUND-NUCLEUS REACTIONS



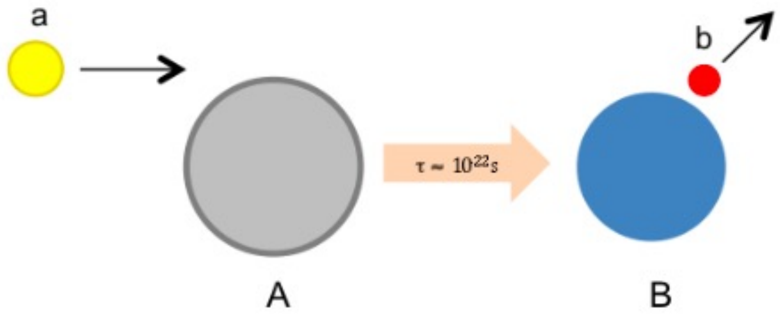
- The **compound nucleus is a relatively long-lived intermediate state of the particle-target composite system.**
- **A large number of collisions between the nucleons leads to a thermal equilibrium inside the compound nucleus.**
- The **time scale** of compound nucleus reactions is **10^{-18} – 10^{-16} s**

- Compound nucleus reactions usually **created if projectile has low energy** (10-20 MeV).
- **Incident particles interact in the volume** of a target nucleus.
- **Products** of the compound nucleus reactions are **distributed near isotropically in angle** (the nucleus loses memory of how it was created – Bohr's hypothesis of independence).
- The **decay mode** of the compound nucleus **does not depend on how** the compound **nucleus is formed**.
- Resonances in the cross-section are typical for the compound nucleus reaction.
- As the energy of the CN increases, more and more particles are evaporated.

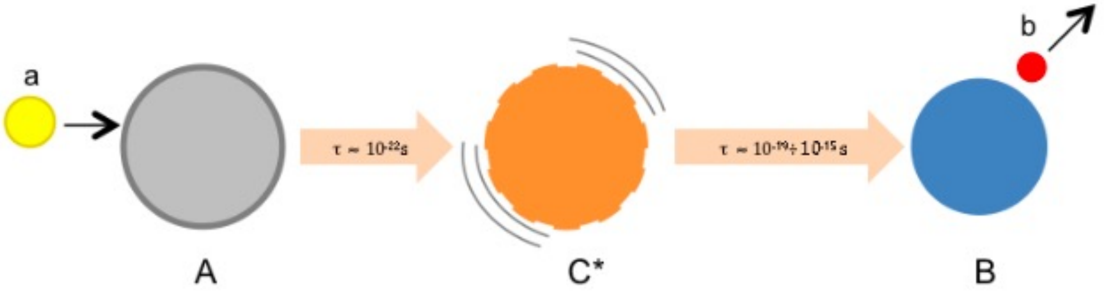
  Evaporated particles usually considered are n, p, α. A heavier particle can be chosen e.g. 3He, 6Li, 9Be but it will have a statistically small probability of formation. Below the particle threshold the CN de-excites by γ-emission.

ANGULAR DISTRIBUTION

DIRECT REACTION



COMPOUND-NUCLEUS REACTION



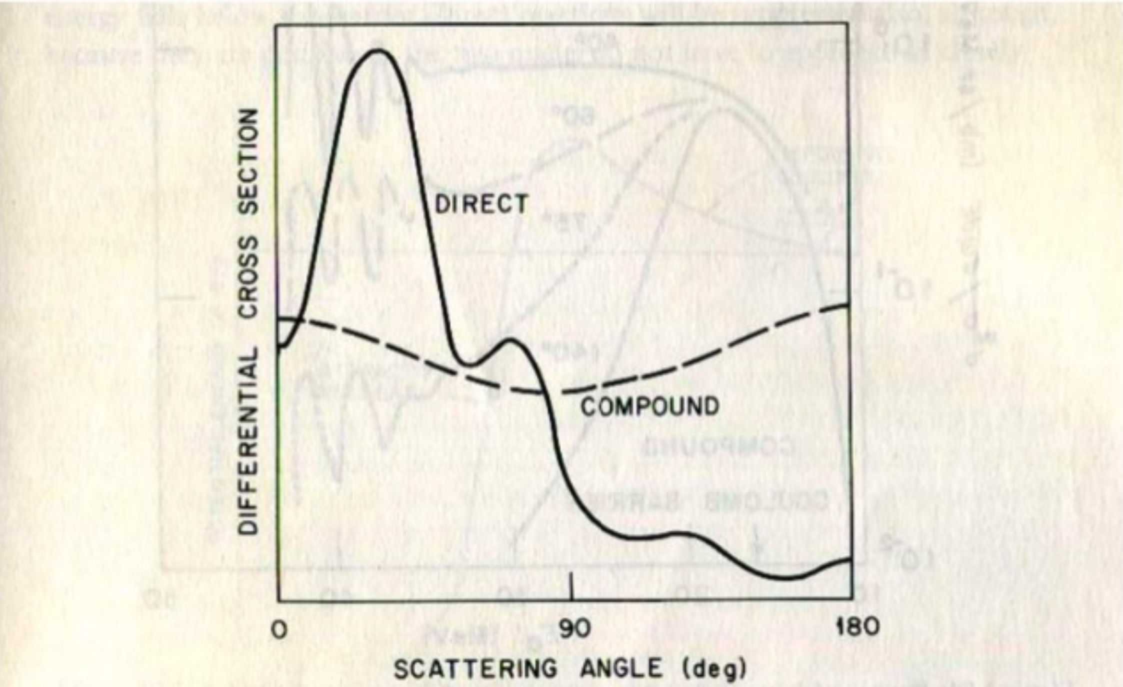


Figure 2.33 Typical angular distributions for direct and compound nucleus reactions induced by light ions with moderate bombarding energies (for example, some (d, p) reaction with 20-MeV deuterons)

NUCLEAR REACTION CODES

FRESCO – http://www.fresco.org.uk/
A flexible universal nuclear reaction code used for coupled-reaction channels calculations in nuclear physics.

TALYS – https://nds.iaea.org/talys/
An opensource nuclear reaction program with the objective to provide a complete and accurate simulation of nuclear reactions up to energies of 200 MeV, through an optimal combination of reliable nuclear models, flexibility and user-friendliness. TALYS can be used for the analysis of basic nuclear reaction experiments or to generate nuclear data for applications.

DWUCK4/5 – https://www.oecd-nea.org/tools/abstract/detail/nesc9872/
A code for zero-range or finite-range Distorted Wave Born Approximation (DWBA) calculations.

PTOLEMY – https://www.phy.anl.gov/theory/research/ptolemy/
A Program for Heavy-Ion Direct-reaction calculations

TWOFNR – https://nucleartheory.eps.surrey.ac.uk/NPG/code.htm
A Program for transfer reaction calcualtions.

OBSERVABLES AND EXPERIMENTAL DETAILS

What can be measured?

