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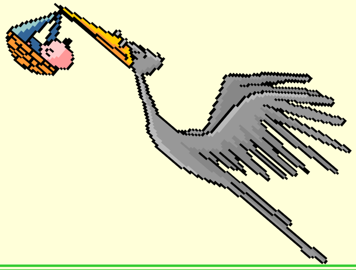
Modelling Fluid Flow with Heat Transfer Mechanism & Plasma Flow Analysis in Engineering Systems



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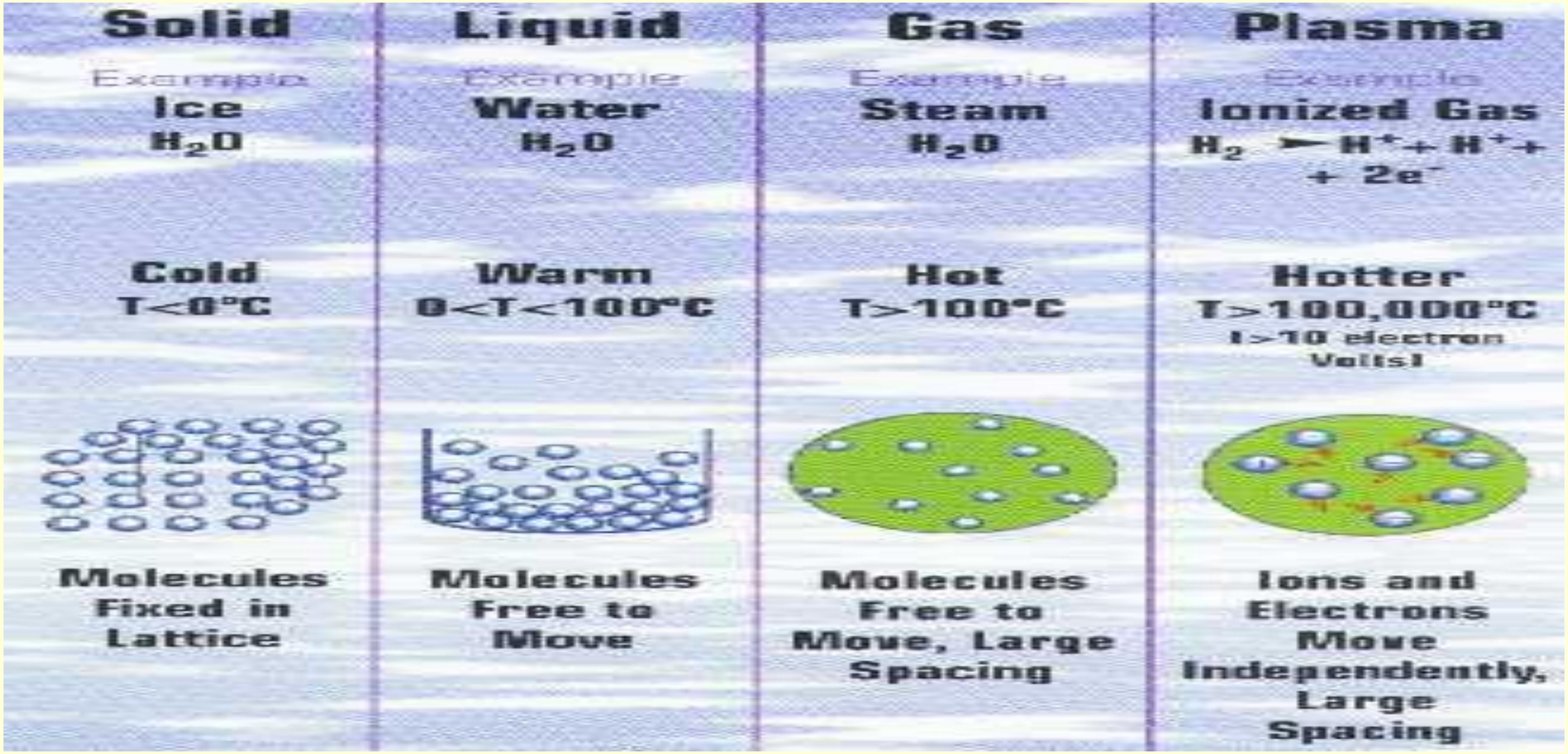
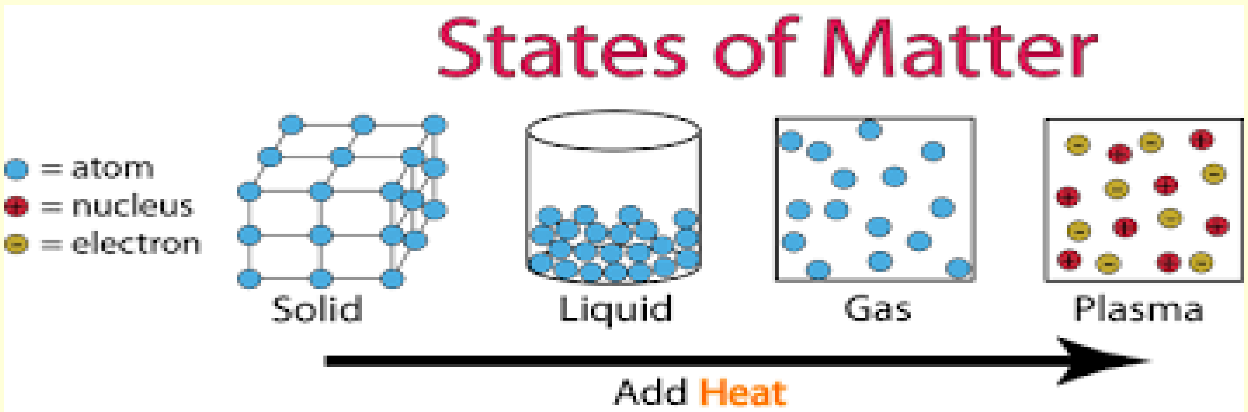
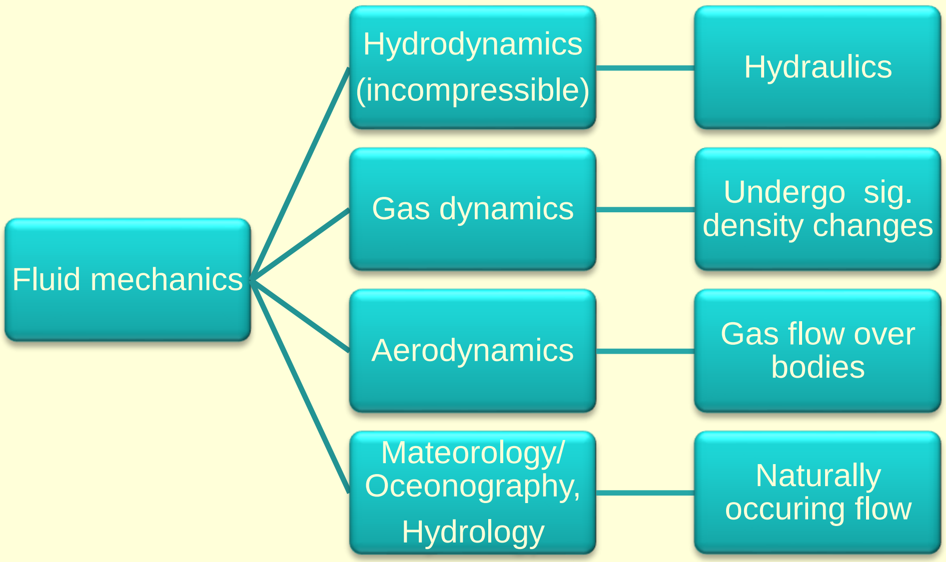
The "Fourth State" of the Matter



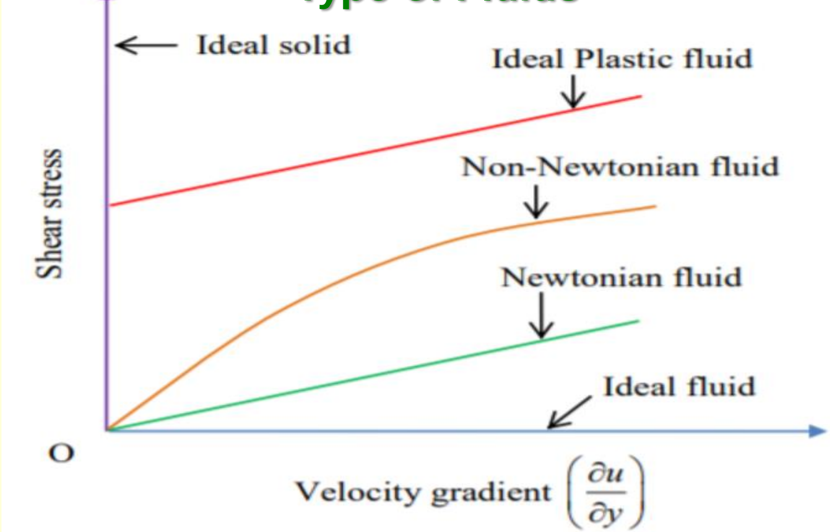


Fluid: Materials that can flow: Liquid, Gas and Plasma.

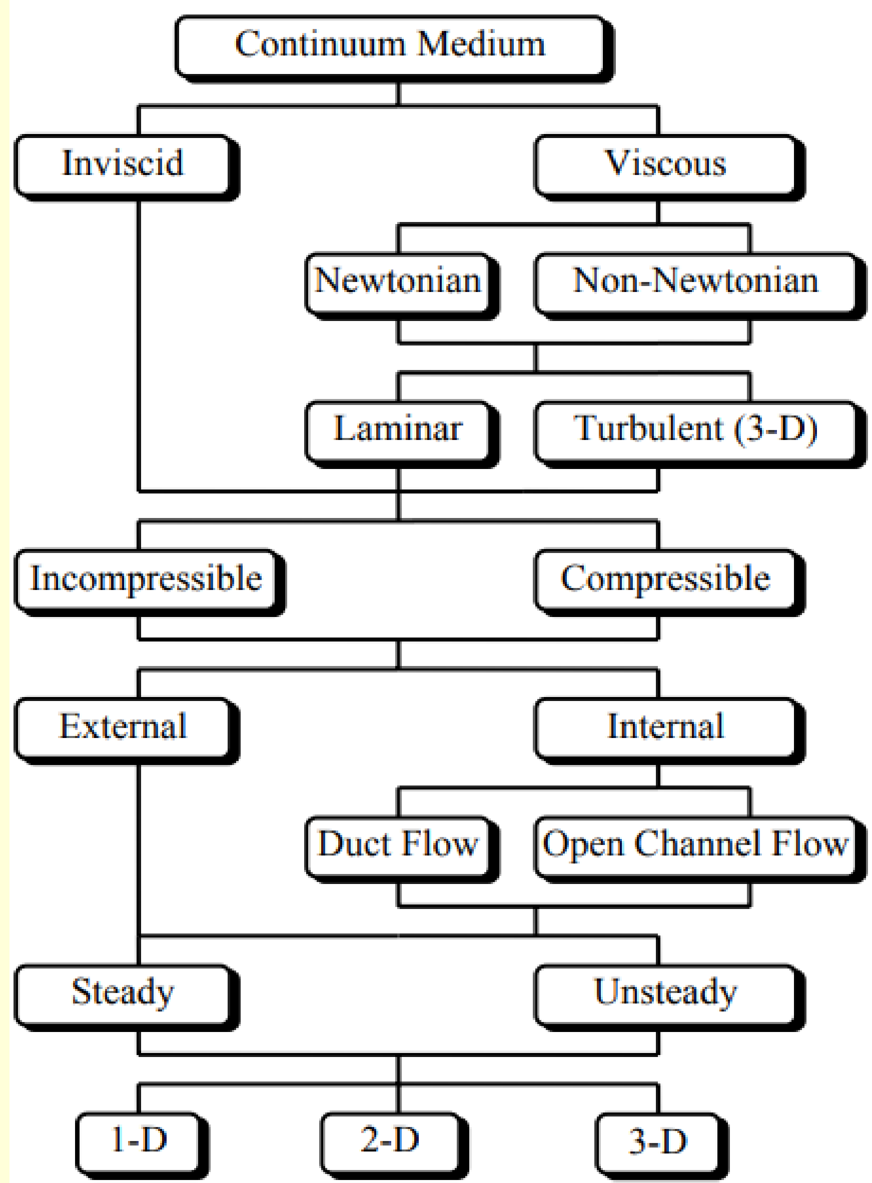
Fluid Mechanics



Type of Fluids



Classification of Fluid Motions



Heat Transfer

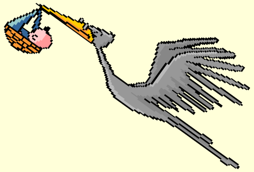
- Heat is a transfer of energy from one object to another due to a difference in temperature
- Temperature is a measure of the molecular energy in an object
- Heat always flows from an object of higher temp (T_H) to one of lower temp (T_L)
- We are often interested in the rate at which this heat transfer takes place

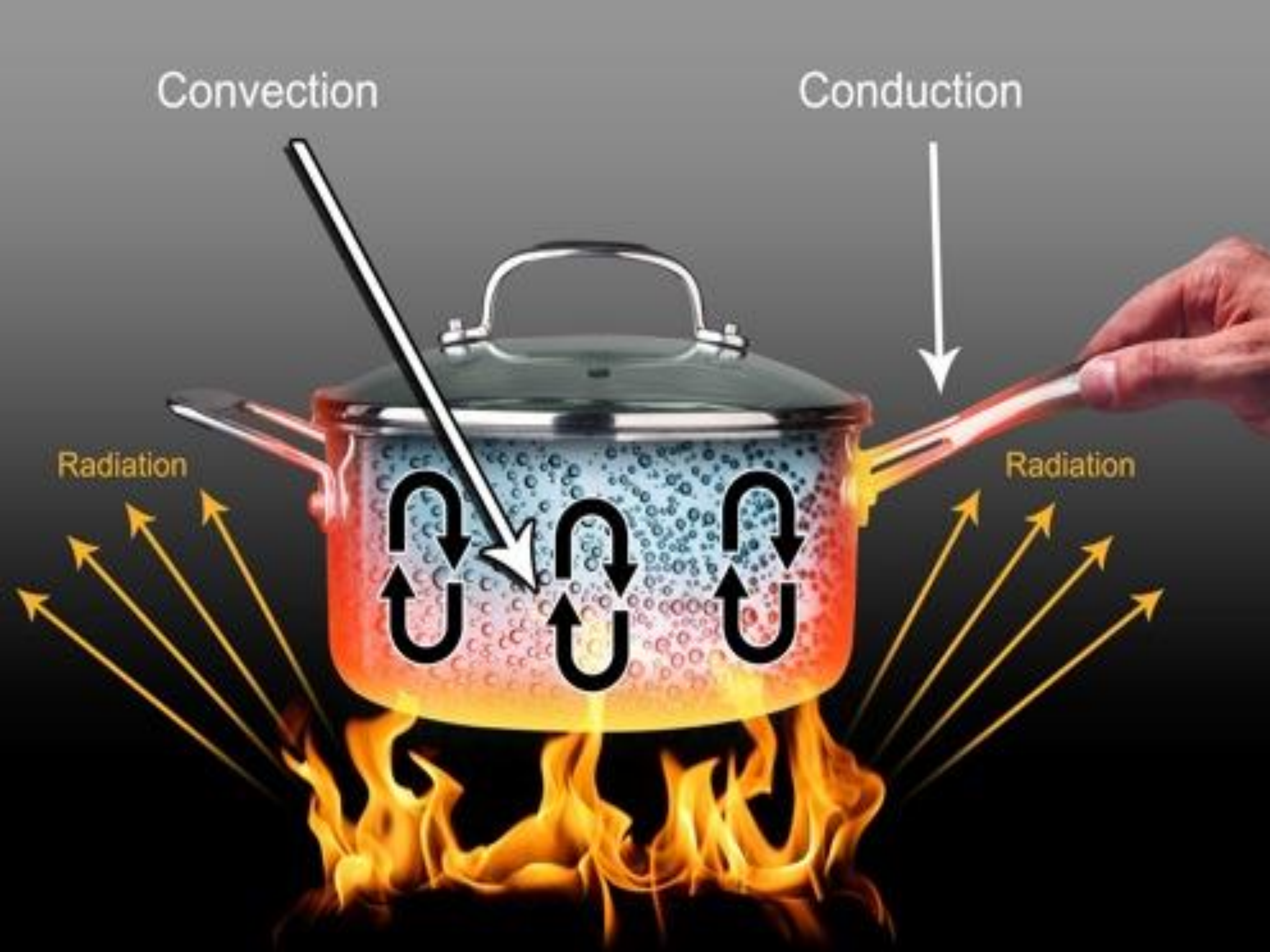
Mechanisms of Heat Transfer

There exist 3 basic mechanisms of heat transfer between different bodies (or inside a continuous body)

- **Conduction** in solids or stagnant fluids
- **Convection** inside moving fluids, but first of all we shall discuss heat transfer from flowing fluid to a solid wall
- **Radiation** (electromagnetic waves) the only mechanism of energy transfer in an empty space

Aim of analysis is to find out relationships between heat flows (heat fluxes) and driving forces (temperature differences)

