



Systematic uncertainty of off-shell corrections and higher-twist contribution in DIS at large x

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CTEQ-JLab Collaboration

**A. Accardi, I. Fernando, X. Jing, S. Li, J. Owens, S. Park,
C.E. Keppel, W. Melnitchouk, P. Monaghan**

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We have to deal with Deuterium target at large- x

Deuterium: nuclear smearing

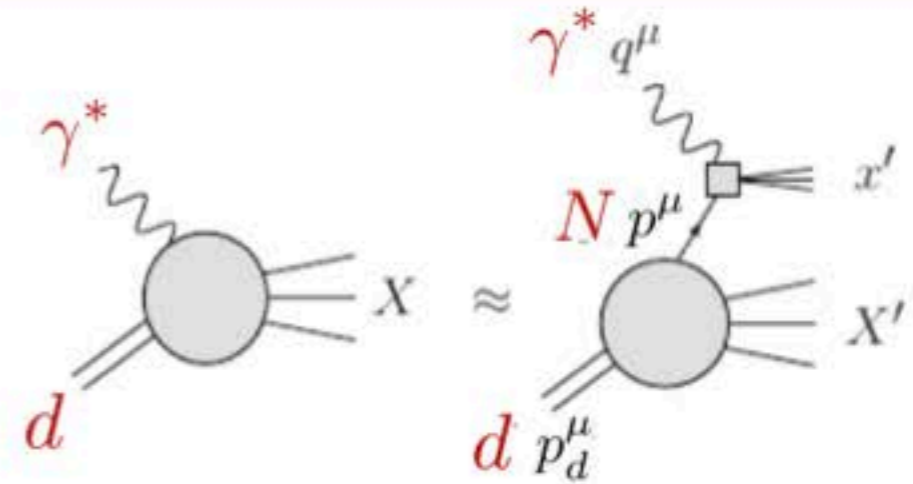
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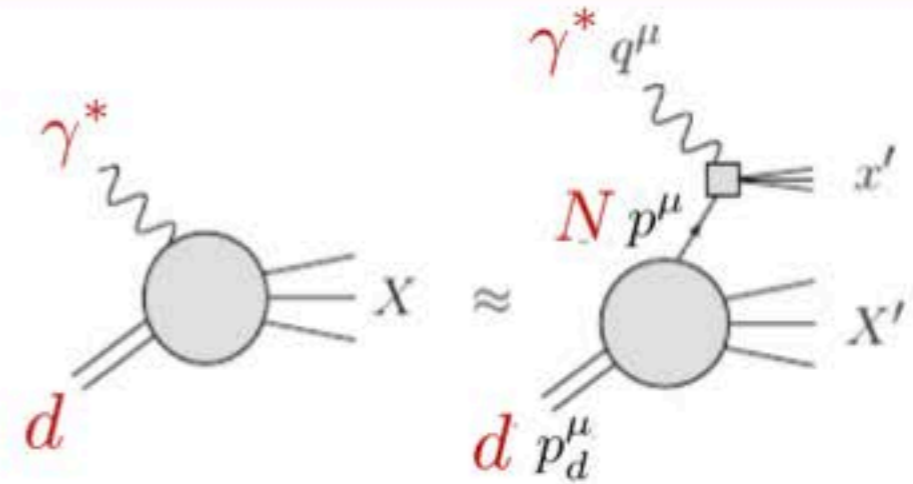
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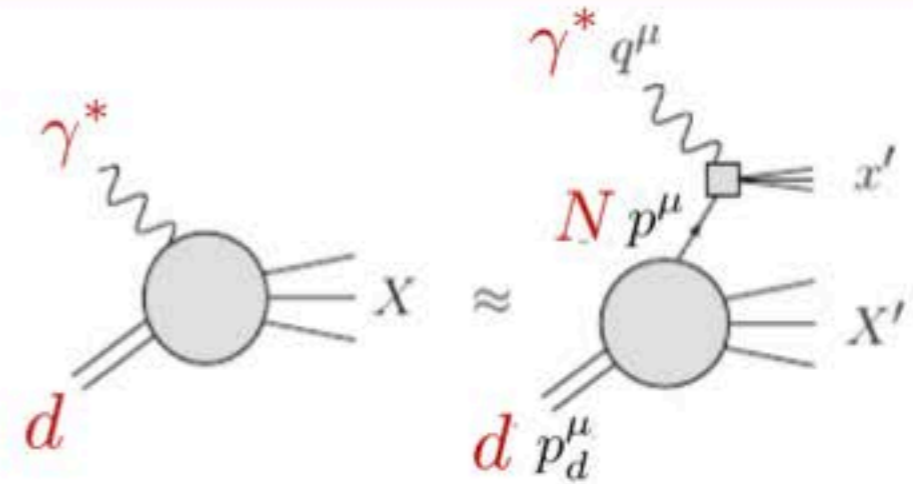
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Smearing function:

$$x_D = \frac{Q^2}{P_D \cdot q}$$

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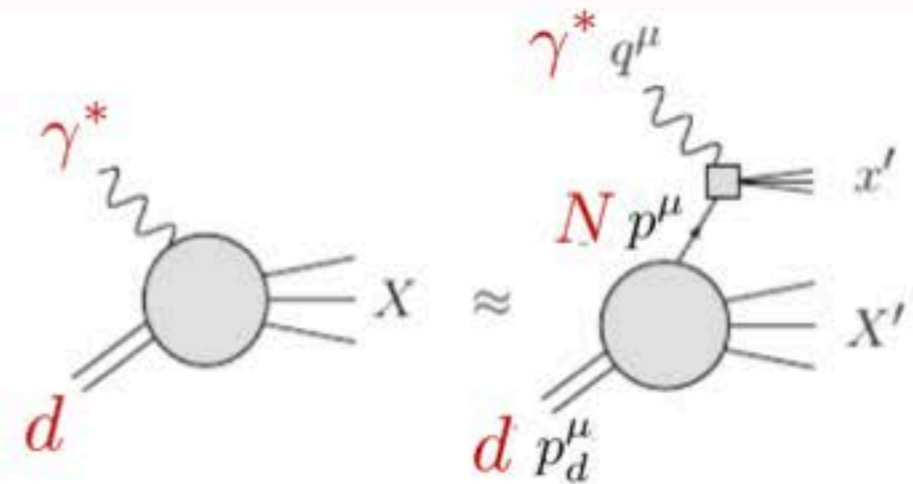
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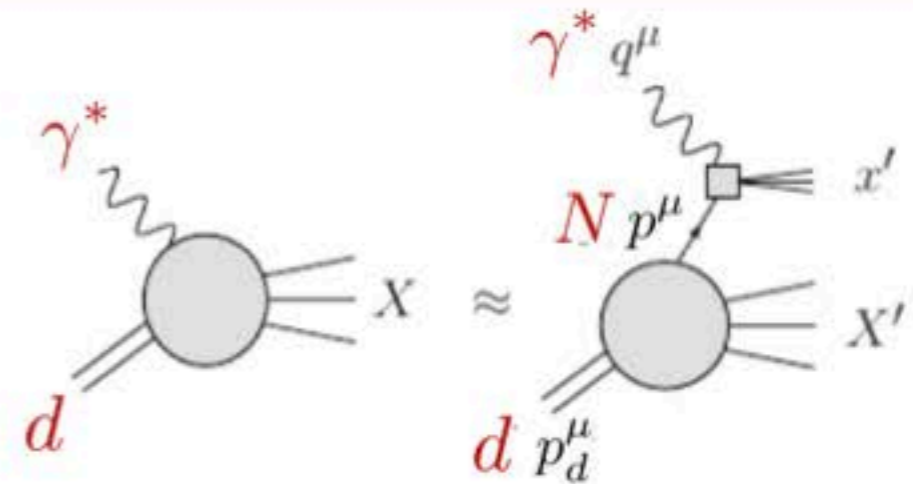
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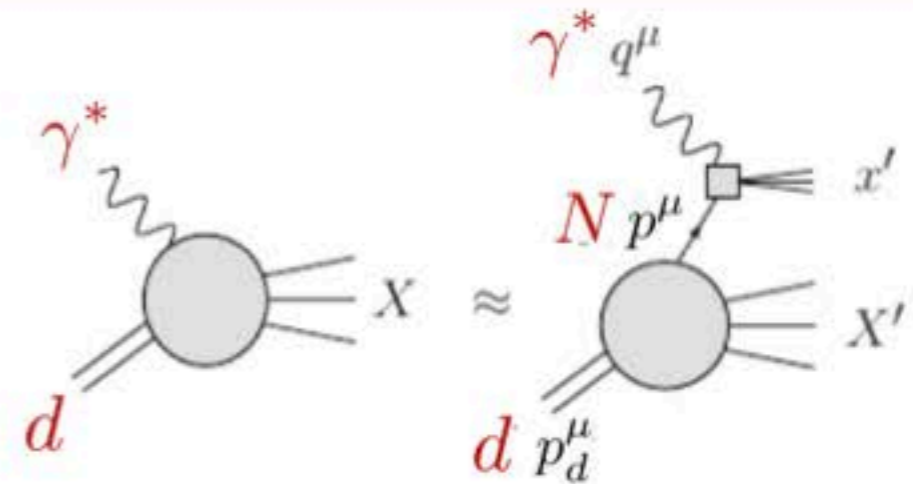
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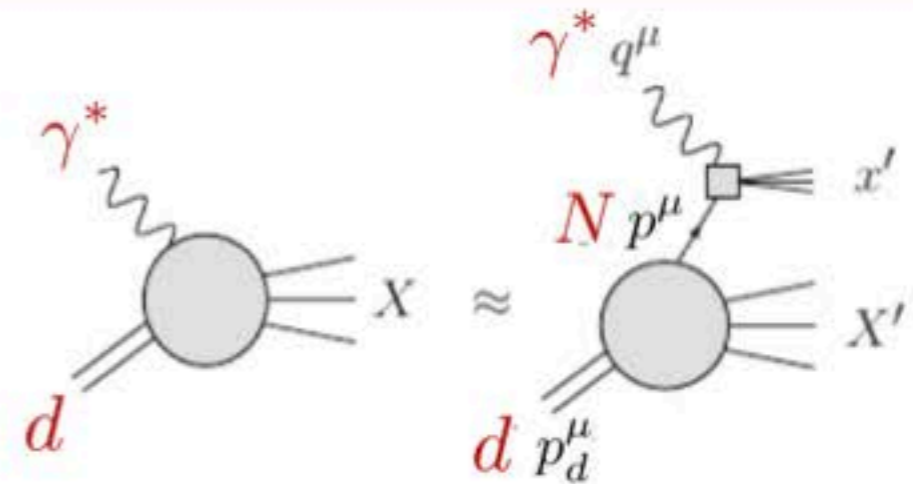
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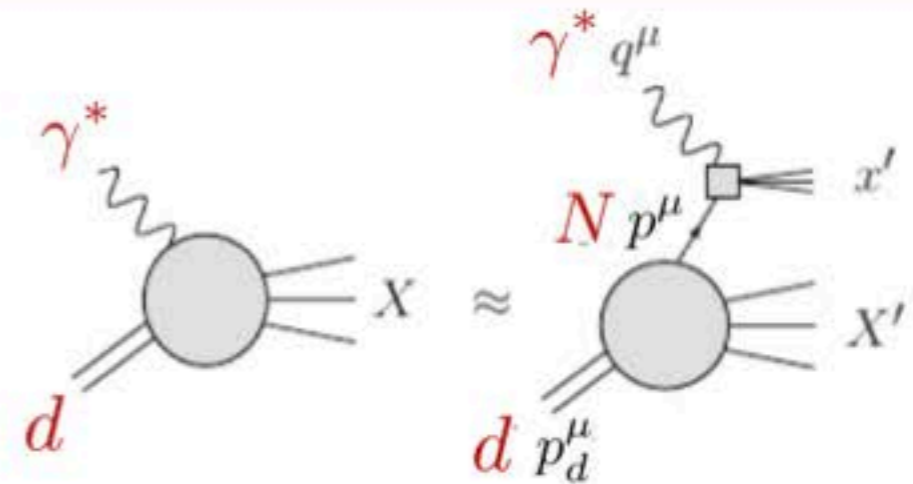
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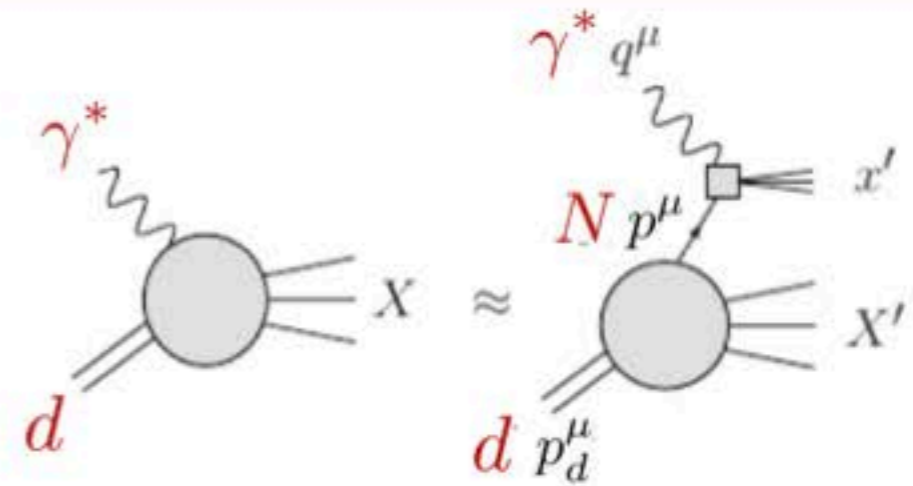
Structure function

of a bound, off-shell nucleon

Deuterium: off-shell corrections

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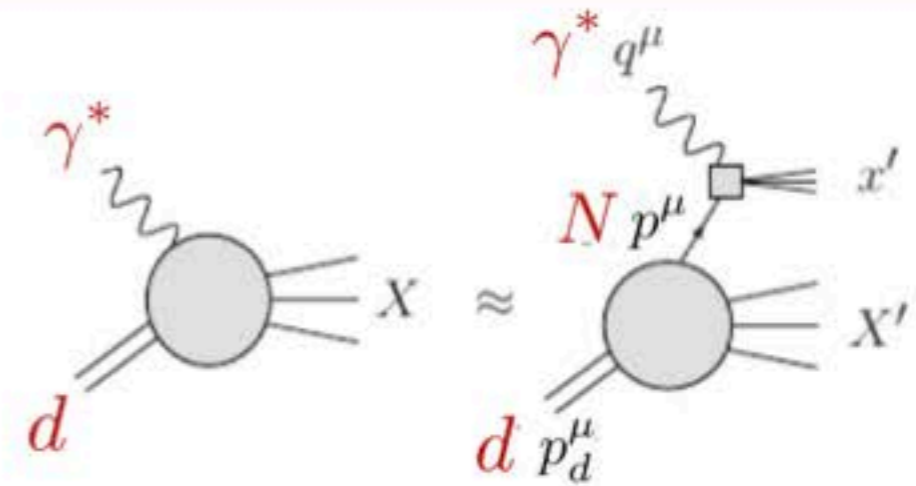
Bound, off-shell nucleon inside the deuteron



Deuterium: off-shell corrections

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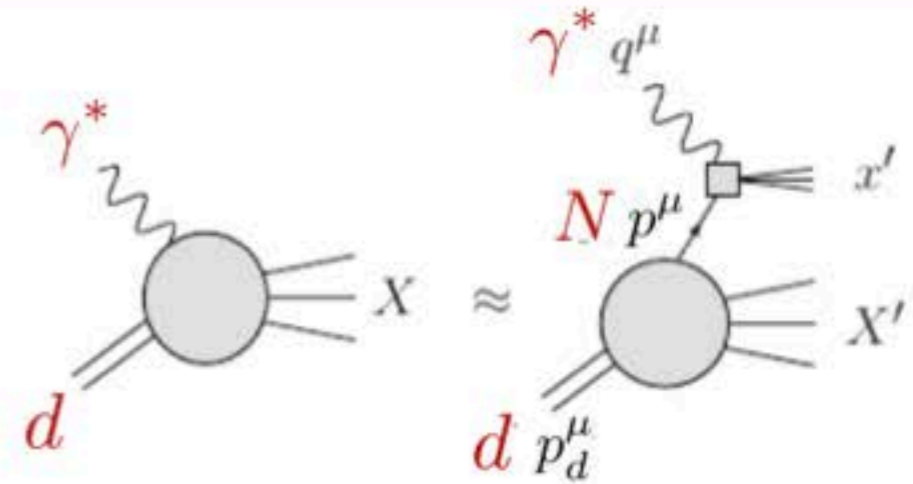


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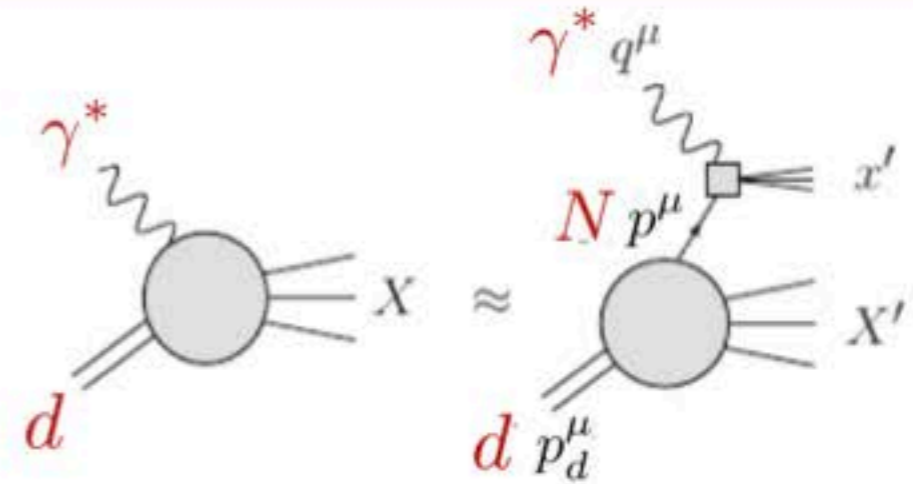


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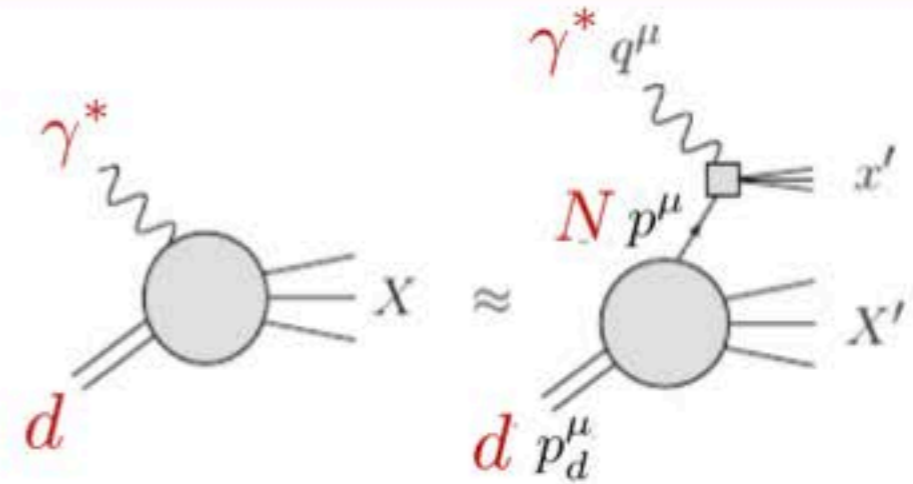
Off-shell expansion (in nucleon virtuality p^2)

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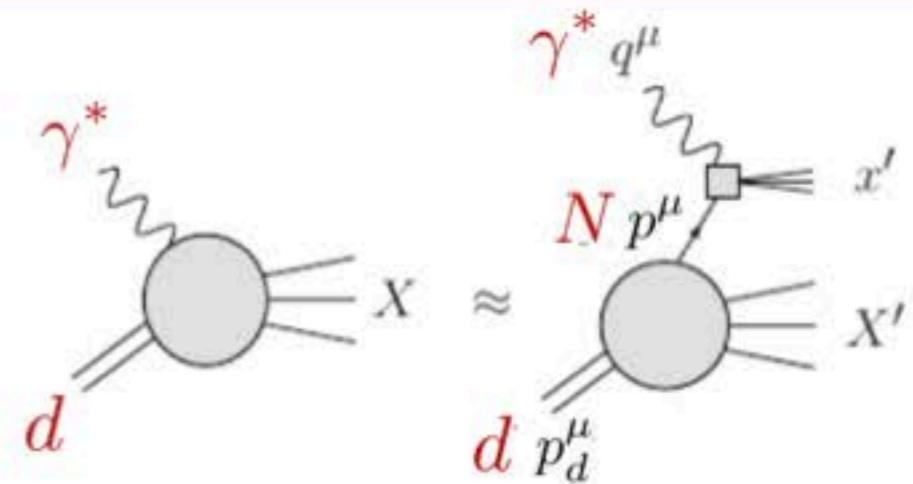
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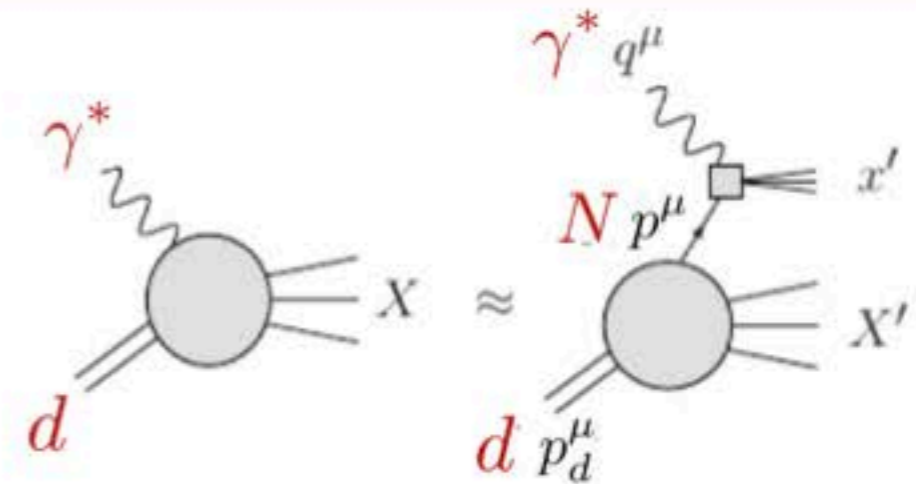
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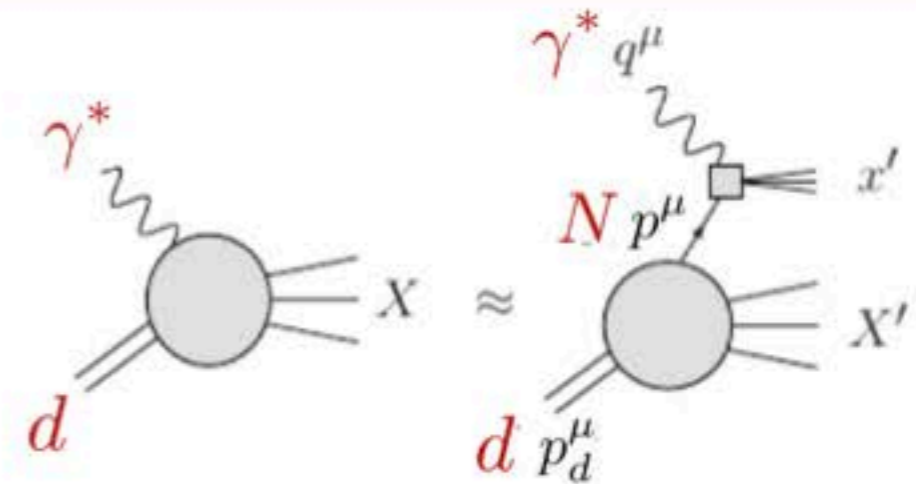
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Off-shell function