Detected scattered charge particles: $Z, A, E, W(\theta)$

Differential cross-section: $\frac{d\sigma}{d\Omega}$

Total reaction cross-section: σ_t

Polarization: spin orientation of residual nucleus and σ dependence

Detect particle and γ radiations

Beam requirements:

- highly collimated and focused to precisely determination of θ and ϕ for the angular distribution,
- sharply defined in energy to observe specific excited states,
- high intensity to collect the necessary statistics,
- time structure (pulsed beam) for time measurements,
- easily selectable to reduce tuning time,
- constant and easily measurable beam intensity to determine the cross section,
- polarised or unpolarised,
- high-vacuum transport system to prevent beam degradation and unwanted products.

OBSERVABLES AND EXPERIMENTAL DETAILS

Target requirements:

- thickness according to experimental goals.
  Thick targets to measure the yield of a reaction through beam attenuation.
  Thin targets when we want to observe b unaffected by interactions in the target itself,
- target can be self-supporting or with backings (which does not contribute to the reaction or affect the passage of particle b),
- problem with heat generated by beam. Cooling system or targets easy to replace,
- polarized target.

Detectors requirements:

- particle detectors to determine the particle energy and type,
- magnetic spectrometer for good energy resolution,
- position-sensitive particle detectors to do angular distribution,
- γ-ray detectors to observe de-excitation of the excited states of residual ncleus,
- - polarimeters to measure the polarization of the particle b
- multi-detectors configurations,
- ...

RADIOACTIVE DECAY

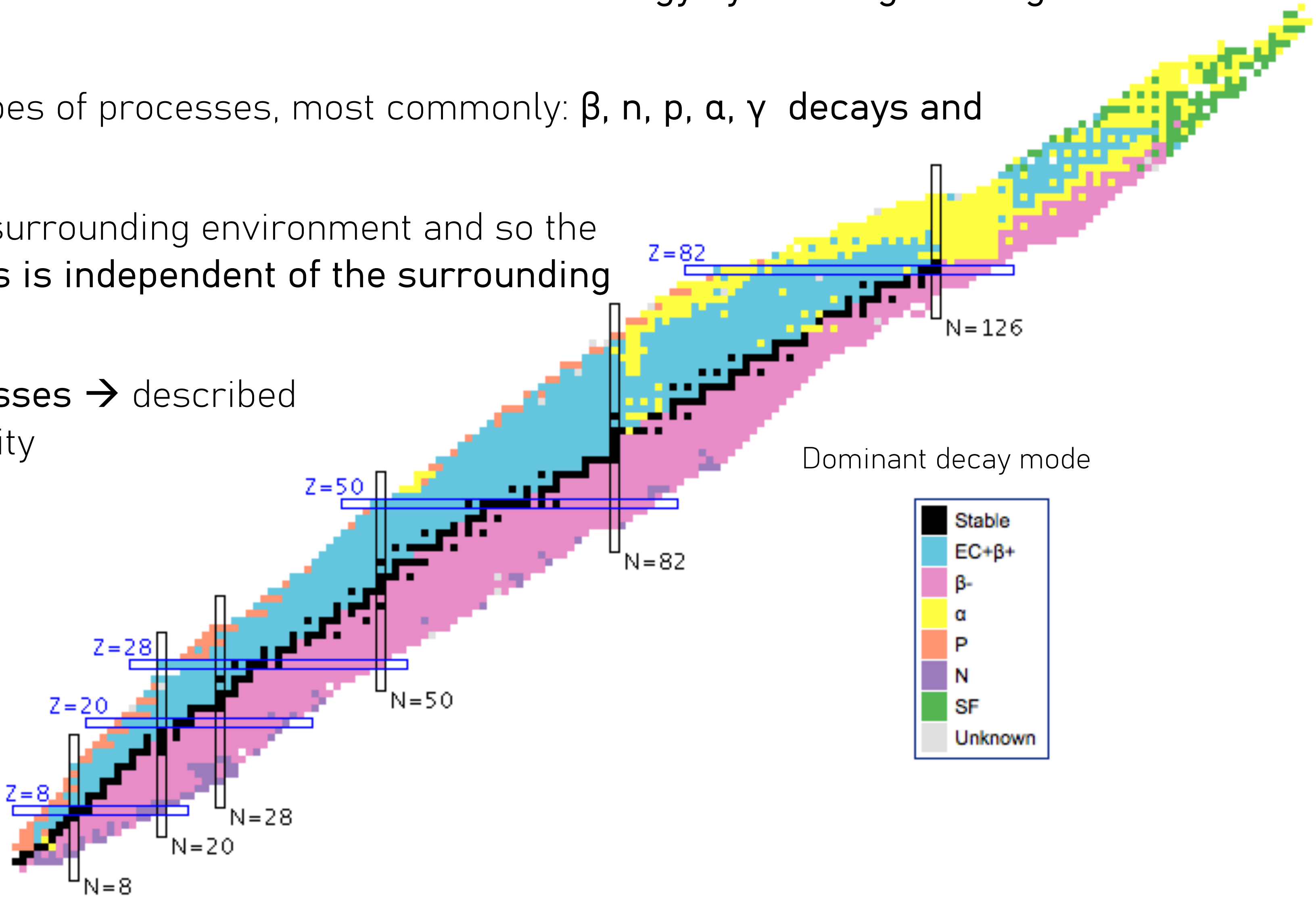
RADIOACTIVE DECAY

Radioactivity is the process by which a nucleus of an **unstable atom loses energy by emitting ionizing radiation to become more stable**.

Unstable nuclei decay through distinct types of processes, most commonly: **β, n, p, α, γ decays and fission**

Nuclei are effectively insulated from the surrounding environment and so the rate at which **radioactive decay proceeds is independent of the surrounding conditions** (e.g. pressure, temperature).

Radioactive decay is a stochastic processes → described using statistical tools describing probability



RADIOACTIVE DECAY

The p**robability per unit time for any radioactive nucleus to decay is a constant**, called the **decay constant** λ.

If there are N radioactive nuclei, then the number that will decay in a time Δt is given by $\Delta N = -\lambda N \Delta t$

The rate of radioactive decay is directly proportional to the number of nuclei present and the decay constant.

$$\frac{dN}{dt} = -\lambda N \Longrightarrow N(t) = N_o e^{-\lambda t}$$

Mean lifetime $\tau = \frac{1}{\lambda}$

Half-life $\boldsymbol{T_{1/2}}$ is the time taken for half the radionuclide to decay: $T_{1/2} = \frac{\ln 2}{\lambda}$

Activity $(\boldsymbol{A})$ is the number of decays that an unstable nuclide undergoes per second: $A(t) = A_0 e^{-\lambda t}$

The **unit** of activity is **Becquerel (Bq):** 1 Bq is 1 disintegration per second.
1 Curie (Ci) = 3.7×10^{10} Bq (historical unit based on the decay from a gram of radium)

RADIOACTIVE DECAY

Example:

^{231}Th undergoes β^-decay with $T_{1/2} = 25.6\ hours$. At $t = 0$, a ^{231}Th sample contains 56000 nuclei.

