CTEQ Spring Meeting 2024

CTEQ

QCD Factorization of Hard Exclusive Processes and Tomographic Image of Proton

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In collaboration with Jianwei Qiu

JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902
PRD 109 (2024) 074023

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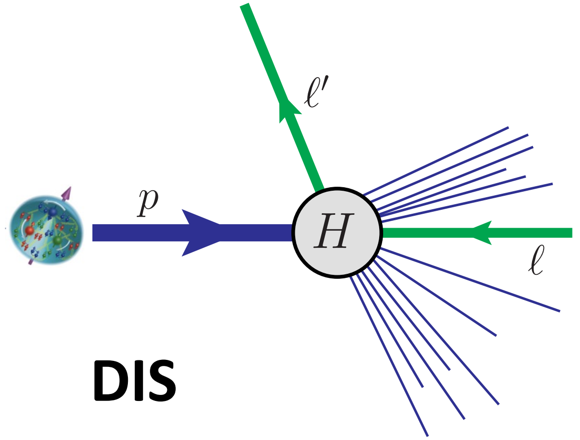
Jun/4/2024

U.S. DEPARTMENT OF ENERGY | Office of Science

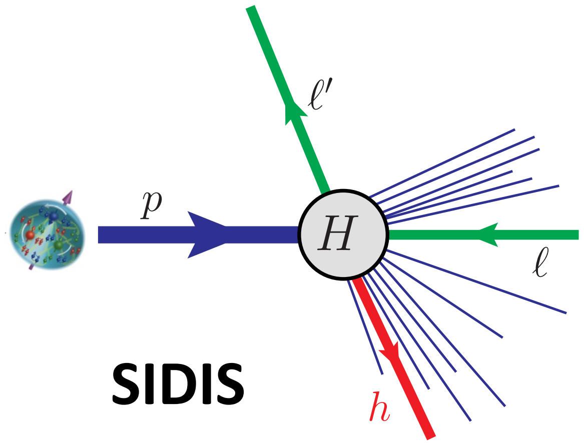
JSA

Hard probes and proton structure

Inclusive scattering

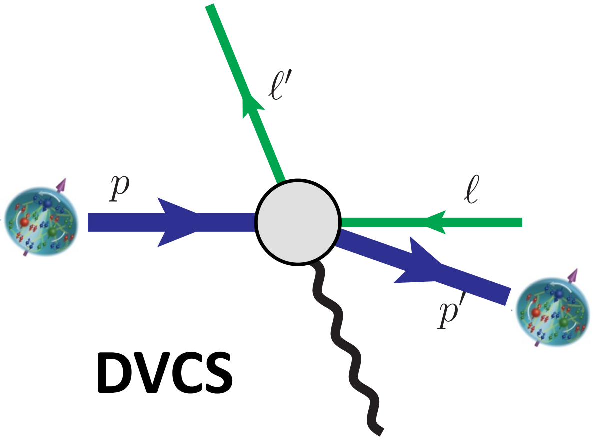


DIS

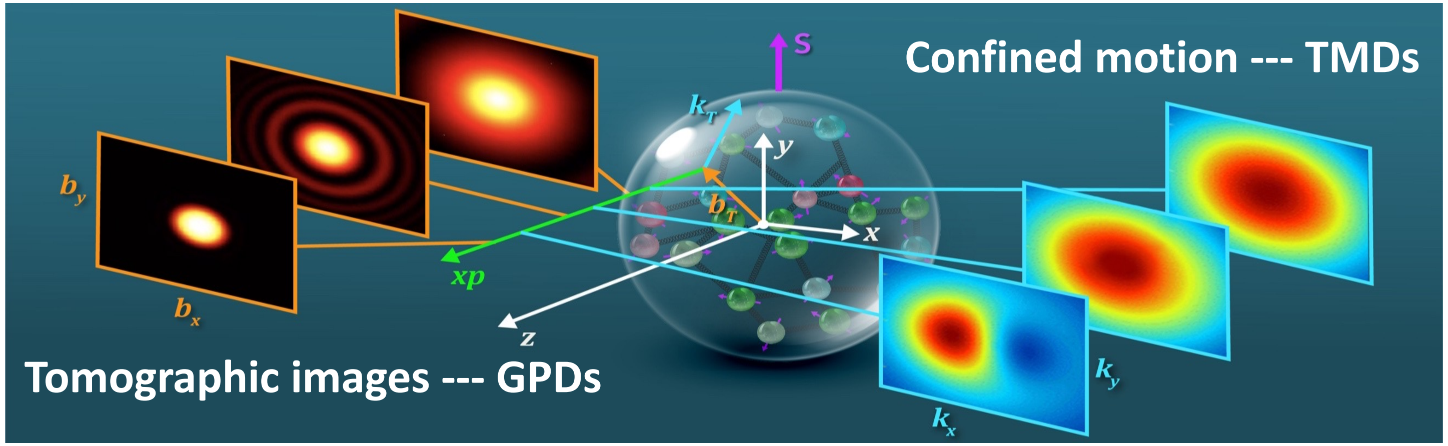


SIDIS

Exclusive diffraction

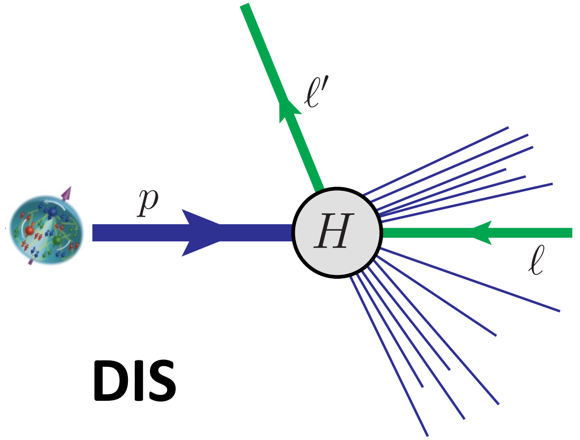


DVCS



Exclusive processes and GPDs

Inclusive scattering



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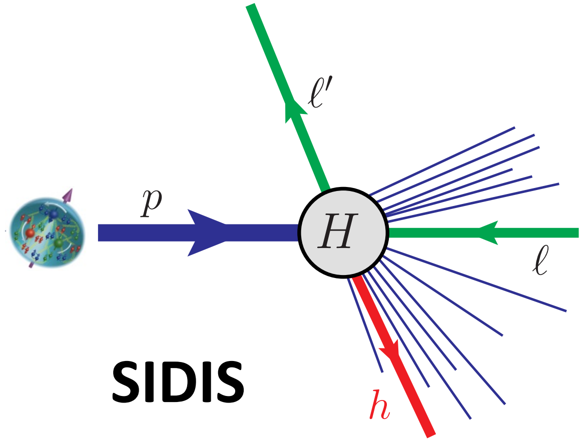

SIDIS

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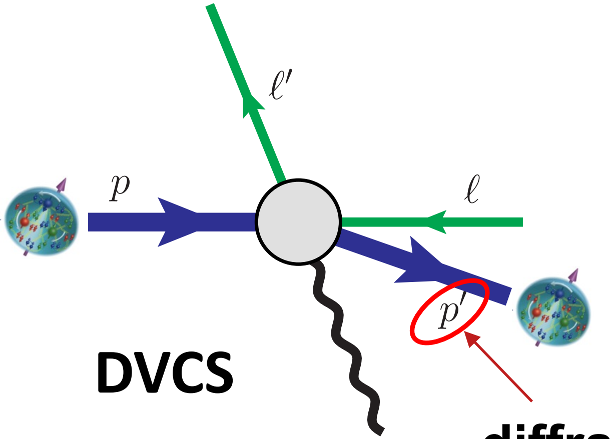
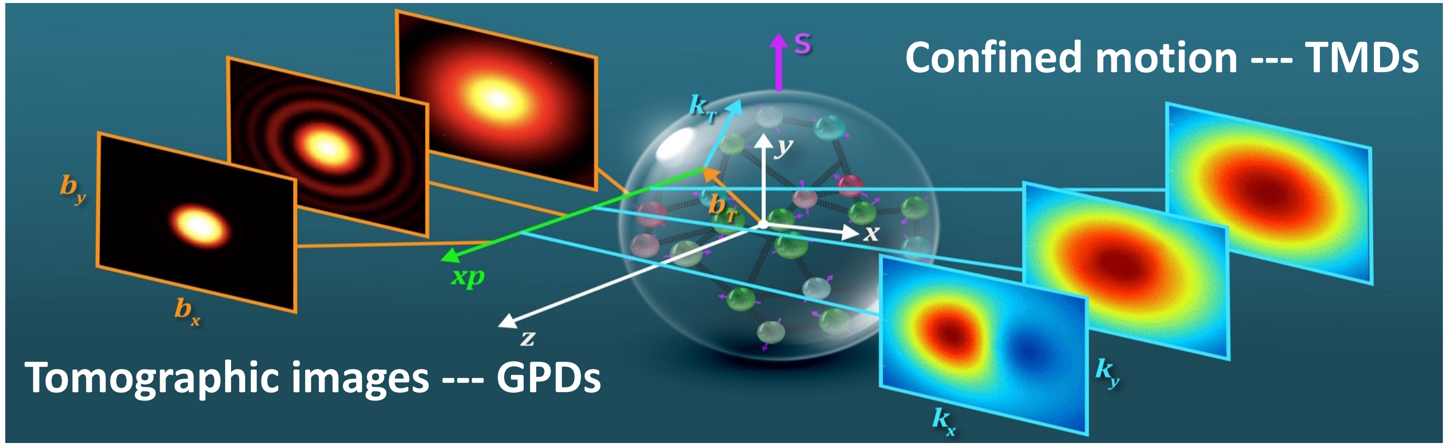
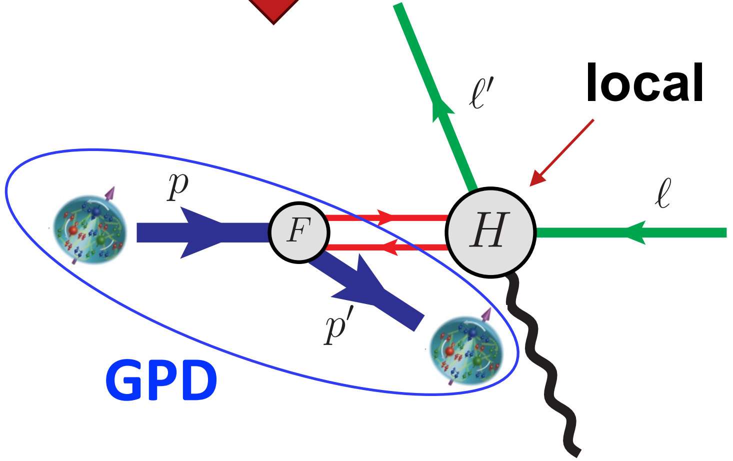


DVCS

diffraction





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Exclusive processes and GPDs

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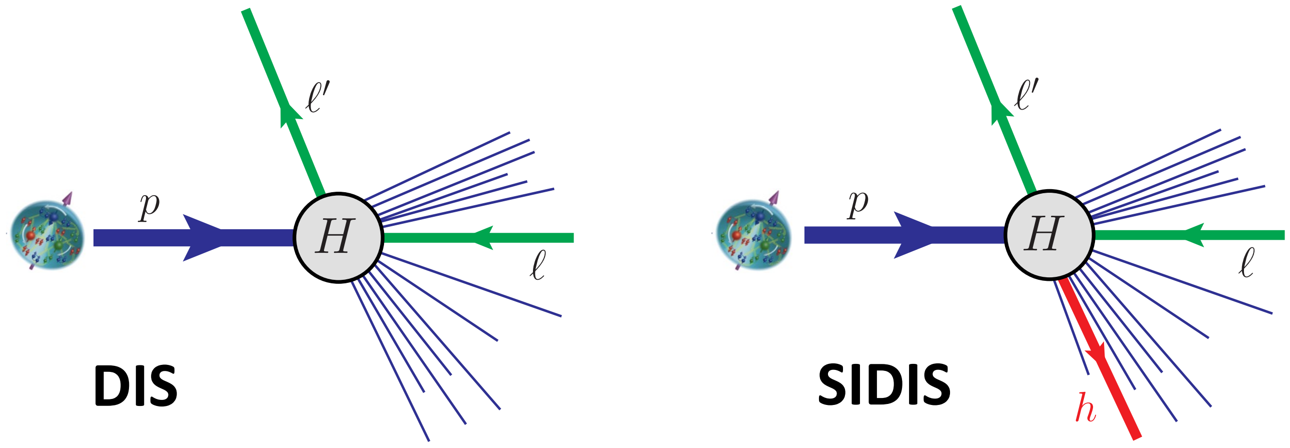

