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Dark matter or “dark matter” ?

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Dark matter is a catchword used to refer to a range of probably related phenomena, most notable of which being:

- Galaxy rotation curves are non-Keplerian if computed according to the "luminous" mass distributions
- Stars' velocity dispersion estimates in elliptical galaxies do not match the predicted velocity dispersion from the observed mass distribution
- The gravitational lensing is stronger than if it were caused by the observed mass
- The CMB spectrum has not only 1st and 2nd peak (corresponding, respectively, to the overall curvature of the universe and visible matter density distribution) but also the 3^{d} peak
- Visible matter alone can't explain the observed structure formation of galaxies and clusters
- Type Ia supernova distance measurements indicate the missing component in the energy density

I. Silly questions

Here comes the first question (not a silly question yet):

What is dark matter's nature and micro structure?

Two views:

1) DM is a corpuscular matter, i.e., it consists of relativistic particles
2) There is no (corpuscular) DM per se, but the gravity must be modified

…There's a relation* between these explanations:

$$\Delta\,(\Phi_{\mathrm{observed}} - \Phi_{\mathrm{luminous}}) = 4\,\pi\,G\,\rho_{\mathrm{DM}}$$

(**in the relativistic case, replace the Poisson eq with Einstein eqs*)

but one must either specify the microscopic structure of ρ_{DM} or explain why $\Phi_{\mathrm{observed}} \neq \Phi_{\mathrm{luminous}}$

Corpuscular interpretation:

Problem #1 Observational

Which particle? (also not a silly question yet)

None of known particles (described by the current Standard Model of particle physics) seems to be a strong candidate for a DM particle

And none from relativistic theories outside or beyond SM:

light bosons (incl. little Higgs and light scalars)

weakly interacting massive particle (WIMPs)

axions

sterile neutrinos

SUSY particles, strings, higher-dimensional effects

geons

primordial black holes

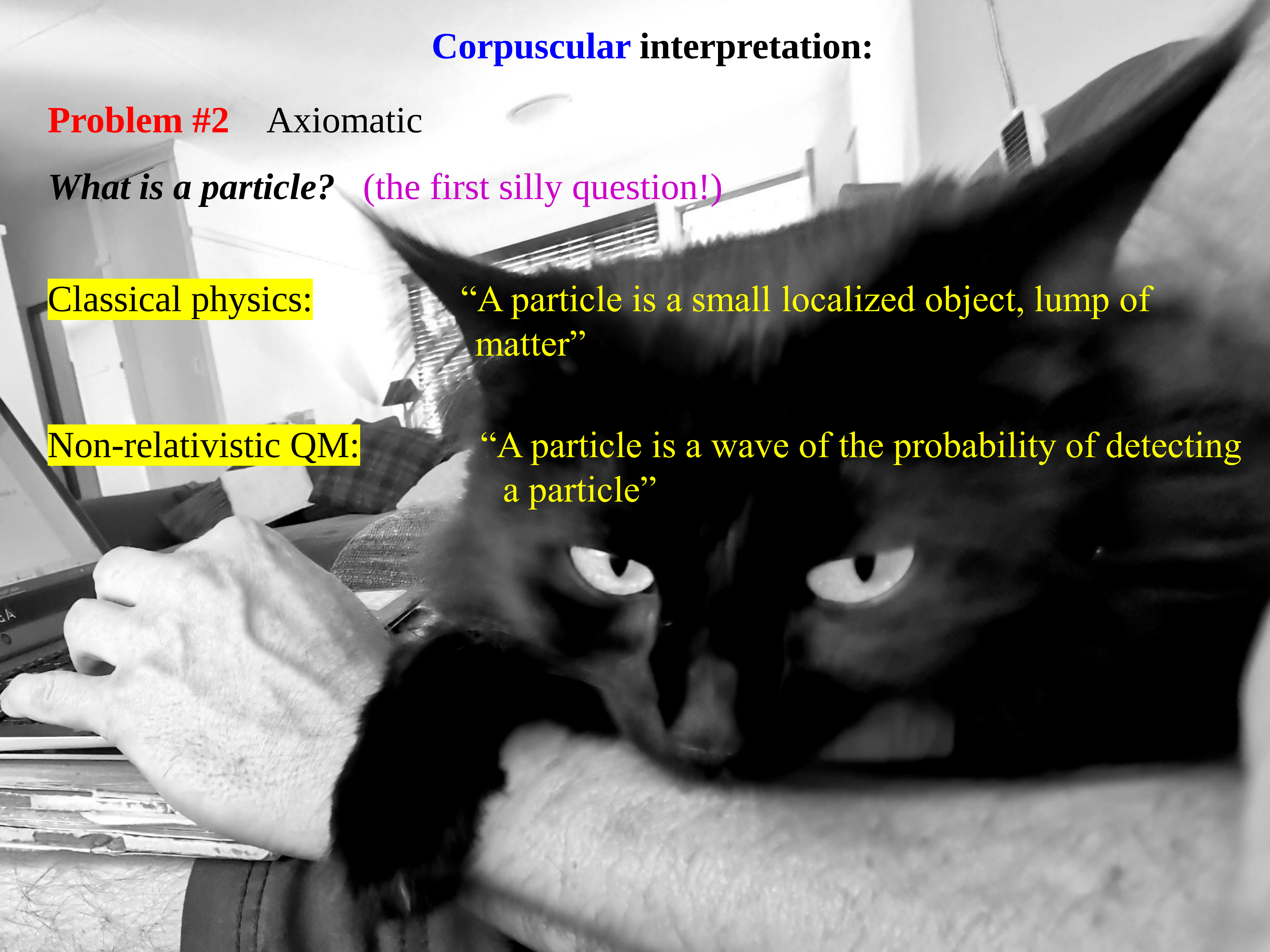
Corpuscular interpretation:

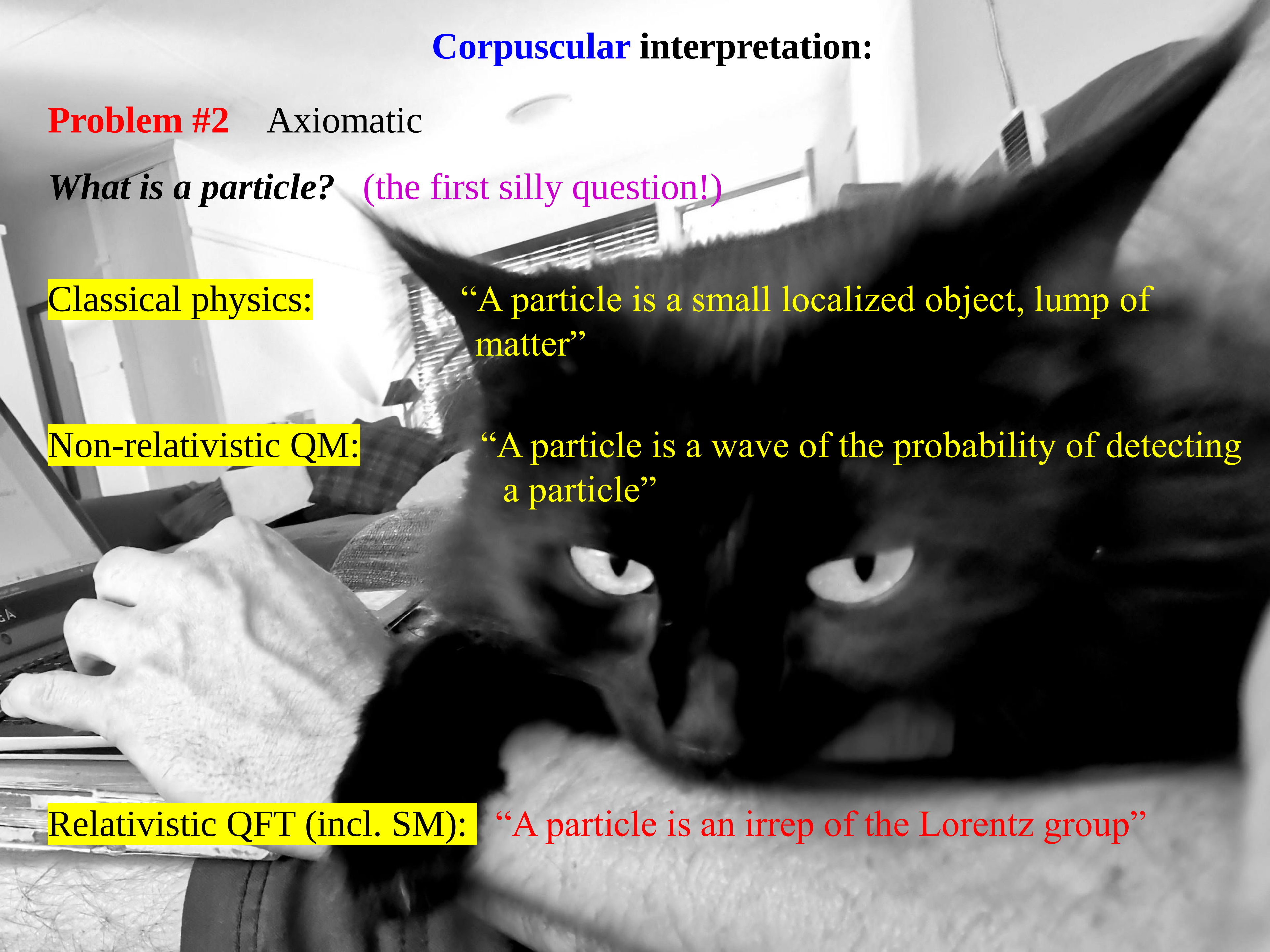
Problem #2 Axiomatic

What is a particle? (the first silly question!)

