Recent work on Bèzier PDF parametrizations in xFitter and L_2 sensitivities

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CTEQ Spring meeting
06/04/2024

Bèzier Pion PDF Parameterizations in xFitter

arXiv:2311.08447,
PhysRevD.109.074027

- Pions arise as a pseudo-Goldstone boson due to spontaneous chiral symmetry breaking.
- A knowledge of the pion could lead to a better understanding of non-perturbative QCD.

Bèzier Curve

$$\mathcal{B}^{(N_m)}(y) = \sum_{l=0}^{N_m} C_l B_{N_m,l}(y)$$

$$B_{N_m,l}(y) \equiv \binom{N_m}{l} y^l (1-y)^{N_m - l}$$

$$\Rightarrow \mathcal{B} = \boldsymbol{T} \cdot \boldsymbol{M} \cdot \boldsymbol{C}$$

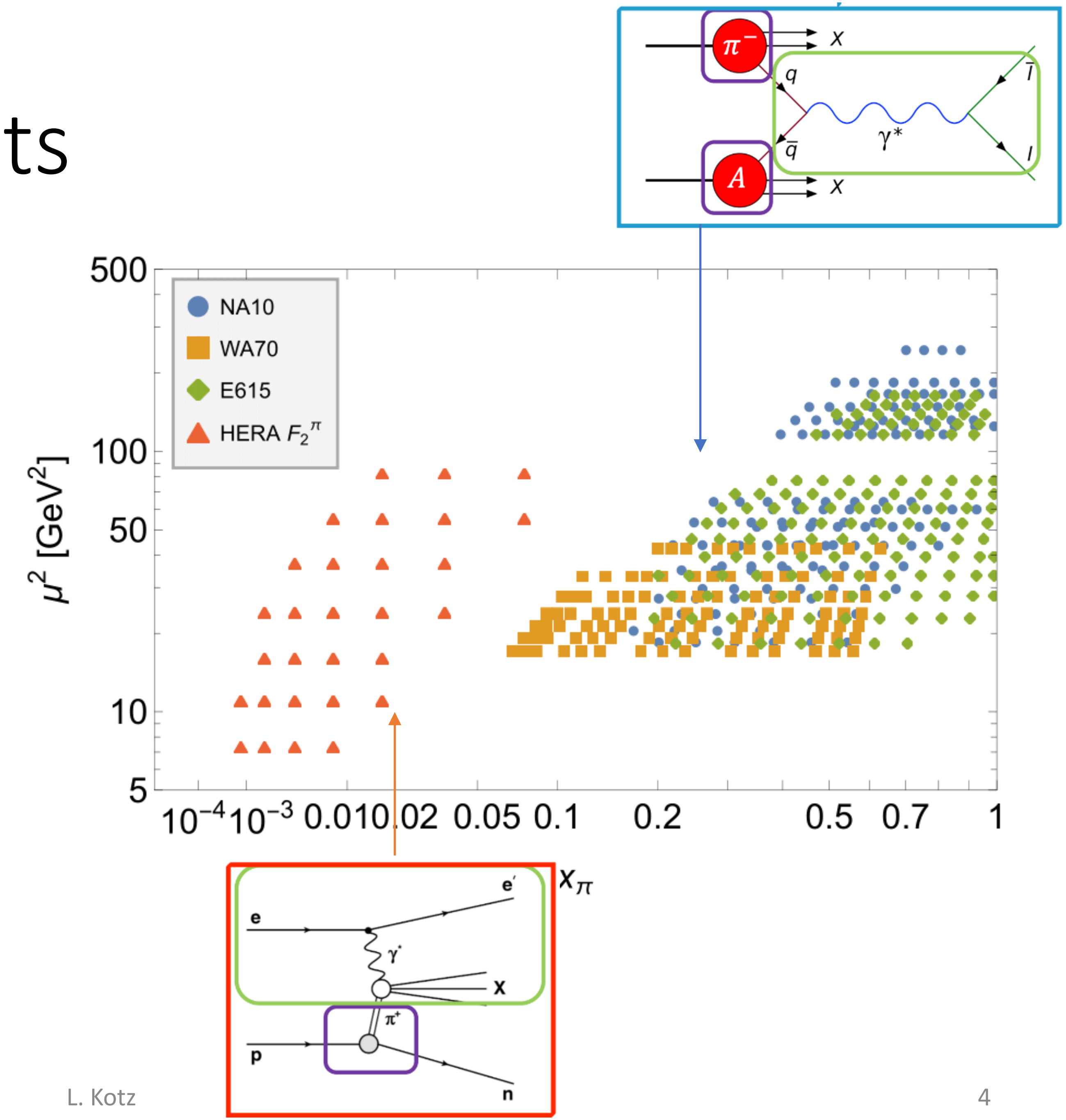
$$\text{or } \boldsymbol{C} = \boldsymbol{M}^{-1} \cdot \boldsymbol{T}^{-1} \cdot \boldsymbol{P}$$

- $\mathcal{B}^{(N_m)}(y)$: Bézier function of N_m^{th}-degree.
- $\boldsymbol{C}$: $N_m + 1$ vector containing Bézier coefficients.
- $B_{N_m,l}(y)$: Bernstein basis polynomial.
- $\boldsymbol{M}$: A fixed $N_m + 1 \times N_m + 1$ matrix containing binomial coefficients. Determined by N_m.
- $\boldsymbol{T}$: A fixed $N_m + 1 \times N_m + 1$ matrix. Determined by the positions of control points.
- $\boldsymbol{P}$: $N_m + 1$ vector containing the values at the control points.

G. Farin (2001)
Kamermans, Mike Pomax: https://pomax.github.io/bezierinfo

Datapoints used in fits

- NA10 & E615: Covers the main kinematic region of $x > 0.2$ and $Q^2 > 10\ \text{GeV}^2$. Constrains valence very well.
- WA70: Provides some sensitivity to the gluon PDFs that the DY data could not provide.
- HERAF_2^π: Constrains the Sea and gluon PDFs at low-x. Uses the HERA prescription.



Performing Fits with a metamorph

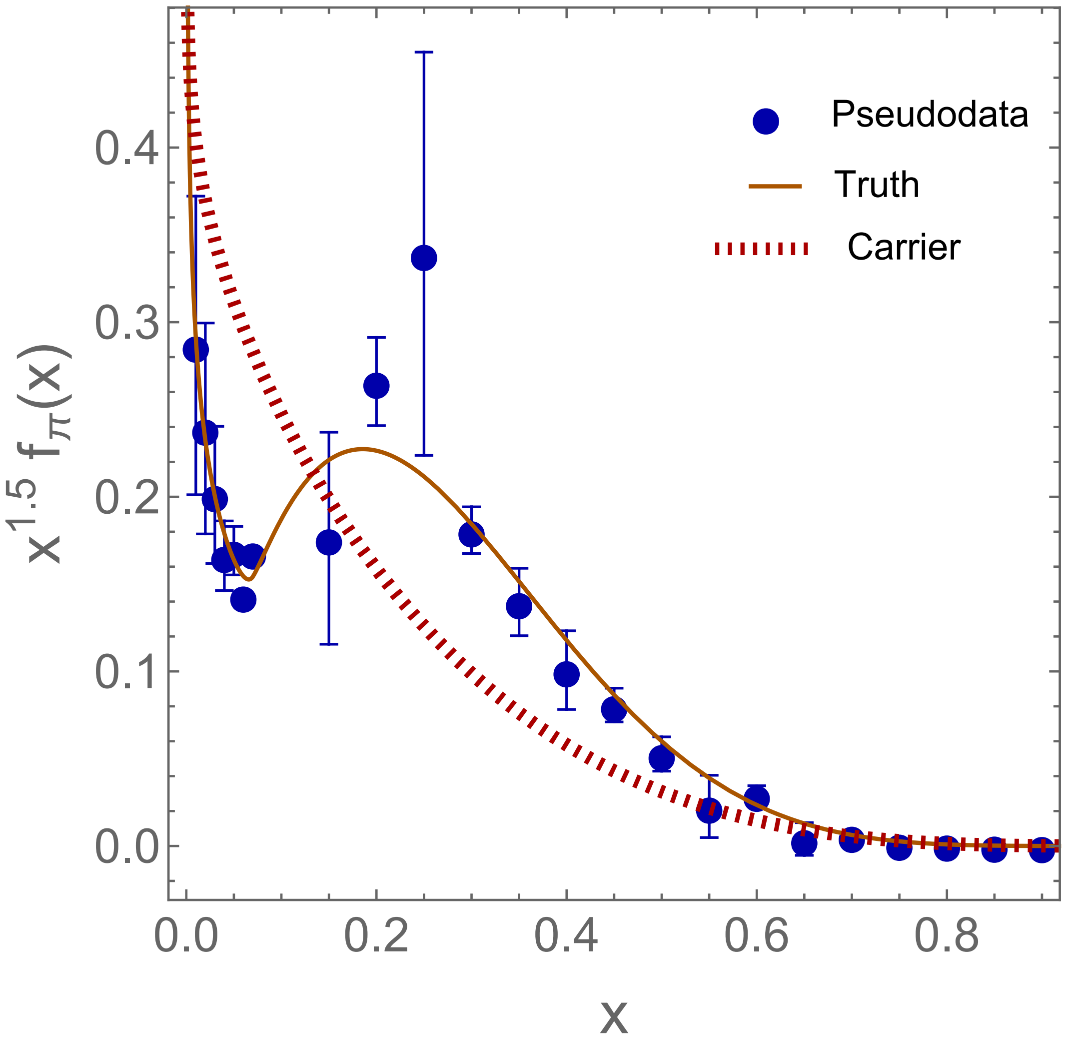
- The functional form of the Fantômas4QCD parameterization is

$$xf(x, Q_0^2) = f_{\text{Carrier}}(x) * f_{\text{Modulator}}(x^{\alpha_x})$$

where

$$f_{\text{Carrier}}(x) \equiv A_f x^{B_f} (1-x)^{C_f}.$$

- The Carrier specifies asymptotic limits of $xf(x, Q_0^2)$ at $x \to 0$ or 1.