(To be fitted)

Latest results from QCD fits in CJ framework

CJ15 fit

Accardi, Brady, et al., PRD 93 (2016)

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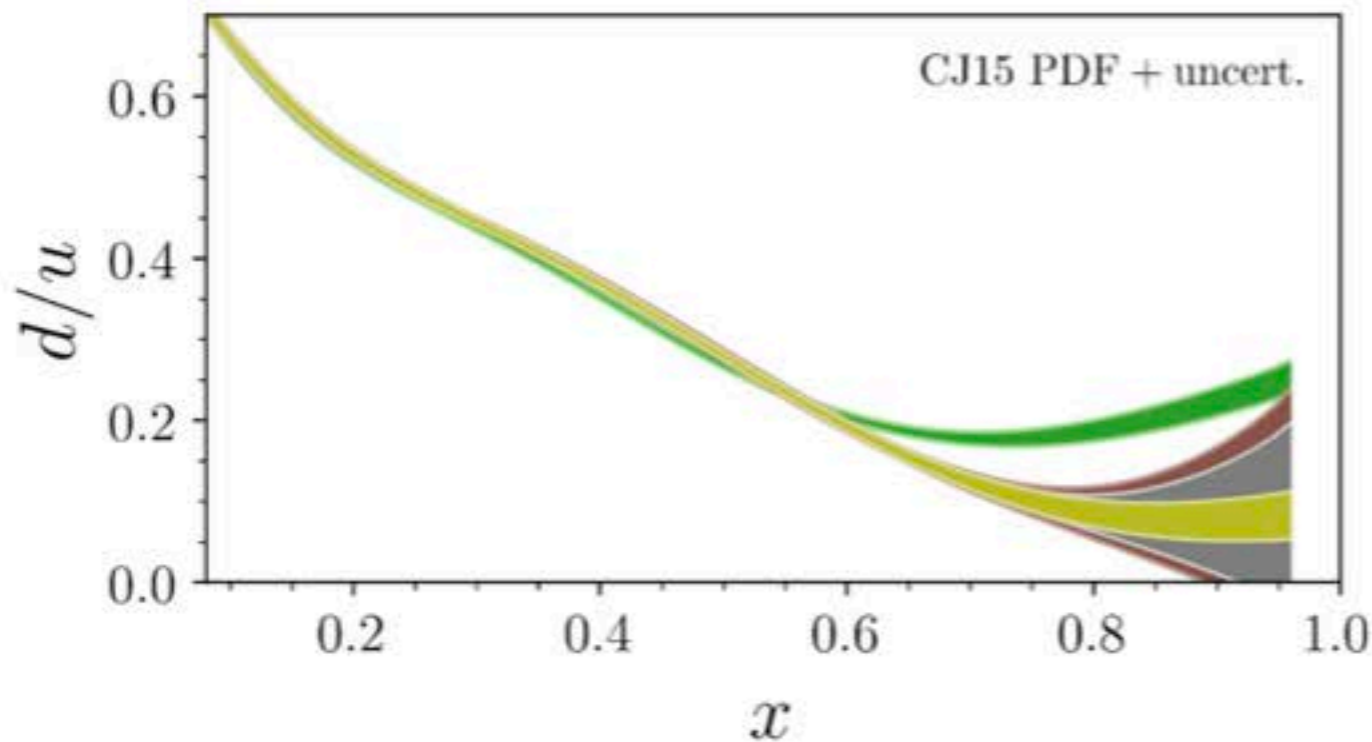
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- Nuclear
- Parametriz.
- Statistical

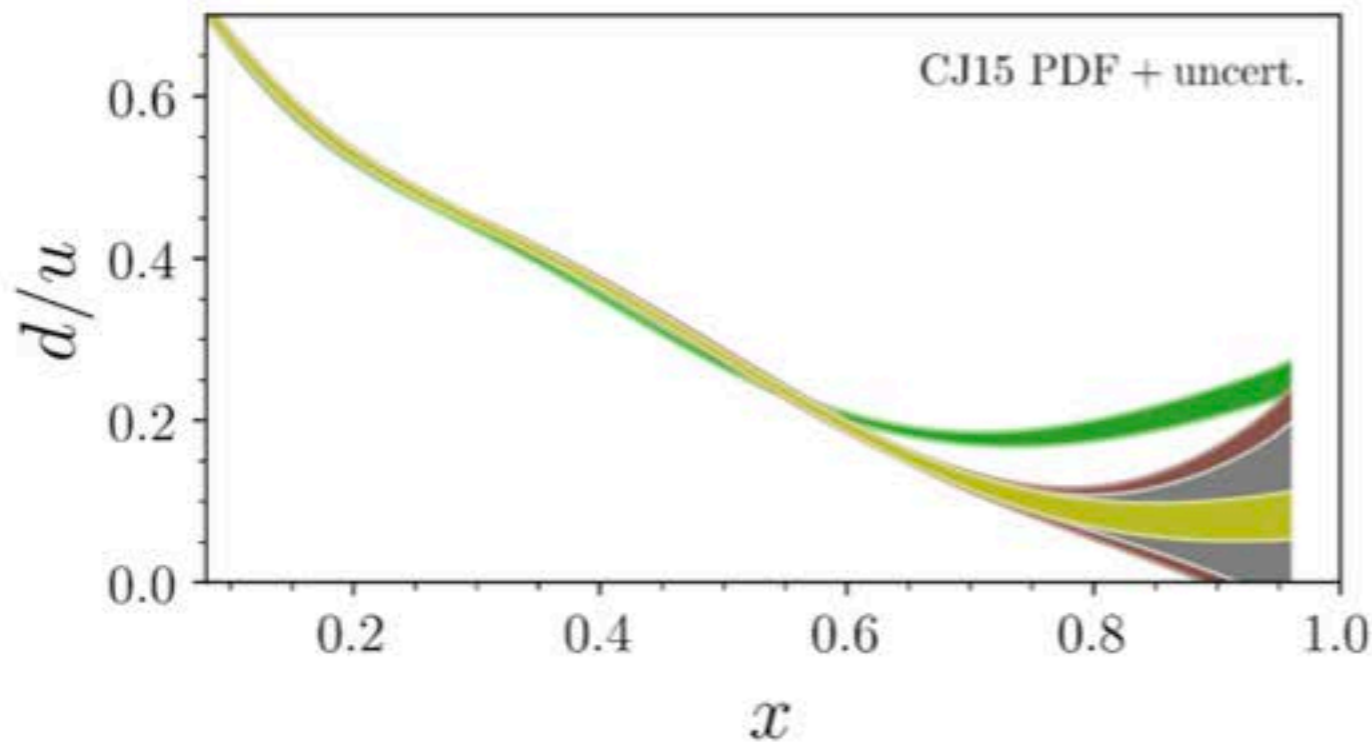
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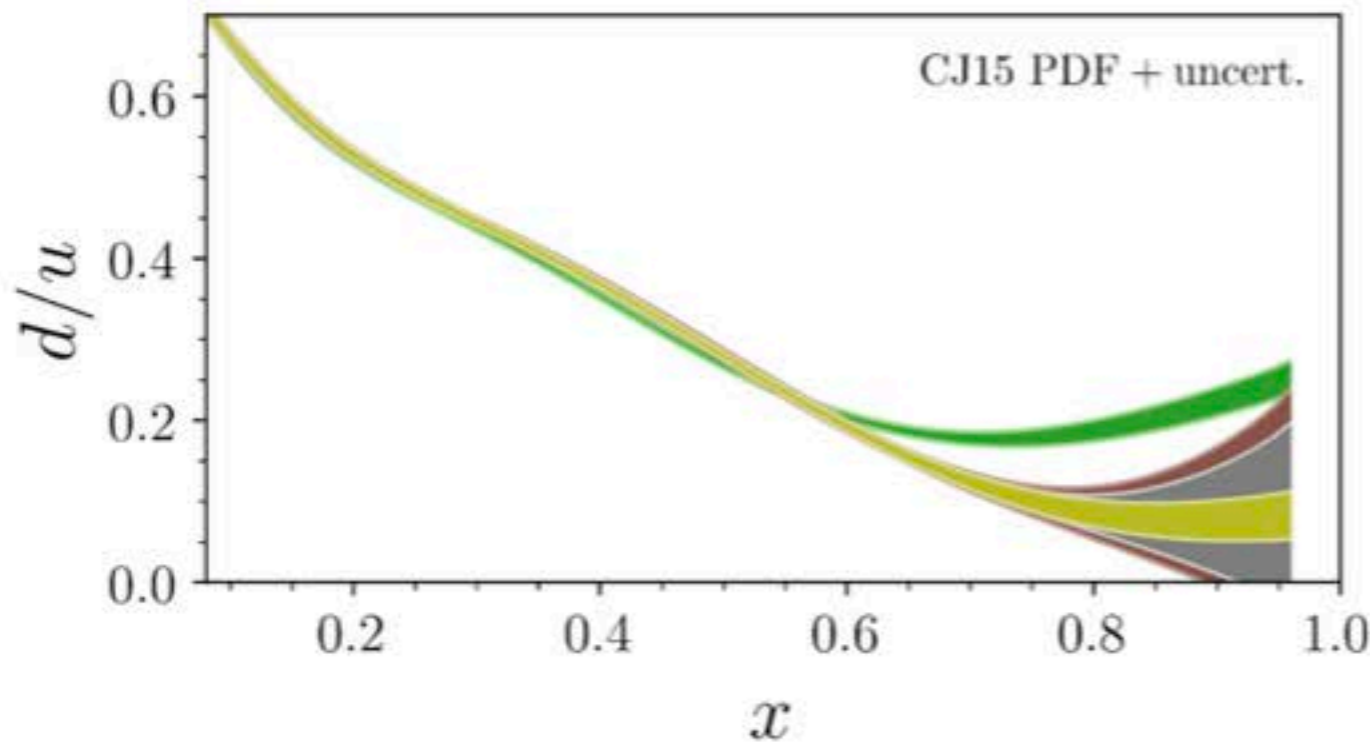
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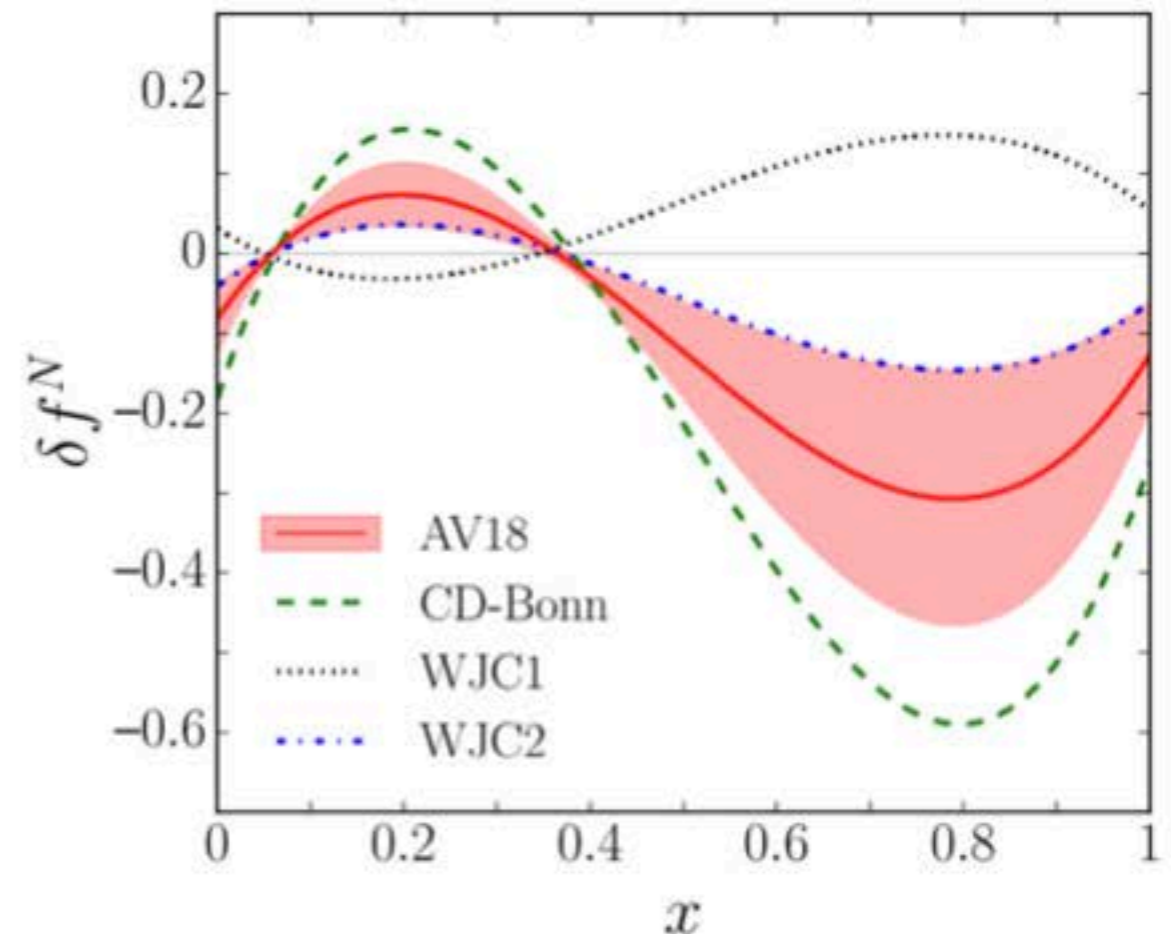
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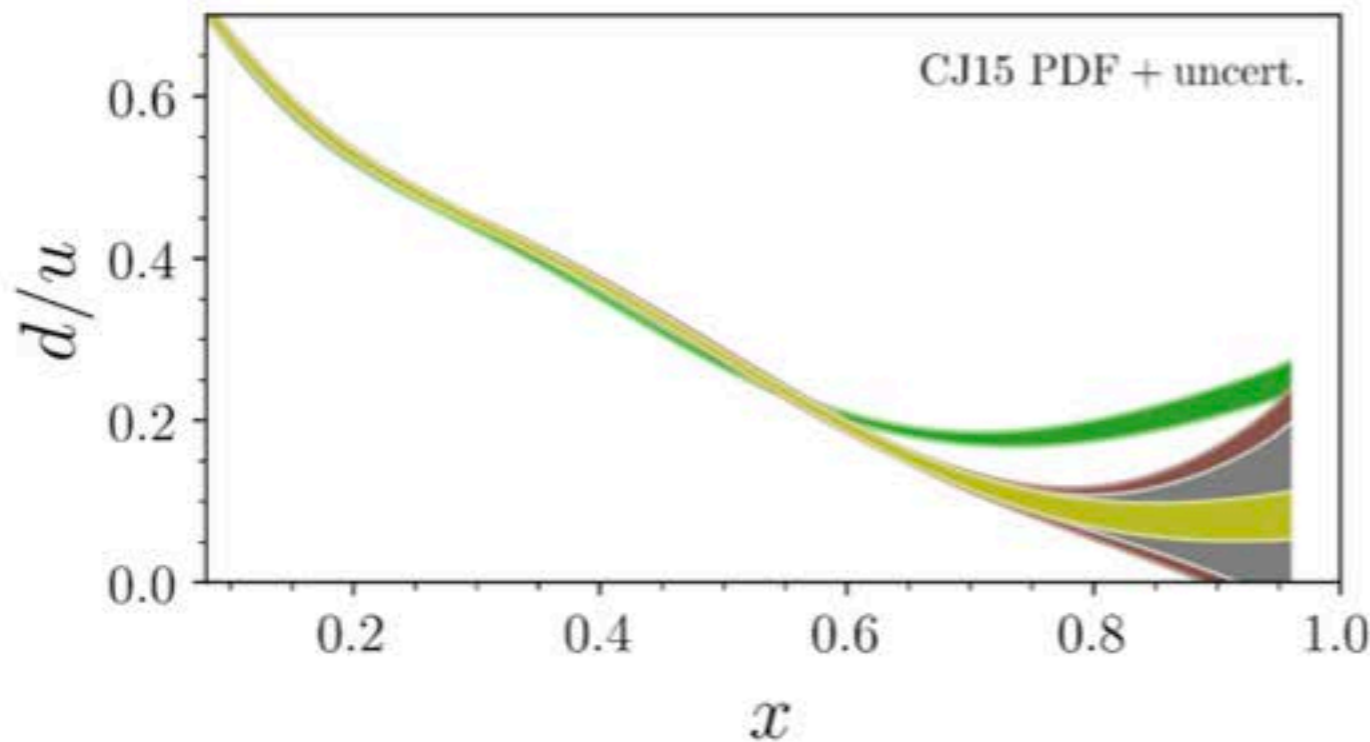
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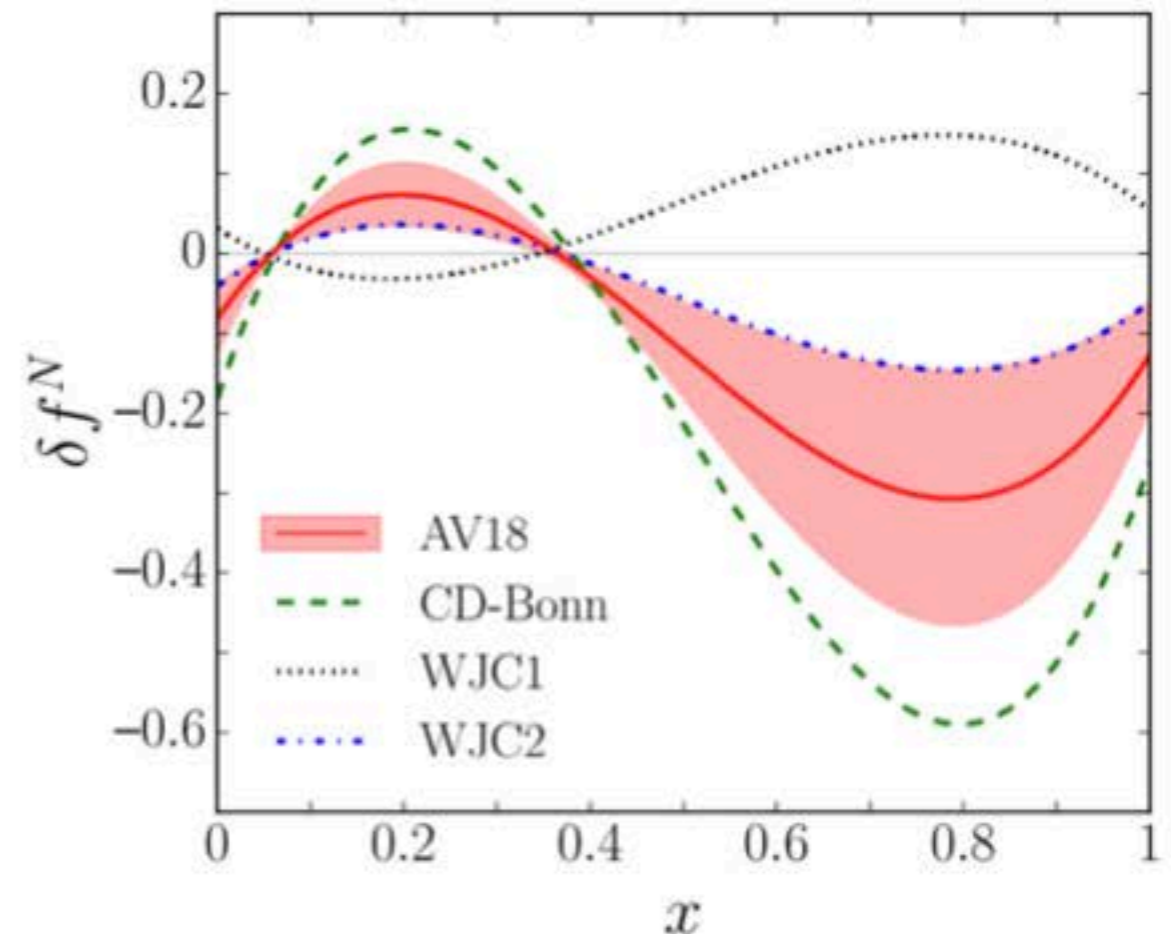
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The most of the recent nuclear potentials does not introduce a bias on the fit



Latest results from QCD fits in CJ framework

CJ22 fit

Accardi, Jing, Owens et al., PRD 107 (2023)

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CJ22 fit

Same off-shell parameterization

More flexible parameterization of sea quarks (NuSea and SeaQuest data)

