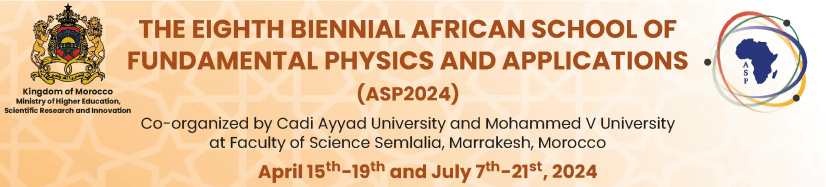
Lectures on Quantum Computing

Mourad Telmini

mourad.telmini@fst.utm.tn

University of Tunis El Manar
Faculty of Science de Tunis, Department of Physics



Outline of the lectures

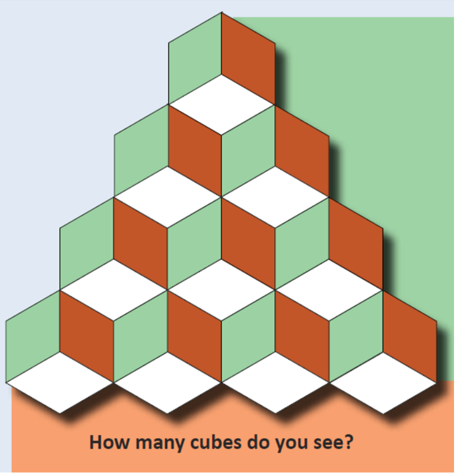
Outline of the lectures

1. Lecture 1 : Introduction to Quantum Computing

2. Lecture 2 : Basics of Quantum Computing

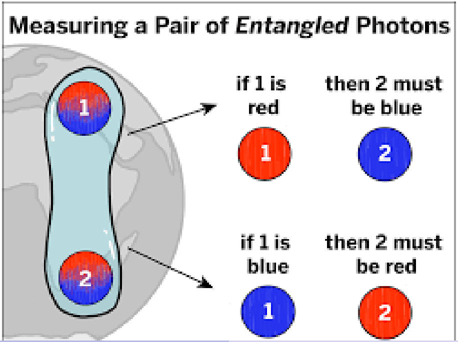
Quantum superposition

While a classical system can only be in a specific state at any time, a quantum system can exist in a superposition of several possible quantum states.



Quantum entanglement

When a system, formed by at least two particles is prepared in a non-separable state, and then it is split as each part moves away, the parts continue to be linked whatever the distance is and the measurment of the state of one part, instantaneously defines the state of the second part.



Hadamard Gate H : superposition

- The Hadamard gate H is fundamental in quantum computation. It is represented by the matrix:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Hadamard Gate H : superposition

- The Hadamard gate H is fundamental in quantum computation. It is represented by the matrix:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- It generates state superpositions by acting on the state $|0\rangle$ or $|1\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$
$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

Hadamard Gate H : superposition

- The Hadamard gate H is fundamental in quantum computation. It is represented by the matrix:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- It generates state superpositions by acting on the state $|0\rangle$ or $|1\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$
$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

- Exercice 1: calculate $H\,|+\rangle$ and $H\,|-\rangle$.

Hadamard Gate H : superposition

- The Hadamard gate H is fundamental in quantum computation. It is represented by the matrix:

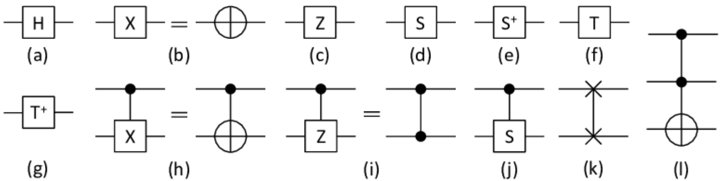
$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- It generates state superpositions by acting on the state $|0\rangle$ or $|1\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$
$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

- Exercice 1: calculate $H|+\rangle$ and $H|-\rangle$.
- Exercice 2: check the identity $H^2 = I$

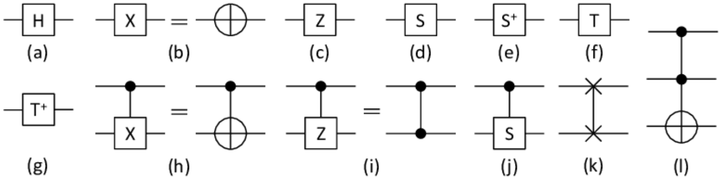
Other single-qubit gates



- X and Z gates :

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

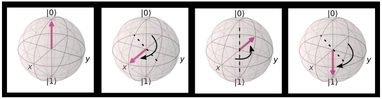
Other single-qubit gates



- X and Z gates :