Performing Fits with a metamorph

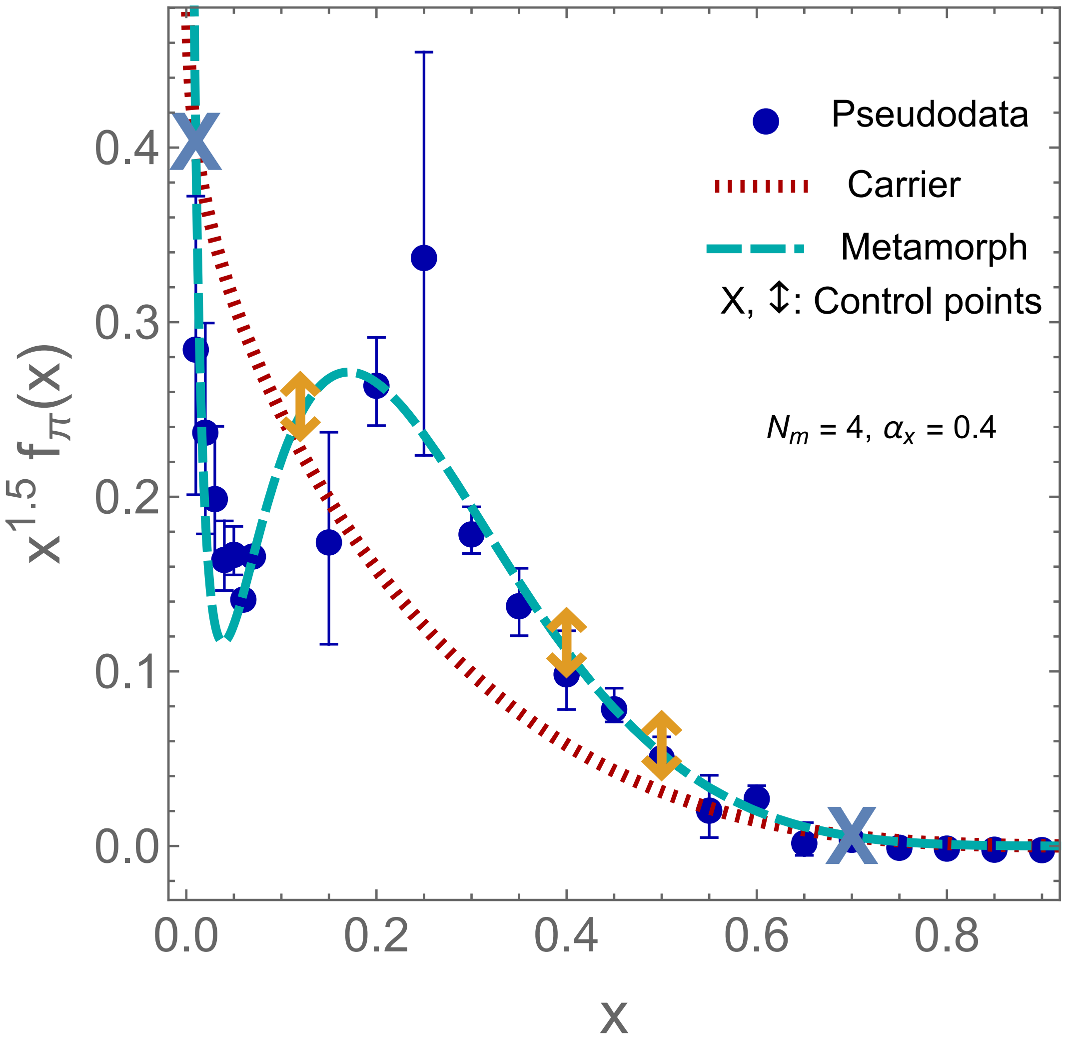
- The functional form of the Fantômas4QCD parameterization is

$$xf(x, Q_0^2) = f_{\text{Carrier}}(x) * f_{\text{Modulator}}(x^{\alpha_x})$$

where we choose

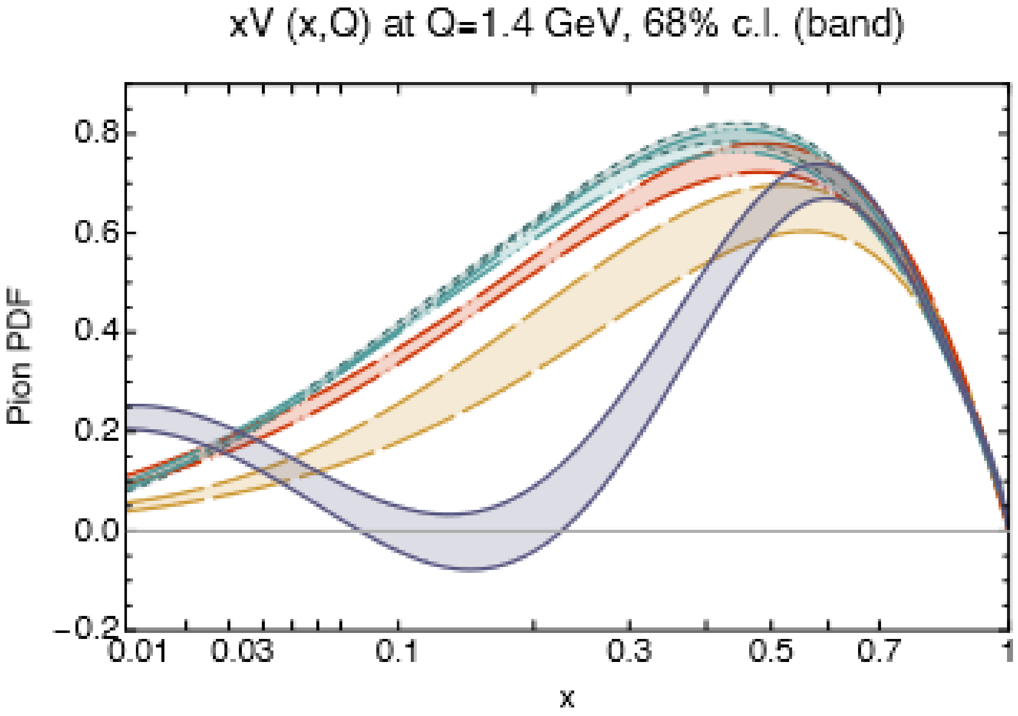
$$f_{\text{Modulator}}(x^{\alpha_x}) = \mathcal{B}^{(N_m)}(x^{\alpha_x}).$$

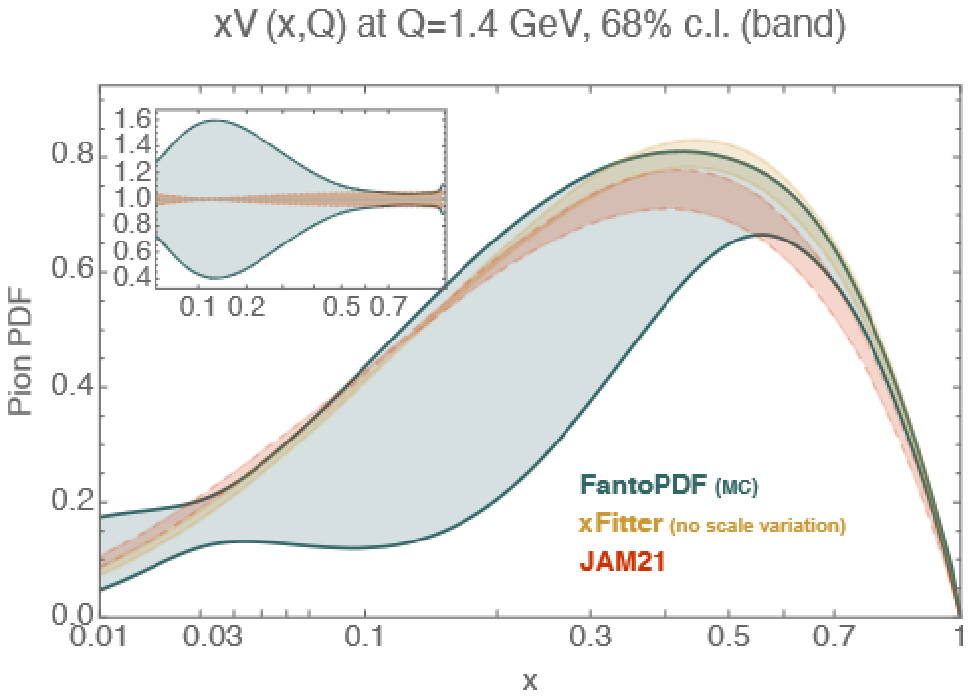
- The Modulator modifies $xf(x, Q_0^2)$ at $0 < x < 1$. α_x is an x-stretching power between 0 and 1.