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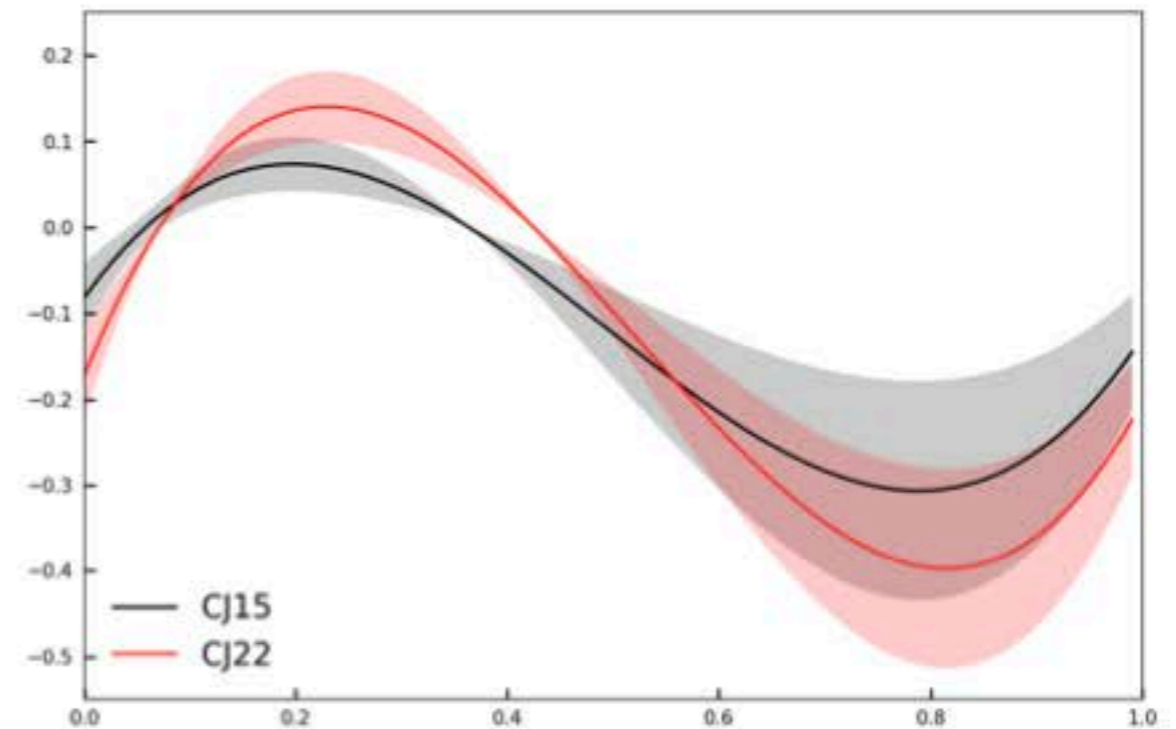
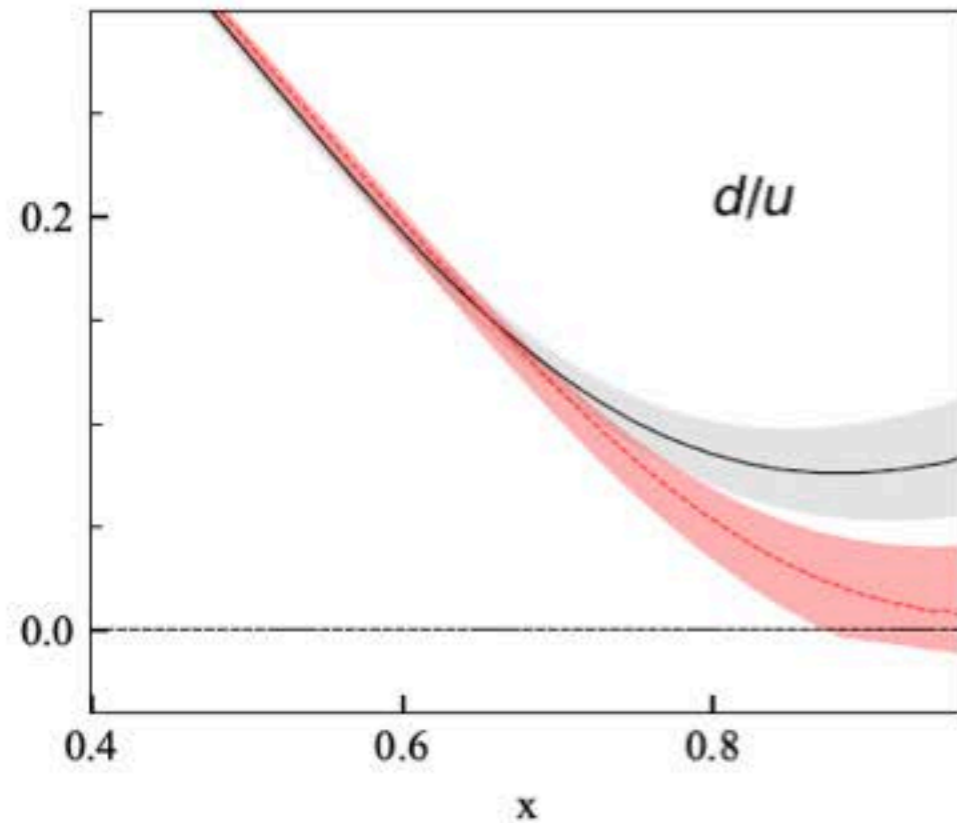
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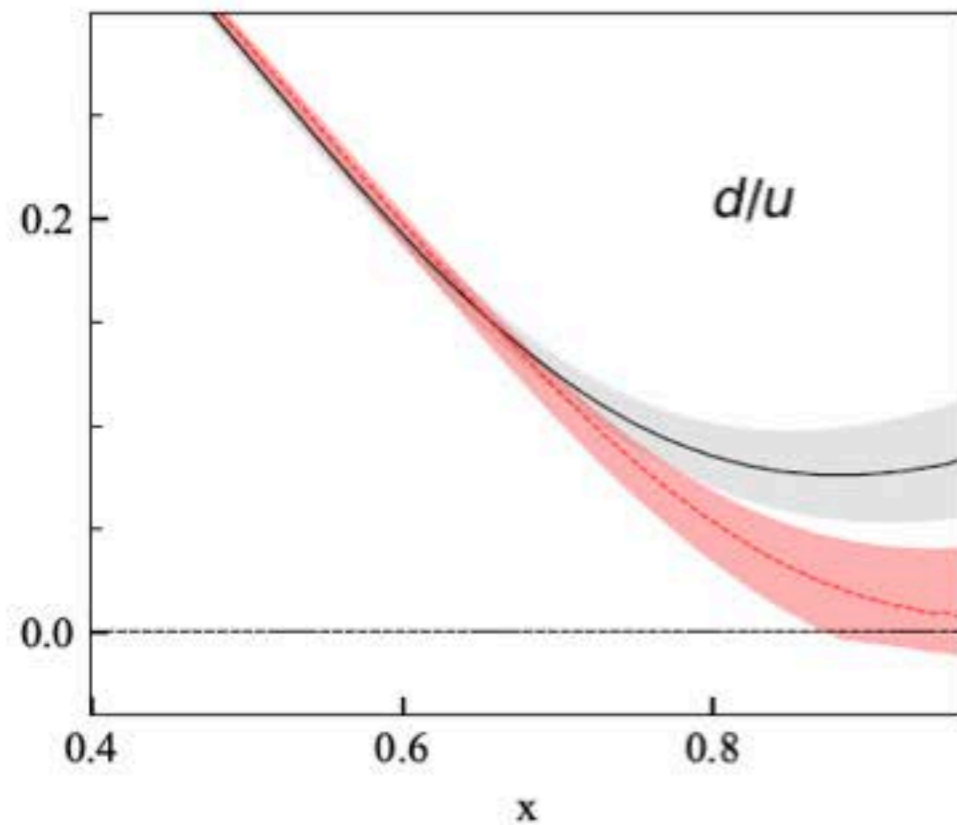
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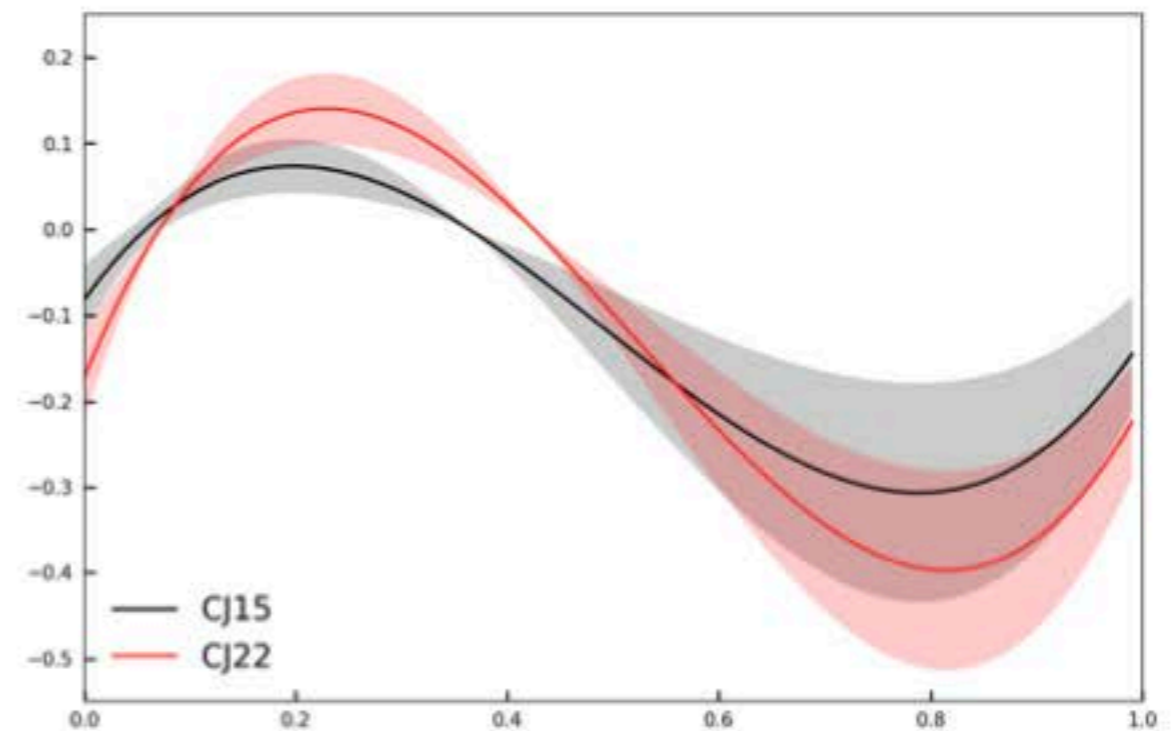
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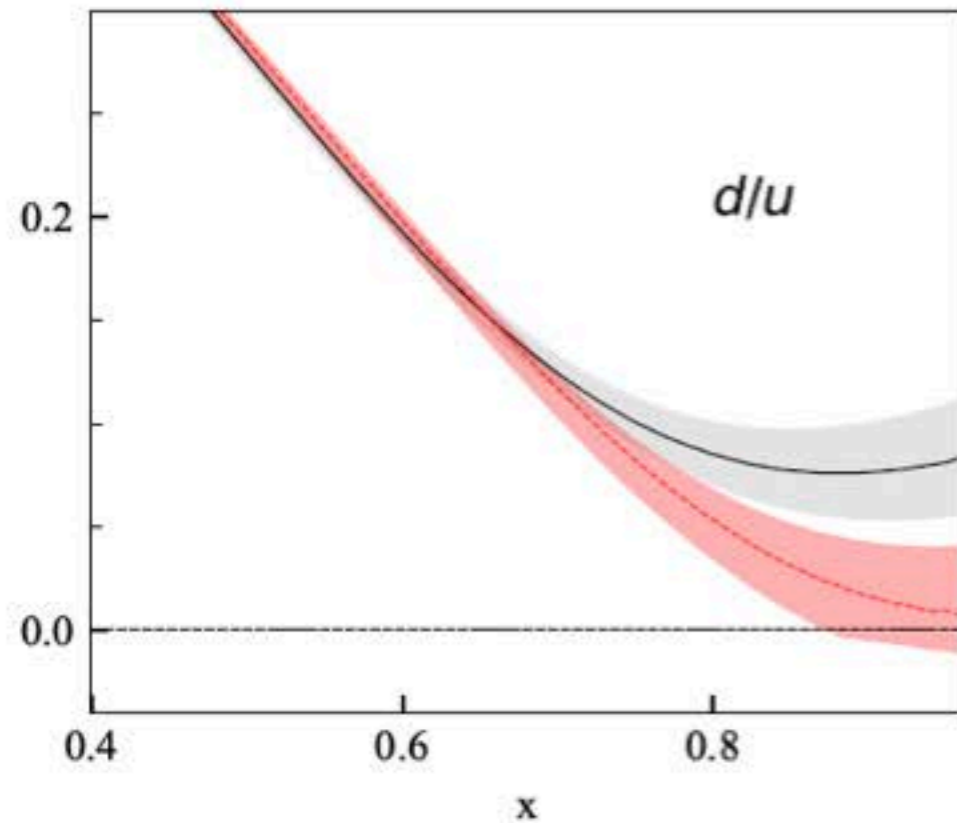
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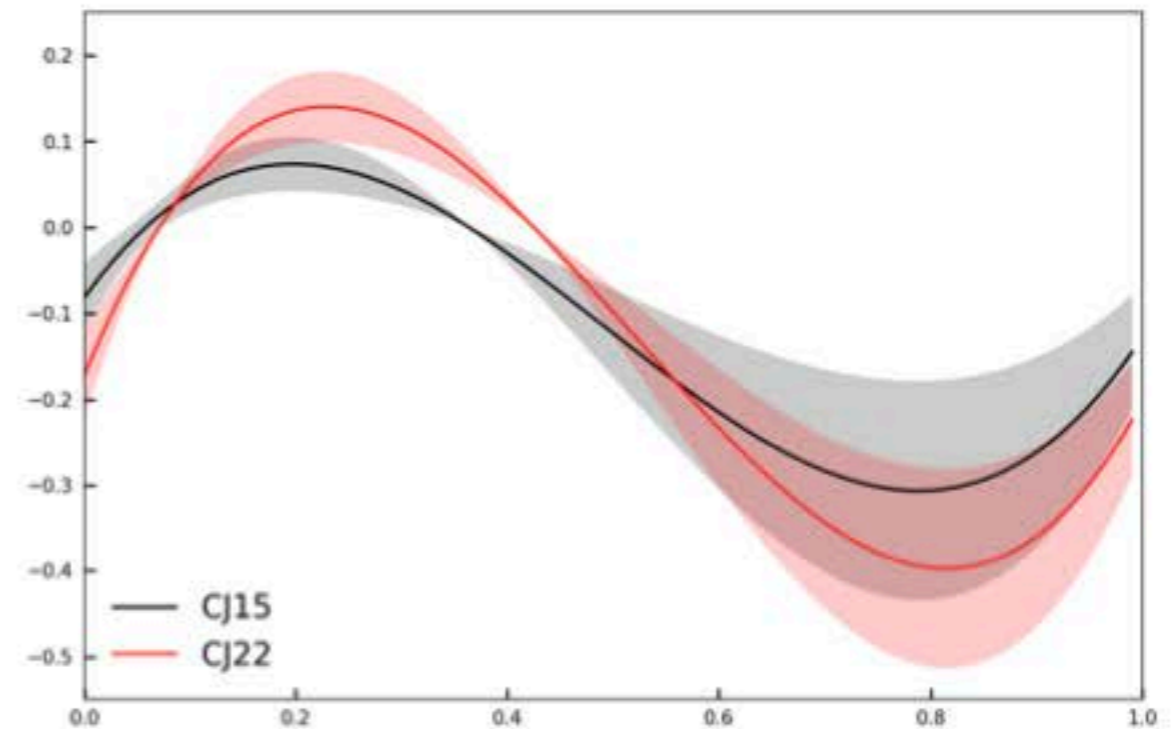
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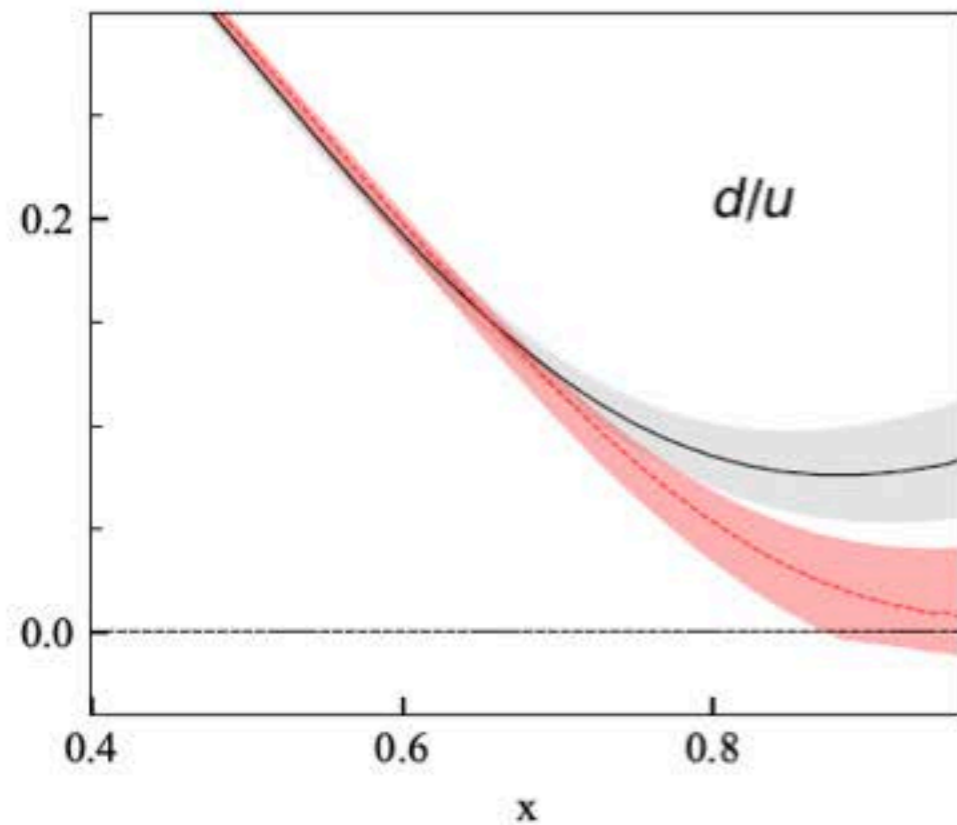
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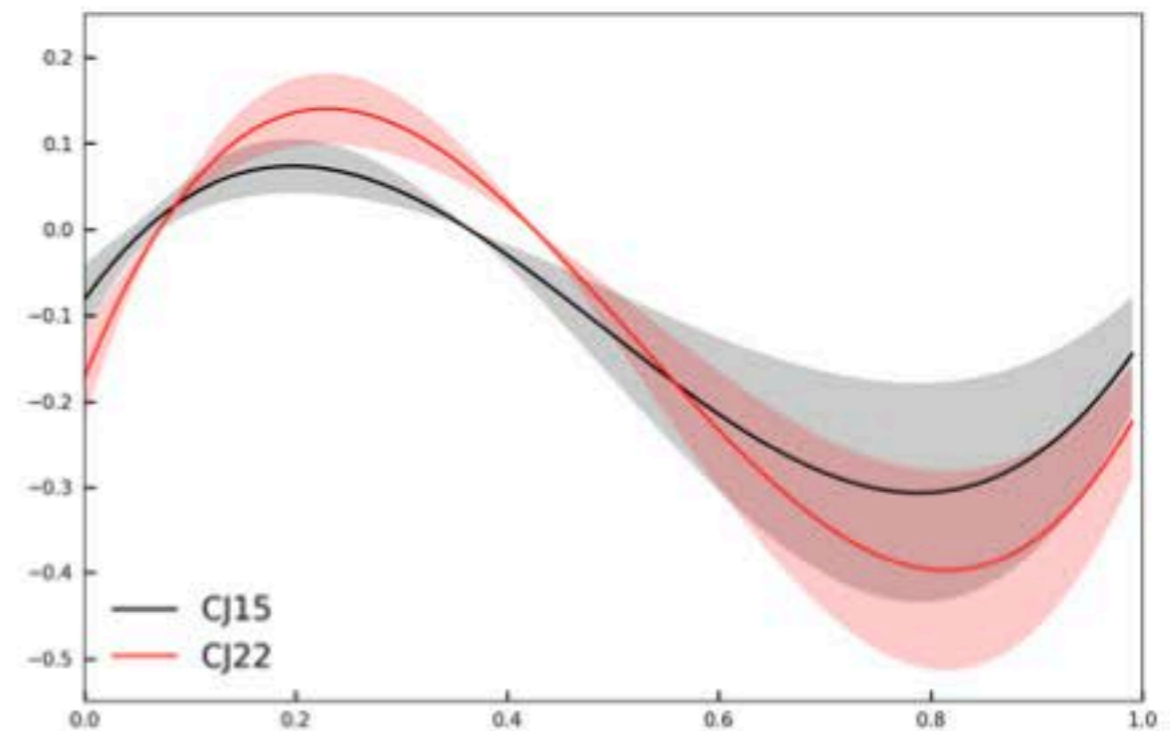
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Difference on d/u is absorbed in something else

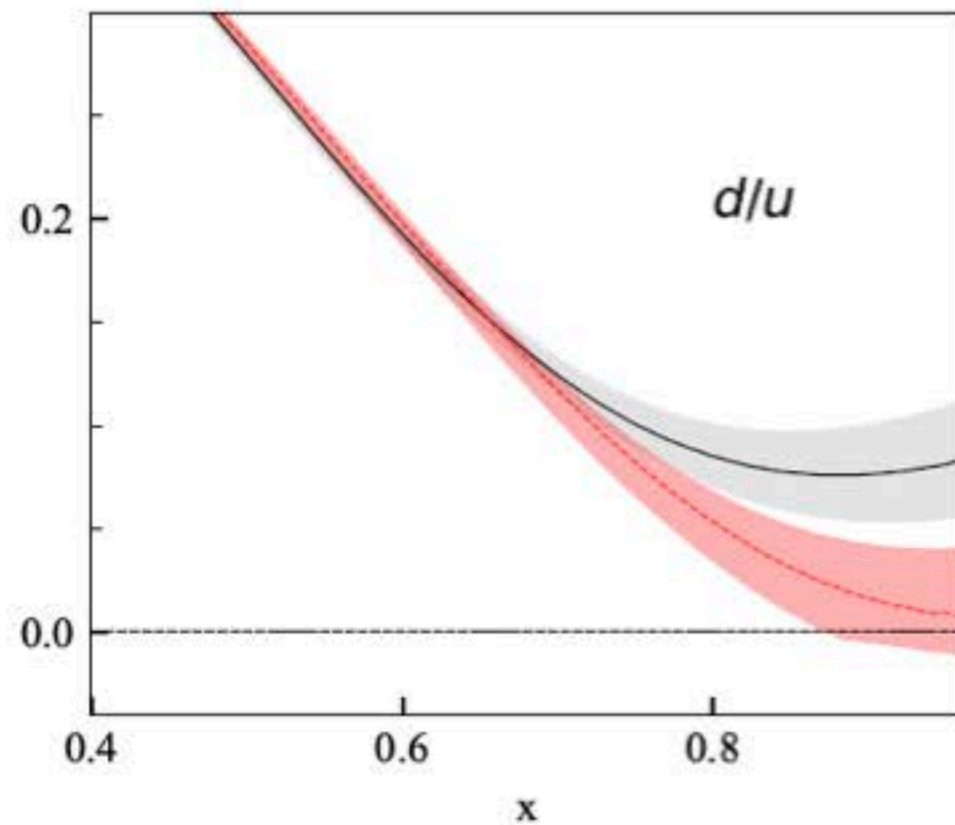
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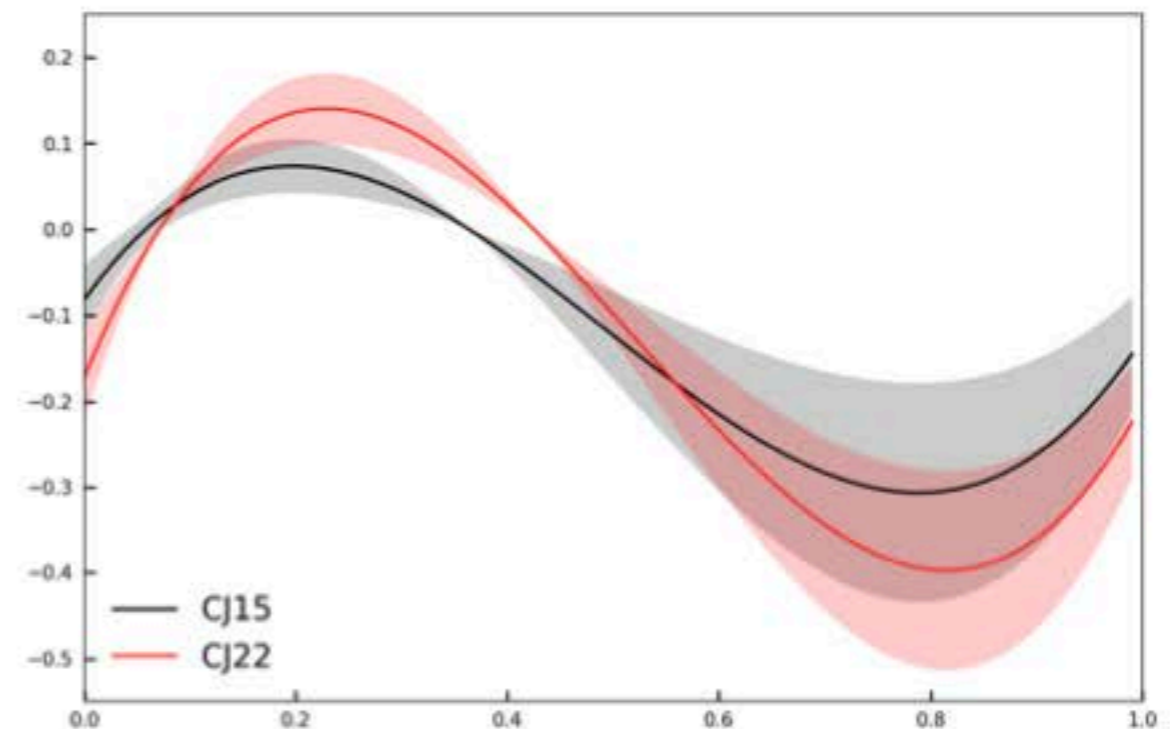
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Difference on d/u is absorbed in something else \rightarrow **Higher Twist**