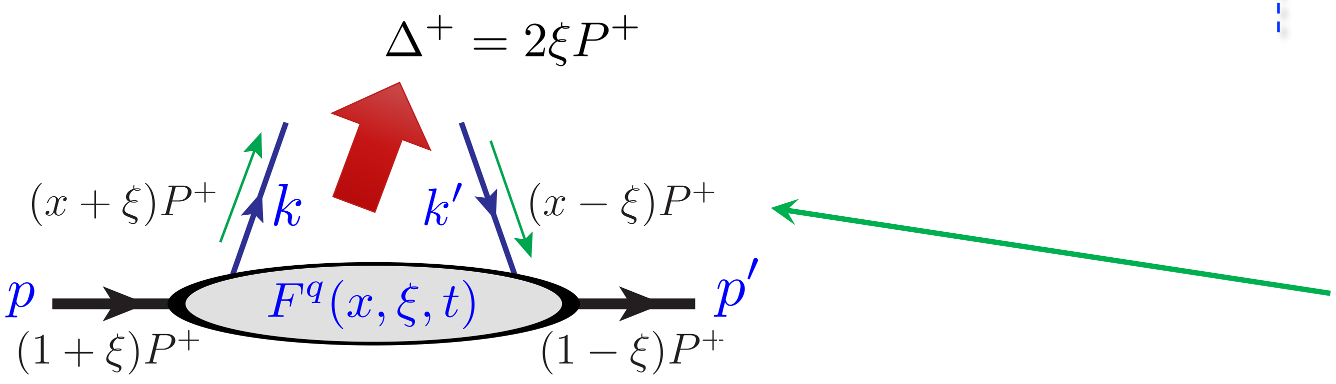

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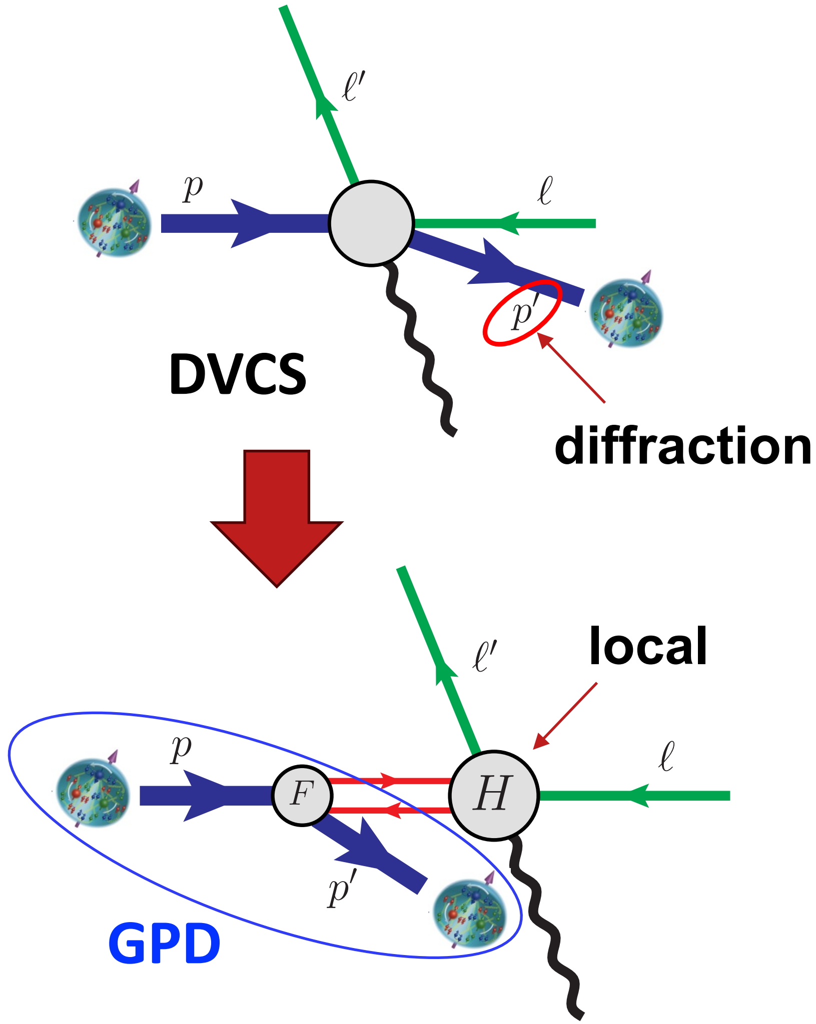
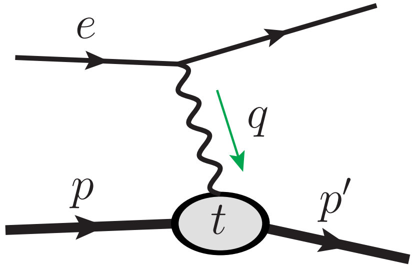


$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}\,(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}\,(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right]$$

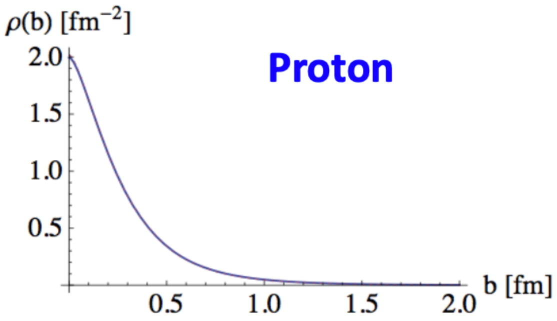
GPD and 3D tomography

- **Diffraction probes form factors and spatial density**



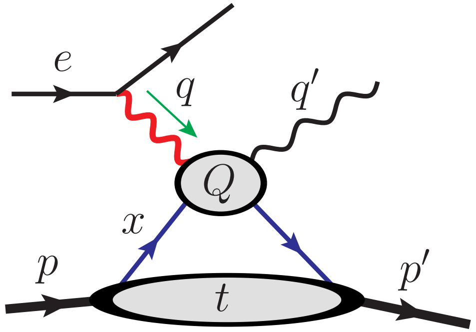
F. T.

$F_{1,2}(t)$



Electric charge radius

- ***Two-scale* diffraction probes 3D tomography**

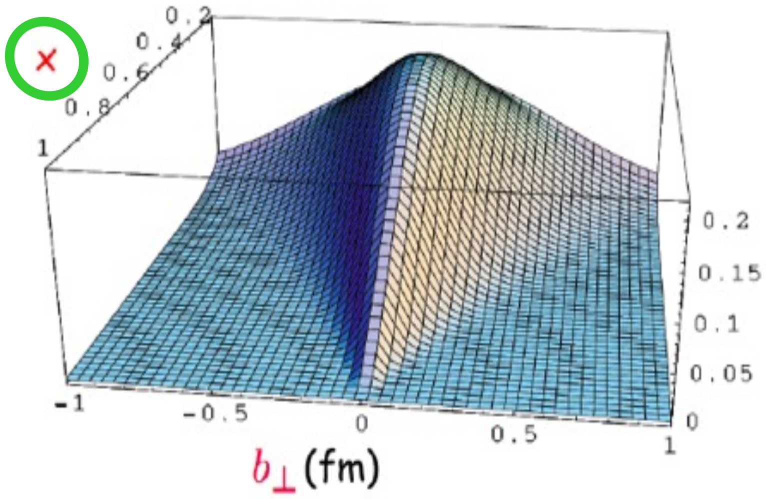


$$f_i(x, \boldsymbol{b}_T) = \int \mathrm{d}^2\boldsymbol{\Delta}_T\, e^{i\boldsymbol{\Delta}_T\cdot\boldsymbol{b}_T} F_i(x, 0, -\Delta_T^2)$$

Parton density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

F. T.

3D image



Proton radius in terms of parton contents

[M. Burkardt, 2000, 2003]

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GPD and Hadronic Property

QCD energy-momentum tensor

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta} \right)^2$$

Gravitational form factor

$$\langle p' \, | T_i^{\mu\nu} | \, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

Connection to GPD moments

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{\left(A_i + \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{\left(B_i - \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

Angular momentum sum rule

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

$$i = q, g$$

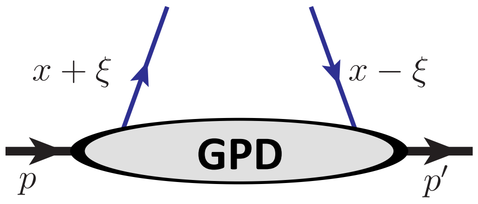
[X.-D. Ji, 1997]

- 3D tomography
- relations to GFF
- angular momentum
- …

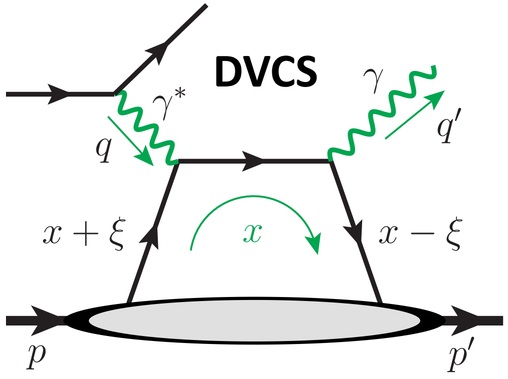
➡ ***x*-dependence!**

x-dependence problem for GPD --- why is it so difficult?

❑ **Amplitude nature: exclusive processes**



x ~ loop momentum



$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

never pin down to some x

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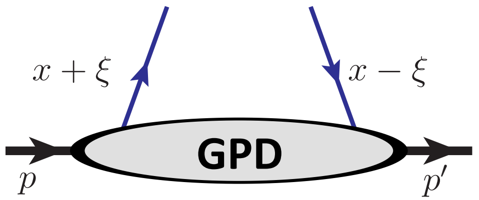
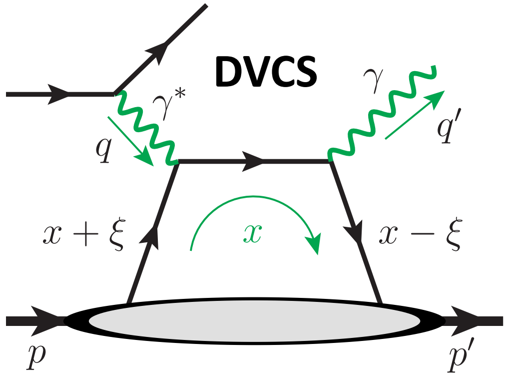
x-dependence problem for GPD --- why is it so difficult?

- **Amplitude nature: exclusive processes**





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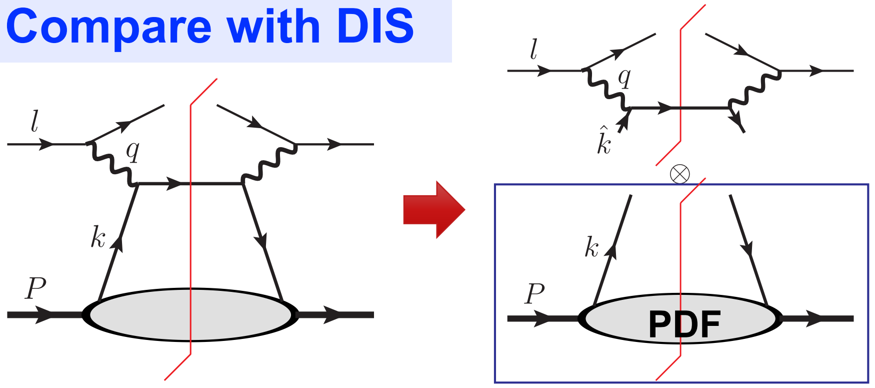
Compare with DIS



cross section: **cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

x-dependence problem for GPD --- why is it so difficult?

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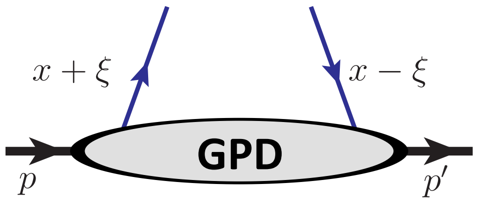

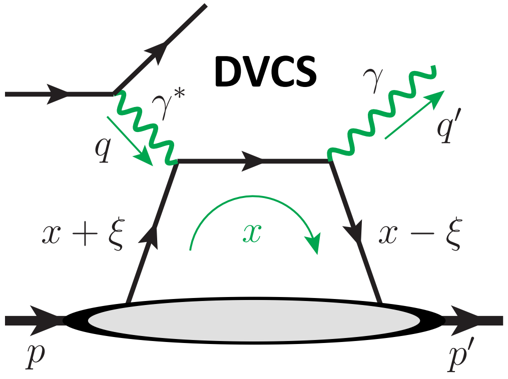

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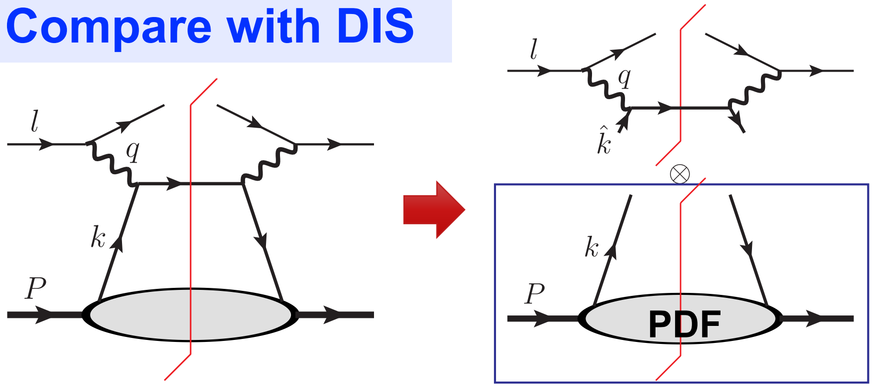