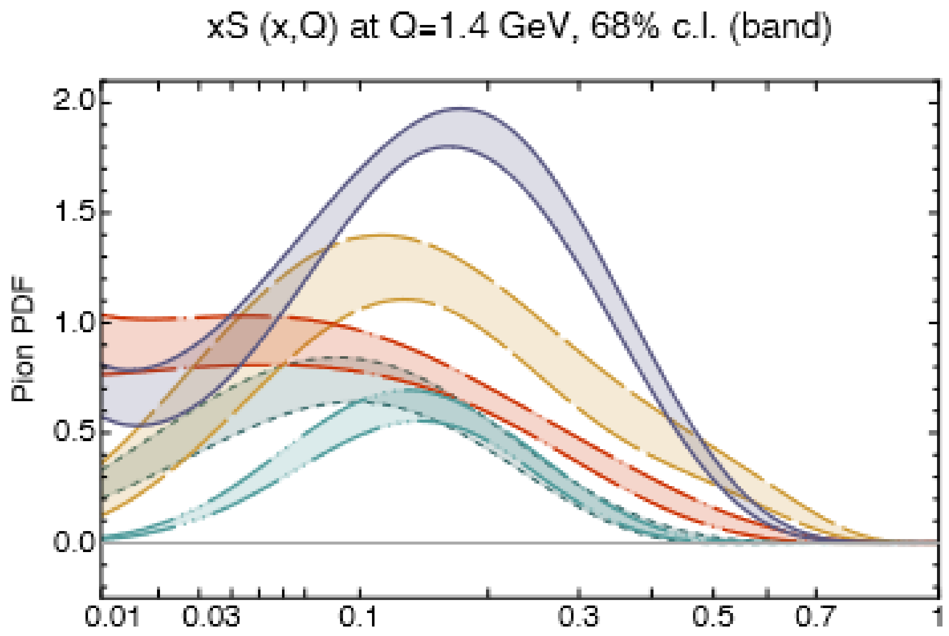


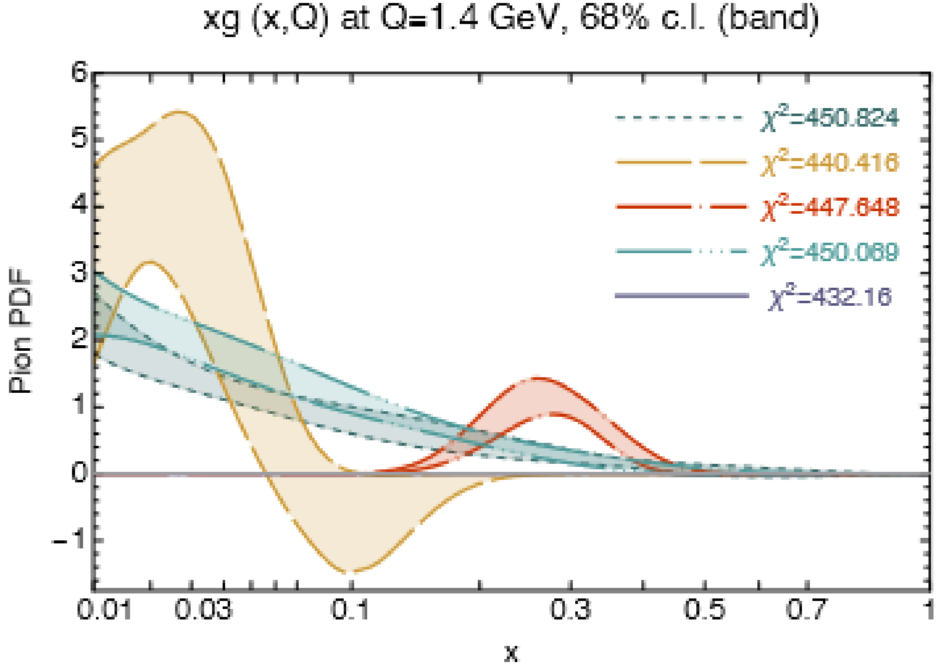
NLO Fantomas PDFs for π^+

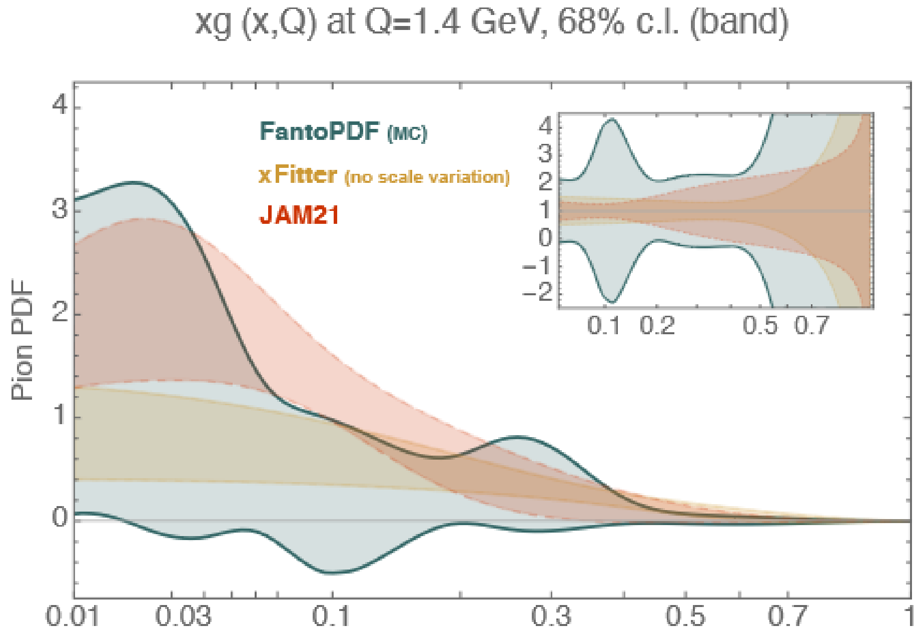
- Various fits were performed using a wide variety of setting combinations.
- Five fits with $\chi^2 < 450$ were chosen to combine.
- The chosen fits are combined using mcgen.
- Compared FantoPDF to xFitter and JAM21 pion fits

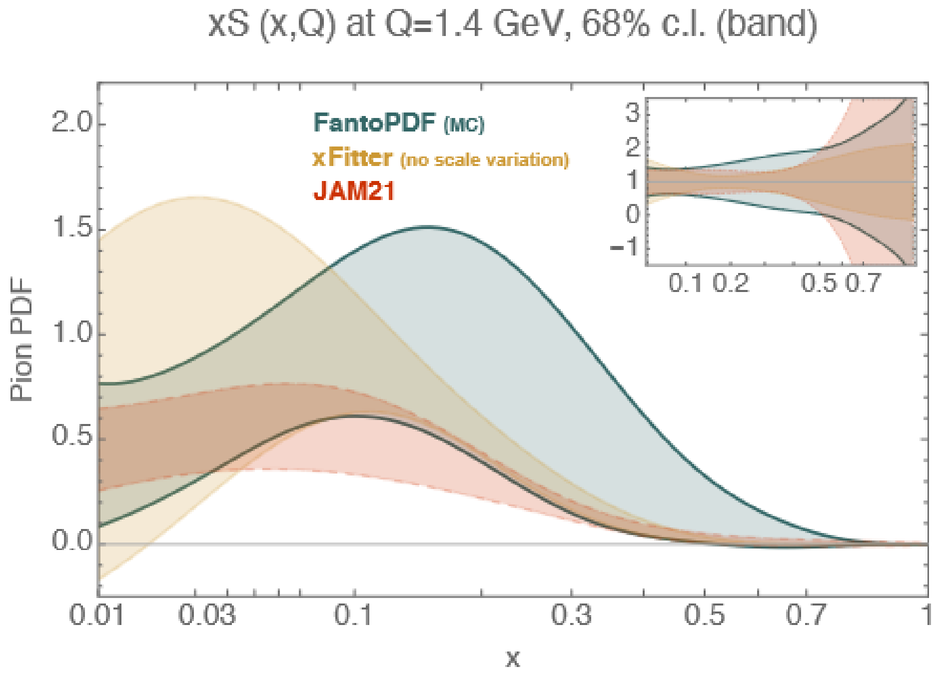





xS (x,Q) at Q=1.4 GeV, 68% c.l. (band)
Pion PDF
2.0
1.5
1.0
0.5
0.0
0.01 0.03 0.1 0.3 0.5 0.7 1


xg (x,Q) at Q=1.4 GeV, 68% c.l. (band)
Pion PDF
x
χ²=450.824
χ²=440.416
χ²=447.648
χ²=450.069
χ²=432.16
6
5
4
3
2
1
0
-1
0.01 0.03 0.1 0.3 0.5 0.7 1


xg (x,Q) at Q=1.4 GeV, 68% c.l. (band)
Pion PDF
FantoPDF (MC)
xFitter (no scale variation)
JAM21
4
3
2
1
0
0.01 0.03 0.1 0.3 0.5 0.7 1
0.1 0.2 0.5 0.7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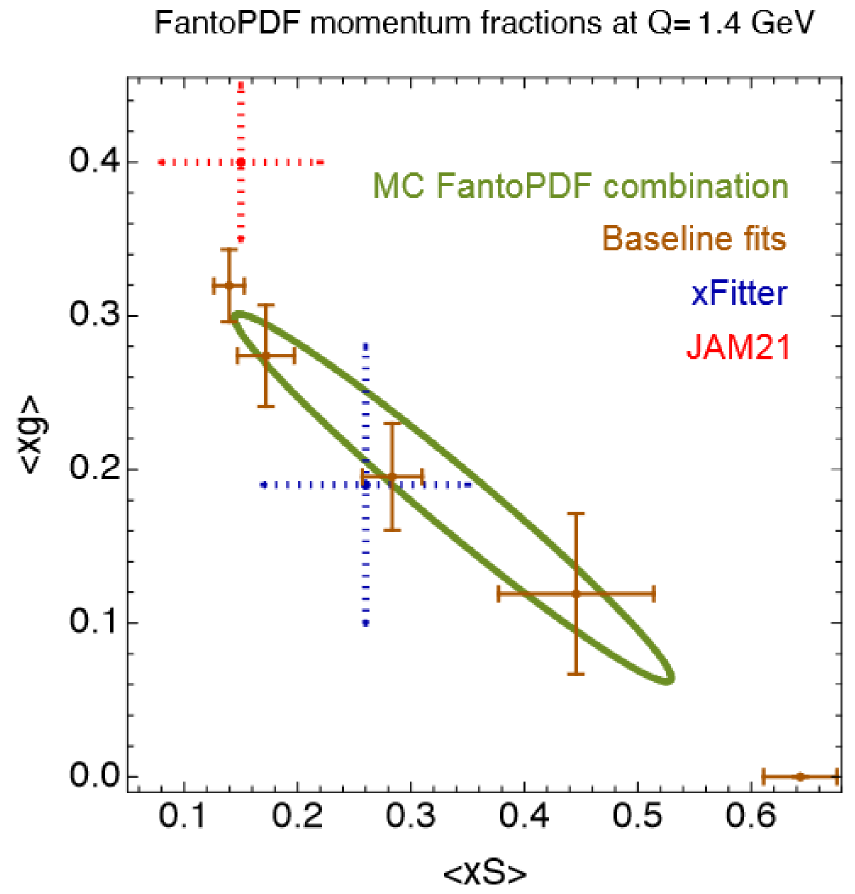
$\langle xf \rangle$ distribution

Name	Q [GeV]	$\langle xV \rangle$	$\langle xS \rangle$	$\langle xg \rangle$
FantoPDF (DY+γ+LN)	$\sqrt{1.9}$	0.49(8)	0.34(19)	0.18(12)
xFitter [9] (DY+γ)	$\sqrt{1.9}$	0.55(6)	0.26(15)	0.19(16)
xFitter w/o scale variation	$\sqrt{1.9}$	0.55(2)	0.26(9)	0.19(9)
JAM'18 [8] (DY)	1.27	0.60(1)	0.30(5)	0.10(5)
JAM'18 [8] (DY+LN)	1.27	0.54(1)	0.16(2)	0.30(2)
JAM'21 [11] (DY+LN)	1.27	0.53(2)	0.14(4)	0.34(6)
JAM'21 [11] (DY+LN) +NLL double Mellin	1.27	0.46(3)	0.15(7)	0.40(5)
CT18 NLO (proton)	$\sqrt{1.9}$	0.443(6)	0.160(10)	0.396(10)
CT18 NNLO (proton)	$\sqrt{1.9}$	0.451(5)	0.157(10)	0.390(10)