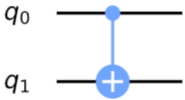
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Exercice : check the identity $X = HZH$



CNOT gate

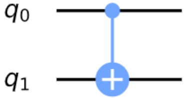
- The CNOT gate is a 2-qubits gate



CNOT gate

- The CNOT gate is a 2-qubits gate
- Its matrix representation is given by the tensor product of I and X matrices:

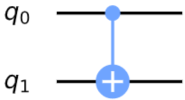
$$(CNOT) = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



CNOT gate

- The CNOT gate is a 2-qubits gate
- Its matrix representation is given by the tensor product of I and X matrices:

$$(CNOT) = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

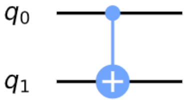


- It is a conditional gate which causes a gate X to act on the second qubit $|q_1\rangle$ (target), if the first qubit $|q_0\rangle$ (control) is in state $|1\rangle$.

CNOT gate

- The CNOT gate is a 2-qubits gate
- Its matrix representation is given by the tensor product of I and X matrices:

$$(CNOT) = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



- It is a conditional gate which causes a gate X to act on the second qubit $|q_1\rangle$ (target), if the first qubit $|q_0\rangle$ (control) is in state $|1\rangle$.
- In this case, the CNOT gate reverses the amplitudes of the target qubit. Otherwise, she does not change her condition.

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.
- a quantum state of the register can be obtained by a tensor product of independent single-qubit states $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$.

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.
- a quantum state of the register can be obtained by a tensor product of independent single-qubit states $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$.
- if the two qubits (q_0, q_1) are in "pure" states $(|0\rangle)$ or $(|1\rangle)$ each, the register can be in one of the four states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.
- a quantum state of the register can be obtained by a tensor product of independent single-qubit states $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$.
- if the two qubits (q_0, q_1) are in "pure" states $(|0\rangle)$ or $(|1\rangle)$ each, the register can be in one of the four states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
- These four states are separable. However, the linearity of quantum mechanics, reflected in the linearity of the algebra, allows for any linear combination of these states to describe a physical state of the register.

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.
- a quantum state of the register can be obtained by a tensor product of independent single-qubit states $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$.
- if the two qubits (q_0, q_1) are in "pure" states $(|0\rangle)$ or $(|1\rangle)$ each, the register can be in one of the four states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
- These four states are separable. However, the linearity of quantum mechanics, reflected in the linearity of the algebra, allows for any linear combination of these states to describe a physical state of the register.
- In particular, states that can not be factorized as a tensor product like $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$

CNOT gate

- the two qubits (q_0, q_1) constitute a quantum register.
- a quantum state of the register can be obtained by a tensor product of independent single-qubit states $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$.
- if the two qubits (q_0, q_1) are in "pure" states $(|0\rangle)$ or $(|1\rangle)$ each, the register can be in one of the four states $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
- These four states are separable. However, the linearity of quantum mechanics, reflected in the linearity of the algebra, allows for any linear combination of these states to describe a physical state of the register.
- In particular, states that can not be factorized as a tensor product like $|q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$
- These states are called non-separable, or entangled states. For example, the Bell state :

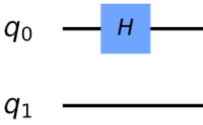
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

How to create entanglement ?

- The major interest of the CNOT gate is to create an entangled state.

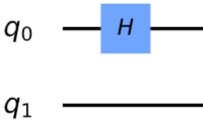
How to create entanglement ?

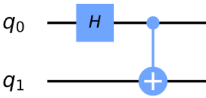
- The major interest of the CNOT gate is to create an entangled state.
- In the first step, to create a superimposed state, we make a gate of H act on $|q_0\rangle$, which produces the state $|+\rangle$ from the initial state $|0\rangle$. This is done by applying a $(H \otimes I)$ gate to the system.

- 

How to create entanglement ?

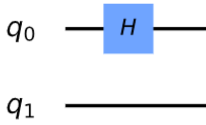
- The major interest of the CNOT gate is to create an entangled state.
- In the first step, to create a superimposed state, we make a gate of H act on $|q_0\rangle$, which produces the state $|+\rangle$ from the initial state $|0\rangle$. This is done by applying a $(H \otimes I)$ gate to the system.

- 

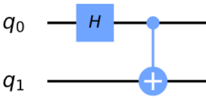
- 

How to create entanglement ?

- The major interest of the CNOT gate is to create an entangled state.
- In the first step, to create a superimposed state, we make a gate of H act on $|q_0\rangle$, which produces the state $|+\rangle$ from the initial state $|0\rangle$. This is done by applying a $(H \otimes I)$ gate to the system.