***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

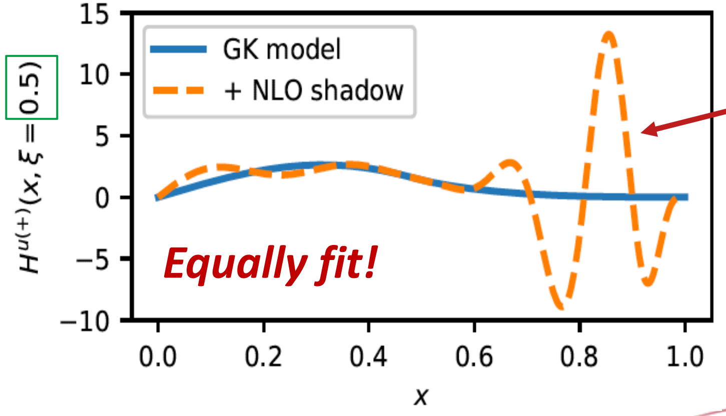
❑ **Sensitivity to x: comes from $C(x,\xi;Q/\mu)$**

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

“moment”

“LO scaling”



Shadow GPD

[Bertone et al. PRD `21]

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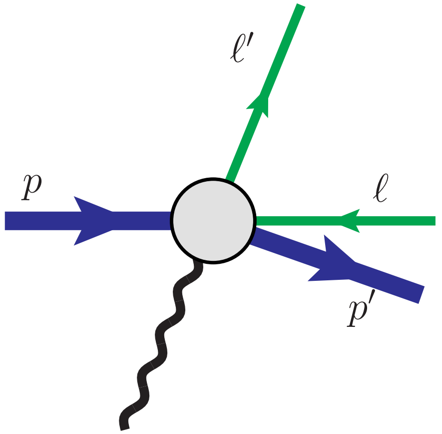
A unified framework of GPD processes

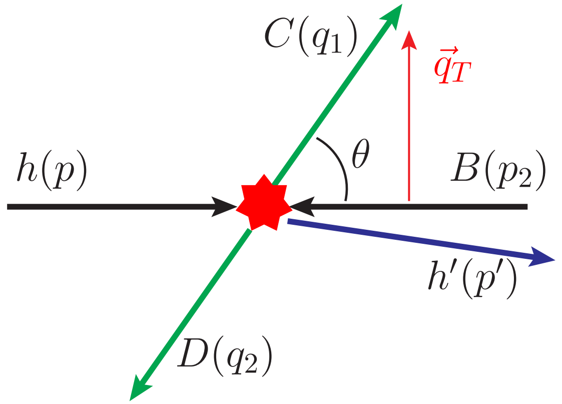
- **Single diffractive hard exclusive process (SDHEP)**

[Qiu & Yu, PRD 107 (2023) 014007]

DVCS in lab frame



$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$



2 → 3: *minimal*
kinematic configuration!

A unified framework of GPD processes

❑ Single diffractive hard exclusive process (SDHEP)

[Qiu & Yu, PRD 107 (2023) 014007]

DVCS in lab frame

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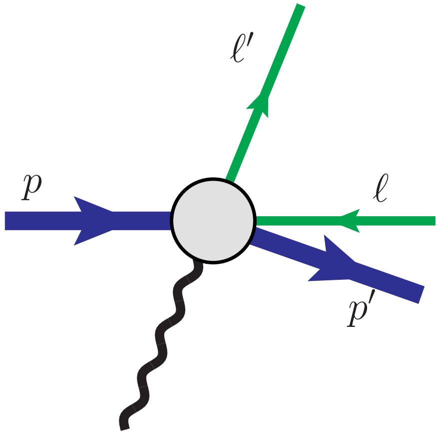
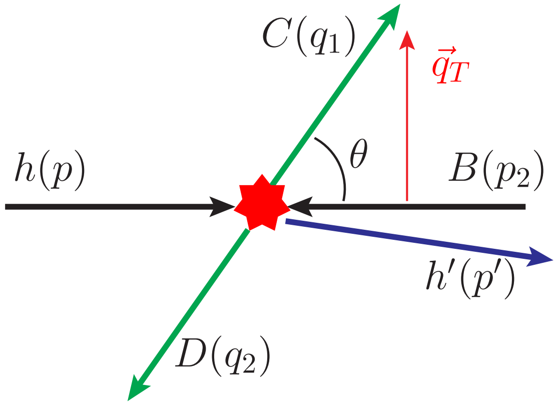




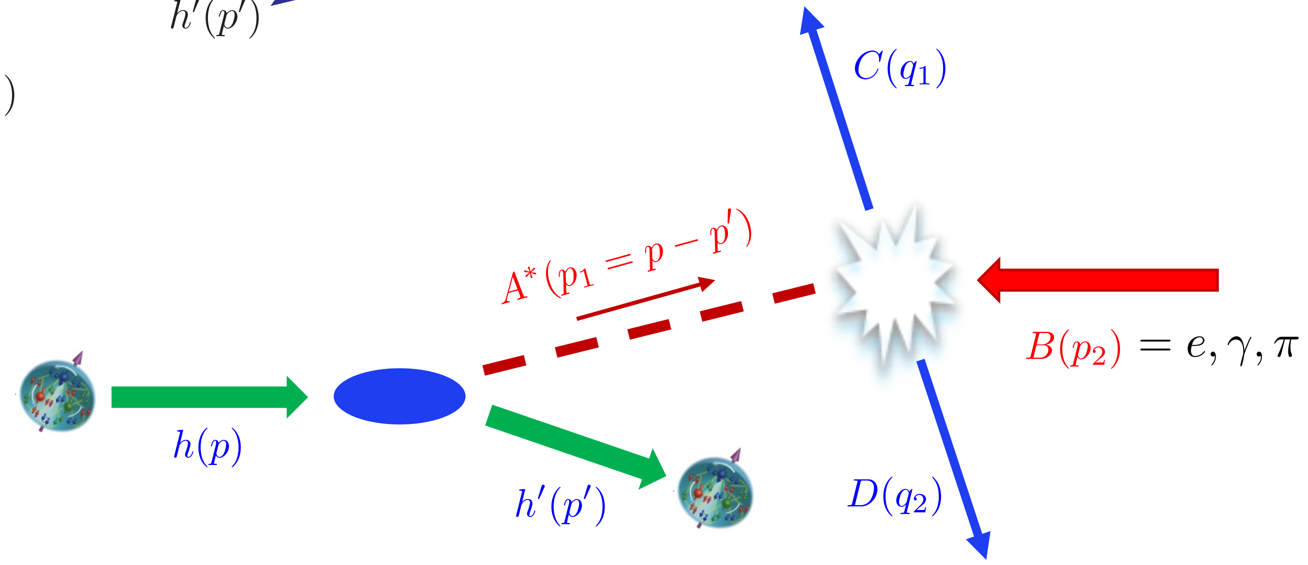
2 → 3: *minimal*
kinematic configuration!

❑ Two-stage process paradigm

Single diffractive: $h(p) \to h'(p') + A^*(p_1 = p - p')$

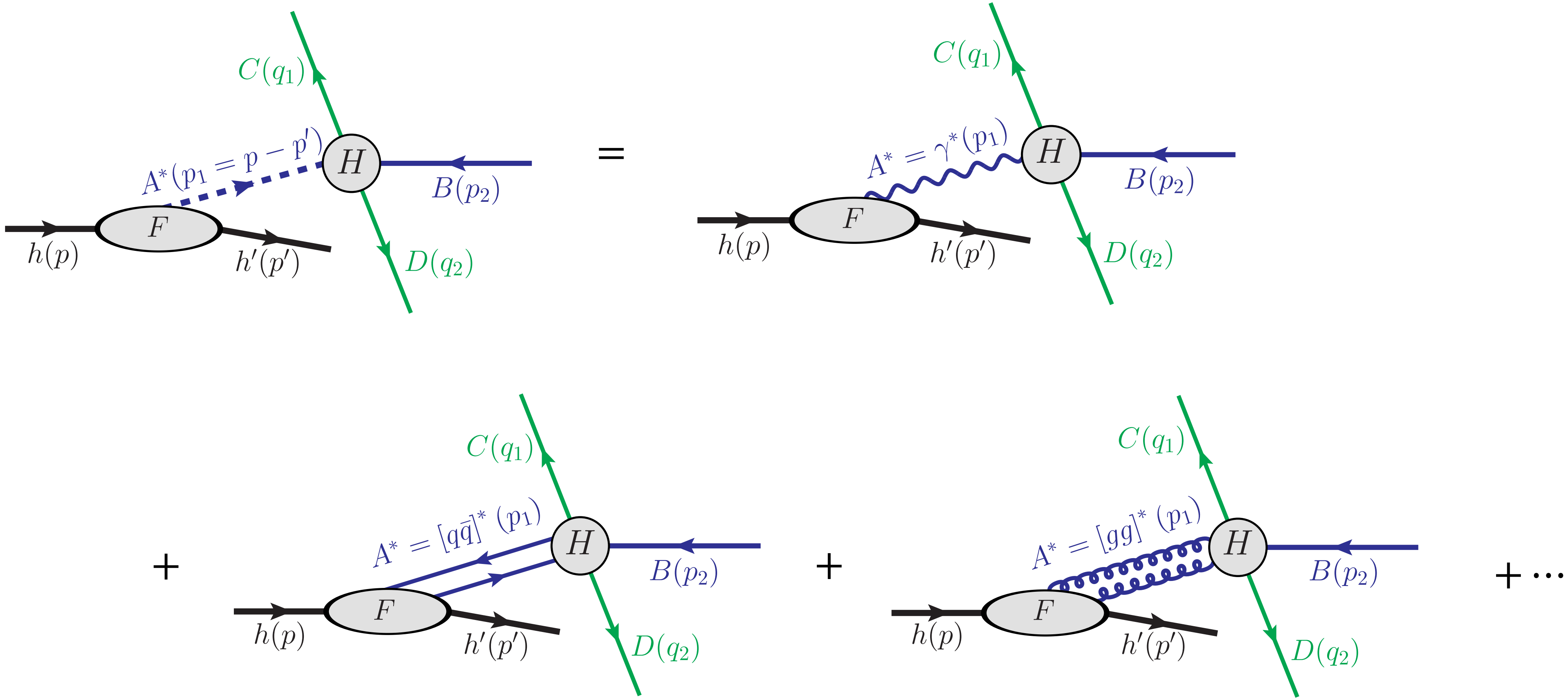
factorize

Hard exclusive: $A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$



Necessary **condition for factorization:** $q_T \gg \sqrt{-t} \simeq \Lambda_{\rm QCD}$ $\quad t = (p - p')^2$

