The Phenomenology of Rare $b \to s\ell^+\ell^-$ Decays

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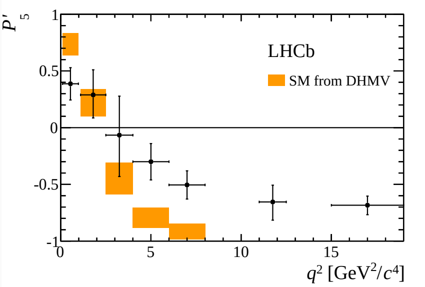
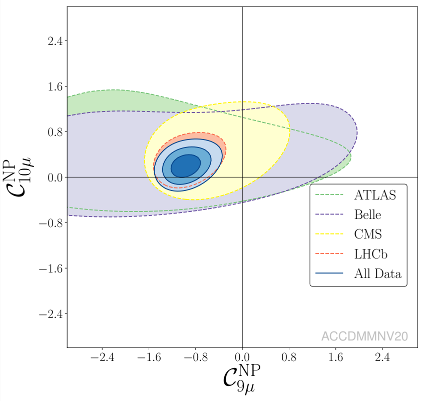
Institute for Particle Physics Phenomenology, Durham

Seminar, University of Birmingham, Oct 30th 2024

Motivation





- deviations between measurements and Standard Model (SM) predictions requires careful interpretation

Possible Explanations

1. QED: mismatch between predictions and measurements, particularly in differential observables
   - unlikely explanation [Isidori/Nabeebaccus/Zwicky 2009.00929]
   - "dangerous hard-collinear logarithms cancel at the differential level in the currently used experimental treatment"
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   - quantify potential hadronic and BSM effects (within the Weak Effective Theory)
   - topic of this presentation

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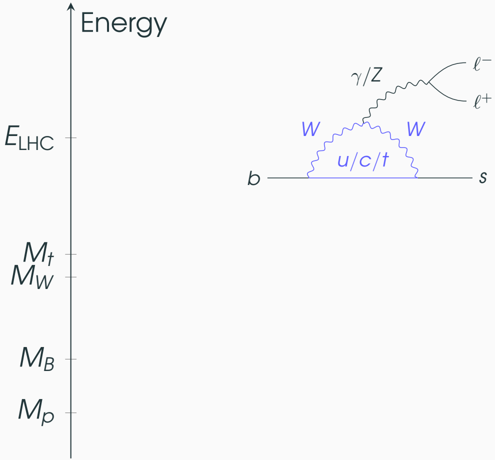
2. QCD: lack of understanding of the Standard Model long-distance dynamics, which mimic beyond the Standard Model (BSM) effects
   - quantify potential hadronic and BSM effects (within the Weak Effective Theory)
   - topic of this presentation
3. BSM: genuine BSM effects in the data?
   - interpret potential BSM effects qualitatively
   - task for model builders (i.e.: not me!)

Interpretation within the Weak Effective Theory

Weak Effective Theory

- widely used tool of theoretical physics
- used to predict observable in SM and interpret measurements w/o assuming a concrete model beyond SM



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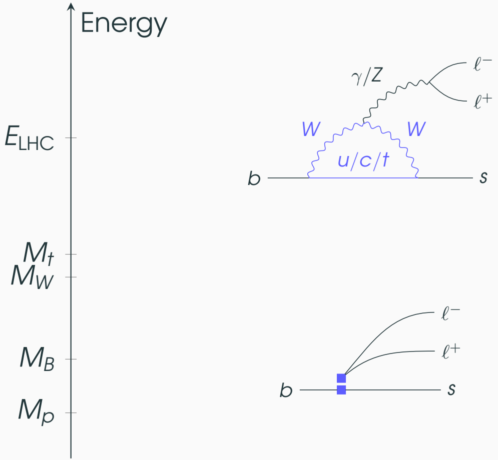

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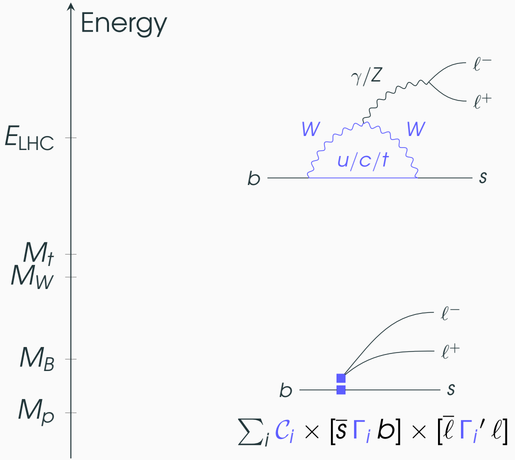
- local operators must respect remaining $\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{SU}(3)_{\mathrm{C}}$ symmetry



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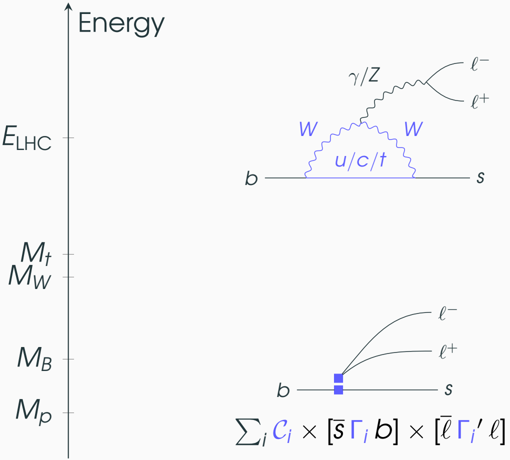
- local operators must respect remaining $\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{SU}(3)_{\mathrm{C}}$ symmetry
- for $b \to s\ell\ell$ we find in general
    - 10 semileptonic $[\bar{s}\Gamma b]\,[\bar{\ell}\Gamma' \ell]$ ops
    - 20 four-quark $[\bar{s}\Gamma b]\,[\bar{c}\Gamma' c]$ ops
    - ...