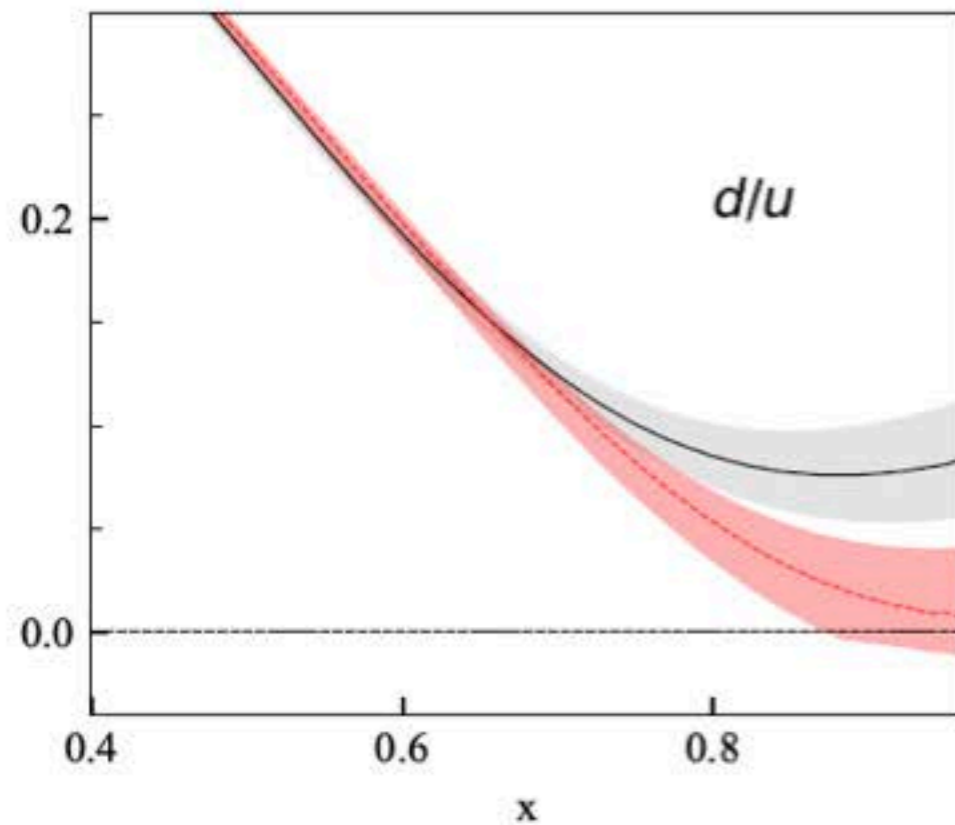
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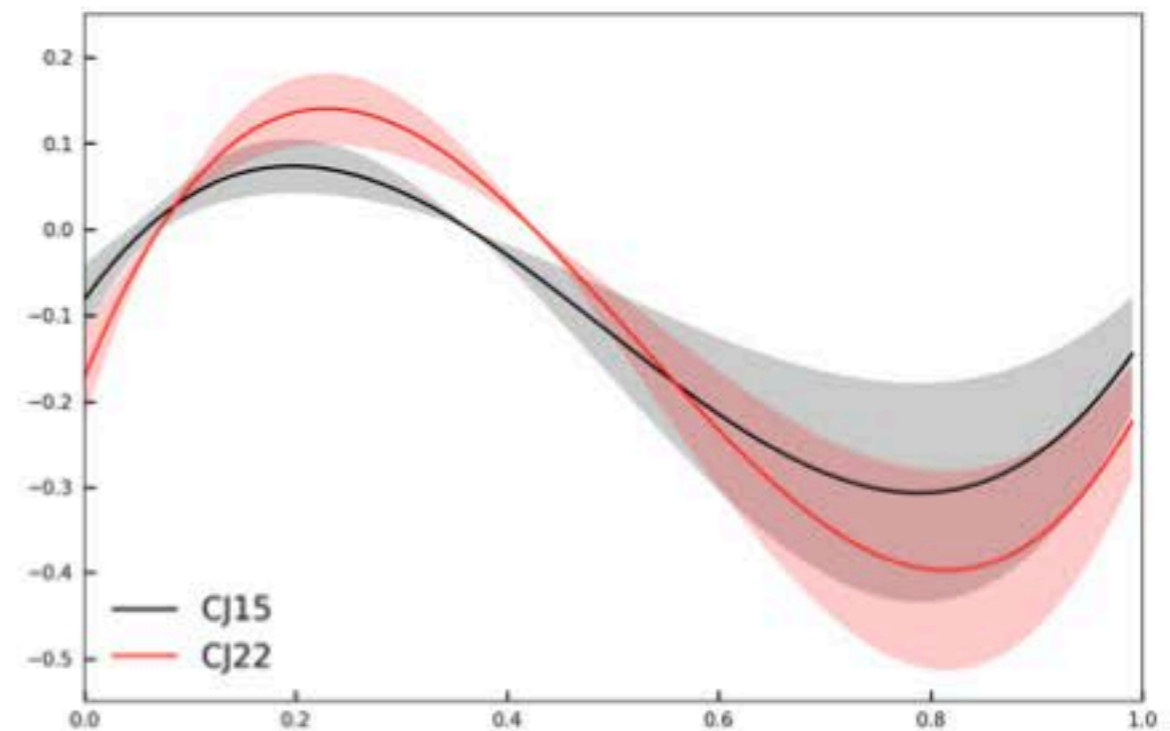
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Is the model for off-shell correction enough flexible?

Polynomial off-shell function

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C, x_0 and x_1 fitted $\Rightarrow x_1 \simeq x_0$

Polynomial model $\Rightarrow \delta f(x) = \sum_n a_{off}^{(n)} x^n$

Alekhin, Kulagin, Petti, PRD 96 (2017)

Polynomial off-shell function

$$\delta f^N = C(x - x_0)(x - x_1)(1 + x_0 - x)$$

KP-like model

Kulagin and Petti, NPA 765 (2006)

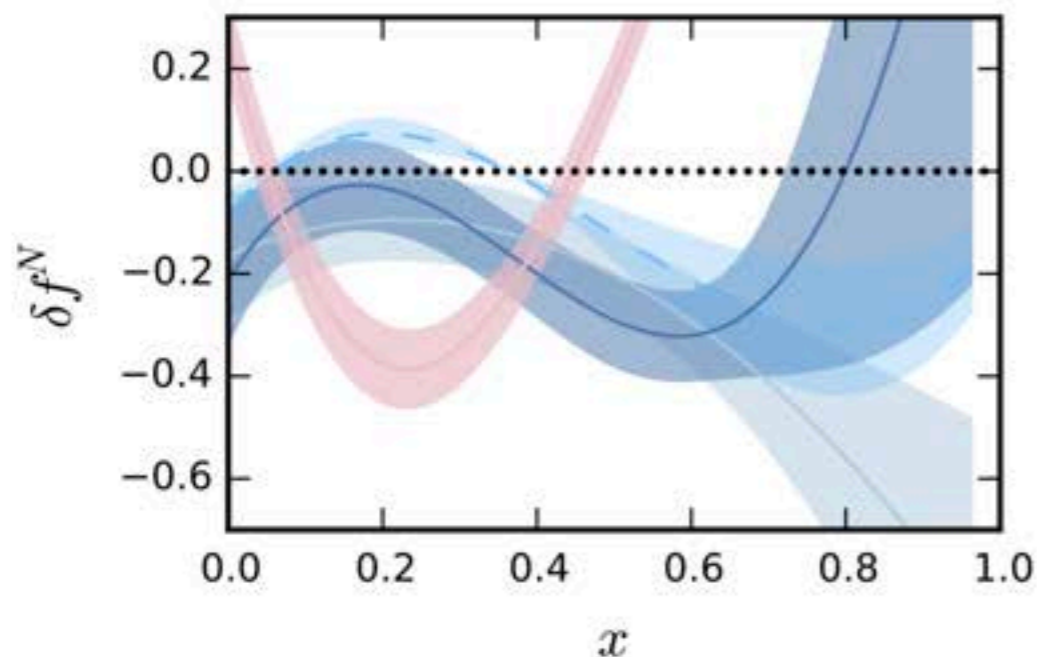
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$$\int_0^1 dx \delta f^N(x) [q(x) - \bar{q}(x)] = 0$$

Release the assumption of the valence sum rule

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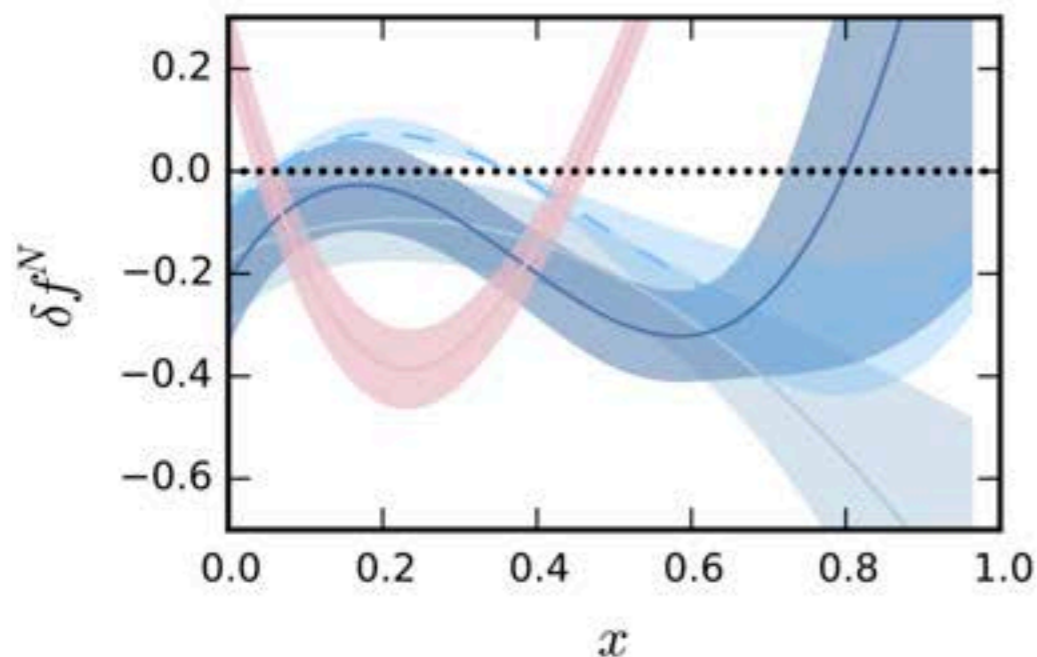
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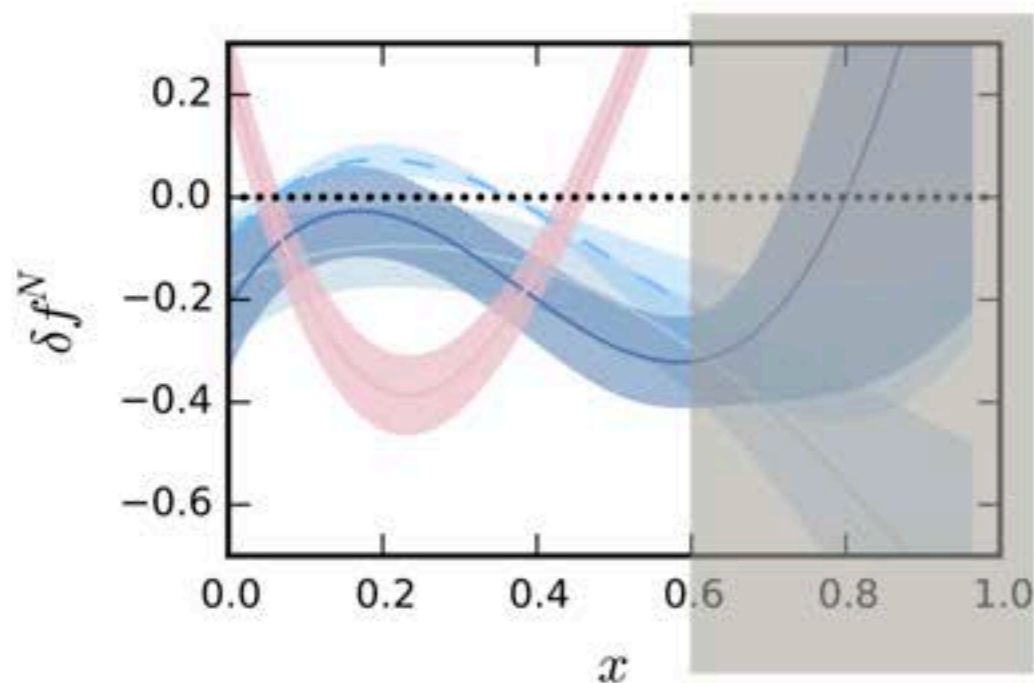
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Constrain power of CJ15
dataset only up to $x = 0.6$

Higher-Twist function

Higher Twist correction

Higher-Twist function

Higher Twist correction

Multiplicative

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left(1 + \frac{C(x)}{Q^2} \right)$$

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Additive

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they are related

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CJ fits

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Are experimental observables independent of the choice of the HT?

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Bias in n/p function

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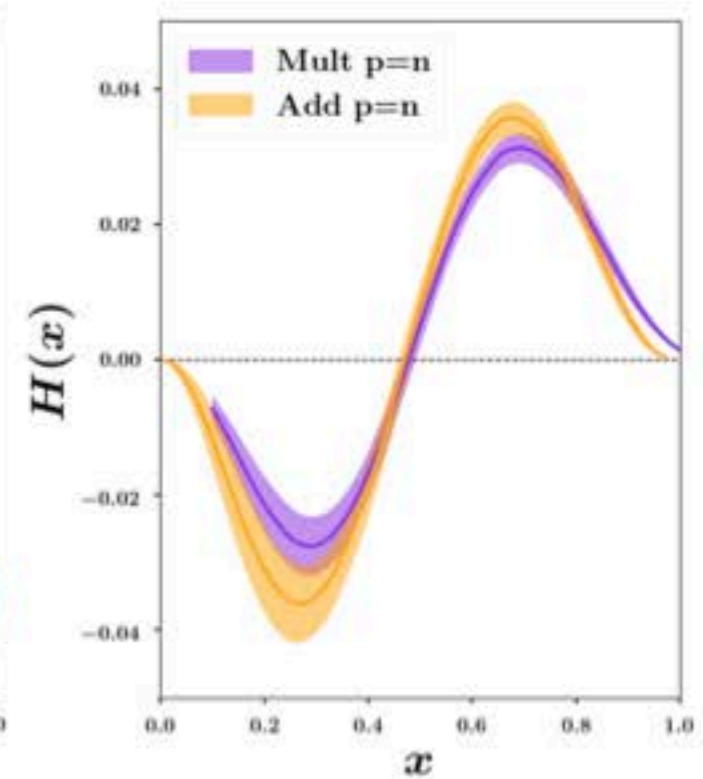
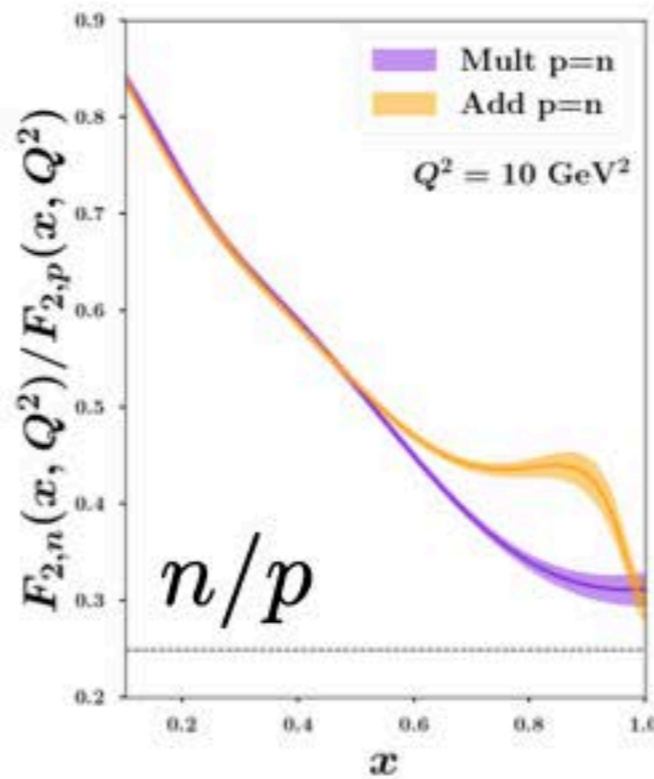
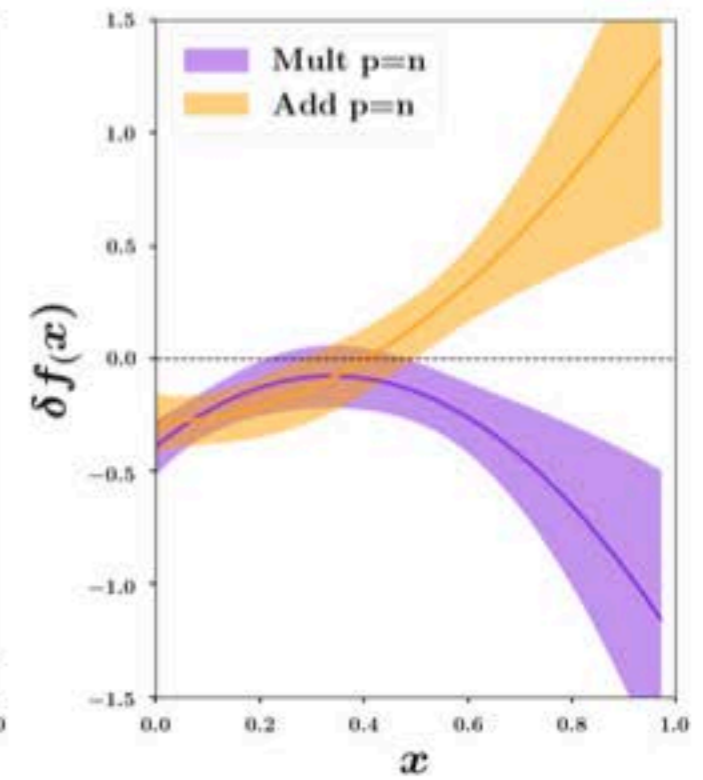
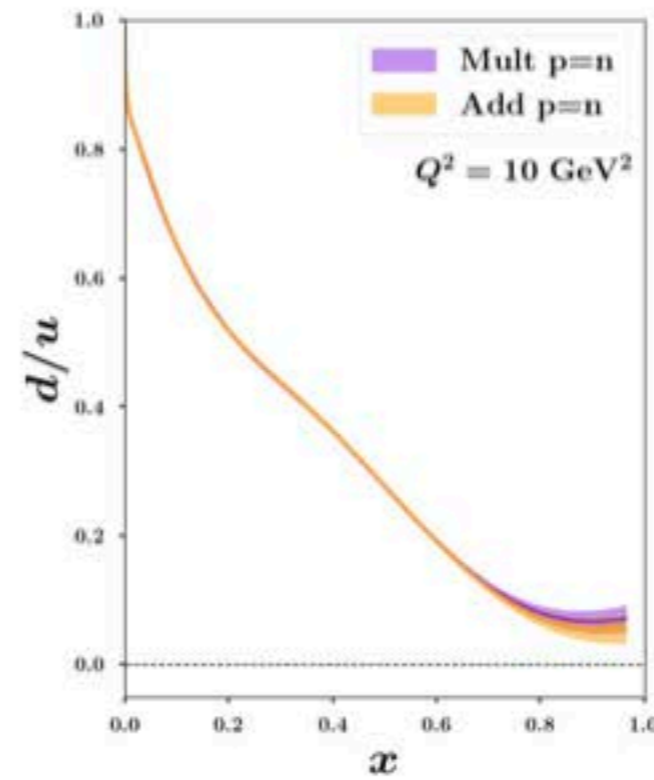
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same as Add

Bias not present!