1. How many ^{231}Th nuclei will remain after 3 days?
2. What is the total rate at which electrons are emitted at $t = 0$ and after 3 days?

$$N(t) = N_0 e^{-\lambda t} \qquad \lambda = \frac{\ln 2}{T_{1/2}} \qquad \frac{dN}{dt} = -\lambda N_0$$

Solution:

1. How many ^{231}Th nuclei will remain after 3 days?

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{92160\ s} = 7.52 \times 10^{-6} s^{-1}$$

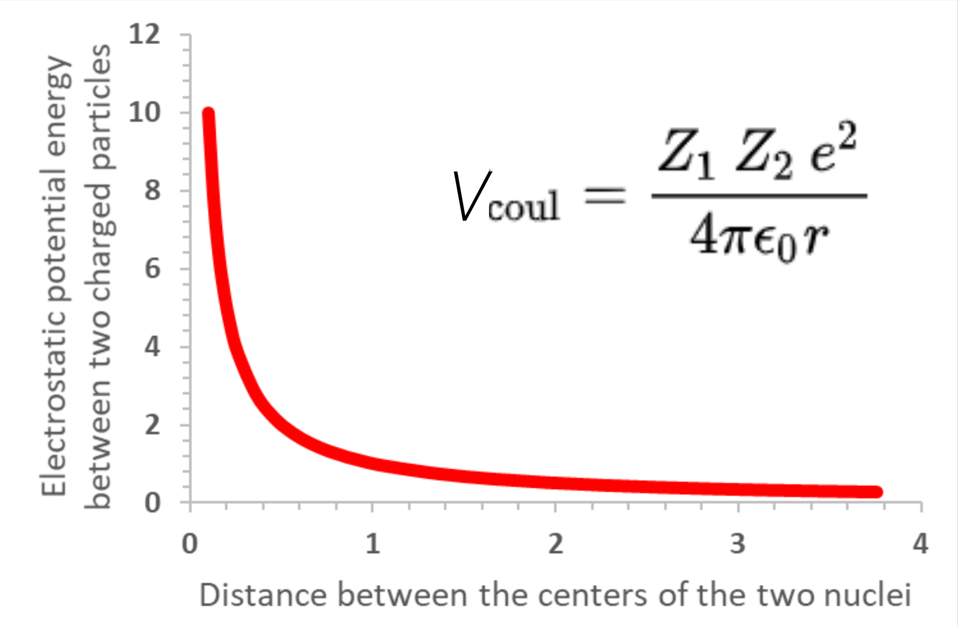
$$N(after\ 3\ days) = N_0 e^{-\lambda t} = 56000 e^{-7.52\times10^{-6}\times259200} = 7971.52\ nuclei$$

2. What is the total rate at which electrons are emitted at $t = 0$ and after 3 days?

Activity at $t = 0$: $\frac{dN}{dt} = -\lambda N_0 = 0.42112\ Bq$

Activity at $t = 3\ days$: $\frac{dN}{dt} = -\lambda N(t) = 0.0599\ Bq$

COULOMB BARRIER



The electrostatic force that two charged particles experience when they are in close proximity to each other.

It is the potential energy barrier that must be overcome for two positively charged atomic nuclei to approach each other closely enough to undergo nuclear reactions.

The repulsion between the positively charged nuclei, can be overcome by sufficient kinetic energy or through the quantum mechanical tunnelling effect.

$$V_{\text{coul}} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

V_{coul} is the electrostatic potential energy between two charged particles (in this case, two atomic nuclei)

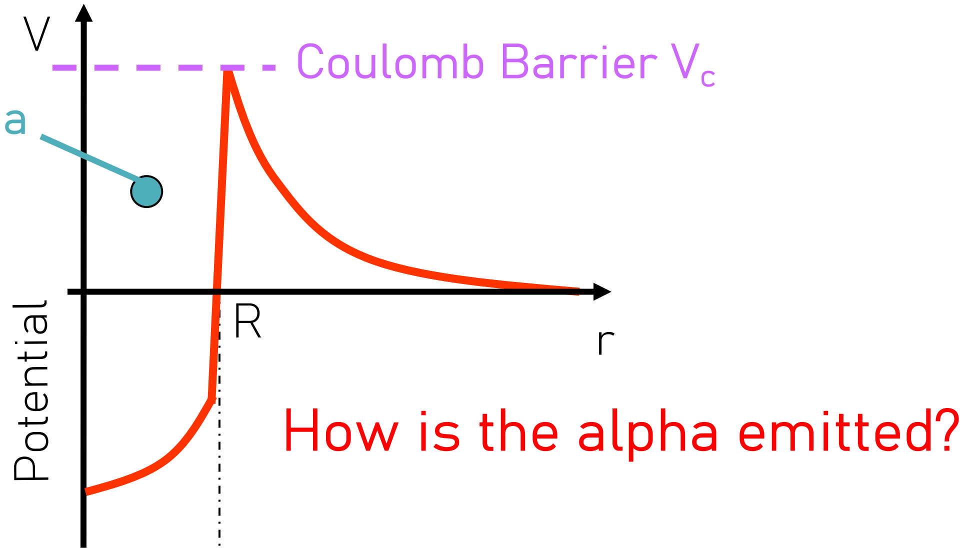
Z_1 and Z_2 are the atomic numbers (i.e., number of protons) of the two nuclei

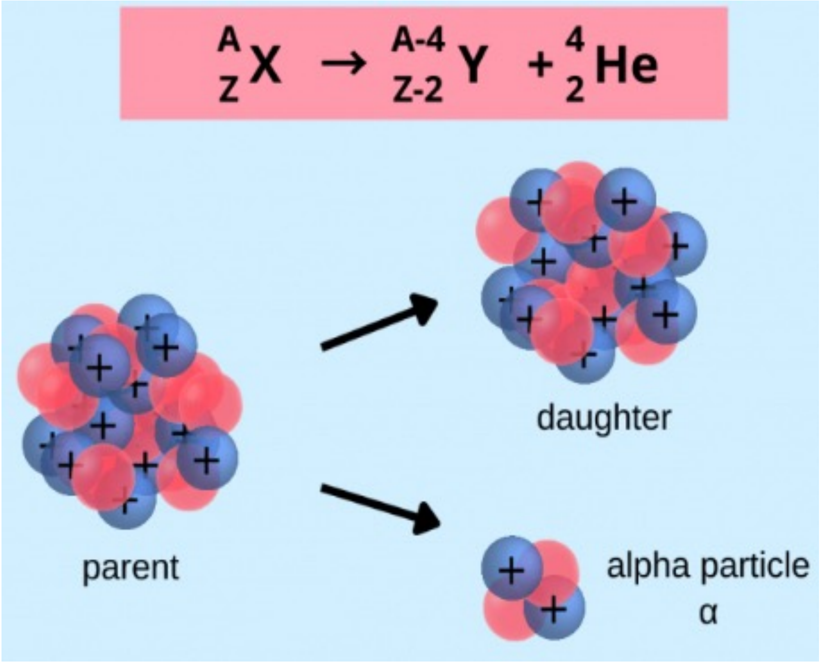
e is the elementary charge

r is the distance between the centers of the two nuclei

ε_0 is the electric constant (also known as the vacuum permittivity).

ALPHA DECAY





https://sciencenotes.org/alpha-particle-definition-symbol-and-charge/

Classically, if a particle does not have enough to overcome the barrier it will remain bound.

Quantum mechanically, there is a finite probability that a particle below the Coulomb barrier will penetrate it.

→ This is due to Heisenberg's uncertainty principle in its position.