- Separation of the Sea and gluon PDFs is highlighted in the momentum distributions

L_2 sensitivities of various experiments for CT and MSHT PDFs

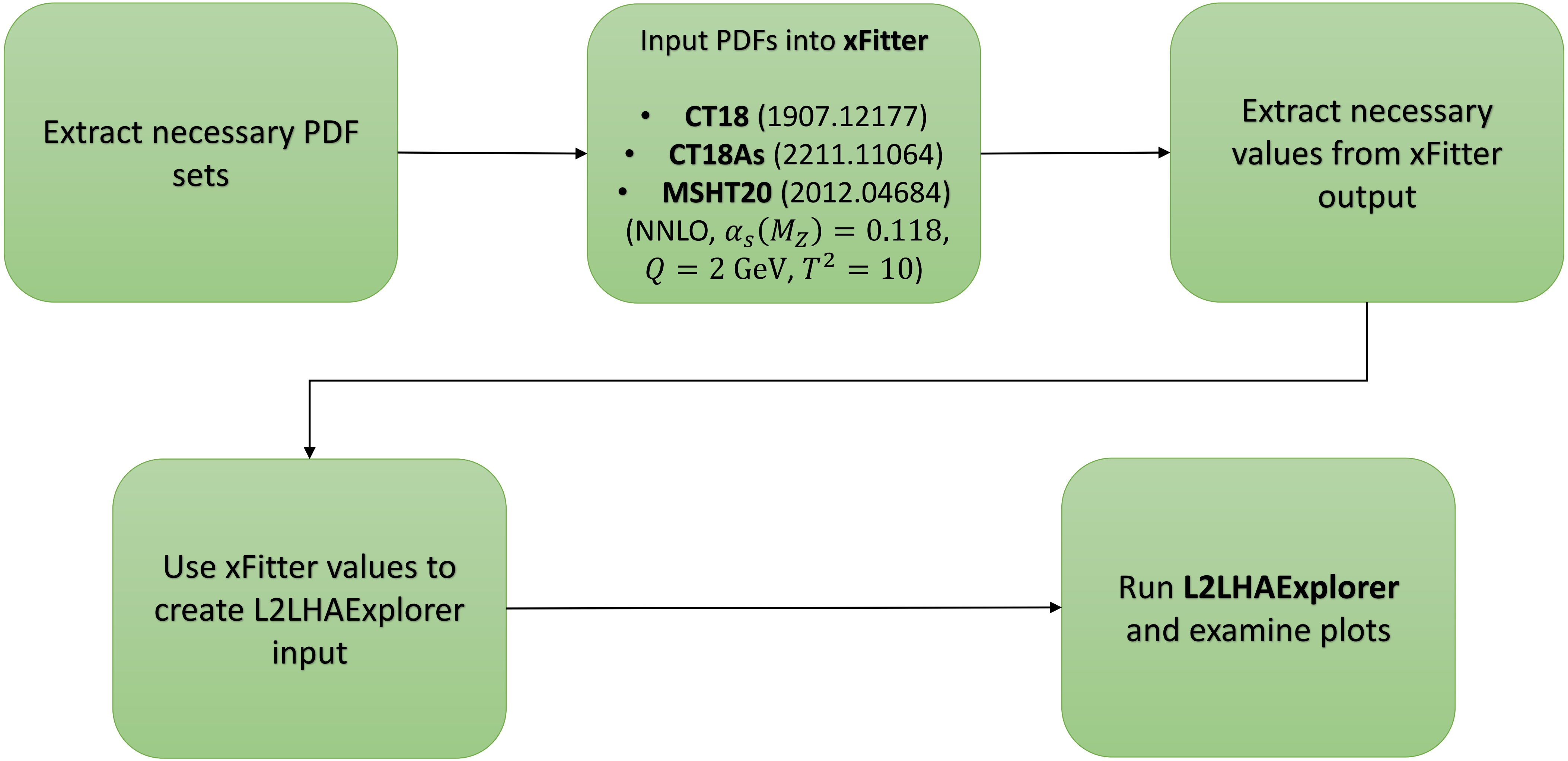
arXiv:2401.11350

- Examining L_2 sensitivities can help determine the potential impact of a dataset on a given PDF set.
- Using xFitter allows for the possibility to examine the potential impact without having to include the dataset into the global analysis first.

L_2 sensitivity

- The L_2 sensitivity method visually displays the potential influence of an experiment on a PDF set given x and $Q.$
- $S^H_{f,L_2}(E) = \frac{(\vec{\nabla}\chi^2_E \cdot \vec{\nabla} f)}{\delta_H f} = \boldsymbol{\delta_H \chi^2_E \times C_H(f, \chi^2_E)}$ arXiv: [1904.00022]()
  - $C_H(f, \chi^2_E) = \frac{1}{4\delta_H f \delta_H \chi^2_E} \sum_{i=1}^{D} (f_{+i} - f_{-i})(\chi^2_{E,+i} - \chi^2_{E,-i})$
    - $\delta_H \chi^2_E$: 1σ uncertainty for χ^2 for experiment E at the 68% C.L.
    - $\delta_H f$: 1σ uncertainty for the PDF f at the 68% C.L.
    - $f_{\pm i}$: $f_0 \pm \delta f$ for the i-th experiment.
    - $\chi^2_{E,\pm i}$: $\chi^2_{E,0} \pm \delta_H \chi^2_E$ for the i-th experiment.
- Positive correlation/$S^H_{f,L_2}(E)$ indicates experiment E favors a smaller PDF to minimize χ^2_E.
- Negative correlation/$S^H_{f,L_2}(E)$ indicates experiment E favors a larger PDF to minimize χ^2_E.

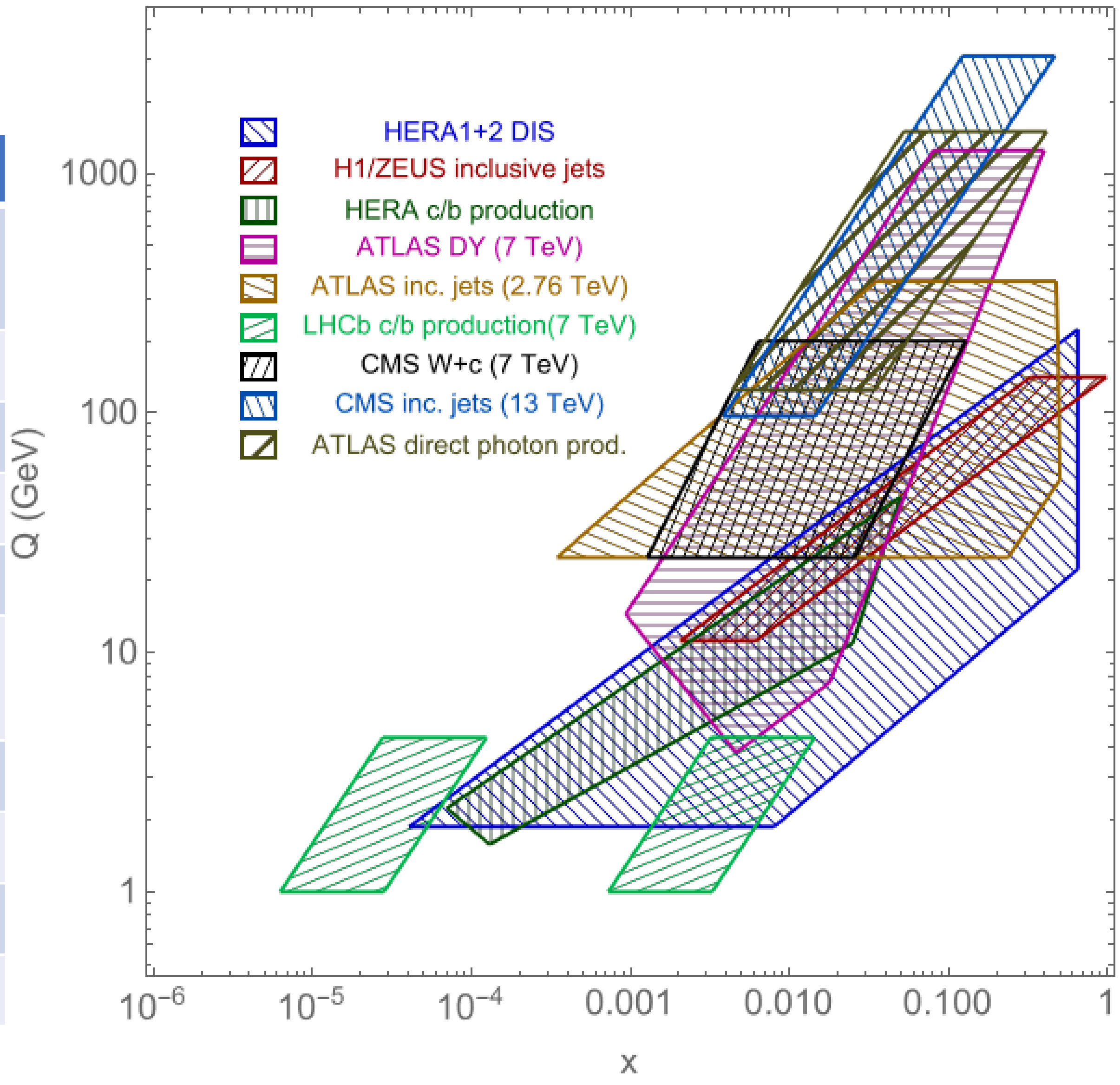
L_2 calculation process using **xFitter** and **L2LHAExplorer**