- The action of this circuit on the 2 qubit system initially in the state $|00\rangle$ is:



$$
\begin{aligned}
(CNOT.(H \otimes I))|00\rangle &= CNOT\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle\right] \\
&= CNOT\left[|\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right] \\
&= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)
\end{aligned}
$$

Quantum algorithms

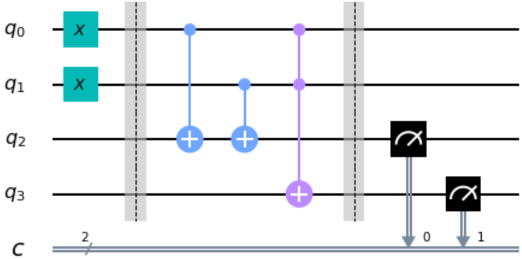
- Several quantum algorithms have been proposed and implemented. Here is a list of the main ones:

Quantum algorithms

- Several quantum algorithms have been proposed and implemented. Here is a list of the main ones:
  - Deutsch's and Deutsch-Jozsa Algorithms
  - Bernstein-Vazirani Algorithm
  - Simon's Algorithm
  - Quantum counting
  - Quantum Teleportation
  - Shor's Algorithm
  - Grover's Algorithm
  - Quantum Key Distribution (Quantum Cryptography)
  - Quantum Fourier Transform (QFT)
  - Super-dense Coding
  - Quantum Phase Estimation (QPE)
  - Variational Quantum Eigensolver (VQE)

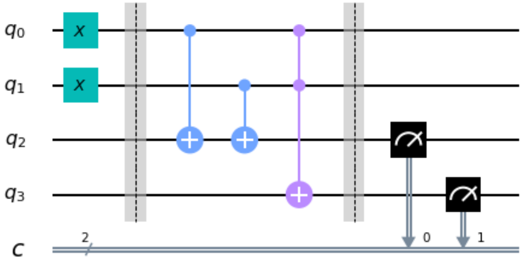
Example 1: Half-adder circuit

- Here again is a half-adder quantum circuit:



Example 1: Half-adder circuit

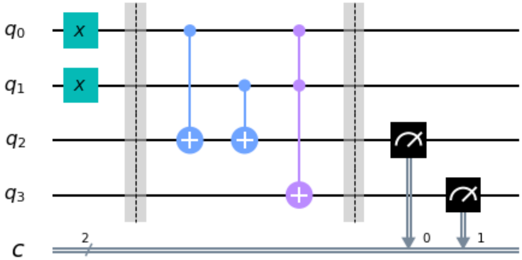
- Here again is a half-adder quantum circuit:



- 4-qubits register + 2 classical registers (for the measurements).

Example 1: Half-adder circuit

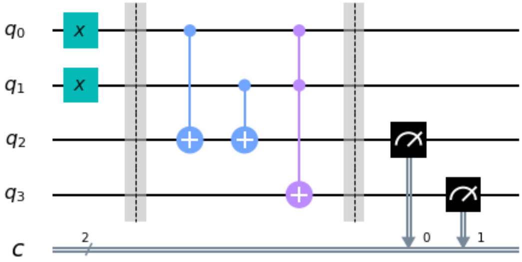
- Here again is a half-adder quantum circuit:



- 4-qubits register + 2 classical registers (for the measurements).
- qubits $|q_0\rangle$ and $|q_1\rangle$ are the entry qubits, while qubits $|q_2\rangle$ and $|q_3\rangle$ represent the sum and the carry, respectively.

Example 1: Half-adder circuit

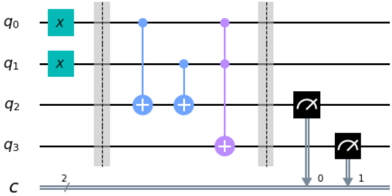
- Here again is a half-adder quantum circuit:



- 4-qubits register + 2 classical registers (for the measurements).
- qubits $|q_0\rangle$ and $|q_1\rangle$ are the entry qubits, while qubits $|q_2\rangle$ and $|q_3\rangle$ represent the sum and the carry, respectively.
- the four qubits are initialized to the state $|0\rangle$. So the the initial state of the register is $|0000\rangle$

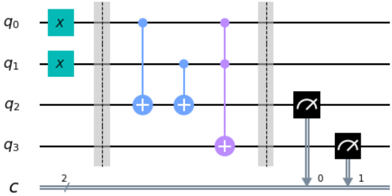
Example 1: Half-adder circuit

- Here again is a half-adder quantum circuit:



Example 1: Half-adder circuit

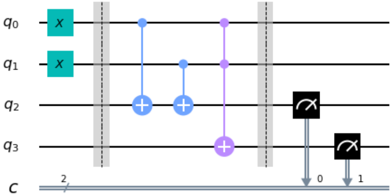
- Here again is a half-adder quantum circuit:



- if the circuit works correctly, it should reproduce the table of classical half-adder table of truth :

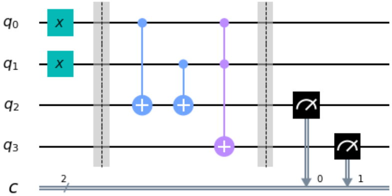
bit 1	bit 2	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Example 1: Half-adder circuit



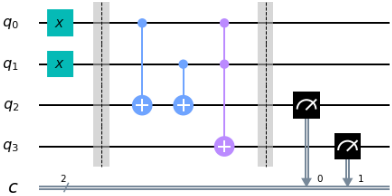
- Case 1: 0+0

Example 1: Half-adder circuit



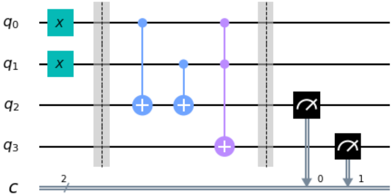
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$

Example 1: Half-adder circuit



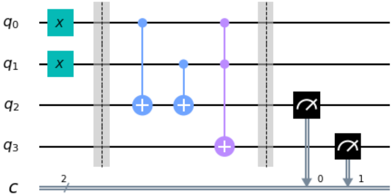
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |0000\rangle$

Example 1: Half-adder circuit



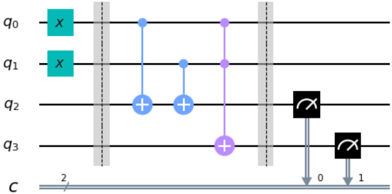
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |0000\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |0000\rangle$

Example 1: Half-adder circuit



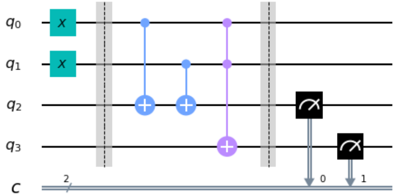
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |0000\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |0000\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |0000\rangle$

Example 1: Half-adder circuit



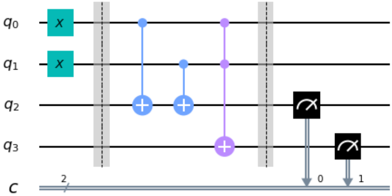
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |0000\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |0000\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |0000\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 0)$ and $|q_3\rangle = |0\rangle$ $(c_3 = 0)$

Example 1: Half-adder circuit



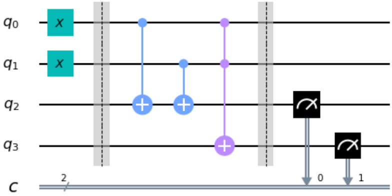
- Case 1: 0+0
- initial state of the register : $|\Psi_0\rangle = |0000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |0000\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |0000\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |0000\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 0)$ and $|q_3\rangle = |0\rangle$ $(c_3 = 0)$
- Outcome : sum=0 and carry=0

Example 1: Half-adder circuit



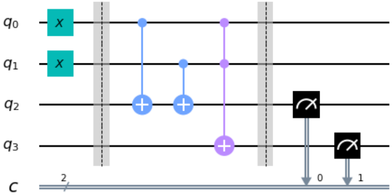
- Case 2: 1+0

Example 1: Half-adder circuit



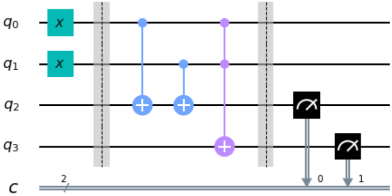
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$

Example 1: Half-adder circuit



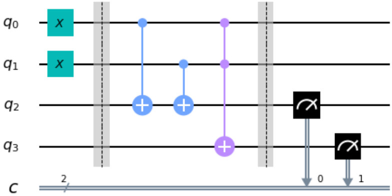
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1010\rangle$

Example 1: Half-adder circuit



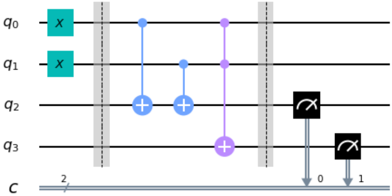
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1010\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1010\rangle$