- in the SM, only the following set of $D = 6$ effective operators contributes:

$$\mathcal{L}_{\text{SM}}^{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_{i=3}^{10} \mathcal{C}_i\, \mathcal{O}_i + \lambda_c \sum_{j=1}^{2} \mathcal{C}_j{}^c\, \mathcal{O}_j{}^c + \lambda_u \sum_{k=1}^{2} \mathcal{C}_k{}^u\, \mathcal{O}_k{}^u\right] \quad \text{with } \lambda_q \equiv V_{qb}V_{qs}^*$$

radiative

$$\mathcal{O}_{7(\prime)} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_{R(L)} b) F_{\mu\nu} \qquad \mathcal{O}_{8(\prime)} = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_{R(L)} T^A b) G^A_{\mu\nu}$$

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four-quark QCD penguins

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- SM contributions to $\mathcal{C}_i(\mu_b)$ known to high accuracy (NNLL)

[Bobeth,Misiak,Urban '99; Misiak,Steinhauser '04, Gorbahn,Haisch '04]
[Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

Tangent 1: Renormalization Group Equations (RGE)

- Wilson coefficients $\mathcal{C}_i$ can be computed in perturbation theory at some high energy scale $\mu_0 \sim M_W \gg m_b$
- however, matrix elements of operators are evaluated (e.g. from lattice QCD) at some low energy scale $\Lambda_{\text{had}} < \mu_1 < m_b$
- mismatch must be resolved to obtain reliable predictions
- Renormalization Group Equations (RGEs) provide means to *evolve* both the Wilson coefficients and the matrix elements from their respective intrinsic scales to one common scale
  $\Rightarrow$ RGE-improved perturbation theory

Tangent 1: Renormalization Group Equations (RGE)

- RGE for multiplicatively-renormalizing quantities:

$$\mu \frac{d}{d\mu}\mathcal{C}(\mu) = \gamma(\alpha_s(\mu))\mathcal{C}(\mu) \qquad \mu \frac{d}{d\mu}\alpha_s(\mu) = 2\beta(\alpha_s(\mu))$$

$$\gamma = \gamma^{(0)}\frac{\alpha_s}{4\pi} + \mathcal{O}\left(\alpha_s^2\right) \qquad \beta = \beta^{(0)}\left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}\left(\alpha_s^3\right)$$

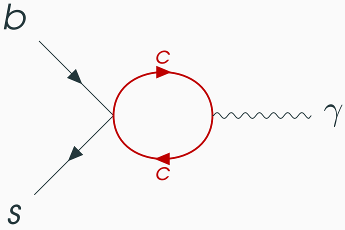
Solution

$$\mathcal{C}(\mu_1) = \underbrace{\mathcal{C}(\mu_0)\left[\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right]^{\left(\frac{\gamma^{(0)}}{2\beta^{(0)}}\right)}}_{\text{LL}} + \underbrace{\mathcal{O}\left(\alpha_s^{n+1}(\mu_0)\ln^n\left(\frac{\mu_1}{\mu_0}\right)\right)}_{\text{NLL}}$$

(*): resums all leading-logarithmic (LL) terms $\alpha_s^n(\mu_0)\ln^n\left(\frac{\mu_1}{\mu_0}\right)$ via

$$\left[\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right]^{\left(\frac{\gamma^{(0)}}{2\beta^{(0)}}\right)} = 1 - \gamma^{(0)}\alpha_s(\mu_0)\ln\left(\frac{\mu_1}{\mu_0}\right) + \mathcal{O}\left(\alpha_s(\mu_0)^2\ln^2\left(\frac{\mu_1}{\mu_0}\right)\right)$$

- *sbcc* 4-quark operators yield UV divergence
  - must be renormalized
  - require $sb\ell\ell$ / $sb\gamma$ counterterm (C_9 / C_7)
- SM operator basis renormalizes multiplicatively
  - γ is promoted to a matrix γ_{ij}
  - operators mix under RGE
- phenomenologically important
  - SM *sbcc* operators contribute $\sim$ 50% of $\mathcal{C}_9^{\text{SM}}(\mu_b)$ at NNLL



Weak Effective Theory: $b \to s\ell\ell$ BSM Operators

- in the presence of NP effects

$$\mathcal{L}^{\text{eff}}_{\text{BSM}} = \mathcal{L}^{\text{eff}}_{\text{SM}} + \frac{4G_F}{\sqrt{2}} \left[\lambda_t \sum_i \mathcal{C}_i \, \mathcal{O}_i \right]$$

semileptonic

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$$\mathcal{O}_S = \frac{\alpha}{4\pi} (\bar{s}P_R b)(\bar{\ell}\ell) \qquad \mathcal{O}_{S'} = \frac{\alpha}{4\pi} (\bar{s}P_L b)(\bar{\ell}\ell)$$

$$\mathcal{O}_P = \frac{\alpha}{4\pi} (\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell) \qquad \mathcal{O}_{P'} = \frac{\alpha}{4\pi} (\bar{s}P_L b)(\bar{\ell}\gamma_5 \ell)$$

$$\mathcal{O}_T = \frac{\alpha}{4\pi} (\bar{s}\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu} \ell) \qquad \mathcal{O}_{T5} = \frac{\alpha}{4\pi} (\bar{s}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5 \ell)$$

- regularly considered in the literature!