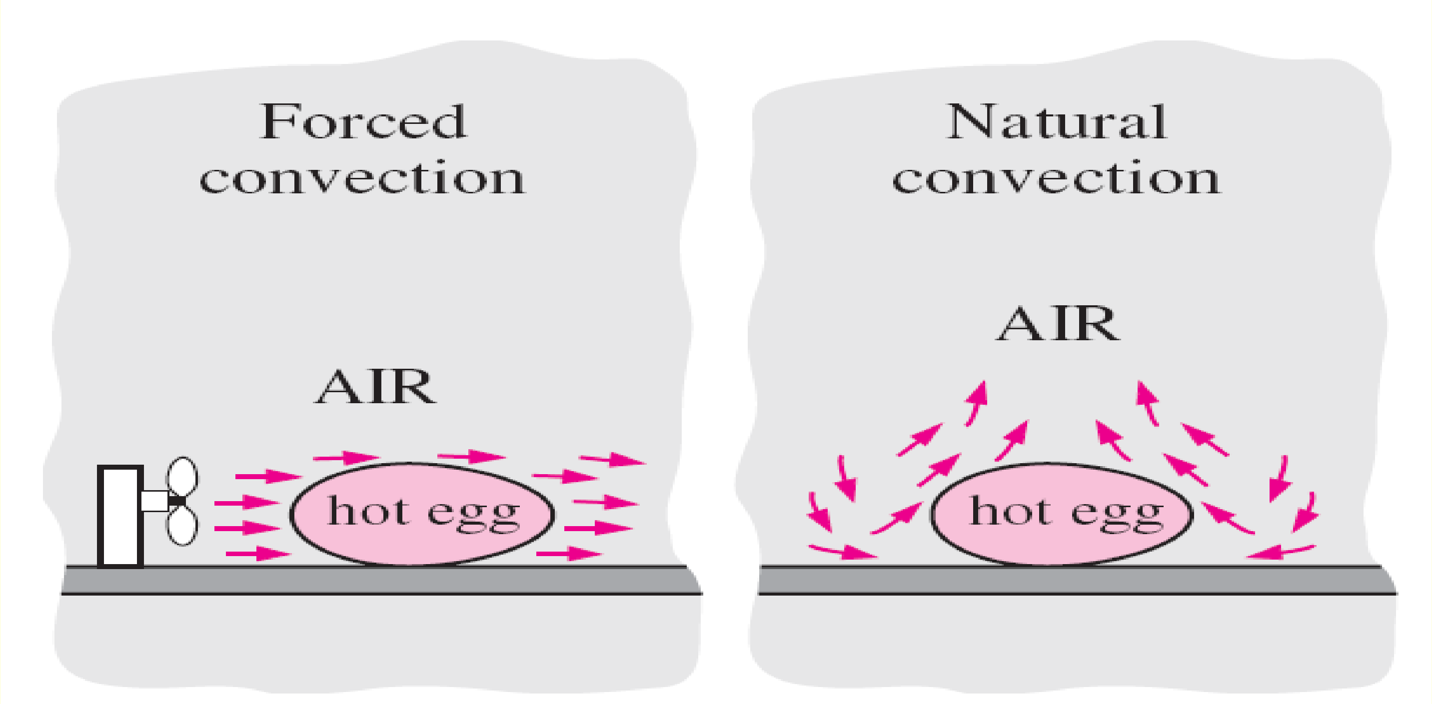




Convection
Conduction
Radiation
Radiation

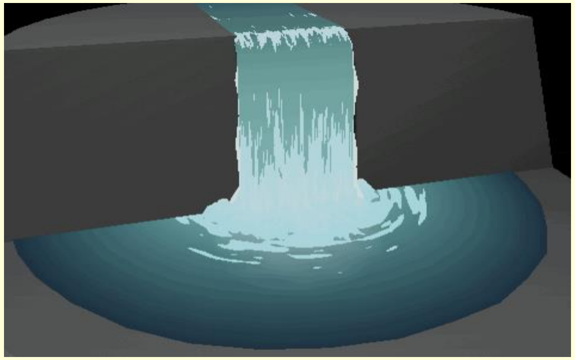
Forced, Free and Mixed Convection

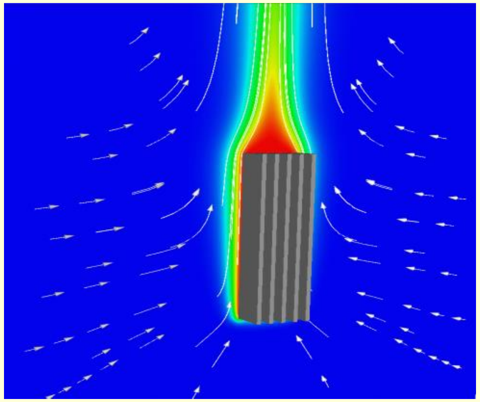
- Convection is called **forced convection** if the fluid is *forced* to flow in a tube or over a surface by external means (pressure gradient) such as a fan, pump, or the wind.
- In contrast, convection is called **free** (or **natural**) **convection** if the fluid motion is caused by buoyancy forces induced by density differences due to the variation of temperature in the fluid.
- Mixed convection occurs when the fluid motion is driven by combined action of both pressure gradient (forced convection) and buoyancy forces (free convection).

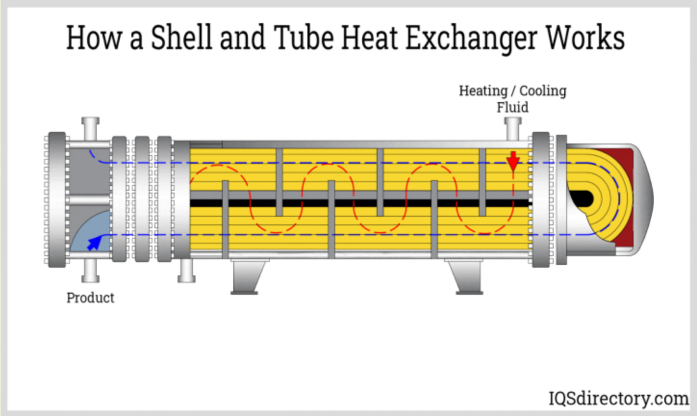


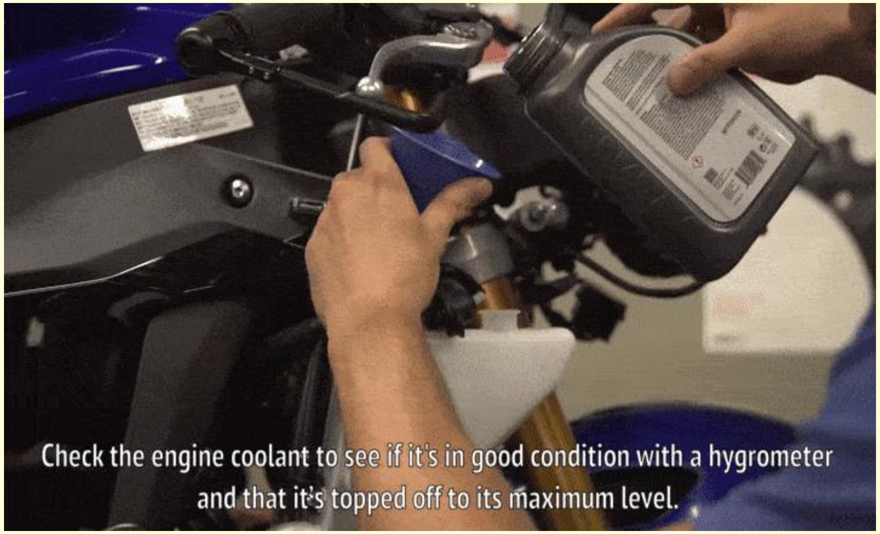
Introduction: Thermal Management

Thermal management refers to the tools and technologies used to maintain a system within its operating temperature range. With electronic devices, thermal management typically dissipates excess heat to prevent overheating.



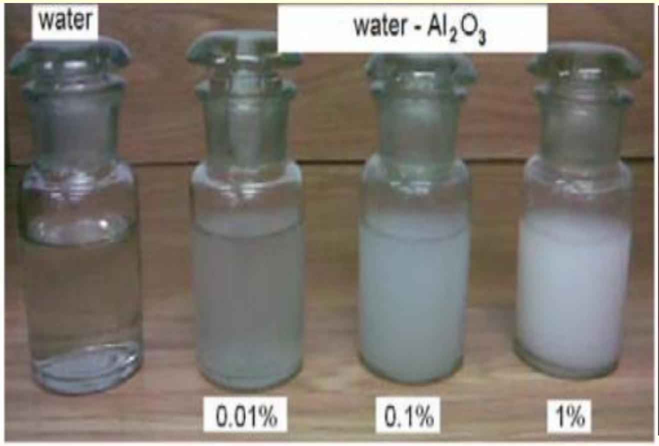






NanoFluids –An Offspring of Nanotechnology

NanoFluids is the mixture of basefluid (liquid) and nanoparticles (solid) of size 10^{-9}



Alumina-water nanofluid

Alumina nanoparticles

Cu nanoparticles

Nanoparticle Materials Include:

- Metals (Al, Cu, Ag, Au, Fe)
- Oxide ceramics (Al_2O_3,CuO,TiO_2)
- Metal carbides (SiC)
- Nitrides (AlN, SiN)
- Nonmetals (graphite, carbon nanotubes)
- Layered (Al+ Al_2O_3, Cu+C)

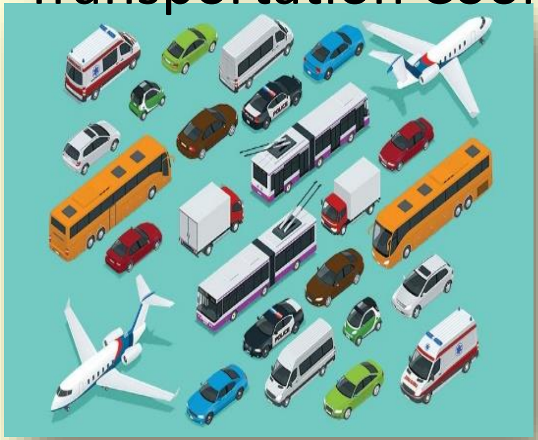
Base Fluids include:

- Water
- Oil & other lubricants
- Ethylene or tri-ethylene glycols & other coolants.
- Polymer solutions
- Bio-fluids
- Other common fluids

Application of Nanofluids

Electronic Cooling

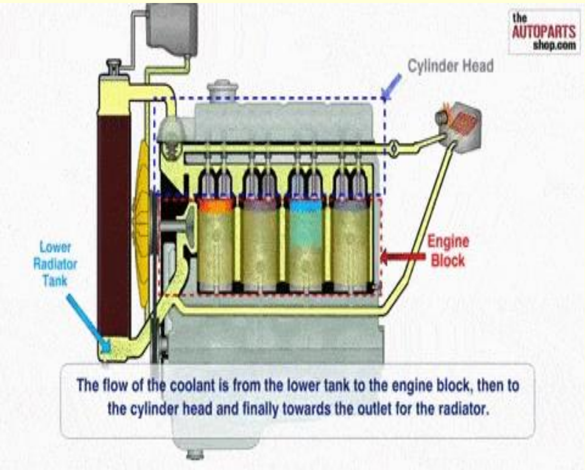
Transportation Cooling



Industrial Cooling

Nuclear Cooling

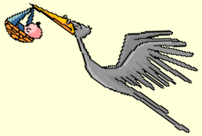
Engine Cooling



Military Application

Experimental vs. Theoretical Analysis

Fluid flow with heat transfer in engineering systems can be studied either *experimentally* (testing and taking measurements) or theoretically (by analysis or calculations).



- The **experimental approach** has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical.

- The **theoretical approach** (including the numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions, approximations, and idealizations made in the analysis.

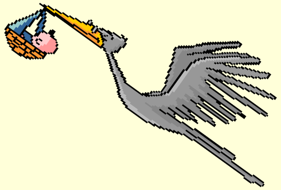
Modelling Procedure

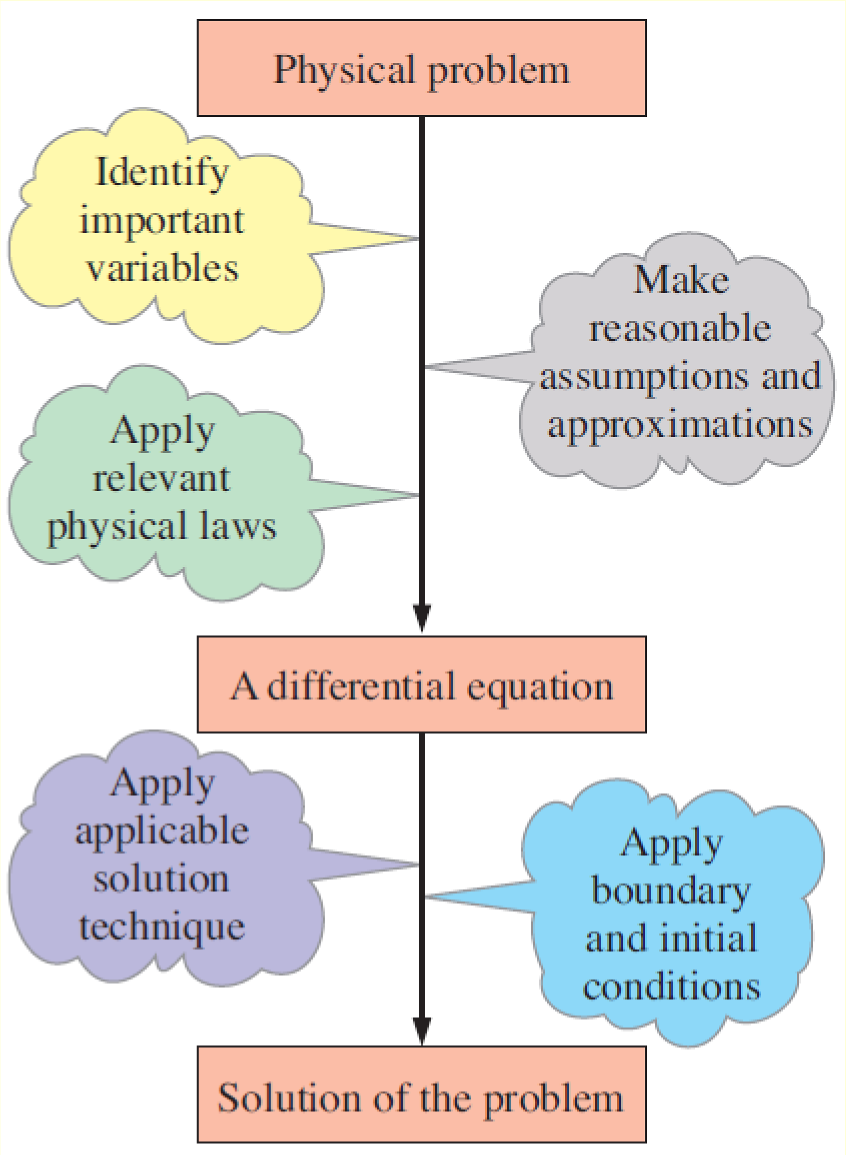
Why do we need differential equations?
The descriptions of most scientific problems involve equations that relate the changes in some key variables to each other.

In the limiting case of infinitesimal or differential changes in variables, we obtain ***differential equations*** that provide precise mathematical formulations for the physical principles and laws by representing the rates of change as ***derivatives***.

Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering.

Do we always need differential equations?
Many problems encountered in practice can be solved without resorting to differential equations and the complications associated with them.

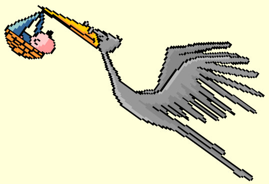




Mathematical modeling of physical problems.

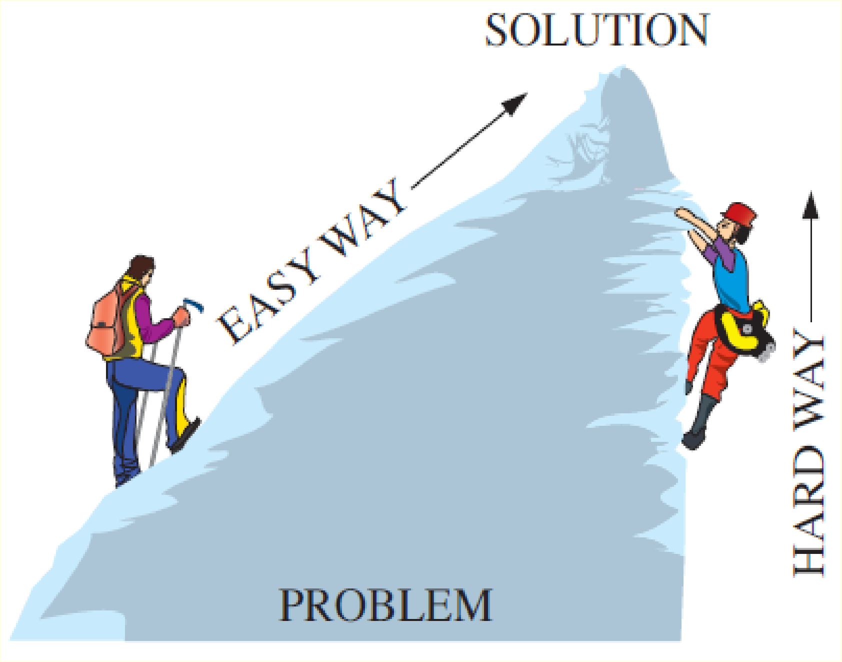
PROBLEM-SOLVING TECHNIQUE

- Step 1: Problem Statement
- Step 2: Schematic
- Step 3: Assumptions and Approximations
- Step 4: Physical Laws
- Step 5: Properties
- Step 6: Calculations
- Step 7: Reasoning, Verification, and Discussion



The assumptions made while solving an engineering problem must be reasonable and justifiable.