Classical physics: "A particle is a small localized object, lump of matter"

Non-relativistic QM: "A particle is a wave of the probability of detecting a particle"





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Classical physics: "A particle is a small localized object, lump of matter"

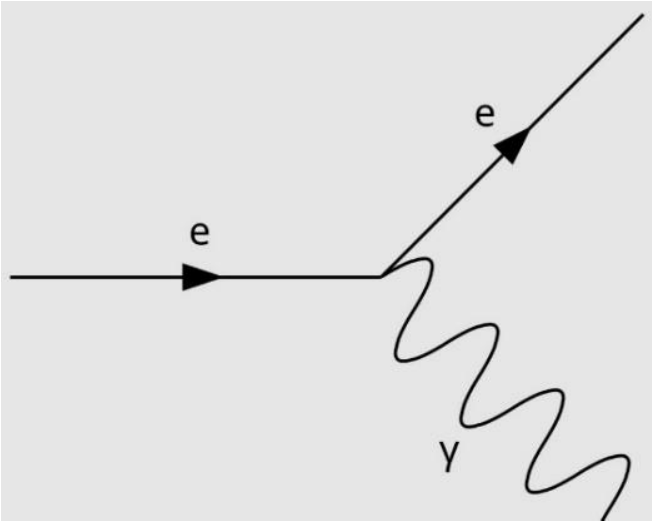
Non-relativistic QM: "A particle is a wave of the probability of detecting a particle"

Relativistic QFT (incl. SM): "A particle is an irrep of the Lorentz group"

Corpuscular interpretation:

Problem #2 Axiomatic

Relativistic QFT's workflow: we introduce quantum field, write the Lagrangian (using Lorentz symmetry and its irreps), define the perturbation theory & Feynman diagram rules $\Longrightarrow$ here comes particle interpretation



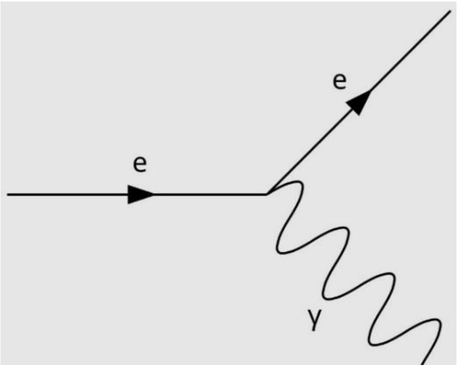
Corpuscular interpretation:

Problem #2 Axiomatic

Relativistic QFT's workflow: we introduce quantum field, write the Lagrangian (using Lorentz symmetry and its irreps), define the perturbation theory & Feynman diagram rules $\Rightarrow$ particle interpretation

(… *this is where fundamental problems begin*)

For example, the diagram "an electron emits a photon":



- one elementary point particle (by definition!) instantaneously produces another particle (*SQ*: where was the photon *before* the emission event)
- spin & statistics: $\frac{1}{2} \rightarrow \frac{1}{2} + 1$, F $\rightarrow$ F + B, EPR paradoxes
- zoom-in issue (UV divergencies): infinite values occur in higher-order perturbation terms (in some theories, renormalization procedures combine and replace infinities with experimentally known numbers)

In relativistic particle theory, the notion of a *particle* is mathematically ill-defined, physically obscure and computationally incomplete

Modified gravity interpretation:

Problem #1 Axiomatic (lots of SQ's)

What is the (gravitational) interaction, how do massive bodies "talk" to each other? Does the general relativity (GR) complete the theory of gravity?

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For starters, GR is not a quantum theory. Can we quantize GR?

Formally, yes (metric tensor as a dynamical variable, Hamiltonian form of EFE (ADM), LQG quantization or quantization with constraints *a la* Dirac)

…but

Such quantized GR is not even renormalizable, therefore infinite values of observables (e.g., graviton-graviton scattering cross-section etc.).

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Moreover, if you think about it:

Spacetime is, by definition, a set of "rods" and clocks for measuring distances and times between events $\Rightarrow$ S. is a part of measuring apparatus

Modified gravity interpretation:

Problem #2 Useless multeity

- Tons of mutually excluding relativistic MG theories
- MG theories solve one or two particular problem but not all
- MG theories don’t have quantum origins (or even any justifications at all, other than “mathematical beauty”)
- MG theories inherit fundamental issues of GR: quantization, non-conserved EMT, self-energy divergences, spacetime singularities

When it comes to the completely relativistic approach to the DM problem, neither particle nor MG pictures are flawless.

Is there a more general theory, which would also include GR as a subset?