The probability for this tunnelling for two like charges colliding at speed v is (Gamow 1928):

$$e^{-\frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h v}}$$

Hence:

1) tunnelling decreases with higher charge
2) Increases with velocity (kinetic energy)

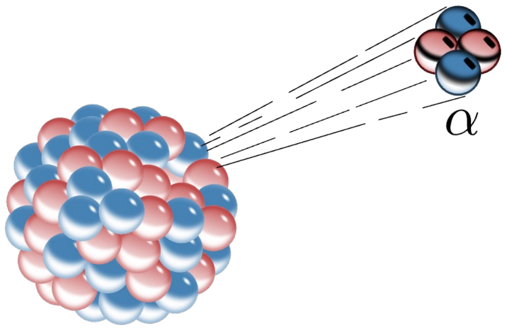
ALPHA DECAY

beyond Bi α emission ends the valley of stability !

Yellow → are α emitter

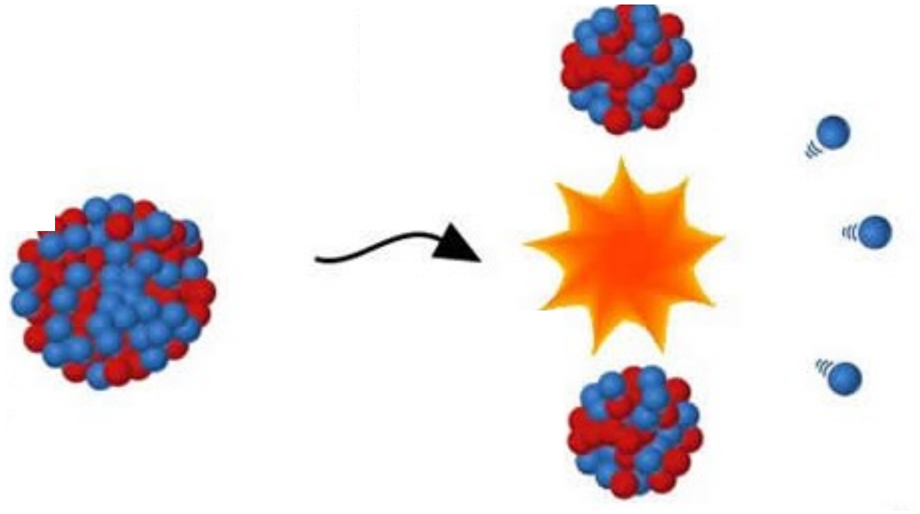
lightest α emitter: ^{144}Nd (Z=60)
(Q_α=1.9 MeV but still $T_{1/2}$=2.3 x 10^{15} yr)

the higher the Q-value the easier the Coulomb barrier can be overcome and the shorter the α-decay half-lives

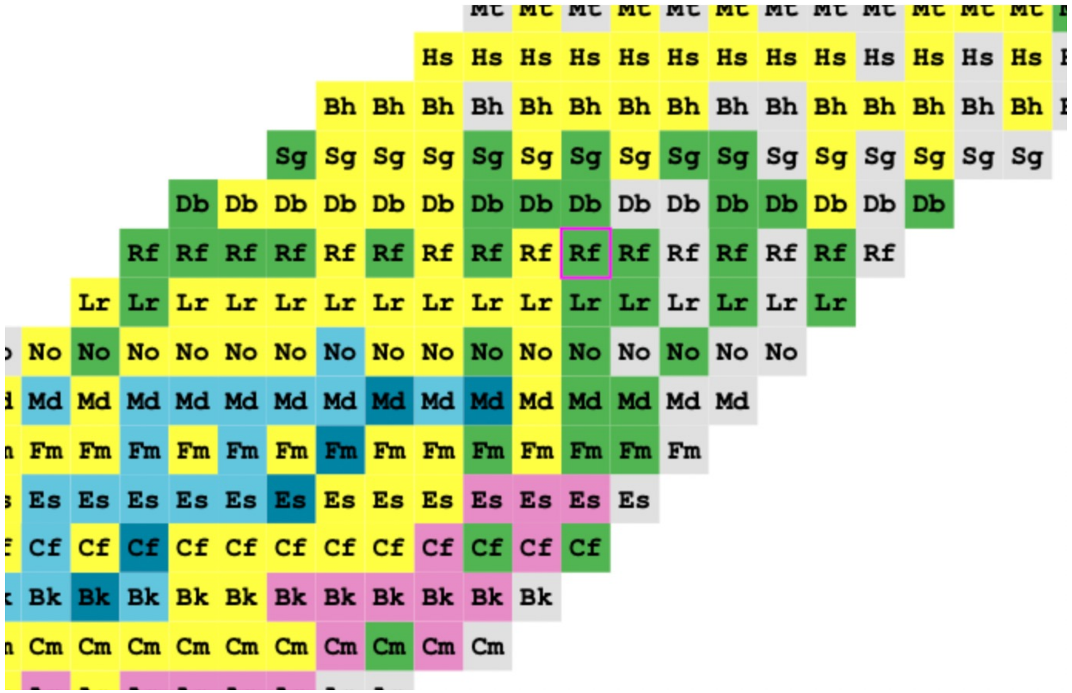


SPONTANEOUS FISSION

Green → spontaneous fission

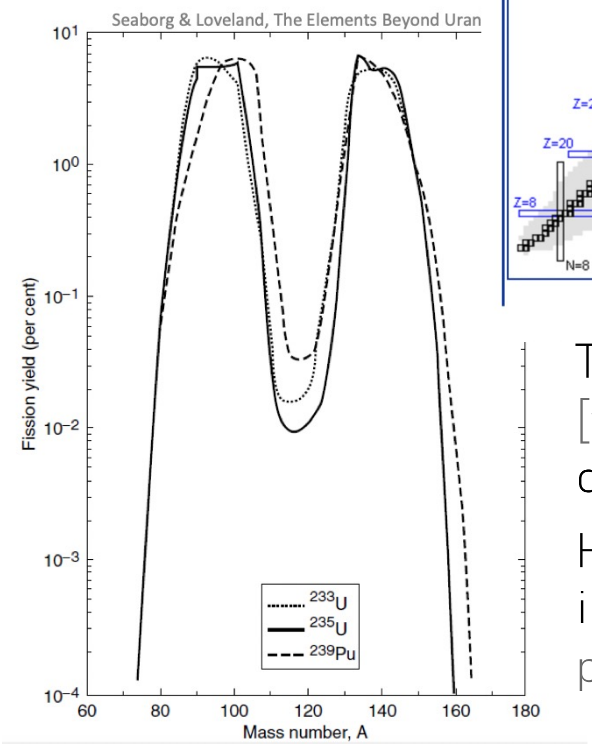


Modified from https://www.arpansa.gov.au/understanding-radiation/what-is-radiation/ionising-radiation/radiation-decay