Results in the CJ fitting framework

Case 1: isospin symmetry



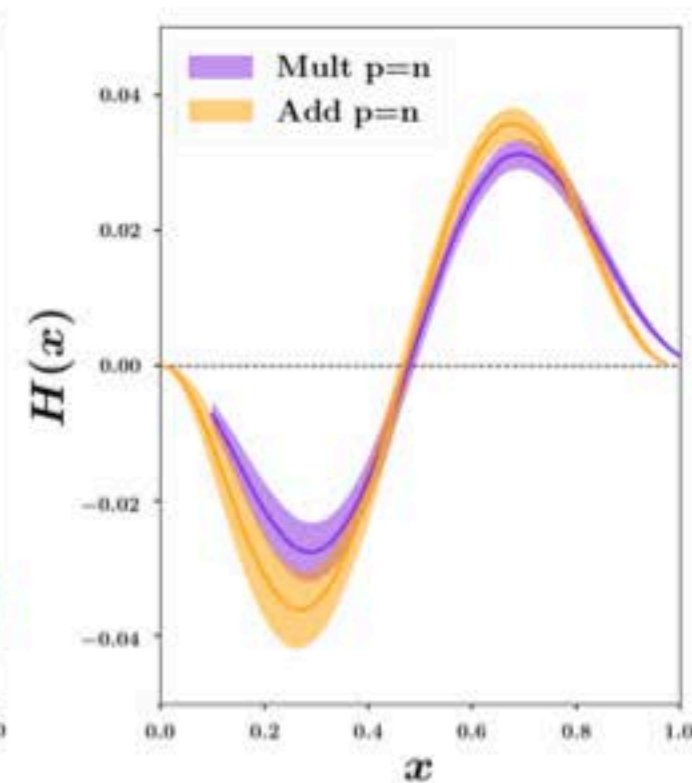
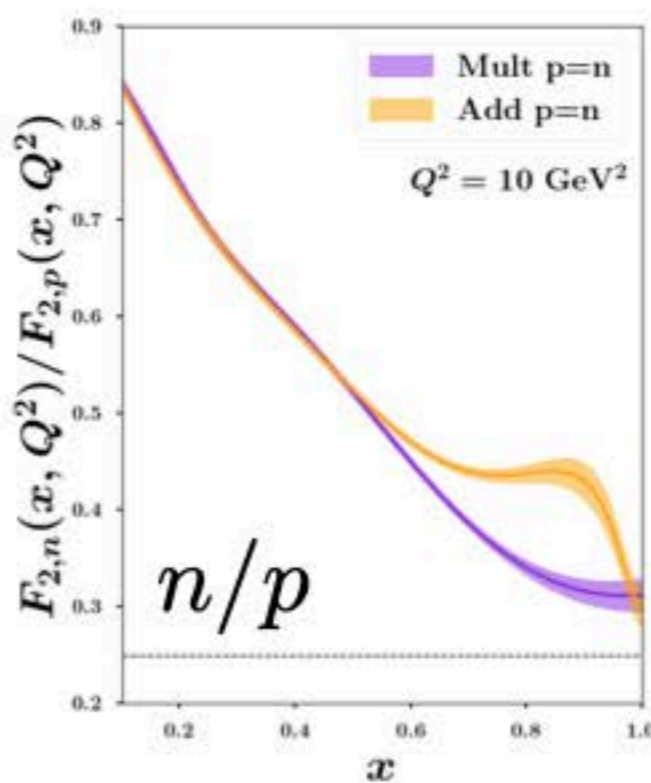
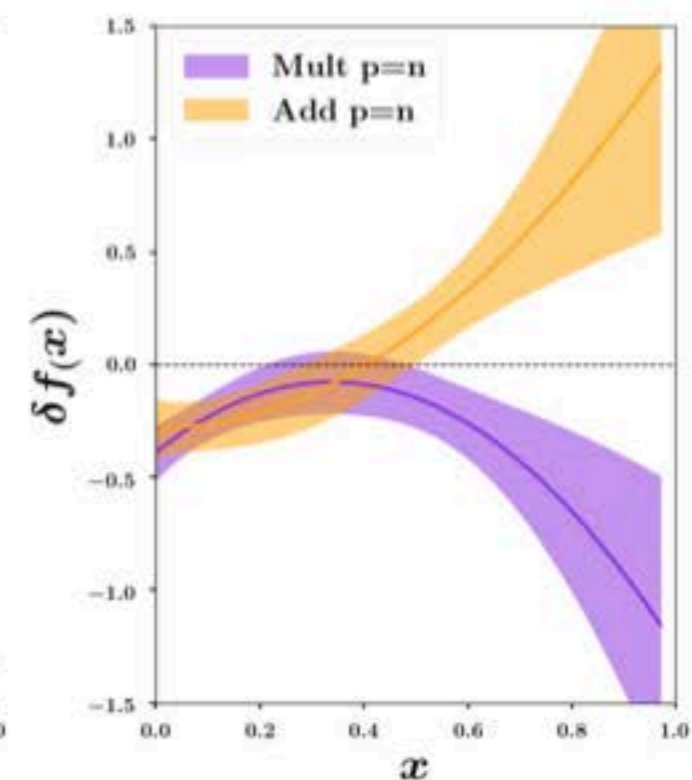
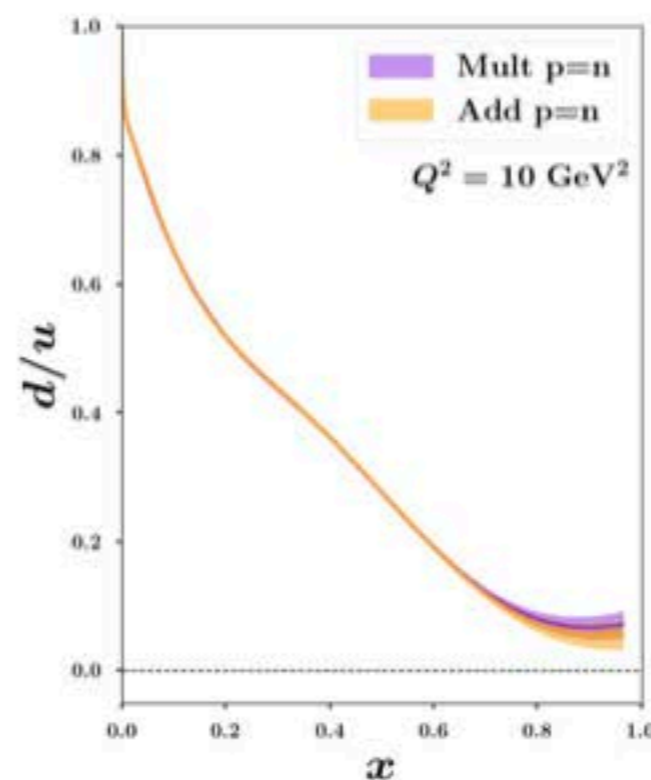
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Add HT

Unnaturally large n/p

BUT smaller d/u than Mult



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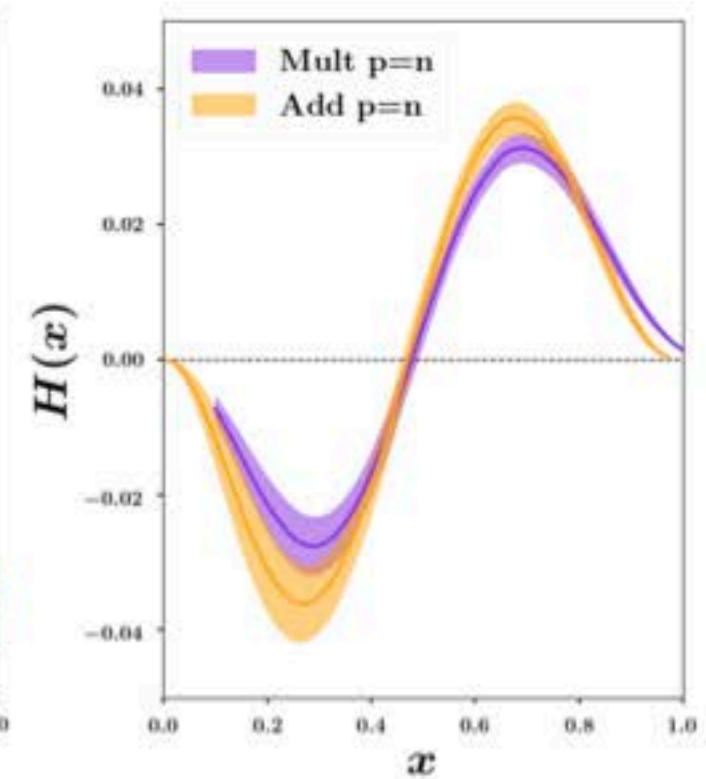
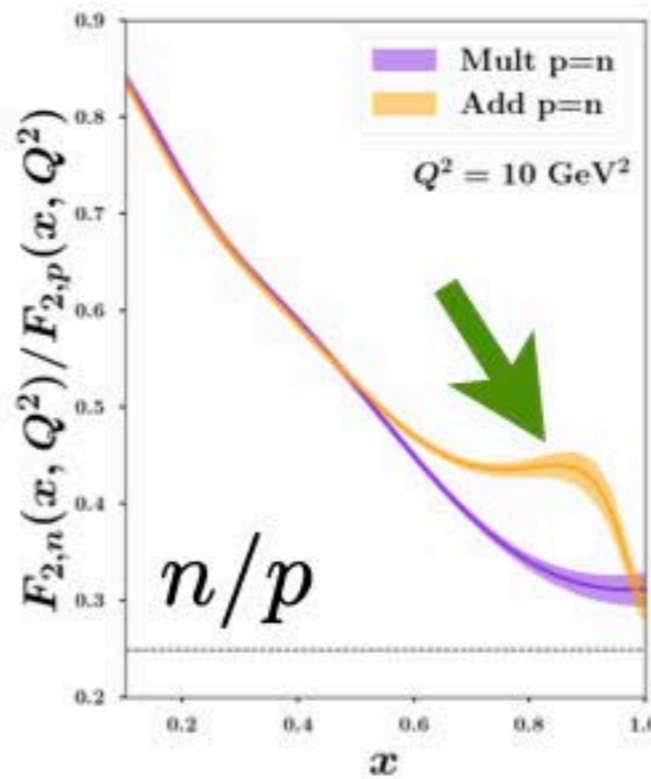
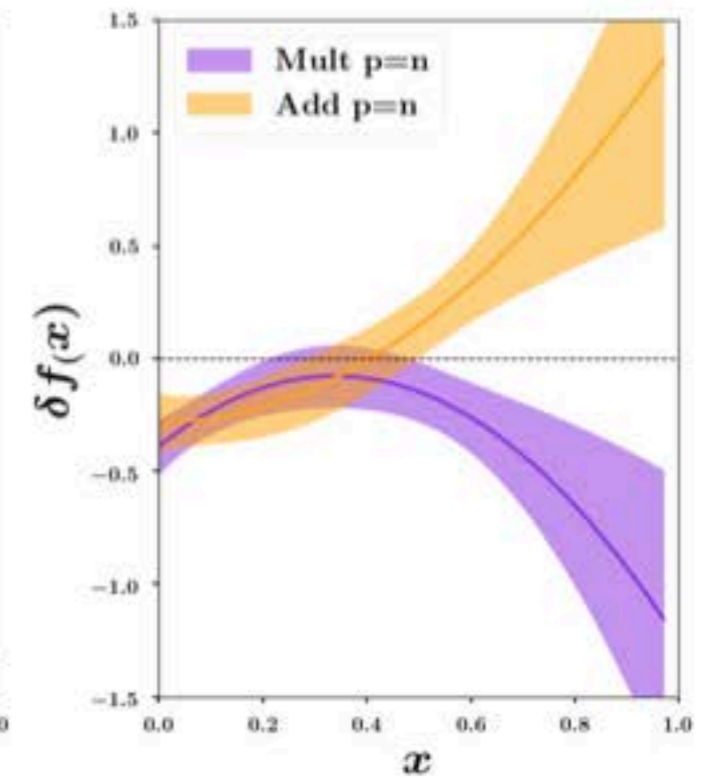
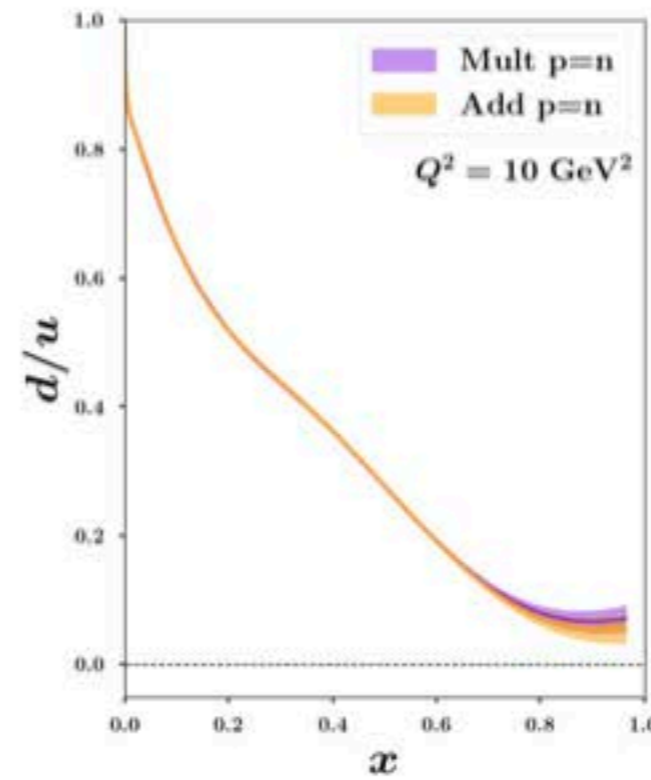
Case 1: isospin symmetry

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Bias identified



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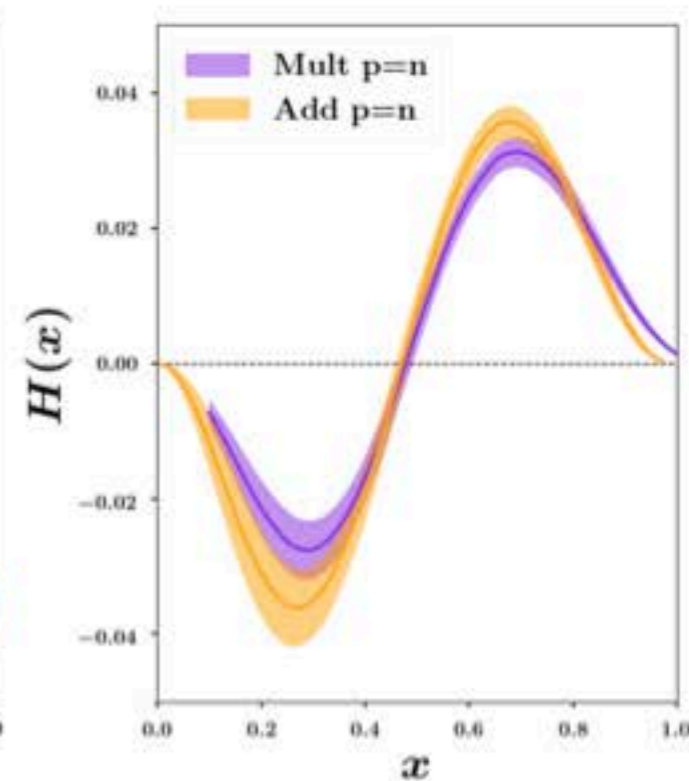
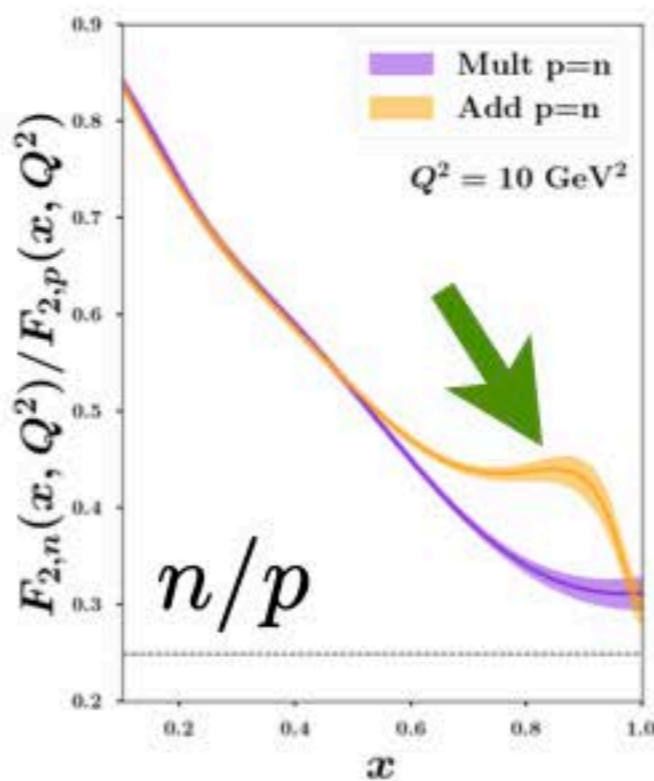
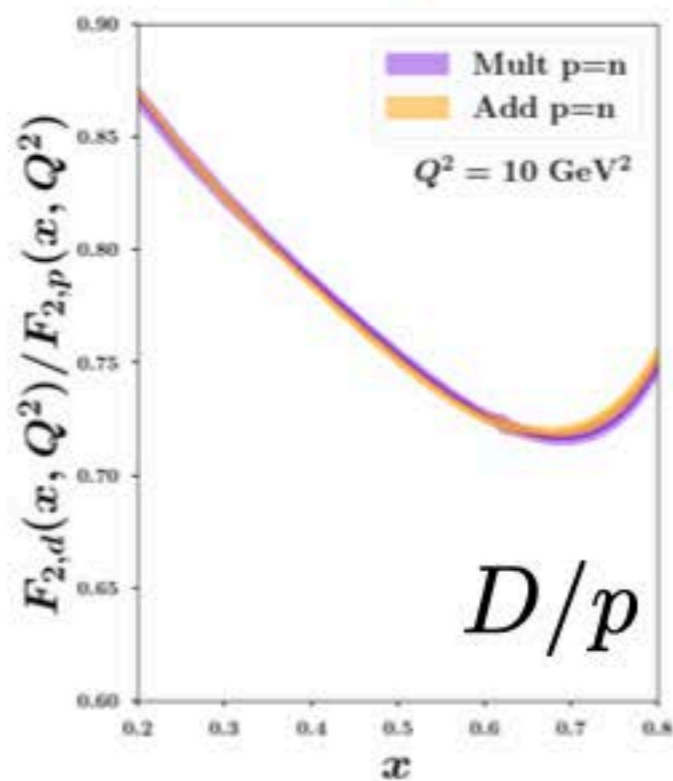
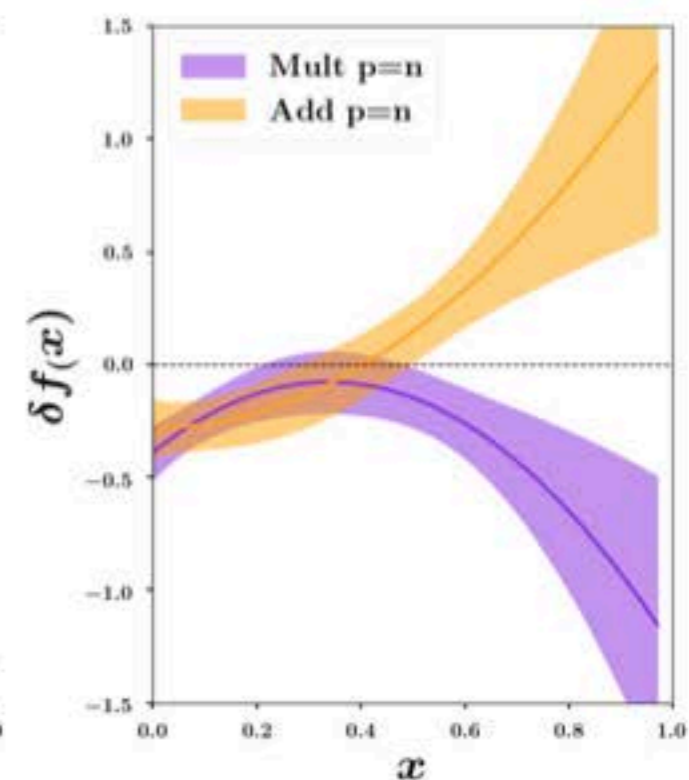
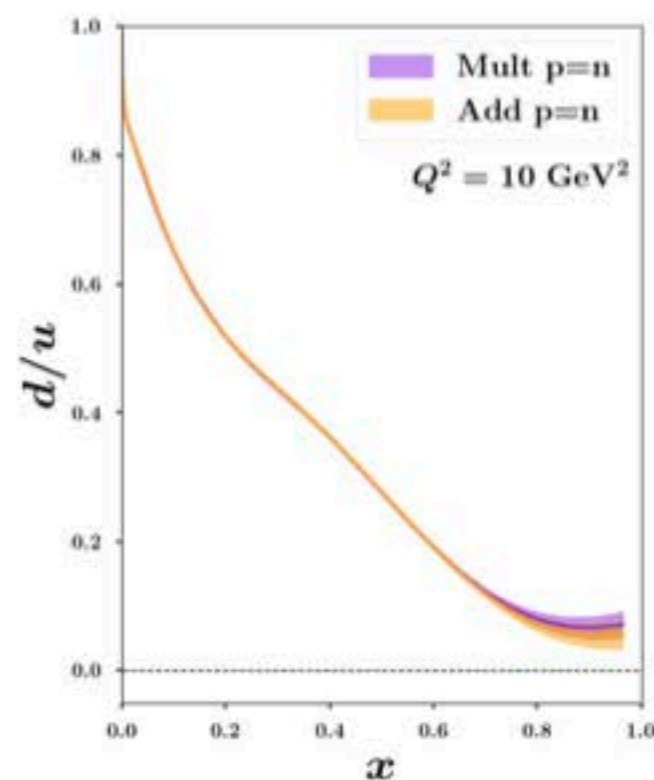
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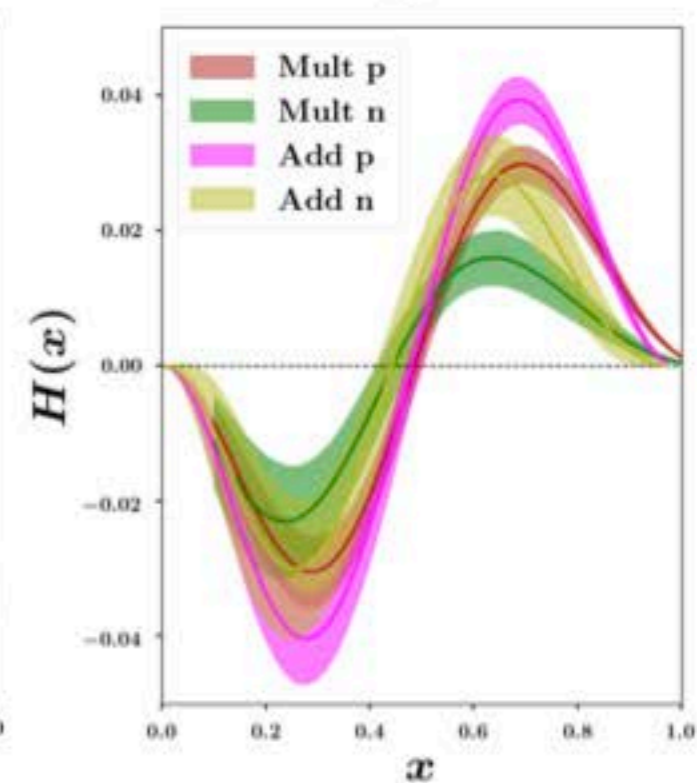
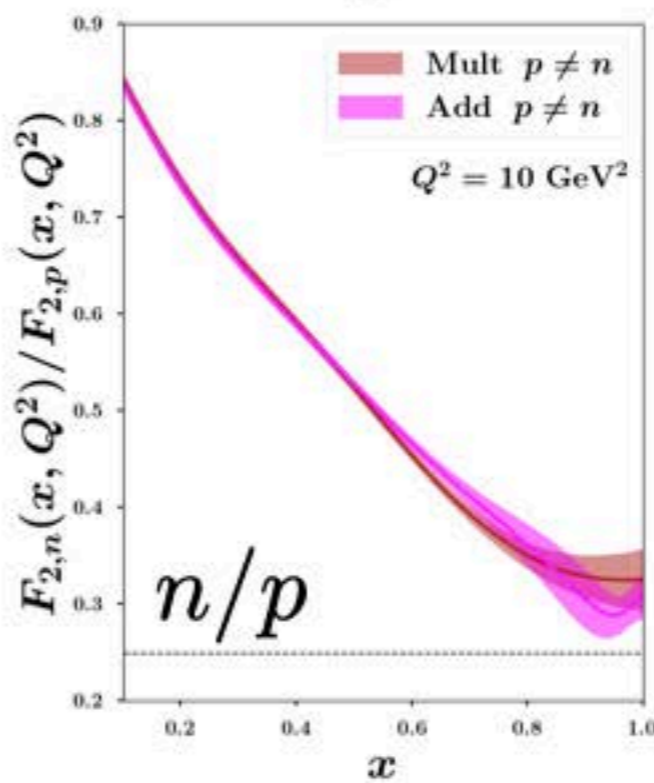
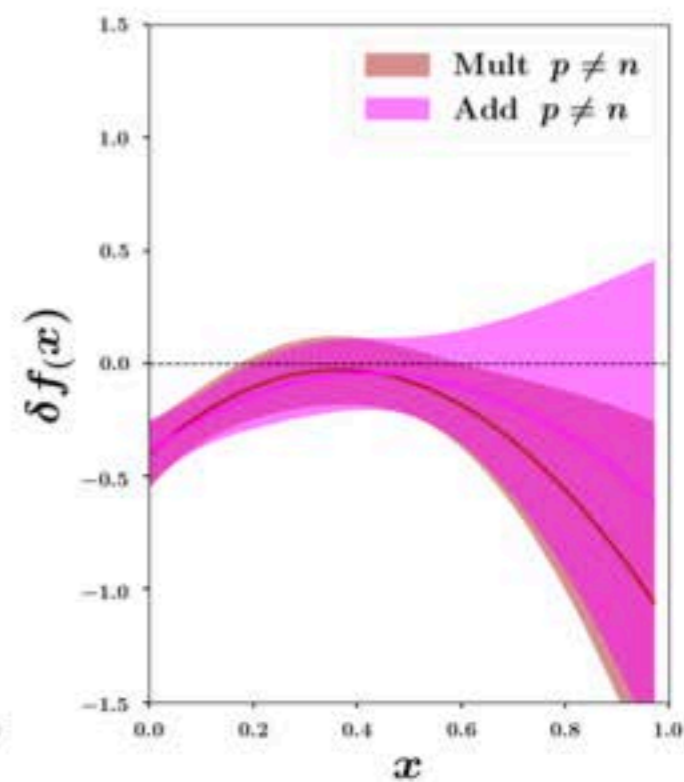
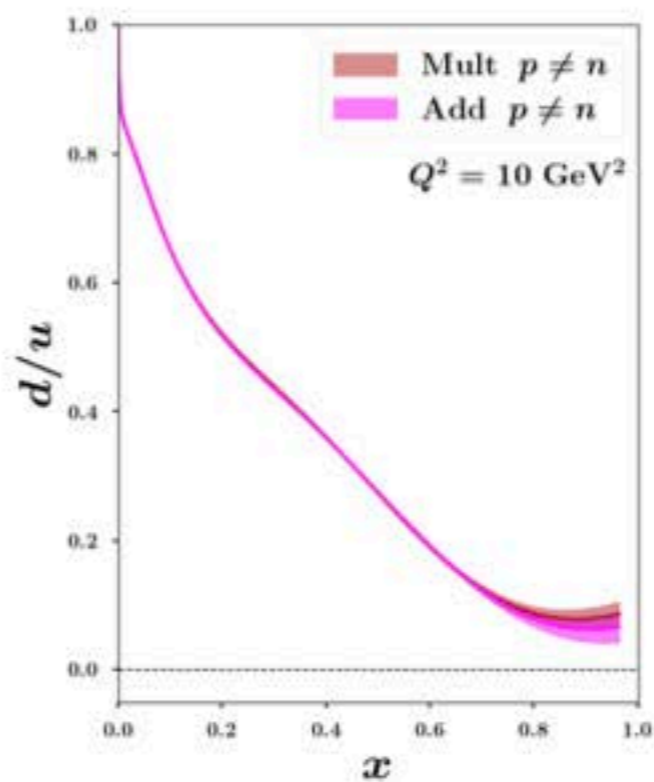
Bias identified

