Clocks!

Quantum Sensing Autumn School for DRD5 / RDQuantum

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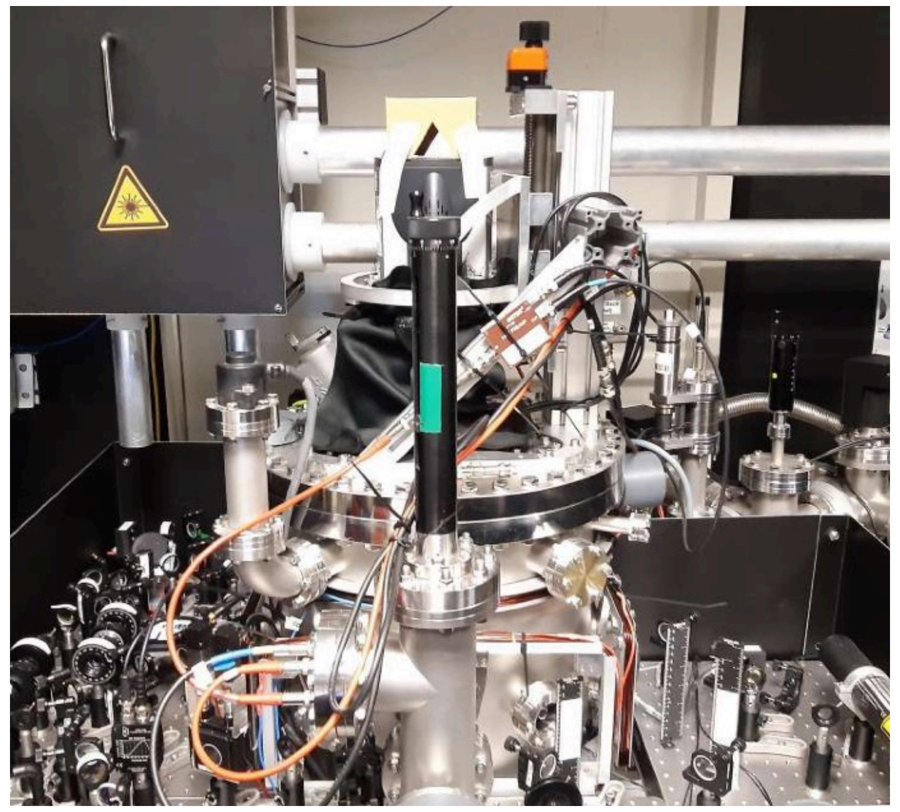
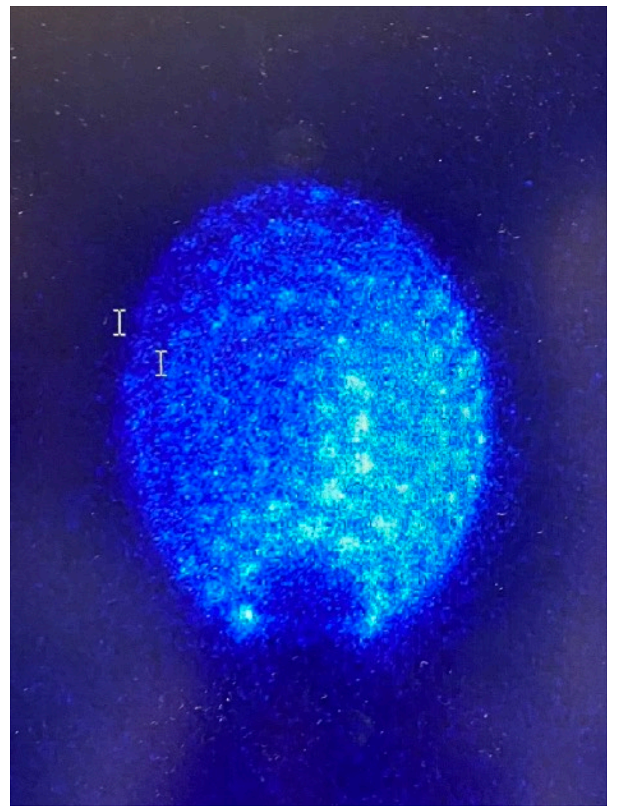
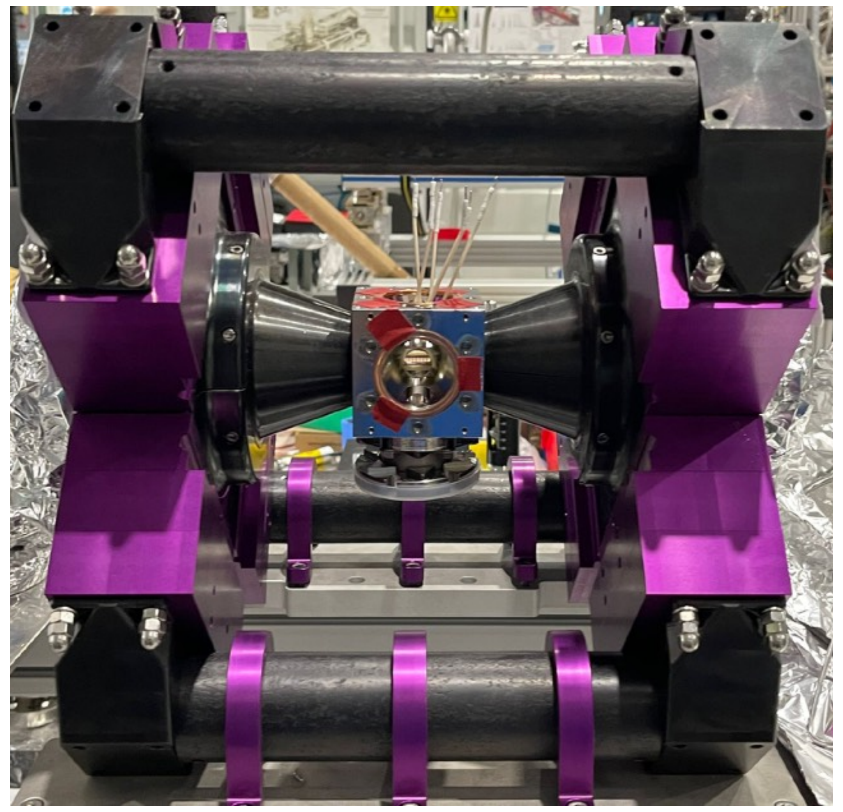
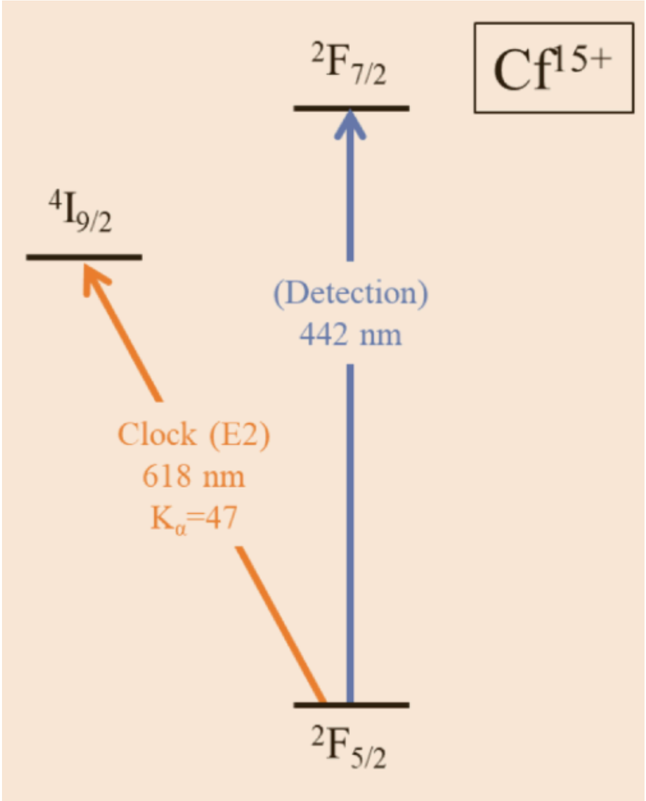
November 5, 2024

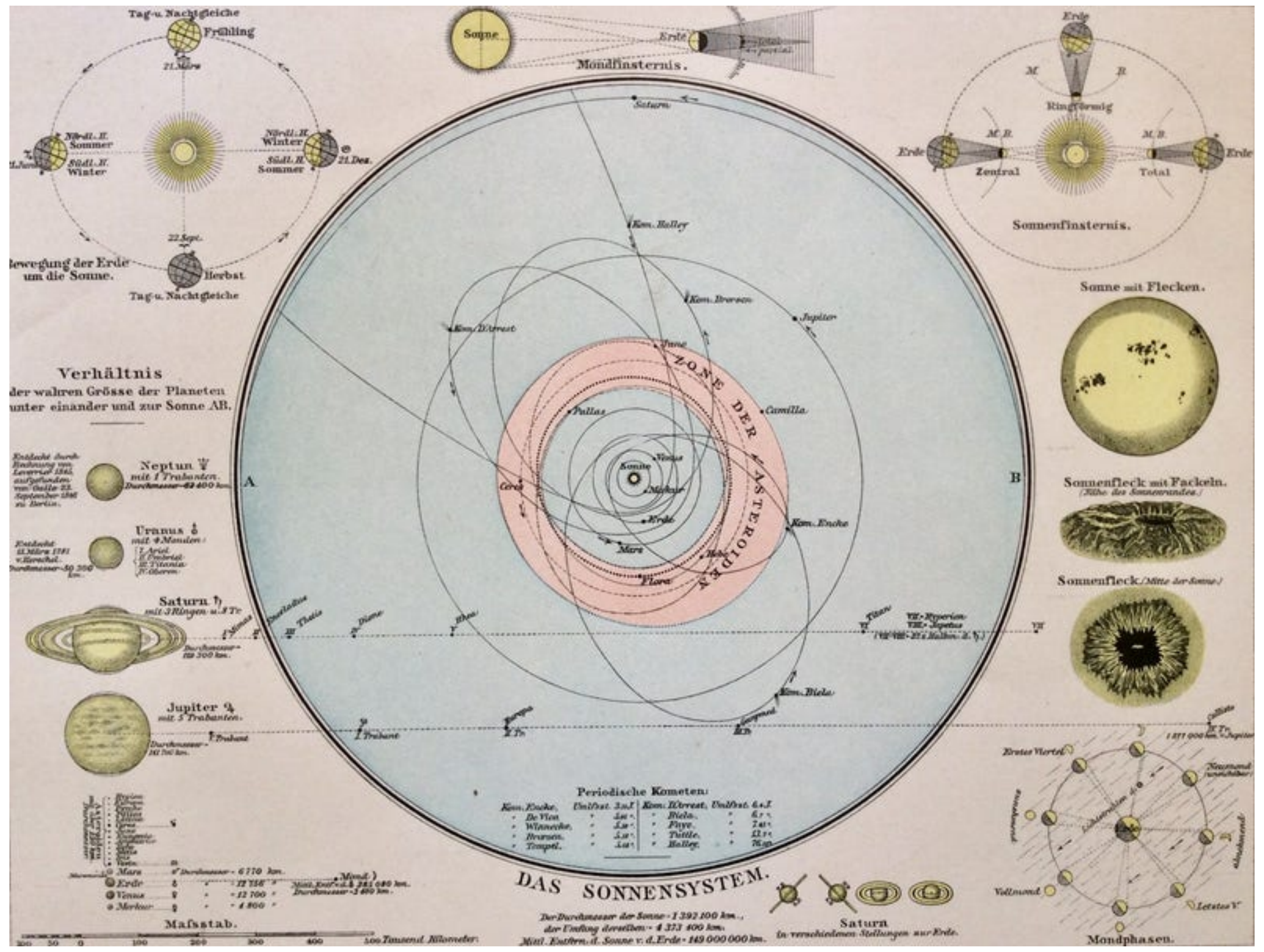

Outline

- Clocks in History
- Clock Basics
- What makes a good clock?
- Physics with a Clock
- Networks of Clocks
- The Future?

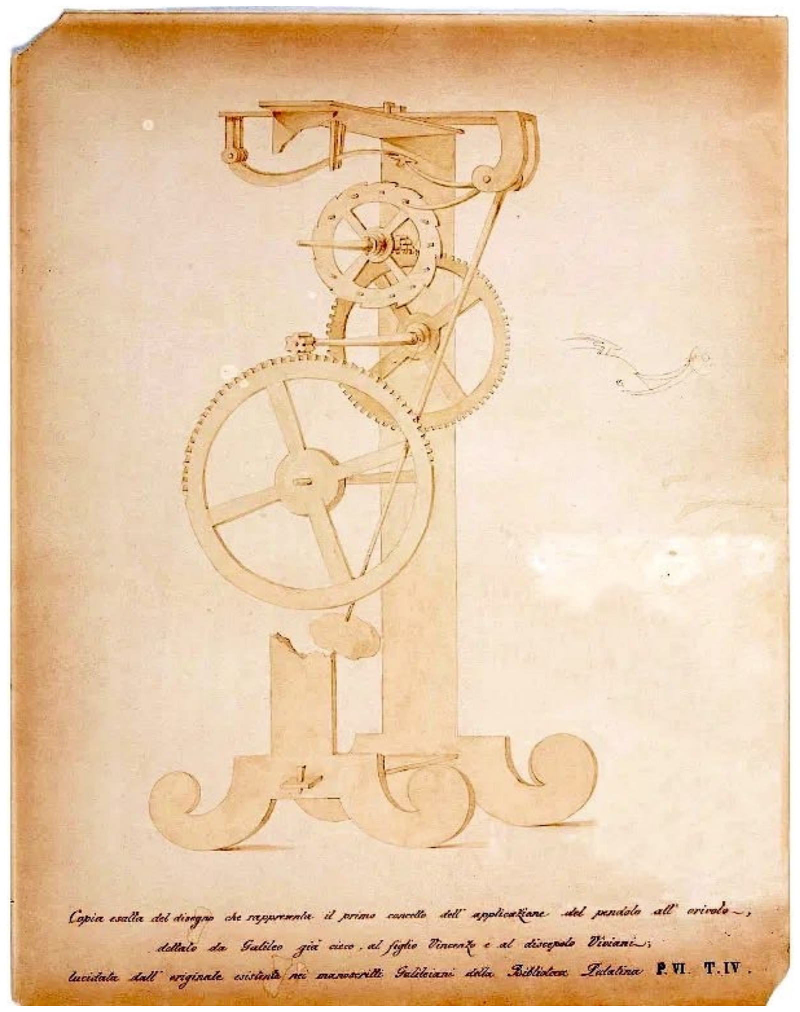
- Disclaimer: not covering essential AMO techniques for clocks, such as laser / sympathetic cooling, trapping techniques

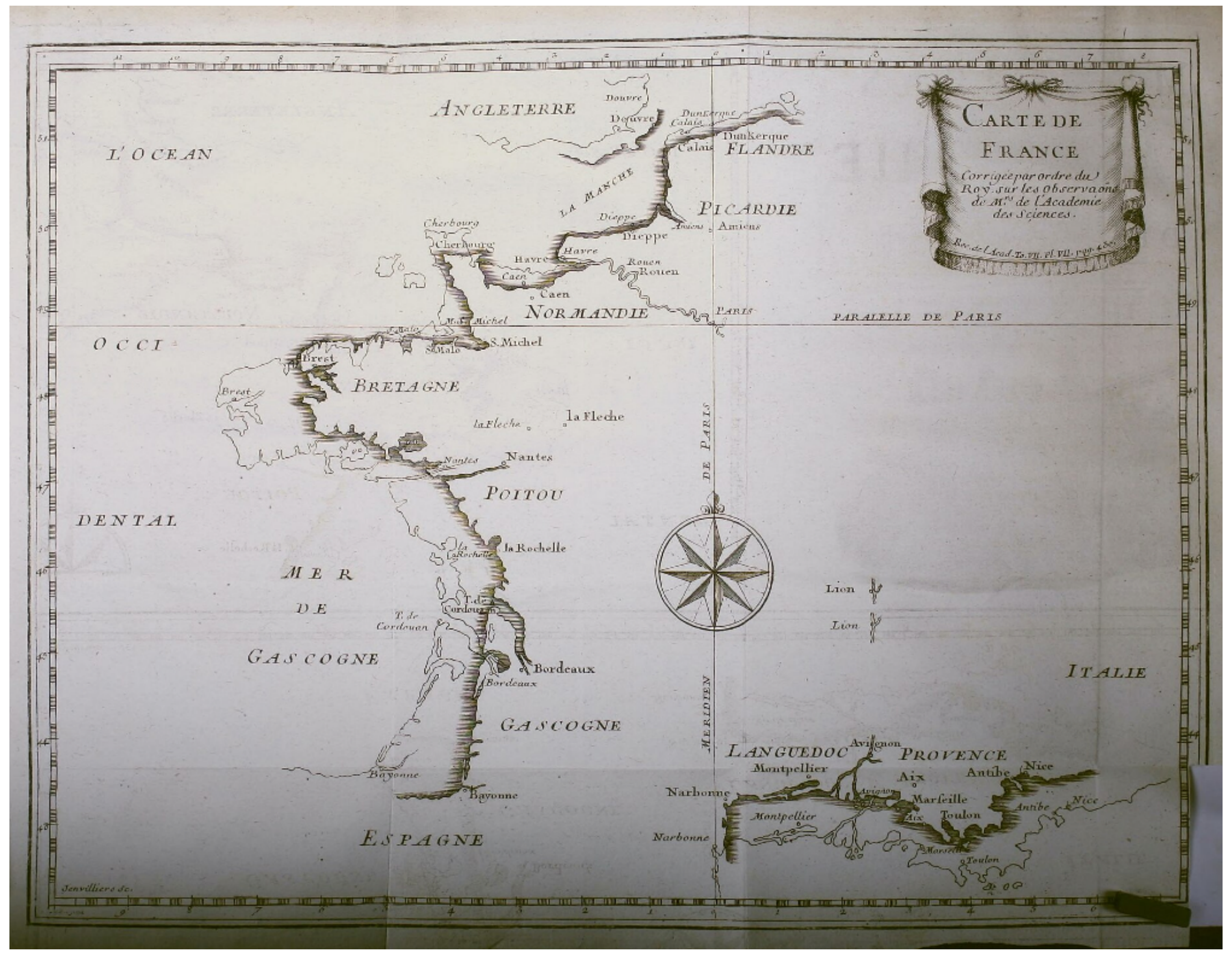
Revolutionary Clocks for Science

- Galileo proposed the pendulum clock - to fix longitude for astronomical observations (1637)
- Huygens made the first pendulum clocks (1656); essential for astronomy, navigation, mapping the earth
- Newton used pendulum clocks for gravity experiments (1687)
- Maxwell proposed using atoms as natural standard units of time (and length) in 1879
- Many advances driven by science needs/discoveries, which in turn enables new science

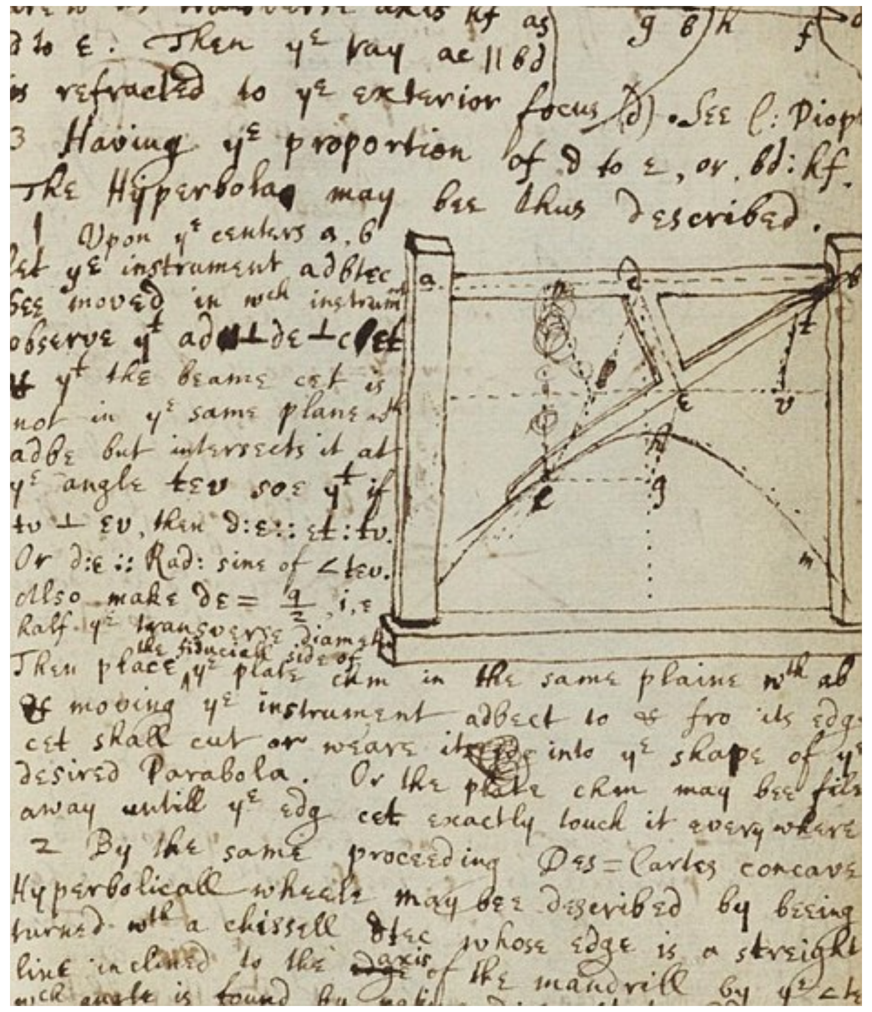


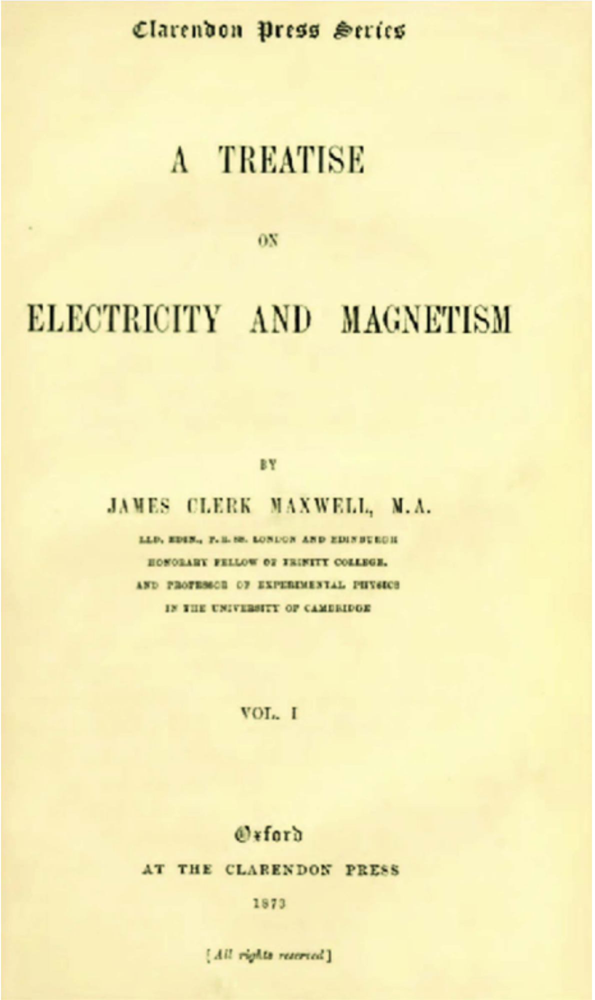
Astronomical predictions for the Solar System


Galileo's pendulum clock design



Remapping of France with pendulum clocks


Newton's tests of gravity


Clarendon Press Series

A TREATISE

ON

ELECTRICITY AND MAGNETISM

BY

JAMES CLERK MAXWELL, M.A.