A step-by-step approach can greatly simplify problem solving.



IMPORTANCE OF DIMENSIONS AND UNITS

- Any physical quantity can be characterized by **dimensions**.
- The magnitudes assigned to the dimensions are called **units**.
- Some basic dimensions such as mass *m*, length *L*, time *t*, and temperature *T* are selected as **primary** or **fundamental dimensions**, while others such as velocity *V*, energy *E*, and volume *V* are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**.
- **Metric SI system**: A simple and logical system based on a decimal relationship between the various units.
- **English system**: It has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily.

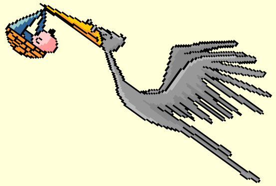


TABLE 1–1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

TABLE 1–2

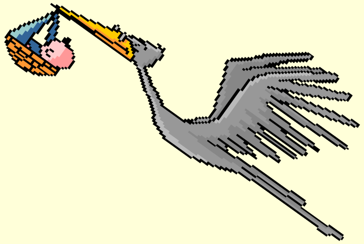
Standard prefixes in SI units

Multiple	Prefix
10^{24}	yotta, Y
10^{21}	zetta, Z
10^{18}	exa, E
10^{15}	peta, P
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n

NEED FOR RELEVANT SOFTWARE PACKAGES AND COMPUTERS

All the computing power and the software packages available today are just ***tools***, and tools have meaning only in the hands of masters.

Note that availability of sophisticated software packages and computers cannot replace adequate training and knowledge in engineering systems. They will simply cause a shift in emphasis in the courses from mathematics to physics. **That is, more time should be spent in the classroom discussing the physical aspects of the problems in greater detail, and less time on the mechanics of the solution procedures.**



An excellent word-processing program does not make a person a good writer; it simply makes a good writer a more efficient writer.

Numerical Methods: properties

Finite differences

- time-dependent PDEs
- seismic wave propagation
- geophysical fluid dynamics
- Maxwell's equations
- Ground penetrating radar

-> robust, simple concept, easy to parallelize, regular grids, explicit method

Finite elements

- static and time-dependent PDEs
- seismic wave propagation
- geophysical fluid dynamics
- all problems

-> implicit approach, matrix inversion, well founded, irregular grids, more complex algorithms, engineering problems

Finite volumes

- time-dependent PDEs
- seismic wave propagation
- mainly fluid dynamics

-> robust, simple concept, irregular grids, explicit method

Other numerical methods

Particle-based methods →
- lattice gas methods
- molecular dynamics
- granular problems
- fluid flow
- earthquake simulations

-> very heterogeneous problems, nonlinear problems

Boundary element methods →
- problems with boundaries (rupture)
- based on analytical solutions
- only discretization of planes

-> good for problems with special boundary conditions (rupture, cracks, etc)

*Pseudo*spectral methods →
- orthogonal basis functions, special case of FD
- spectral accuracy of space derivatives
- wave propagation, ground penetrating radar

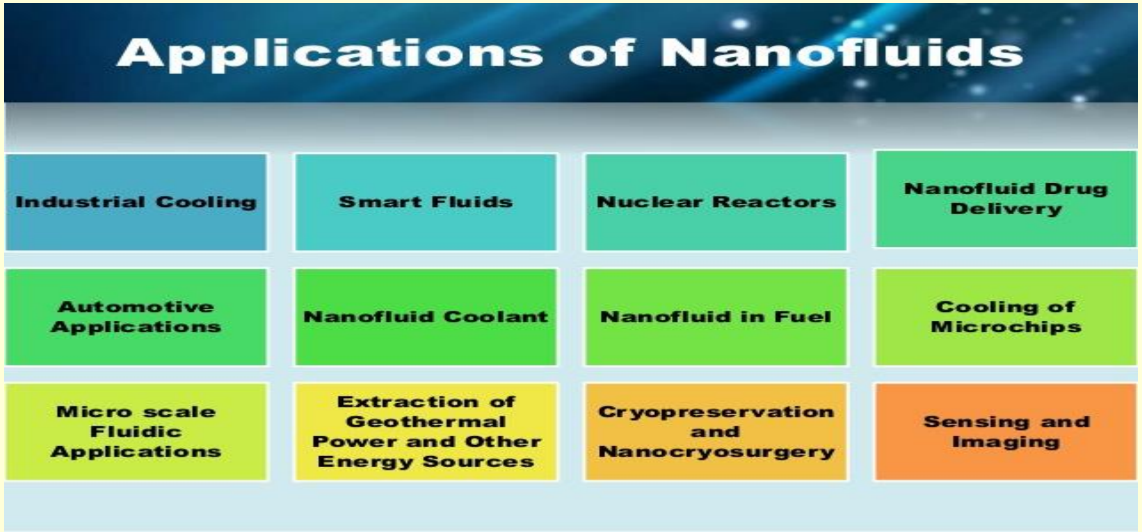
-> regular grids, explicit method, problems with strongly heterogeneous media

Model Illustration I

Nanofluid Heat Transfer Enhancement in Engineering Systems

Internal Flow Problem

Microchannel Flow with Heat Transfer



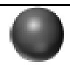
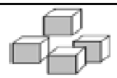
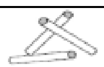
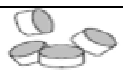
(DEMONSTRATION USING MAPLE SOFTWARE)

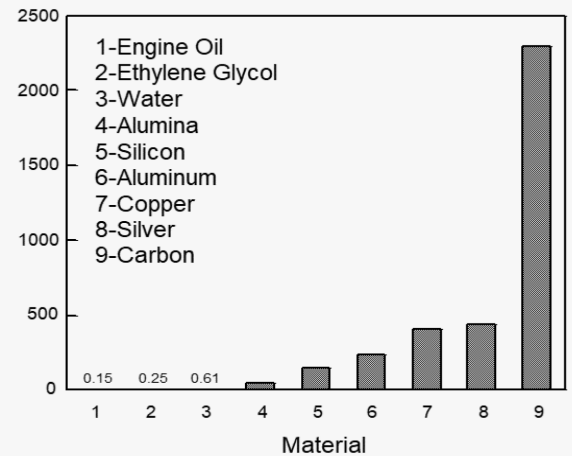
Nanofluids Composition Properties and Relations

Thermophysical Properties of Basefluid and Nanoparticles

Materials	ρ(kg/m^3)	C_p (J/kgK)	k (W/mK)	β (K^{-1})	σ (S/m)
Pure water	997.1	4179	0.613	21×10^{-5}	5.5×10^{-6}
Copper (Cu)	8933	385	401	1.67×10^{-5}	58×10^{6}
Alumina (Al_2O_3)	3970	765	40	0.85×10^{-5}	35×10^{6}

Structure of Different Nanoparticle Shapes

Nanoparticles Shape	Shape Structure	Shape Factor (m)
Spherical		3
Bricks		3.7
Cylindrical		4.9
Platelets		5.7
Blades		8.6



Thermal conductivity of typical materials

$$\gamma = \frac{\sigma_s}{\sigma_f}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$

$$\sigma_{nf} = \sigma_f\left[1 + \frac{3(\gamma-1)\phi}{(\gamma+2)-(\gamma-1)\phi}\right]$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + (m-1)k_f - (m-1)\phi(k_f - k_s)}{k_s + (m-1)k_f + \phi(k_f - k_s)}$$

CONSERVATION LAWS: FLUID DYNAMICS BASIC EQUATIONS

- **Navier-Stokes equations**

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\frac{1}{\rho_{nf}}\nabla p + \frac{\mu_{nf}}{\rho_{nf}}\nabla^2\mathbf{V} + \mathbf{g} + \frac{1}{\rho_{nf}}\mathbf{j}\times\mathbf{B} \quad (1)$$

- **Continuity** $\nabla\cdot\mathbf{V} = 0$ **(2)**

- **Energy equation with the Joule heating**

$$(\rho C_p)_{nf}\left(\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)T\right) = k_{nf}\nabla^2 T + \mu_{nf}\Phi + \frac{j^2}{\sigma_{nf}} + q''' \quad (3)$$

- **Ampere's law**

$$\mathbf{j} = \mu^{-1}\nabla\times\mathbf{B} \quad (vacuum: \mu_0 = 4\pi\ 10^{-7} = 1.257\ 10^{-6}\ H/m) \quad (4)$$

- **Faraday's law** $\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$ **(5)**

- **Ohm's law*** $\mathbf{j} = \sigma_{nf}(\mathbf{E} + \mathbf{V}\times\mathbf{B})$ **(6)**

***Eqs.(4-6) are usually grouped together as Maxwell Equations**

Micro-Channel and Heat Transfer

- A decrease in channel diameter increases the heat transfer coefficient, thus the small diameter of micro-channel boosts the compactness and effectiveness of heat removal, since

$$h \approx \frac{k}{D}$$

where h=heat transfer coefficient, D=diameter, k = thermal conductivity.

- Micro-channels small aspect ratio increased heat dissipation rates and reduced temperature gradients across electronic components.

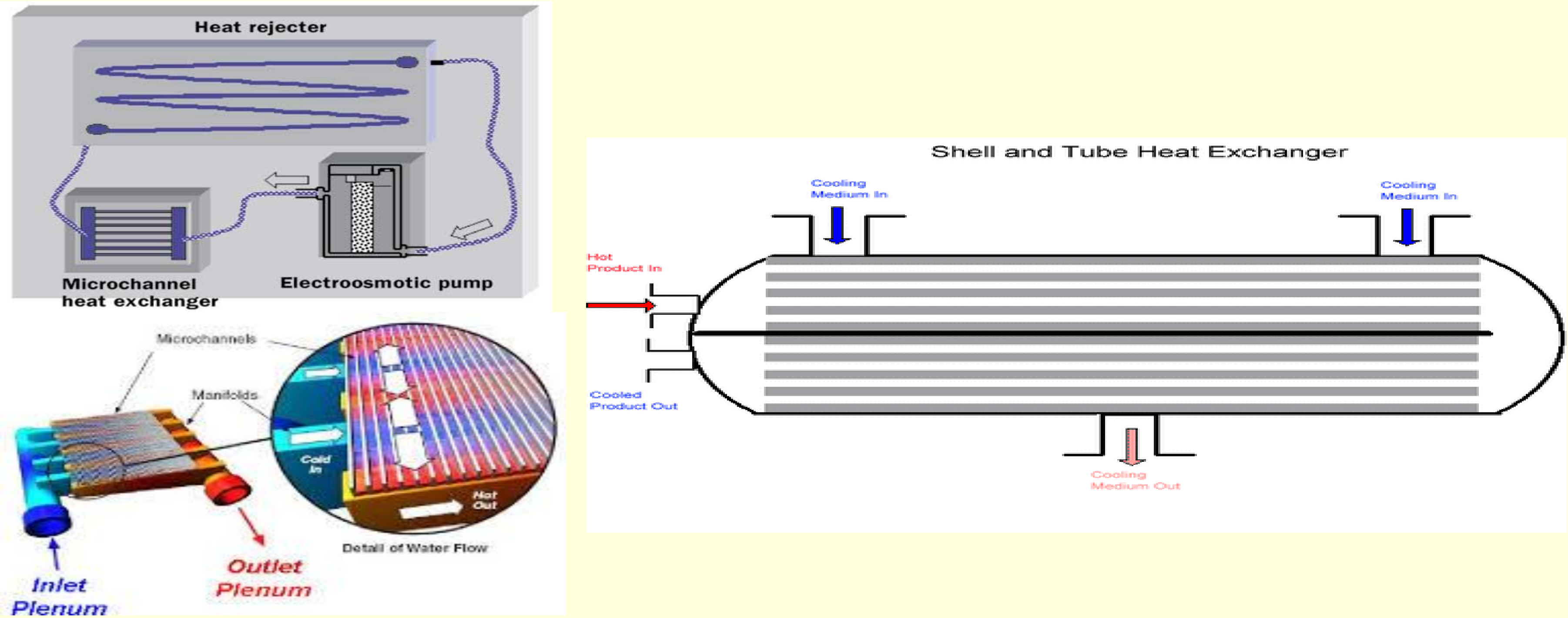


Figure 1: Micro-channels illustration (Heat Exchanger)

Mathematical Model - I

Thermal Analysis of Unsteady Mixed Convection of Cu-Water Nanofluid in a Microchannel

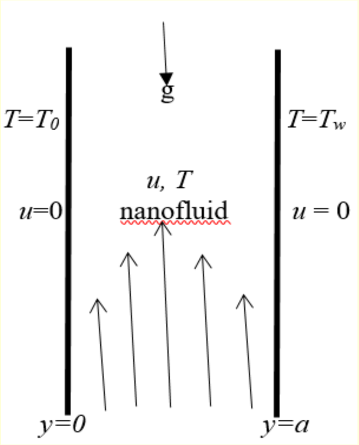


Figure 1 : Problem Geometry

Assumptions

- Unsteady flow
- Mixed Convection
- Two dimensional
- Incompressible

Parameters

Gr = Grashof number
Pr = Prandtl number
Ec = Eckert number
A= Pressure gradient parameters
ϕ=Nanoparticles volume fraction

$$w=\frac{ua}{\upsilon_f},\ \theta=\frac{T-T_0}{T_w-T_0},\ \eta=\frac{y}{a},\ X=\frac{x}{a},\ \tau=\frac{t\upsilon_f}{a^2},\ \upsilon_f=\frac{\mu_f}{\rho_f},\ Gr=\frac{\beta_f(T_w-T_0)a^3}{\upsilon_f^2}$$

$$A=-\frac{\partial\overline{p}}{\partial X},\ \overline{p}=\frac{a^2\rho_f p}{\mu_f^2},\ \alpha_f=\frac{k_f}{(\rho C_p)_{nf}},\ \Pr=\frac{\upsilon_f}{\alpha_f},\ Ec=\frac{\upsilon_f^2}{C_p(T_w-T_0)a^2}$$

$$A_1=\frac{\rho_{nf}}{\rho_f},\ A_2=\frac{\mu_{nf}}{\mu_f},\ A_3=\frac{k_{nf}}{k_f},\ A_4=\frac{(\rho C_p)_{nf}}{(\rho C_p)_f},\ A_5=\frac{(\rho\beta)_{nf}}{(\rho\beta)_f}$$

DIMENSIONLESS MODEL EQUATIONS

$$\frac{A_1}{A_2}\frac{\partial w}{\partial \tau}=A+\frac{\partial^2 w}{\partial \eta^2}+\frac{A_5}{A_2}Gr\theta$$

$$\frac{A_4}{A_3}\Pr\frac{\partial \theta}{\partial \tau}=\frac{\partial^2 \theta}{\partial \eta^2}+\frac{A_2}{A_3}\Pr Ec\left(\frac{\partial w}{\partial \eta}\right)^2$$

Initial and Boundary Conditions

$$w(\eta,0)=0,\quad \theta(\eta,0)=0$$

$$\left.\begin{array}{ll} w(0,\tau)=0, & \theta(0,\tau)=0 \\ w(1,\tau)=0, & \theta(1,\tau)=1 \end{array}\right\}\ \text{for}\ \ \tau>0$$

$$C_f=A_2\left.\frac{\partial w}{\partial \eta}\right|_{\eta=0,1}$$ **(Skin Friction)**

$$Nu=-A_3\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0,1}$$ **(Nusselt Number)**

Numerical Procedure (Method of Lines)

Finite difference technique based of method of lines in which the central differencing approach is utilized for the spatial derivatives discretization is employed. The initial boundary value problem (IBVP) is converted to initial value problem (IVP). Fourth order Runge-Kutta integration method is employed to tackle the resulting IVP.