Off-shell compensates n/p bias



Results in the CJ fitting framework

Case 2: isospin breaking

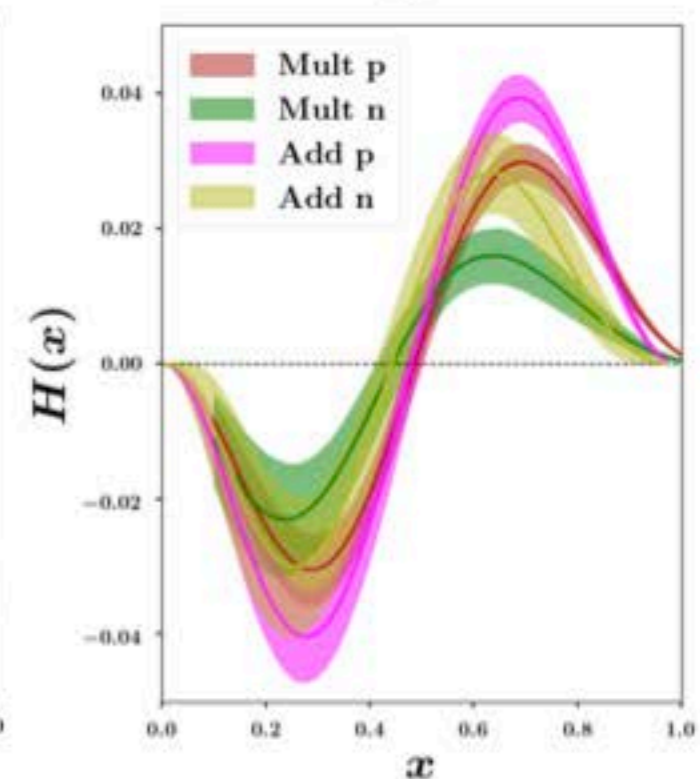
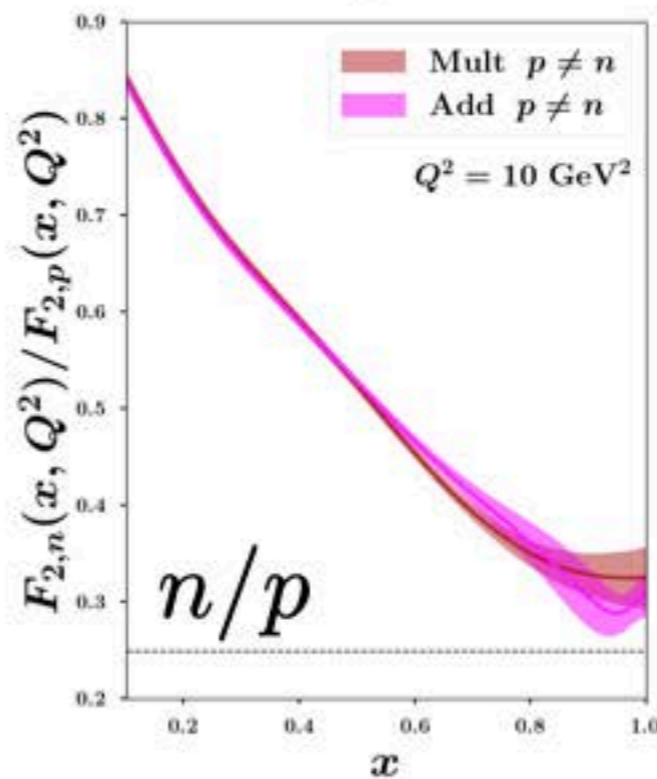
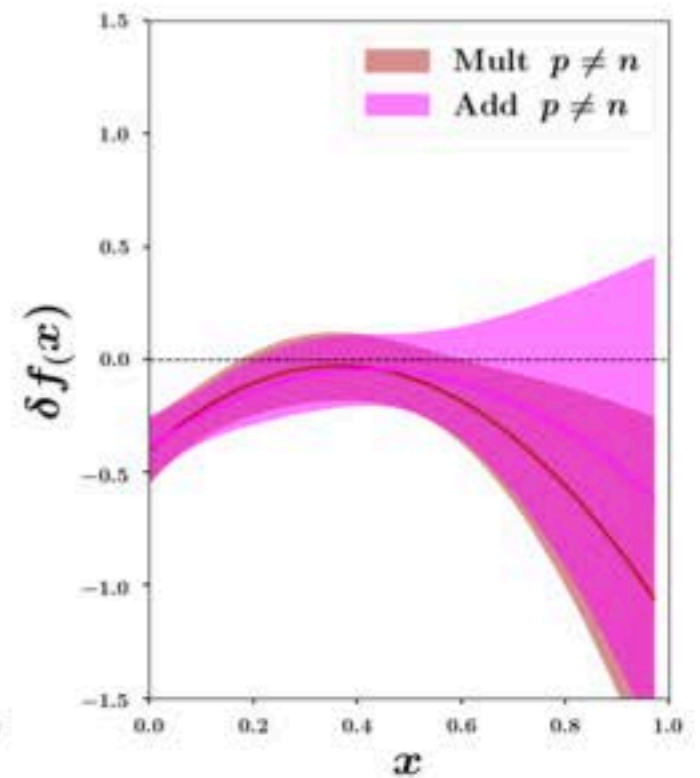
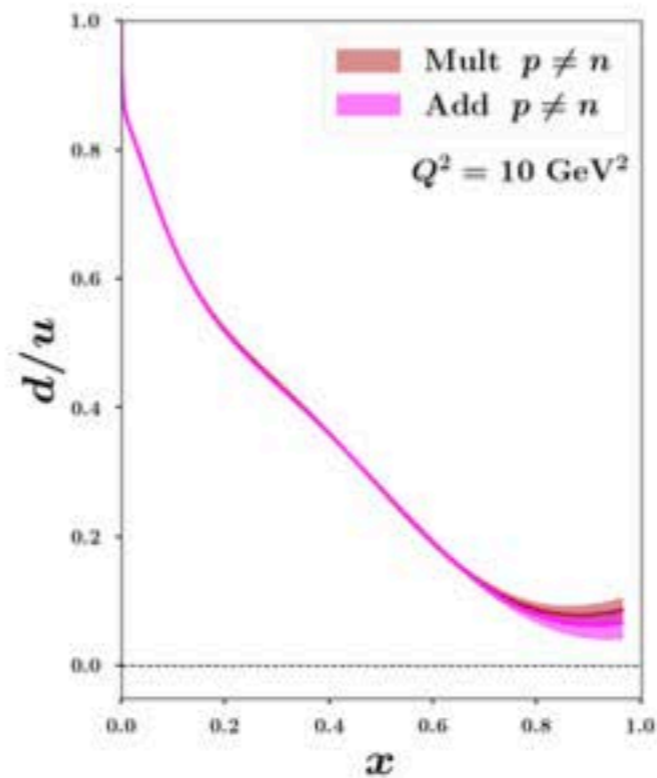


Results in the CJ fitting framework

Case 2: isospin breaking

Compatible n/p

$$H_n(x) \simeq \frac{1}{2}H_p(x)$$



Results in the CJ fitting framework

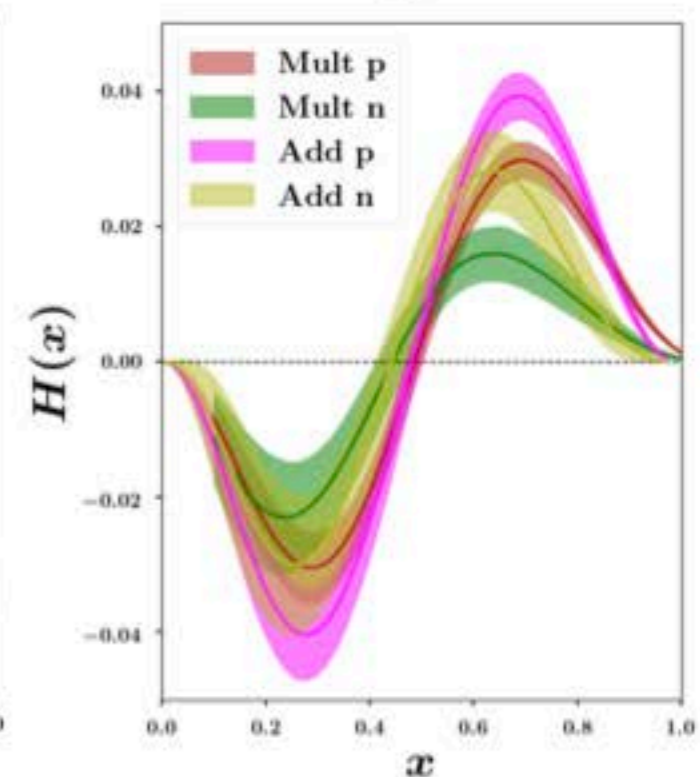
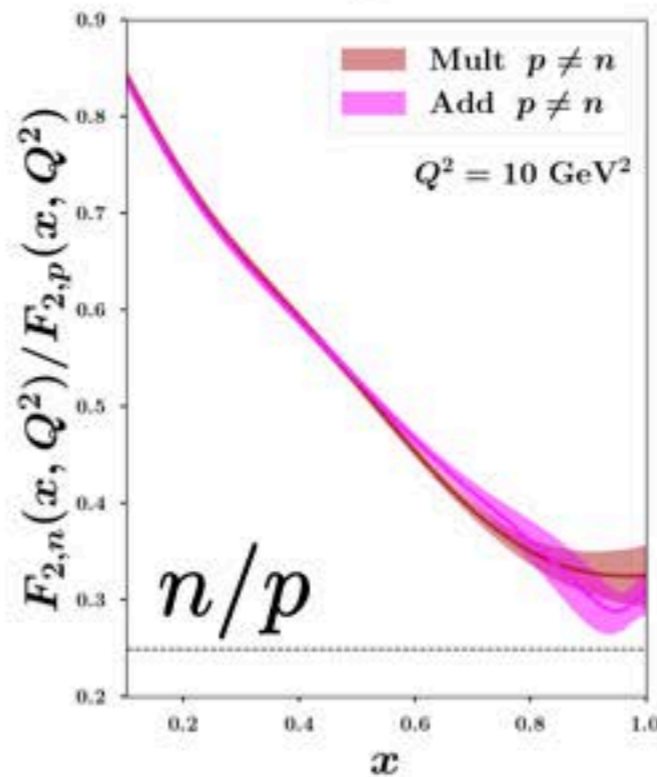
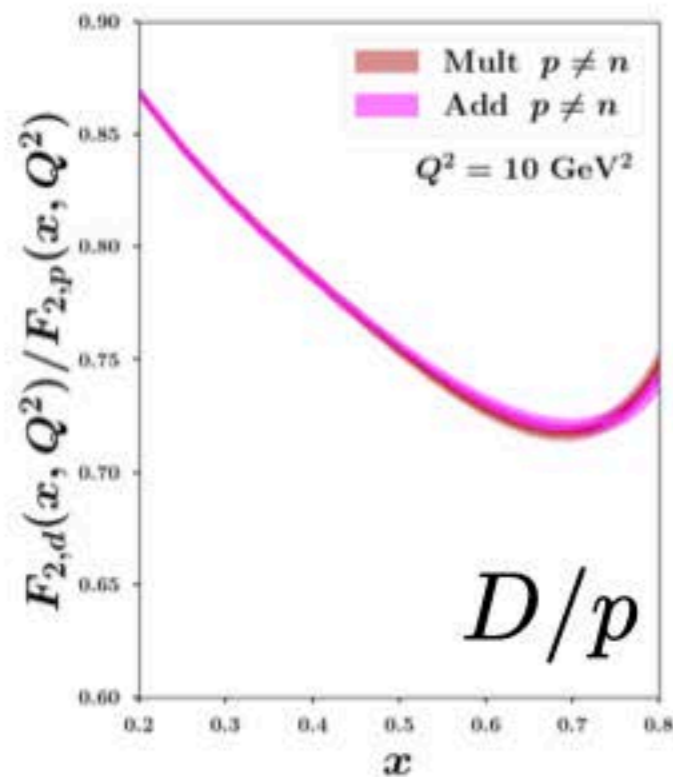
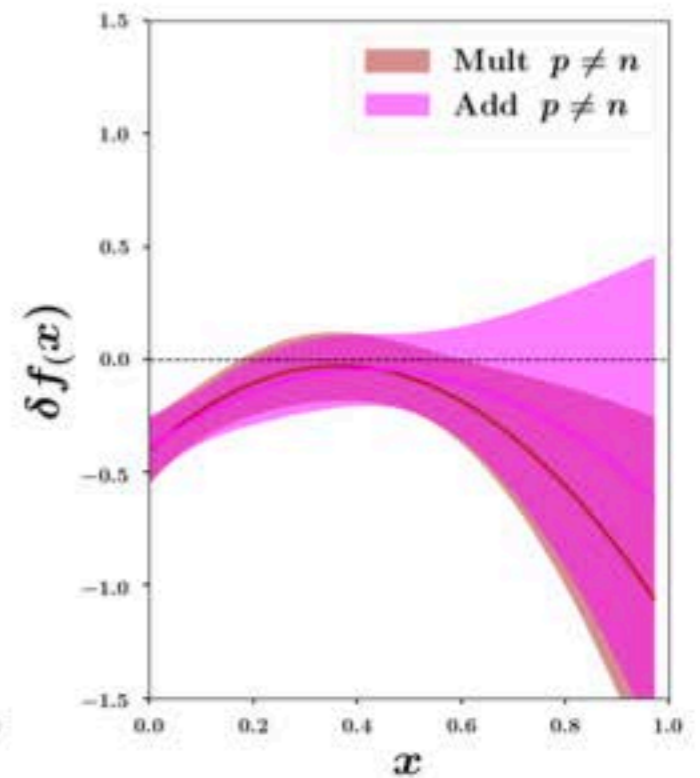
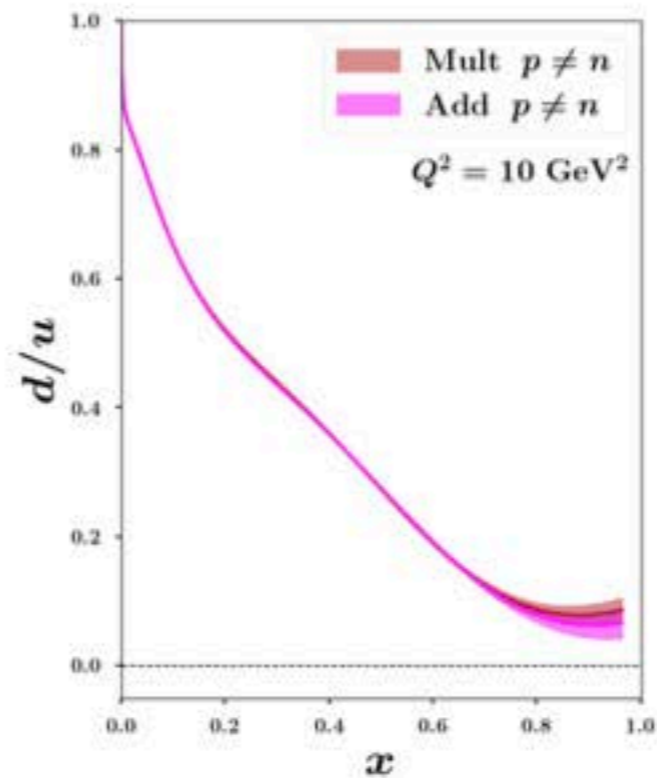
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Bias removed

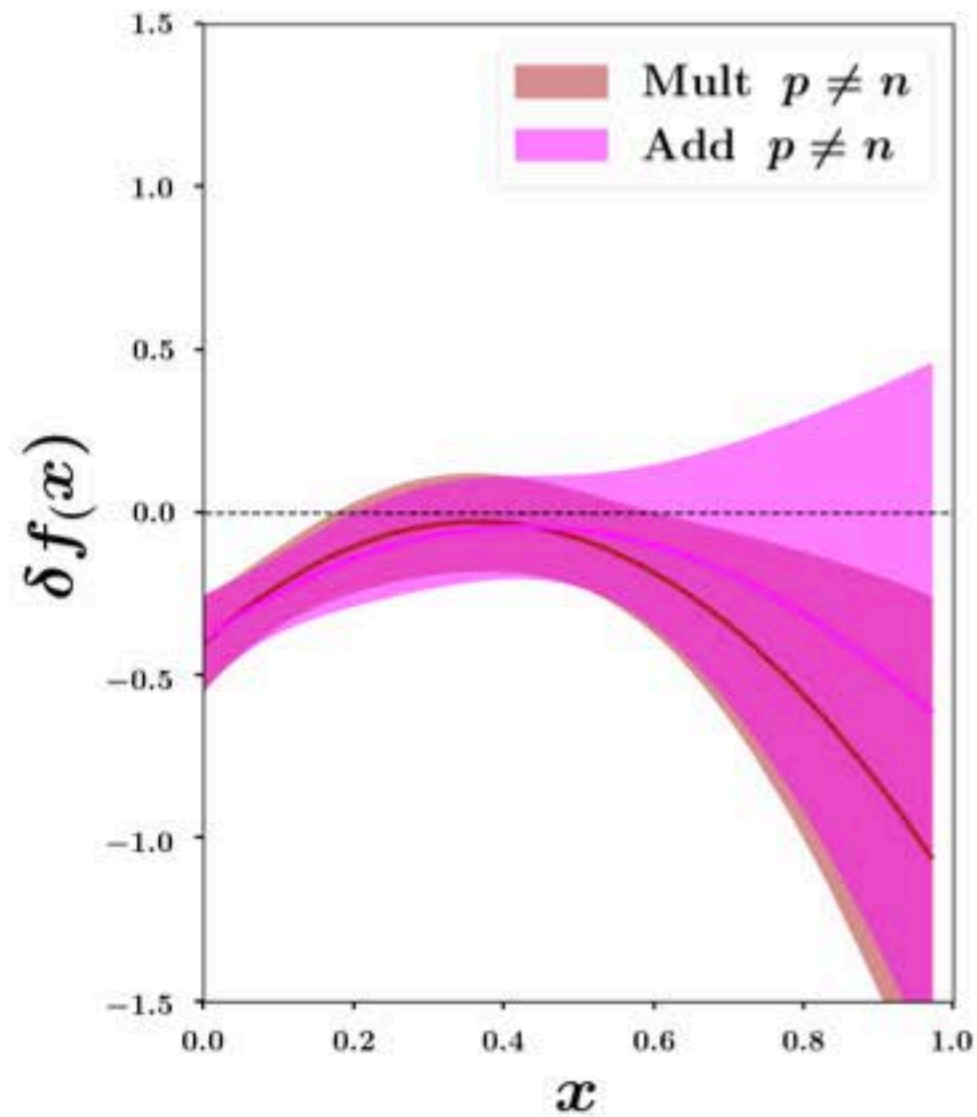
No need of compensation by off-shell
Theory calculation confirmed!