Weak Effective Theory: $b \to s\ell\ell$ BSM Operators

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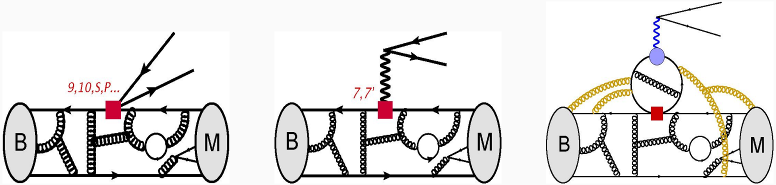
  - add further 2×18 operators with $q = c, u$
  - add further "QCD-penguin" operators with $q = d, s, b$
  - these operators are routinely ignored in the literature! [except by Jäger,Kirk,Lenz,Leslie '17]
  - for a truly model-independent analysis of data, would need to fit coefficients of all operators!

Weak Effective Theory: Summary

- WET makes calculations in the SM possible in the first place
  - separates long-distance ($[\bar{s}\Gamma b][\ldots]$) physics from short-distance physics ($\mathcal{C}$)
- "divides and conquers"
  - SM WET contributions under excellent theory control
  - precision of SM predictions hinges on accurate control of hadronic matrix elements
- accounts transparently and model-independently for the effects of physics beyond the SM
  - treats Wilson coefficients $\mathcal{C}$ as generalized couplings and fits them from data
  - provides an excellent interface to model builders

From the WET to the Observables

Anatomy of exclusive $b \to s\ell^+\ell^-$ decay amplitudes



$$\mathcal{A}_\lambda^X = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

nomenclature of the essential hadronic matrix elements $\quad q^2 = m_{\ell\ell}^2$

- $\mathcal{F}_\lambda$ local form factors of dimension-three $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu\gamma_5 b$ currents
- $\mathcal{F}_\lambda^T$ local dipole form factors of dimension-three $\bar{s}\sigma^{\mu\nu} b$ currents
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

all three needed for consistent description to leading-order in α_e

Local Form Factors

- local form factors are conceptually "easy"
  - yet a substantial source of uncertainties
- lattice QCD provides results typically at large q^2 for $B \to K$, $B \to K^*$, $B_s \to \phi$
  - caveat: K^* is broad state, non-zero width can have $\mathcal{O}(10\%)$ effects [Descotes-Genon,Khodjamirian,Virto '19]
  - new lattice results down to $q^2 = 0$ for $B \to K$ form factors [HPQCD '22]
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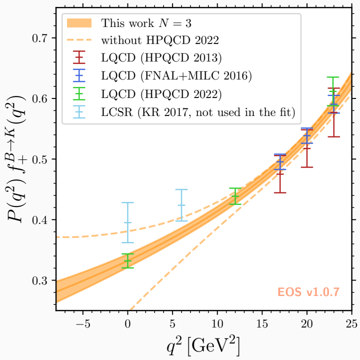
  - caveat: systematic uncertainties hard to quantify
- IPPP group recently revisited dispersive bounds for all local $b \to s$ form factors



[Gubernari,Reboud,DvD,Virto '23]

Tangent 3: Dispersive Bound

consider auxilliary quantity: moment of cross section $\chi = \int ds\, \omega(s) \sigma(e^+ e^- \to X_{b\bar{s}})$

exclusive picture

- moment of cross section is sum of positive-definite terms
- involves squares of $\bar{B}K$, $\bar{B}K^*$ form factors $\chi \sim \int dq^2 |F(q^2)|^2 + \text{pos. terms}$

inclusive picture

- moment of cross section can be computed "perturbatively"

- focussing on one exclusive final state (e.g, $\bar{B}K$), pertubartive results for χ limits form factor parameter space
- using apt parametrization

$$F(q^2) = \frac{1}{\sqrt{\chi} \dots} \sum_n a_n z(q^2)^n$$

the bound takes the form $\sum_n |a_n|^2 < 1$

Local Form Factors: Summary of Recent Analysis

- global analysis finds good compatibility between LCSR and lattice QCD results
- dispersive bounds have been applied
  - reduce extrapolation error
  - turn hard-to-quantify systematic unc. into parametric unc.

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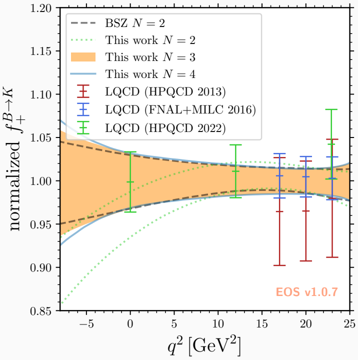
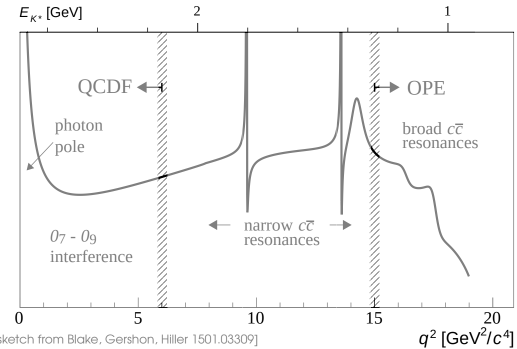
- commonly used BSZ parametrization surprisingly efficient
  - dispersive bound and BSZ very compatible for $q^2 \geq 0$, no need to swap params as of yet
  - theory will also require local form factors at $q^2 < 0$, where BSZ underestimates uncertainties



[Gubernari,Reboud,DvD,Virto '23]

Non-Local Form Factors: Spectrum

$$\mathcal{H}_\lambda = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{em}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

source of dominant systematic uncertainties in theoretical predictions!
perturbative treatment does not reflect hadronic spectrum!