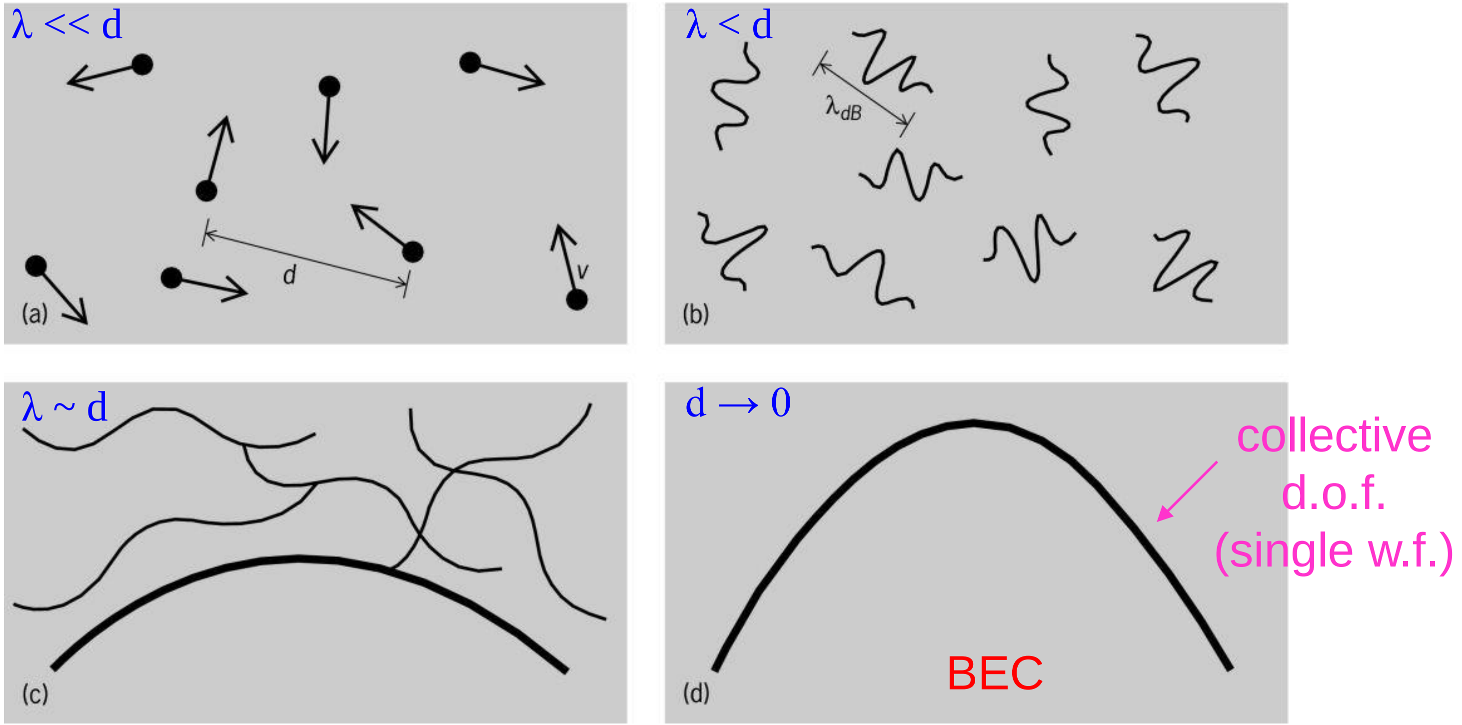
II. Quantum Liquids: Introduction

Q: What is quantum Bose liquid?

A: It is a quantum fluid that consists of the particles that obey the quantum Bose-Einstein statistics (not Fermi–Dirac, not Maxwell–Boltzmann); these particle are called bosons;
spin-statistics theorem: bosons must have integer spin;
examples: BEC, superfluids, etc.

Example 1: **Bose-Einstein condensate**

QM: same-species Bose particles are indistinguishable; particle-wave duality



BEC is an extended continuous quantum object (not anymore a cloud of particles)

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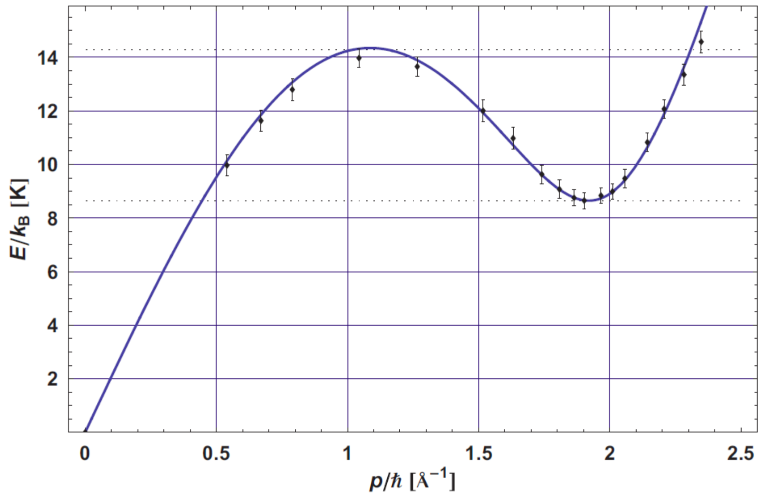
Example 2: **Superfluid**

Superfluid is a quantum Bose liquid in which dissipative fluctuations are suppressed

no dissipations $\Rightarrow$ no friction/drag force $\Rightarrow$ macroscopically behaves like a perfect or ideal fluid



Landau shape for the excitations' spectrum is a criterion

Modern theory of quantum Bose liquids

(BEC and superfluids)

$$\left[-i\hbar\,\partial_t - \frac{\hbar^2}{2m}\vec{\nabla}^2 + V_{\text{ext}}(\vec{x},\,t) + F(|\Psi|^2)\right]\Psi = 0 \qquad \langle\Psi|\Psi\rangle = N$$

where

$$F(\rho) = b\ln(\rho/\bar{\rho}) + \kappa_1\frac{\rho}{\bar{\rho}} + \kappa_2\left(\frac{\rho}{\bar{\rho}}\right)^2 \qquad \rho = n = |\Psi|^2$$

logarithmic term (vacuum?)