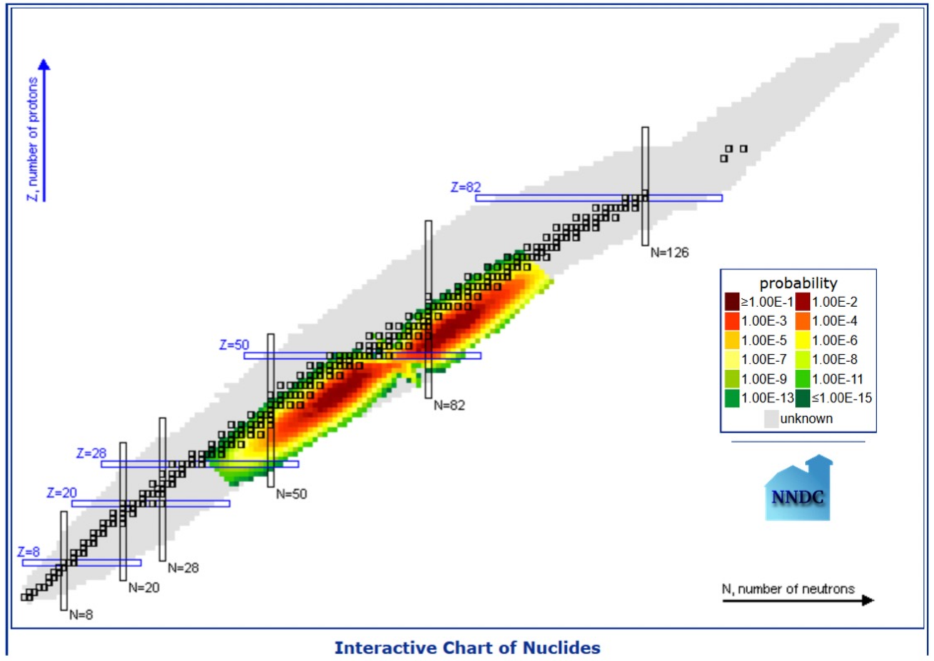


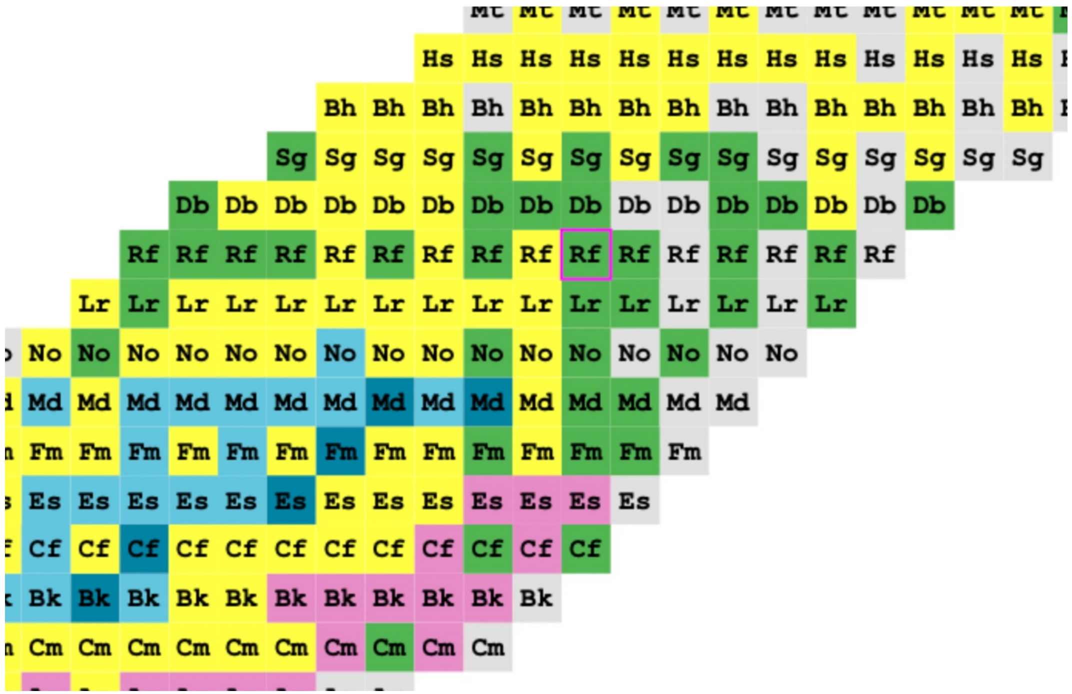
- **Spontaneous fission** can occur only in **very heavy elements with A>92**.
- Different to the nuclear fission that occurs in a nuclear reactor which is induced by neutron bombardment of the fuel.
- **Spontaneous fission occurs because of quantum tunnelling**.
- The results of spontaneous fission are the same as that for induced fission, with the **element splitting into two lighter nuclei and releasing neutrons in the process**.

SPONTANEOUS FISSION



^{252}Cf spontaneous fission





To first order, the fission fragments will be those that maximize the total energy release [which can be estimated from the liquid drop model] → This would always result in a case of a symmetric distribution of the matter.

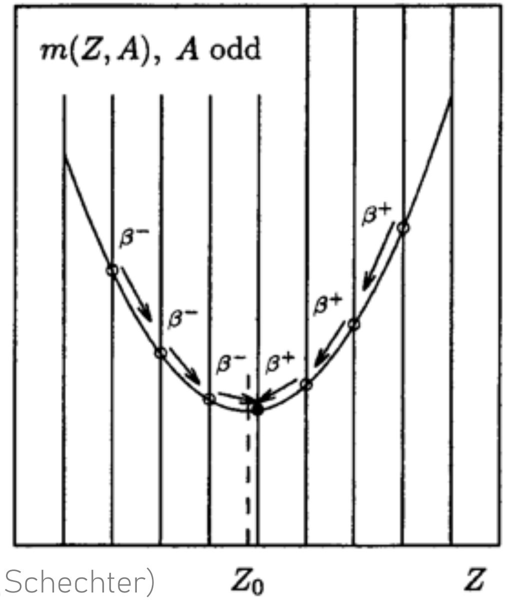
However, often the **observed mass distributions are instead quite asymmetric**: the individual fragments approximately **maintain the original A/Z** (because neutrons and protons separate in the same way), but **often high-A and low-A peaks exist**

BETA DECAY

Any nucleus with a given mass number A will be converted into the most stable proton/neutron combination with mass number A by β decays