SDHEP: Two-stage paradigm and channel expansion



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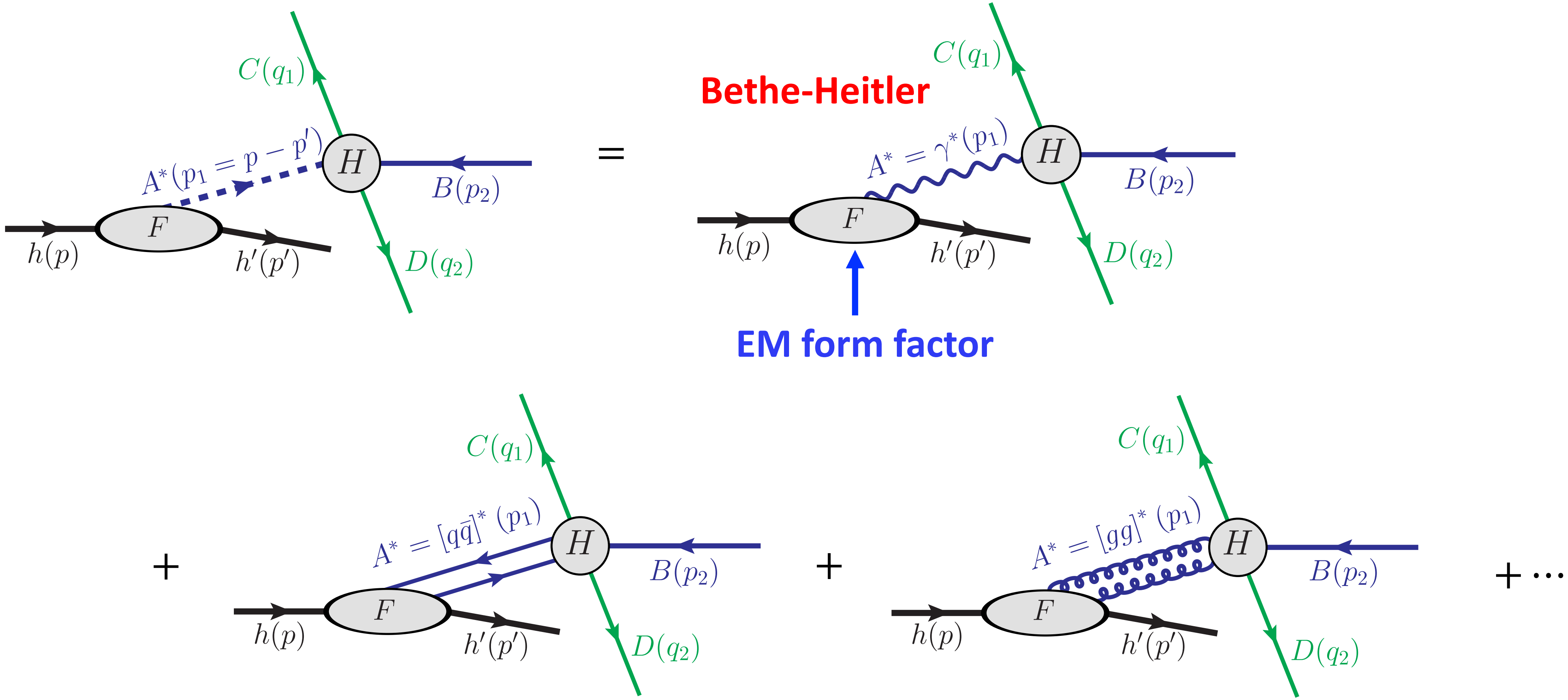
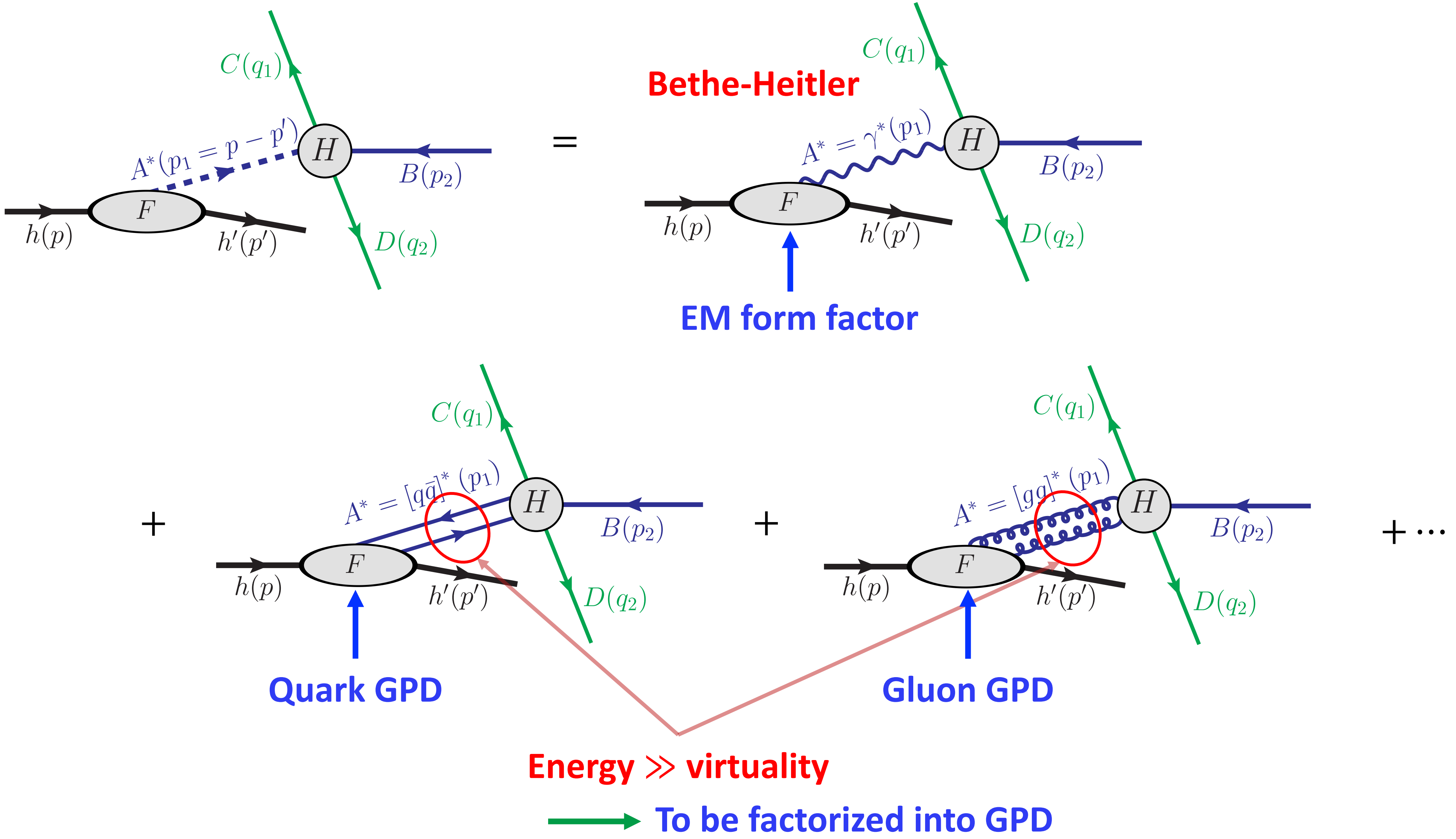
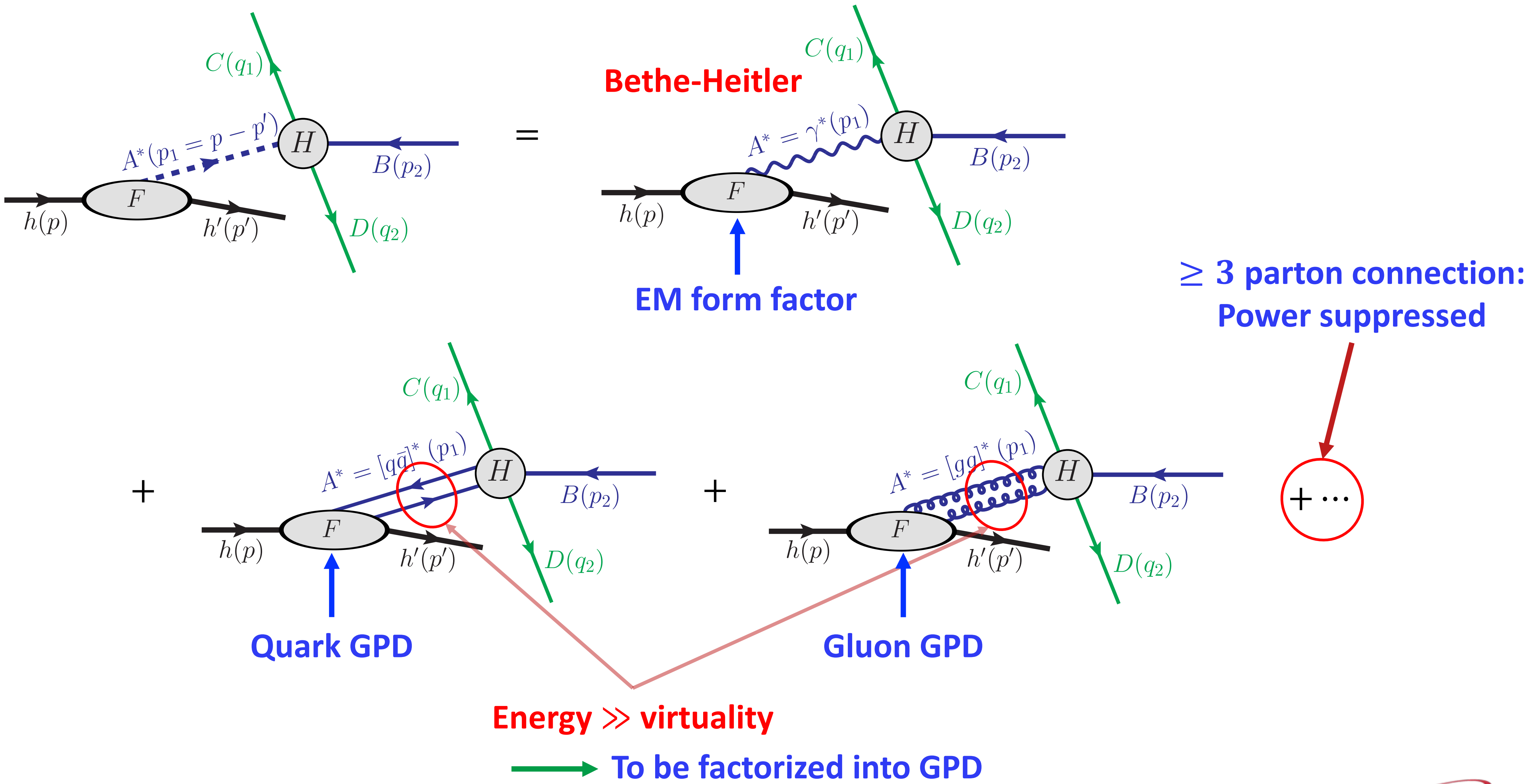
SDHEP: Two-stage paradigm and channel expansion



SDHEP: Two-stage paradigm and channel expansion



SDHEP: Two-stage paradigm and channel expansion (twist expansion)



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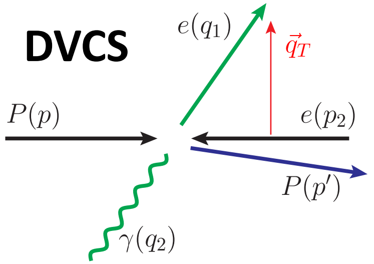
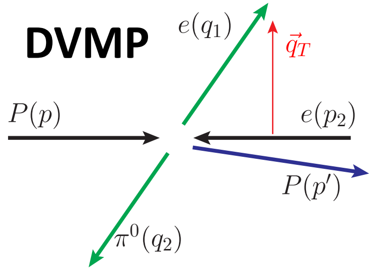
Classification of SDHEPs

❑ Electro-production (JLab, EIC, …)

DVCS

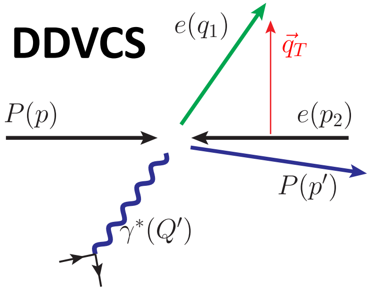


DVMP



add virtuality →

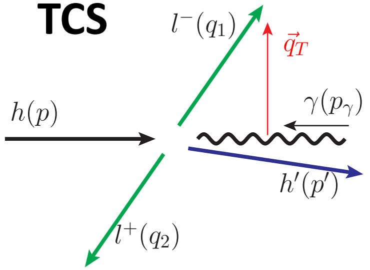
DDVCS

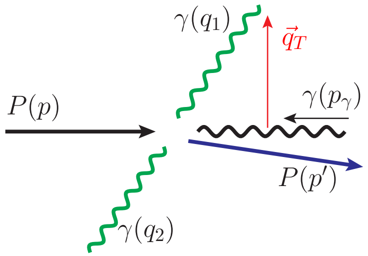


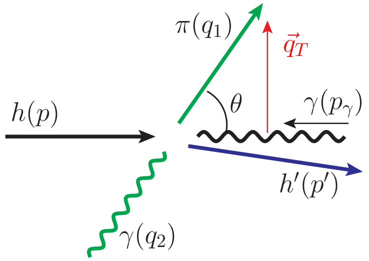
DHQP…

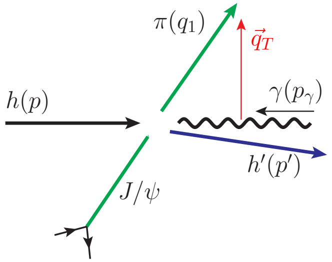
❑ Photo-production (JLab, EIC, …)

TCS









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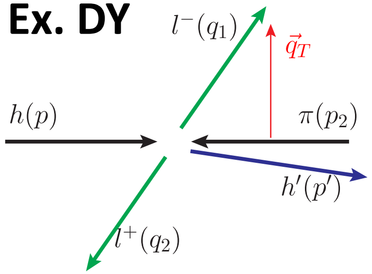
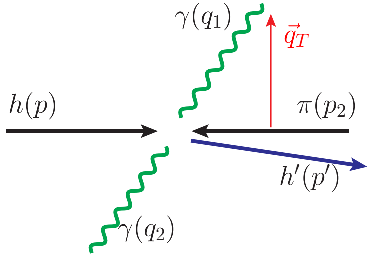
❑ Meso-production (AMBER, J-PARC, …)

Ex. DY





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Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

Classification of SDHEPs

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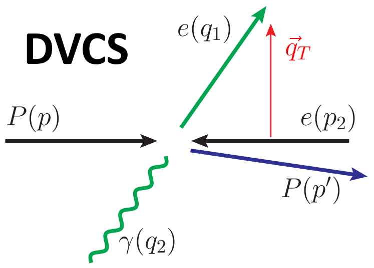

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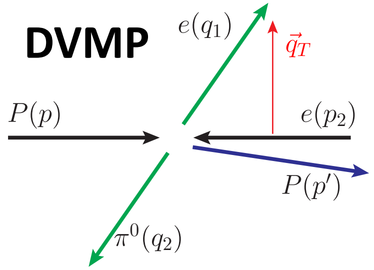

add virtuality

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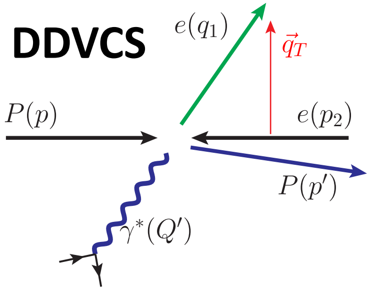

DHQP…

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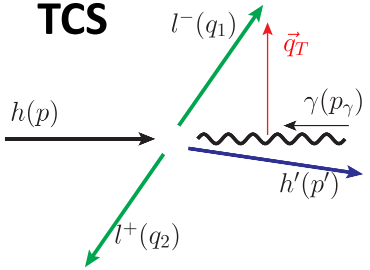

