Single ion heat engine

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Outline

1. Four-stroke Otto Engine
   - Exact analytical description
   - Efficiency at maximum power (classical and quantum)

2. Proposal with single trapped ion
   - Description of the set-up
   - Monte Carlo simulation of efficiency

Introduction

Miniaturization:

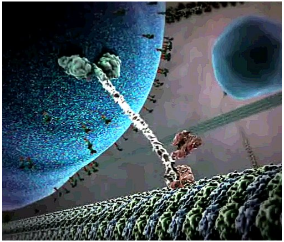
is about building smaller devices — Feynman 1959, Drexler 1981

⟶ fundamental limit = atomic structure of matter

Two basic strategies:

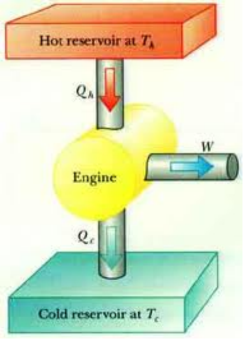
Follow engineers

Follow nature



Macroscopic heat engine

$\longrightarrow$ convert thermal energy into mechanical work = motion



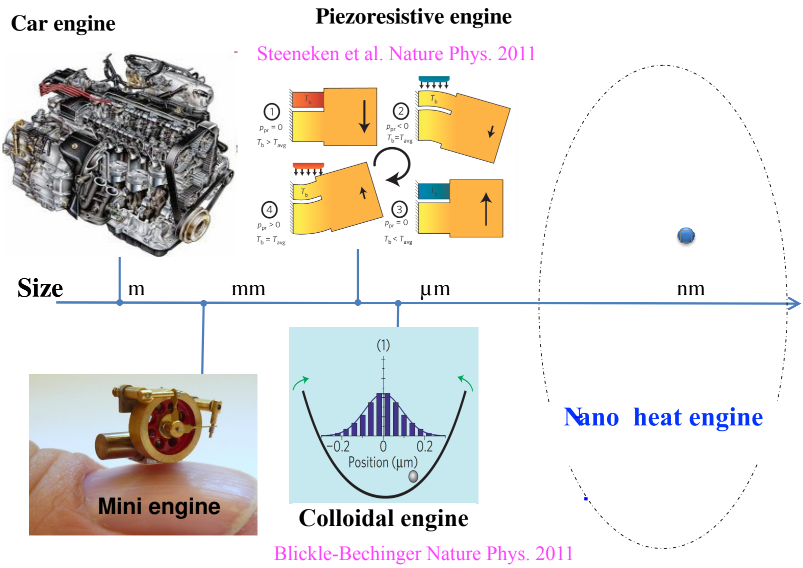
Carnot efficiency:

$$\eta = \frac{\text{work produced}}{\text{heat absorbed}} \leq 1 - \frac{T_1}{T_2} = 1 - \frac{\beta_2}{\beta_1}$$

(James Watt 1783: $\eta \simeq 5 - 7\%$)

What about nano heat engines?

Downscaling of heat engines



Ion trap heat engine

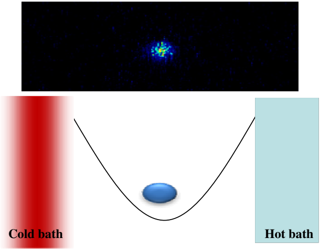
Proposal:

- single ion (nano) heat engine in Paul trap
- with potential to reach quantum regime

Quantum heat engines been studied theoretically for 50 years

Scovil, Schulz-Dubois PRL 1959

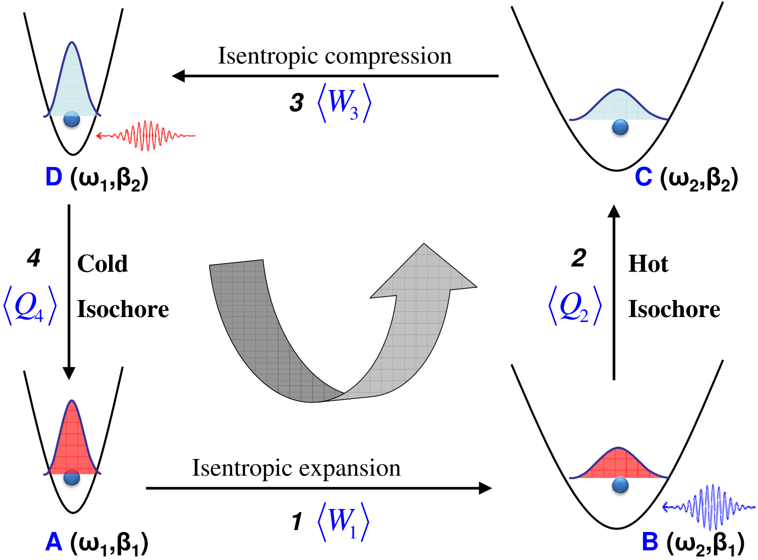
→ none has been built to date



Linear Paul trap:

- excellent preparation and control
- can be cooled to ground state
- allow for reservoir engineering

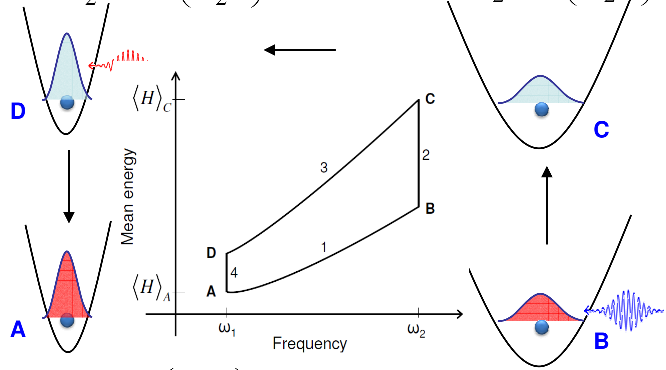
Quantum Otto (four-stroke) cycle



Quantum Otto cycle: theory

$$\langle H\rangle_{D} = \frac{\hbar\omega_1}{2} Q_2^* \coth\left(\frac{\beta_2\hbar\omega_2}{2}\right)$$

$$\langle H\rangle_{C} = \frac{\hbar\omega_2}{2} \coth\left(\frac{\beta_2\hbar\omega_2}{2}\right)$$



$$\langle H\rangle_{A} = \frac{\hbar\omega_1}{2} \coth\left(\frac{\beta_1\hbar\omega_1}{2}\right)$$

$$\langle H\rangle_{B} = \frac{\hbar\omega_2}{2} Q_1^* \coth\left(\frac{\beta_1\hbar\omega_1}{2}\right)$$

Quantum Otto cycle: theory

Efficiency:

$$\eta = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{\langle Q_2 \rangle}$$
$$= 1 - \frac{\omega_1}{\omega_2} \frac{\coth(\beta_1 \hbar\omega_1/2) - Q_2^* \coth(\beta_2 \hbar\omega_2/2)}{Q_1^* \coth(\beta_1 \hbar\omega_1/2) - \coth(\beta_2 \hbar\omega_2/2)}$$

⟶ exact quantum expression

Adiabaticity parameter Q^*: Husimi 1953

⟶ depends on the driving

For adiabatic process: $Q^*_{1,2} = 1$

For sudden switch: $Q^*_{1,2} = (\omega_1^2 + \omega_2^2)/(2\omega_1\omega_2)$

Efficiency at maximal power

Power output:

$$P = \frac{\text{work done per cycle}}{\text{duration of a cycle}} = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{t_{cycle}}$$

Maximization: for a given ω_1, maximize with respect to ω_2

High temperature (classical) regime:

- adiabatic process: Curzon-Ahlborn 1975

$$\eta = 1 - \sqrt{\frac{\beta_2}{\beta_1}} = 1 - \sqrt{\frac{kT_1}{kT_2}} \leq 1$$

- sudden frequency switch: Rezek-Kosloff 2006

$$\eta_{ss} = \frac{1 - \sqrt{\beta_2/\beta_1}}{2 + \sqrt{\beta_2/\beta_1}} \leq 0.5$$

Efficiency at maximal power

Low temperature (quantum) regime:

- adiabatic process:

$$\eta^q = 1 - \sqrt{\hbar\omega_1\beta_2/2} = 1 - \sqrt{\frac{\hbar\omega_1}{2kT_2}}$$

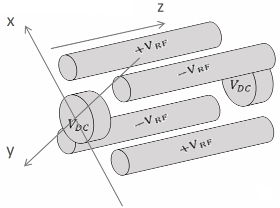
- sudden frequency switch:

$$\eta^q_{ss} = \frac{1 - \sqrt{\hbar\omega_1\,\beta_2/2}}{2 + \sqrt{\hbar\omega_1\,\beta_2/2}}$$