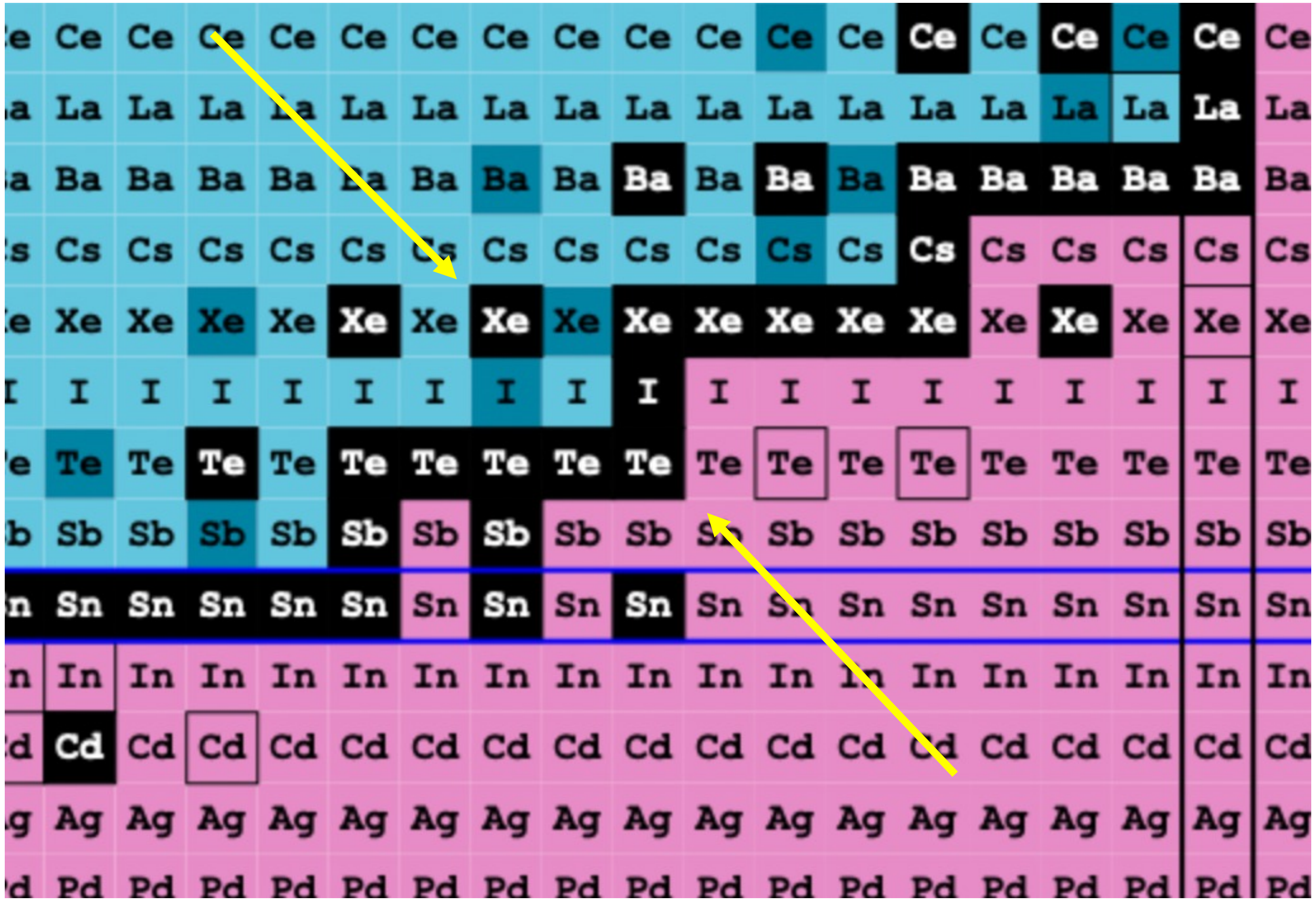
$$\beta^-: n \rightarrow p + e^- + \bar{\nu}_e$$
$$\beta^+: p \rightarrow n + e^+ + \nu_e$$
$$EC:\ e^- + p \rightarrow n + \nu_e$$

- If energetically possible it usually happens (except if another decay mode dominates)



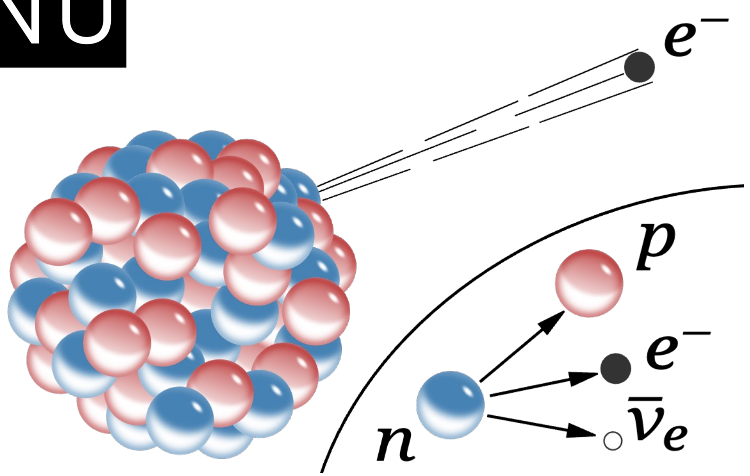
(Bertulani & Schechter)

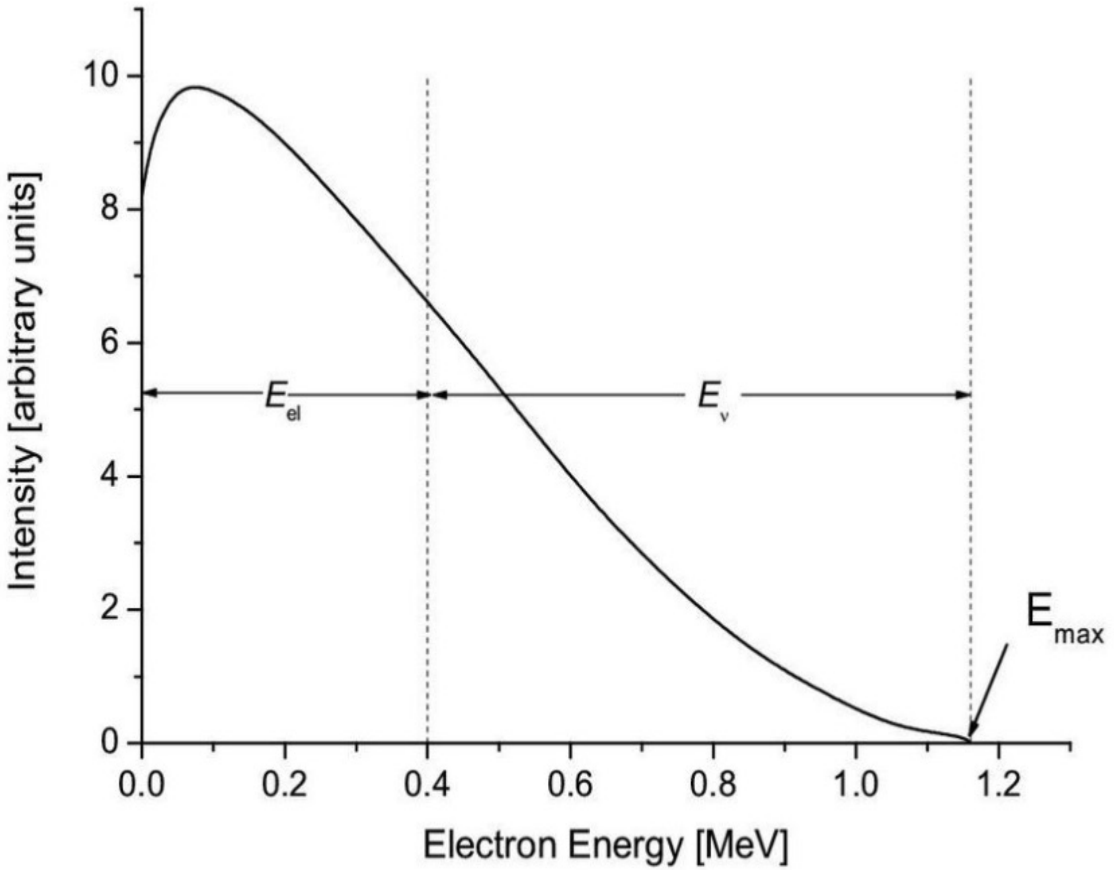
Blue: excess of protons, undergo β^+ decay



Pink: excess of neutrons, undergo β^- decay

NEUTRINO





If β-decay were a two-body process, then all β-particles would have a unique energy.