Gross-Pitaevskii term (2-body)

Ginzburg-Sobyanin term (3-body)

Advantages of including the logarithmic term:

- ✓ Supported by statistical mechanics (strong interaction: when $K/U \ll 1$)
- ✓ Non-perturbative
- ✓ Takes into account vacuum effects
- ✓ Fits experimental data (+ resolves some puzzles)

08:09

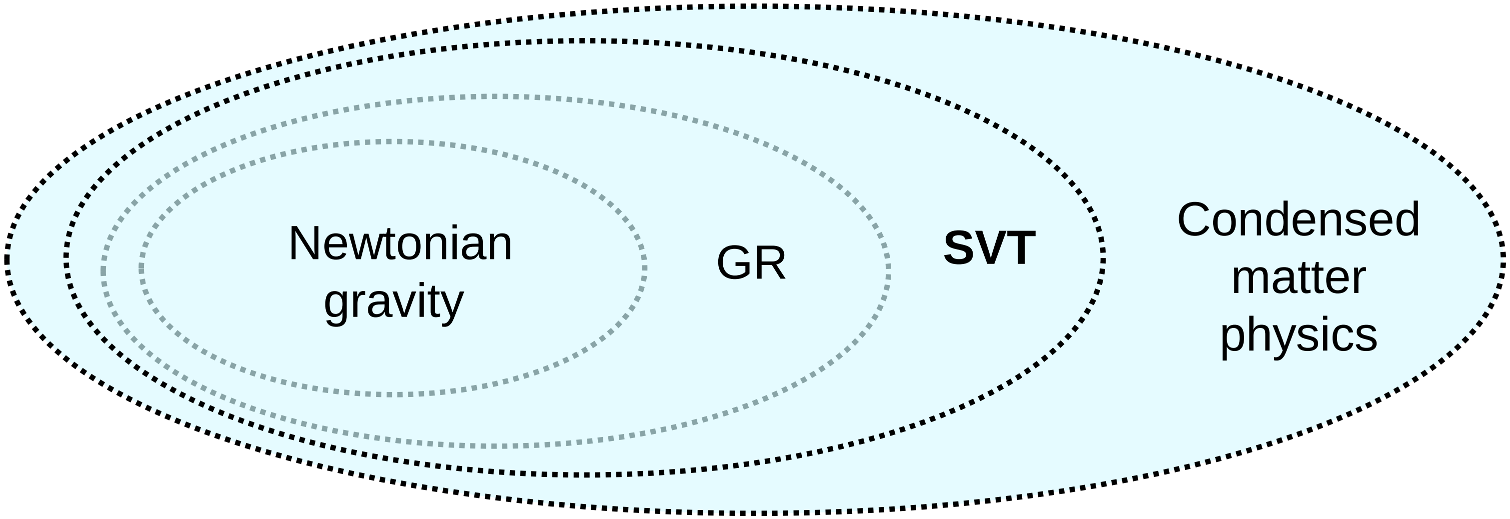
III. Superfluid Vacuum Theory

Superfluid vacuum theory (SVT) is a theory of physical vacuum, where vacuum is the superfluid in 3D Euclidean space, which generates 4D curved spacetime and associated phenomena

SVT is a viable candidate for a post-relativistic theory, because:

- it is quantum by construction (no anomalies or divergencies found)
- it generalizes GR
- it is a part of physics we already know (condensed matter)

SVT is also a framework for models of quantum gravity



The core of SVT:

Superfluid-spacetime correspondence

Propagation of small perturbations inside a 3D non-relativistic inviscid quantum fluid (superfluid), described by logarithmic SE for wavefunction $\Psi = \sqrt{\rho}\exp(i\mathcal{S})$

is analogous to:

propagation of probe particles along geodesics of a 4D pseudo-Riemannian spacetime, whose (- + + +) metric is given by:

$$g_{\mu\nu} = (g^{\mu\nu})^{-1} \propto \frac{\rho}{\sqrt{A_b}} \begin{pmatrix} -N^2 & \vdots & -\mathcal{D}\partial_j\mathcal{S} \\ \cdots & \cdot & \cdots \\ -\mathcal{D}\partial_i\mathcal{S} & \vdots & \delta_{ij} \end{pmatrix}$$

where

$$N^2 \equiv A_b - \mathcal{D}^2(\boldsymbol{\nabla}\mathcal{S})^2 = \frac{\mathcal{D}}{2}\left(\partial_t\mathcal{S} + \frac{1}{\hbar}V\right) - \frac{\mathcal{D}}{\hbar}b\left[1 + \frac{1}{2}\ln(\rho/\rho_0)\right] - \frac{\mathcal{D}^2}{4}\frac{\Delta\rho}{\rho}$$

$$A_b = A - \frac{\mathcal{D}}{\hbar}b, \;\; A = \frac{\mathcal{D}^2}{4}\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}\rho}{\rho}\right), \;\; B = \frac{\mathcal{D}^2}{4}\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}\delta\rho}{\rho}\right),$$