Let $w_i = w(\eta_i, \tau),\ \theta_i = \theta(\eta_i, \tau)$ where $\eta_i = i\Delta\eta,\ \ \Delta\eta = \dfrac{1}{\mathrm{N}},\ i = 0 \ldots N$

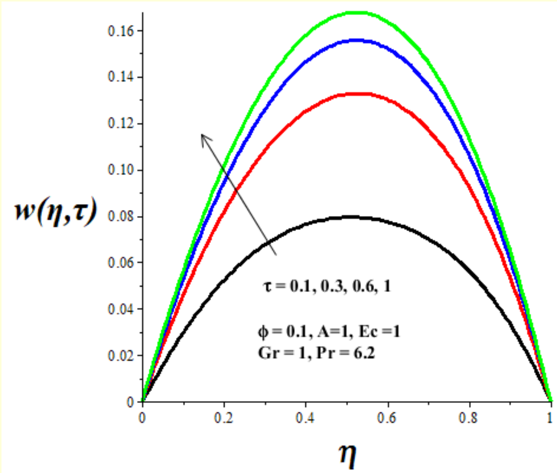
$$\frac{A_1 A_5}{A_3} \Pr \frac{d\theta_i}{d\tau} = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta\eta)^2} + \frac{A_2}{A_3} \Pr Ec \left(\frac{w_{i+1} - w_{i-1}}{2\Delta\eta} \right)^2$$

$$\frac{A_1}{A_2} \frac{dw_i}{d\tau} = A + \frac{w_{i+1} - 2w_i + w_{i-1}}{(\Delta\eta)^2} + \frac{A_1 A_4}{A_2} Gr\theta_i$$

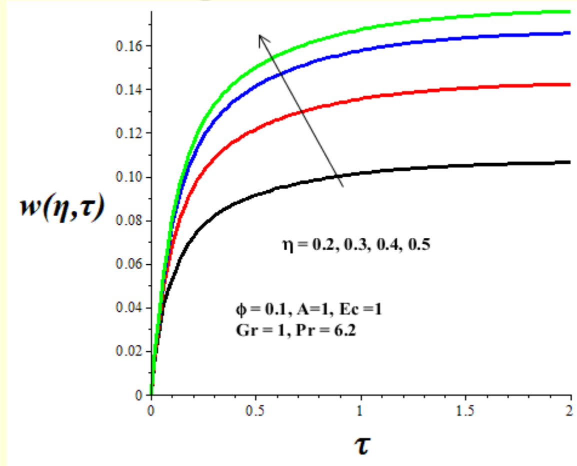
with $w_0 = 0, \theta_0 = 0,\ w_N = 0,\ \theta_N = 1$ and the initial conditions given as

$w_i(0) = 0,\ \theta_i(0) = 0,\ \forall\, i$

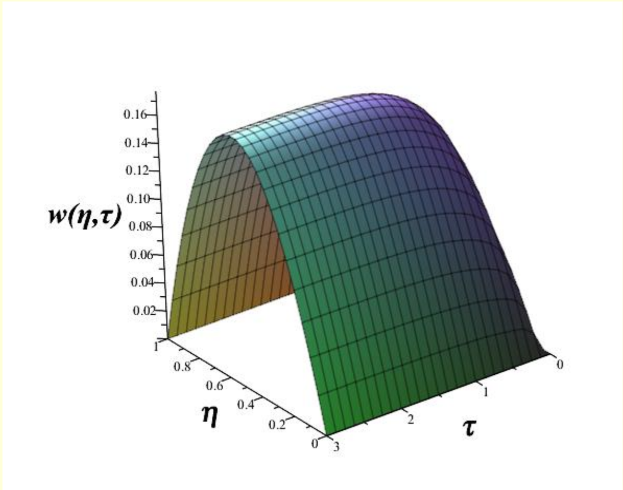
Graphical Results



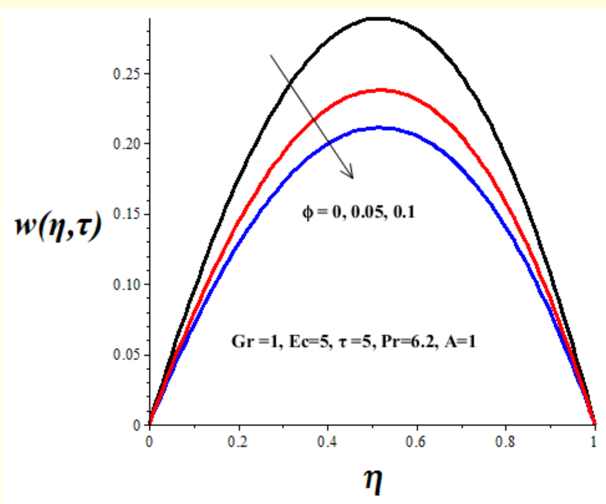
Velocity Profiles with (τ)



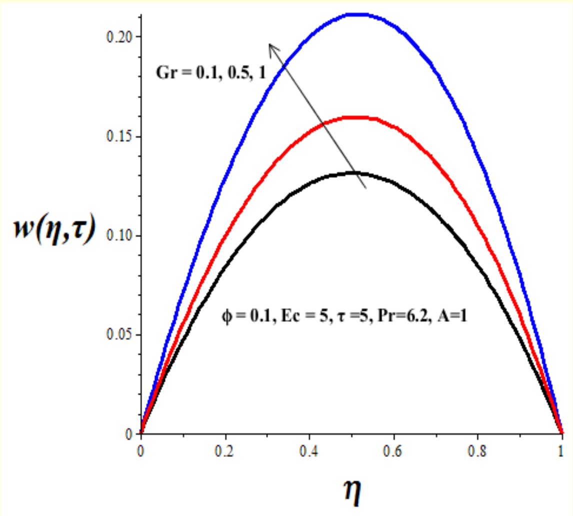
Velocity Profiles with (τ)



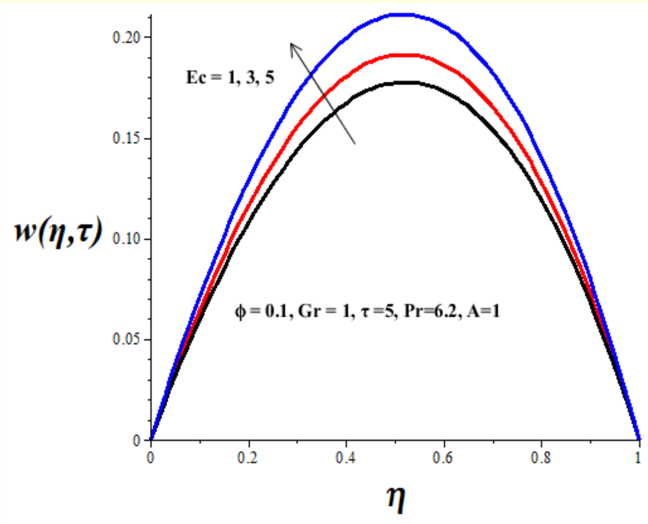
Velocity Profiles with (η,τ)



Velocity Profiles with (ϕ)

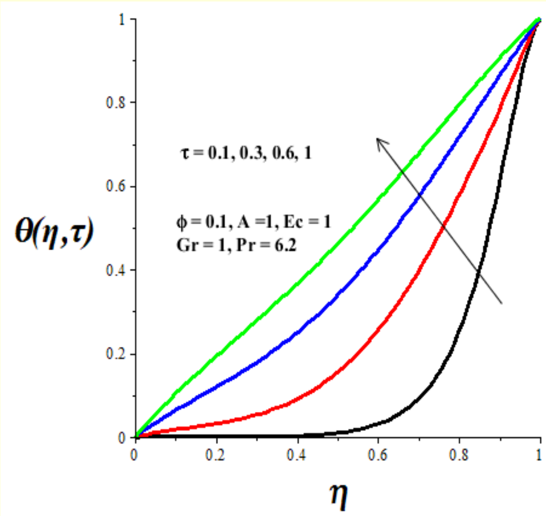


Velocity Profiles with (Gr)

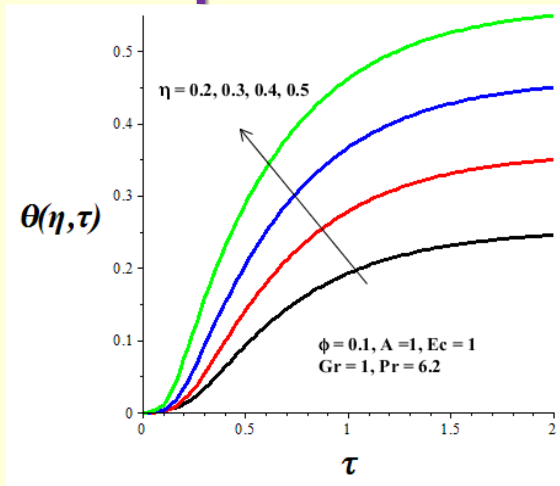


Velocity Profiles with (Ec)

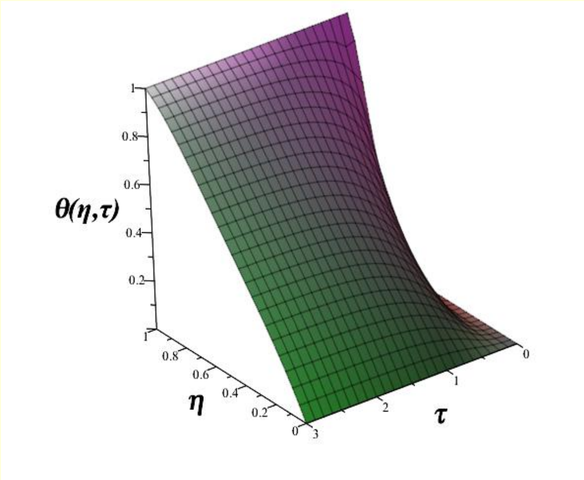
Graphical Results



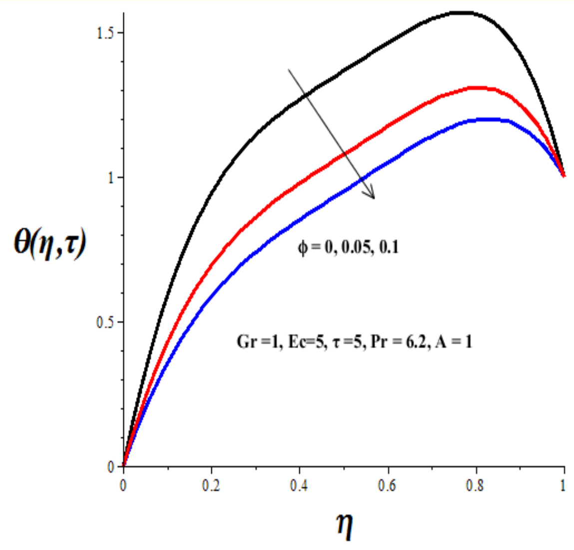
Temperature Profiles with (τ)



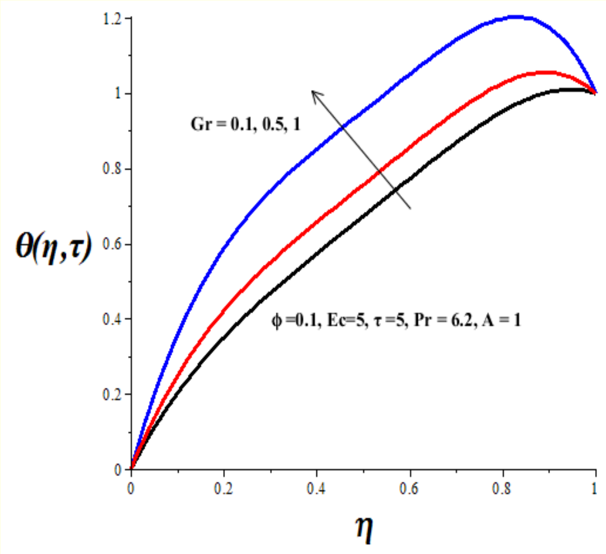
Temperature Profiles with (τ)



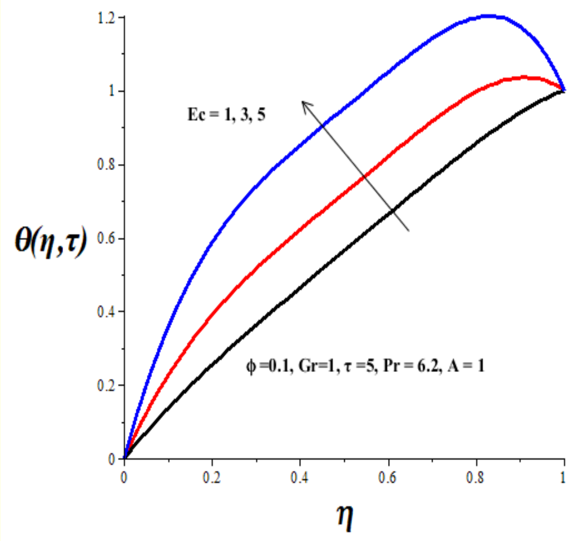
Temperature Profiles with (η,τ)



Temperature Profiles with (ϕ)

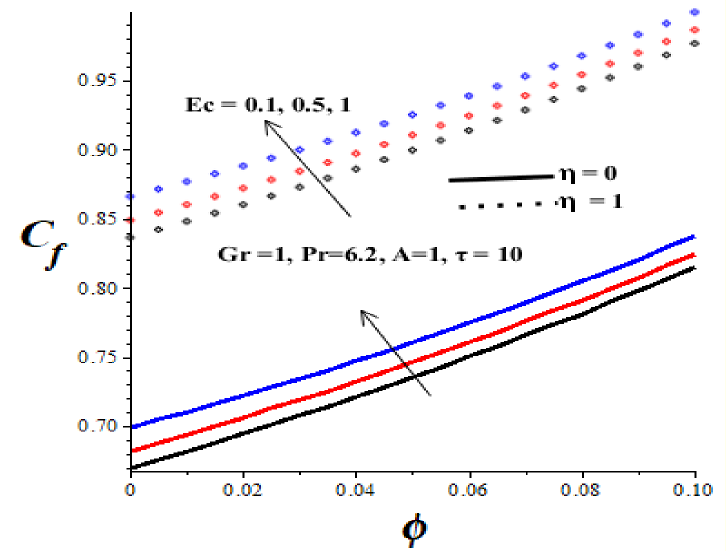


Temperature Profiles with (Gr)

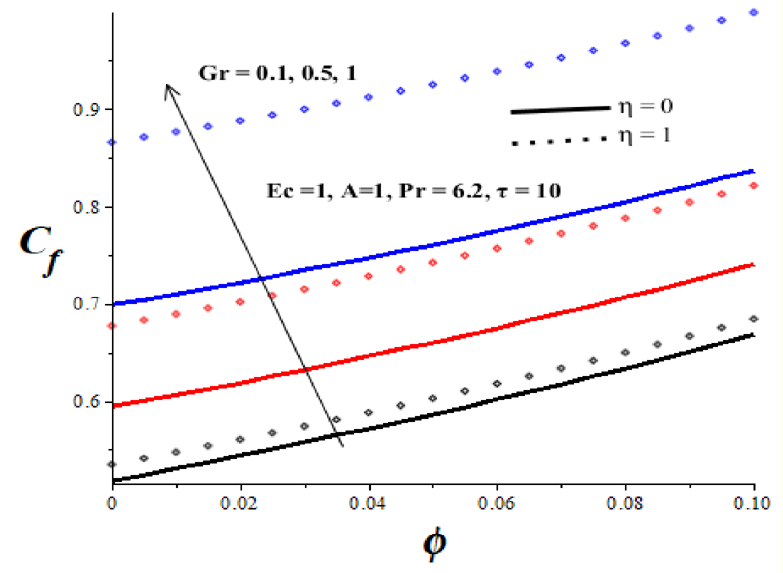


Temperature Profiles with (Ec)

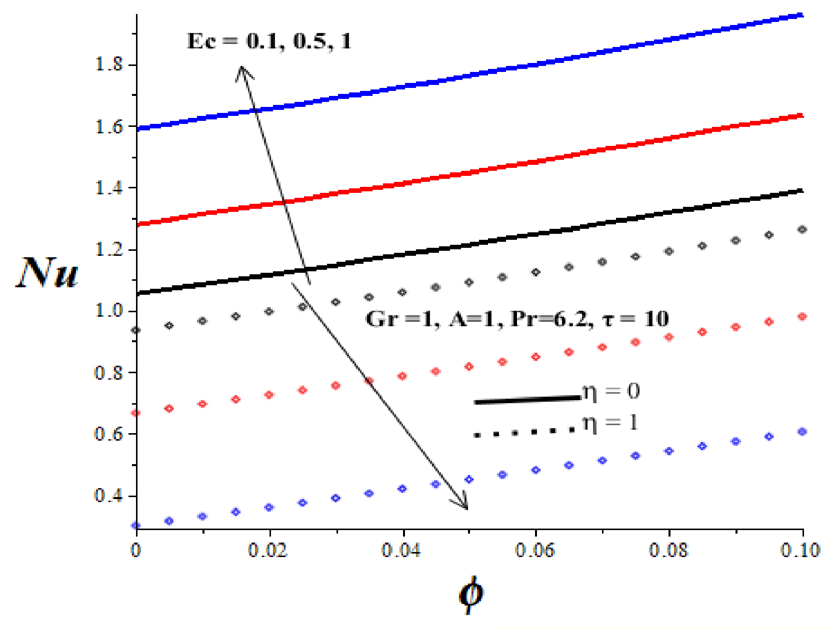
Graphical Results



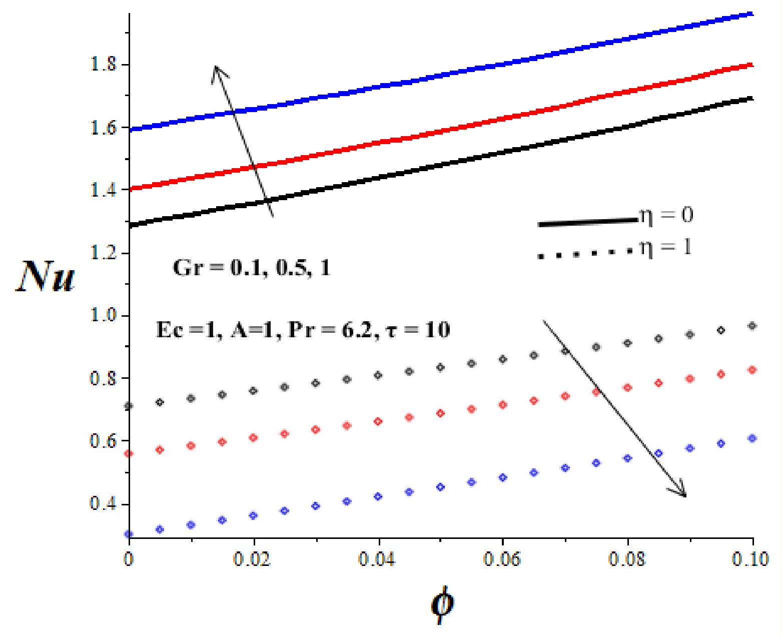
Skin Friction with ϕ, Ec



Skin Friction with Gr



Nusselt number with ϕ, Ec.