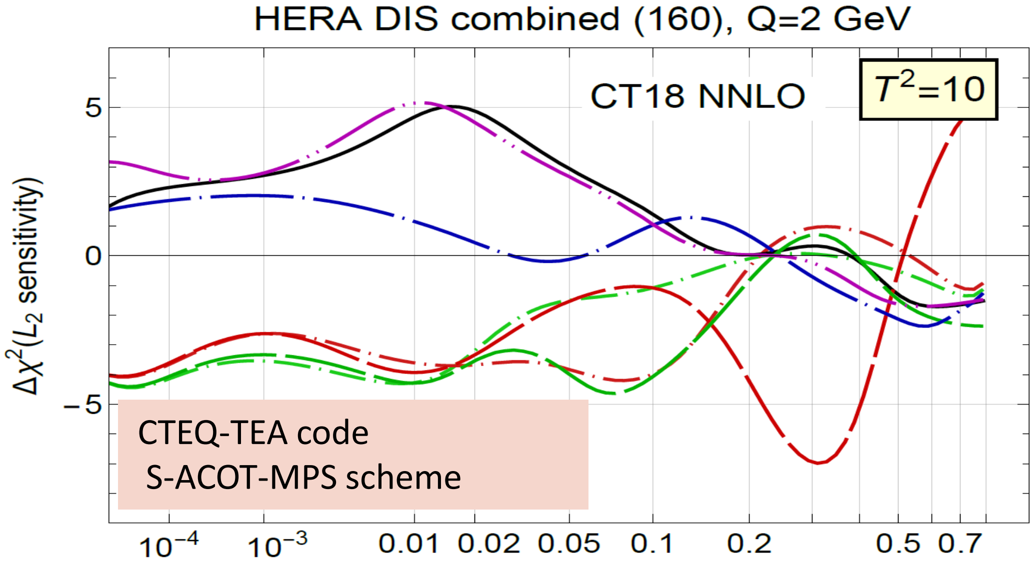
Experiments Studied

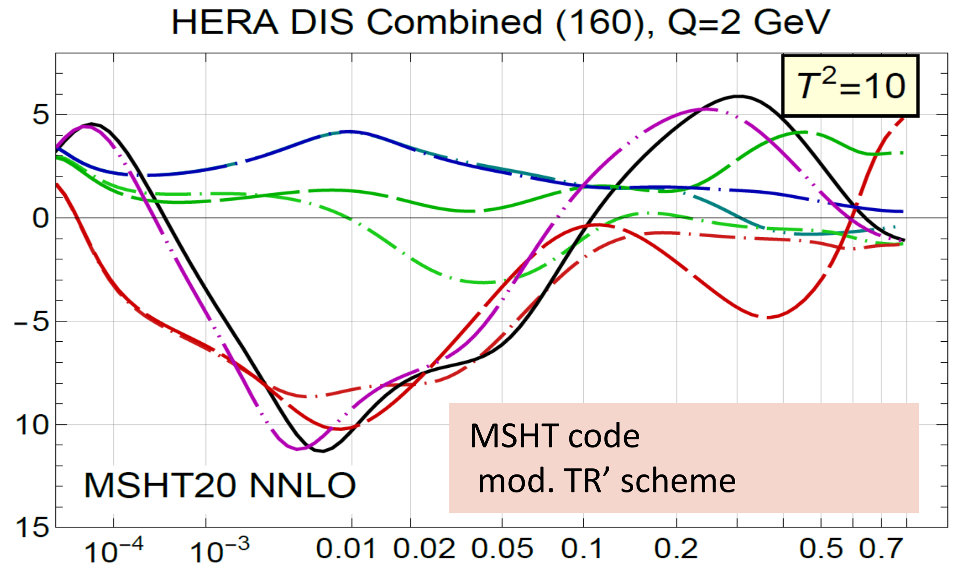
	CT18	CT18As	MSHT20
ATLAS direct γ production - 8 and 13 TeV	×	×	×
ATLAS DY - 7 TeV	×	×	✓
ATLAS inc. jet - 2.76 TeV	×	×	✓
CMS inc. jet - 13 TeV	×	×	×
CMS $W + c$ - 7 TeV	×	×	×
H1+ZEUS c and b production	×	×	✓
H1 jet			
HERA I+II DIS	✓	✓	✓
LHCb c and b – 7 TeV	×	×	×
ZEUS jet			

Approximate {x,Q} for all experiments

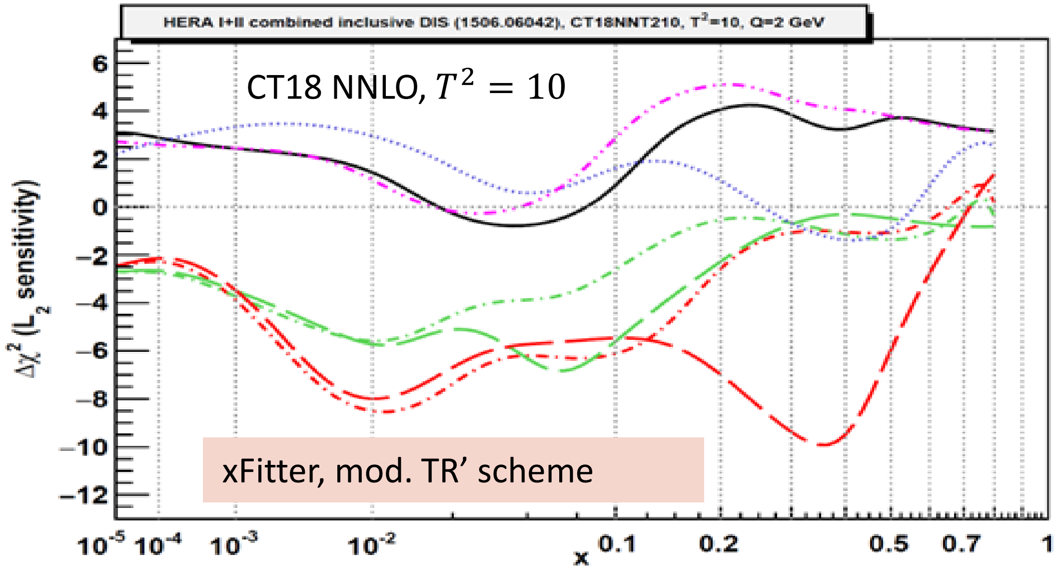


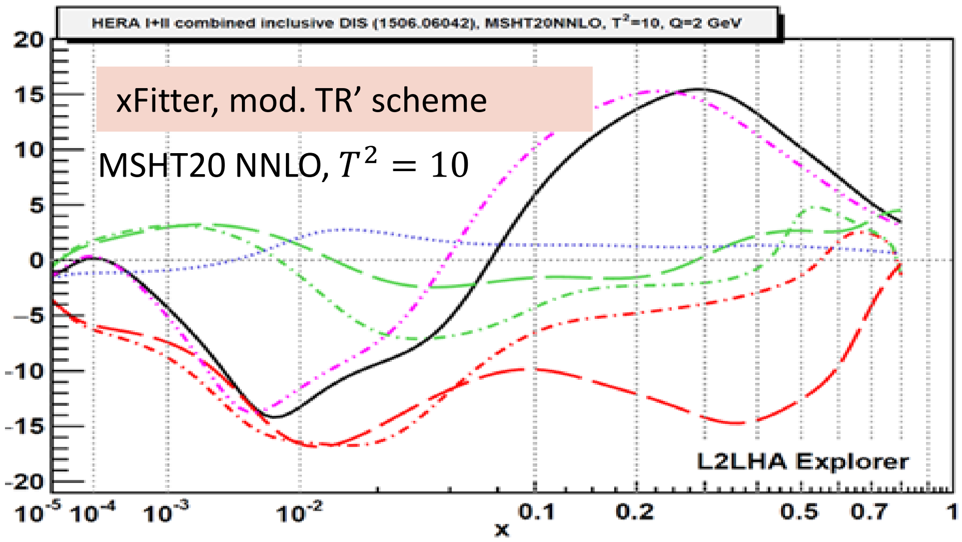
HERA I+II combined inclusive DIS [in CT18 and MSHT20]



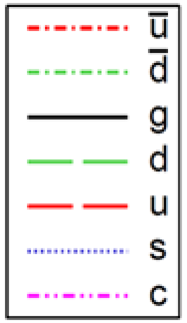


Upper row:
From Jing et al.,2306.03918





Lower row:
From L. Kotz, 2401.11350

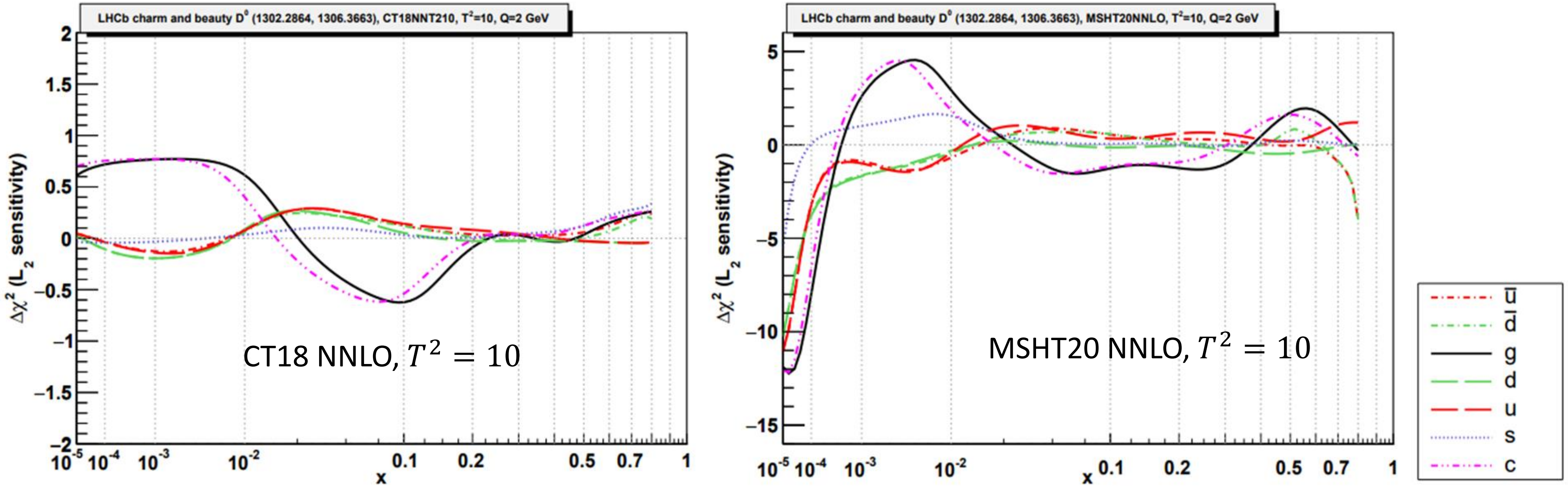


Left column: differences in χ^2 definition and heavy-quark scheme. Same PDFs and m_Q.