For example, from mass differences ${}^{210}_{83}Bi$ β-decay would have β^--kinetic energy of $T_\beta({}^{210}_{83}Bi) = 1.16\ MeV$, yet a **continuous energy distribution is observed**.

The existence of the neutrino ν, was first postulated in 1930 by Wolfgang Pauli to explain the apparent violation of conservation laws in β^--decay (spin not conserved if considered $n \rightarrow p + e^-$ only - $j_n = j_p = j_e = \frac{1}{2} \rightarrow 0 \leq j_n \leq 1 \neq \frac{1}{2}$)

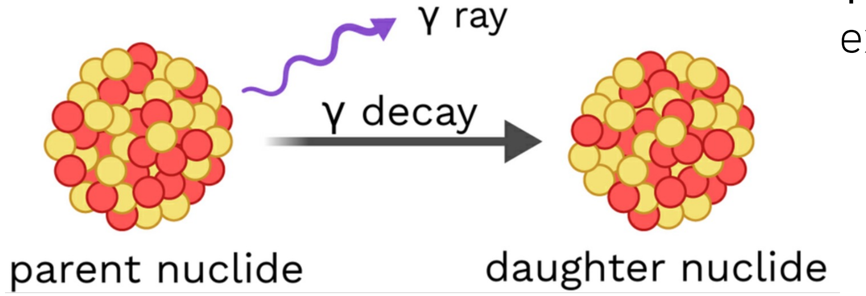
The new particle or **neutrino** (little neutral one) was named by Enrico Fermi who worked out the β^--decay theory in 1933 → Without a third particle in the decay process several conservation laws are violated.

GAMMA DECAY

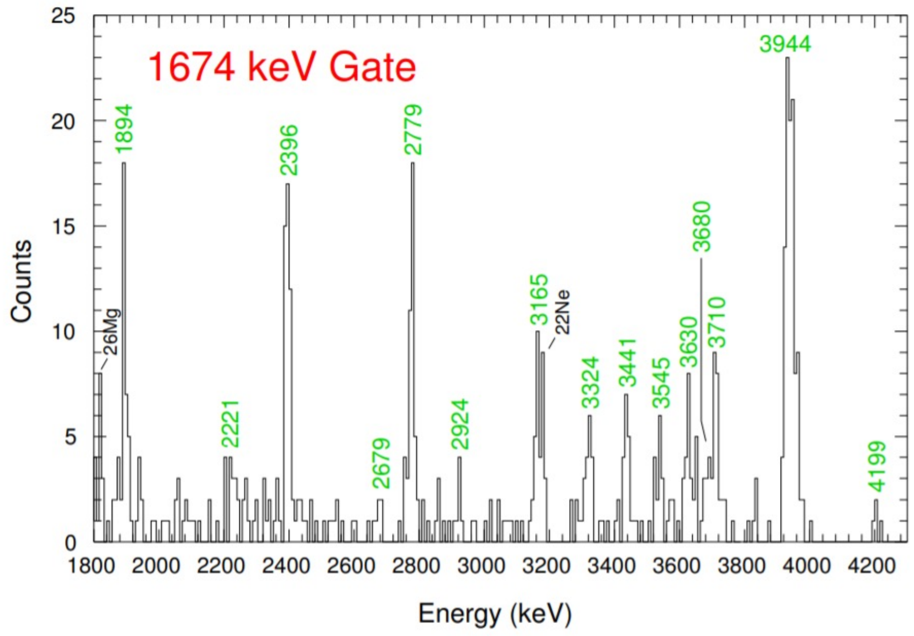
γ decay is a de-excitation from an excited state to a lower energy state, preceded by some decay or reaction. Z & A are unchanged

γ ray **energies** can span anywhere **from several keV to several MeV**

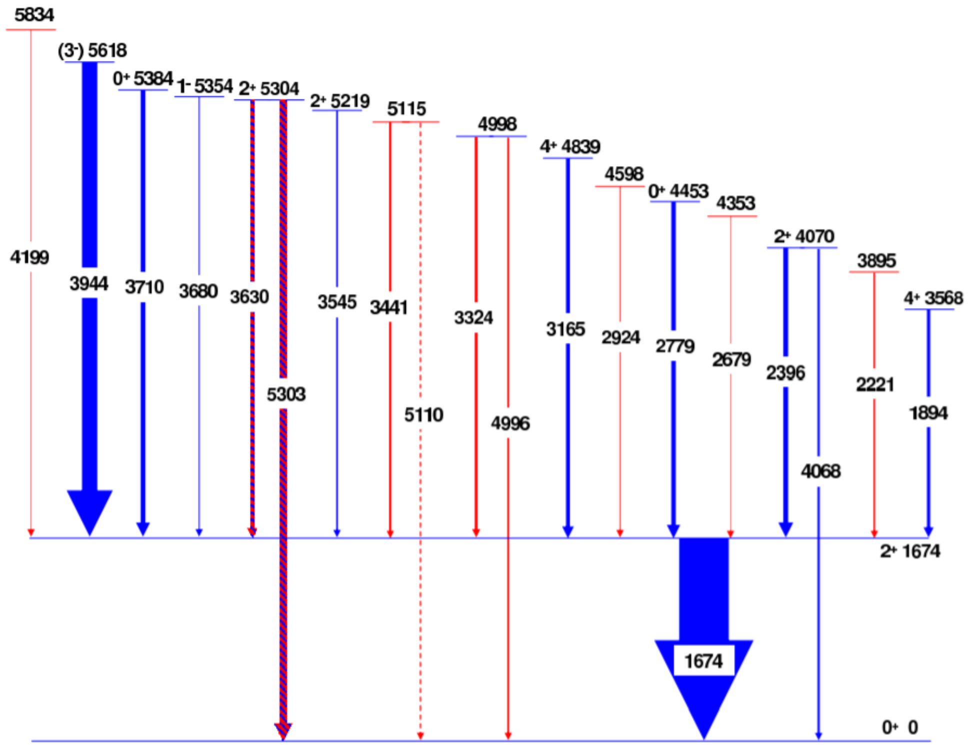
γ decay lifetimes are **typically extremely short** ($\tau \lesssim$ femtoseconds) except for isomeric states.



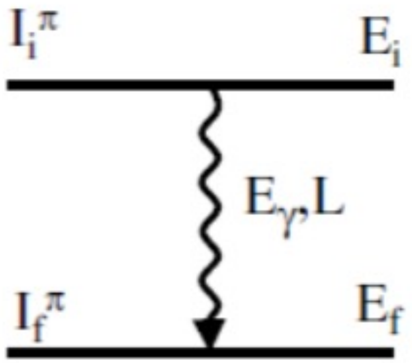
https://theory.labster.com/gamma-decay-msc/



A highly excited nuclear state with many different decay probabilities to lower levels will decay via transitions with largest decay strength i.e. with the smallest multipole.