Nusselt number with Gr

Model Illustration II

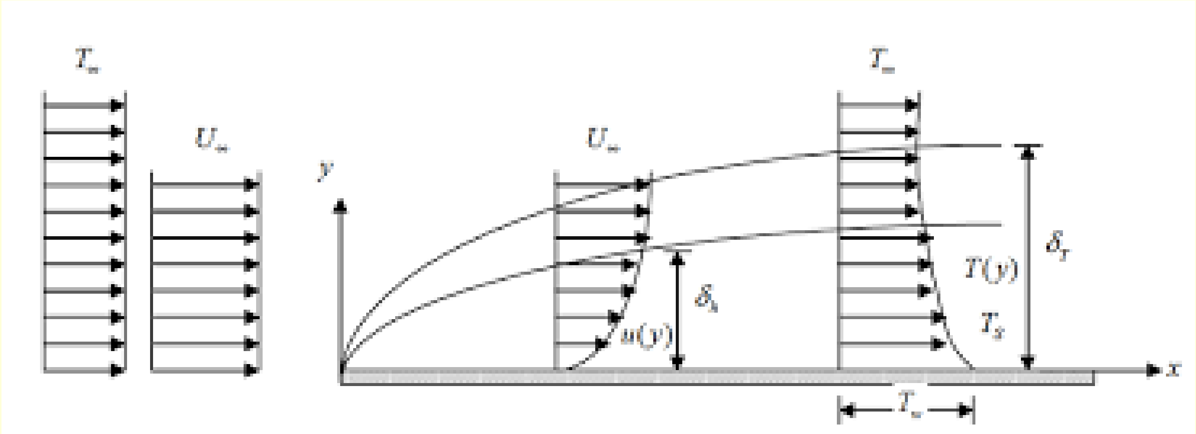
Nanofluid Heat Transfer Enhancement in Engineering Systems

External Flow Problem
Thermal Boundary Layer Problem

(DEMONSTRATION USING MAPLE SOFTWARE)

Momentum & Thermal Boundary Layer Flow

Boundary layer flow is the thin region on the surface of the body where viscosity effects are important in the development of momentum and thermal boundary layer



Application of BL

- Airfoils
- Hull of Ship
- Turbine blades
- Industrial cooling of hot surfaces

Blasius* (1908) developed an exact solution for laminar flow over a flat surface with no pressure variation. Blasius theoretically predict:
Momentum boundary layer thickness = $\delta(x)$
Velocity profile = u/U vs y/$\delta(x)$,
Wall shear stress = $\tau_w(x)$
*(first graduate student of Prandtl)

$$\eta = \frac{y}{\delta(x)} = \frac{y\sqrt{\mathrm{Re}_x}}{x} = \frac{y}{\sqrt{\upsilon x/U}}$$

$$\delta(x) \approx \frac{x}{\sqrt{\mathrm{Re}_x}}, \quad \mathrm{Re}_x = \frac{Ux}{\upsilon}$$

Similarity variable

Mathematical Model - II

Heated Surface Cooling Using NanoFluid

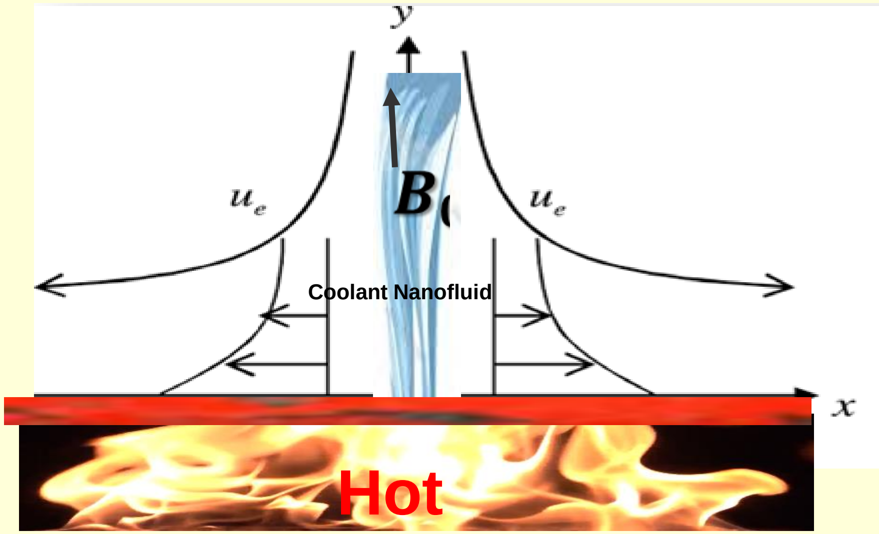
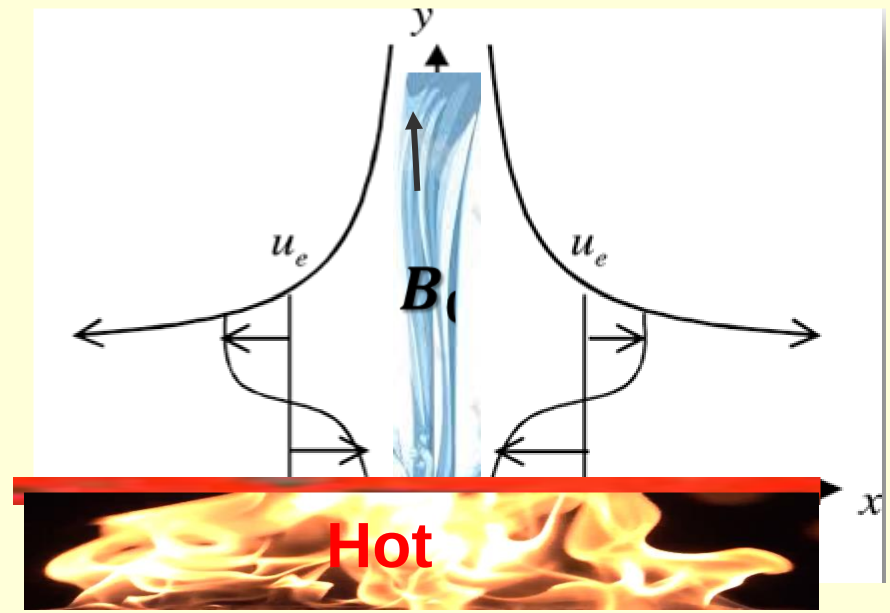


Figure 1 : Schematic diagram of the problem



DIMENSIONLESS MODEL EQUATIONS

$$\frac{d^3F}{d\eta^3} + A_1\left[F\frac{d^2F}{d\eta^2} - \left(\frac{dF}{d\eta}\right)^2 + 1\right] - B_1M\left(\frac{dF}{d\eta} - 1\right) = 0$$

$$\frac{d^2\theta}{d\eta^2} + A_2 \Pr F\frac{d\theta}{d\eta} + A_3 \Pr Ec\left(\frac{d^2F}{d\eta^2}\right)^2 + A_4 \Pr EcM\left(\frac{dF}{d\eta} - 1\right)^2 = 0$$

Boundary conditions

$$F(0) = 0, \quad \frac{dF}{d\eta}(0) = \lambda + S\frac{d^2F}{d\eta^2}(0), \quad \frac{d\theta}{d\eta}(0) = Bi[\theta(0) - 1]$$

$$\frac{dF}{d\eta}(\infty) = 1, \quad \theta(\infty) = 0$$

Assumptions

- Steady flow
- Two dimensional
- Incompressible

Dimensionless Variables & Parameters

Local similarity variables and parameters

$$B_1 = \frac{\sigma_{nf}\mu_f}{\sigma_f\mu_{nf}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}$$

$$\Pr = \frac{\mu_f C_{pf}}{k_f}$$

$$Ec = \frac{U_\infty^2}{C_{pf}(T_f - T_\infty)}$$

$$Bi = \frac{h_f}{k_f}\sqrt{\frac{\upsilon_f}{b}}$$

$$U_w = ax$$

$$A_1 = \frac{\rho_{nf}\mu_f}{\rho_f\mu_{nf}}$$

$$A_2 = \frac{k_f(\rho C_p)_{nf}}{k_{nf}(\rho C_p)_f}$$

$$A_3 = \frac{k_f\mu_{nf}}{k_{nf}\mu_f}$$

$$A_4 = \frac{\sigma_{nf}k_f}{\sigma_f k_{nf}}$$

$$U_\infty = bx$$

$$\eta = y\sqrt{\frac{b}{\upsilon_f}}$$

$$\psi = x\sqrt{b\upsilon_f}\,F(\eta)$$

$$\lambda = \frac{a}{b},$$

$$S = \beta\sqrt{\frac{b}{\upsilon_f}}$$

$$M = \frac{\sigma_f B_0^2}{\rho_f b}$$

Parameters

M = *Magnetic field parameter*

Bi = *Biot number*

Pr = *Prandtl number*

λ = *Stretching /Shrinking parameter*

Ec = *Eckert number*

S = *Slip parameter*

ϕ = *Nanoparticles volume fraction*

Other engineering quantities of interest are defined as:

- Skin friction ($C_f\sqrt{\mathrm{Re}\,x} = (1-\phi)^{-2.5}\frac{d^2F}{d\eta^2}(0)$
- Nusselt number ($Nu\,\mathrm{Re}^{-1/2} = \frac{k_{nf}}{k_n}\frac{d\theta}{d\eta}(0)$
- Heat Transfer Enhanceme

$$HTE = \frac{Nu(\phi \neq 0) - Nu(\phi = 0)}{Nu(\phi = 0)} \times 100.$$

Numerical Approach

The set of nonlinear differential equations together with the corresponding boundary conditions have been solved numerically using shooting iteration technique together with Runge-Kutta fourth-order integration scheme.

We first transform the nonlinear BVP to IVP by letting:

$$x_1 = F, x_2 = F', x_3 = F'', x_4 = \theta, x_5 = \theta'$$

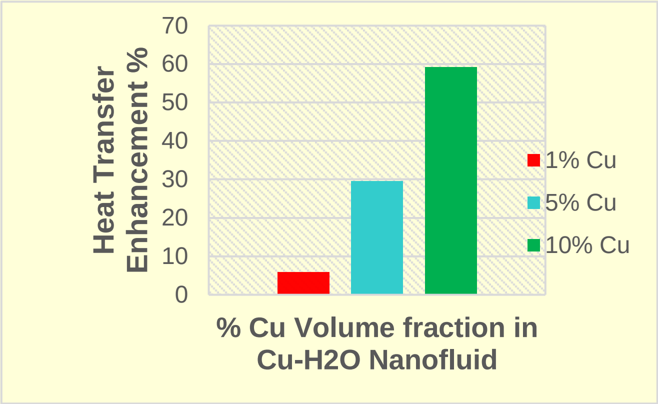
then

$$\begin{aligned}
x_1' &= x_2, \\
x_2' &= x_3, \\
x_3' &= B_1 M(x_2 - 1) - A_1\left(x_1 x_3 - x_2^2 + 1\right), \\
x_4' &= x_5, \\
x_5' &= -A_2 \Pr x_1 x_5 - A_3 \Pr Ec(x_3)^2 - A_4 \Pr EcM(x_2 - 1)^2.
\end{aligned}$$

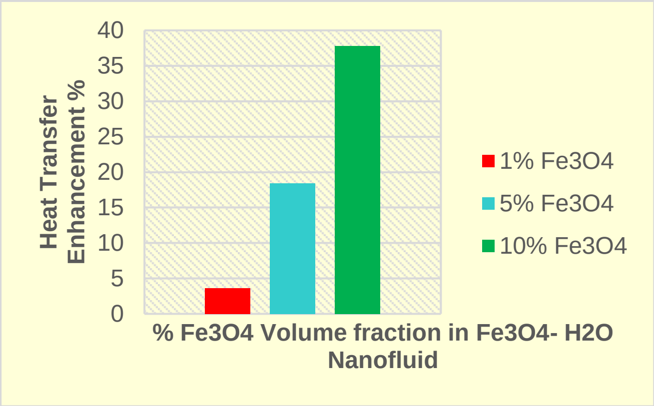
with the initial conditions

$$x_1(0) = 0, x_2(0) = \lambda + Sx_3(0), x_3(0) = a_1, x_4(0) = a_2, x_5(0) = Bi(a_2 - 1).$$

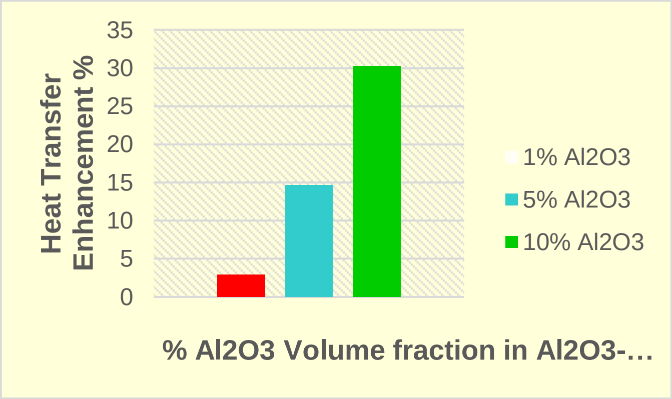
Numerical Results



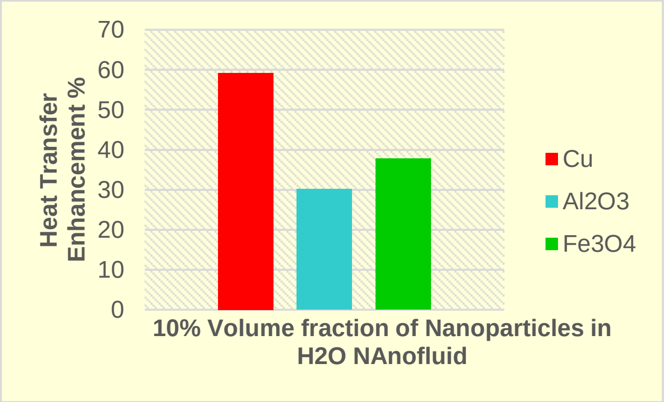
Heat transfer enhancement of different Cu-water volume fraction



Heat transfer enhancement of different Fe_3O_4-water volume fraction

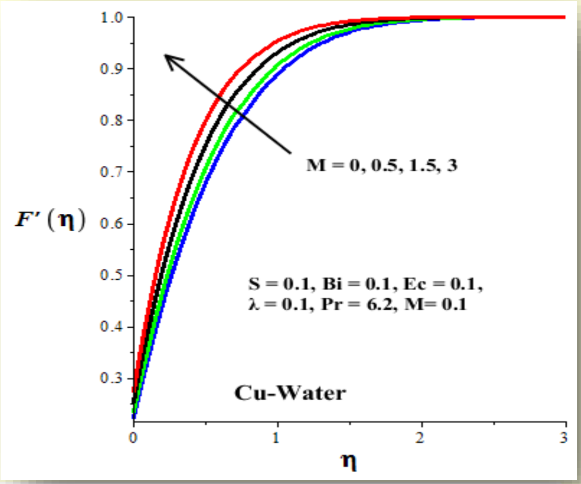


Heat transfer enhancement of different Al_2O_3-water volume fraction

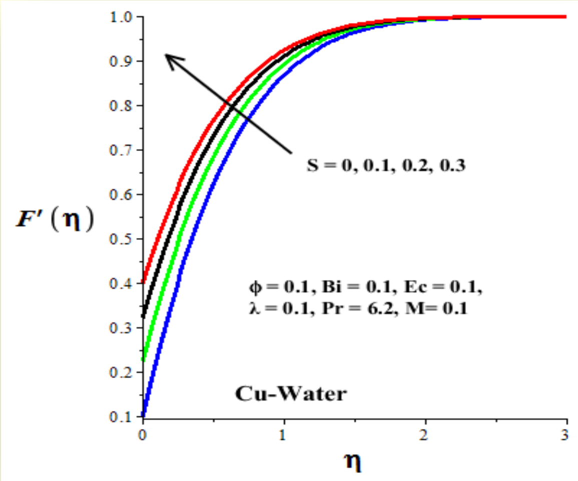


Heat transfer enhancement of different nanofluids at 10% volume fraction

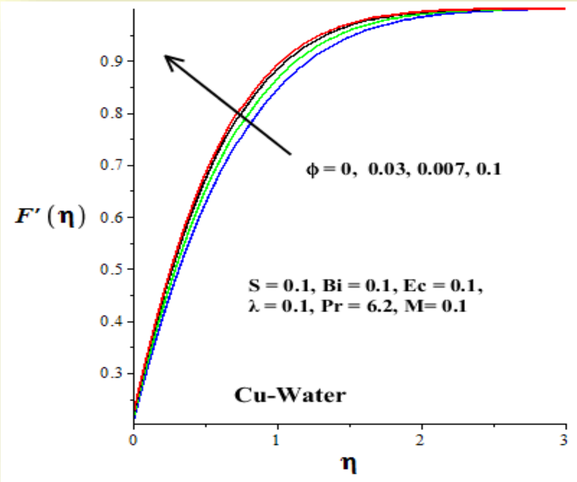
Numerical Results



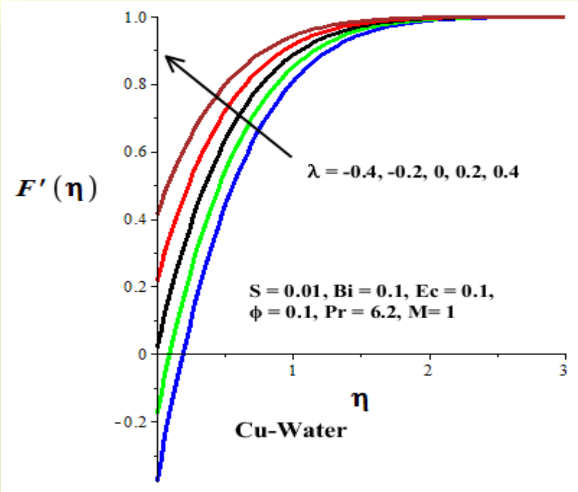
Velocity Profiles with (M)