Example 1: Half-adder circuit



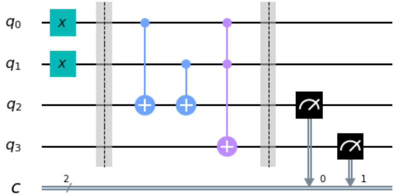
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1010\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1010\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1010\rangle$

Example 1: Half-adder circuit



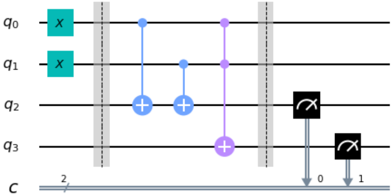
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1010\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1010\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1010\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 1)$ and $|q_3\rangle = |0\rangle$ $(c_3 = 0)$

Example 1: Half-adder circuit



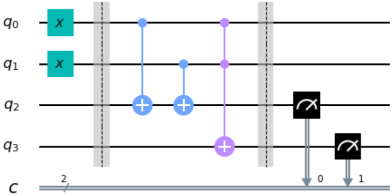
- Case 2: 1+0
- initial state of the register : $|\Psi_0\rangle = |1000\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1010\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1010\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1010\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 1)$ and $|q_3\rangle = |0\rangle$ $(c_3 = 0)$
- Outcome : sum=1 and carry=0

Example 1: Half-adder circuit



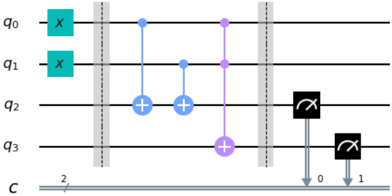
- Case 2: 1+1

Example 1: Half-adder circuit



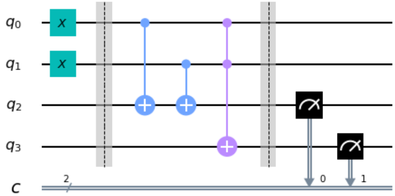
- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$

Example 1: Half-adder circuit



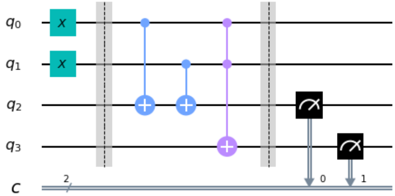
- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1110\rangle$

Example 1: Half-adder circuit



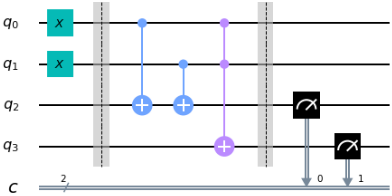
- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1110\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1100\rangle$

Example 1: Half-adder circuit



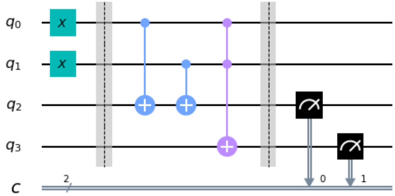
- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1110\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1100\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1101\rangle$

Example 1: Half-adder circuit



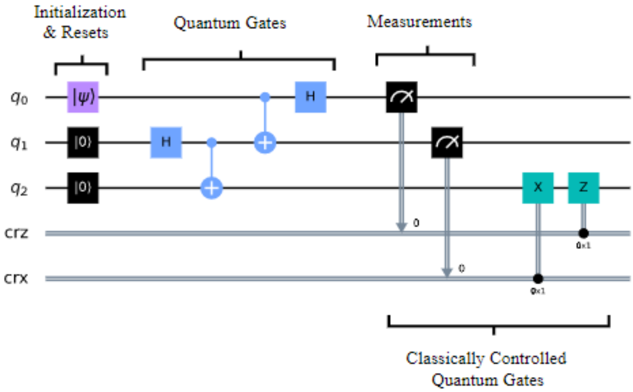
- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1110\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1100\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1101\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 0)$ and $|q_3\rangle = |1\rangle$ $(c_3 = 1)$