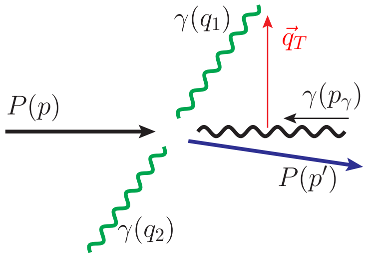

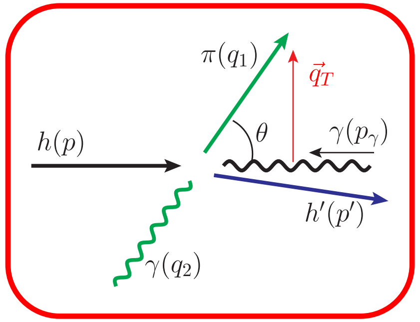

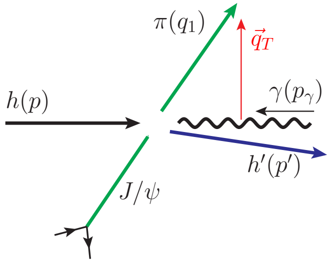

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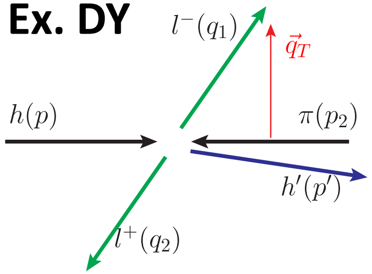
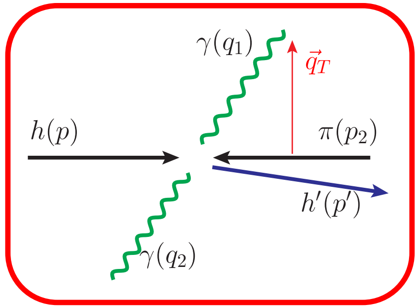
Ex. DY





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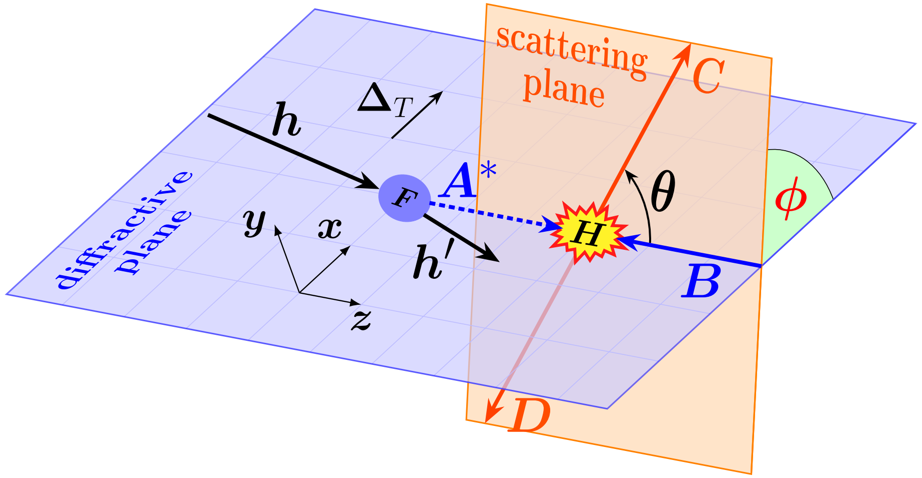
Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

Where does the x-sensitivity come from?



❑ x-sensitivity ⇔ 2 → 2 hard scattering

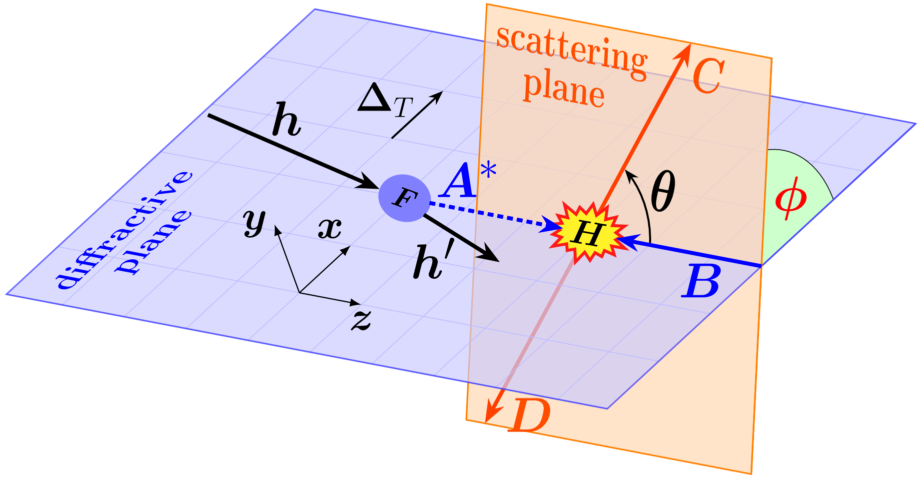
Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⬅ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⬌ x
3. ϕ ⬅ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

Where does the x-sensitivity come from?



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Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⟵ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\,\sin\theta$ ⟷ x
3. ϕ ⟵ (A^*B) spin states

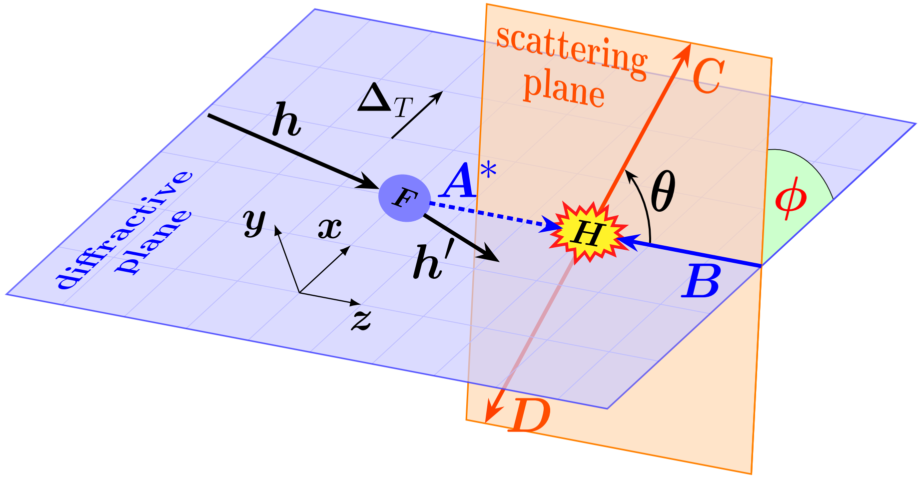
$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⟶ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

➡ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

Where does the x-sensitivity come from?



❑ x-sensitivity ⇔ 2 → 2 hard scattering

Kinematics:

1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⟵ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⟷ x
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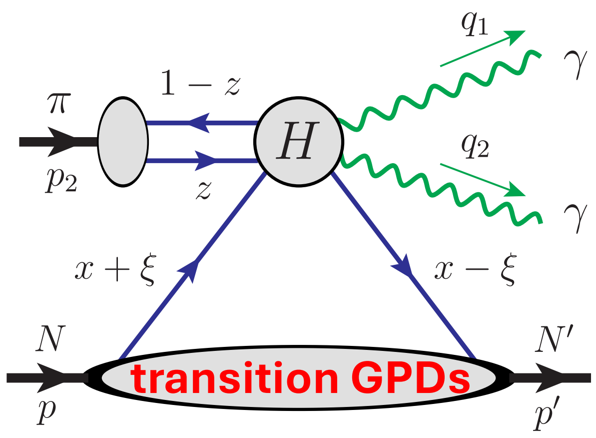
➢ **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⟹ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

⟹ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

➢ **Enhanced sensitivity** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟹ $d\sigma/dQ \sim \left|C(x;Q) \otimes_x F(x,\xi,t)\right|^2$

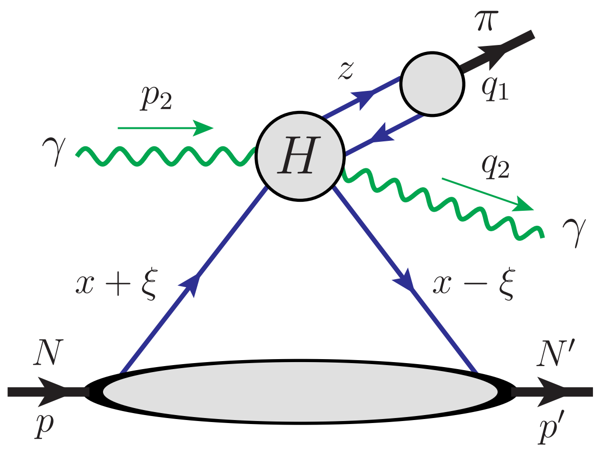
Scaling *breaking* at LO

Two new example processes with enhanced x-sensitivity



J-PARC, AMBER

Qiu & Yu, JHEP 08 (2022) 103
Qiu & Yu, PRD 109 (2024) 074023

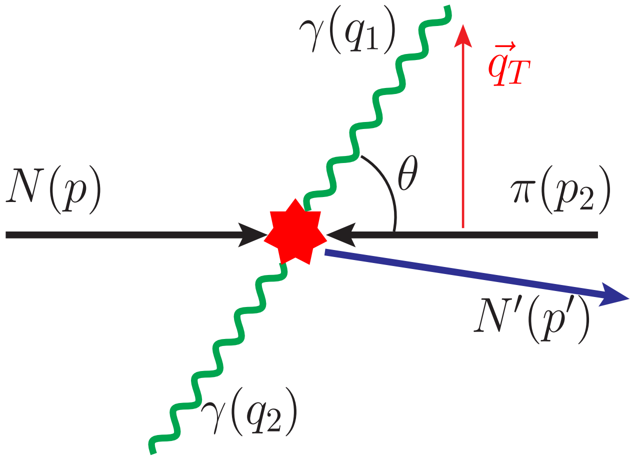


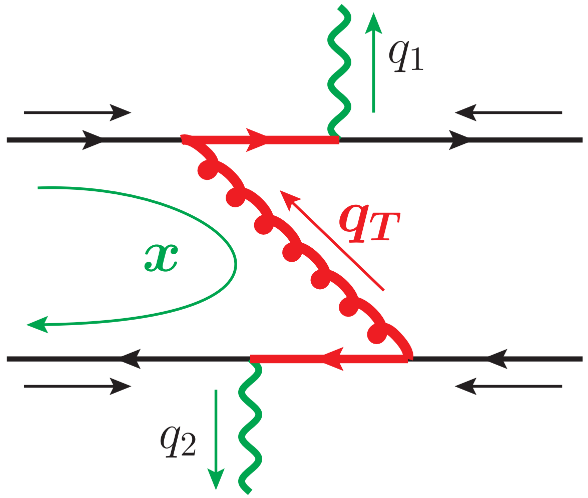
JLab Hall D

G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRD 107 (2023), 014007
Qiu & Yu, PRL 131 (2023), 161902

Enhanced x-sensitivity: (1) diphoton mesoproduction

[Qiu & Yu, JHEP 08 (2022) 103;
PRD 109 (2024) 074023]





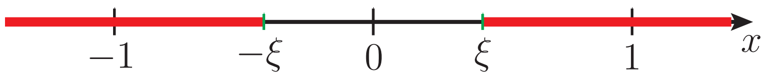
In addition to

$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \xi + i\epsilon}$$

$i\mathcal{M}$ **also contains**

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho(z;\theta) + i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2) - z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2)\, z}{1 - z - \tan^2(\theta/2)\, z}\right] \in (-\infty, -\xi] \cup [\xi, \infty)$$



Enhanced x-sensitivity: (1) diphoton mesoproduction

[Qiu & Yu, PRD 109 (2024) 074023]

❑ **Diphoton process:** $N\pi \to N'\gamma\gamma:\ (1)\ p\pi^- \to n\gamma\gamma;\ (2)\ n\pi^+ \to p\gamma\gamma$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = 2\pi\left(\alpha_e\alpha_s\frac{C_F}{N_c}\right)^2\frac{1}{\xi^2 s^3}\cdot\Bigg[(1-\xi^2)\sum_{\alpha=\pm}\left(|\mathcal{M}_\alpha^{[\widetilde{H}]}|^2+|\widetilde{\mathcal{M}}_\alpha^{[H]}|^2\right)-\left(\xi^2+\frac{t}{4m^2}\right)\sum_{\alpha=\pm}|\widetilde{\mathcal{M}}_\alpha^{[E]}|^2$$
$$-\frac{\xi^2 t}{4m^2}\sum_{\alpha=\pm}|\mathcal{M}_\alpha^{[\widetilde{E}]}|^2-2\xi^2\sum_{\alpha=\pm}\mathrm{Re}\left(\widetilde{\mathcal{M}}_\alpha^{[H]}\widetilde{\mathcal{M}}_\alpha^{[E]*}+\mathcal{M}_\alpha^{[\widetilde{H}]}\mathcal{M}_\alpha^{[\widetilde{E}]*}\right)\Bigg]$$