VOL. I

Oxford

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1873

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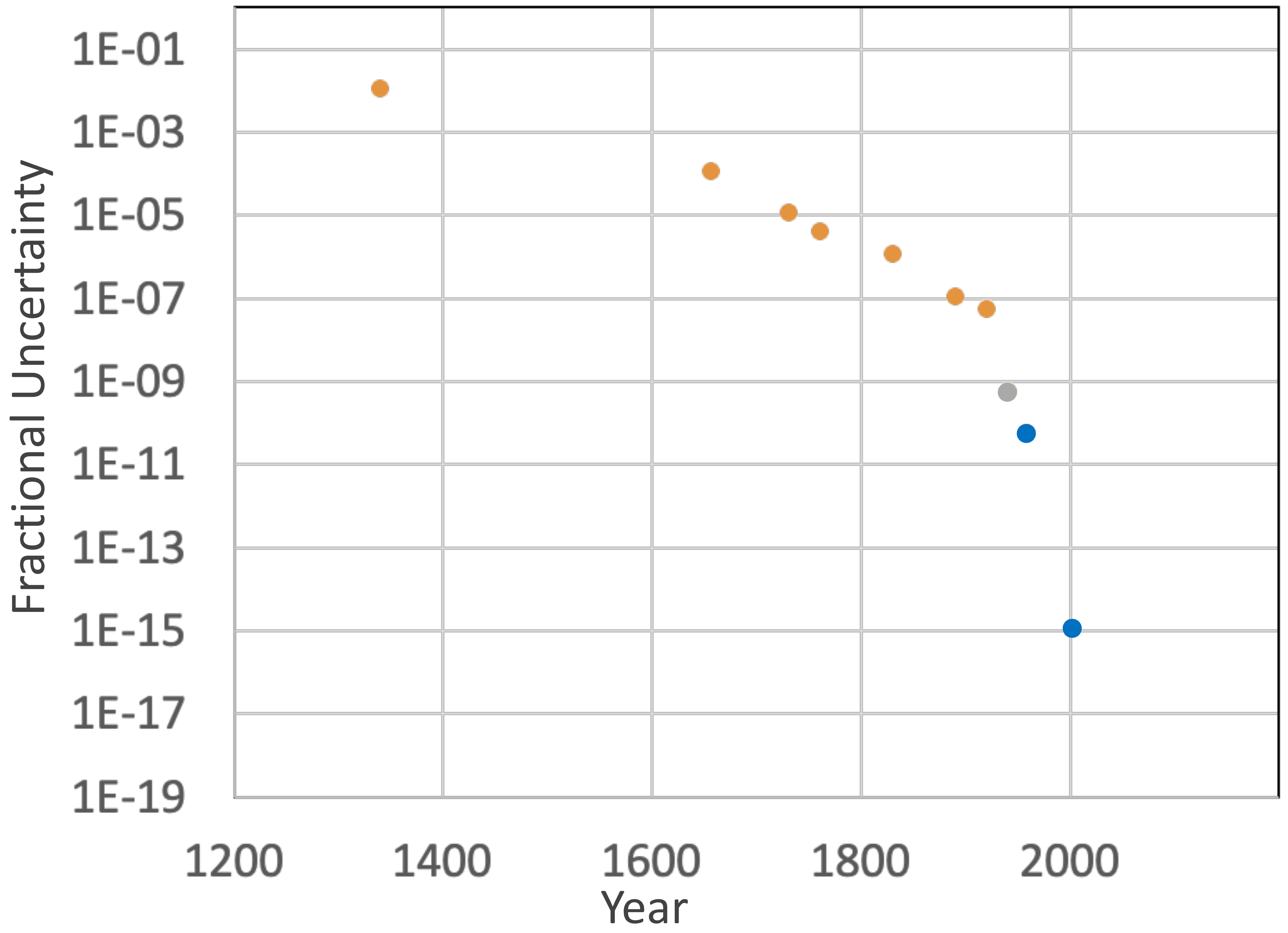
Maxwell: atoms as clocks?

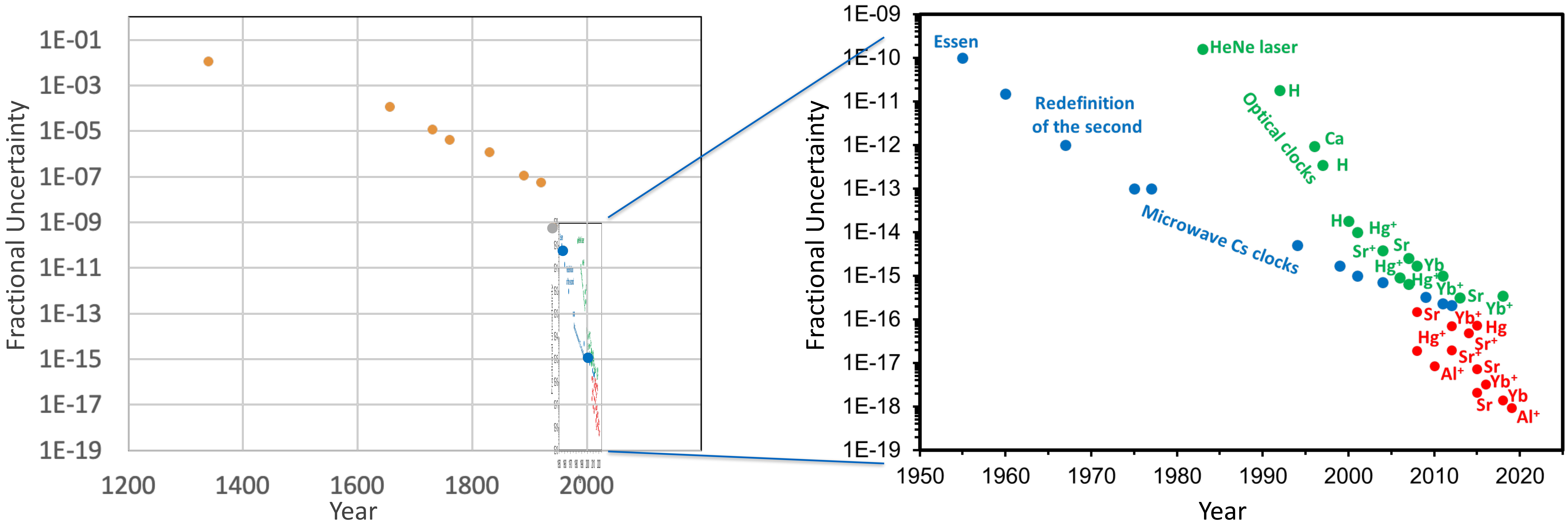
Fractional Uncertainty of Clocks vs. Year

- Mechanical
- Crystal
- Atomic (microwave)



Fractional Uncertainty of Clocks vs. Year

- Mechanical
- Crystal
- Atomic (microwave)
- Atomic (optical in Hz)
- Atomic (optical w/ estimated uncertainty)



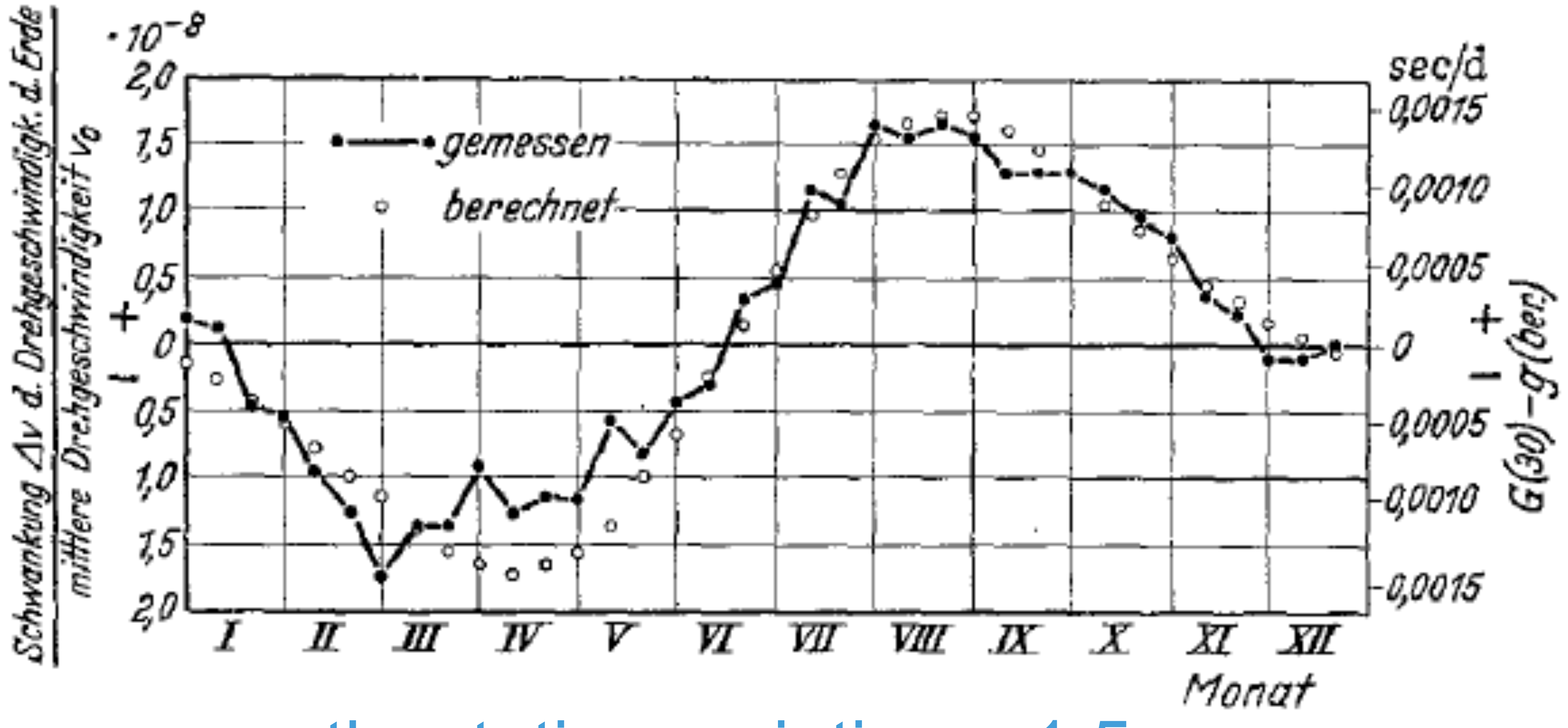
[R. Godun, NPL]

What Science can you do with an Optical Atomic Clock?

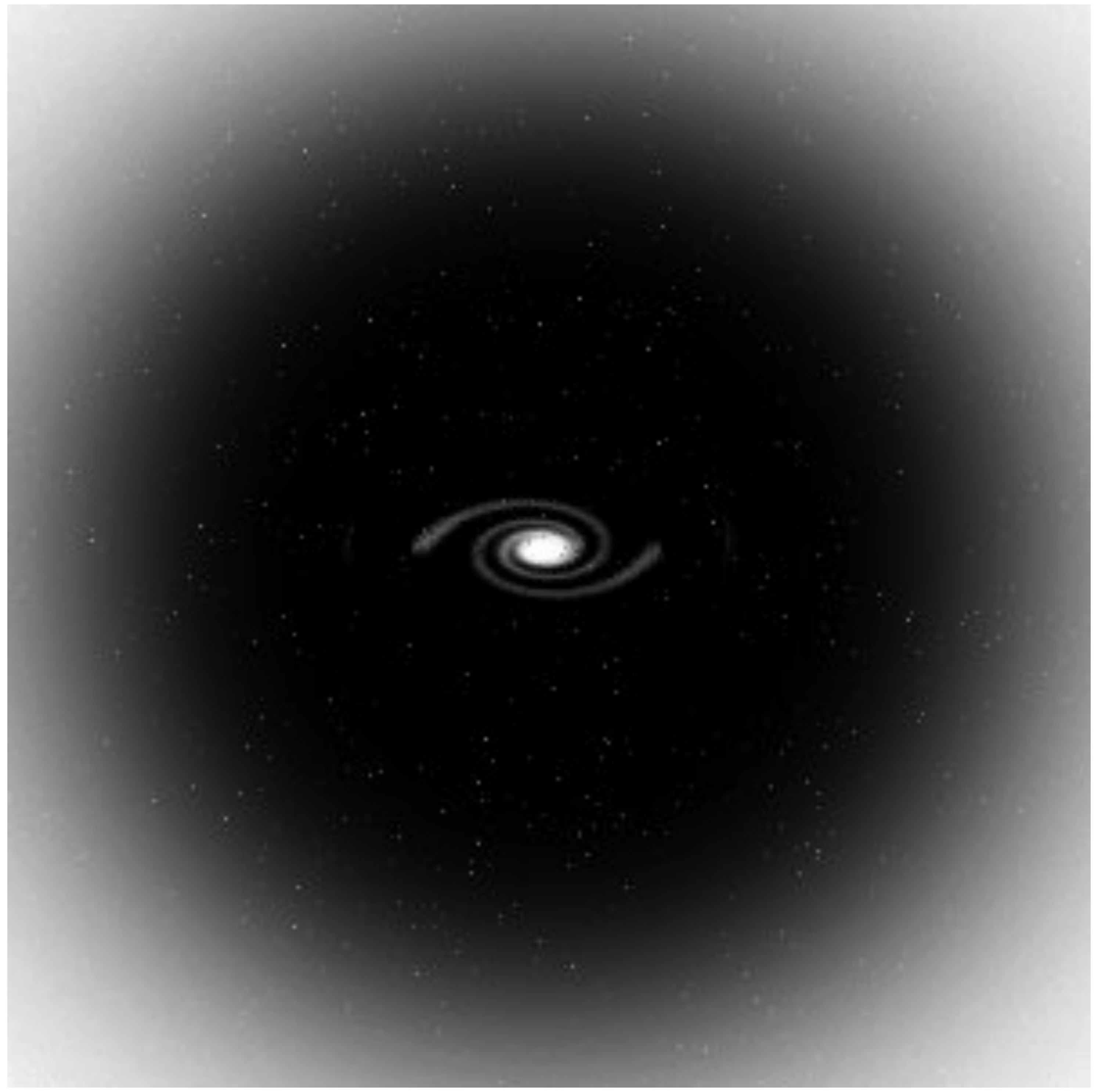
Q: What (science) can you do with such a good clock??

A: Discover new stuff!

- Ultra-light Dark Matter, topological defects Dark Matter
- Local Lorentz Invariance
- 5th Forces
- Gravitational redshift
- Change in fundamental constants
- Dark Energy
- Gravitational Waves
- Equivalence Principle
- …



earth rotation variation: ±1.5 ms



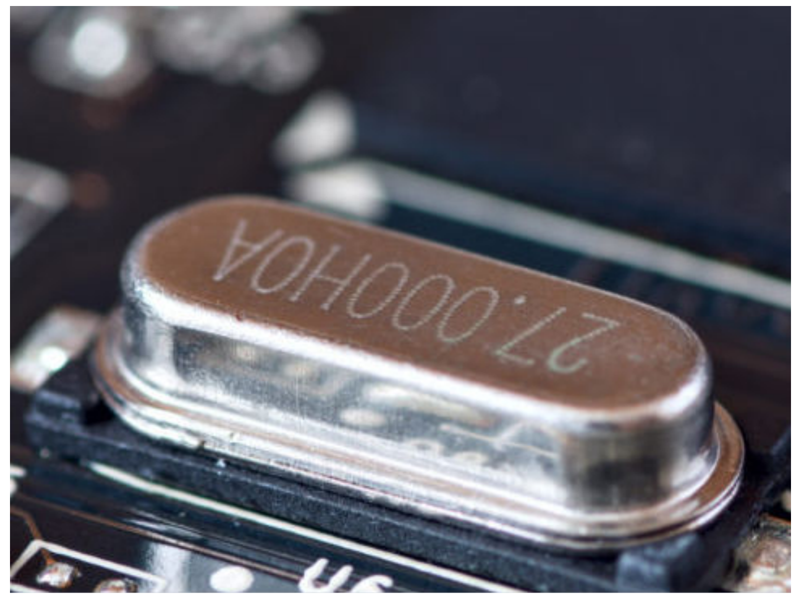
[A. Scheibe and U. Adelsberger, Z. Phys. 127, 416 (1950)]

Clock Basics

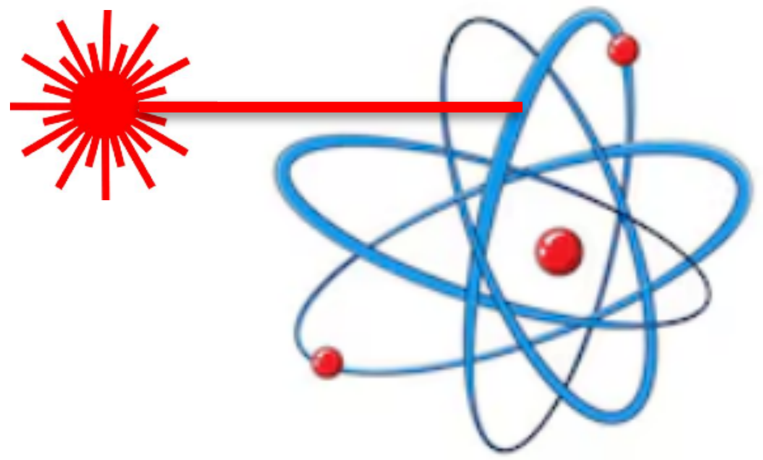
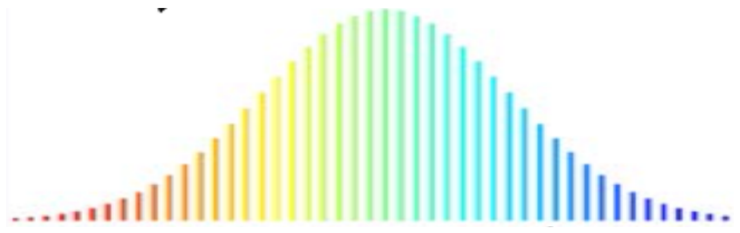
- Clock parts: oscillator and counter
- Both parts are important, true especially for atomic clocks
- Earth is imperfect oscillator, same for man-made resonators
- Many different atoms available, and they are all the same


minutes


seconds



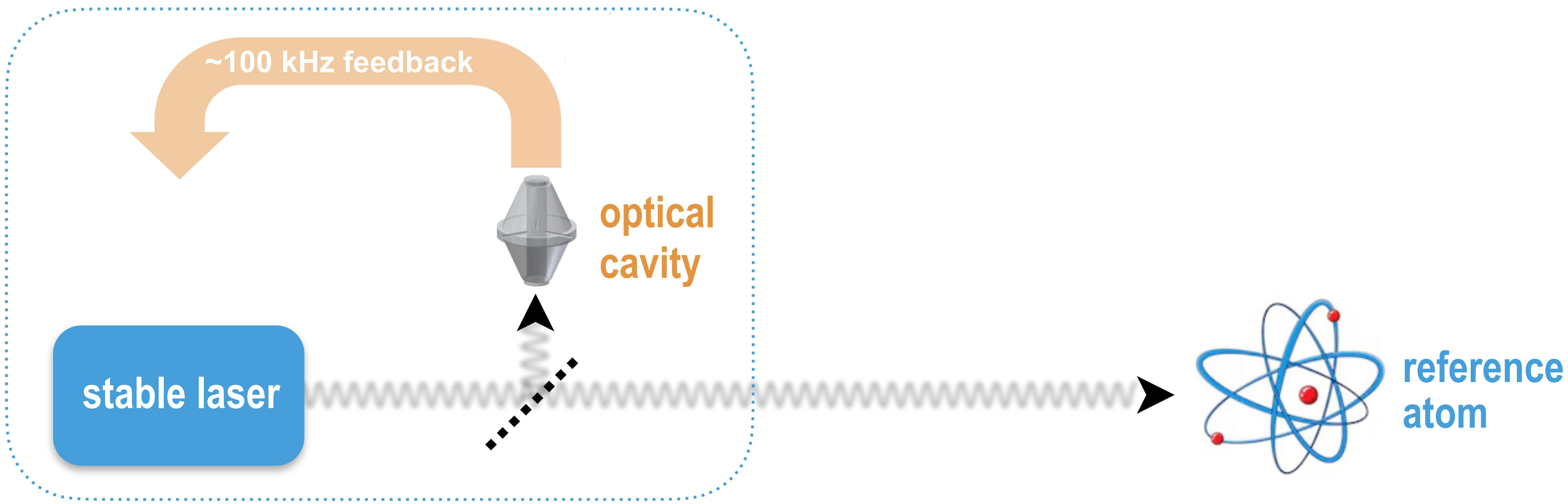

microseconds

< nanoseconds

Optical Atomic Clocks

- Laser for oscillator, usually a narrow frequency around atomic transition
- Ultra-stable laser needed (e.g. referenced to a cavity)