Example 1: Half-adder circuit

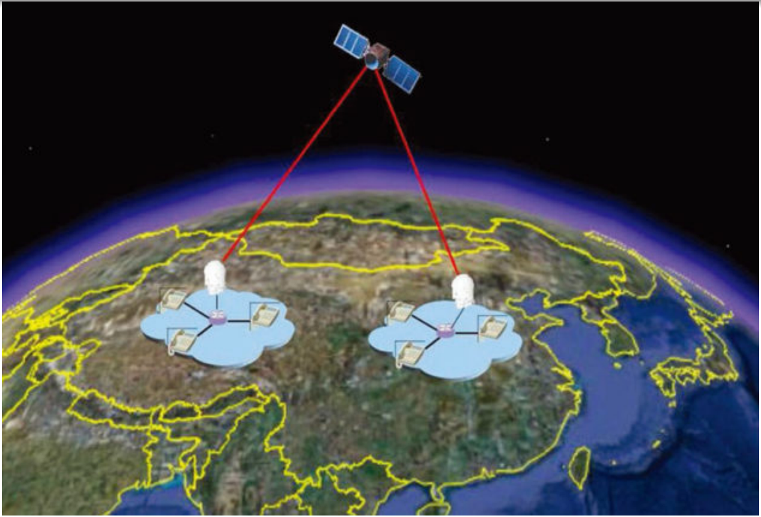


- Case 2: 1+1
- initial state of the register : $|\Psi_0\rangle = |1100\rangle$
- after first CNOT gate : $|\Psi_1\rangle = |1110\rangle$
- after second CNOT gate : $|\Psi_2\rangle = |1100\rangle$
- after CCNOT gate : $|\Psi_3\rangle = |1101\rangle$
- Measurements: $|q_2\rangle = |0\rangle$ $(c_2 = 0)$ and $|q_3\rangle = |1\rangle$ $(c_3 = 1)$
- Outcome : sum=1 and carry=0

Example 2: Teleportation circuit

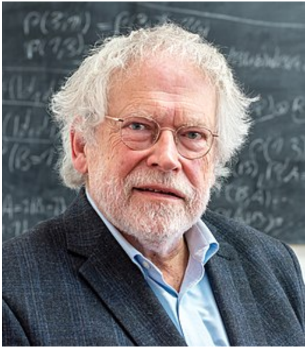


Quantum Satellite (2017)

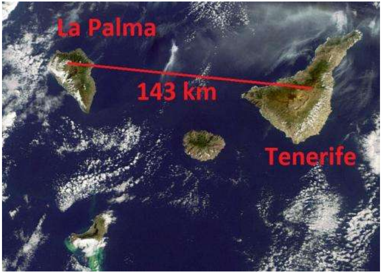


The Chinese satellite Micius has helped break the quantum teleportation distance record, transmitting entangled photons across a distance of 1,200 kms.

Implementation of teleportation algorithm



Anton Zeilinger, Nobel Prize winner 2022



Canary islands experiment : Quantum teleportation over 143 kms

Quantum Teleportation

- Alice wants to send a qubit q_0 to Bob (more precisely the quantum state of the qubit q_0). However, the quantum non-cloning theorem prohibits this operation if the only parties involved are the sender (Alice) and the receiver (Bob).
- If $|\chi\rangle$ is any state of system A (for example a qubit), it is not possible to clone it, *i.e.* to copy it to a system B (for example another qubit).

Quantum Teleportation

- Alice wants to send a qubit q_0 to Bob (more precisely the quantum state of the qubit q_0). However, the quantum non-cloning theorem prohibits this operation if the only parties involved are the sender (Alice) and the receiver (Bob).
- If $|\chi\rangle$ is any state of system A (for example a qubit), it is not possible to clone it, *i.e.* to copy it to a system B (for example another qubit).
- This result is known as "Quantum non-cloning theorem".

Quantum non-cloning theorem

- This theorem addresses the following question: Is it possible to duplicate (copy / clone) the quantum state of a system?

Quantum non-cloning theorem

- This theorem addresses the following question: Is it possible to duplicate (copy / clone) the quantum state of a system?
- To answer this question, we suppose the system A in the state $|\chi_A\rangle$ and we want to copy this state in a system B, previously in a state $|\phi_B\rangle$.

Quantum non-cloning theorem

- This theorem addresses the following question: Is it possible to duplicate (copy / clone) the quantum state of a system?
- To answer this question, we suppose the system A in the state $|\chi_A\rangle$ and we want to copy this state in a system B, previously in a state $|\phi_B\rangle$.
- For this, we consider the system AB including the two parts A and B. If the cloning operation is possible, then there exists a unit transformation (quantum gate) which transforms the state $|\chi_A \otimes \phi_B\rangle$ into the state $|\chi_A \otimes \chi_B\rangle$.

$$U : |\chi_A \otimes \phi_B\rangle \longrightarrow |\chi_A \otimes \chi_B\rangle$$

Quantum non-cloning theorem

- This theorem addresses the following question: Is it possible to duplicate (copy / clone) the quantum state of a system?
- To answer this question, we suppose the system A in the state $|\chi_A\rangle$ and we want to copy this state in a system B, previously in a state $|\phi_B\rangle$.
- For this, we consider the system AB including the two parts A and B. If the cloning operation is possible, then there exists a unit transformation (quantum gate) which transforms the state $|\chi_A \otimes \phi_B\rangle$ into the state $|\chi_A \otimes \chi_B\rangle$.