Einstein Field Eqs: Valid in the "phononic" (low-momenta) limit of SVT

$$T_{\mu\nu}^{(\mathrm{ind})} \equiv \kappa^{-1}\left[R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g)\right]$$

They must be interpreted not as the diff equations for the unknown metric, but rather as an expression for the induced stress-energy tensor of the effective matter to which the small fluctuations and test particles couple:

$$\text{SF dynamics} \xrightarrow{\textit{LogSE}} \Psi \xrightarrow{\textit{SF-spacetime correspondence}} g_{\mu\nu} \rightarrow R_{\mu\nu} \xrightarrow{\textit{EFE}} T_{\mu\nu}$$

In practice: you know EMT and want to derive Ψ, hence the reverse order; the reversed SF-spacetime correspondence fulfils the quantization of gravity

Thus, in SVT, spacetime is an emergent phenomenon, EFE and related concepts represent an approximate description;
empty space's symmetries are no longer relevant (we don't have empty space)

Two types of observers:

- R(elativistic) O. operates with small vacuum excitations only, and "sees" only spacetime $g_{\mu\nu}$ and matter $T_{\mu\nu}^{(\mathrm{ind})} \equiv \kappa^{-1}\left[R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g)\right]$
- F(ull) O. observes a quantum SF in Euclidean space

Derivation of gravitational potential and speed of light

(*skip tech details*)

GR: in the weak gravity limit, a lapse function can be written in the form

$$N^2 = c_{(0)}^2 + 2\Phi$$

therefore, we can associate:

$$\Phi \sim -\frac{b}{4m} \ln\left(|\Psi|^2/\rho_0\right) - \frac{[b]}{2m}$$

gravitational potential

$$c_{(0)}^2 \sim \frac{\mathcal{D}\,\omega}{2} - \frac{b_0}{2m} = \frac{1}{2m}(\hbar\omega - b_0)$$

speed of light (squared)

It is worth summarizing the built-in ambiguities: (i) the physical metric is derived up to a conformal factor, due to the remaining choice of a physical frame (units), (ii) for a given conformal frame, values Φ and $c_{(0)}$ are defined up to a factor, due to time coordinate transformation, (iii) the coupling b is defined up to a factor, due to the $U(1)$ symmetry of the original wave equation, metric signature choice and coordinate transformations, and (iv) values Φ and $c_{(0)}^2$ are defined up to, respectively, additive function and additive constant, due to some terms being neglected because of various small density perturbation approximations.

$$ds^2 \approx -c_{(0)}^2\left[1+\frac{2\Phi(r)}{c_{(0)}^2}\right]dt^2+\frac{dr^2}{1+2\Phi(r)/c_{(0)}^2}+R^2(r)d\Omega_{(2)}^2$$

$$\Phi = -\frac{1}{m}V_{\text{eff}}(\boldsymbol{r},\, t) = \frac{1}{m}\left(b_0 - \frac{q}{r^2}\right)\ln\left(\frac{|\Psi_{\text{vac}}(\boldsymbol{r},\, t)|^2}{\bar{\rho}}\right)$$

Generic ansatz for the amplitude of SF's wavefunction

$$|\Psi_{\text{vac}}| = \sqrt{\bar{\rho}}\left(\frac{r}{\bar{\ell}}\right)^{\chi_0/2} P(r)\, \exp\left(-\frac{a_2}{2\bar{\ell}^2}r^2 + \frac{a_1}{2\bar{\ell}}r + \frac{a_0}{2}\right)$$

Or (when approximating $P(r)$):

$$|\Psi_{\text{vac}}|^2 \approx \bar{\rho}\, \exp\left[-\frac{a_2}{\bar{\ell}^2}r^2 + \frac{a_1}{\bar{\ell}}r + \chi\ln\left(\frac{r}{\bar{\ell}}\right) + a_0\right]$$

where a's are some constants, and $\bar{\ell} = (m/\bar{\rho})^{1/3}$ or $\hbar/\sqrt{m b_0}$ is a characteristic length scale chosen.

If an exact solution is known (req. boundary conditions, state's specification, etc), then the a-constants are known exactly.
Our case: they are not known theoretically but can be bound empirically.