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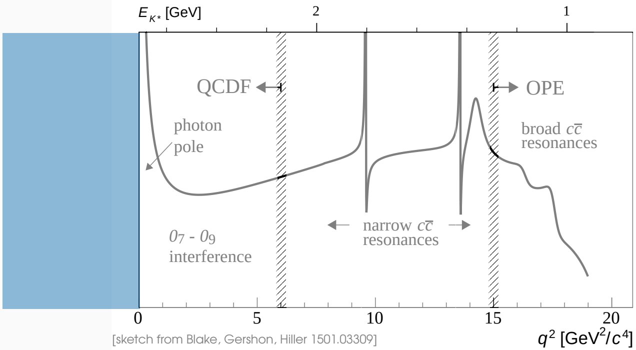

[sketch from Blake, Gershon, Hiller 1501.03309]

- $O_{1,2}^c \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

- leading contributions expressed through local form factors $\mathcal{F}_\lambda$
- correction suppressed by $1/(q^2 - 4m_c^2)$ can by systematically obtained

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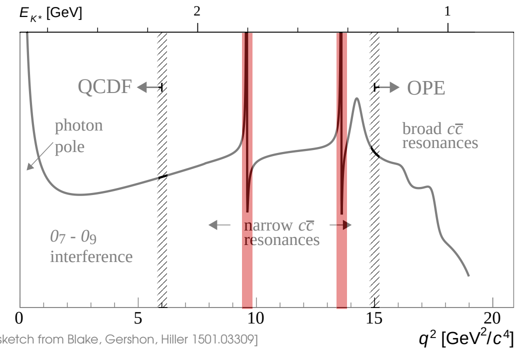

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- $O^c_{1,2} \sim [\bar{s}\Gamma b]\,[\bar{c}\Gamma' c]$

- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by $B \to K^*\psi(\to \mu^+\mu^-)$ decays
- experimental measurements provide additional information about $\mathcal{H}_\lambda$

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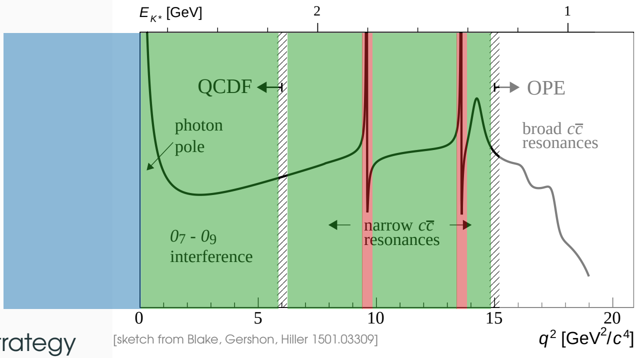

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[Bobeth,Chrzaszcz,DvD,Virto '17]

new strategy

- compute $\mathcal{H}_\lambda$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$ using suitable parametrization
- include information from hadronic decays to narrow charmonia J/ψ and $\psi(2S)$

Tangent 4: QCD Factorization (QCDF)

- the literature frequently discusses "the QCDF" approach to the non-local form factors [Beneke,Feldmann,Seidel '01&'04]
  - more correctly labelled: 1-loop, perturbative approach to non-local form factors

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  - QCDF is not "dealing" with the charm loop contributions; it is agnostic of their treatment
- slightly more technical
  - QCDF is used to express exclusive form factors for small q^2 in terms of nonlocal B and $K^{(*)}$ matrix elements (LCDAs)
  - this calculation encounters universal divergences $\Rightarrow$ not predictive for an individual form factor
  - universal divergences cancel in ratios

Preparing $b \to s\ell\ell$ predictions for the era of the High-Luminosity LHC

Reduce systematical theory uncertainties

- check previous computations of the nonlocal form factors at subleading power ✓

  [Gubernari,DvD,Virto '20]

  - previous results incomplete, missing terms cancel known contributions
  - subleading-power terms are negligible at spacelike q^2

- improve the parametrization to control the extrapolation error ✓

  [Gubernari,DvD,Virto '20; Gubernari,Reboud,Virto '22; Gubernari,Reboud,Virto '23]

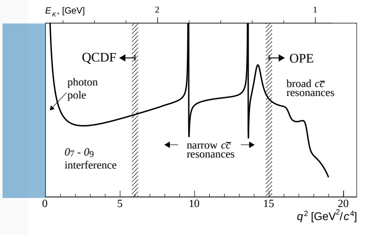
  - use dispersively-bounded parametrization for both local and non-local form factors

- challenge implicit theory assumptions in the nonlocal form factors
  - determine WET Wilson coefficients of $sbcc$ operators from data ongoing

  [Kirk,McPartland,Reboud,DvD,Virto]