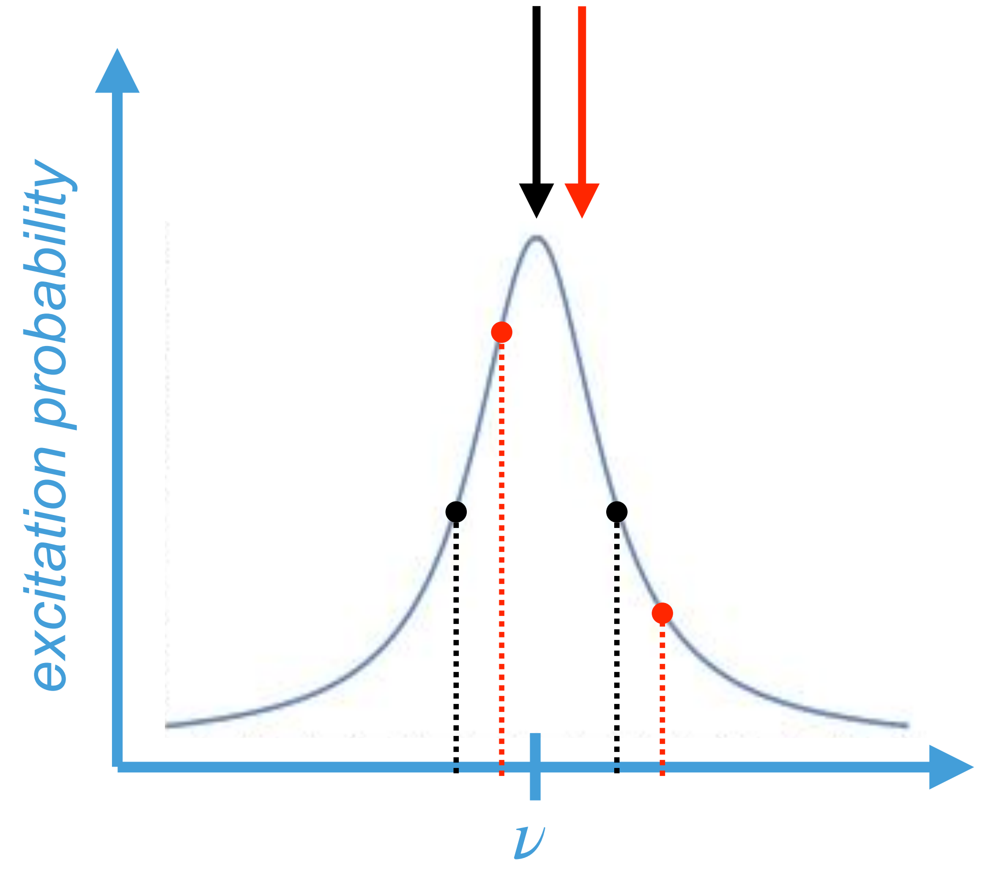


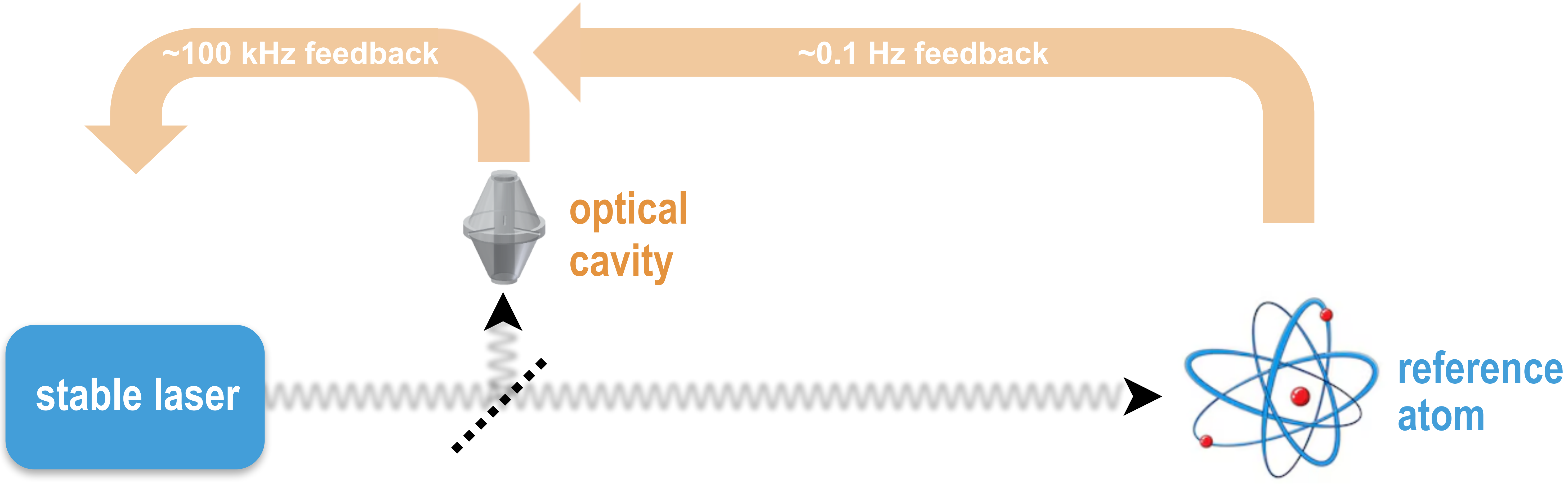
ultra-stable, narrow laser:

$$\Delta\nu \approx \mathcal{O}(1\ \text{Hz})$$

Optical Atomic Clocks

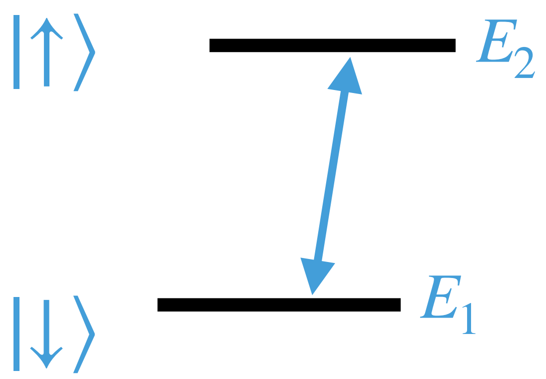
- Laser for oscillator, usually a narrow frequency around atomic transition
- Ultra-stable laser needed (e.g. referenced to a cavity)
- Locked to atomic transition: frequency detuned/offset (e.g. $\pm\Delta\nu$) to find max





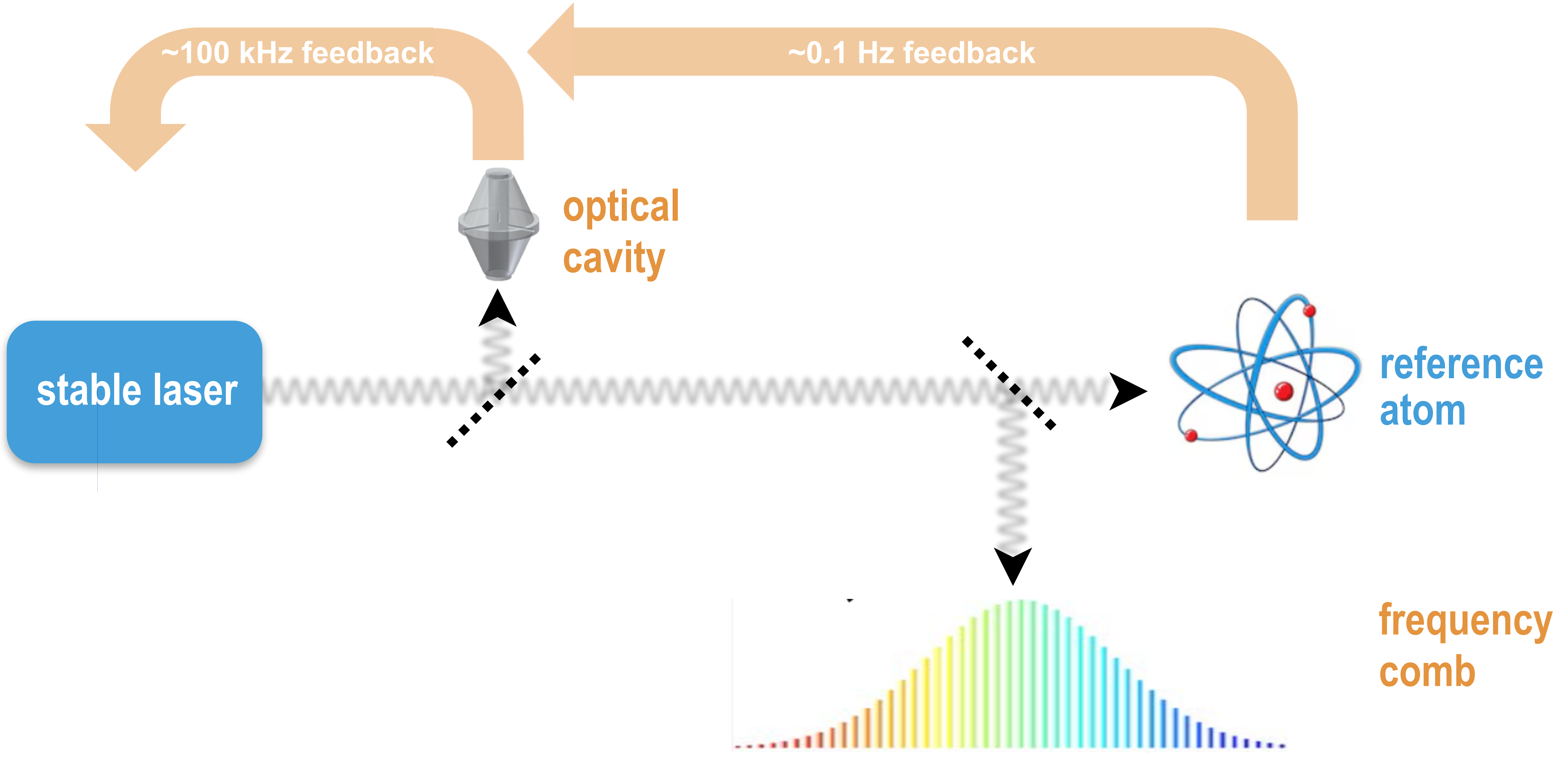
optical ν :

$$\nu = \frac{E_2 - E_1}{h} \approx \mathcal{O}(10^{15}\ \text{Hz})$$



Optical Atomic Clocks

- Laser for oscillator, usually a narrow frequency around atomic transition
- Ultra-stable laser needed (e.g. referenced to a cavity)
- Locked to atomic transition: frequency detuned/offset (e.g. $\pm\Delta\nu$) to find max
- Frequency comb used to link optical frequencies to countable microwave



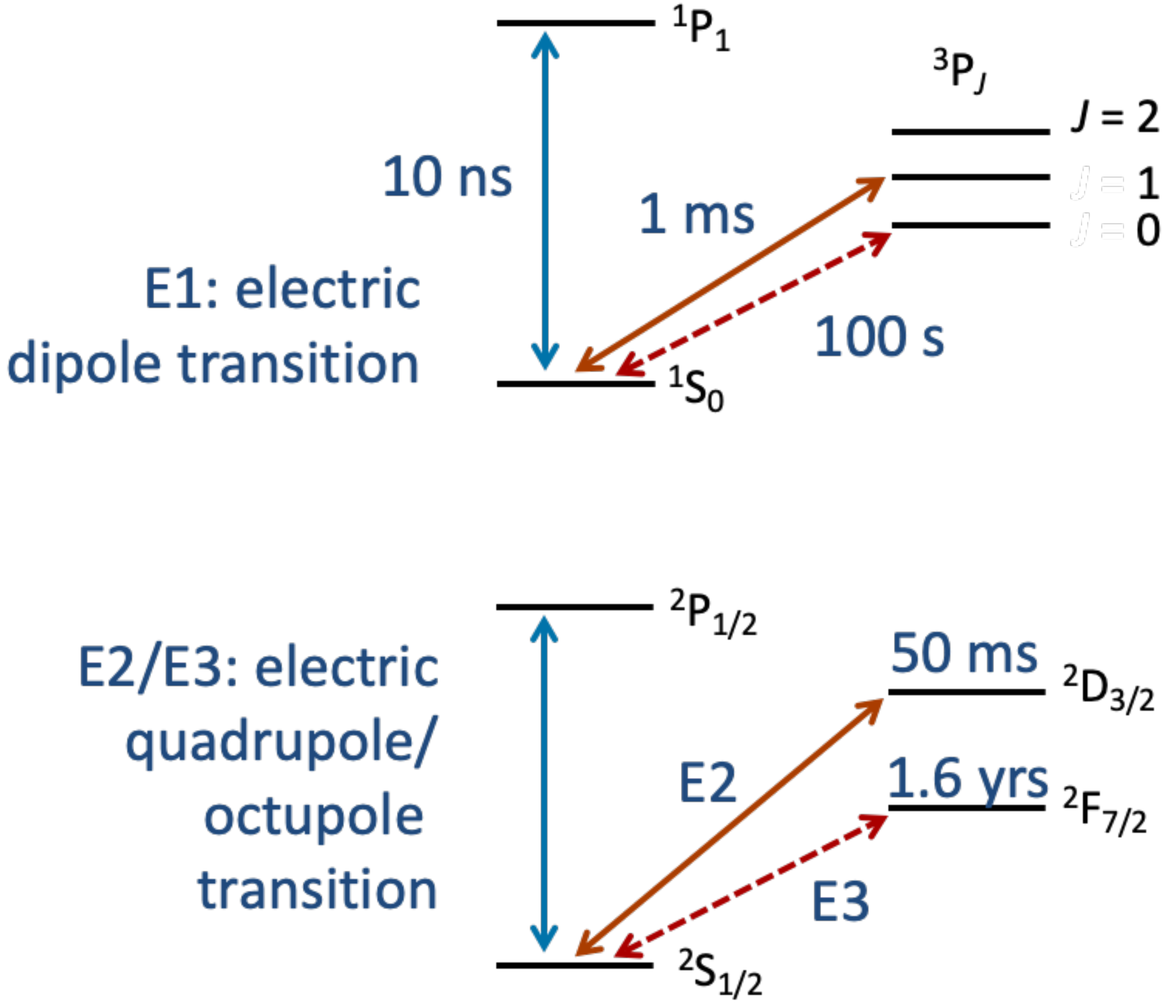
What Makes a Good Atomic Clock?

- Choice of atom
- How to stabilise the oscillator
- Arrangement of atoms (trap, fountain, …)
- Measurement choices & measuring ratios
- How to characterise the clock (Allan deviation, noise sources, etc)
- How to count really fast (Frequency Comb)

[https://www.bbc.com/future/article/20220721-the-super-clocks-that-define-what-time-it-is]

Selecting an Atomic Reference

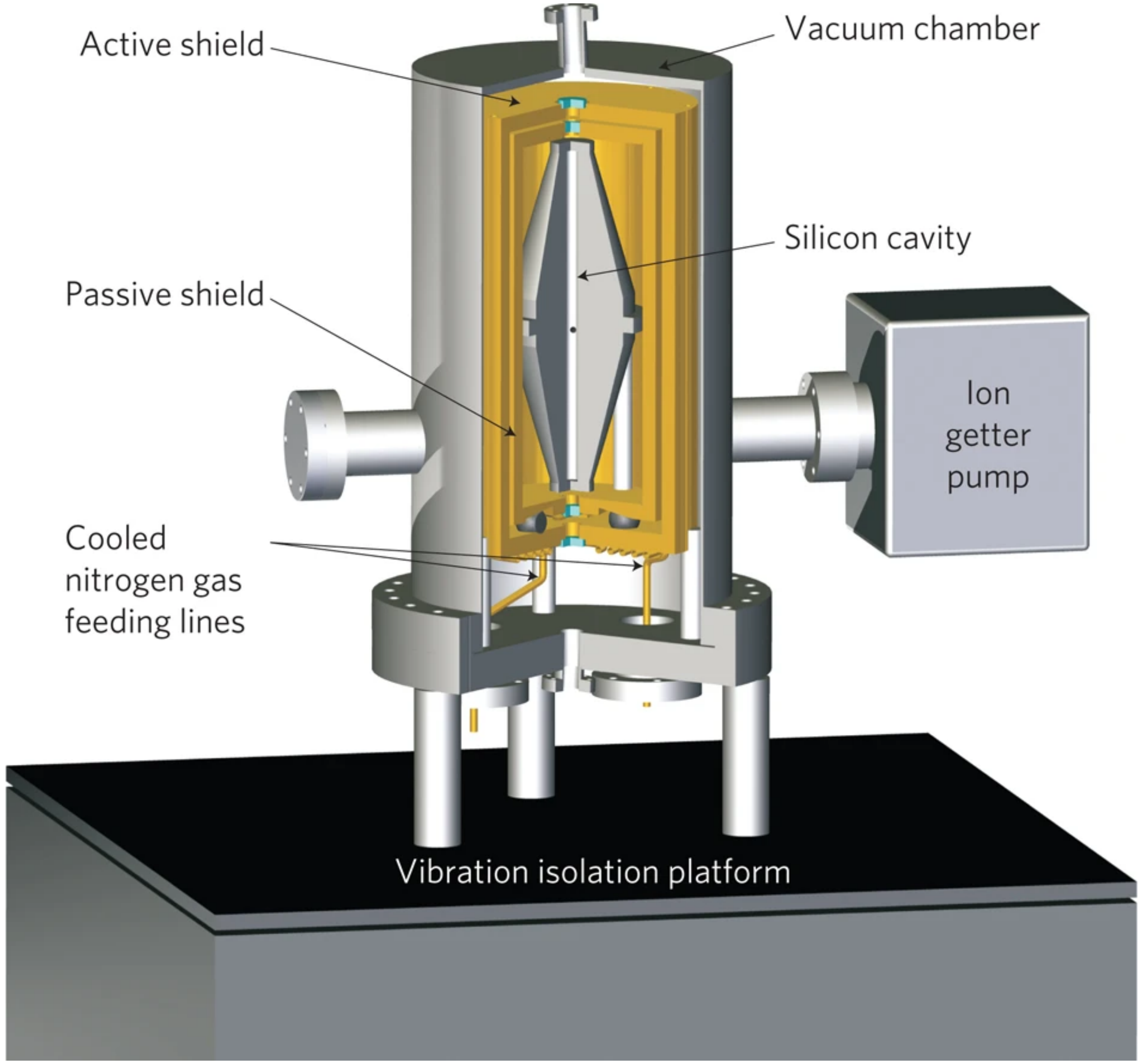
- Two requirements for the atom/transition:
  - long observation/interrogation time
  - narrow transition (can find the peak more precisely)
- Long-lived transitions are favourable
  - one minute or more… “forbidden” transitions
  - possible in some atoms with dipole transitions
  - more common for quadrupole/octupole transitions
- Other considerations:
  - convenient optical wavelengths
  - ease of preparation/operation
  - availability of atoms
  - sensitivity to external effects (low or high)

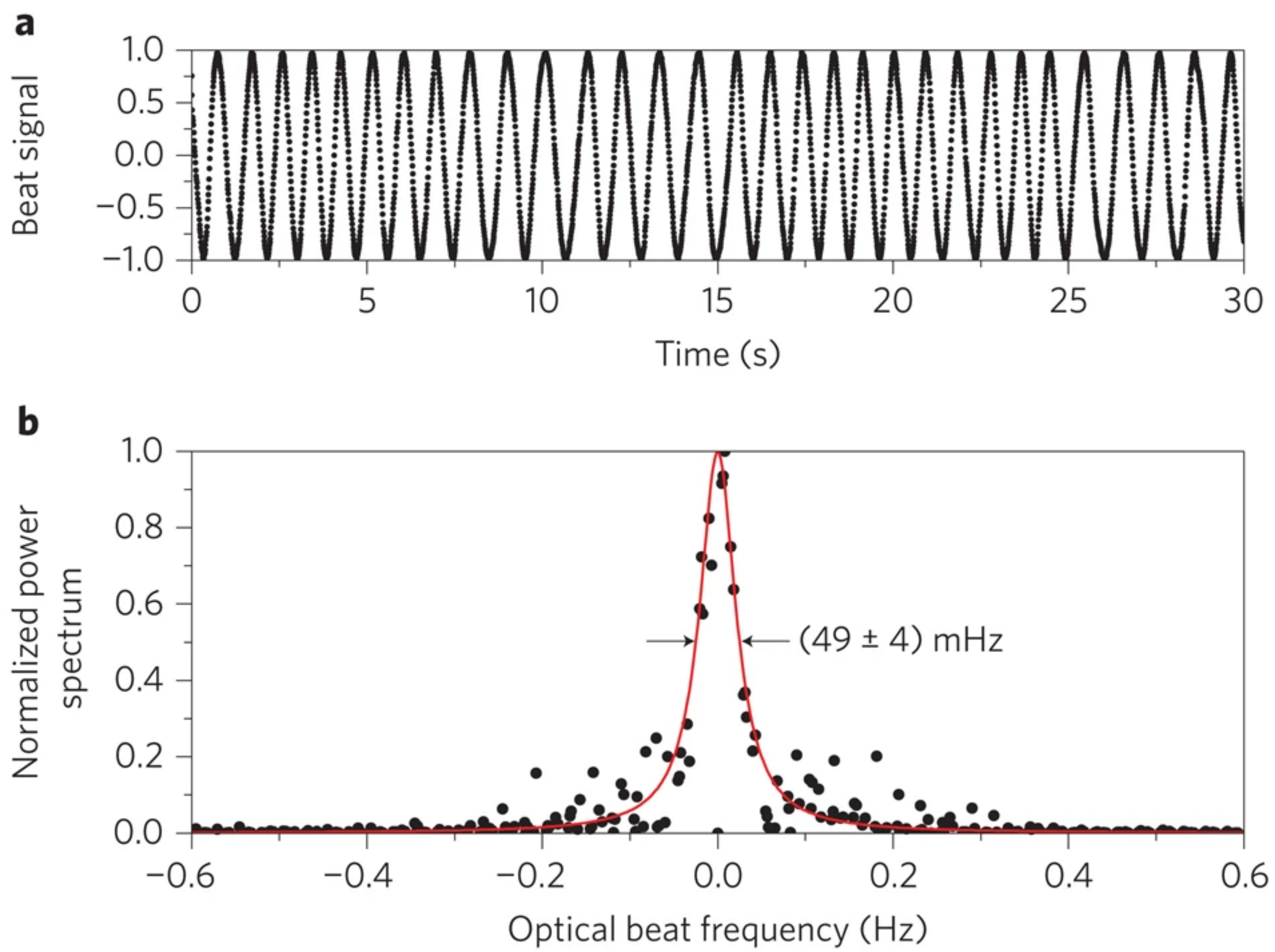


Strontium Level Diagrams

[Lisdat: Optical Clocks: An Introduction]