$$U : |\chi_A \otimes \phi_B\rangle \longrightarrow |\chi_A \otimes \chi_B\rangle$$

- If such a gate exists, then it must be able to copy any state from A to B, *i.e.* for 2 states χ_{1A} and χ_{2A} :

$$U|\chi_{1A} \otimes \phi_B\rangle = |\chi_{1A} \otimes \chi_{1B}\rangle$$
$$U|\chi_{2A} \otimes \phi_B\rangle = |\chi_{2A} \otimes \chi_{2B}\rangle$$

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:
  - On the one hand we have

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \phi_B | U^\dagger U | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} \otimes \phi_B | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \phi_B | \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \end{aligned}$$

  because U is unitary and $\langle \phi_B | \phi_B \rangle = 1$.

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:
  - On the one hand we have

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \phi_B | U^\dagger U | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} \otimes \phi_B | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \phi_B | \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \end{aligned}$$

    because U is unitary and $\langle \phi_B | \phi_B \rangle = 1$.
  - On the other hand :

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \chi_{1B} | \chi_{2A} \otimes \chi_{2B} \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \chi_{1B} | \chi_{2B} \rangle \end{aligned}$$

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:
  - On the one hand we have

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \phi_B | U^\dagger U | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} \otimes \phi_B | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \phi_B | \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \end{aligned}$$

  because U is unitary and $\langle \phi_B | \phi_B \rangle = 1$.
  - On the other hand :

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \chi_{1B} | \chi_{2A} \otimes \chi_{2B} \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \chi_{1B} | \chi_{2B} \rangle \end{aligned}$$

- Since $\langle \chi_{1A} | \chi_{2A} \rangle = \langle \chi_{1B} | \chi_{2B} \rangle$, on déduit que $X = X^2$. There are therefore 2 cases:

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:
  - On the one hand we have

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \phi_B | U^\dagger U | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} \otimes \phi_B | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \phi_B | \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \end{aligned}$$

    because U is unitary and $\langle \phi_B | \phi_B \rangle = 1$.
  - On the other hand :

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \chi_{1B} | \chi_{2A} \otimes \chi_{2B} \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \chi_{1B} | \chi_{2B} \rangle \end{aligned}$$

- Since $\langle \chi_{1A} | \chi_{2A} \rangle = \langle \chi_{1B} | \chi_{2B} \rangle$, on déduit que $X = X^2$. There are therefore 2 cases:
  - $X = 1 \Rightarrow |\chi_1\rangle = |\chi_2\rangle$.

Quantum non-cloning theorem

- Let us calculate the quantity X scalar product of the left (then right) members of the previous equality:
  - On the one hand we have

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \phi_B | U^\dagger U | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} \otimes \phi_B | \chi_{2A} \otimes \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \phi_B | \phi_B \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \end{aligned}$$

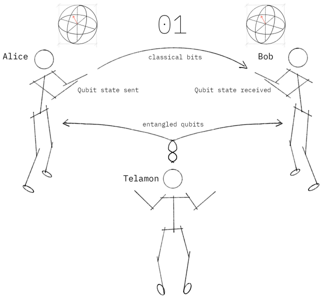
    because U is unitary and $\langle \phi_B | \phi_B \rangle = 1$.
  - On the other hand :

$$\begin{aligned} X &= \langle \chi_{1A} \otimes \chi_{1B} | \chi_{2A} \otimes \chi_{2B} \rangle \\ &= \langle \chi_{1A} | \chi_{2A} \rangle \langle \chi_{1B} | \chi_{2B} \rangle \end{aligned}$$

- Since $\langle \chi_{1A} | \chi_{2A} \rangle = \langle \chi_{1B} | \chi_{2B} \rangle$, on déduit que $X = X^2$. There are therefore 2 cases:
  - $X = 1 \Rightarrow |\chi_1\rangle = |\chi_2\rangle$.
  - $X = 0 \Rightarrow |\chi_1\rangle \perp |\chi_2\rangle$.

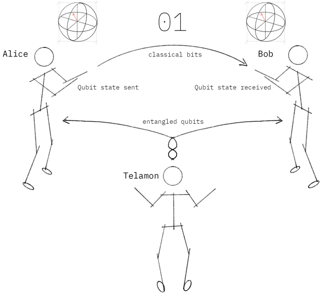
Teleportation protocol

- Alice and Bob call on a third partner (Telamon) who sends each a qubit that is part of a pair of entangled qubits (q_1 for Alice and q_2 for Bob).