Nucleon transition GPDs

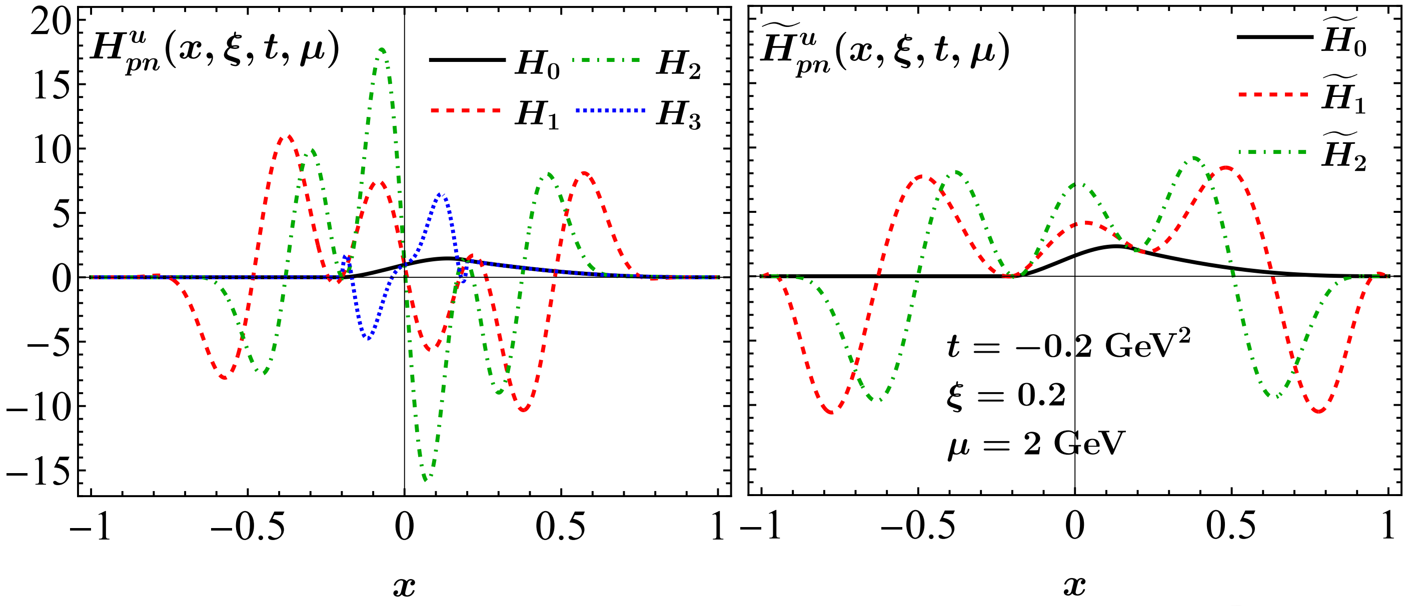
$$H^u_{pn} = H^u_p - H^d_p,\ \text{etc.}$$

GPD models = GK model + shadow GPDs

Goloskokov & Kroll, \`05, \`07, \`09

Bertone et al. \`21
Moffat et al. \`23

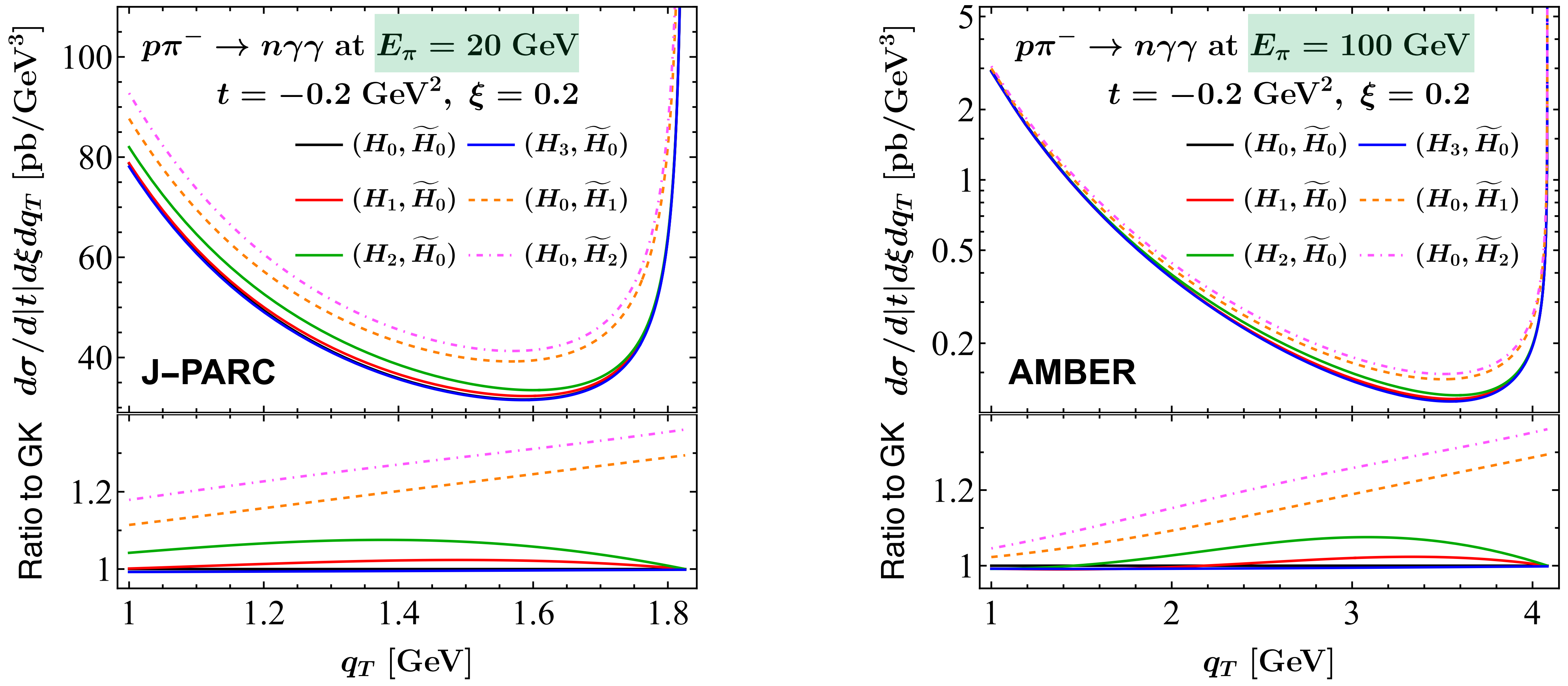
$$\int_{-1}^{1}\frac{dx\,S(x,\xi)}{x-\xi\pm i\epsilon}=0$$



Jefferson Lab

Enhanced x-sensitivity: (1) diphoton mesoproduction (at J-PARC or AMBER)

[Qiu & Yu, PRD 109 (2024) 074023]



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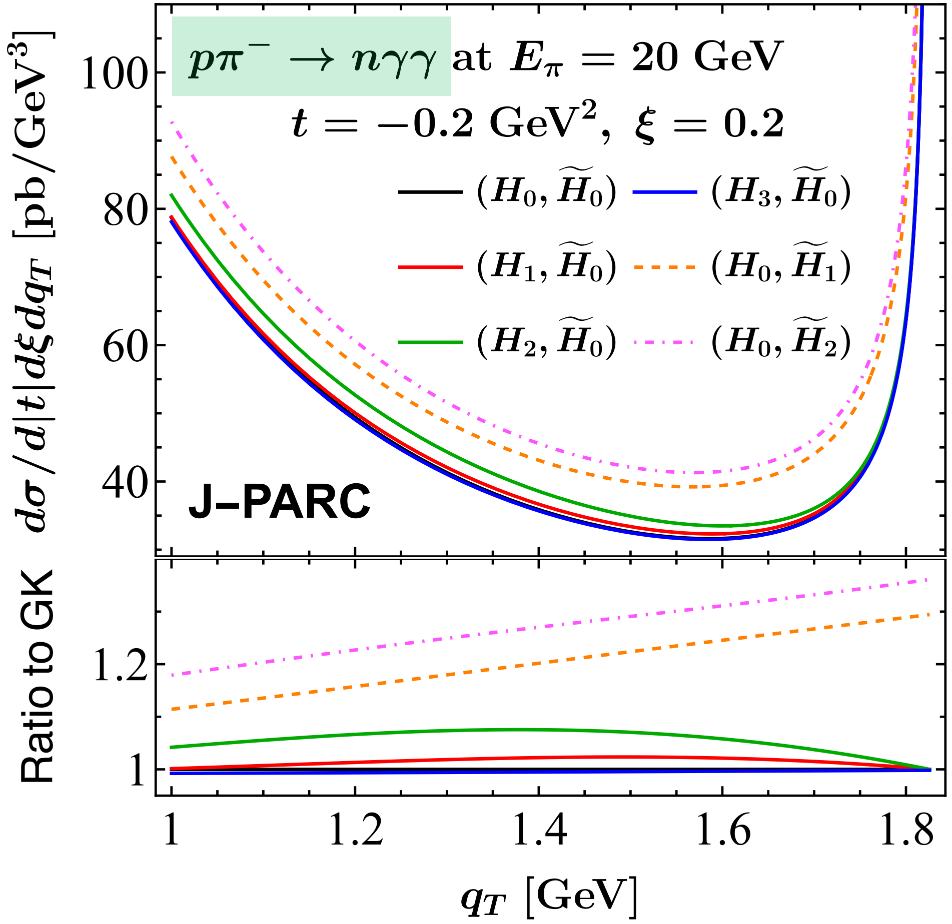
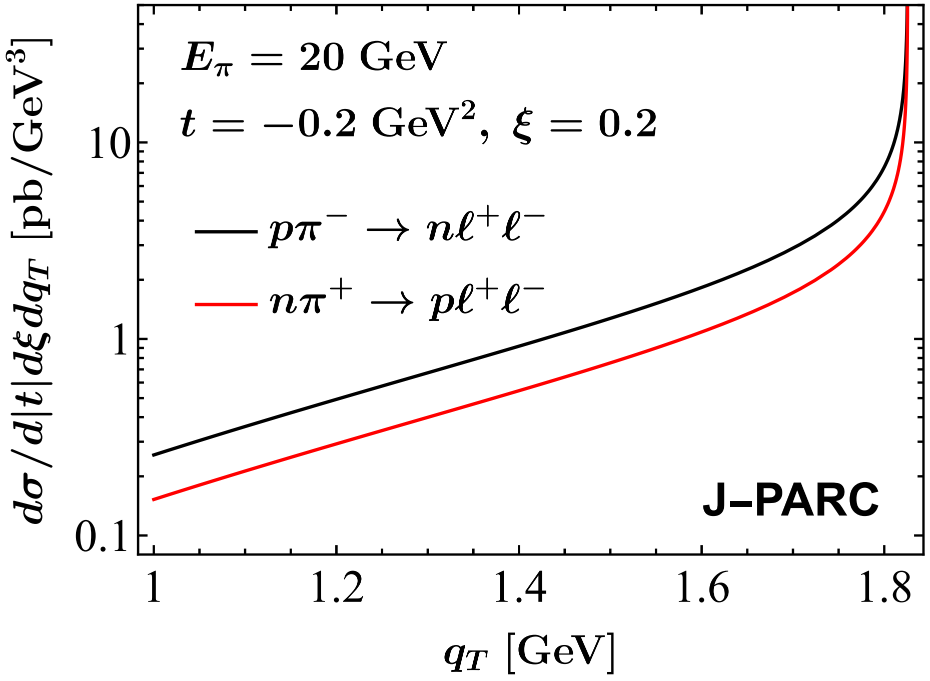
Enhanced x-sensitivity: (1) diphoton mesoproduction (at J-PARC or AMBER)

[Qiu & Yu, PRD 109 (2024) 074023]



Exclusive Drell-Yan dilepton mesoproduction

$$N + \pi \to N' + \gamma^* [\to \ell^+ + \ell^-]$$



- Blind to shadow GPDs
- Lower rate

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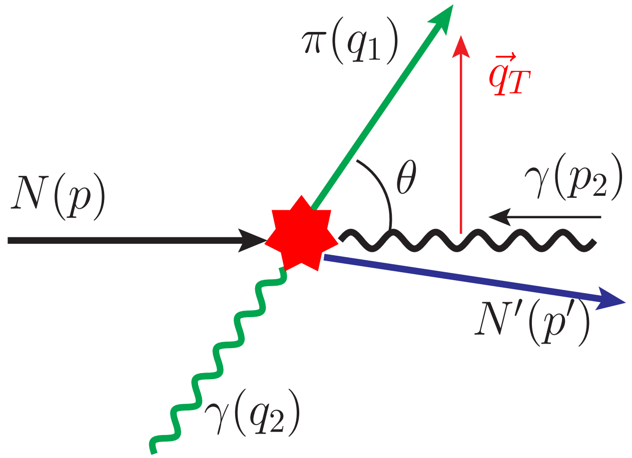
Enhanced x-sensitivity: (2) γ-π pair photoproduction

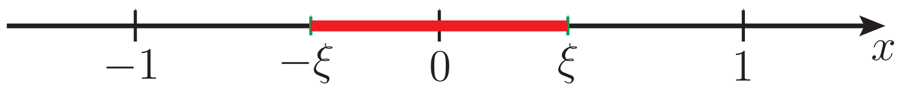
[Qiu & Yu, PRL 131 (2023) 161902]