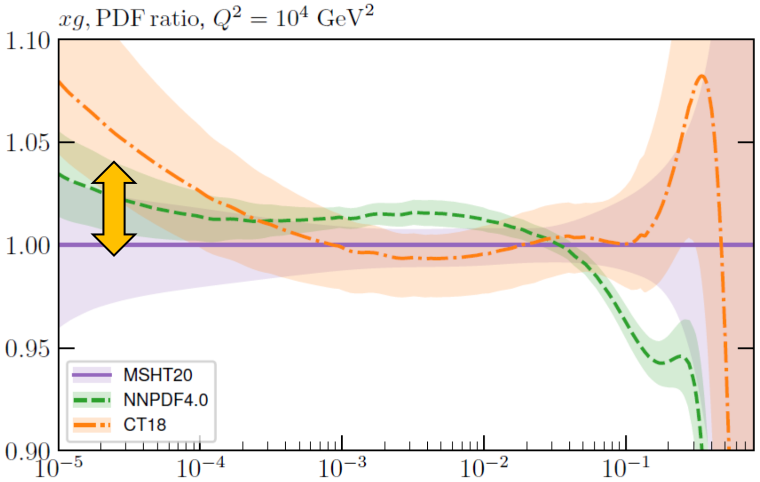
Right column: differences in χ^2 definition only. Same PDFs and m_Q.

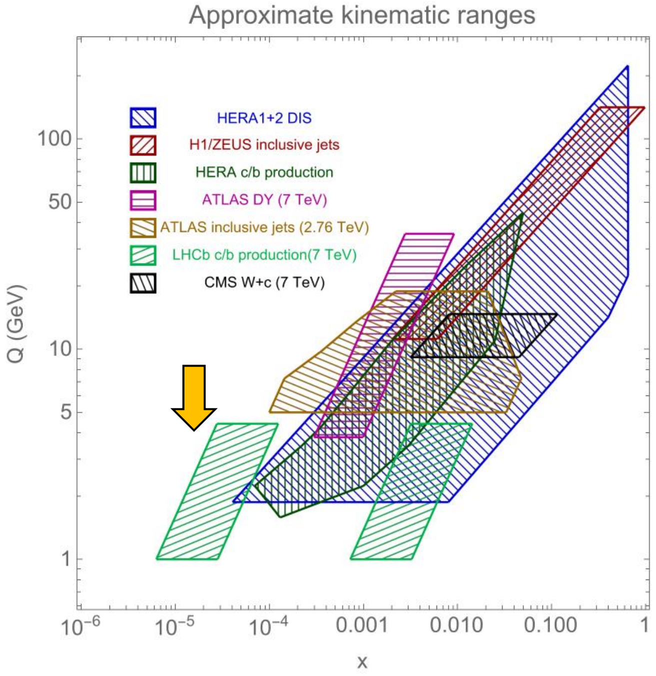
LHCb c and b @7 TeV; $p_T^{\text{meson}} \geq 2$ GeV [Not in CT18 or MSHT20]

From L. Kotz, 2401.11350

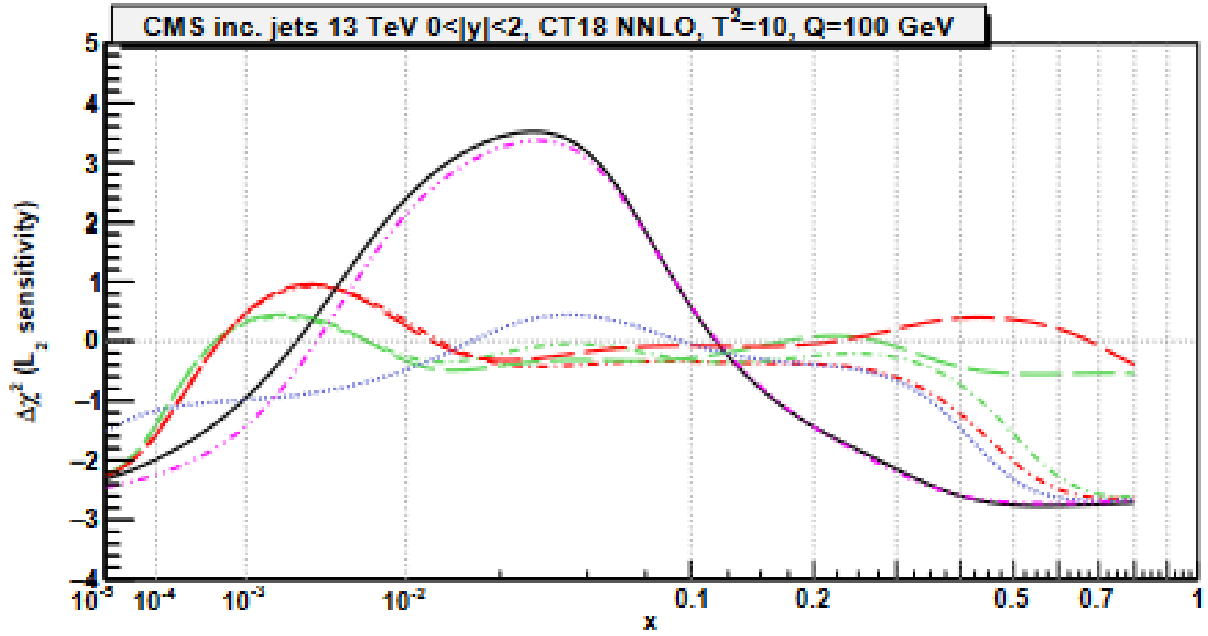


- LHCb c and b production prefers about the same gluon for CT18/CT18As.
- A larger gluon is preferred for MSHT20 at $x < 10^{-4}$.



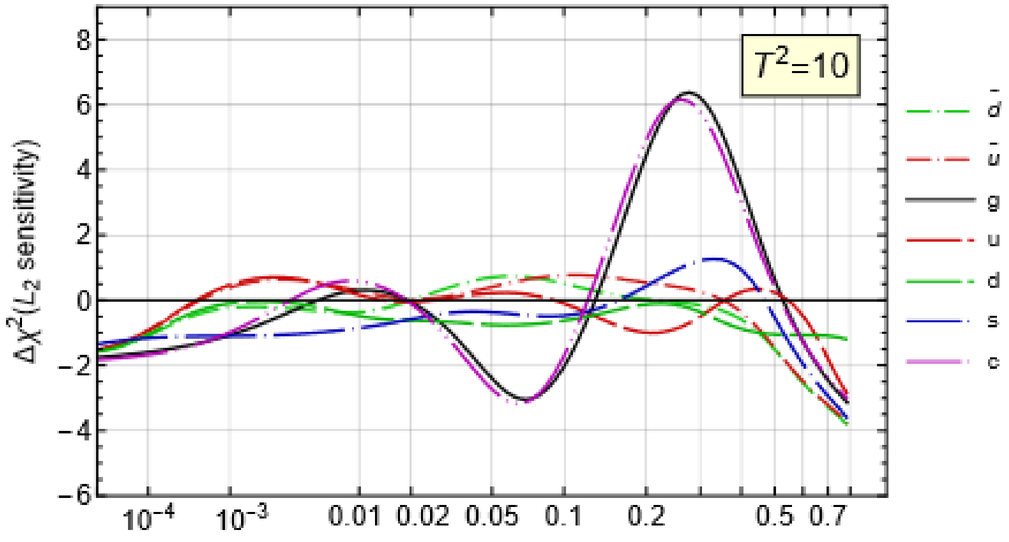


CMS inclusive jets $\sqrt{s} = 13$ TeV (CT18/CT18As)

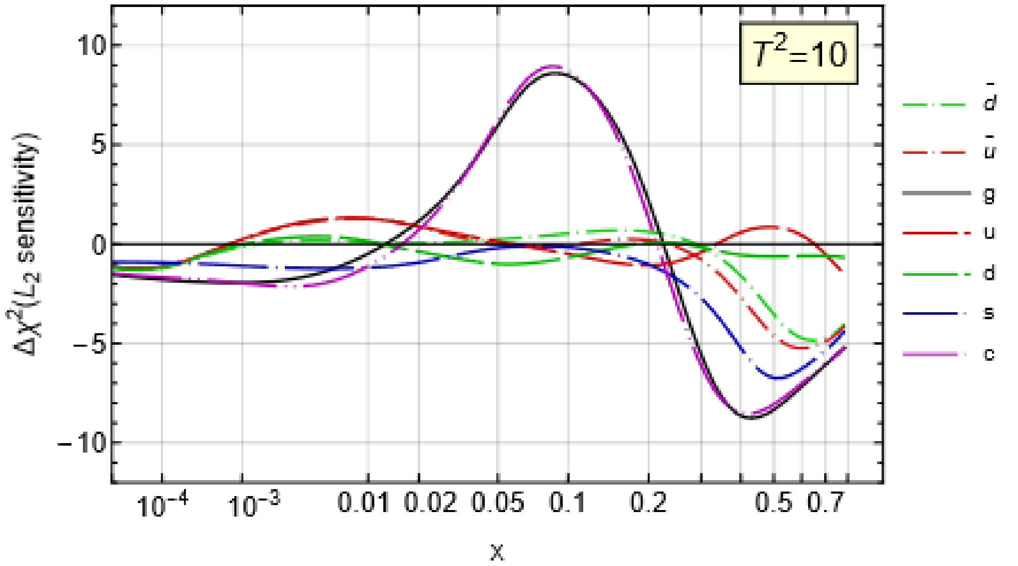


- Expect to see the position of the L_2 peak to shift a factor of $\frac{\sqrt{s}}{13\text{ TeV}}$.
- The position of the peak for 13 TeV follows the expected shift with $\sqrt{s} = 8$ TeV. However, not for 7 TeV.
- CMS 13 TeV jet data may be more compatible with the 8 TeV data than the 7 TeV data for CT18.