Compute Light-Cone OPE

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$

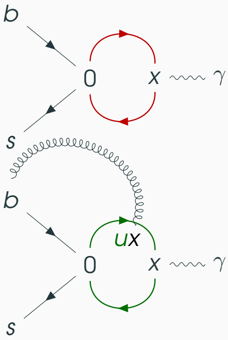


- expansion in operators at light-like distances $x^2 \simeq 0$

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

- employing light-cone expansion of charm propagator

[Balitsky, Braun 1989]



$$\int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\, J^\mu_{\text{em}}(x), [C_1 O_1^c + C_2 O_2^c](0)\,\}$$

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}}\, [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ \,(\text{coeff \#2}) \times [\,\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,]$$

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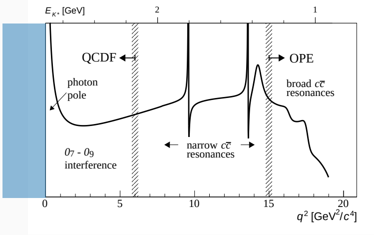

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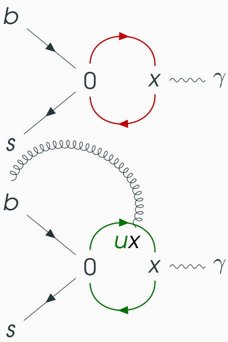

$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}} + \text{coeff \#2} \times \tilde{\mathcal{V}}$$

- leading part identical to QCD Fact. results

[Beneke, Feldmann, Seidel '01&'04]

- subleading coefficient computed previously

[Khodjamirian, Mannel, Pivovarov, Wang '10]

- we find full agreement, also cast result in convenient form

[Gubernari,Virto,DvD '20]

- next step: determine "subleading form factor" $\tilde{\mathcal{V}}$

Compute Soft gluon matrix elements

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\mathrm{GeV}^2)$	GvDV2020	KMPW2010
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\mathrm{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\mathrm{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\mathrm{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\mathrm{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\mathrm{GeV}$	—

reduction by a factor of ~ 200

- new structures in three-particle LCDAs account for factor 10 (due to cancellations!)
- updated inputs that enter the sum rules account for further factor 10
- similar relative uncertainties, but absolute uncertainties reduced by $\mathcal{O}(100)$

Compute Developments

- ongoing project at IPPP to compute leading non-local contributions for full BSM basis of *sbcc* operators
  - first step to full control of non-local form factors in the WET
  - ✓ one-loop calculation
  - w.i.p. two-loop calculation; working on reduction of master integrals
  - w.i.p. identifying observables that provide constraints on full basis (e.g., $\overline{B} \to KJ/\pi$ or B lifetime)

- ongoing project in Siegen to better classify non-local operators
  - of particular interest: contributions with hard-collinear gluon
  - relevant to “internal” charm loop

Extrapolate Parametrisations

- Taylor expand $\mathcal{H}_\lambda$ in q^2/M_B^2 around 0 [Ciuichini et al. '15]
  - + simple to use in a fit
  - – incomaptible with analyticity properties, does not reproduce resonances
  - – expansion coefficients unbounded!

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- use information from hadronic intermediate states in a dispersion relation [Khodjamirian et al. '10]

$$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(s_0) = \frac{q^2 - s_0}{\pi} \int ds \frac{\mathrm{Im}\,\mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \dots$$

  - + reproduces resonances
  - - hadronic information above the threshold must be modelled
  - - complicated to use in a fit, relies on theory input in single point s_0

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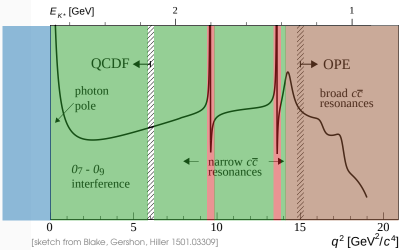
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point s_0
- expand the matrix elements in variable $z(q^2)$ that develops branch cut at $q^2 = 4M_D^2$

  [Bobeth,Chrzaszcz,DvD,Virto '17]

  - \+ resonances can be included through explicit poles (Blaschke factors)
  - \+ easy to use in a fit
  - \+ compatible with analyticitiy properties
  - \- expansion coefficients unbounded!

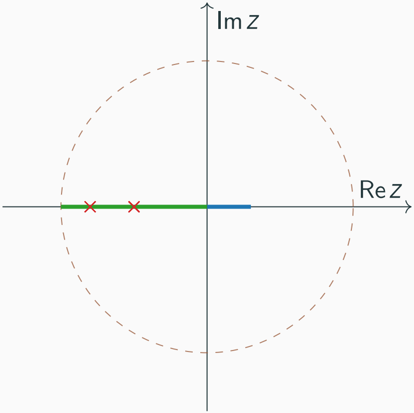
- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$

[Bobeth/Chrzaszcz/DvD/Virto '17]



[sketch from Blake, Gershon, Hiller 1501.03309]

- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]
  - branch cut is mapped onto unit circle in z
  - real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
  - data and theory live insides the unit circle