Substituting this ansatz into the expression of induced potential, we obtain the latter as a sum of seven terms:

$$\begin{aligned}\Phi(r) &= \Phi_{\text{smi}}(r) + \Phi_{\text{RN}}(r) + \Phi_{\text{N}}(r) \\ &\quad + \Phi_{\text{gal}}(r) + \Phi_{\text{mgl}}(r) + \Phi_{\text{dS}}(r) + \Phi_0\end{aligned}$$

where…

$$ds^2 \approx -c_{(0)}^2\left[1+\frac{2\Phi(r)}{c_{(0)}^2}\right]dt^2+\frac{dr^2}{1+2\Phi(r)/c_{(0)}^2}+R^2(r)d\Omega_{(2)}^2$$

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Induced gravitational potential has a multi-scale structure:

$$\Phi(r) = \Phi_{\rm smi}(r)+\Phi_{\rm RN}(r)+\Phi_{\rm N}(r) + \Phi_{\rm gal}(r)+\Phi_{\rm mgl}(r)+\Phi_{\rm dS}(r)+\Phi_0$$

where

KNOWN / PREDICTED

$$\Phi_{\rm smi}(r) = -\frac{\chi\, q}{m}\frac{\ln(r/\bar\ell)}{r^2} = -\zeta_{\chi q}c_b^2\frac{L_{\rm smi}^2\ln(r/\bar\ell)}{r^2}$$
← Strong Gravity (*hierarchy problem resolved*)

$$\Phi_{\rm RN}(r) = -\frac{a_0\, q}{m}\frac{1}{r^2} = -\zeta_{a_0 q}c_b^2\frac{L_{\rm RN}^2}{r^2},$$
← Reissner-Nordstrom (*abelian charge generation*)

$$\Phi_{\rm N}(r) = -\frac{a_1 q}{m\bar\ell}\frac{1}{r} = -\frac{GM}{r},$$
← Newton / Schwarzschild (*grav mass generation*)

$$\Phi_{\rm gal}(r) = \frac{\chi\, b_0}{m}\ln(r/\bar\ell) = c_b^2\,\chi\ln(r/\bar\ell),$$
← Galactic scale (*flat rotation curves, acts as DM*)

$$\Phi_{\rm mgl}(r) = \frac{a_1 b_0}{m\bar\ell}r = \zeta_{a_1}c_b^2\frac{r}{L_{\rm mgl}},$$
← Large/Extra Galactic scale (*crossover to non-flat RCs, contributes to expansion*)

$$\Phi_{\rm dS}(r) = -\frac{a_2 b_0}{m\bar\ell^2}r^2 = -c_b^2\frac{r^2}{L_{\rm dS}^2},$$
← de Sitter (*accelerating expansion, acts as DE*)

$$\Phi_0 = \frac{1}{m}\left(a_0 b_0+\frac{a_2 q}{\bar\ell^2}\right)$$
← Gauge term?

where

$$c_b = \sqrt{\frac{b_0}{m}},\quad GM = \frac{a_1 q}{m\bar\ell},\quad L_{\rm smi} = \sqrt{\frac{|\chi\, q|}{b_0}},\quad L_{\rm mic} = \sqrt{\frac{|a_0 q|}{b_0}},\quad L_{\rm mgl} = \frac{\bar\ell}{|a_1|} = \frac{|q|}{mGM},\quad L_{\rm cos} = \frac{\bar\ell}{\sqrt{a_2}},\quad L_\chi = \frac{\chi\bar\ell}{a_1} = \frac{\chi q}{mGM}$$

IV. Example: Alternative to DM

Galactic rotation curves

Rotation curves (RC) are rotation velocities as functions of a distance from a gravitating center.

According to the Newton's theory of gravity, rotation curves of free-falling non-relativistic test particles must have a Keplerian form <=> velocity must be inversely proportional to the square root of distance.

However, numerous astronomical observations, of both stars and luminous gas in galaxies, show significant deviations from this behaviour:

- RC's are Keplerian-like (Newton-driven) in inner regions of galaxies
- RC's are essentially non-Keplerian in the middle and mid-outer regions (sometimes having flat shape, but not necessarily so)

Is this non-Keplerian behaviour due to DM or SVT-induced multi-scale gravity?
If latter, then what does SVT predict for galactic RC?

$$v(R) \approx \sqrt{v_{\rm N}^2 + \frac{b_0}{m} R \frac{d}{dR} \left\{ \chi \ln\left(\frac{R}{\bar{\ell}}\right) + \ln\left[\left(k_2 R^2 + k_1 R + 1\right)^2 \right] \right\} + \tilde{a}_1 R - \tilde{a}_2 R^2}$$