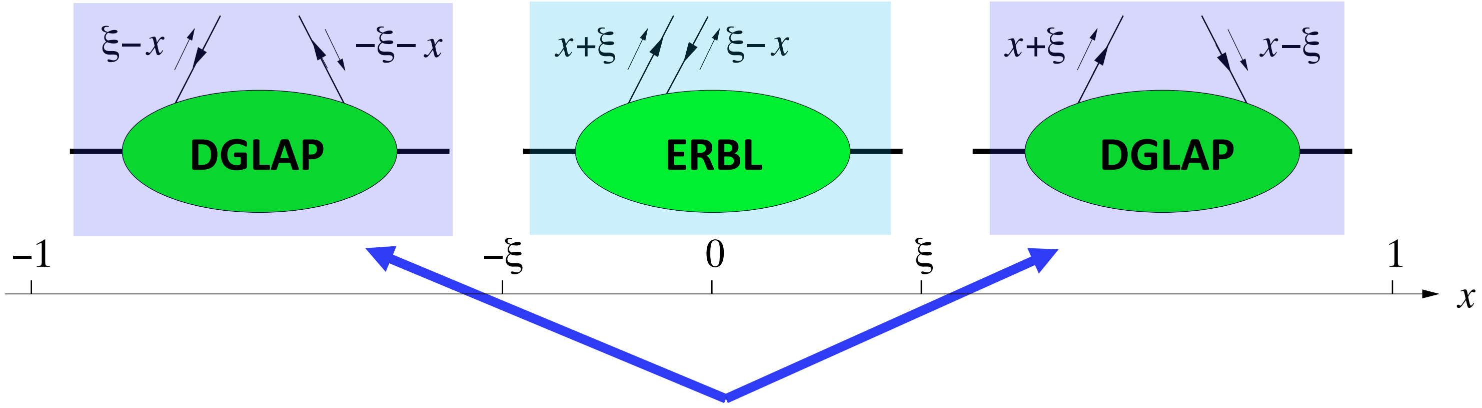
$i\mathcal{M}$ **also contains the special integral**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$



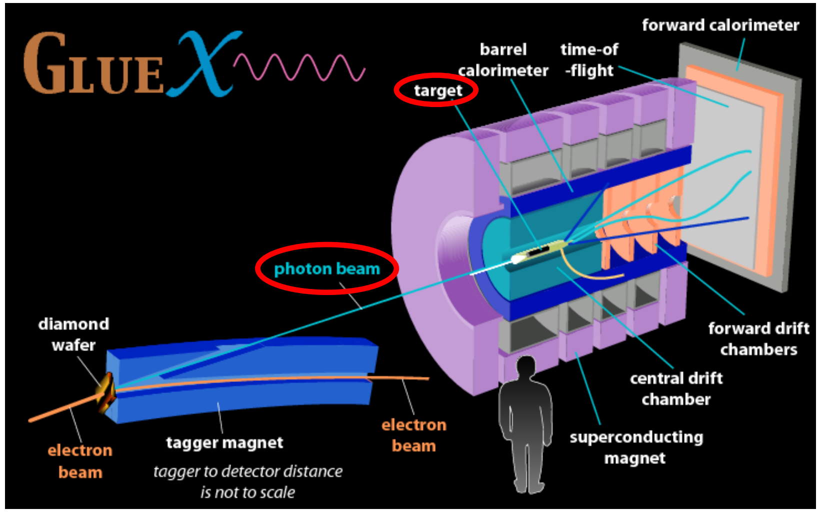
Complementary sensitivity

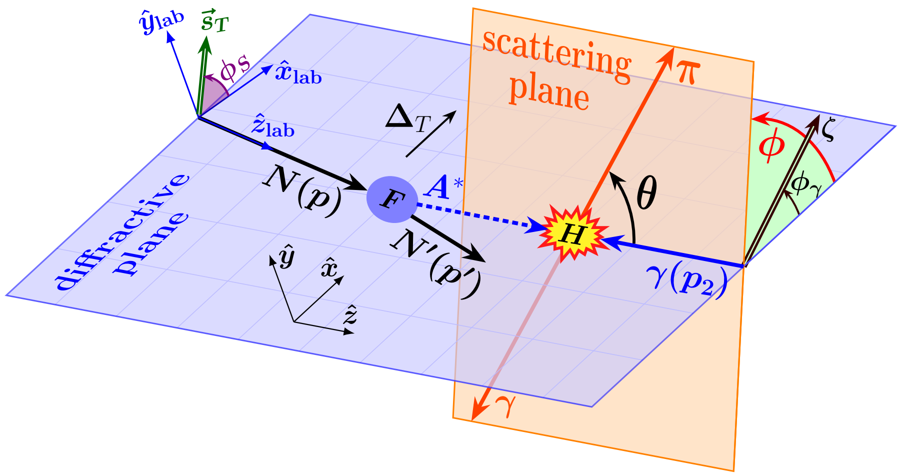


$N\,\pi \to N'\,\gamma\,\gamma$

Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

[Qiu & Yu, PRL 131 (2023) 161902]





Polarization asymmetries

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1 + \lambda_N\lambda_\gamma A_{LL} + \zeta A_{UT}\cos 2(\phi-\phi_\gamma) + \lambda_N\zeta A_{LT}\sin 2(\phi-\phi_\gamma)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi(\alpha_e\alpha_s)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

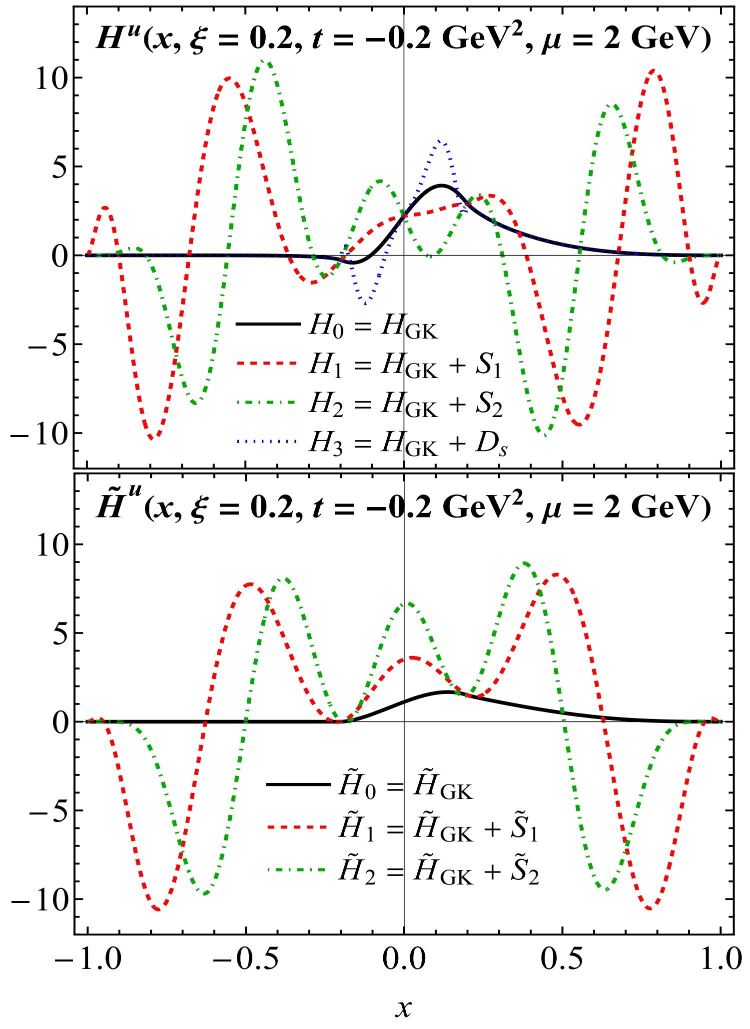
Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel

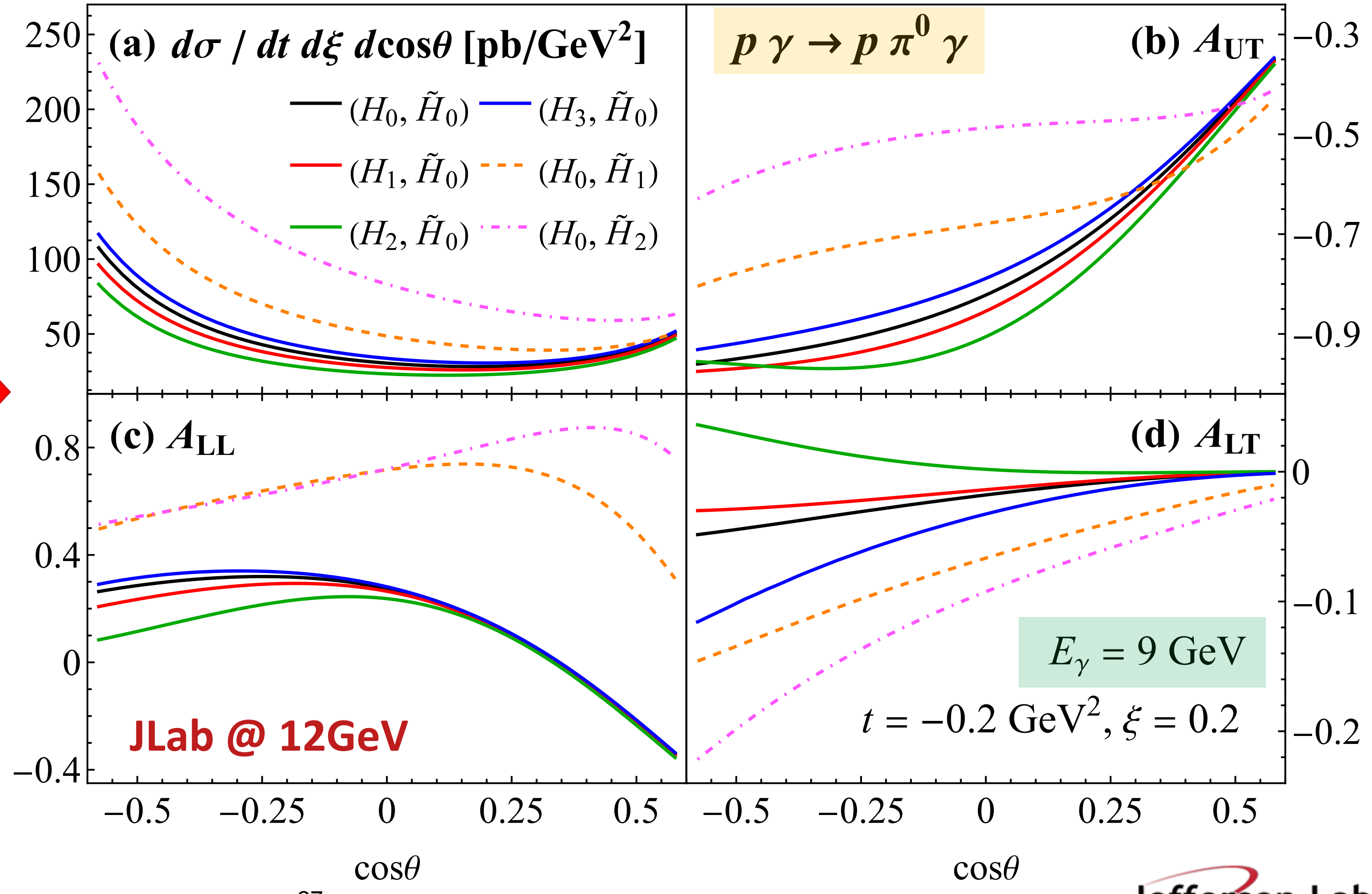
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23





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Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

GPD models = GK model + shadow GPDs

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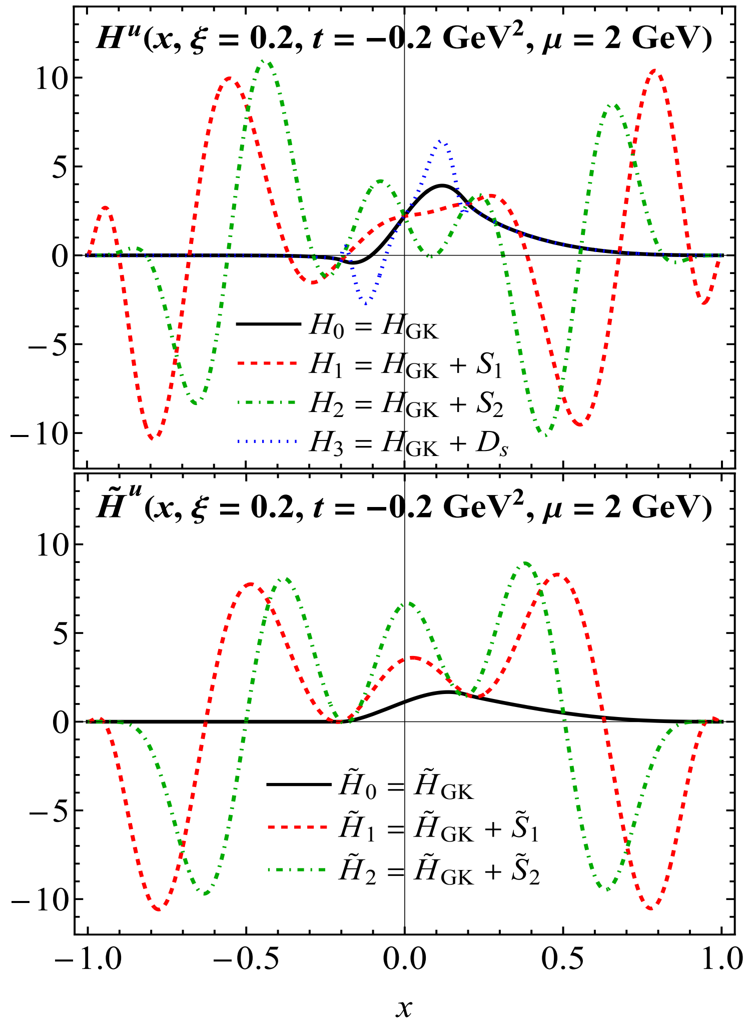