Results in the CJ fitting framework

After removing the bias

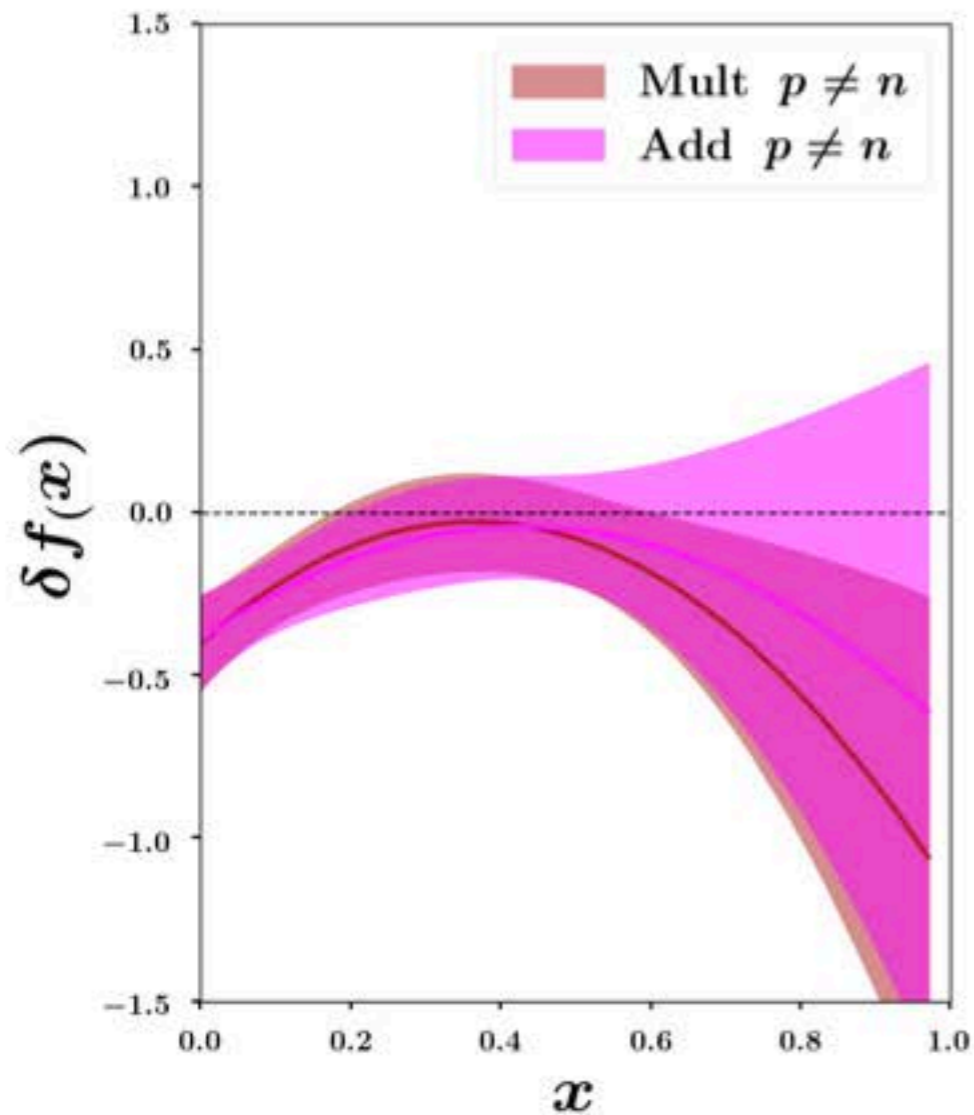
$$\delta f(x) \simeq 0$$



Results in the CJ fitting framework

After removing the bias

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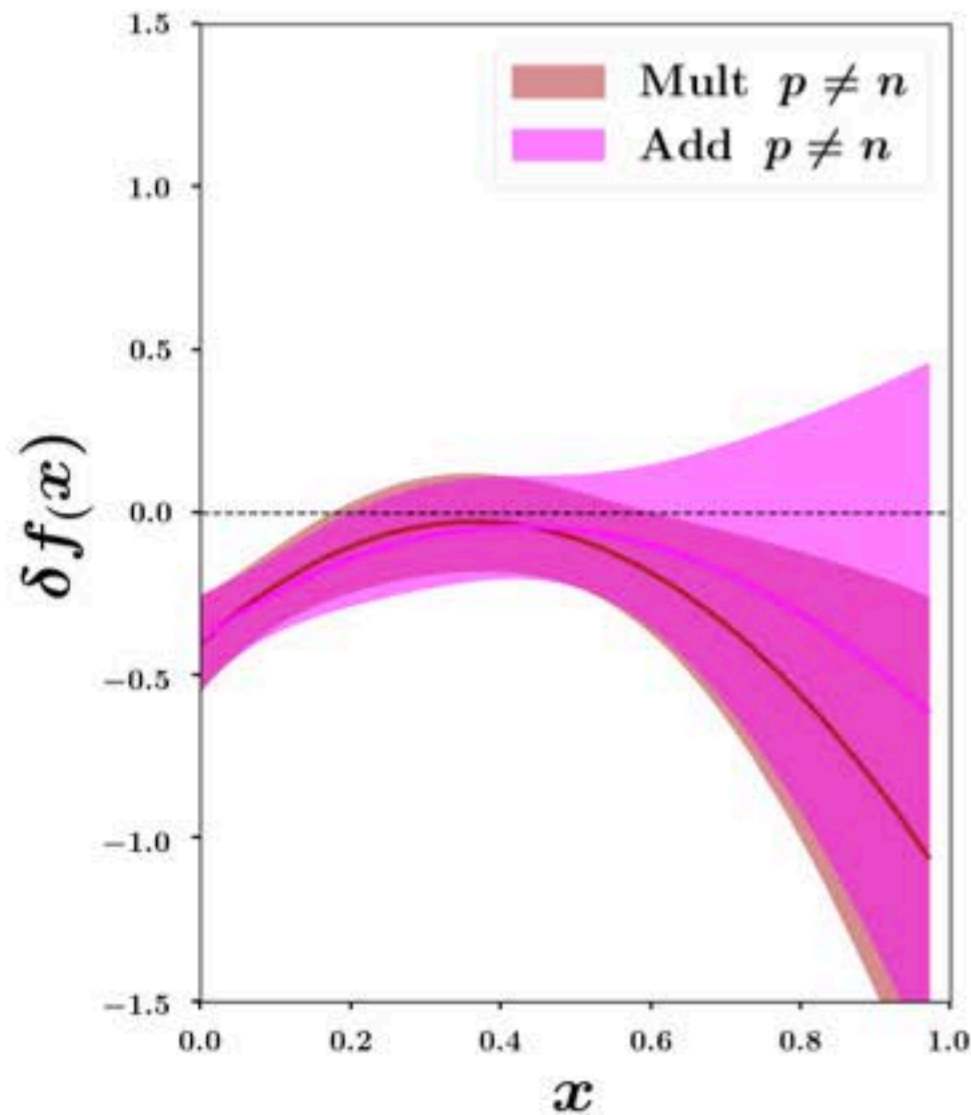


Is the nucleon inside the deuterium
almost on-shell?

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After removing the bias

$$\delta f(x) \simeq 0$$



Is the nucleon inside the deuterium almost on-shell?

Need $A=3$ data to assess flavour dependence of off-shell function

MARATHON data
Adams, et al., PRL 128 (2022)

Other extractions of the off-shell correction

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AKP

Alekhin, Kulagin, Petti, PRD 107 (2023)

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AKP results

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Add HT ($p=n$) as baseline choice

H_2, H_T parametrized

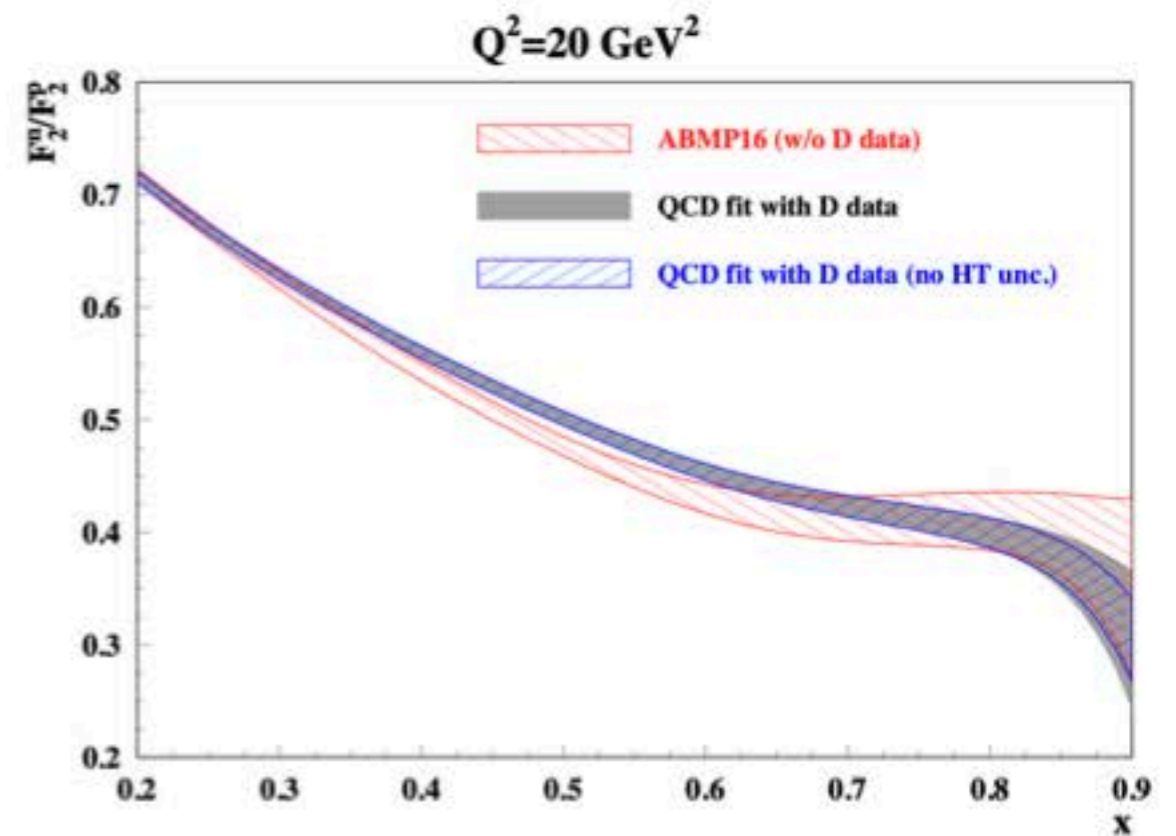
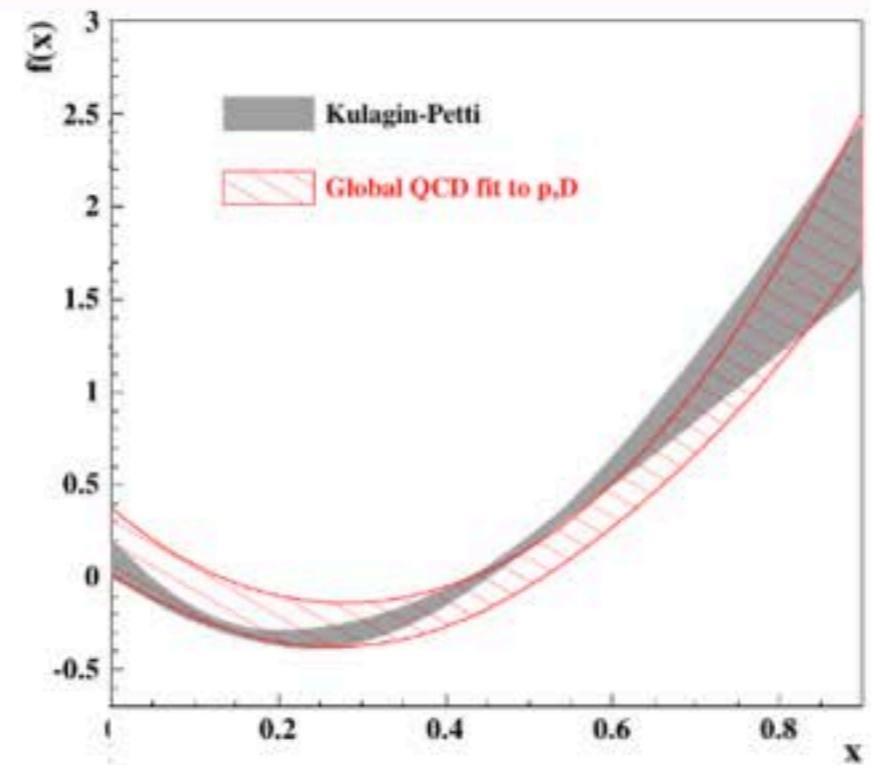
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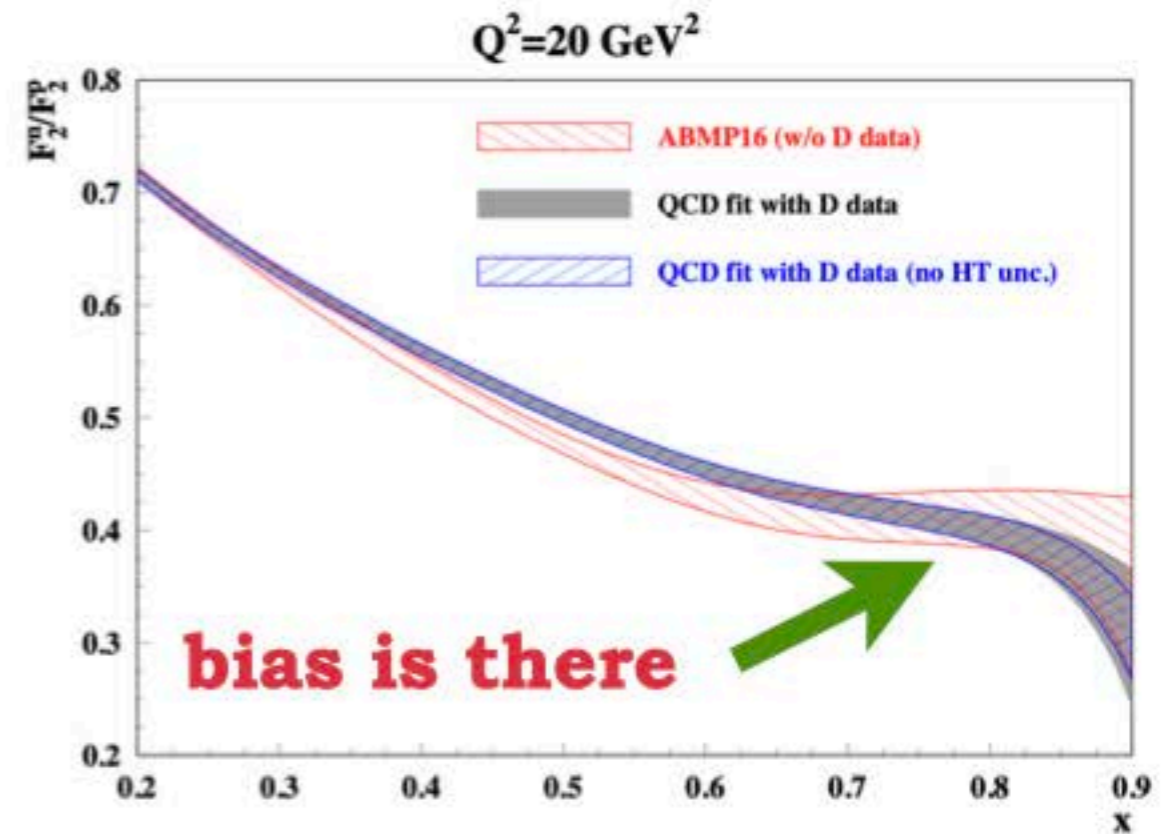
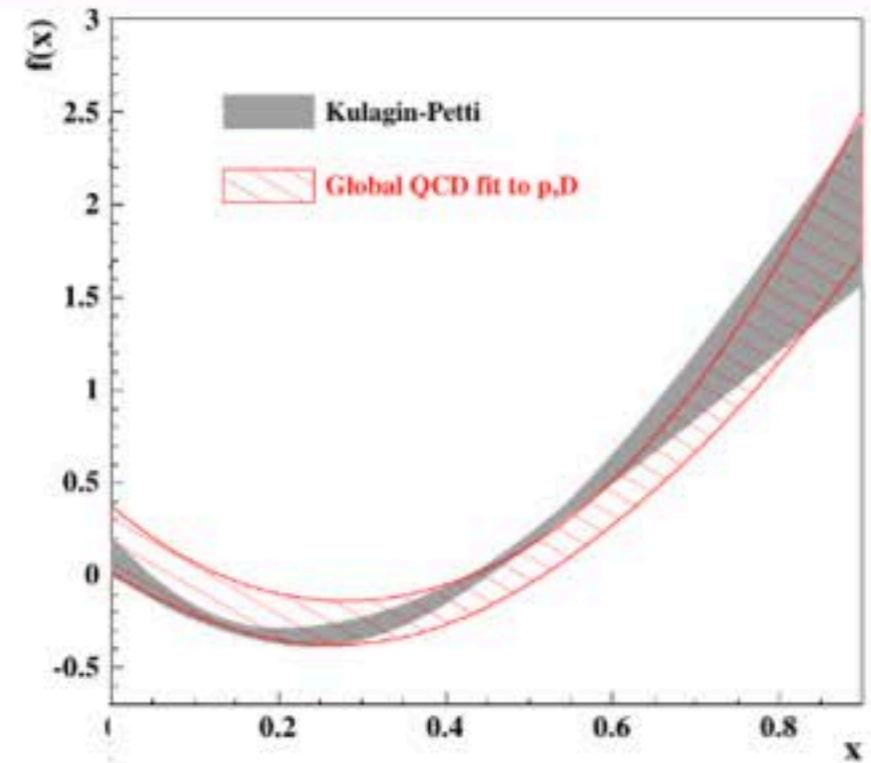
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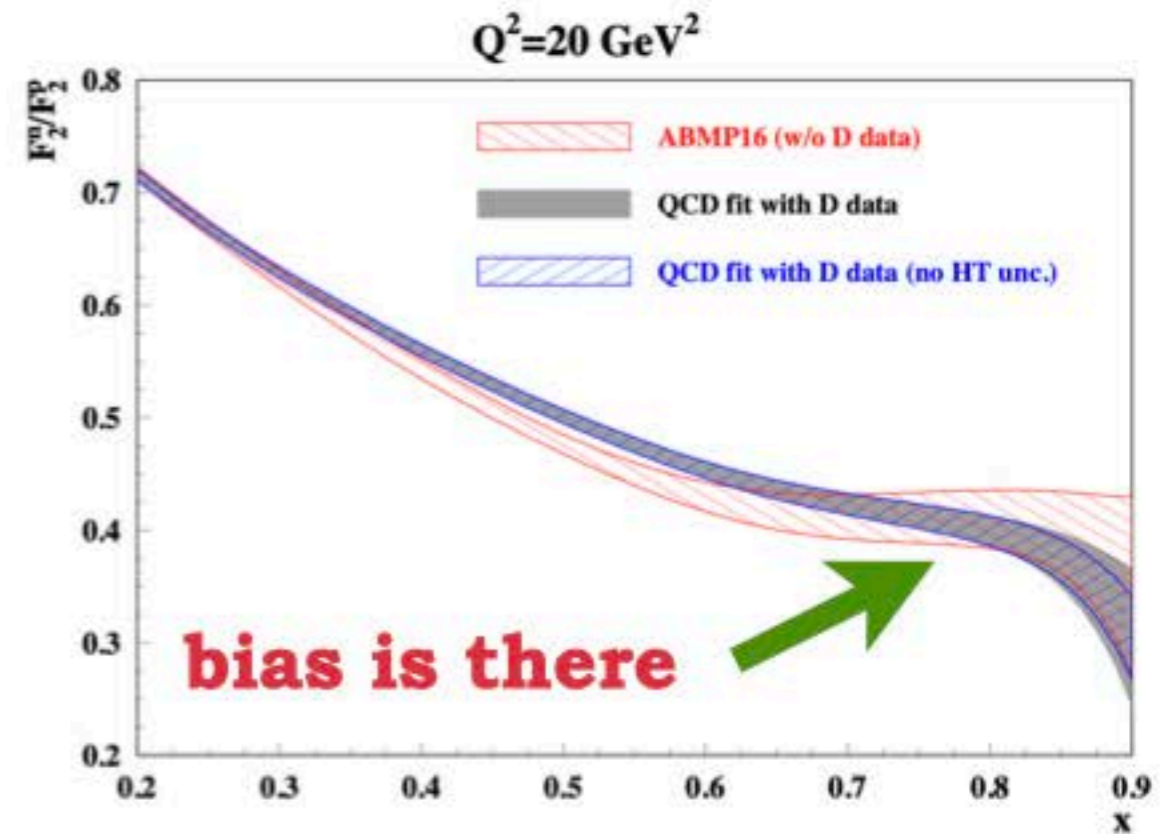
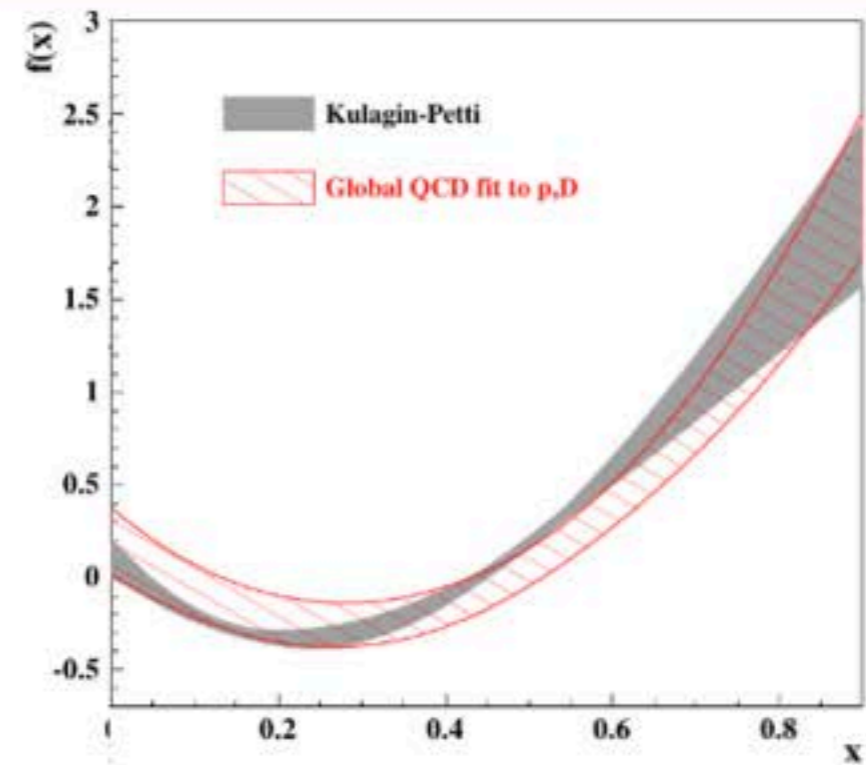
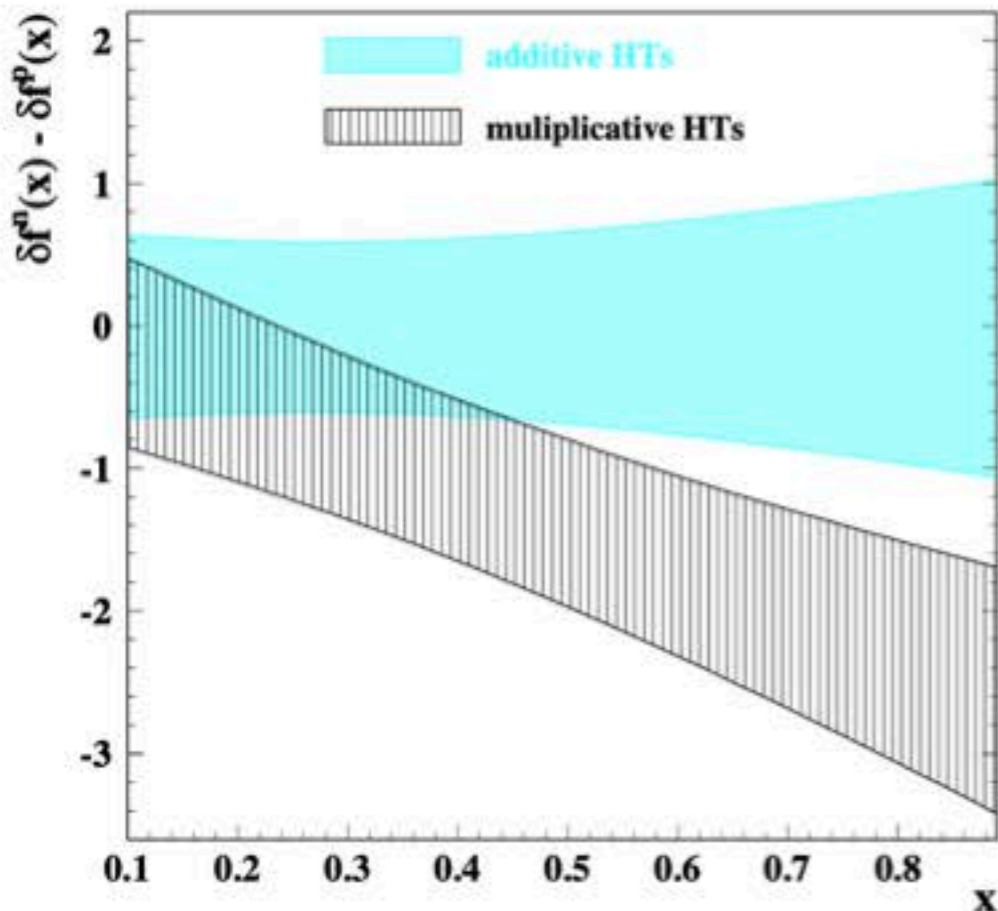
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Alekhin, Kulagin, Petti, PRD 107 (2023)