Flat regime | Approx. Flat | Linear | Quadratic

where

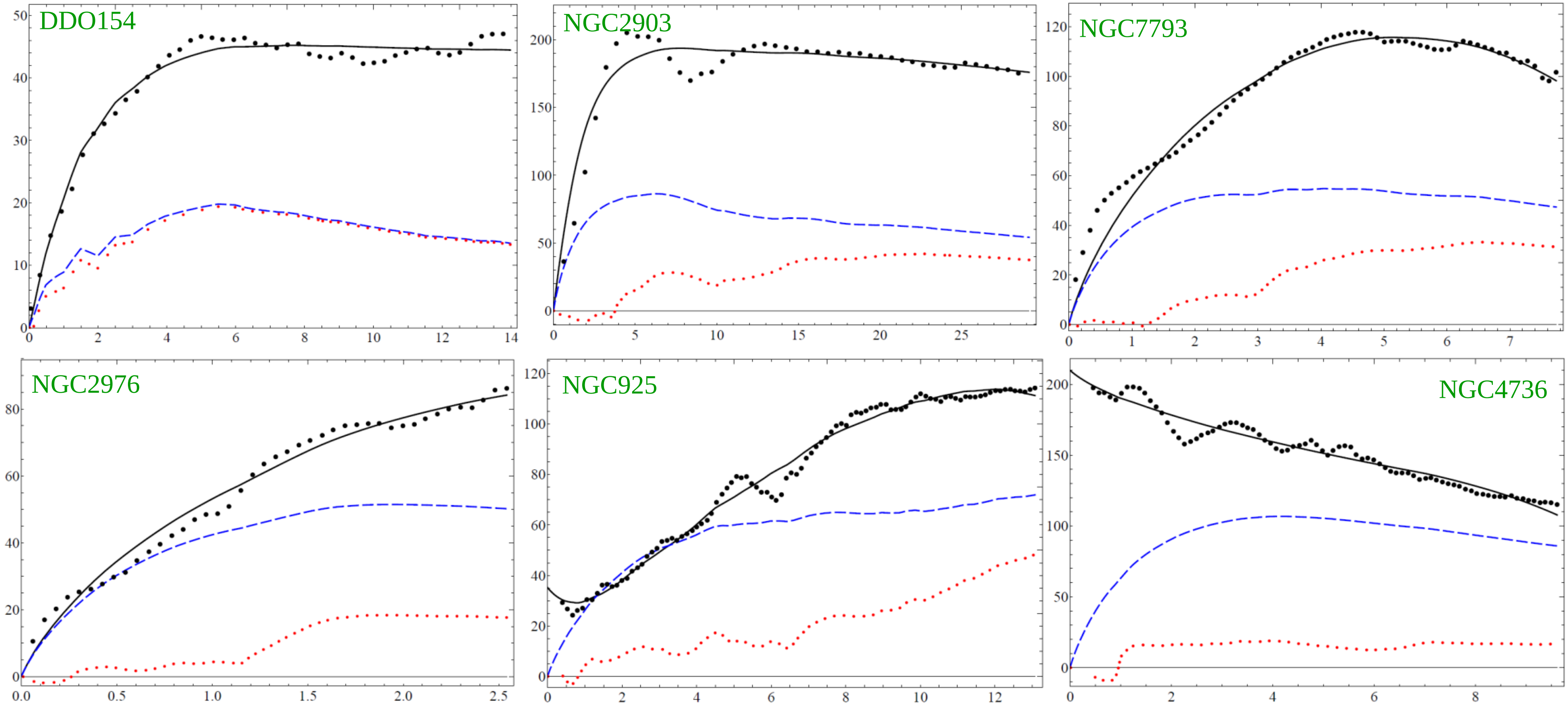
$$v_{\rm N}^2 = \frac{4}{3} v_{\rm HI}^2 + v_\star^2 = \frac{4}{3} v_{\rm HI}^2 + \frac{G M_\star}{2 h_R^3} R^2 B\left(\frac{R}{2h_R}\right),$$

$$v_\chi^2 = \frac{1}{2} R \frac{d}{dR} \left[c_{(0)}^2 \beta_\chi^2 \ln\left(\frac{R}{\bar{\ell}}\right) \right] = \frac{\chi b_0}{m},$$

where $M_\star$ and h_R are, respectively, the total gravitational mass and surface brightness' scale length of the stellar disc, $B(x) = I_0(x)K_0(x) - I_1(x)K_1(x)$, $I_n(x)$ and $K_n(x)$ are modified Bessel functions of the first and second kind, respectively, and the mass ratio between helium and neutral hydrogen (HI) is assumed to be $1/3$. Notice that the contribution (4) is independent of R, which proves our earlier statement about the relation between the FRC phenomenon and logarithmic term in the induced metric. Thus, a rotation curve would be asymptotically flat if $a_1 = a_2 = 0$ identically, while the Keplerian term $v_{\rm N}^2$ rapidly decreases as R grows, thus making the term (4) to predominate in Eq. (2) at large R. However, if either or both of a_1 and a_2 are not zero then we have what we call the *asymptotically non-flat behaviour* of a rotation curve.

Fitted 15 galaxies from the THINGS catalogue. Here's some examples:

Velocity [km /s] vs. distance [kpc]



Least-squares fits (**solid** curves), black **dots** are mean values from THINGS, dotted curves are contribution of gas, dashed curves are total contribution of gas & stellar disc

Notice: SVT fits not only flat-type galaxies with but also other non-Keplerian ones

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