Single-Crystal Silicon Optical Cavity



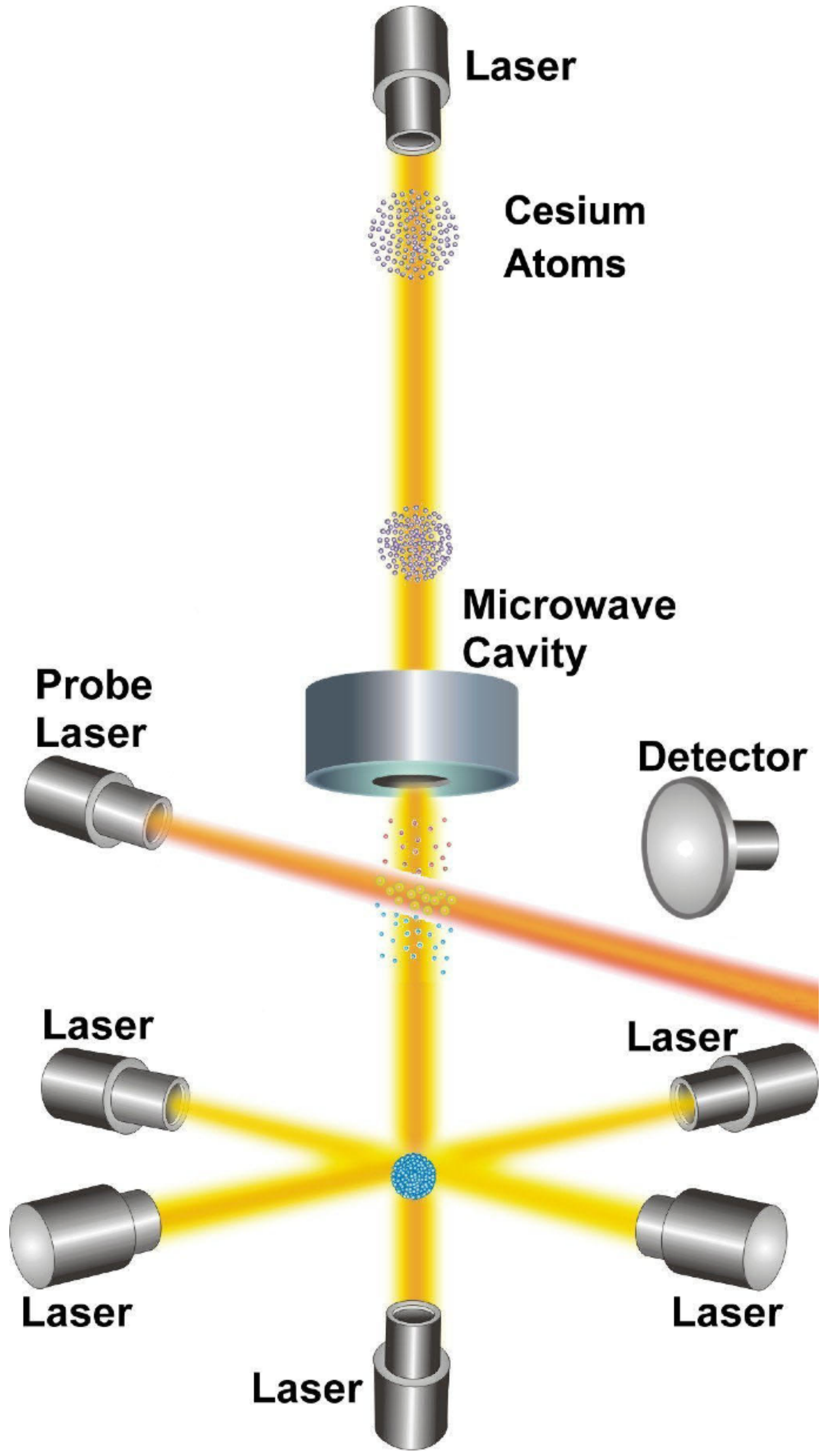


Normalized fast Fourier transform of the beat signal recorded with a HP 3561A FFT analyser (37.5 mHz resolution bandwidth, Hanning window). A Lorentzian fit is indicated by the red line. The combined result of five consecutive recordings of the beat signal (black dots) is displayed here, demonstrating the robustness of this record-setting linewidth.

[T. Kessler et al.. Nature Photon **6**, 687–692 (2012)]

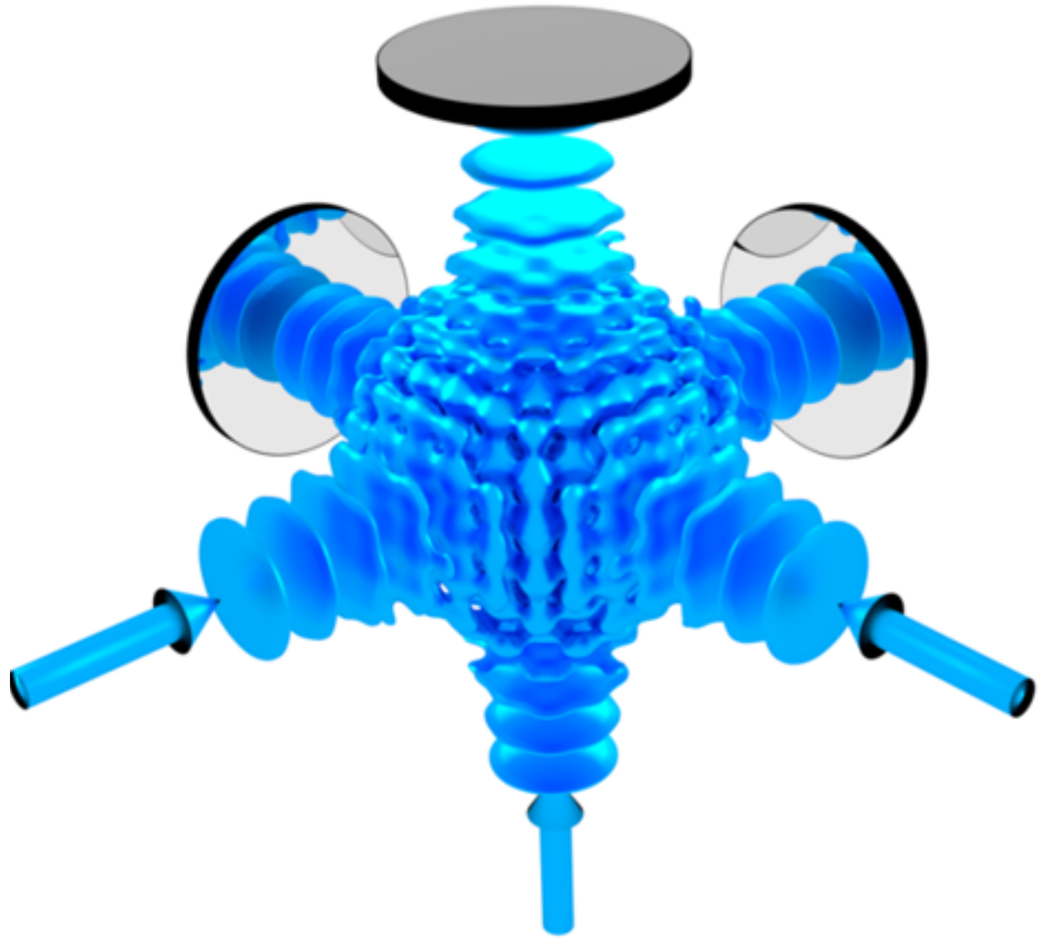
Manipulation and Trapping of Atoms

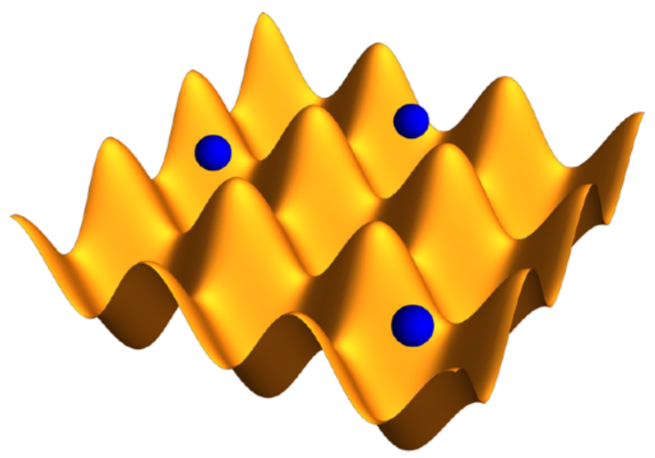
- Fountains (or ovens): e.g. Cesium Fountain Clock
- Neutral atoms in optical lattice: e.g. Strontium Lattice Clock
- Ions held in electromagnetic trap:



Atomic Fountain (Cesium)

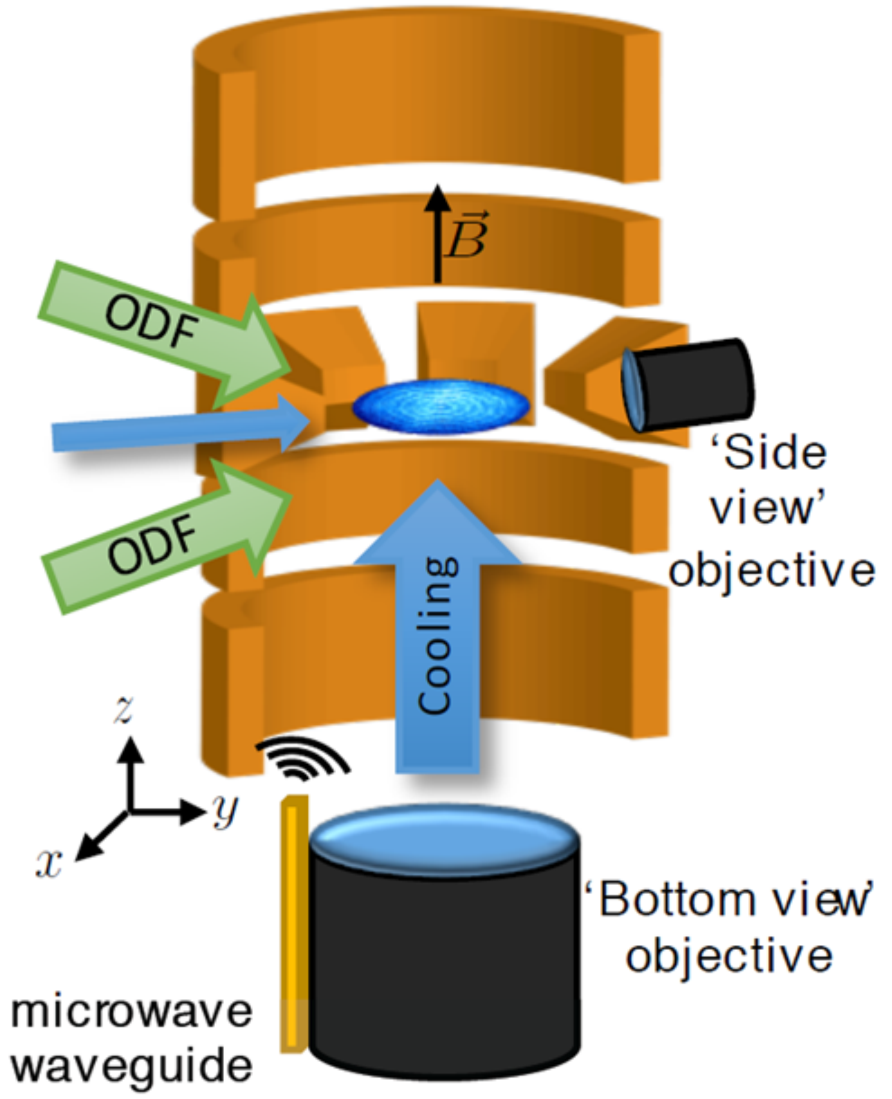
[https://www.nist.gov/news-events/news/1999/12/nist-f1-cesium-fountain-clock]

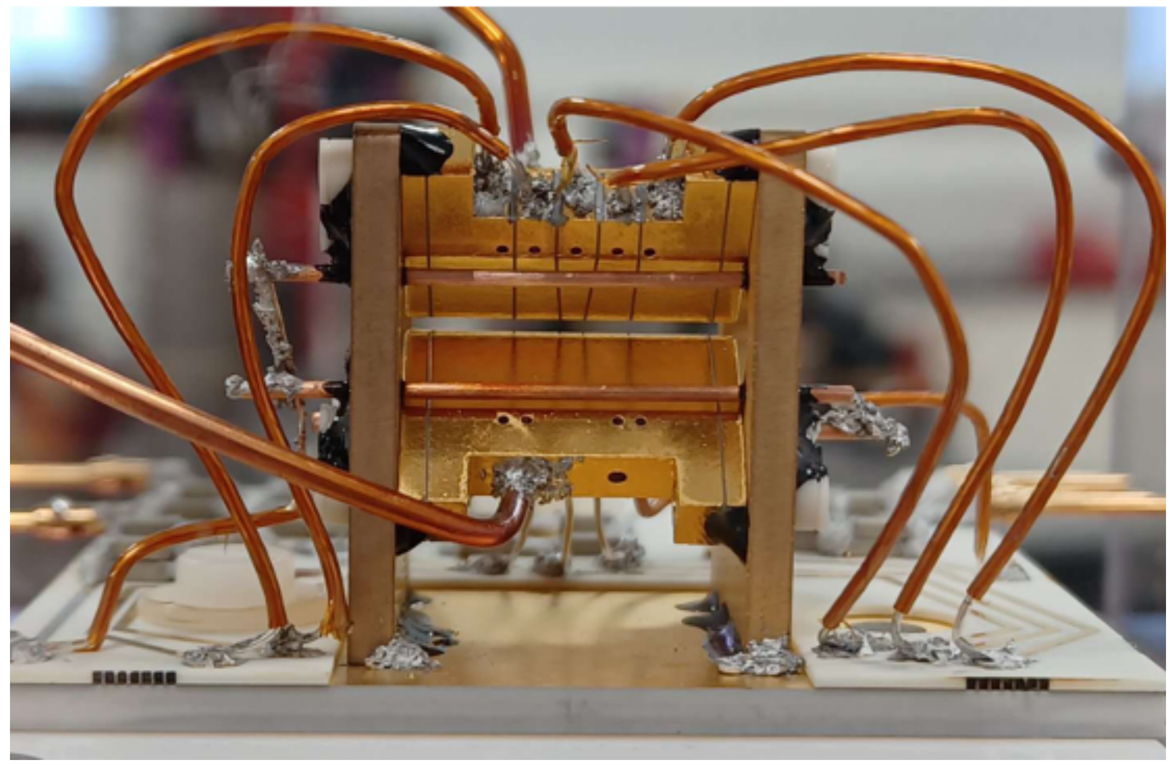




2D and 3D Lattice Potentials

[Wikipedia, https://www.amop.phy.cam.ac.uk/]



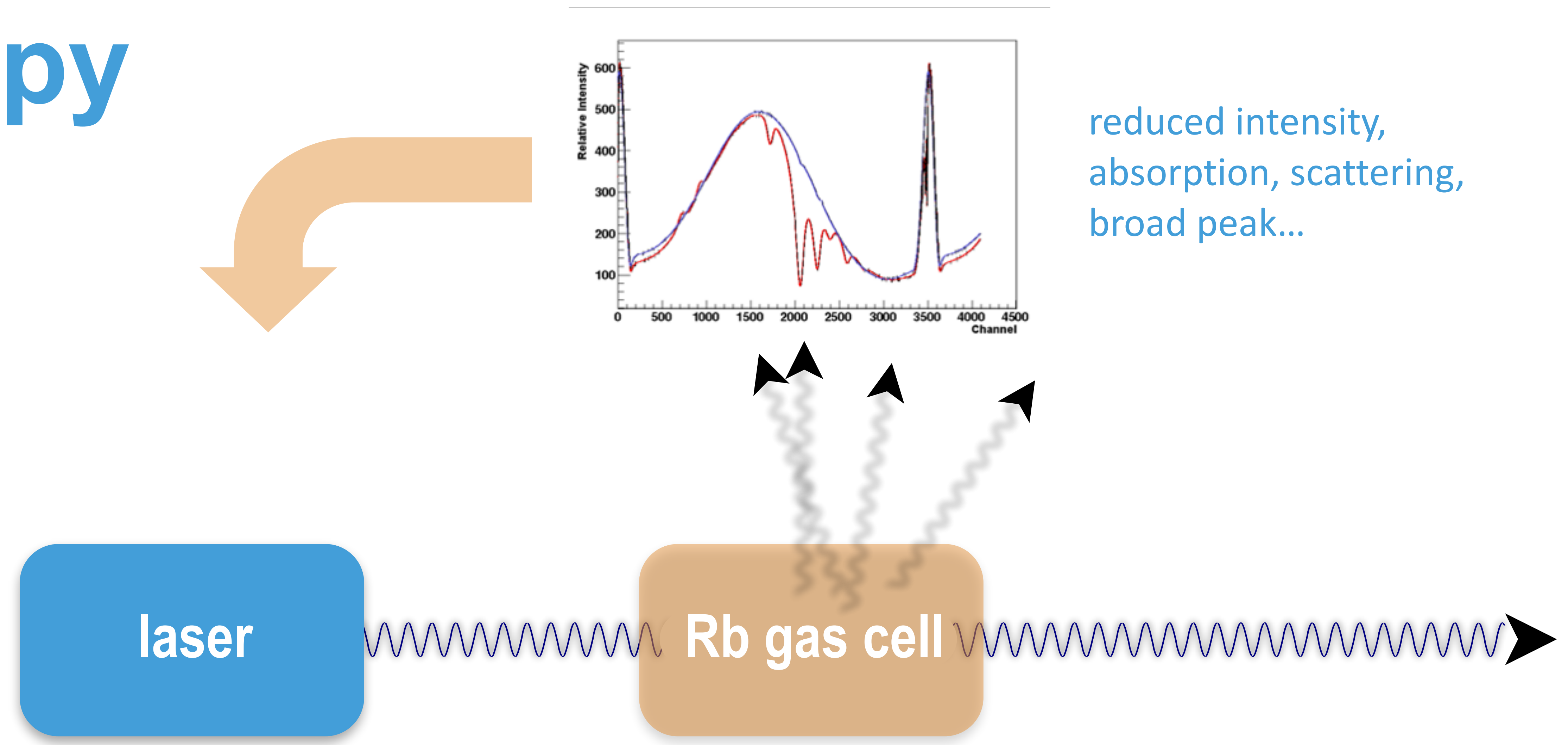


Penning and Paul Traps

[Barontini/PTB, https://www.nist.gov/image/penningtrappng]

Interacting with Atoms: Spectroscopy

- Incoherent: absorption spectroscopy for laser stabilisation
  - Signal is continuous (timing is not an issue)
  - Absorption/scattering method $\Longrightarrow$ short lifetime $\Longrightarrow$ broad linewidth



- Coherent: interactions via Rabi oscillations
  - Very long lifetime—usually limited by other effects
  - Setup, query and read-out is not continuous
  - Oscillation between ground and excited states (rotation around Bloch sphere)

Bloch vector: $\frac{\mathrm{d}}{\mathrm{dt}}\vec{b} = \vec{\Omega}_{\mathrm{eff}} \times \vec{b}$

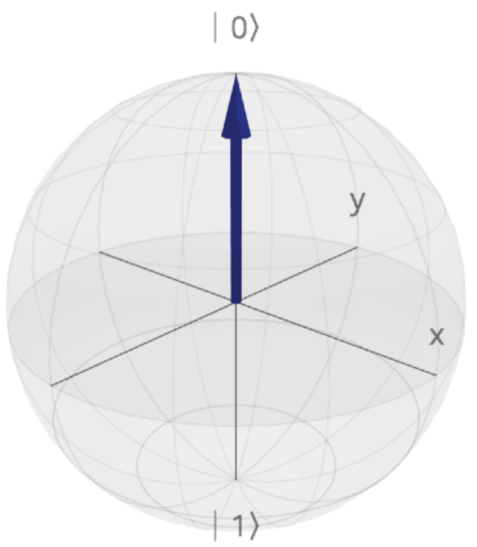
$\Omega = \langle g|\, \vec{d} \cdot \vec{E}_0 \,|e\rangle$