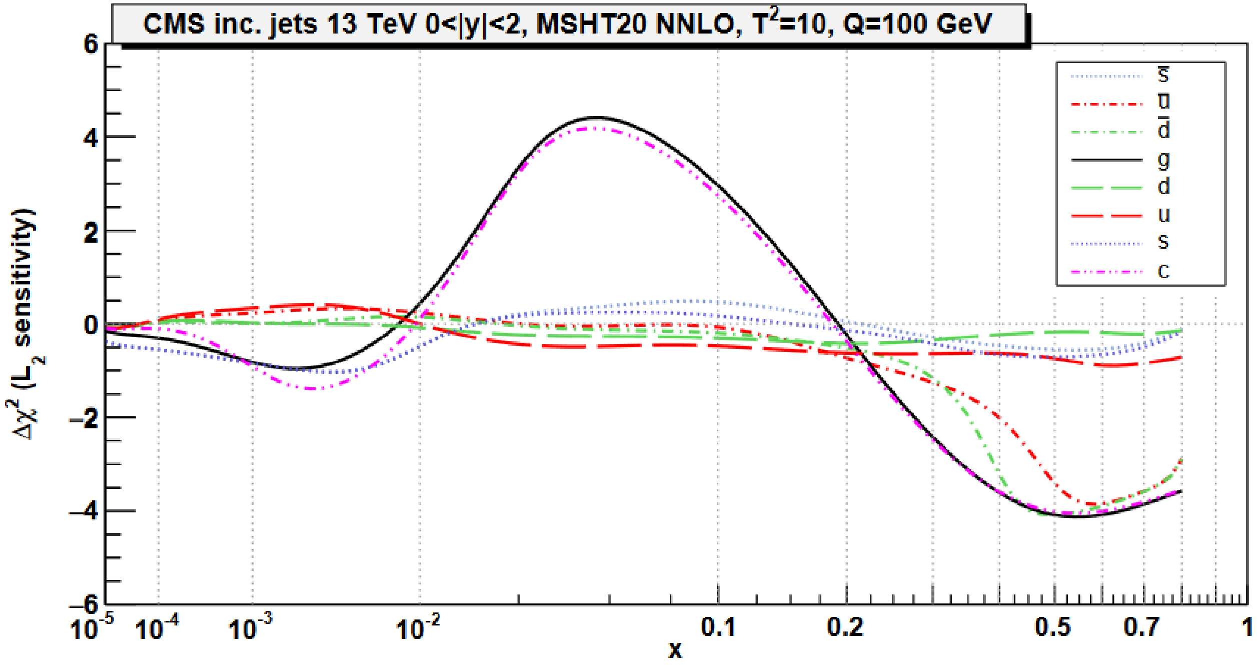
CT18 NNLO
CMS 7 TeV jets (542), Q=100 GeV



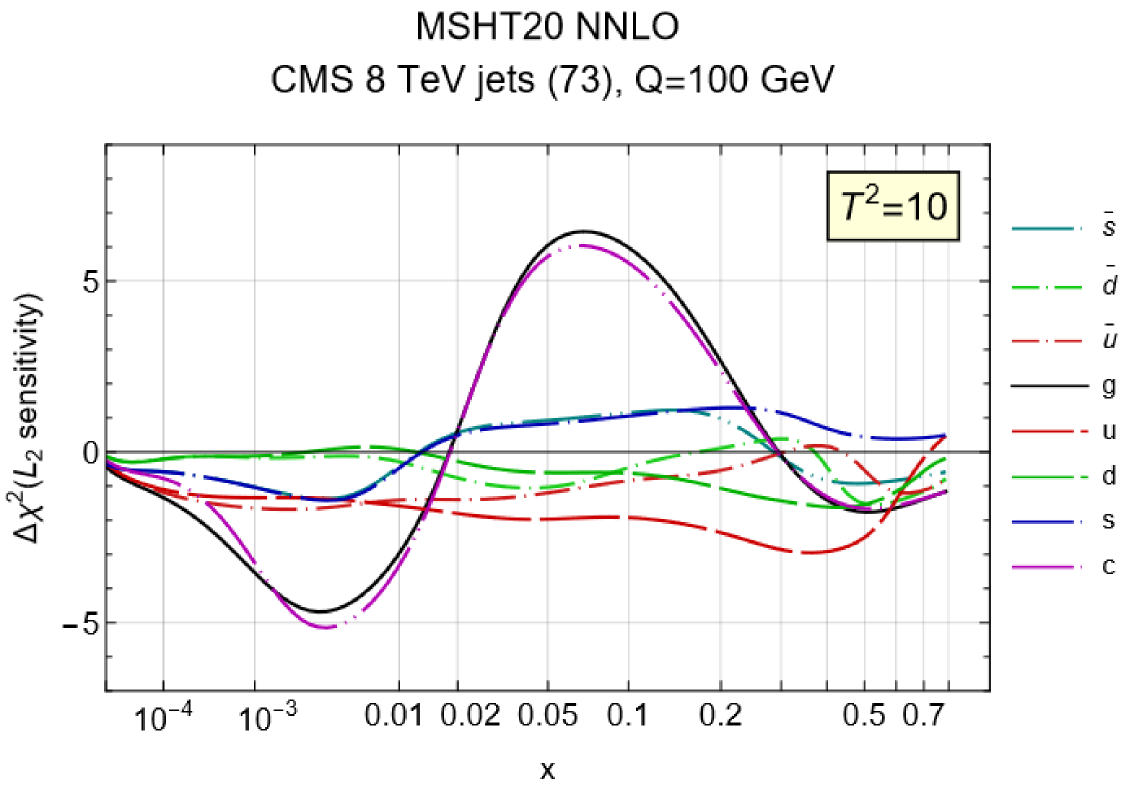
CT18 NNLO
CMS 8 TeV jets (545), Q=100 GeV



CMS inclusive jets $\sqrt{s} = 13$ TeV (MSHT20)



No CMS 7 TeV on hepforge available



- Expect to see the position of the L_2 peak to shift a factor of $\frac{\sqrt{s}}{13\ \mathrm{TeV}}$.
- The position of the peak for 13 TeV follows the expected shift with $\sqrt{s} = 8$ TeV. However, not for 7 TeV.
- CMS 13 TeV jet data may be more compatible with the 8 TeV data than the 7 TeV data for MSHT20. However, it is inconclusive as the $\sqrt{s} = 7$ TeV plot is not available.

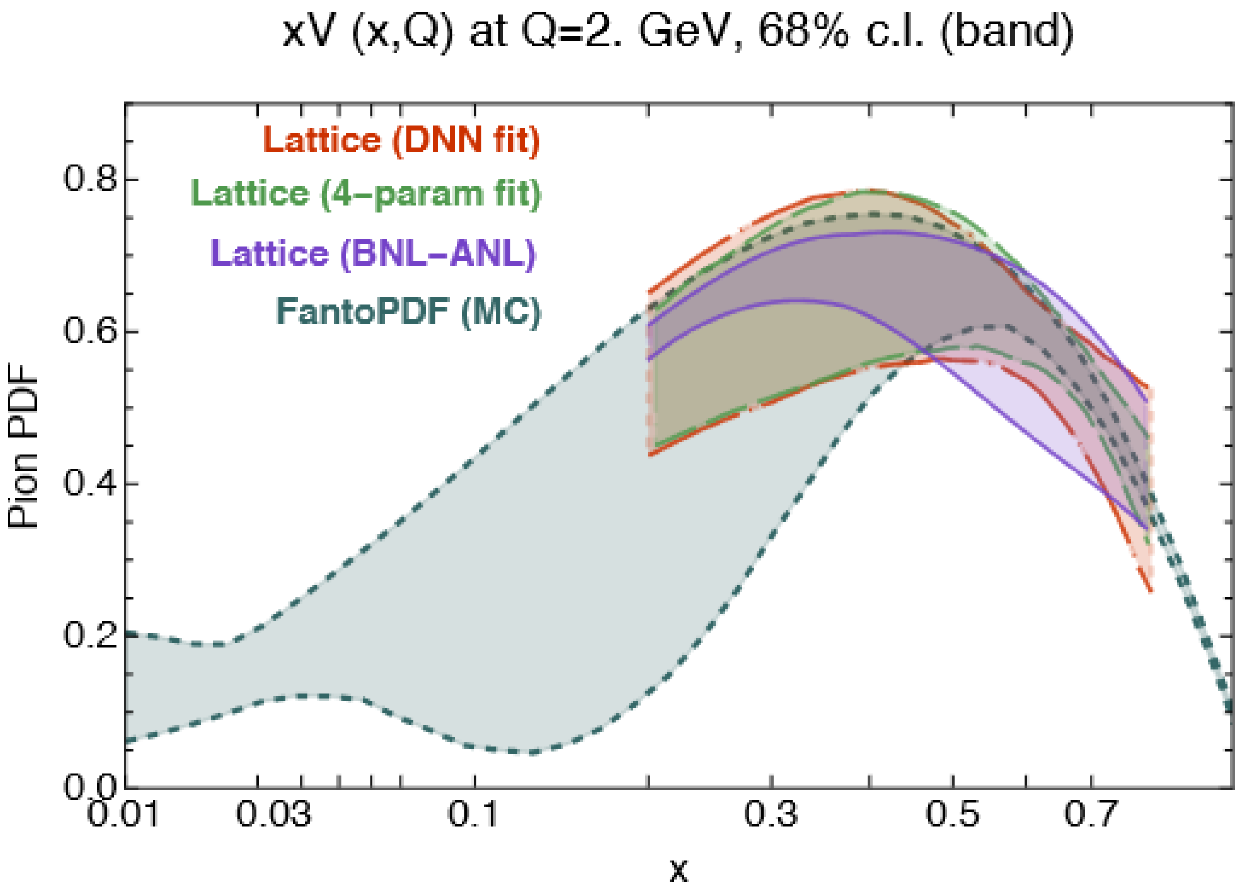
Conclusion

- Despite having similar momentum fractions, Fantômas π^+ PDFs have overall larger uncertainties than other studies found.
- FantoPDF demonstrates the versatility of the Fantômas parameterization and future projects using this parameterization are currently underway.
- Examining L_2 sensitivities reveal that xFitter may treat theoretical calculations differently than CT and MSHT groups.
- CMS 13 TeV jet sensitivities indicates that it may not be compatible with 8 TeV data for CT18, CT18As, and MSHT20 PDF sets.
- Inclusion of LHCb c and b production data may lead to smaller gluon in CT PDFs and a larger gluon in MSHT20.