GAMMA DECAY AND SELECTION RULES



$$E_\gamma = E_i - E_f$$

$$\left|I_i - I_f\right| \le L \le I_i + I_f$$

$$\Delta\pi(EL) = (-1)^L$$

$$\Delta\pi(ML) = (-1)^{L+1}$$

- Selection rules to infer spins and parities of states.
- Gamma-decay selection rules result from the conservation laws of angular momentum and parity.
- A photons carries angular momentum L and parity π.
- Conservation laws require:

$$\ell_i = \ell_f + L \text{ and } \pi_i = \pi_f \pi$$

with ℓ_i, π_i*and* ℓ_f, π_f for the initial and final states

MULTIPOLARITY OF GAMMA DECAY

Character/multipolarity	Symbol	L	$\mid I_i - I_f \mid$	$\pi = \pi_i \pi_f$
Electric dipole	$E1$	1	≤ 1	-1
Magnetic dipole	$M1$	1	≤ 1	$+1$
Electric quadrupole	$E2$	2	≤ 2	$+1$
Magnetic quadrupole	$M2$	2	≤ 2	-1
Electric octupole	$E3$	3	≤ 3	-1
Magnetic octuploe	$M3$	3	≤ 3	$+1$

The type of γ-ray decay is identified by its multipolarity L and character π.

Transitions with increasing L have decreasing transition rates and increasing lifetimes.

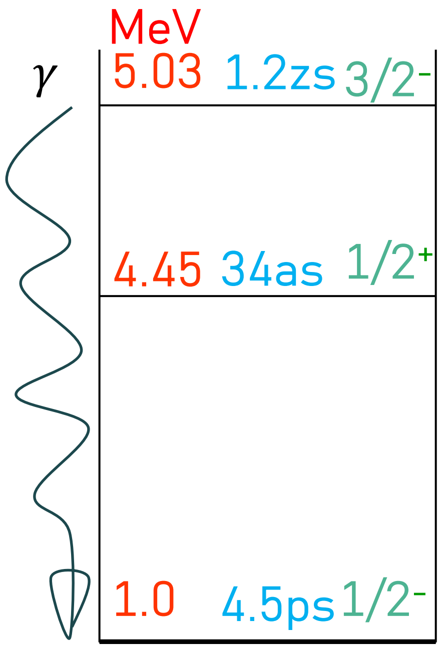
Photons of transitions with different L have different angular distributions.

Photons of transitions of different character (magnetic/electric) have different linear polarizations.

MULTIPOLARITY OF GAMMA DECAY

Exercise:

1) What are the possible types of gamma-rays that are allowed between the 5.03 and 1.0 MeV states?



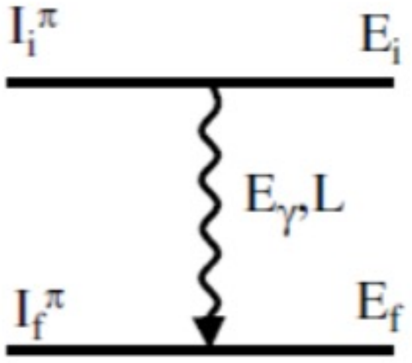
$$E_\gamma = E_f - E_i = 5.03 - 1.0 = 4.03\ MeV$$

$$\left|\frac{1}{2} - \frac{3}{2}\right| \le L \le \left|\frac{1}{2} + \frac{3}{2}\right| \Rightarrow L = 1, 2$$

$$\pi(E1) = (-) \qquad \pi(E2) = (+)$$
$$\pi(M1) = (+) \qquad \pi(M2) = (-)$$

$$\pi = \pi_i \pi_f = (-)(-) \Rightarrow \pi = +$$

Allowed: $E2\ and\ M1$



$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \le L \le I_i + I_f$$

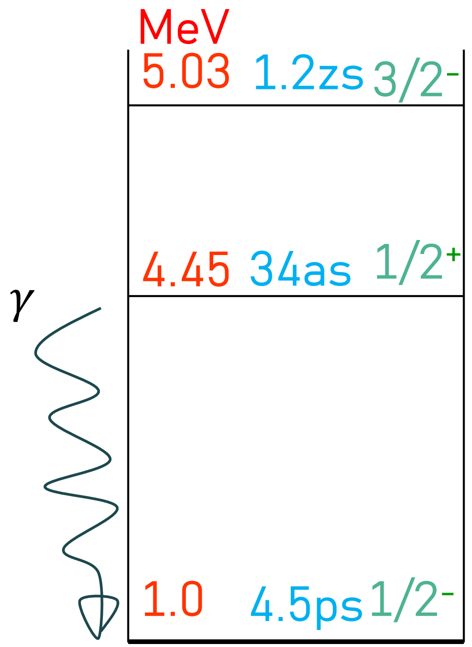
$$\Delta\pi(EL) = (-1)^L$$

$$\Delta\pi(ML) = (-1)^{L+1}$$

MULTIPOLARITY OF GAMMA DECAY

Exercise:

2) What are the possible types of gamma-rays that are allowed between the 4.45 and 1.0 MeV states?



$$E_\gamma = E_f - E_i = 4.45 - 1.0 = 3.45\ MeV$$

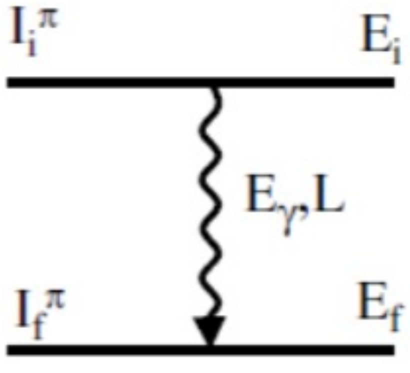
$$\left|\frac{1}{2} - \frac{1}{2}\right| \leq L \leq \left|\frac{1}{2} + \frac{1}{2}\right| \Rightarrow L = 0, 1$$

$$\pi(E0) = (+) \qquad \pi(E1) = (-)$$
$$\pi(M0) = (-) \qquad \pi(M1) = (+)$$

$$\pi = \pi_i \pi_f = (+)(-) \Rightarrow \pi = -$$

Allowed: $E1$

Note: No magnetic monopoles in nature



$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq L \leq I_i + I_f$$

$$\Delta\pi(EL) = (-1)^L$$

$$\Delta\pi(ML) = (-1)^{L+1}$$


THANK YOU FOR LISTENING

REFERENCES AND FURTHER MATERIAL

Textbooks:

K. S. Krane: Introductory nuclear physics - Editor: Wiley

G.R. Satchler: Introduction to nuclear reactions - Editor: Macmillan Education LTD

Bogdan Povh, Klaus Rith, Christoph Scholz, Frank Zetsche: Particles and Nuclei – Editor: Springer

Lectures and Publications:

Lecture Notes in Nuclear Structure Physics (B.A. Brown)

Z. Meisel, Lectures PHYS7501, Ohio University

M. Wiedeking, Lectures, Nuclear physics and related topics

I. Usman, Lecture Nuclear Reaction, Structure and Astrophysics, African School of Physics, NMU 2022

A. Moro, Models for nuclear reactions with weakly-bound systems

Videos:

the valley of stability - https://www.youtube.com/watch?v=UTOp_2ZVZmM