Velocity Profiles with (S)

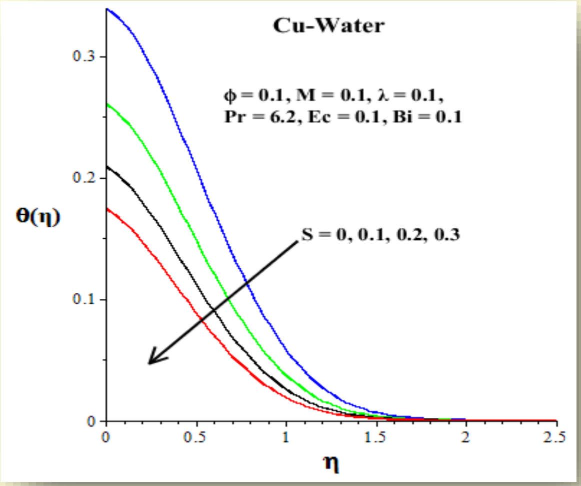


Velocity Profiles with (ϕ)

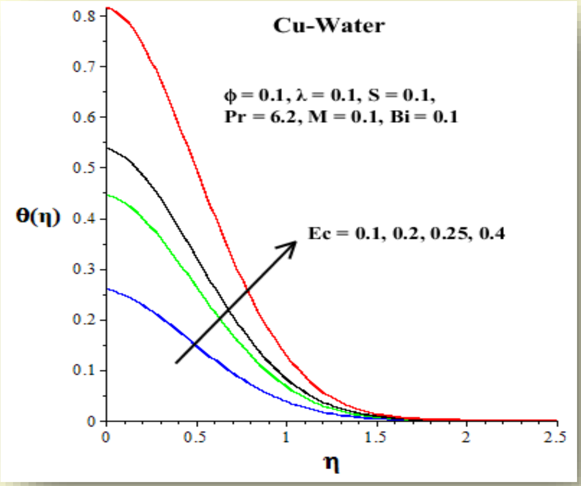


Velocity Profiles with (λ)

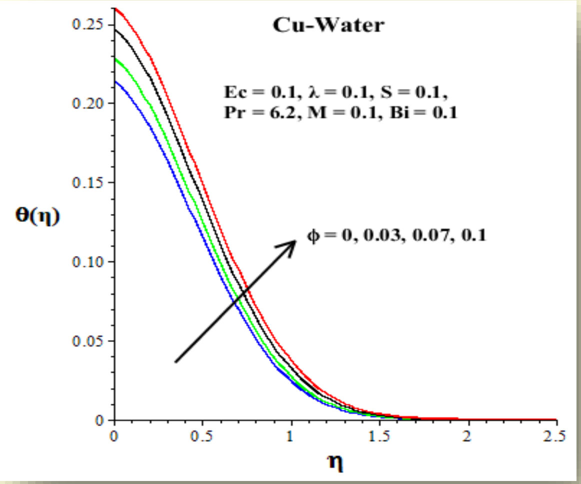
Numerical Results



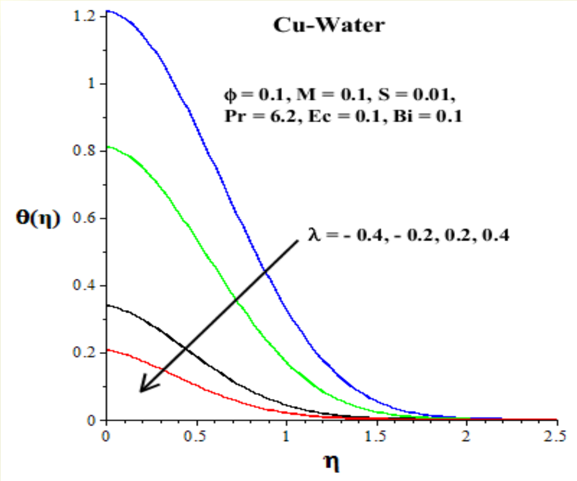
Temperature Profiles with (S)



Temperature Profiles with (Ec)

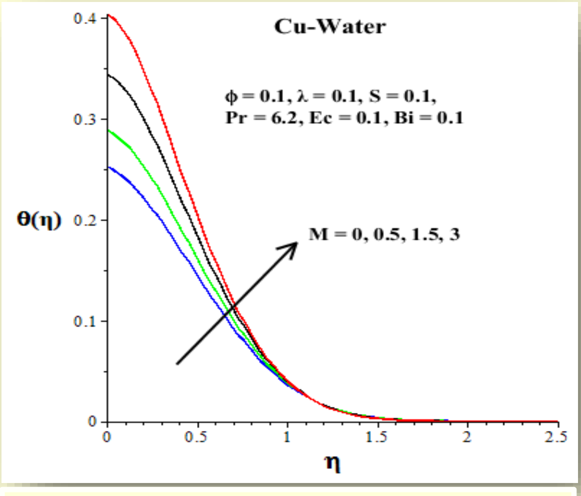


Temperature Profile with (ϕ)

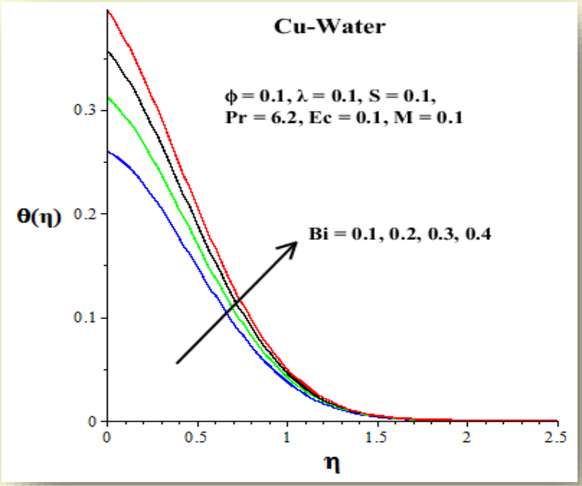


Temperature Profile with (λ)

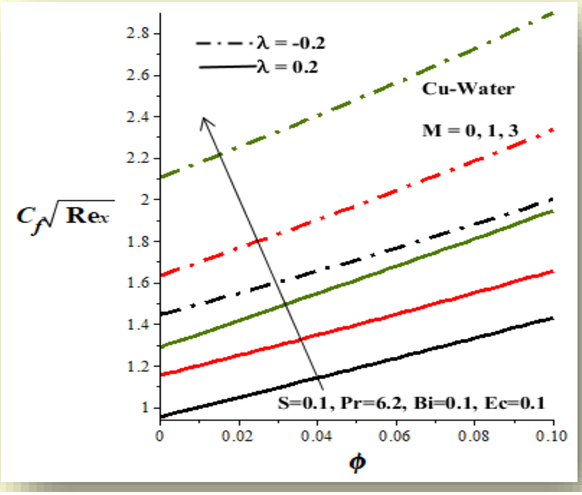
Numerical Results



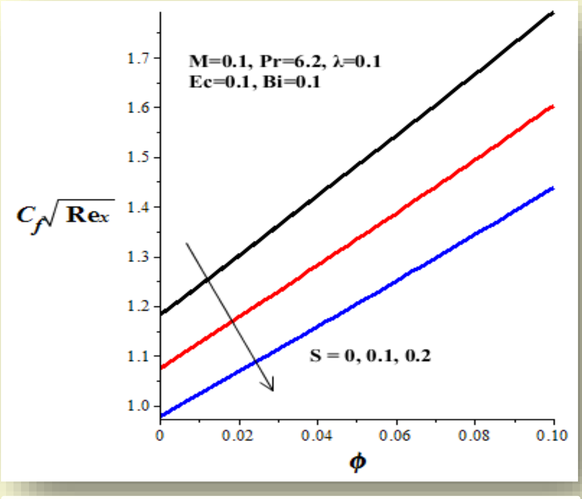
Temperature Profile with (M)



Temperature Profile with (Bi)

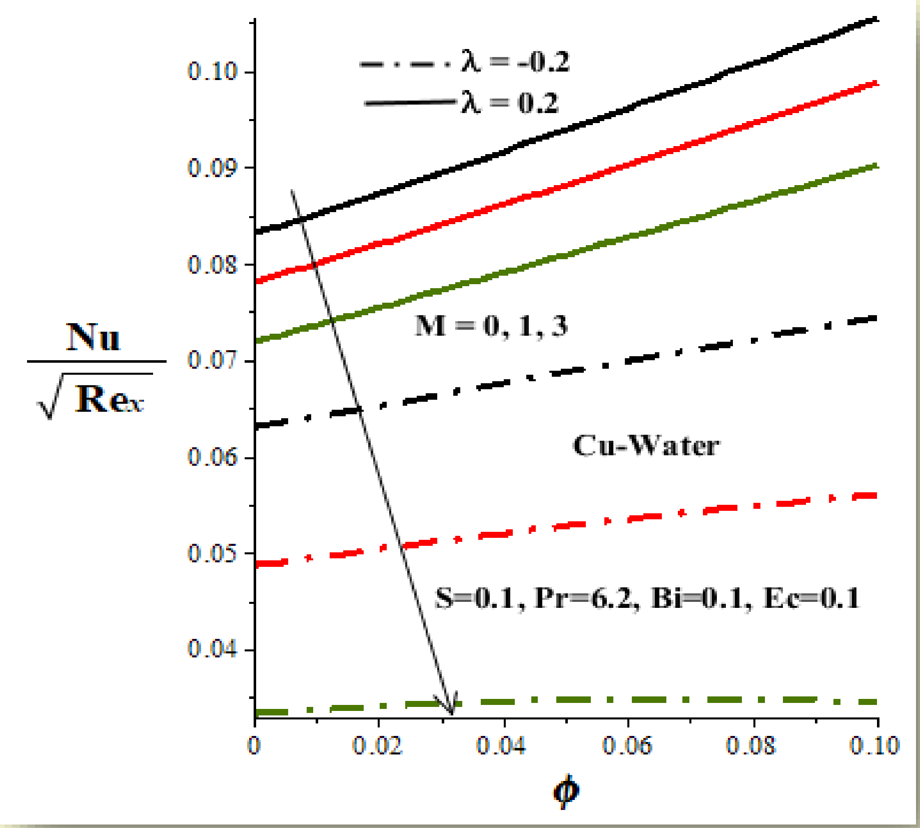


Skin Friction (Cf) Profile with (λ) and (M)

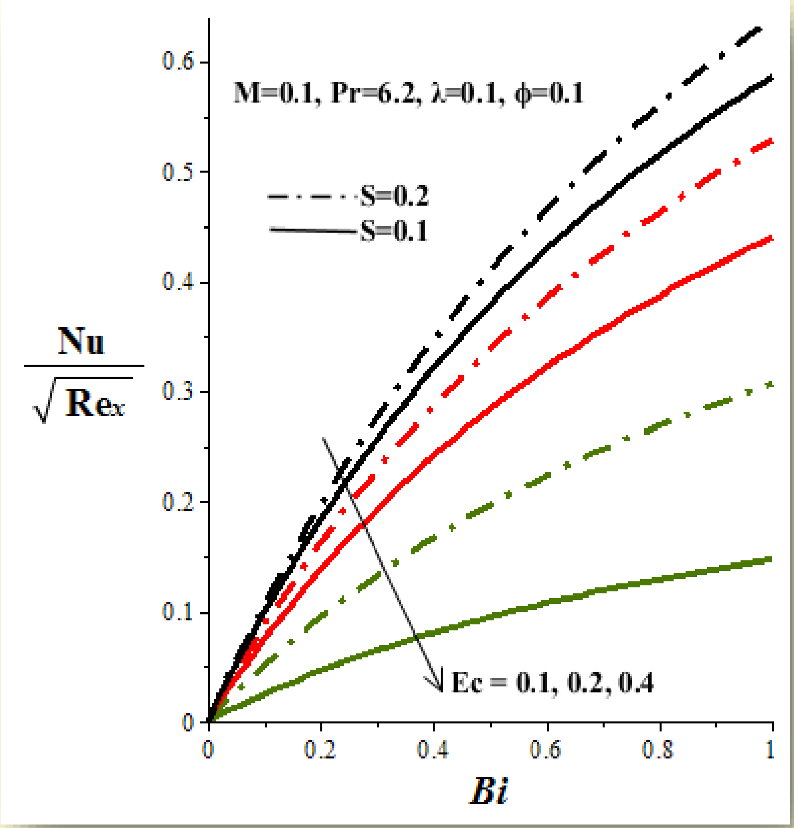


Skin Friction (Cf) Profile with (S)

Numerical Results



Nusselt Number Profile with (λ) and (M)



Nusselt Number Profile with (S) and (Ec)

PLASMA AS FLUID

- **Definition**
- **Engineering Applications**
- **Basic Fundamental Equations**
- **Analytical Illustrations**

Standard Definition of Plasma

- "Plasma" named by Irving Langmuir in 1920's
- The standard definition of a plasma is as the 4th state of matter (solid, liquid, gas, plasma), where the material has become so hot that (at least some) electrons are no longer bound to individual nuclei. Thus a plasma is electrically conducting, and can exhibit collective dynamics.
- I.e., a plasma is an ionized gas, or a partially-ionized gas.
- Implies that the potential energy of a particle with its nearest neighboring particles is weak compared to their kinetic energy (otherwise electrons would be bound to ions). → Ideal "weakly-coupled plasma" limit. (There are also more-exotic strongly-coupled plasmas, but we won't discuss those.)
- Even though the interaction between any pair of particles is typically weak, the collective interactions between many particles is strong.
  2 examples: Debye Shielding & Plasma Oscillations.

Fluid Description of Plasma

Plasma phenomena can be explained by a fluid model, in which the identity of the individual particle is neglected, and only the motion of fluid elements is taken into account

The theoretical study of plasma as a fluid is governed by the concept of magnetohydrodynamics (MHD) which involved a combination of conservation equations of conducting fluid mass, charges and momentum coupled with state equation and Maxwell equations of electromagnetism

Plasma may involve the dynamics **positively charged ion fluid** and **negatively charged electron fluid**. In a partially ionized gas, the dynamics of **fluid of neutral atoms** may also be involved. The neutral fluid will interact with the ions and electrons only through collisions. The ion and electron fluids will interact with each other even in the absence of collisions due to the generation of the electric and magnetic fields

Plasma Applications

Applications of Plasma range from **energy production by thermonuclear fusion to laboratory astrophysics, creation of intense sources of high-energy particle and radiation beams, and fundamental studies involving high-field quantum electrodynamics**.

Plasma is being used in many high tech industries.

- It is used in **making many microelectronic or electronic devices such as semiconductors**.
- It can help make features on chips for computers.
- Plasma is also used in making transmitters for microwaves or high temperature films.

Plasma research is leading to profound new insights on the inner workings of the Sun and other stars, and fascinating astrophysical objects such as black holes and neutron stars. The study of plasma is **enabling prediction of space weather, medical treatments, and even water purification**.