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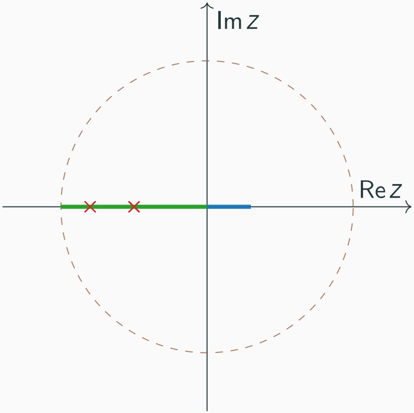
  - branch cut is mapped onto unit circle in z
  - real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
  - data and theory live insides the unit circle
- expand in z
  + resonances J/ψ, $\psi(2S)$ can be included (via explicit poles/Blaschke factors)
  + easy to use in a fit to theory and data
  + compatible with analyticity
  - expansion coefficients unbounded!



matrix elements $\mathcal{H}^{(\lambda)}$ arise from nonlocal operator [Gubernari,DvD,Virto '20]

$$\mathcal{H}^\mu \sim \langle K|O^\mu(Q;x)|B\rangle \qquad O^\mu(Q;x) \sim \int d^4y\, e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

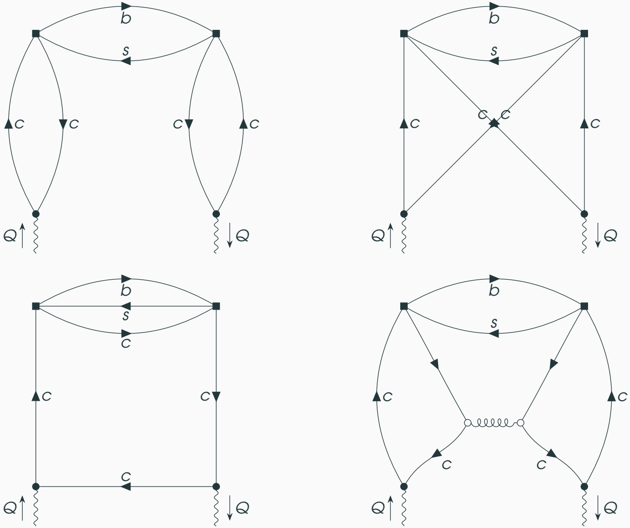
construct four-point operator to derive a dispersive bound

- define matrix element of “square” (i.e., hermitian) operator

$$\int d^4x\, e^{iQ\cdot x}\, \langle 0|\, T\{O^\mu(Q;x) O^{\dagger,\nu}(Q;0)\}\, |0\rangle \equiv \left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right] \Pi(Q^2)$$

- $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}, c\bar{c}c\bar{c}, \ldots$)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}, b\bar{s}g, b\bar{s}c\bar{c}, \ldots$)

Extrapolate Cuts of Π



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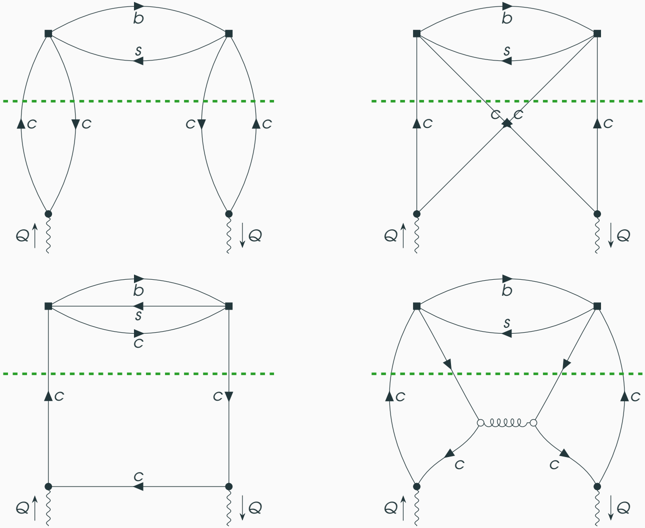
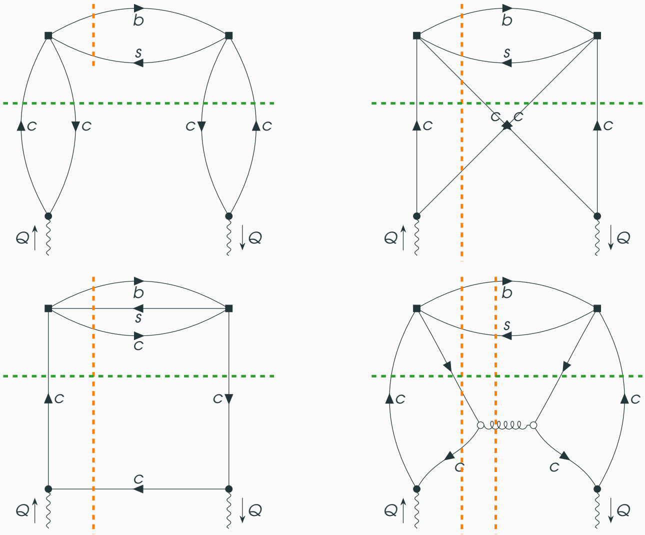
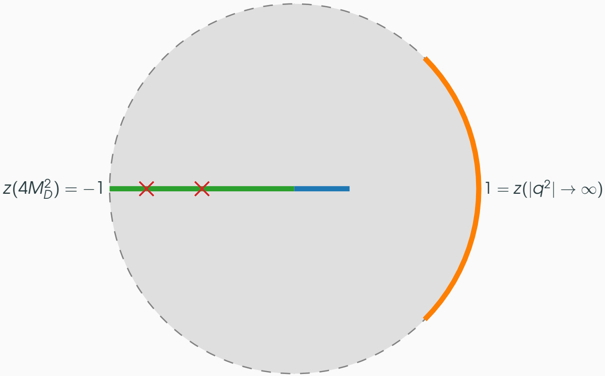


- unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)

Extrapolate Cuts of Π



- unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
- $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)



light-cone OPE

SL phase space

$J/\psi, \psi(2S)$

$\bar{s}b$ cut

Extrapolate Dispersion relation for Π

dispersive representation of the $b\bar{s}$ contribution to a derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi} \int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s-Q^2} > 0 \qquad \text{if } Q^2 < 0$$

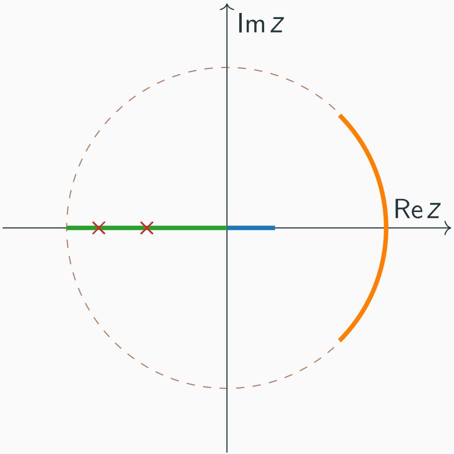
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow \chi^{\mathrm{OPE}}(Q^2)$
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the nonlocal form factors $|\mathcal{H}_\lambda|^2$
  $\rightarrow \chi^{\mathrm{had}}(Q^2)$

- global quark hadron duality suggests that $\chi^{\mathrm{OPE}}(Q^2) = \chi^{\mathrm{had}}(Q^2)$
- parametrize $\mathcal{H}_\lambda \propto \sum_n a_{\lambda,n} f_n$ with orthonormal functions f_n
  $\Rightarrow$ dispersive bound: $\quad \chi^{\mathrm{OPE}} \geq \sum_n |a_{\lambda,n}|^2$
- *first application* of such a bound to nonlocal form factors
- technically more challenging than for local form factors

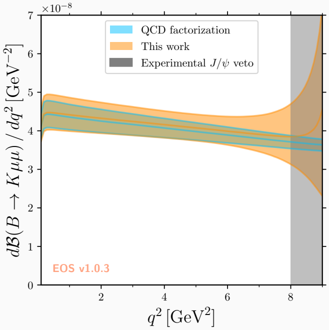
- expand in z
  - $f_n(z)$ orthogonal on arc
  - \+ accounting for behaviour on arc produces dispersive bound on each parameter ✓

[Gubernari/DvD/Virto '20]

- turns (so far!) hardly quantifiable systematic theory uncertainties into parametric uncertainties
- implemented in EOS
  - open source software at `github.com/eos/eos`
  - Python 3 interface, available via `pip` as `eoshep`

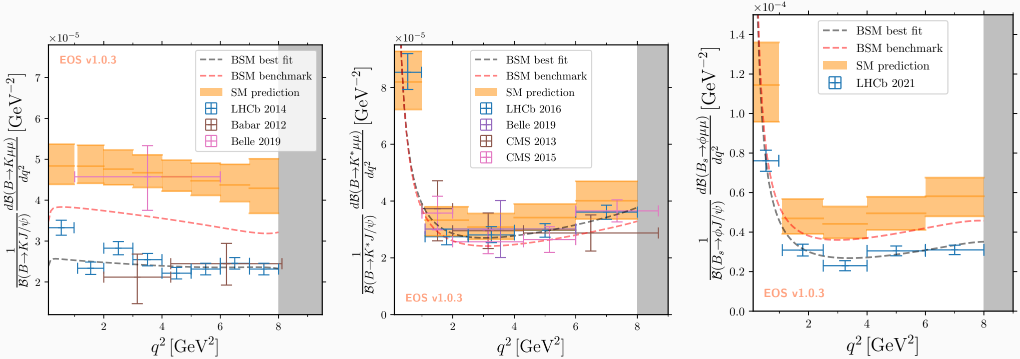


SM Predictions: Comparing to Previous Works



- predictions mutually compatible; slight change to the slope in $B_s \to \phi$ due to local FFs
- our uncertainties larger, but systematically improvable

SM Predictions: Challenging Data



- substantial tensions in $\mathcal{B}(B \to K\mu^+\mu^-)$ and $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$, lower in $\mathcal{B}(B \to K^*\mu^+\mu^-)$

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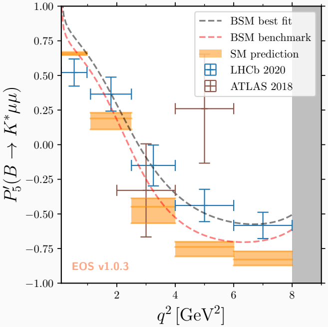
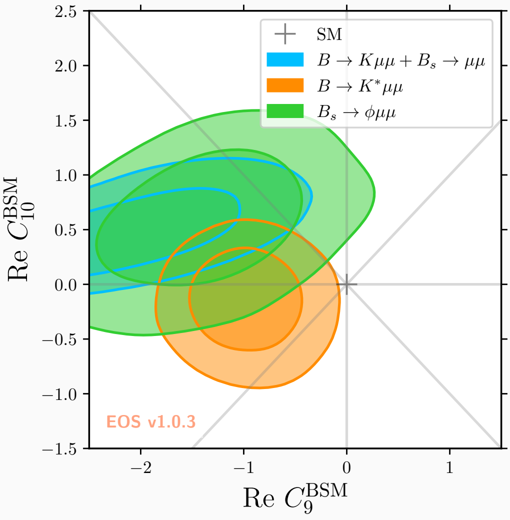