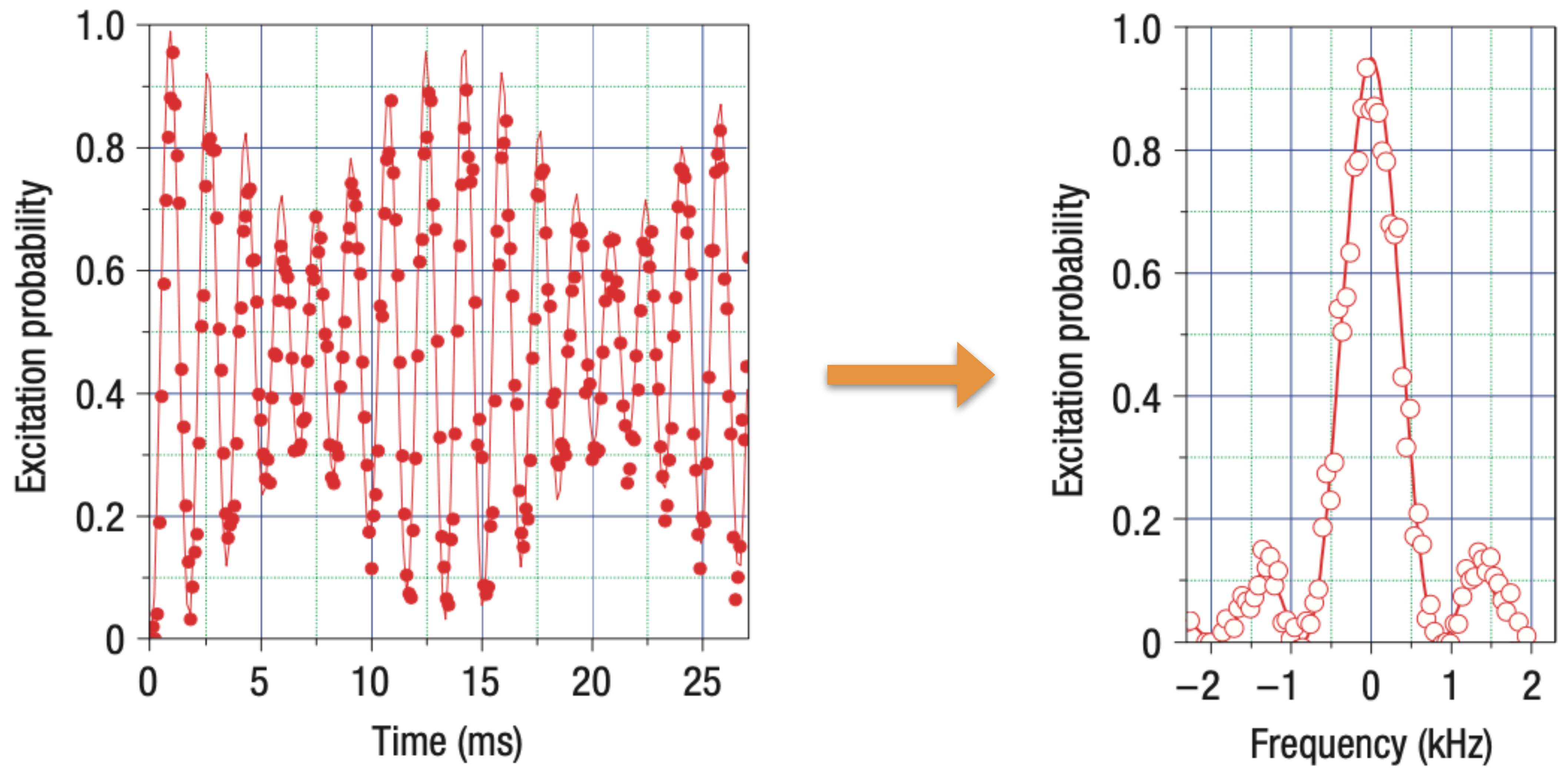
$\delta = \omega - \omega_0$

$\delta \ll \Omega$

Rabi Frequency: $\vec{\Omega}_{\mathrm{eff}} = \begin{pmatrix} \Omega \\ 0 \\ -\delta \end{pmatrix}$

Pulse: $\vec{\Omega}_{\mathrm{eff}} \approx \begin{pmatrix} \Omega \\ 0 \\ 0 \end{pmatrix}$ free evolution: $\vec{\Omega}_{\mathrm{eff}} = \begin{pmatrix} 0 \\ 0 \\ -(\pm\delta) \end{pmatrix}$

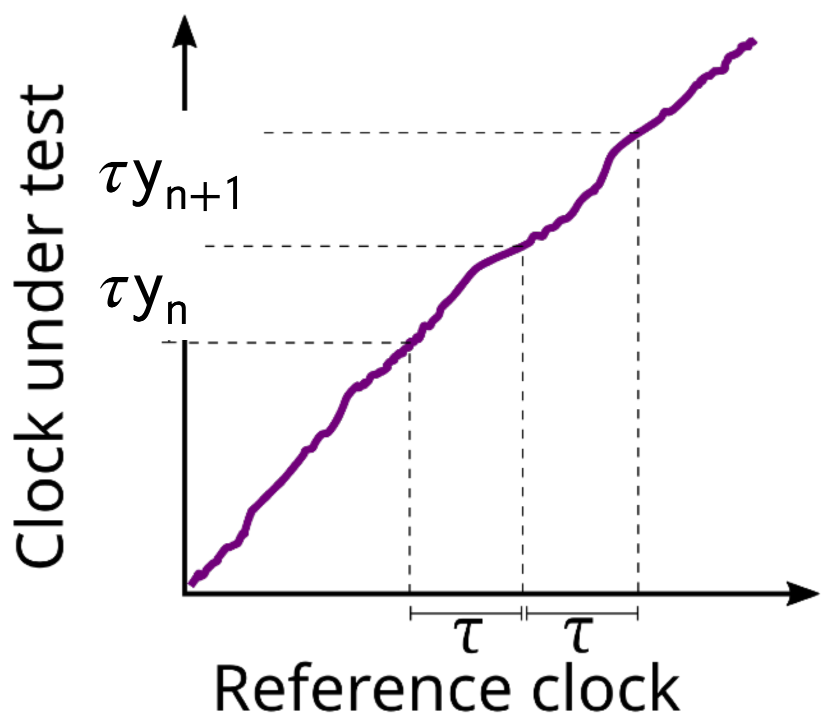




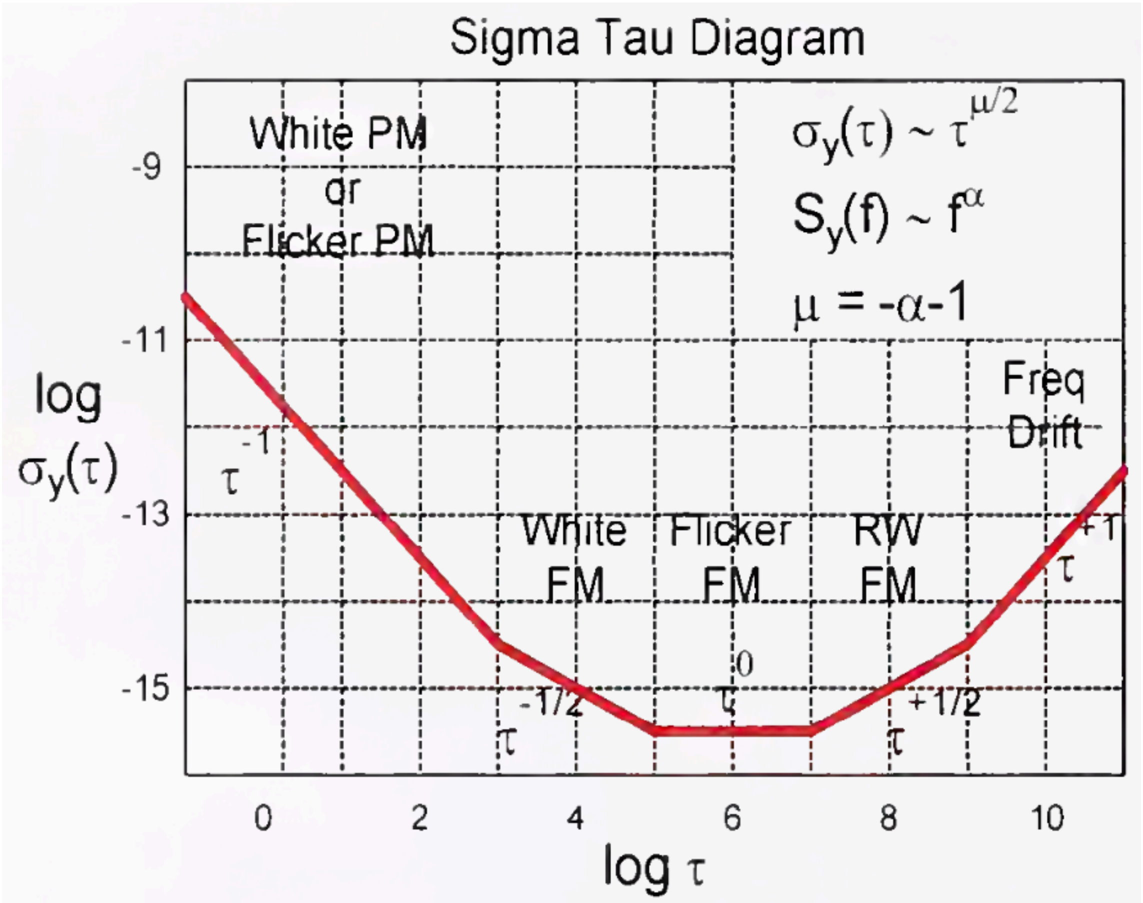
[Akatsuka et al, Nature Phys. 4, 954 (2008); Lisdat: Optical Clocks: An Introduction]

Clock Performance: Allan Deviation

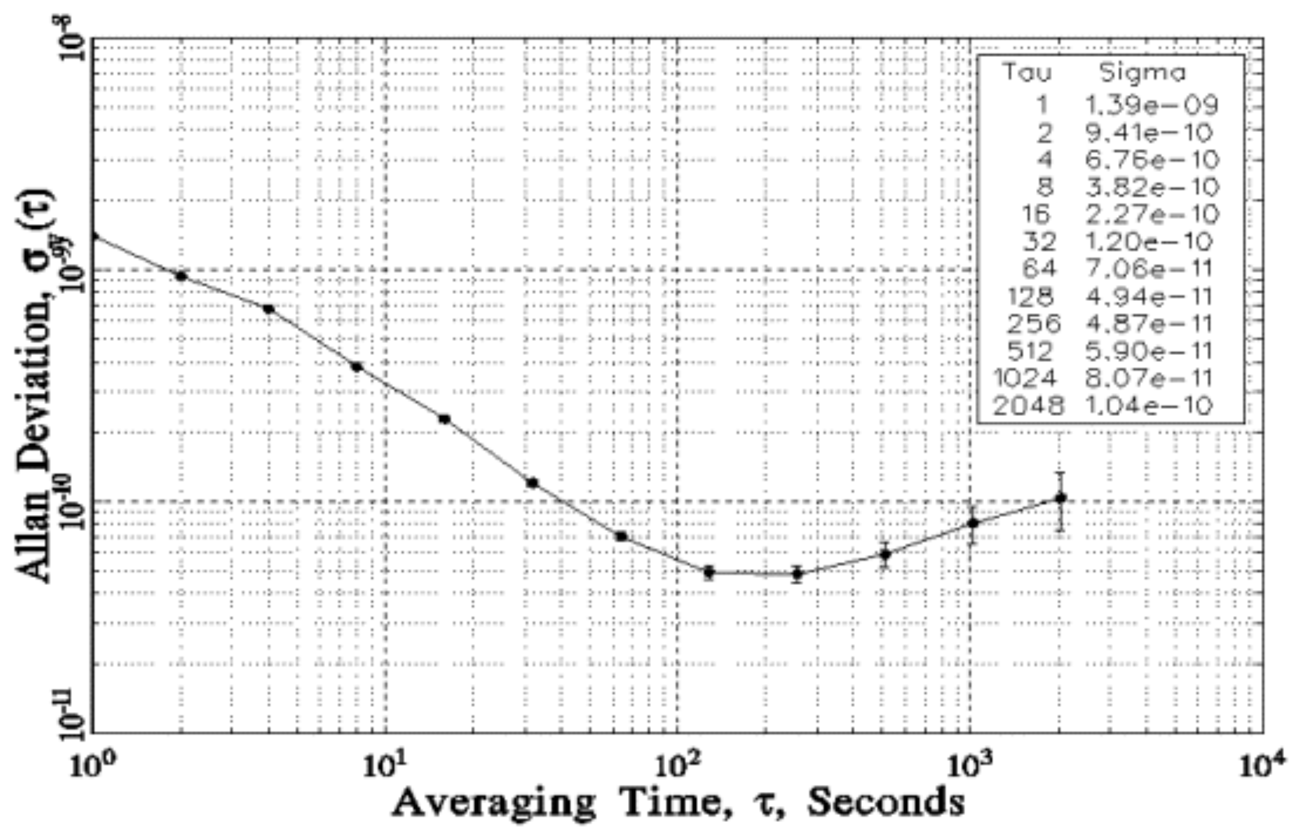
- Allan or two-sample variance better suited to oscillators than e.g. standard deviation
- Used to measure frequency stability in clocks, characterise (non-systematic) noise



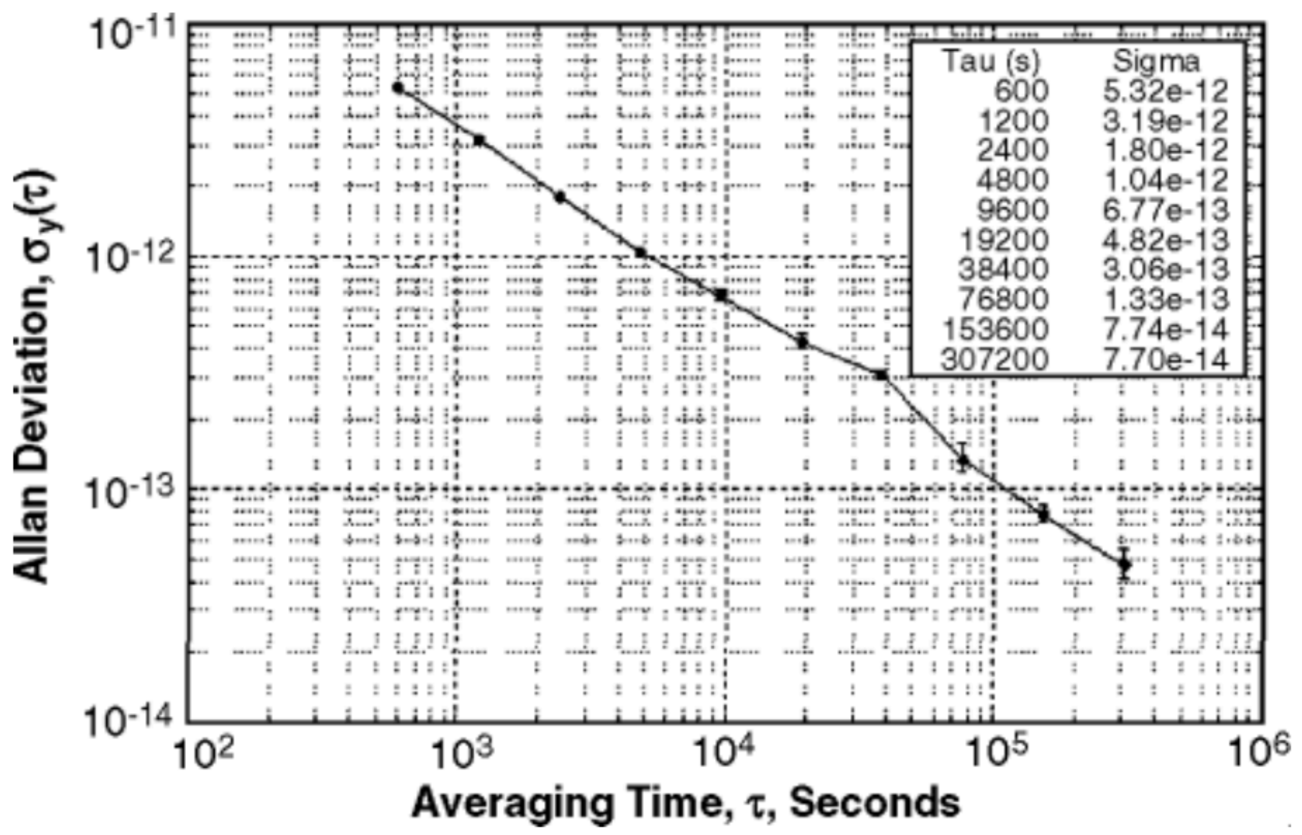
$$\sigma_y(\tau) = \sqrt{\sigma_y^2(\tau)} = \frac{1}{2}\langle(\bar{y}_{n+1} - \bar{y}_n)^2\rangle$$



- Slope of the curve (log scale) gives different exponent-dependent noise source



Example Allan deviation plot



Allan deviation for early GPS signals

[Lombardi, Fundamentals of Time and Frequency; NIST Pub 1065, Handbook of Frequency Stability Analysis, 2008; Wikipedia]

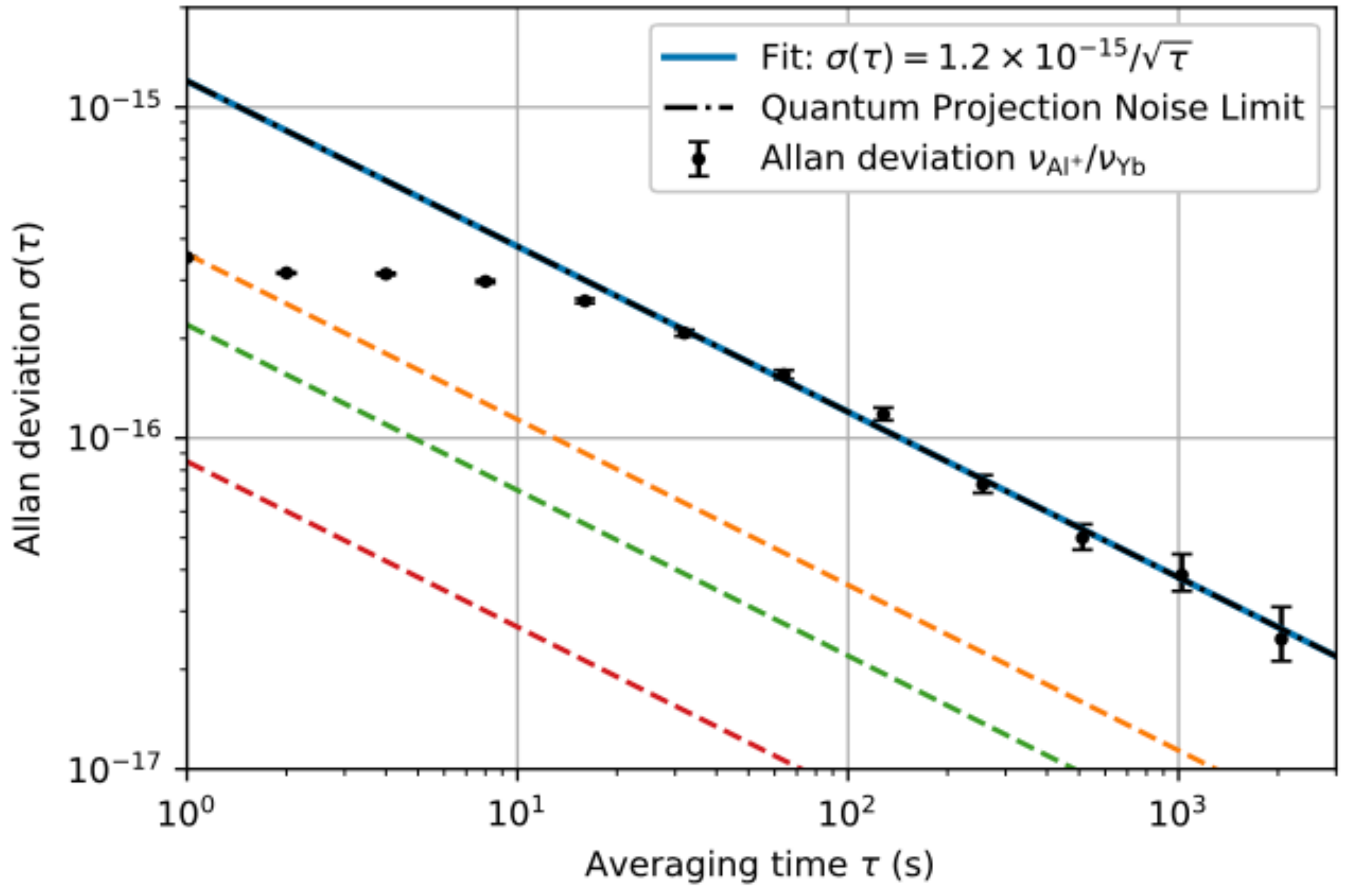
Optical Clock Uncertainties

- Typically many systematic noise sources
  - Blackbody radiation: local heat sources (e.g. for room temperature clocks)
  - Stark/Zeeman shift: know your fields very well
  - Motional uncertainty, background gas: better vacuum
  - Laser-induced effects: improve laser stability
  - Micromotion (ion motion in sync with trap AC field): better trap design, better control of AC field
- Quantum Projection Noise (shot noise)
  - After laser interrogation, you want to read out the state (either ground or excited, 0 or 1)
  - Binary, discrete nature of quantum leads to uncertainty $\sigma(\tau) = 1/f\sqrt{NT_p\tau}$, for frequency f, probe time T_p and measurement time τ
  - Can be reduced further using squeezing/entanglement ($\sim 1/N$), or more atoms!

NIST $^{27}Al^{+}$ Quantum-Logic Clock

TABLE I. Fractional frequency shifts ($\Delta\nu/\nu$) and associated systematic uncertainties for the $^{27}Al^{+}$ quantum-logic clock.