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Fit to $A=3$ data $\delta F_p \delta F_n$



JAM results

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JAM *Fit including $A=3$ data* δf_u δf_d

JAM Collaboration, PRL 127 (2021)

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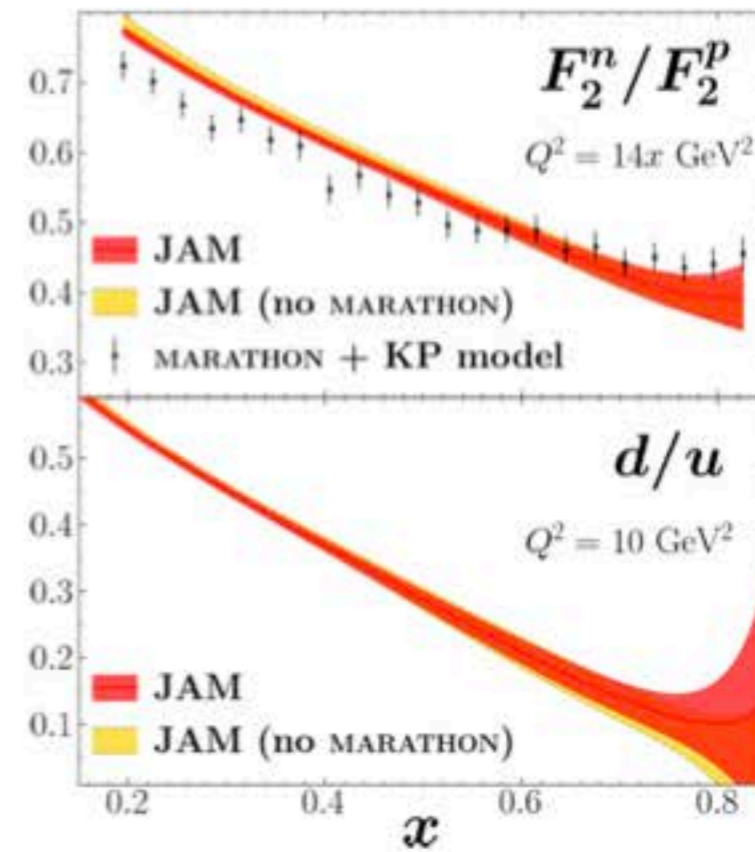
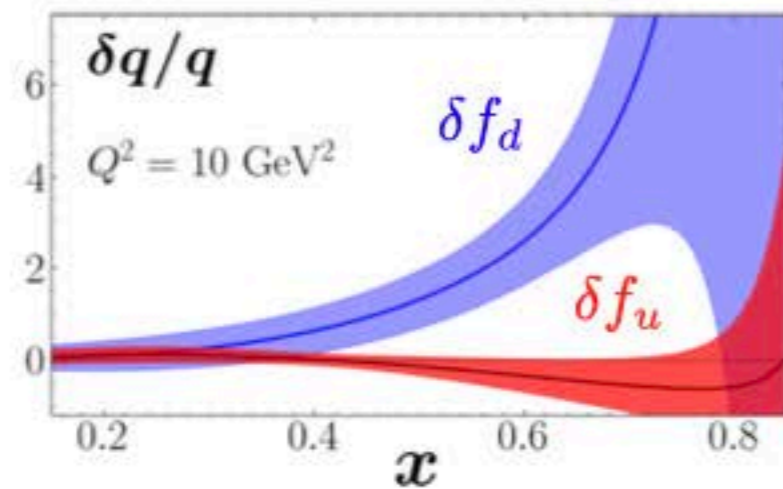
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JAM Collaboration, PRL 127 (2021)

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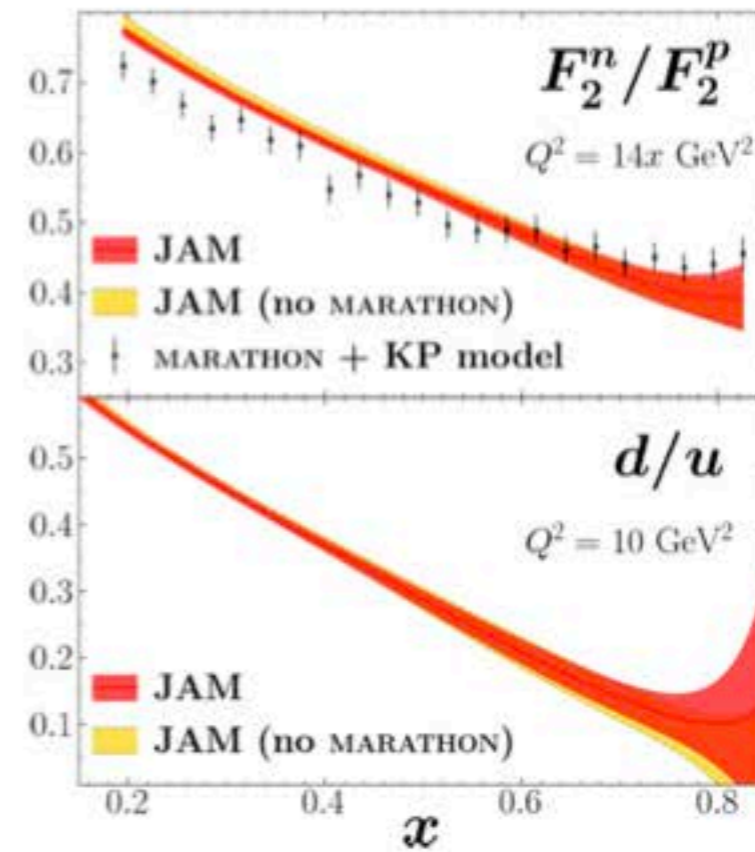
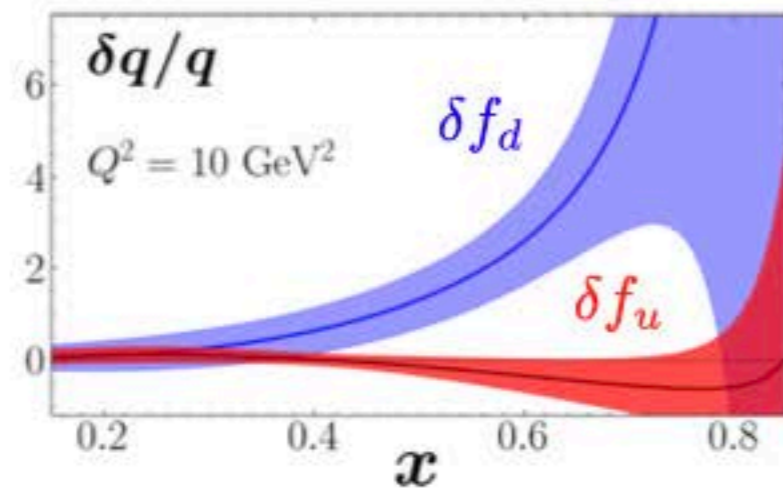


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JAM Collaboration, PRL 127 (2021)

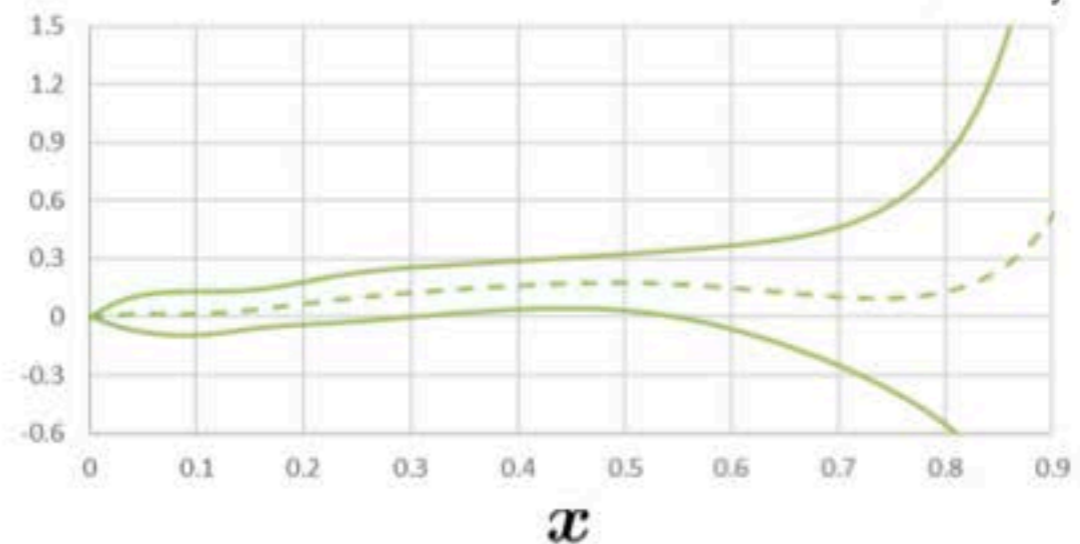
Mult HT (p=n) as default choice



$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u + d}$$

Isoscalar offshell function (JAM)

Courtesy of C. Cocuzza



Need more information

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Experimental data differential on the off-shell proton virtuality p^2 would allow us to pin down the off-shell correction in a more clean way



Conclusions

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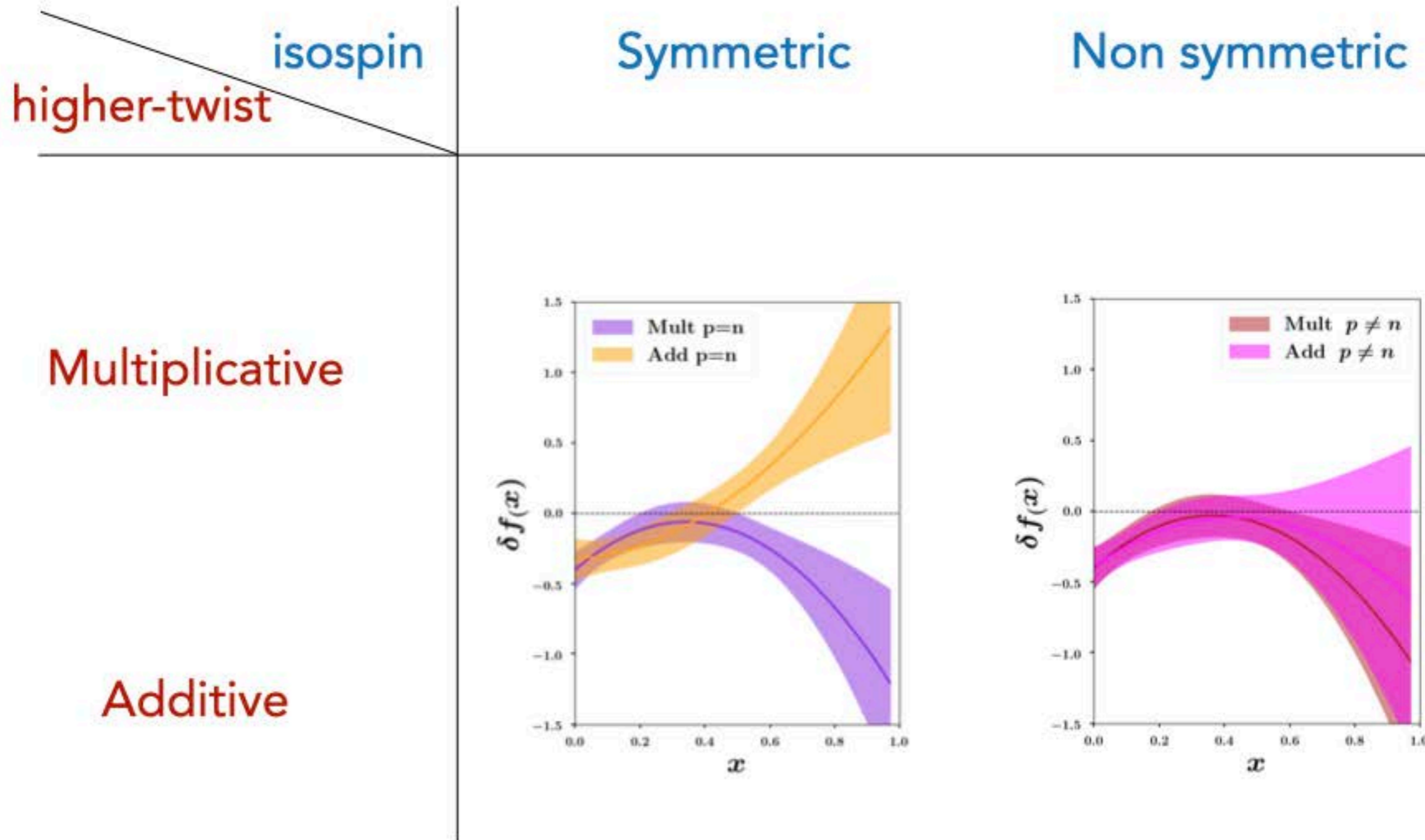
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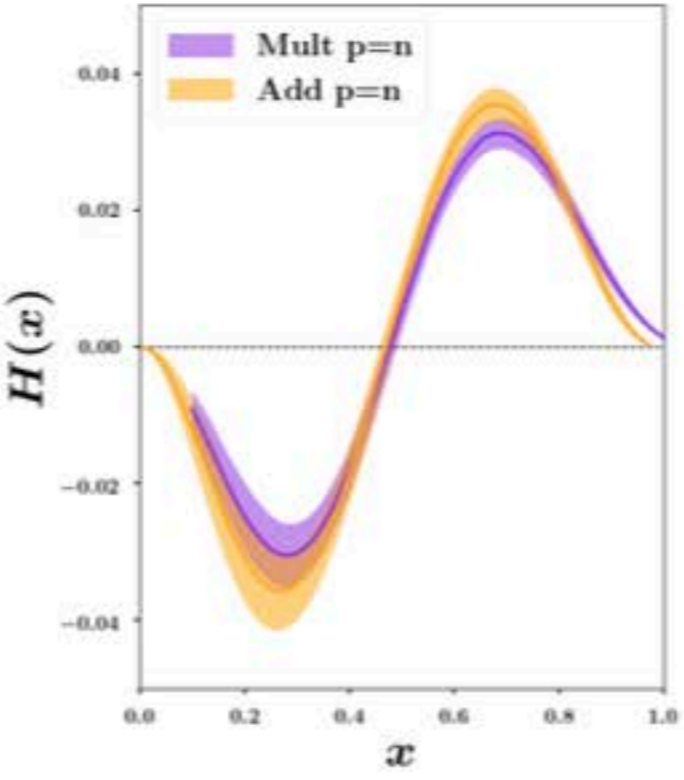
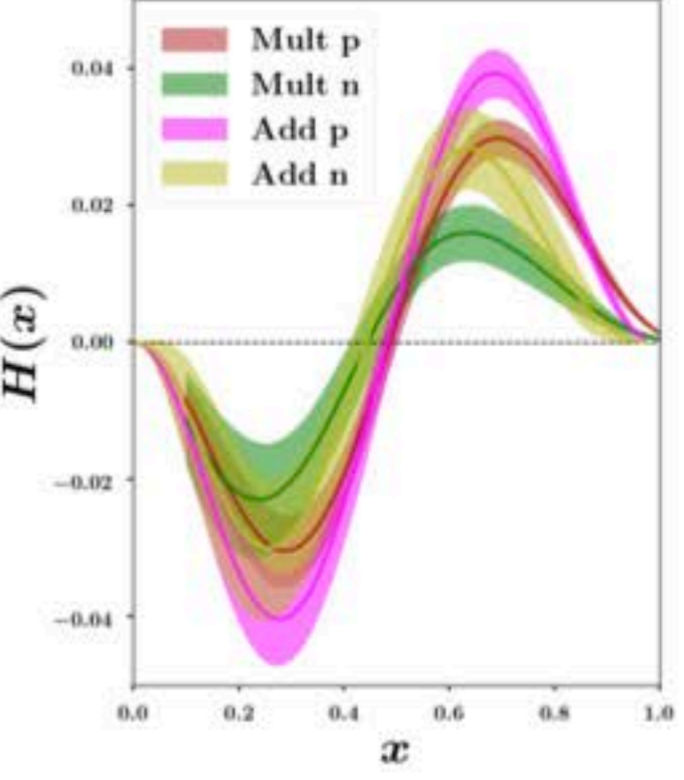
JAM A=3 fit not in agreement with AKP. Average result compatible with CJ

Backup

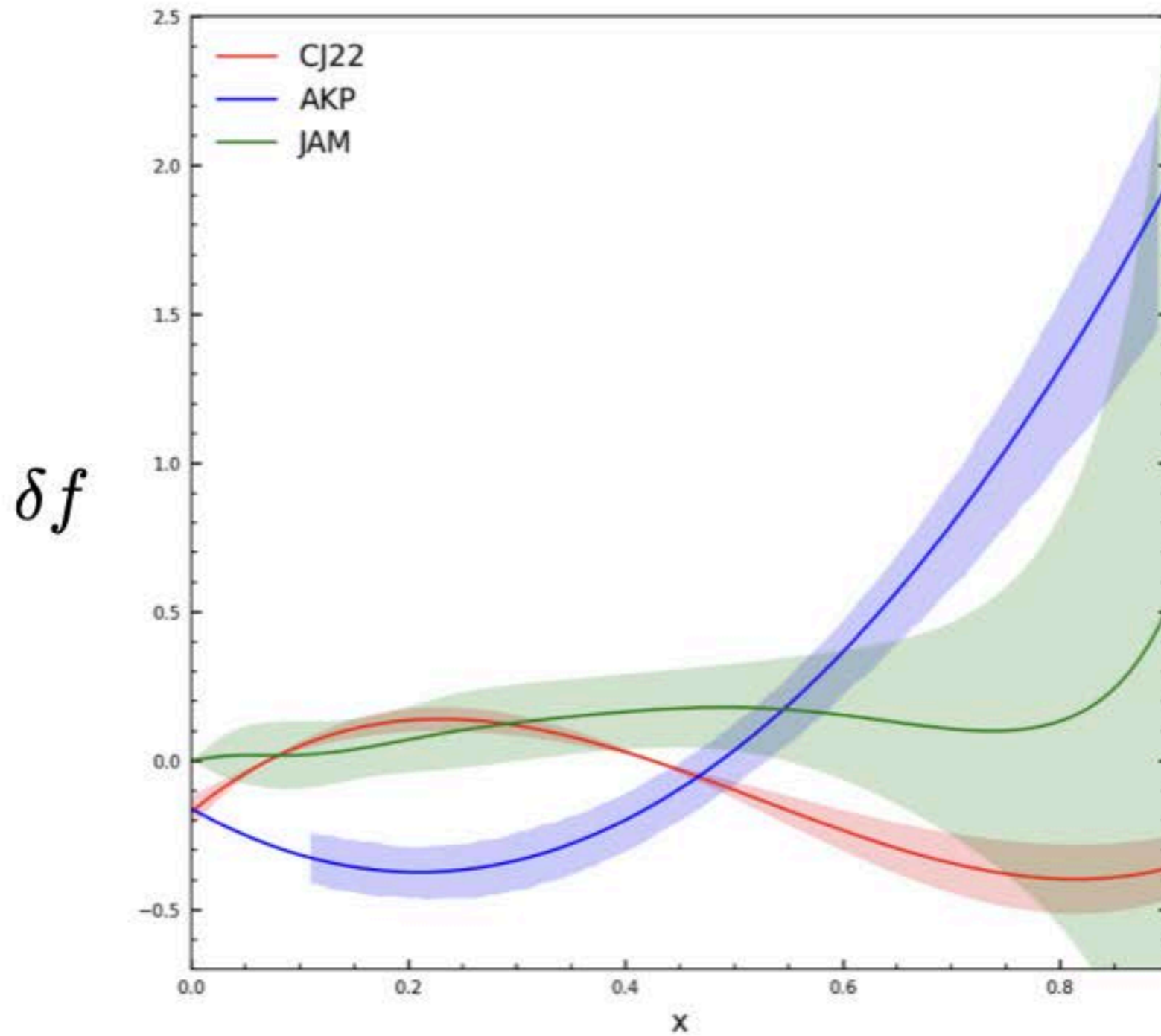
Off-shell table



Higher-Twist table

higher-twist	isospin	Symmetric	Non symmetric
<p data-bbox="159 915 647 997">Multiplicative</p> <p data-bbox="249 1453 557 1528">Additive</p>	<p data-bbox="1001 676 1580 741">$\tilde{H} = F_{2,N}(x, Q^2)H_{\text{Mult}}(x)$</p> 	<p data-bbox="1852 676 2483 741">$\delta\tilde{H} = F_{2,N}(x, Q^2)\delta H_{\text{Mult}}(x)$</p> 	

AKP vs CJ



Some implementation differences

Theoretical choices \longrightarrow

Corrections (increasing-x) \downarrow

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST $x < 0.1$	(same)
smearing	Paris	AV18	AV18 $x > 0.1$	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----