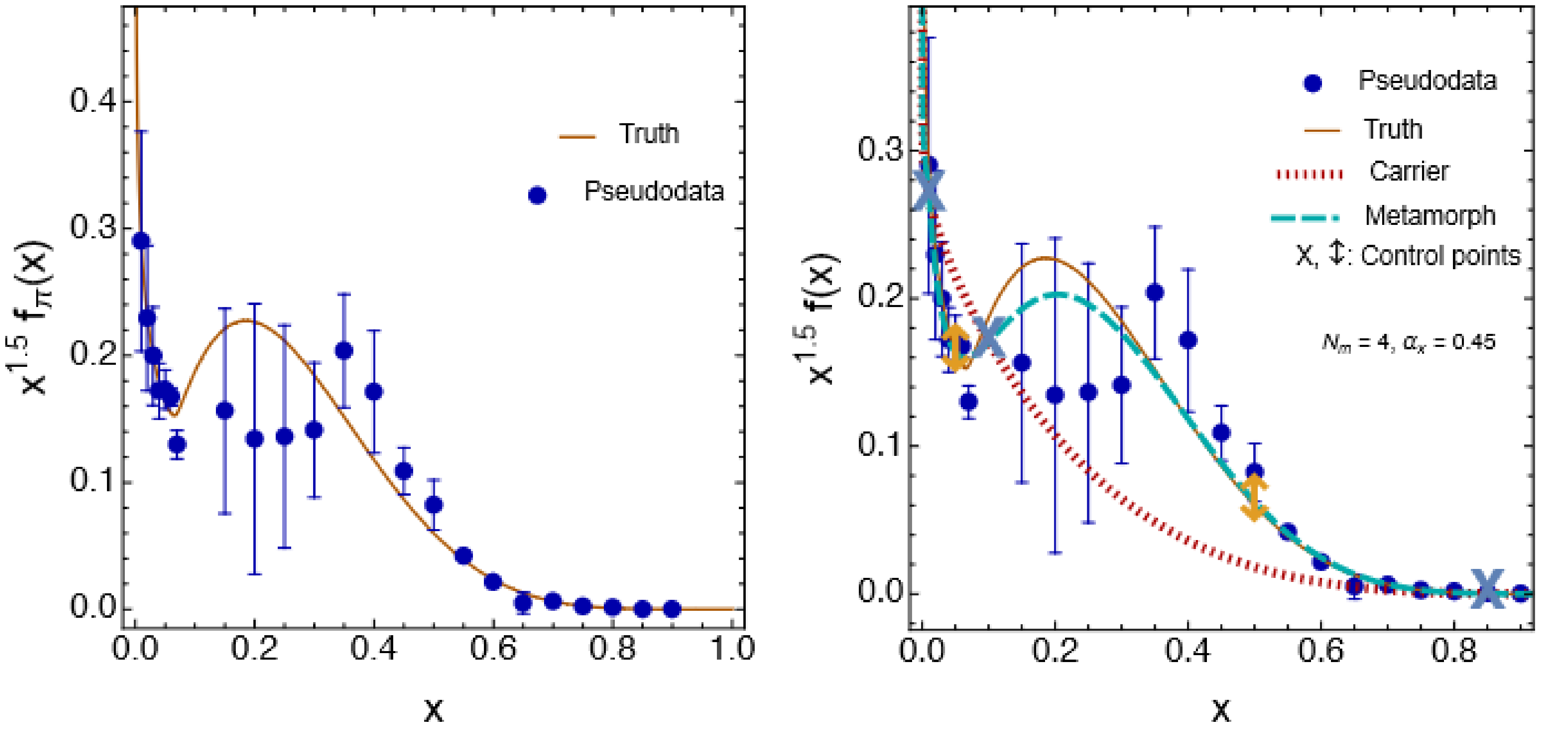
Extra Slides

Comparison with lattice results

Name	Q[GeV]	$\langle x(u+\bar{u})_{\pi^+}\rangle$	$\langle xg\rangle$
FantoPDF	2	0.331(25)	0.24(10)
HadStruct [19]	2	0.2541(33)	–
Ref. [20]	3.2	0.216(19)(8)	–
ETM [46]	2	0.261(3)(6)	–
ETM [91]	2	$0.601(28)\vert_{u+d}$	0.52(11)
Ref. [94]	2	–	0.37(8)(12)
Ref. [92]	2	–	0.61(9)
Ref. [93]	2	–	0.364(38)(36)
ZeRo Coll. [95]	2	0.245(15)	–
Ref. [96]	7	0.02	–

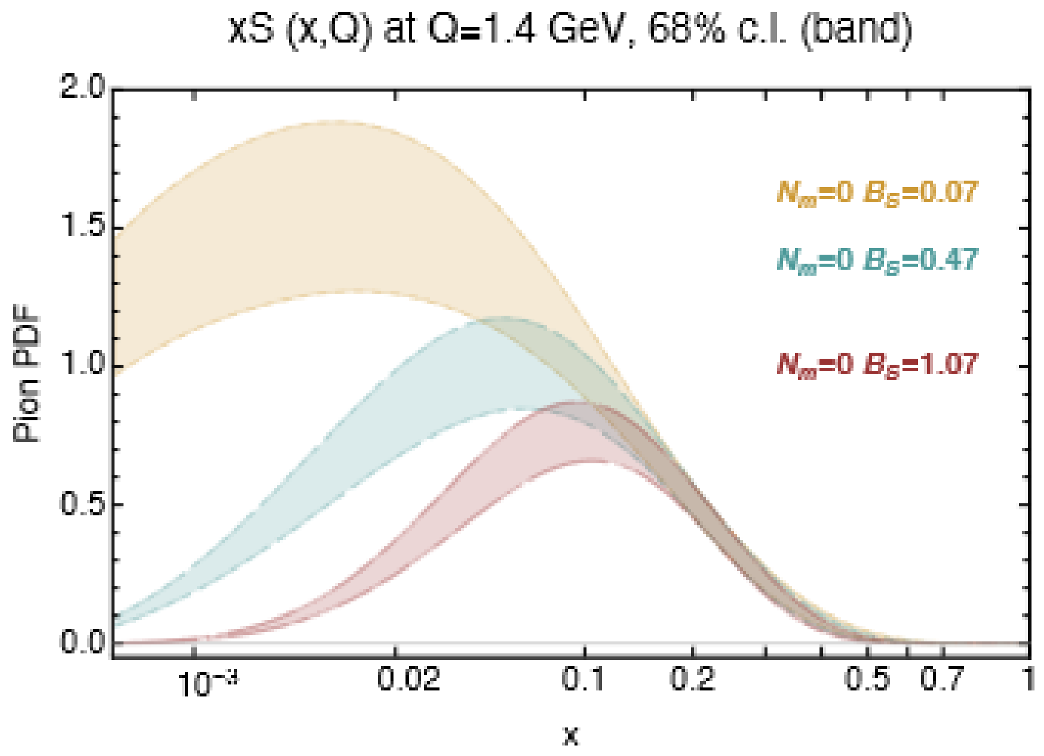


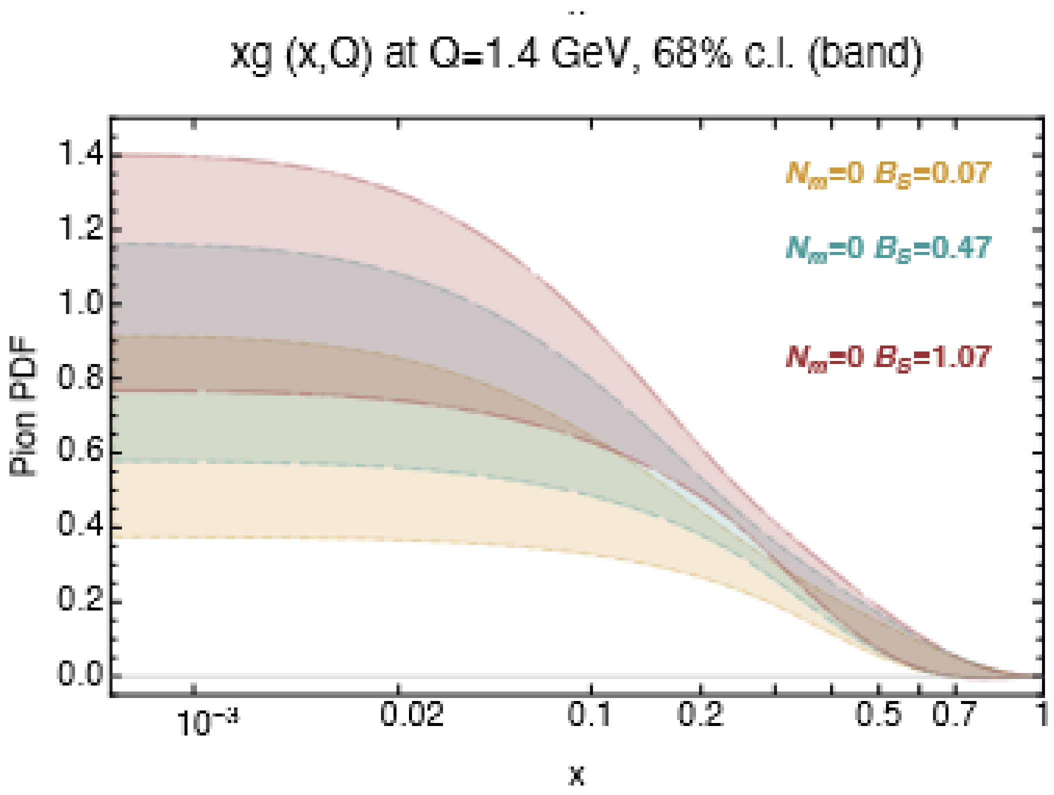
Example of Fantômas fitting



$$xf(x) = A_f x^{B_f} (1-x)^{C_f} \times f_{mod}(x)$$

$N_m = 0$ (DY+γ)	χ^2 [$d.o.f = 379 - 5$]	$\langle xV \rangle$	$\langle xS \rangle$	$\langle xg \rangle$
$B_S = 0.07$	445.70	0.556	0.268	0.177
$B_S = 0.27$	445.38	0.557	0.239	0.204
$B_S = 0.47$	445.29	0.558	0.217	0.225
$B_S = 0.67$	445.36	0.559	0.199	0.243
$B_S = 0.87$	445.52	0.559	0.184	0.257
$B_S = 1.27$	445.76	0.559	0.172	0.269


xS (x,Q) at Q=1.4 GeV, 68% c.l. (band)
N_m=0 B_S=0.07
N_m=0 B_S=0.47
N_m=0 B_S=1.07
Pion PDF
x


xg (x,Q) at Q=1.4 GeV, 68% c.l. (band)
N_m=0 B_S=0.07
N_m=0 B_S=0.47
N_m=0 B_S=1.07
Pion PDF
x