Examples of naturally occurring plasmas:

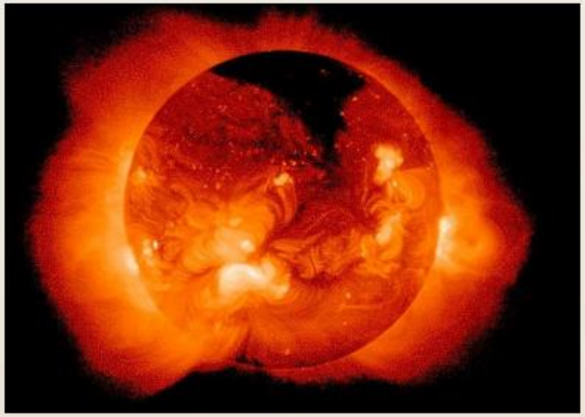
- (99% of the visible universe is a plasma)

Gas Nebula

Lightning

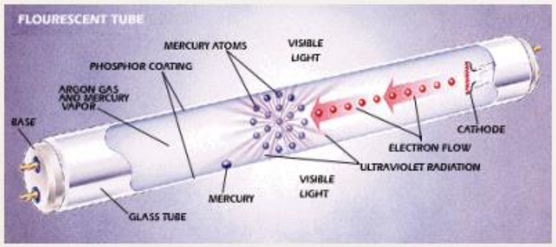
Flames

Aurora Borealis

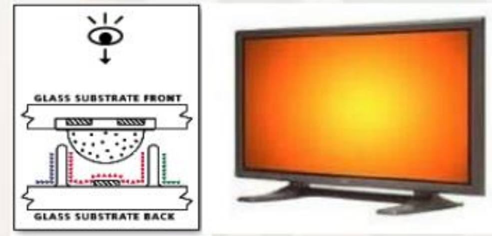


Solar Corona

Examples of man-made plasmas:

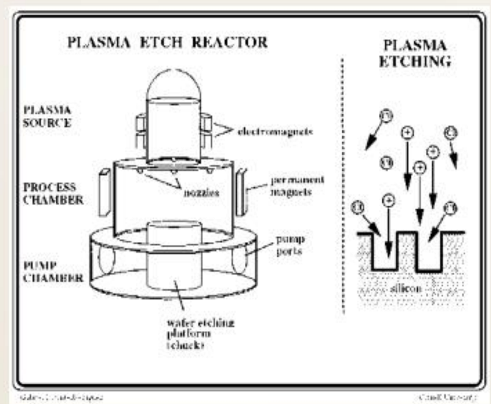


Fluorescent lamps
(glow discharge)

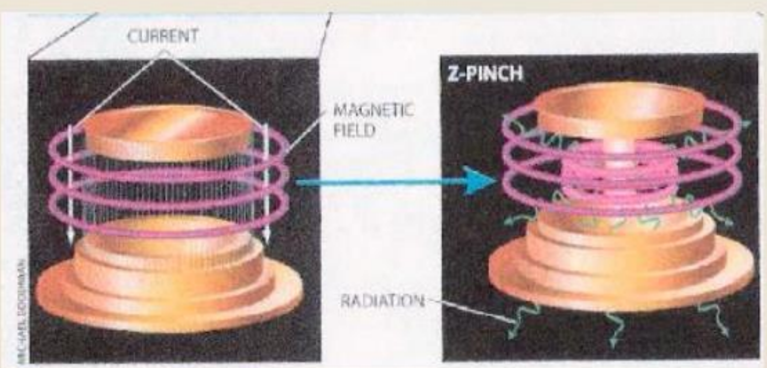


Flat panel plasma display

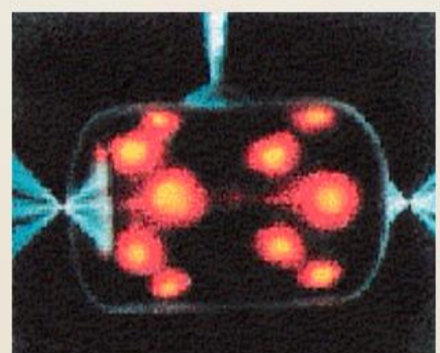
Plasma torch



Plasma etching reactor
(plasmas play important role in the manufacturing of integrated circuits)



Z-pinch



Laser-created plasmas

Magneto-Fluid Dynamics

- Magneto-fluid dynamics (MFD) is the study of the flow of electrically conducting fluids in a magnetic field.
- MFD is derived from three words; magneto – magnetic field, fluids, and dynamics – movement.
- It covers phenomena where electrically conducting ionized fluids, with velocity field **V,** and the magnetic field **B** are coupled.
- Any movement of a conducting material in a magnetic field generates electric currents **j**, which in turn induce
  - their own magnetic fields, and
  - **j** x **B** forces on the medium known as *Lorentz* force.

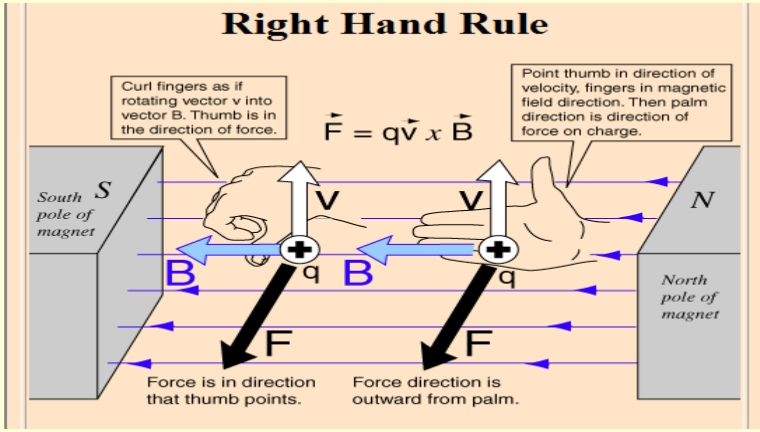
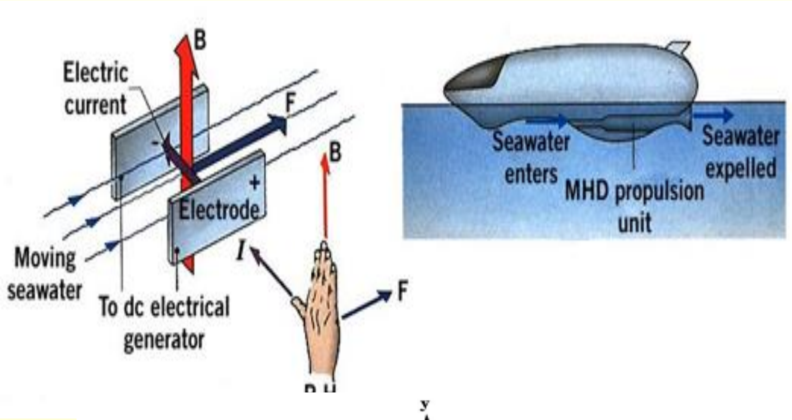
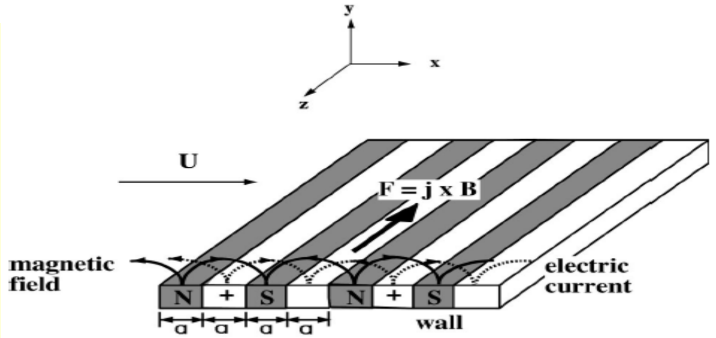


Fig.1: The Right Hand Rule





Hannes Alfvén (1908-1995), winning the Nobel Prize in Physics for his work on Magnetohydrodynamics.

Some Plasma Properties

Mass density $\rho_m = n_e m_e + n_i m_i$

Charge density $\sigma = q_e n_e + q_i n_i$

Mass velocity $V = (n_e m_e v_e + n_i m_i v_e) / \rho_m$

Current density $\mathbf{j} = q_e n_e v_e + q_i n_i v_i = q_e n_e (v_e - v_i)$

Total pressure $p = p_e + p_i$

where the subscripts *i* and *e* represent the ions and electrons, respectively.

Magnetohydrodynamics (MHD) Equations for Plasma

(1) $$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (nV) = 0,$$ (Mass Conservation Equation)

(2) $$\frac{\partial \sigma}{\partial t} + \nabla \cdot (n\mathbf{j}) = 0,$$ (Charge Conservation Equation)

(3) $$\underbrace{\rho_m\left(\frac{\partial V}{\partial t} + \mathrm{V}\cdot\nabla V\right)}_{\substack{\text{\textit{rate} of change of} \\ \text{total momentum density}}} = \underbrace{\sigma E}_{\substack{\textit{Electric} \\ \text{body force}}} + \underbrace{\mathbf{j}\times B}_{\substack{\textit{Magnetic}\text{ force} \\ \text{on current}}} - \underbrace{\nabla P}_{\text{Pr}\textit{essure}},$$ (Momentum Equation)

or

$$\rho_m\left(\frac{\partial V}{\partial t} + \mathrm{V}\cdot\nabla V\right) = \sigma(E + V\times B) - \nabla P + \rho_m F, \quad F = -\upsilon_{jk}(v_j - v_k),$$

Maxwell Equations

(4) $$\nabla\times B = \mu_0\mathbf{j} + \frac{1}{c^2}\frac{\partial E}{\partial t}, \quad \nabla\times E = -\frac{\partial B}{\partial t},$$

$$\nabla\cdot B = 0, \ \nabla\cdot(\in_0 E) = \sigma, \quad E + V\times B = \eta\mathbf{j} + \frac{\mathbf{j}\times B - \nabla p_e}{ne},$$

where B is the magnetic field strength, E is the electric field, n is the particle density and η is the resistivity.

Equation of state (EOS)

- An equation of state is a thermodynamic equation describing the state of matter under a given set of physical conditions.

$$p = p(n, T), \qquad \varepsilon = \varepsilon(n, T)$$

- Isothermal EOS for slow time variations, where temperatures are allowed to equilibrate. In this case, the fluid can exchange energy with its surroundings.

$$\boxed{p = nkT, \qquad \nabla p = kT\nabla n} \qquad n_g\,(\mathrm{cm}^{-3}) \approx 3.250 \times 10^{16}\, p\,(\mathrm{Torr})$$

→ The energy conservation equation needs to be solved to determine *p* and *T*.

- Adiabatic EOS for fast time variations, such as in waves, when the fluid does not exchange energy with its surroundings

$$\boxed{p = Cn^{\gamma}, \qquad \frac{\nabla p}{p} = \gamma \frac{\nabla n}{n}} \qquad \gamma = \frac{C_p}{C_v} \quad \text{(specific heat ratio)}$$

→ The energy conservation equation is not required.

- Specific heat ratio vs degree of freedom (f) $\qquad \gamma = 1 + \dfrac{2}{f}$

Equation of state

Usually adopt the ideal gas law $\boxed{P=nkT}$

In thermal equilibrium, each internal degree of freedom has energy (kT/2). Thus, internal energy density for an ideal gas with *m* internal degrees of freedom

$$e = nm(kT/2).$$

Combining, $\boxed{P = (\gamma\text{-}1)e}$ where $\gamma = (m+2)/m$

For monoatomic gas (H), γ=5/3 (*m*=3)
diatomic gas (H_2), γ=7/5 (*m*=5)

Also common to use isothermal EOS $\boxed{P = C^2\rho}$ where C=isothermal sound speed when (radiative cooling time) << (dynamical time)

In some circumstances, an ideal gas law is not appropriate, and must use more complex (or tabular) EOS (e.g. for degenerate matter)

Flux conservation:

Given by Maxwell's equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0$$ (constraint rather than evolutionary equation)

From Ohm's Law, the current and electric field are related by

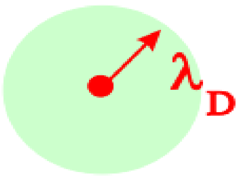
$$\mathbf{J} = \sigma \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right)$$

For a fully conducting plasma, $\sigma \rightarrow \infty$

So c**E** = -(**v** x **B**).

$$\boxed{\frac{\partial B}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0}$$

DEBYE NUMBER



- Consider a sphere of radius the Debye length λ_{D}.
  It contains $N_{\mathrm{D}} \equiv \frac{4}{3}\pi\lambda_{\mathrm{D}}^3 n_{\mathrm{e}}$ electrons: the Debye number.

- The Debye number is the number of electrons in the “coat” shielding any ion in the plasma.

- The Debye number is a measure of the importance of collective effects in the plasma.

- If $N_{\mathrm{D}} < 1$ there are no collective effects. The “plasma” is merely a collection of individual particles.

- If $N_{\mathrm{D}} > 1$ it is a true plasma and cooperative effects are important.

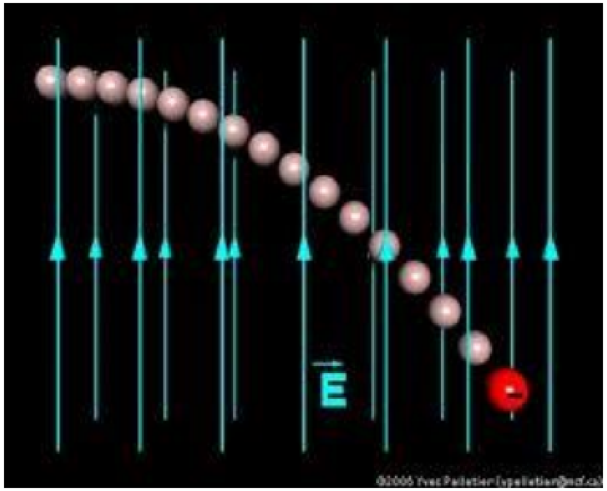
- Usually $N_{\mathrm{D}} \gg 1$, with N_{D} ranging from 10^4 (laboratory) to 10^{32} (cluster of galaxies).

Motions of a charged particle in uniform electric field

- Equation of motion of a charged particle in fields

$$m\frac{d\boldsymbol{v}}{dt} = q[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v}\times\boldsymbol{B}(\boldsymbol{r},t)], \qquad \frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}(t)$$

- Motion in constant electric field



✓ For a constant electric field $\boldsymbol{E} = \boldsymbol{E}_0$ with $\boldsymbol{B} = 0$,

$$\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0 t + \frac{q\boldsymbol{E}_0}{2m}t^2$$

✓ Electrons are easily accelerated by electric field due to their smaller mass than ions.

✓ Electrons (Ions) move against (along) the electric field direction.

✓ The charged particles get kinetic energies.

Motions of a charged particle in uniform magnetic field

- Motion in constant magnetic field

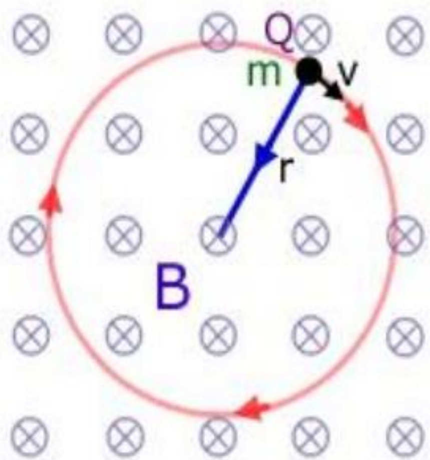
$$m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}$$

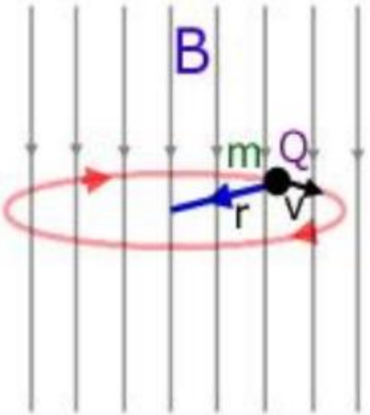
- For a constant magnetic field $\boldsymbol{B} = B_0\boldsymbol{z}$ with $\boldsymbol{E} = 0$,

$$m\frac{dv_x}{dt} = qB_0v_y$$

$$m\frac{dv_y}{dt} = -qB_0v_x$$

$$m\frac{dv_z}{dt} = 0$$





- Cyclotron (gyration) frequency

$$\frac{d^2v_x}{dt^2} = -\omega_c^2v_x$$

$$\boxed{\omega_c = \frac{|q|B_0}{m}}$$

Motions of a charged particle in uniform magnetic field

- Particle velocity

$$v_x = v_\perp \cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp \sin(\omega_c t + \phi_0)$$
$$v_z = v_{z0}$$

- Particle position

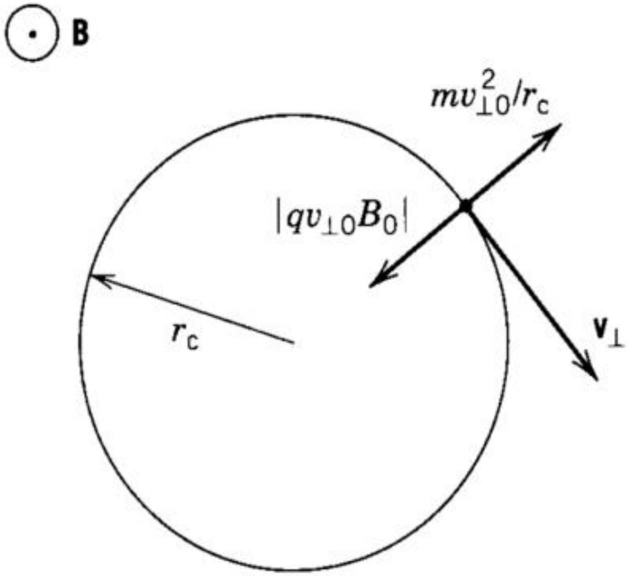
$$x = x_0 + r_c \sin(\omega_c t + \phi_0)$$
$$y = y_0 + r_c \cos(\omega_c t + \phi_0)$$
$$z = z_0 + v_{z0} t$$