- substantial tensions in $\mathcal{B}(B \to K\mu^+\mu^-)$ and $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$, lower in $\mathcal{B}(B \to K^*\mu^+\mu^-)$
- tension in angular distribution in $B \to K^*\mu^+\mu^-$ remains

Updated BSM Interpretation



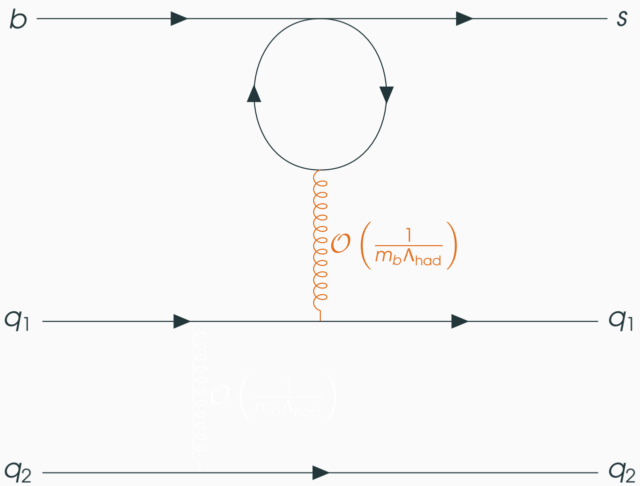
- no global fit yet
  - large # of nuisance params makes global fit difficult
  - instead, three individual fits
  - mutually compatible results!
  - compatible with previous analyses
- fits use all available data, incl. angular obs.
- substantial tensions in $B \to K$ and $B_s \to \phi$, slightly lower in $B \to K^*$

$\Lambda_b \to \Lambda\mu^+\mu^-$

- $\Lambda_b \to \Lambda\mu^+\mu^-$ provides complementary constraints on Wilson coefficients C_9 vs $C_{9'}$ vs C_{10} vs $C_{10'}$, etc.
- however, theory is not as well developed

	$\overline{B} \to \overline{K}\mu^+\mu^-$	$\Lambda_b \to \Lambda\mu^+\mu^-$
local FFs from lattice QCD	✓	✓
local FFs: dispersive bounds	✓	✓ (recently)
QCDF for non-local FFs	✓	✗
soft-gluon correction	✓	✗

QCDF for Baryons?



- hard collinear spectator scattering
- 1st scattering does not suffice!

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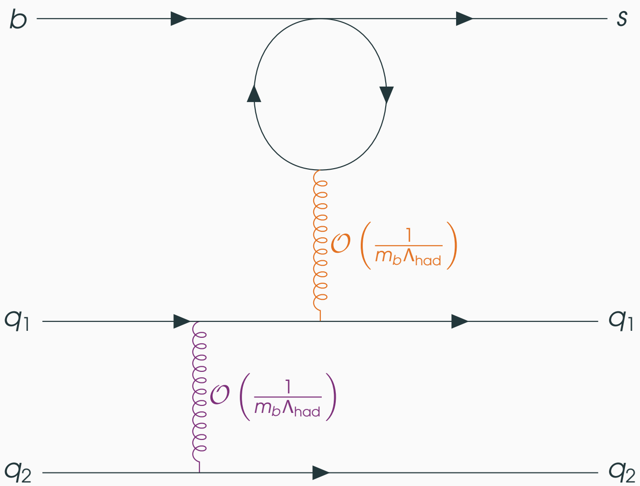


- hard collinear spectator scattering
- 1st scattering does not suffice!
- 2nd scattering alignment?
- 2nd scattering LCDA?

Summary

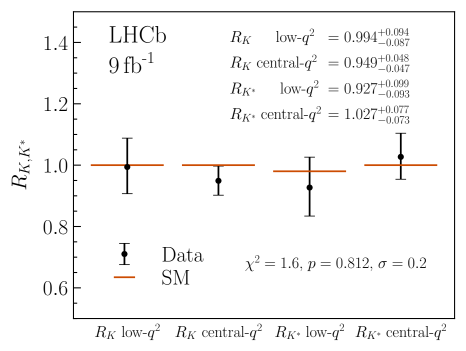
Summary and Outlook

- phenomenology of rare *B* decays is a complicated business
  - WET under good control
  - local form factors see revitalized interest from lattice QCD
  - non-local form factors now under reasonable theory control
- new approach to (B)SM predictions corroborates earlier results qualitatively
  - larger uncertainties reduce significance of the anomalies somewhat
  - uncertainties very conservative and systematically improvable
- still: a lot to do for phenomenologists, amongst others:
  - performing a truly global fit in the new approach
  - extending analysis to $\Lambda_b \to \Lambda$ transitions

Backup Slides

The elephant in the room

Joint LHCb measurement of R_K and R_{K^*}



[LHCb 2212.09153]

- lepton-flavour-nonuniversality in $b \to s\ell^+\ell^-$ is gone!
  - not the longest standing anomaly by far!
  - not the only one, either!
- I prefer to think of it as a precision measurement of $\mathcal{B}(B \to K^{(*)}e^+e^-)$
  - gives rise to a new anomaly
  - $\mathcal{B}(B \to Ke^+e^-)$ deviates from SM prediction by roughly the same amount as $\mathcal{B}(B \to K\mu^+\mu^-)$!