Effect	Shift (10^{-19})	Uncertainty (10^{-19})
Excess micromotion	−45.8	5.9
Blackbody radiation	−30.5	4.2
Quadratic Zeeman	−9241.8	3.7
Secular motion	−17.3	2.9
Background gas collisions	−0.6	2.4
First-order Doppler	0	2.2
Clock laser Stark	0	2.0
AOM phase chirp	0	<1
Electric quadrupole	0	<1
Total	−9336.0	9.4



[Brewer at al, PRL 123, 033201 (2019)]

Uncertainties from Shifts and Broadening

- Doppler broadening: velocity distribution of atoms can cause spectral lines to shift
- An issue at low pressure, high temperature, small wavelengths
- Can be addressed by reducing the thermal motion: laser or sympathetic cooling

- Collisional broadening: electronic interactions with nearby particles can cause frequency shifts
- Suppressed by cryogenic temperatures, or with better vacuum
- Lattice potentials can be used to eliminate collisions and first-order Doppler shifts

- Stark shift: shift in atomic energy levels as a result of external electric fields
- Effects minimised by eliminating constant external fields (linear Stark shift)

- Zeeman shift: the split of energy levels in the presence of magnetic fields
- Can be reduced by local shielding, or by choosing transitions insensitive to first-order Zeeman shift

Micromotion & Black Body Radiation

- Micromotion is excess ion motion synchronous with trap AC field
  - Can change atomic transition line shapes
  - Induce significant second-order Doppler shifts
  - Limit confinement time (for room temp traps)
  - Stark shifts in atomic transitions from the AC field
  - Addressed by better trap design, better control of AC field

- Black-Body Radiation changes the energy levels in clock transitions
  - From heat: major source of uncertainty in room-temperature clocks
  - Shift atom's energy levels via the radiation's electric field (Stark shift)
  - Control by reducing local temperatures, heat sources, windows, etc
  - Advances in calculating the effects (theory)

5028 J. Appl. Phys., Vol. 83, No. 10, 15 May 1998

effects from micromotion

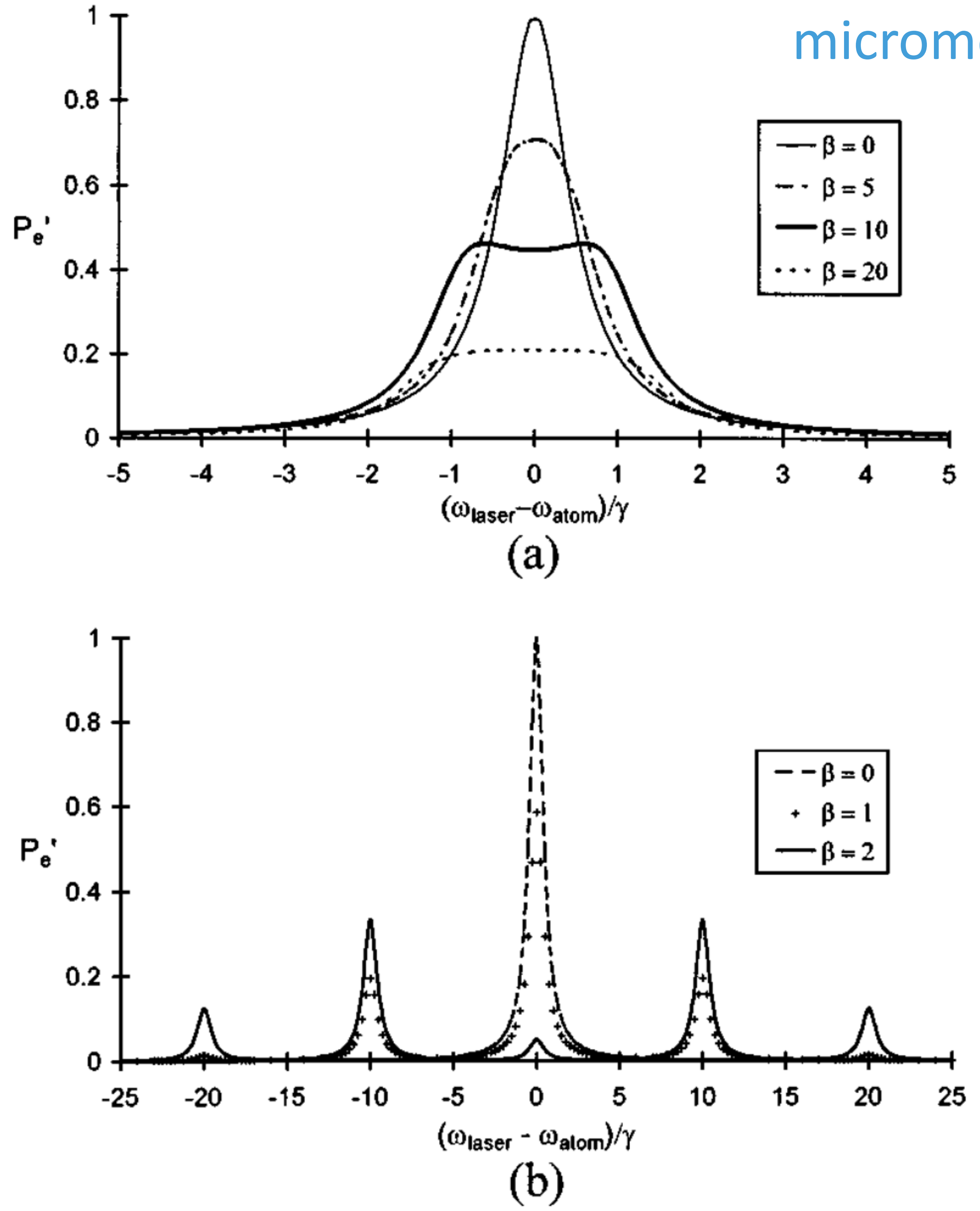
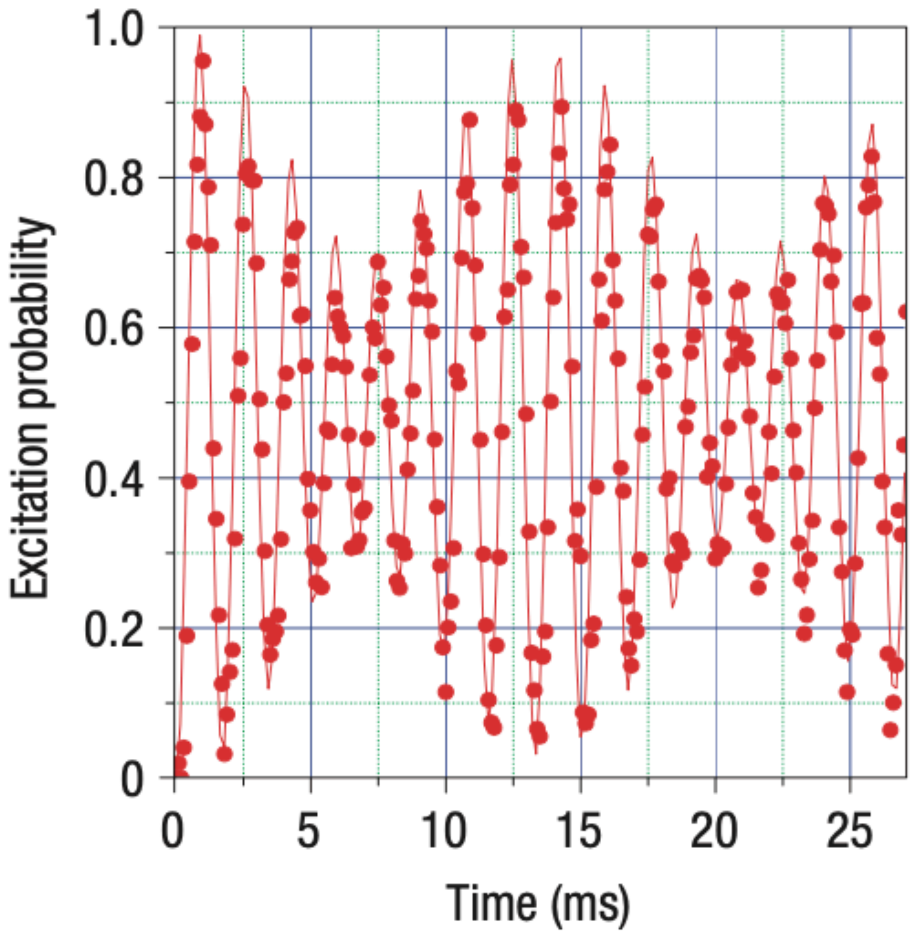


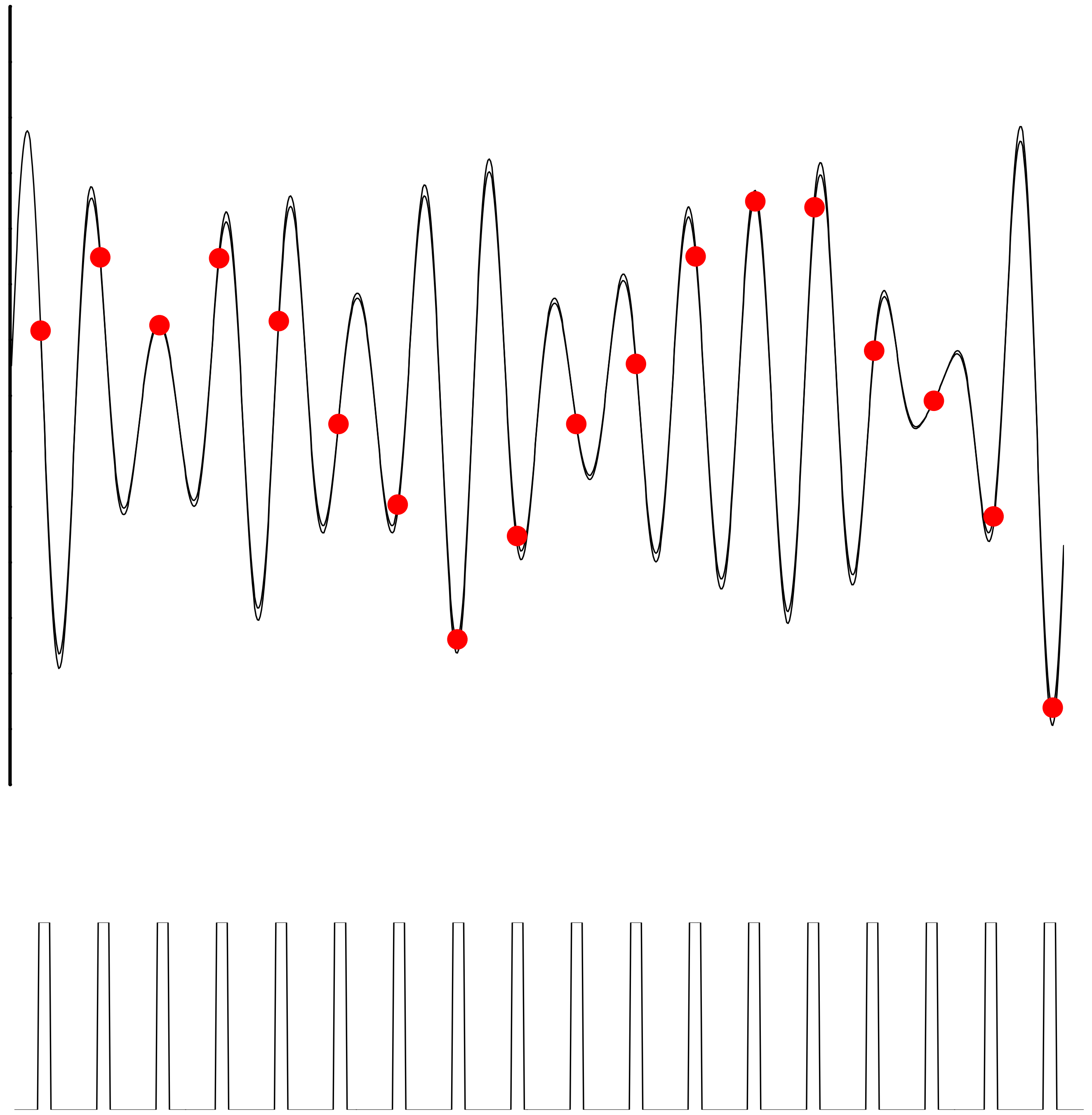
FIG. 2. Effect of micromotion on the spectrum of P_e (excited state population). We plot $P_e' = P_e[\hbar\gamma/(2\mathscr{P}|E_0|)]^2$ for various values of β. For both graphs, we assume that the ion is driven below the saturation limit. (a) $\Omega/\gamma = 0.1$. For $\beta = 10$, heating occurs in the regions $<0.6<(\omega_{\text{laser}} - \omega_{\text{atom}})/\gamma<0$ and $(\omega_{\text{laser}} - \omega_{\text{atom}})/\gamma>0.6$. (b) $\Omega/\gamma = 10$. For $\beta>0$, heating can occur when the laser frequency is tuned near, but above the center of, any of the sideband frequencies.

[Berkeland et al., Journal of Applied Physics, Vol 83, 10 (1998); Safranova et al., Phys. Rev. A 87, 012509]

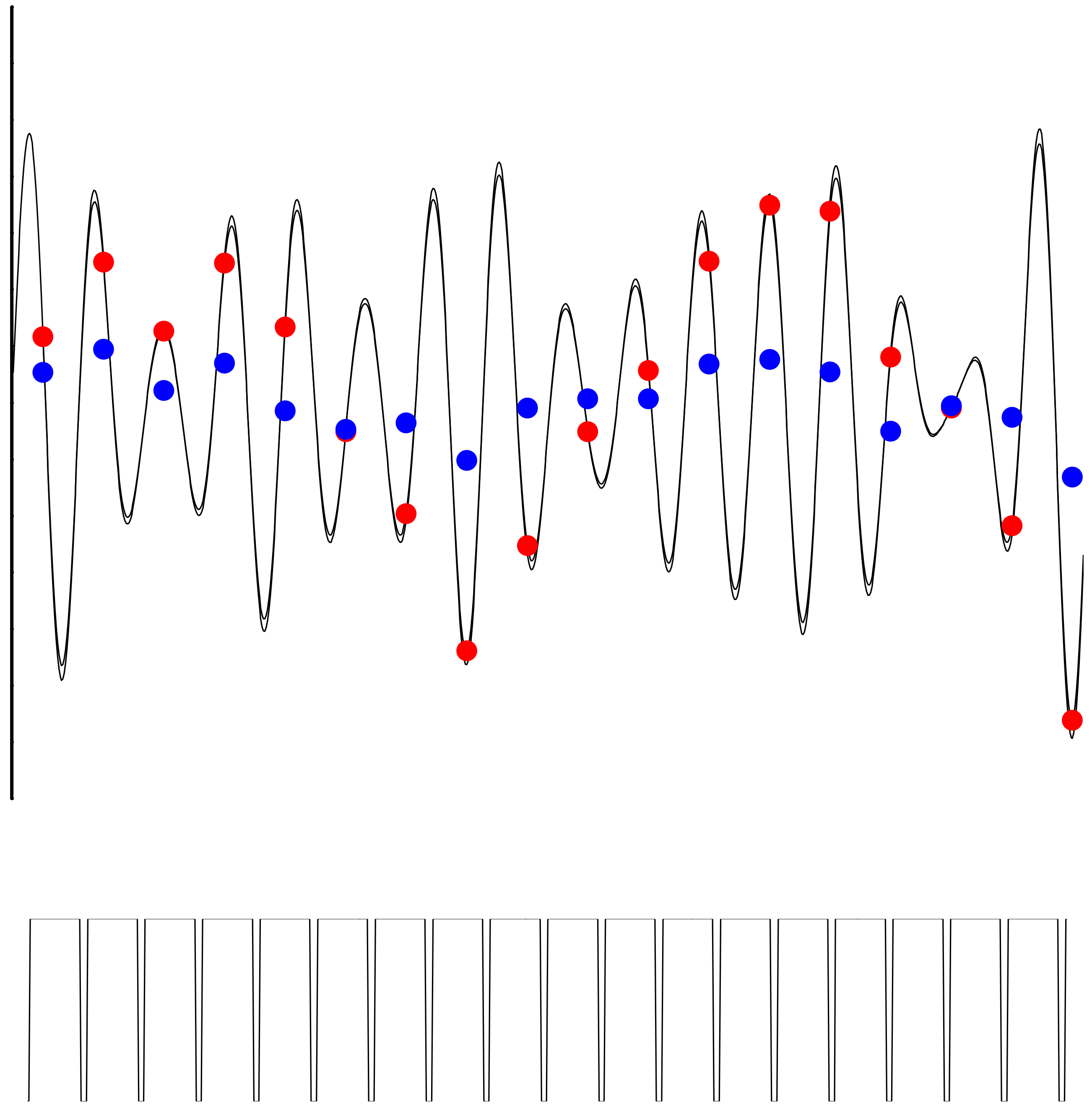
Deadtime-Related Uncertainty

- It takes some unavoidable time for interrogation (Rabi/Ramsey)
- The data acquisition also add dead time… net effect (Dick's effect) is added uncertainty





probe time + long dead time

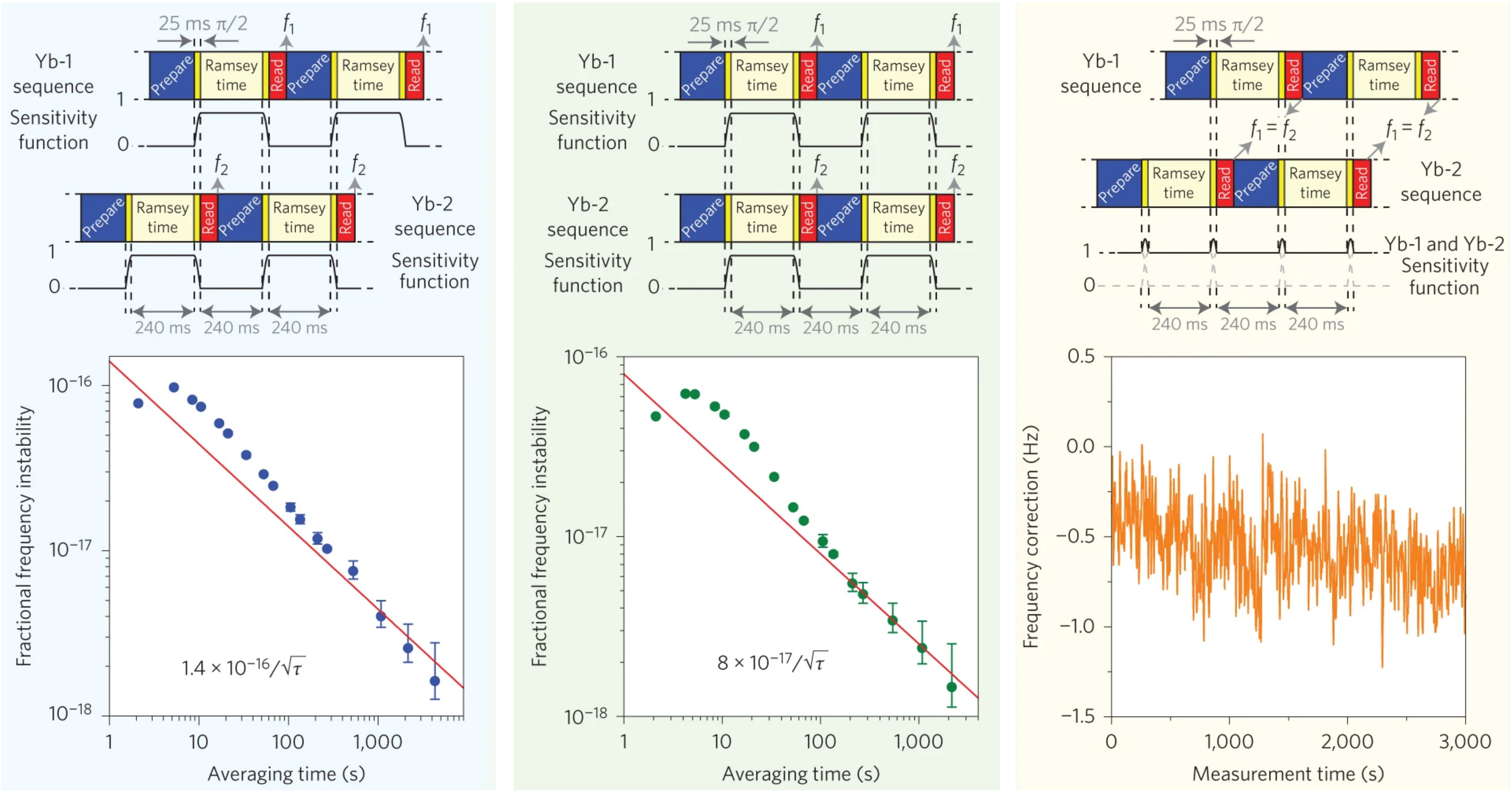


probe time + short dead time

[Santarelli et. al., IEEE Trans. Ultrason. Ferroelectr. Freq. Control 45, 887 – 894 (1998); M. Schioppo et al., Nature Phot. 11, 48 (2017)]

Deadtime Mitigation

- Example of recent efforts to overcome the effect



asynchronous readout | synchronous readout | readout w/ active feedback corrections

[M. Schioppo et al., Nature Phot. 11, 48 (2017)]

What about the Counter?

- Ok, so we have the oscillator and have characterised it, how do we count the clock “ticks”?
- The problem… we can count electronically up to microwave frequencies, but not the 100’s of THz optical
- Al^+ clock example from earlier: 1121015393207859 Hz!
- Frequency combs solve this problem, by translating/connecting the optical frequencies to countable microwave

Frequency Comb

- Q: How to precisely measure optical frequencies? The clock ticks? Compare to other clocks?