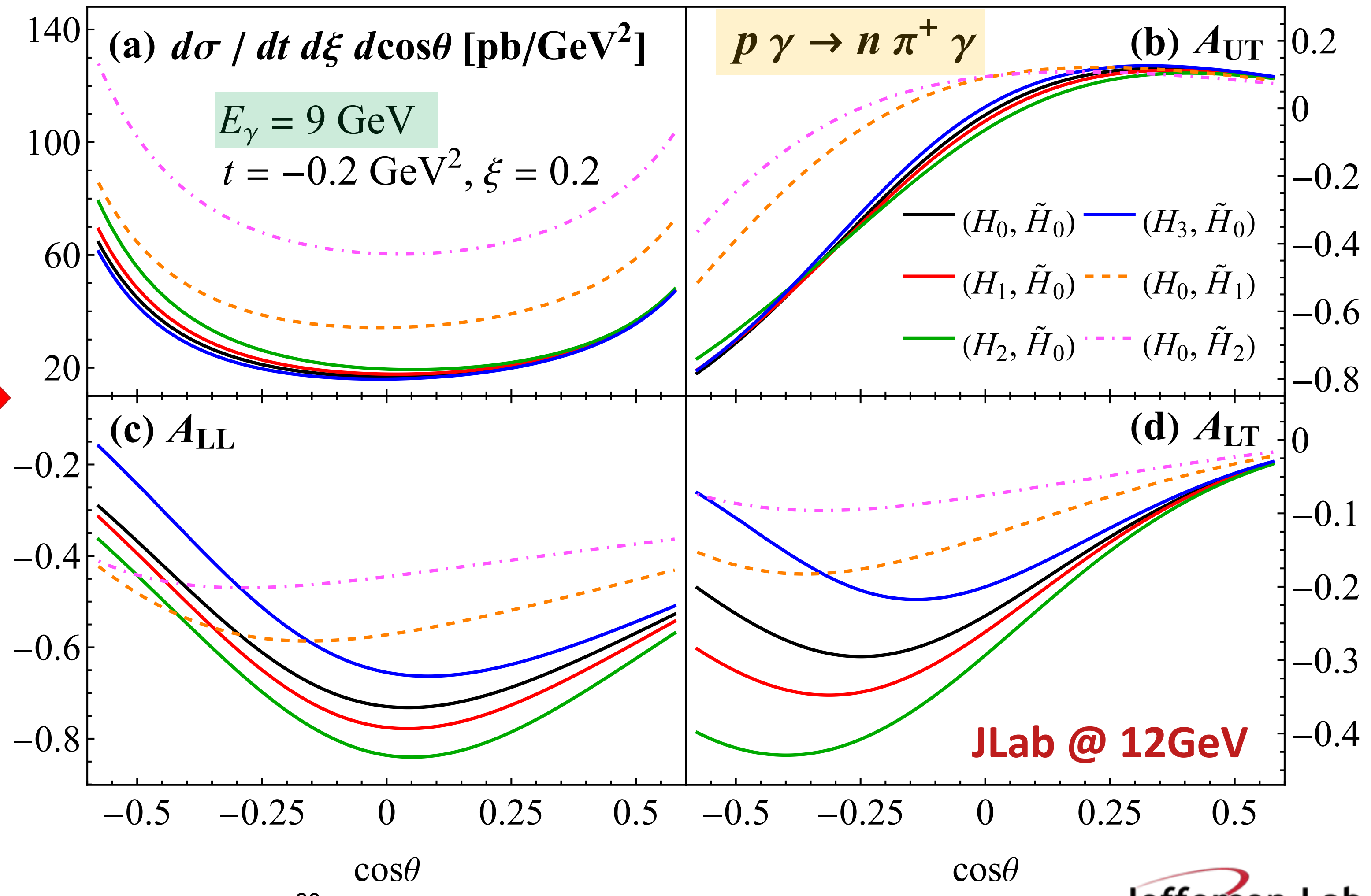

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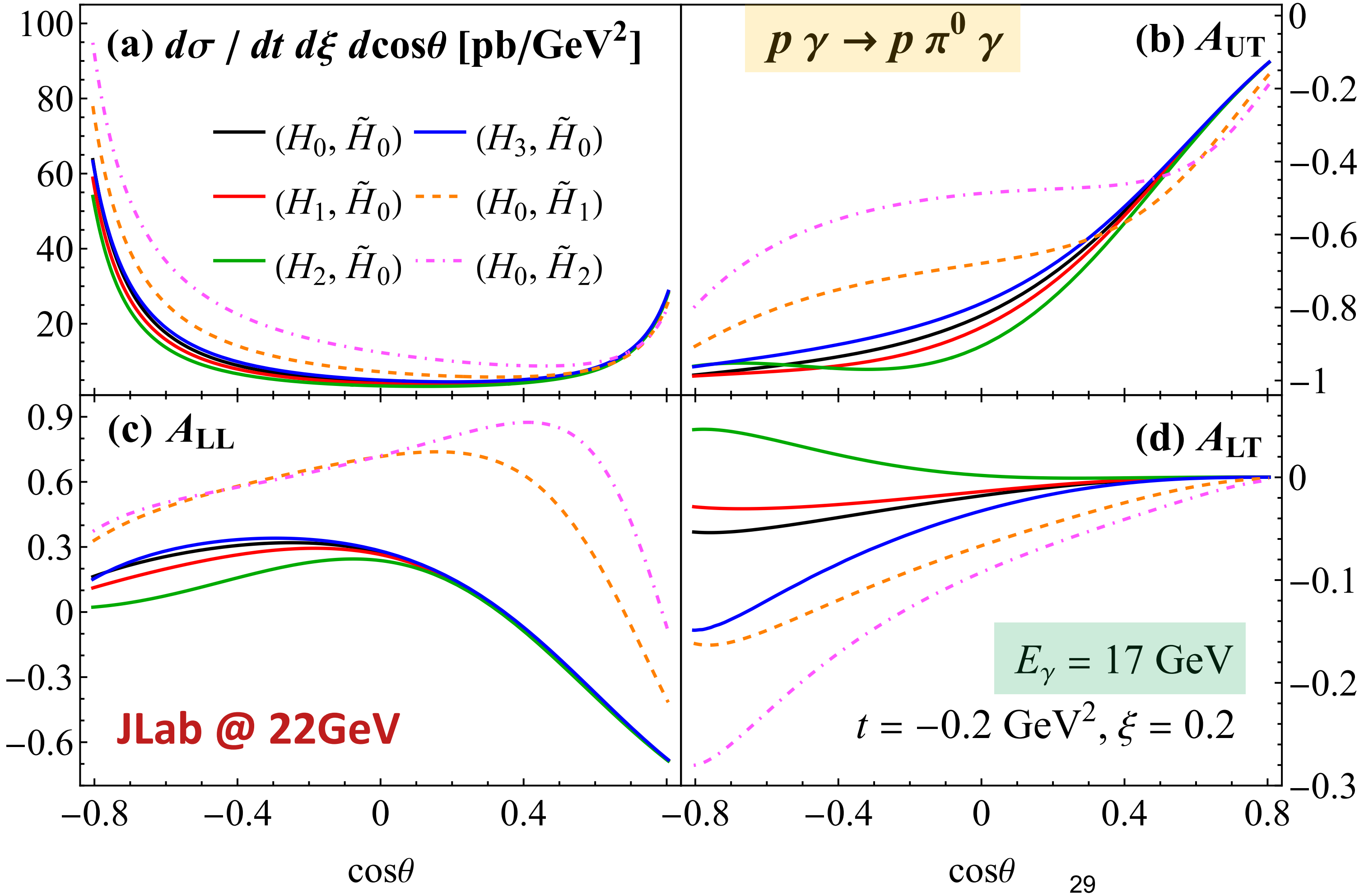
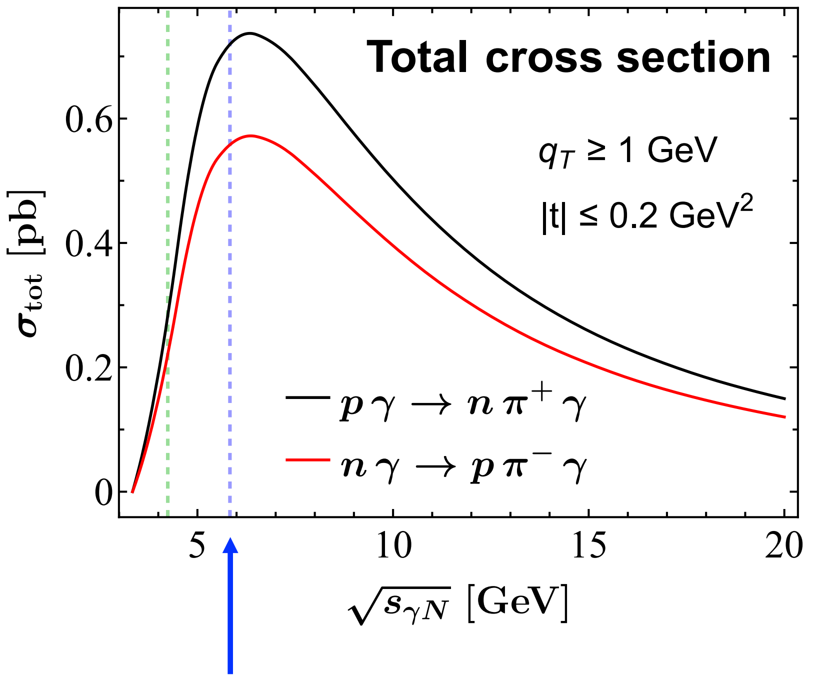
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at upgraded JLab energy)

GPD models = GK model + shadow GPDs ⟵ $\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x - \xi \pm i\epsilon} = 0$

Goloskokov, Kroll, \`05, \`07, \`09
Bertone et al. \`21
Moffat et al. \`23



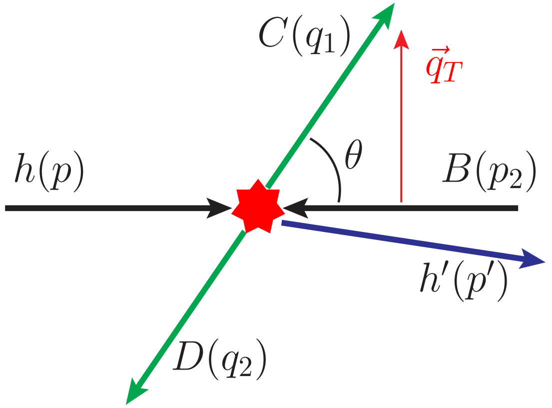


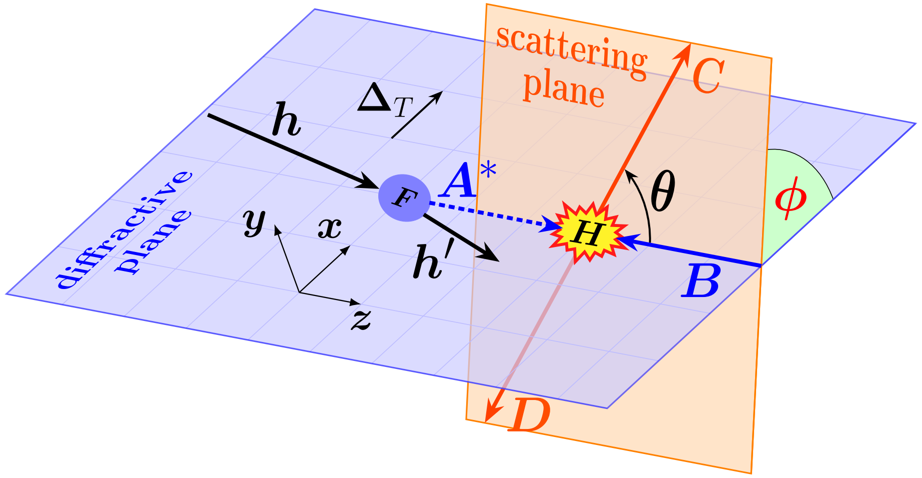
JLab @ 22GeV [arXiv:2306.09360]

Summary

- ❑ A new perspective to frame GPD processes
  - ➢ Single diffractive hard exclusive processes
  - ➢ Unified description: kinematics and factorization
  - ➢ Two new processes to give enhanced *x*-sensitivity
- ❑ Still not there yet
  - ➢ Need more processes, more observables
  - ➢ Need input from lattice QCD
  - ➢ Need global fit

--- A long challenging but exciting way to go!





Thank you!

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