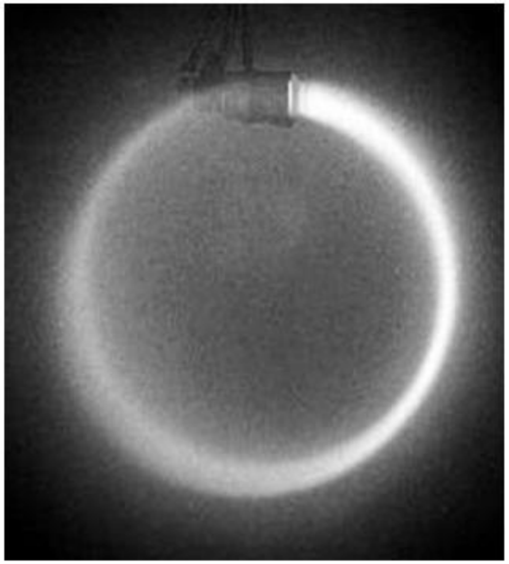
- Guiding center

$$(x_0, y_0, z_0 + v_{z0} t\ )$$

- Larmor (gyration) radius

$$\boxed{r_c = r_\mathrm{L} = \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B_0}}$$

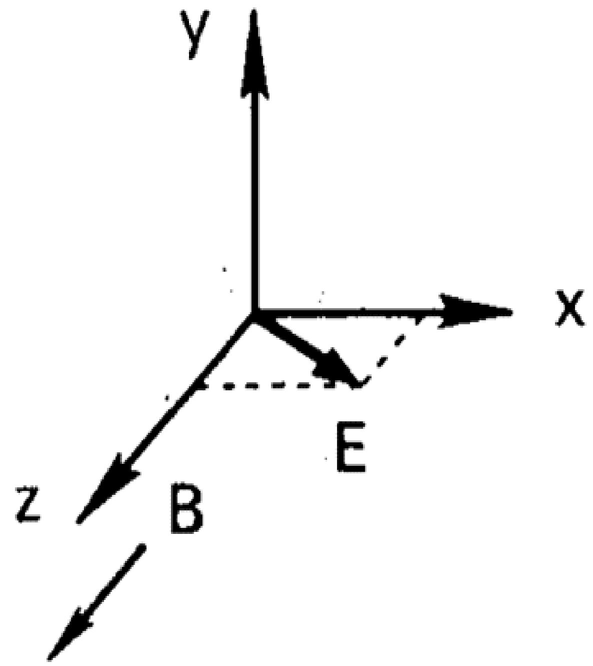




Motions of a charged particle in uniform E and B fields

- Equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



- Parallel motion: $\boldsymbol{B} = B_0\mathbf{z}$ and $\boldsymbol{E} = E_0\mathbf{z}$,

$$m\frac{dv_z}{dt} = qE_z$$

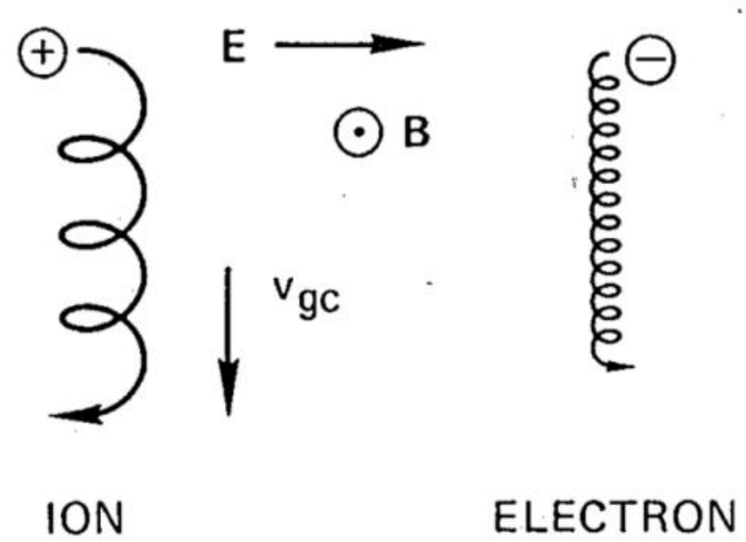
$$v_z = \frac{qE_z}{m}t + v_{z0}$$

→ Straightforward acceleration along B

$\boldsymbol{E}\times\boldsymbol{B}$ drift

- Transverse motion: $\boldsymbol{B} = B_0\boldsymbol{z}$ and $\boldsymbol{E} = E_0\boldsymbol{x}$,

$$m\frac{dv_x}{dt} = qE_0 + qB_0v_y$$
$$m\frac{dv_y}{dt} = -qB_0v_x$$



- Differentiating,

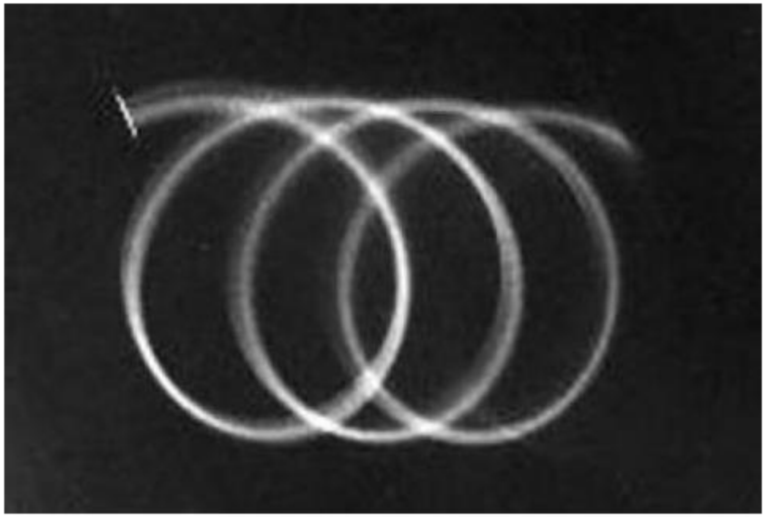
$$\frac{d^2v_x}{dt^2} = -\omega_c^2 v_x$$
$$\frac{d^2v_y}{dt^2} = -\omega_c^2\left(\frac{E_0}{B_0} + v_y\right)$$

$$\boxed{\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}}$$

- Particle velocity

$$v_x = v_\perp\cos(\omega_c t + \phi_0)$$
$$v_y = -v_\perp\sin(\omega_c t + \phi_0) - \frac{E_0}{B_0}$$

v_{gc}



Diffusion and mobility

- The fluid equation of motion including collisions

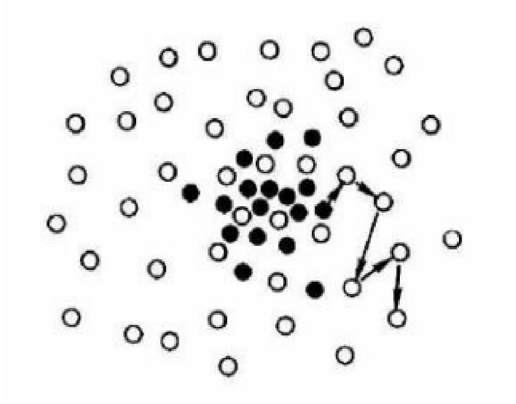
$$mn\frac{d\boldsymbol{u}}{dt} = mn\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right] = qn\boldsymbol{E} - \boldsymbol{\nabla}p - mn\nu_m\boldsymbol{u}$$

- In steady-state, for isothermal plasmas

$$\boldsymbol{u} = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - \boldsymbol{\nabla}p) = \frac{1}{mn\nu_m}(qn\boldsymbol{E} - kT\boldsymbol{\nabla}n)$$

$$= \frac{q}{m\nu_m}\boldsymbol{E} - \frac{kT}{m\nu_m}\frac{\boldsymbol{\nabla}n}{n} = \pm\mu\boldsymbol{E} - D\frac{\boldsymbol{\nabla}n}{n}$$

Drift Diffusion



Diffusion is a random walk process.

- In terms of particle flux

$$\boldsymbol{\Gamma} = n\boldsymbol{u} = \pm n\mu\boldsymbol{E} - D\boldsymbol{\nabla}n$$

$$\boxed{\mu = \frac{|q|D}{kT} \quad : \quad \text{Einstein relation}}$$

$$\boxed{\mu = \frac{|q|}{m\nu_m} \quad : \quad \text{Mobility}}$$

$$\boxed{D = \frac{kT}{m\nu_m} \quad : \quad \text{Diffusion coefficient}}$$

Ambipolar diffusion

- The flux of electrons and ions out of any region must be equal such that charge does not build up. Since the electrons are lighter, and would tend to flow out faster in an unmagnetized plasma, an electric field must spring up to maintain the local flux balance.

$$\Gamma_i = +n\mu_i E - D_i \nabla n \qquad\qquad \Gamma_e = -n\mu_e E - D_e \nabla n$$

- Ambipolar electric field for $\Gamma_i = \Gamma_e$

$$E = \frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{n}$$

- The common particle flux

$$\Gamma = -\frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e}\nabla n = -D_a \nabla n$$

- The ambipolar diffusion coefficient for weakly ionized plasmas

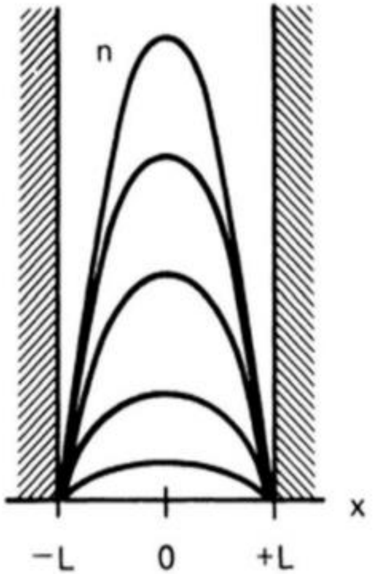
$$D_a = \frac{\mu_e D_i + \mu_i D_e}{\mu_i + \mu_e} \approx D_i + \frac{\mu_i}{\mu_e} D_e \approx D_i\left(1 + \frac{\mathrm{T_e}}{\mathrm{T_i}}\right)$$

Decay of a plasma by diffusion in a slab

- Diffusion equation

Derived from the continuity equation

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = 0 \quad \Leftarrow \quad n_i \approx n_e = n, \quad D = D_a$$



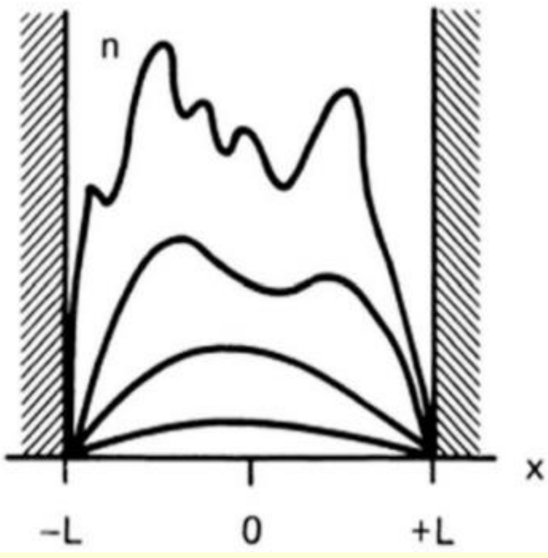
- In Cartesian coordinates,

$$\boxed{\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}}$$

- Find $n(x,t)$ under the boundary conditions [H/W]

$$n(x = \pm L, t) = 0$$

$$n(x, t = 0) = n_0(1 - (x/L)^2)$$



- In general

$$n = n_0 \left(\sum_l a_l e^{-t/\tau_l} \cos \frac{(l + \frac{1}{2})\pi x}{L} + \sum_m b_m e^{-t/\tau_m} \sin \frac{m\pi x}{L} \right)$$

$$\tau_l = \left[\frac{L}{(l + \frac{1}{2})\pi} \right]^2 \frac{1}{D}$$

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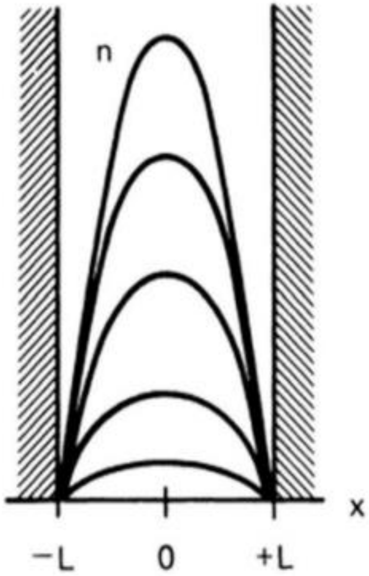

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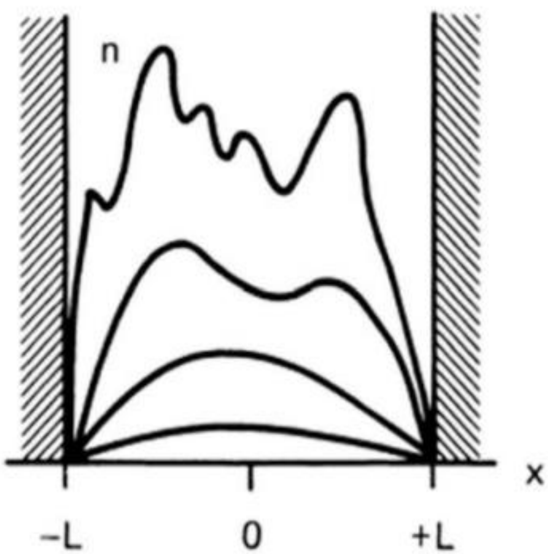

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$$\tau_l = \left[\frac{L}{(l + \frac{1}{2})\pi} \right]^2 \frac{1}{D}$$

Useful Constants and Formulae

Table 1: Commonly used physical constants

Name	**Symbol**	**Value (SI)**	**Value (cgs)**
Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}	1.38×10^{-16} erg K^{-1}
Electron charge	e	1.6×10^{-19} C	4.8×10^{-10} statcoul
Electron mass	m_e	9.1×10^{-31} kg	9.1×10^{-28} g
Proton mass	m_p	1.67×10^{-27} kg	1.67×10^{-24} g
Planck constant	h	6.63×10^{-34} J s	6.63×10^{-27} erg-s
Speed of light	c	3×10^{8} m s^{-1}	3×10^{10} cm s^{-1}
Dielectric constant	ε_0	8.85×10^{-12} F m^{-1}	—
Permeability constant	μ_0	$4\pi \times 10^{-7}$	—
Proton/electron mass ratio	m_p/m_e	1836	1836
Temperature = 1eV	e/k_B	11 604 K	11 604 K
Avogadro number	N_A	6.02×10^{23} mol^{-1}	6.02×10^{23} mol^{-1}
Atmospheric pressure	1 atm	1.013×10^{5} Pa	1.013×10^{6} dyne cm^{-2}

Table .2: Formulae in SI and cgs units

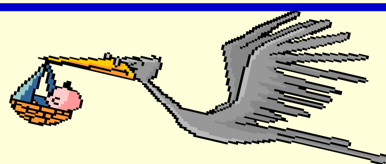
Name	**Symbol**	**Formula (SI)**	**Formula (cgs)**
Debye length	λ_D	$\left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2}$ m	$\left(\frac{k_B T_e}{4\pi e^2 n_e}\right)^{1/2}$ cm
Particles in Debye sphere	N_D	$\frac{4\pi}{3}\lambda_D^3$	$\frac{4\pi}{3}\lambda_D^3$
Plasma frequency (electrons)	ω_{pe}	$\left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{1/2}$ s^{-1}	$\left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2}$ s^{-1}
Plasma frequency (ions)	ω_{pi}	$\left(\frac{Z^2 e^2 n_i}{\varepsilon_0 m_i}\right)^{1/2}$ s^{-1}	$\left(\frac{4\pi Z^2 e^2 n_i}{m_i}\right)^{1/2}$ s^{-1}
Thermal velocity	$v_{te} = \omega_{pe}\lambda_D$	$\left(\frac{k_B T_e}{m_e}\right)^{1/2}$ m s^{-1}	$\left(\frac{k_B T_e}{m_e}\right)^{1/2}$ cm s^{-1}
Electron gyrofrequency	ω_c	eB/m_e s^{-1}	eB/m_e s^{-1}
Electron–ion collision frequency	ν_{ei}	$\frac{\pi^{3/2} n_e Z e^4 \ln\Lambda}{2^{1/2}(4\pi\varepsilon_0)^2 m_e^2 v_{te}^3}$ s^{-1}	$\frac{4(2\pi)^{1/2} n_e Z e^4 \ln\Lambda}{3 m_e^2 v_{te}^3}$ s^{-1}
Coulomb logarithm	$\ln\Lambda$	$\ln\frac{9N_D}{Z}$	$\ln\frac{9N_D}{Z}$

CONCLUSION

Both theoretical and experimental research are very essential in fluid and plasma physics.

- **Complex fluid flow with heat transfer characteristics and plasma problems can be easily investigated theoretically using modelling and computational approach.**
- **Modelling helps to make things better, faster, safer and cheaper through simulation of complex phenomena and the reduction of the flood of data with visualisation**
- An important feature of the application of modelling and computations to fluid and plasma physics problem is that, it enables us to make scientific predictions that are to draw on the basis of logic and with the aid of mathematical methods, correct conclusions whose agreement with reality is then confirmed by experience, experiment and practice leading to innovation and national development





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THE END

THANK YOU VERY MUCH

ALL THE BEST AND GOD BLESS