- *Frequency comb:* mode-locked laser w/ stabilised rep rate (f_{rep}) and carrier-envelope-offset (f_0)

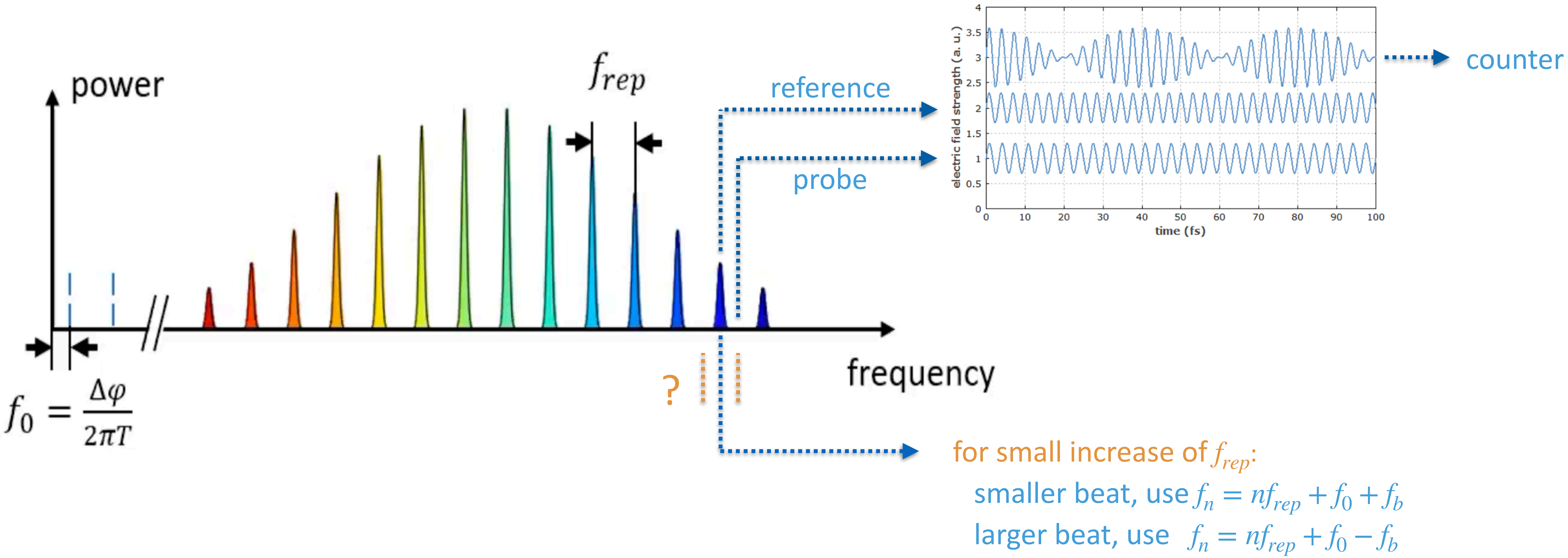
$$f_n = nf_{rep} + f_0$$

- Repetition frequency f_{rep} typically larger than CEO-frequency f_0 (e.g. 200 MHz vs. 50 MHz)
- Both frequencies are tunable, defined in relation to a frequency reference for high-end applications

[https://www.nobelprize.org/prizes/physics/2005/popular-information/]

Frequency Comb: Operation

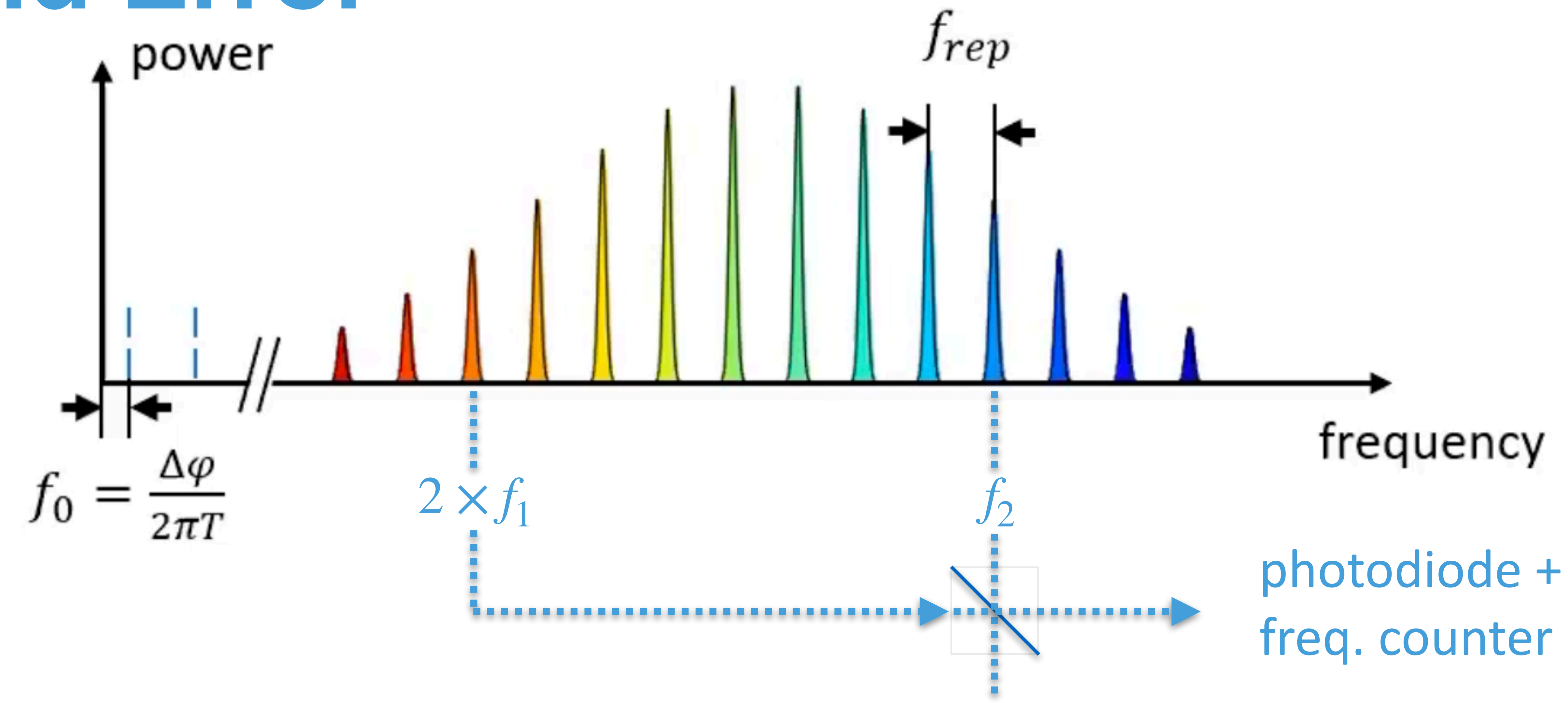
- For a comb with f_0 and f_{rep}, an unknown frequency can be measured wrt one tooth by creating a beat note
- The integer n can be found using a wavemeter with an accuracy better than $f_{rep}/2$ (e.g. 40 MHz)
- Beat notes with additional teeth avoided by extracting only the relevant tooth (e.g. with a grating)
- One can slightly increase f_{rep} to find out whether the sign of the beat frequency is + or -



Frequency Comb: Calibration and Error

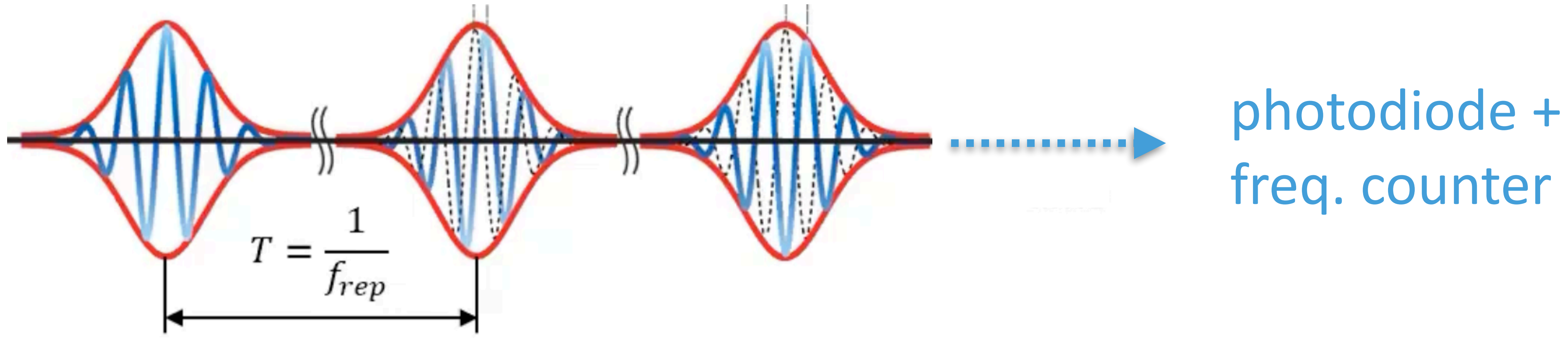
- f_0: If comb spectrum is broad enough to cover both a frequency f_1 and also $2f_1$, then

$$\left.\begin{aligned} 2f_1 &= 2(nf_{rep} + f_0) \\ f_2 &= 2nf_{rep} + f_0 \end{aligned}\right\} \quad f_1 - f_2 = f_0$$



- add $f - 2f$ interferometer to monitor vs. time

- f_{rep}: count comb output with high-bandwidth photodiode



- *Error propagation:* The factor n can be large (10^6) so the error on f_{rep} dominates the measurement error

- Error on f_0 (e.g. from the BDU) can be up to 5-6 orders of magnitude larger than that of f_{rep}

$$f = nf_{rep} + f_0 + f_b$$

$$\sigma^2(f) = n^2\sigma^2(f_{rep}) + \sigma^2(f_0) + \sigma^2(f_b)$$

$$\frac{\Delta f}{f} \approx 10^{-16} \implies \frac{\Delta f_{rep}}{f_{rep}} \approx 10^{-16} \implies \frac{\Delta f_0}{f_0} \lessapprox 10^{-10}$$

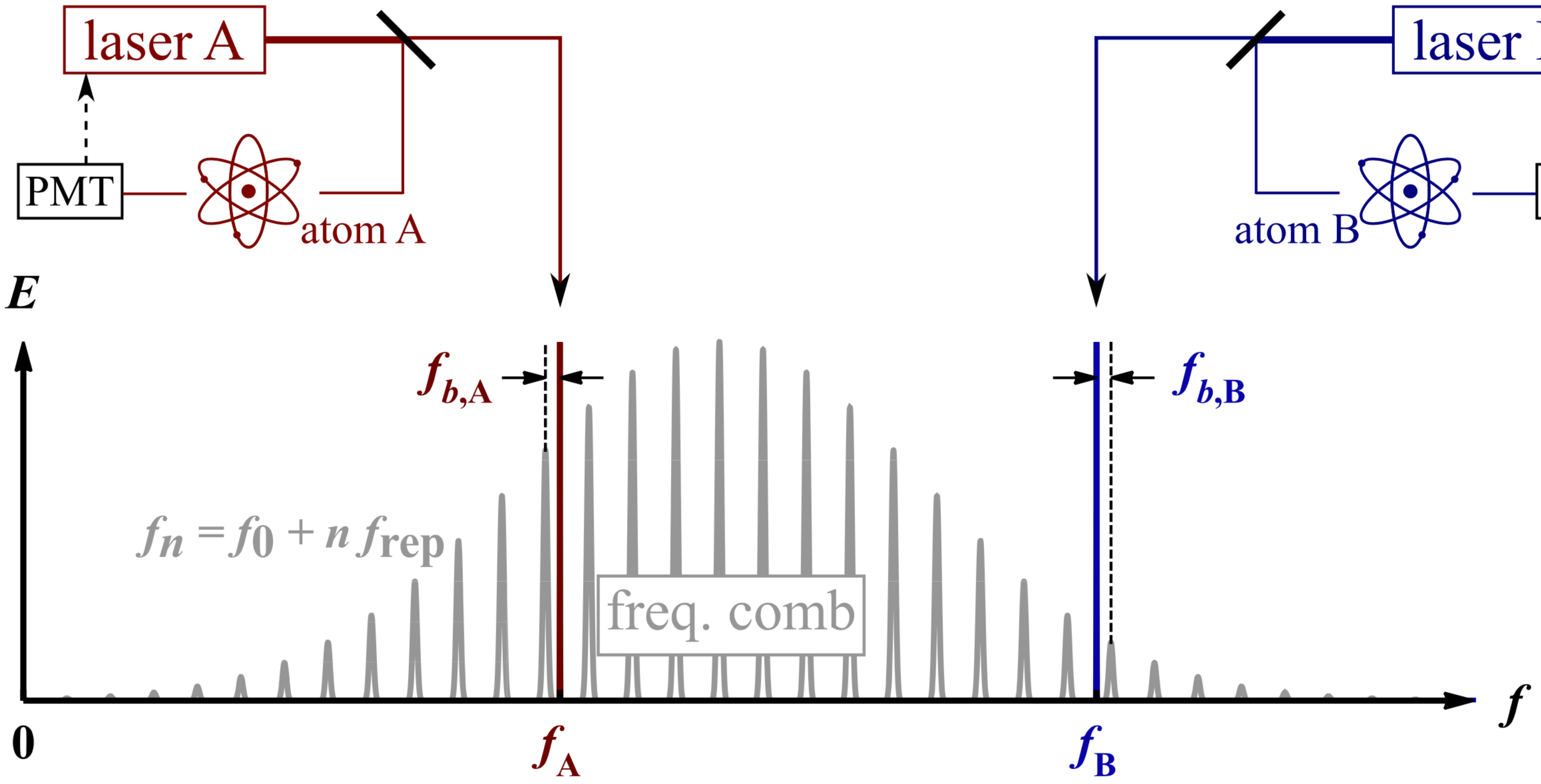
[U. Schwanke]

Why Measure a Frequency Ratio?

- Unitless: Ratios are not limited by the definition/accuracy of the SI second
- Transportable: Frequencies depend on gravity (et al); ratios can be used for comparison
- Measurable: Even at optical frequencies, can use a frequency comb to measure with accuracy and stability limited by the clocks themselves
- Comparative: Can be used to compare different atomic species and/or different reference frames
- Sensitive: Can be engineered to maximise sensitivity to fundamental constants, symmetries of nature

- …also with 18 digits of precision, no predictions from theory!

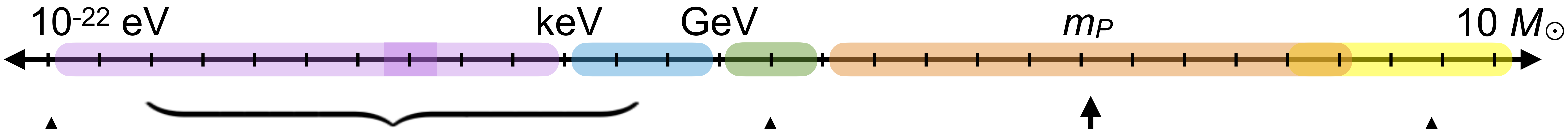
Typical Experiment Setup



[Arvanitaki et al., Phys. Rev. D 91, 015015 (2015)]

Particle Physics Example: Mass Scales for Dark Matter

Quantum Sensing needed for Light/Ultralight Dark Matter



Light and Ultra-light Dark Matter
Axions, ALPs
Quantum Sensing

Composite Dark Matter
WIMPzillas, Q-balls, bound states
CMB, Cosmic Rays

probably not below:
de Broglie wavelength too long

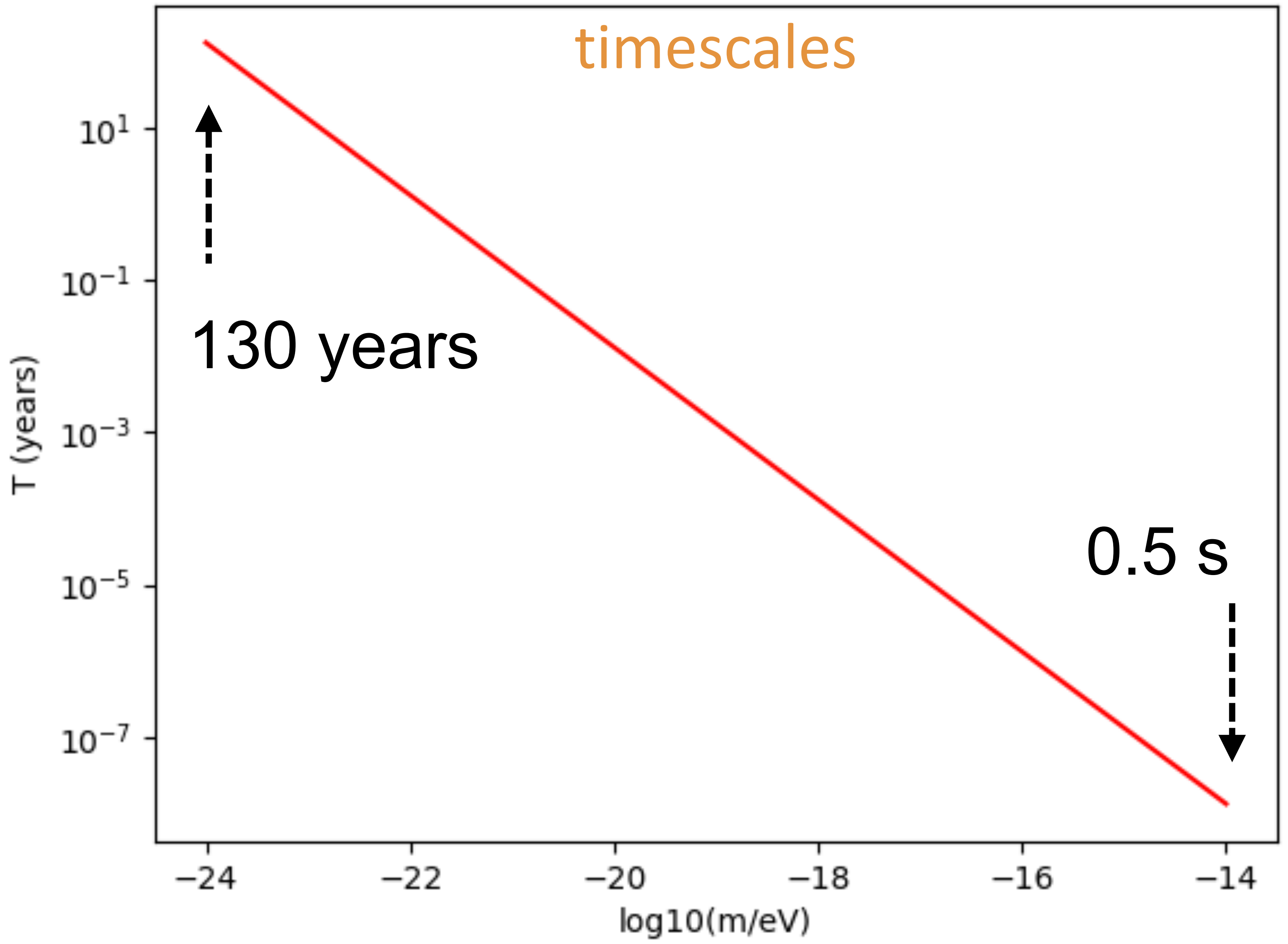
Standard Model Particle Scale
Weakly Interacting Massive Particles
Collider Searches

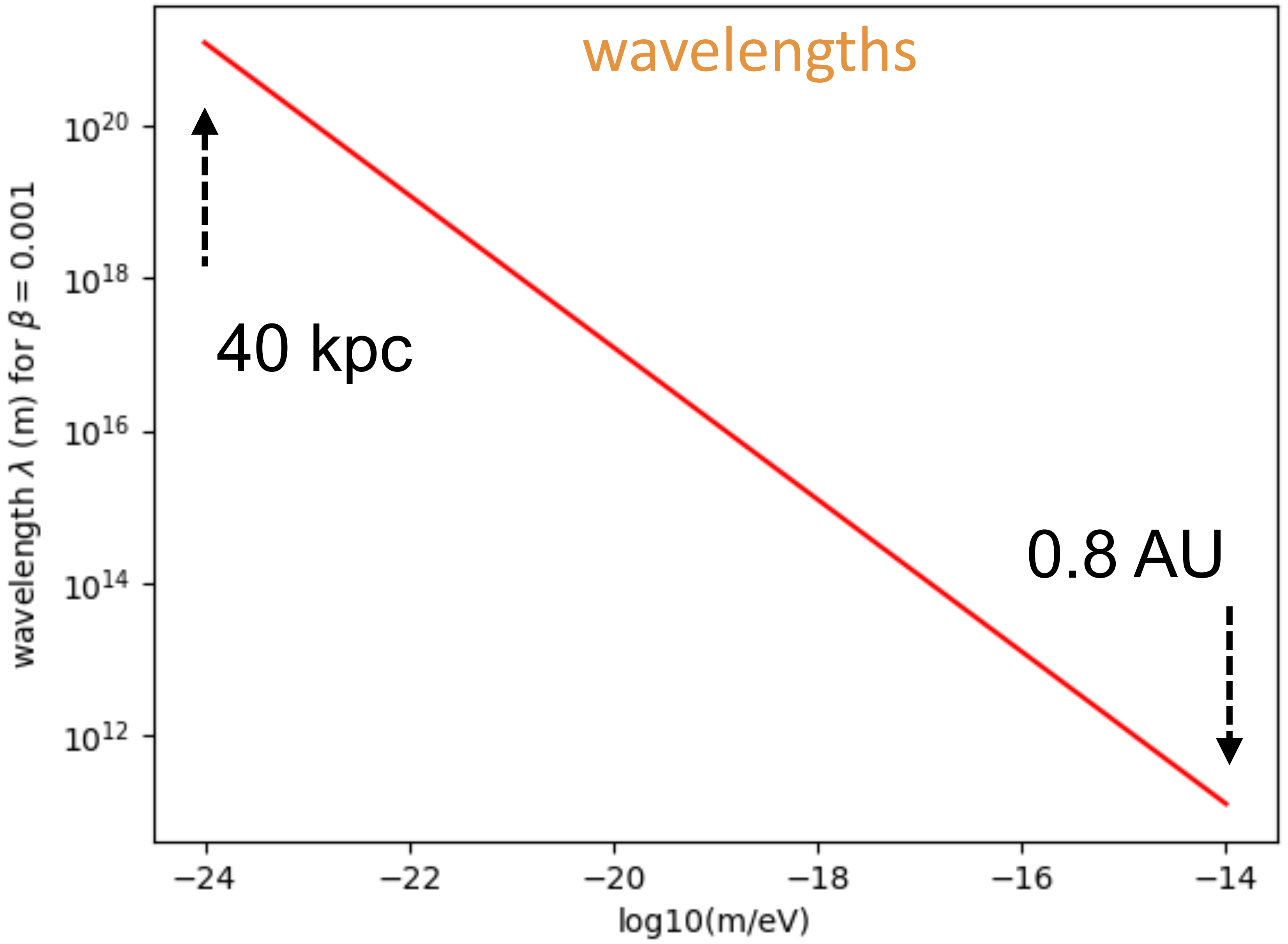
Superheavy Dark Matter
Primordial Black Holes
Gravitational Lensing

[T. Lin, arXiv:1904.07915]

Wave-like Dark Matter

- Dark Matter with clock experiments—oscillation timescales and wavelengths





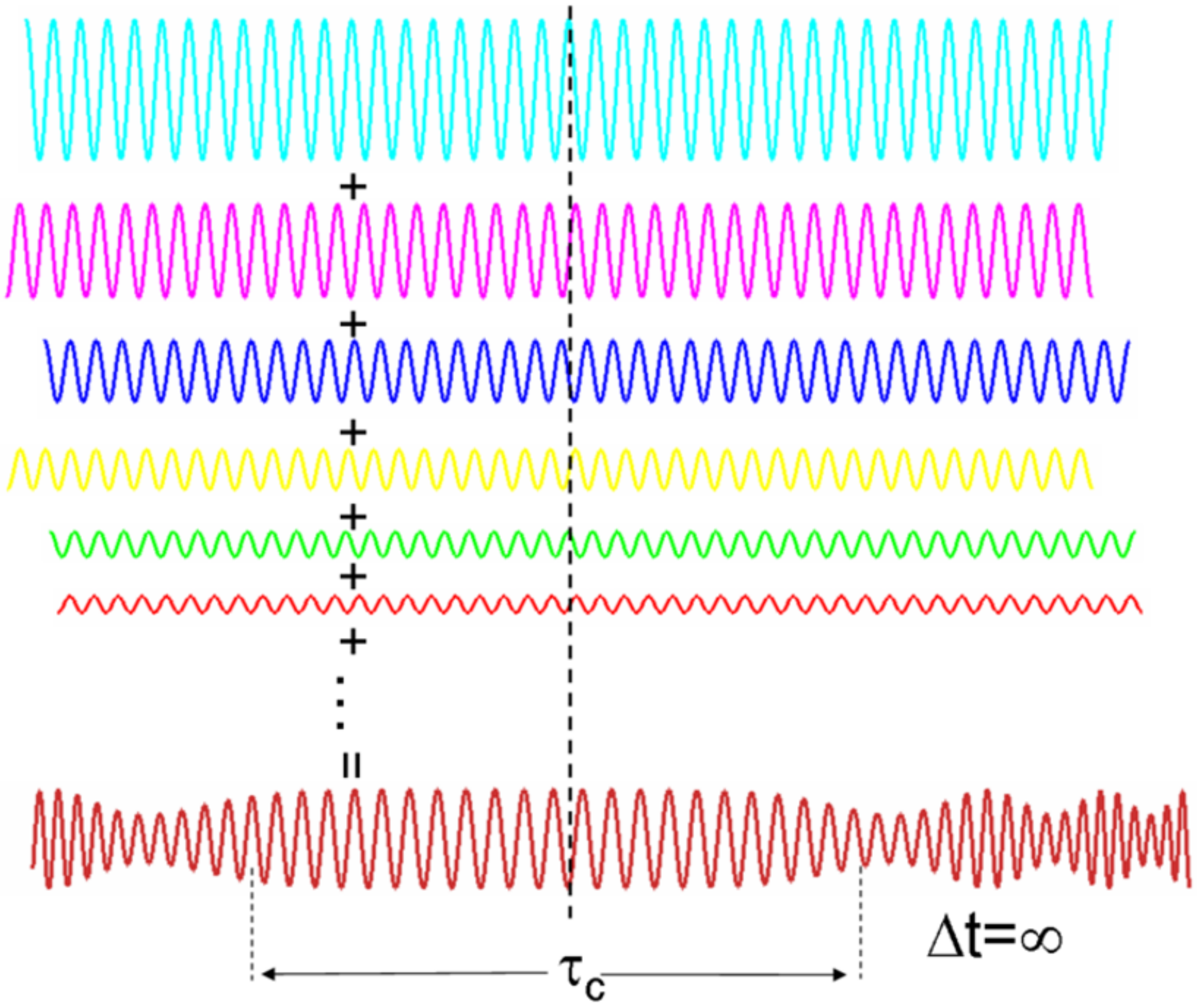
$$E = \hbar\omega = mc^2 = \frac{h}{T} \implies T = \frac{2\pi\hbar}{mc^2}$$

$$\lambda = \frac{h}{P}, \quad pc = \beta\gamma mc^2 \implies \lambda = \frac{2\pi\hbar c}{\beta\gamma mc^2}$$

- Below $m \approx 10^{-21}$ eV, the long wavelength is in tension with the existence of astrophysical DM halos

Ultra-Light Dark Matter as Semi-Coherent Waves?