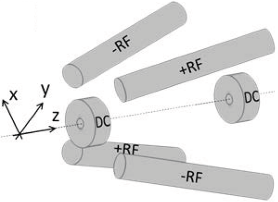
⟶ quantum extensions of Curzon-Ahlborn and Rezek-Kosloff

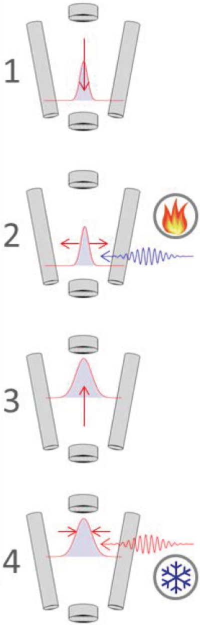
Proposed trap geometry

Usual Paul trap:



Conical Paul trap:

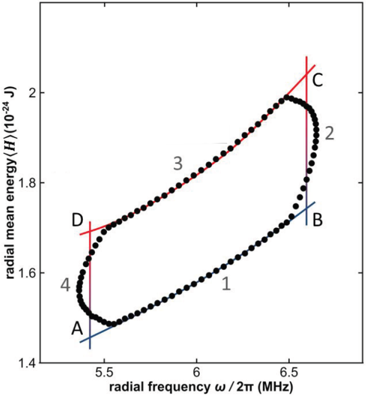


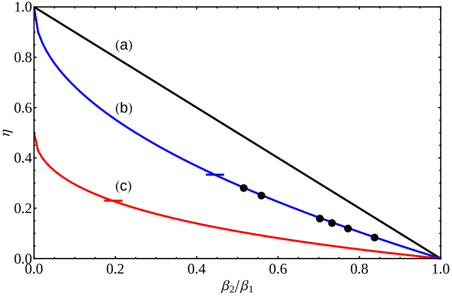


→ $\omega = \omega(z)$

Numerical simulation

Semiclassical Monte Carlo (with realistic parameters):



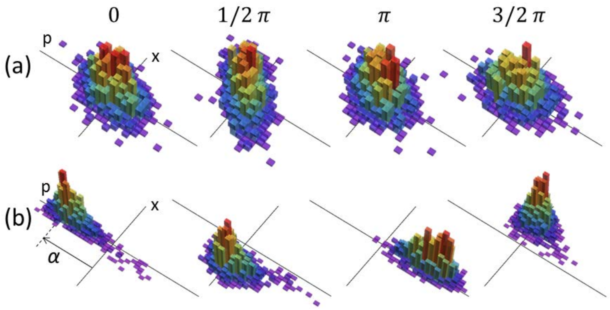


(T between 20mK and 200mK)

⟶ maximum efficiency at maximum power of about 30%

Numerical simulation

Otto cycle in axial direction - energy stored in radial direction:



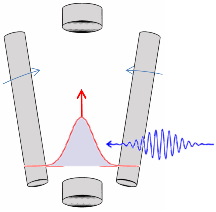
⟶ can in principle be transferred to other trapped ions or nanomechanical oscillators
(or extracted if run as a heat pump)

Some numbers

Single ion Otto engine

Max efficiency: $\simeq 30 - 35\%$

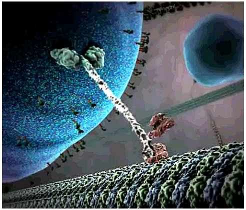
Power: $\simeq 10^{-20} J/s$
$(10^4 - 10^5 \text{kT/s})$



Kinesin

Max efficiency: $\simeq 40 - 50\%$

Power: $\simeq 10^{-18} J/s$
(10^3kT/s)



Quantum regime?

Discreteness of energy spectrum: $\langle H \rangle \neq kT$ if $T \leq 100\mu$K

$\longrightarrow$ quantum regime requires sub-Doppler cooling

$\longrightarrow$ would allow study of quantum coherence and correlation

Plan:

- build and test classical nanoengine (in progress)
- do quantum simulation (new trap design?)

Summary

- concrete proposal for a single ion Otto engine

  ⟶ can run at maximum power over a wide range of parameters (e.g. temperature)

- runs at the nanoscale and (potentially) in the quantum regime

  ⟶ protopype to study similar harmonic engines (e.g. nanomechanical oscillator)