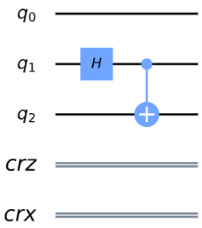
Teleportation protocol

- Alice and Bob call on a third partner (Telamon) who sends each a qubit that is part of a pair of entangled qubits (q_1 for Alice and q_2 for Bob).
- Telamon uses a special pair which is a Bell pair, in which both qubits are in a Bell entangled state.

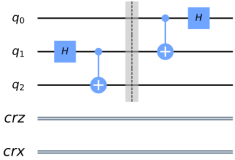


Teleportation protocol

- The translation in terms of quantum circuit of this preparation step is the first piece :
- The qubit q_1 passes a Hadamard gate which creates a $|+\rangle$ state. Then apply a $CNOT$ gate on the other qubit q_2 controlled by q_1.

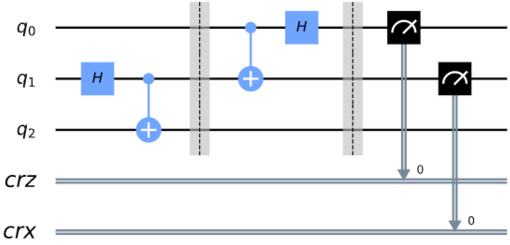


- Alice applies a $CNOT$ gate to q_1 controlled by q_0. Next, a Hadamard gate on q_0 that she wants to send to Bob.



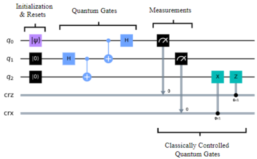
Teleportation protocol

Then, Alice measures the 2 qubits q_1 and q_0 and records the results in two standard bits. Then she sends these 2 classic bits to Bob through a classic channel.



Teleportation protocol

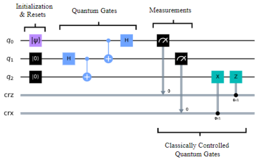
- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

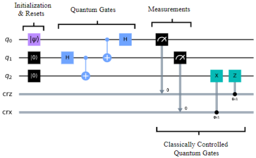
- 00 $\longrightarrow$ Nothing to do



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

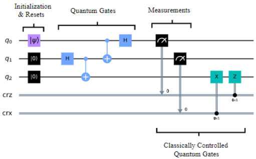
- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

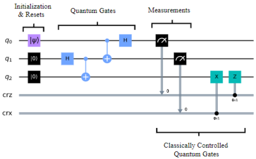
- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

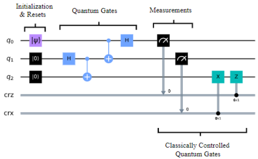
- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

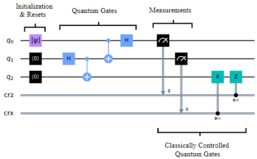
- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate



Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate

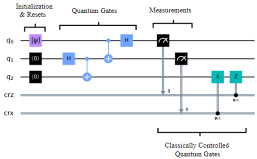


- Note here that this information transfer is purely classical.

Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate

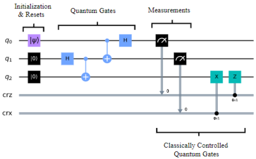


- Note here that this information transfer is purely classical.
- That's it ! At the end of the protocol, Alice's qubit q_0 was teleported to Bob. Let us insist on the fact that it is not the qubit itself which has been teleported, but a state of the qubit.

Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate

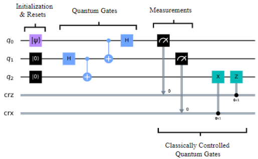


- Note here that this information transfer is purely classical.
- That's it ! At the end of the protocol, Alice's qubit q_0 was teleported to Bob. Let us insist on the fact that it is not the qubit itself which has been teleported, but a state of the qubit.
- Specifically, Bob reconstructed the quantum state that Alice sent him, thanks to the invaluable help of Telamon and his entangled qubits.

Teleportation protocol

- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:

- 00 $\longrightarrow$ Nothing to do
- 01 $\longrightarrow$ Apply X gate
- 10 $\longrightarrow$ Apply Z gate
- 11 $\longrightarrow$ Apply ZX gate



- Note here that this information transfer is purely classical.
- That's it ! At the end of the protocol, Alice's qubit q_0 was teleported to Bob. Let us insist on the fact that it is not the qubit itself which has been teleported, but a state of the qubit.
- Specifically, Bob reconstructed the quantum state that Alice sent him, thanks to the invaluable help of Telamon and his entangled qubits.
- So: No entanglement, no Quantum Teleportation!

Follow up

- Topical sessions on Quantum computing : 20 July 2024 by Nicholas Bornman