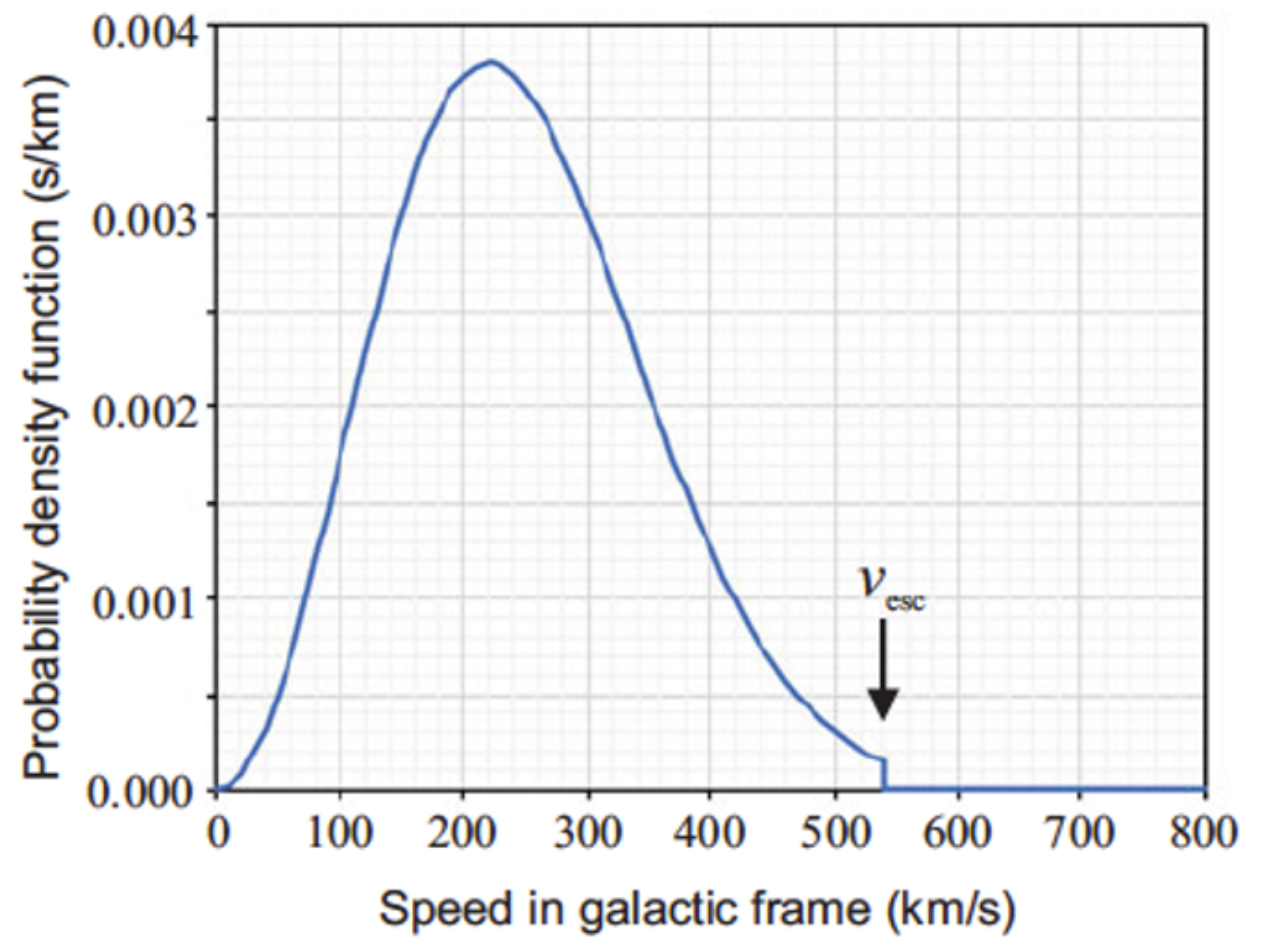
- Simple case: DM is coherent and frequency completely determined by mass (previous page)
- This picture is too simple… the frequency is also a function of velocity (Milky Way, local rest frame)
- The result is a superposition of DM waves and a time dependence
- Amplitude of DM signal only approx. constant within coherence time τ_c



[fig: Wikipedia]

Dark Matter and the Milky Way

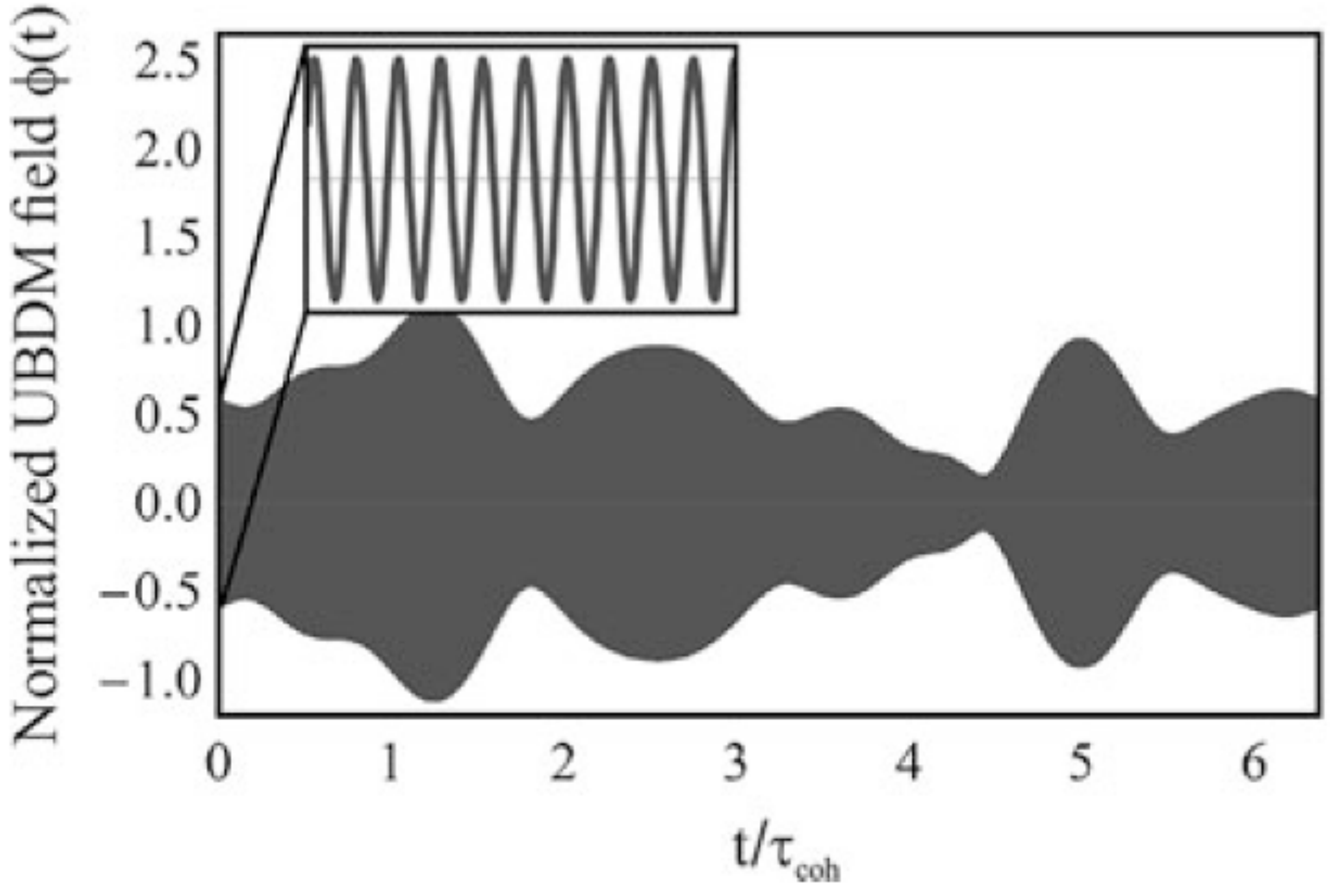
- DM makes up most of the Milky Way mass, and density at the solar circle is 0.3-0.4 GeV cm^{-3}
- Velocity dispersion is large, around 300 km/s (DM is virialised, not thermalised)
- Velocity distribution has cutoff at 544 km/s
- Sun moves with 220 km/s toward constellation Cygnus
- Earth moves around Sun at ~30 km/s
- Mean velocity ≈ velocity dispersion ≈ 300 km/s = 10^{-3} c

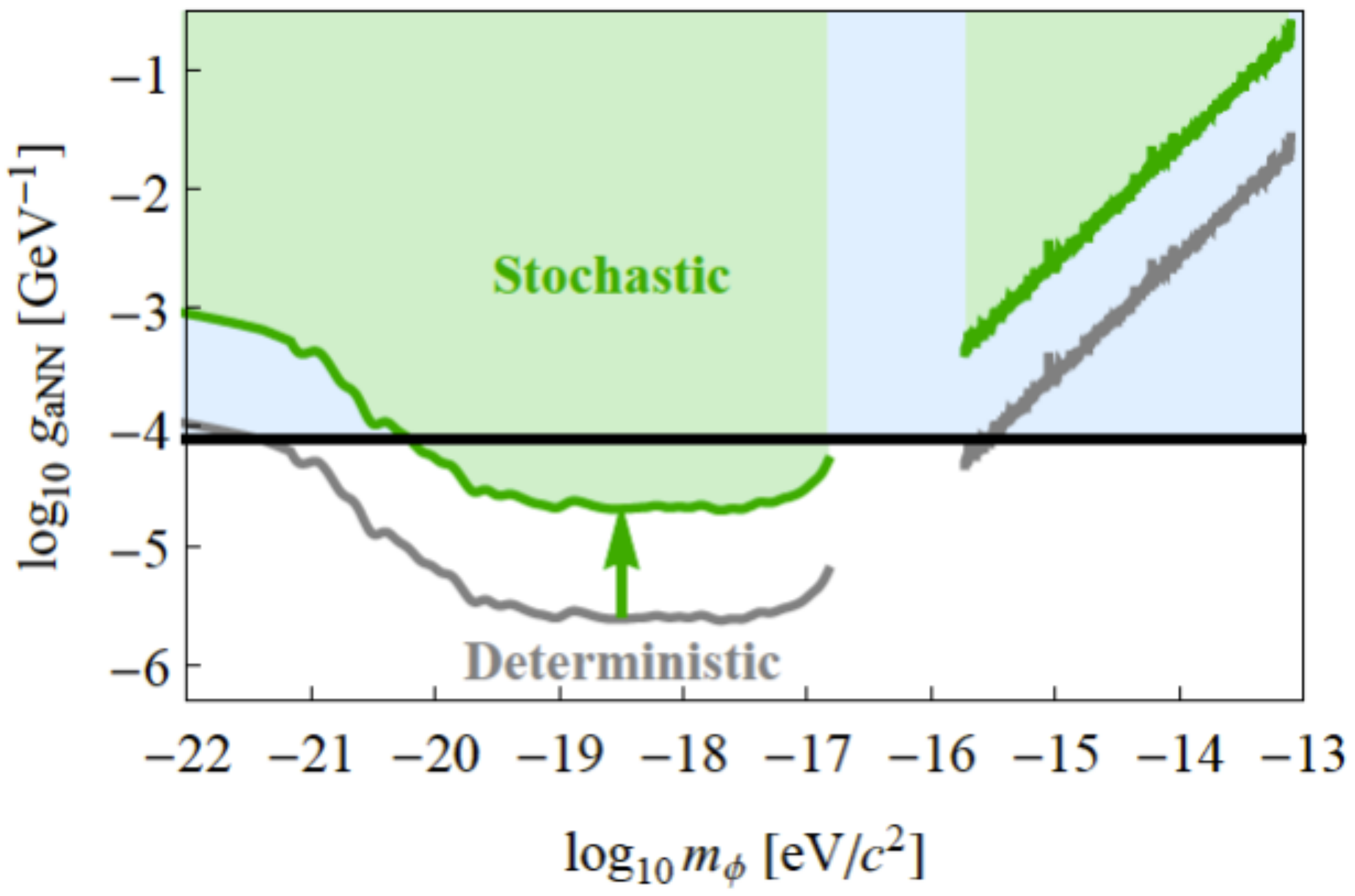


[U. Schwanke; G. Centers et al., Nature Communications 12, 7321 (2021)]

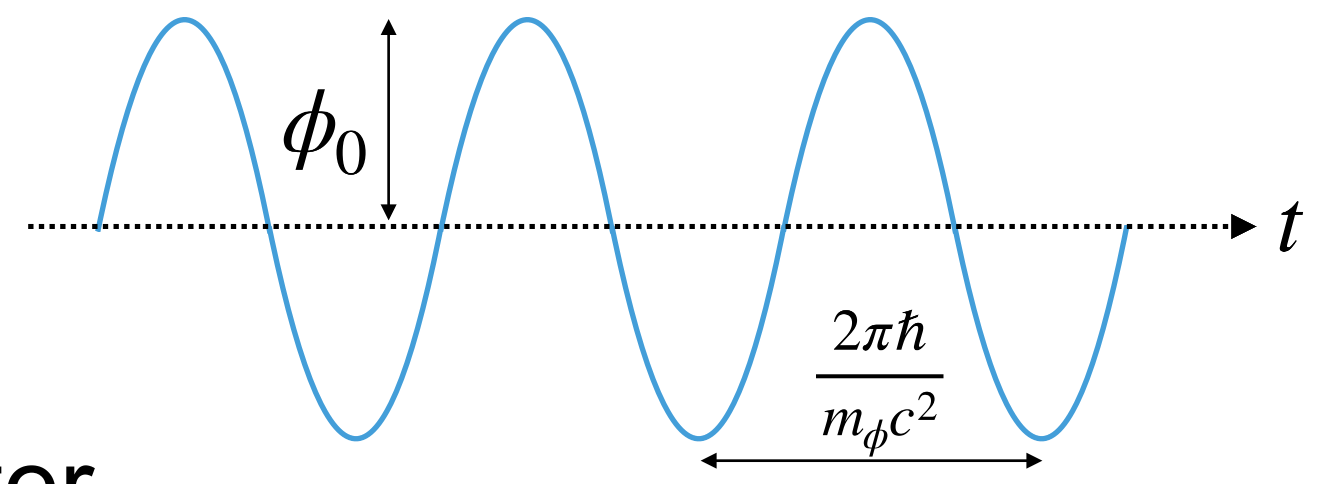
Dark Matter Coherence Time vs T_{data}

- Field amplitude varies stochastically with time
- For $T_{data} < \tau_c$: oscillation occurs at same frequency, but field amplitude not same as average DM density
- For $T_{data} > \tau_c$: assumption of a single frequency fails, but the amplitude is given by the average DM density
- Impact on clock experiments: Limits are weakened by about a factor of 3-10





Ultra-Light Dark Matter: Phenomenology



- Starting with Standard Model Lagrangian, adding new DM interaction (field)
- Bosonic: Ultra-light DM must be bosonic in nature
- Non-relativistic ($\sim 10^{-3}c$): so it neither leaves the galaxy or clumps near the center
- Oscillating classical field: coherent, practically monochromatic → wave-like

$$\phi(t) \approx \phi_0 \cos(m_\phi c^2 t/\hbar)$$

$$\mathcal{L}_{int} = \frac{4\pi\phi}{M_{pl}}\left(\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - d_{m_e}m_e\bar{e}e - \frac{d_g\beta_3}{2g_3}G^A_{\mu\nu}G^{A\mu\nu} - \sum_{i=u,d}(d_{m_i} + \gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right)$$

- Coupling to Lagrangian is linear (in ϕ) for lowest order interaction w/ scalar field (or quadratic with $\phi \leftrightarrow -\phi$ symmetry)

$$\mathcal{L}_{DM} = \frac{\phi}{\Lambda_\gamma}\frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{\phi}{\Lambda_e}m_e\bar{\psi}\psi \qquad \mathcal{L}_{DM} = \frac{\phi^2}{(\Lambda'_\gamma)^2}\frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{\phi^2}{(\Lambda'_e)^2}m_e\bar{\psi}\psi$$

- At the effective new physics energy scales Λ_α and Λ_e, α and m_e appear to oscillate

$$\frac{d\alpha}{\alpha} \approx \frac{\phi_0\cos(m_\phi t)}{\Lambda_\gamma}, \quad \frac{dm_e}{m_e} \approx \frac{\phi_0\cos(m_\phi t)}{\Lambda_e} \qquad \frac{d\alpha}{\alpha} \approx \frac{\phi_0^2\cos^2(m_\phi t)}{(\Lambda'_\gamma)^2}, \quad \frac{dm_e}{m_e} \approx \frac{\phi_0^2\cos^2(m_\phi t)}{(\Lambda'_e)^2}$$

[Damour & Donoghue, PRD 82, 084033 (2010); Arvanitaki et al., Phys. Rev. D 91, 015015 (2015); Safranova et al., RMP 90, 025008 (2018)]

Beyond The Standard Model

- Many possible new interactions for light Dark Matter that lead to oscillations in fundamental parameters

Bosons

Scalar (Spin = 0)
Axions and other Goldstone Bosons

Pseudoscalar (Spin = 1)
Investigate anomaly-free couplings

Electro-magnetism

$$\left(\frac{a}{f_a} F\tilde{F}\right)$$
$$\left(\sim \frac{a}{f_a}\vec{E}.\vec{B}\right)$$

Nuclear Spin

$$\left(\frac{\partial_\mu a}{f_a}\bar{N}\gamma^\mu\gamma_5 N\right)$$

General Axions

Nuclear Force

$$\left(\frac{a}{f_a} G\tilde{G}\right)$$

QCD Axions

Electro-magnetism

$$\left(\epsilon F' F\right)$$

Kinetic Mixing

Nuclear Spin

$$\left(\frac{F'_{\mu\nu}}{f_a}\bar{N}\sigma^{\mu\nu}N\right)$$

Dipole Moment

Nuclear Current

$$\left(g A'_\mu J^\mu_{B-L}\right)$$

B-L

[Y. Stadnik]

Clock Transitions and Sensitivities

Clock	Kα	Kμ
Yb^+ (467 nm)	-5.95	0
Sr (698 nm)	0.06	0
Cs (32.6 mm)	2.83	1
CaF (17 μm)	0	0.5
N_2^+ (2.31 μm)	0	0.5
Cf^{15+} (618 nm)	47	0
Cf^{17+} (485 nm)	-43.5	0

- Atomic transition scale set by Rydberg constant $R_\infty = \alpha^2 m_e c/4\pi\hbar$, also fine structure constant α and proton-to-electron mass ratio μ

  Hyperfine transitions: $\nu_{\mathrm{hf}} = A\cdot\mu\alpha^2 F_{\mathrm{hf}}(\alpha)\cdot R_\infty$

  Optical transitions: $\nu_{\mathrm{opt}} = B\cdot F_{\mathrm{opt}}(\alpha)\cdot R_\infty$

  Vibrational transitions: $\nu_{\mathrm{vib}} = C\cdot\mu^{1/2}\cdot R_\infty$

- Transitions have different sensitivities to variations in α or μ, given by K_α and K_μ

$$K_\alpha = \frac{\partial \ln\left(\frac{\nu}{cR_\infty}\right)}{\partial \ln \alpha} = \begin{cases} 2 + \partial \ln F_{hf}/\partial \ln \alpha & \text{for hyperfine transitions} \\ \partial \ln F_{opt}/\partial \ln \alpha & \text{for optical transitions} \end{cases}$$

- Measure ratios of frequencies of two transitions ($R = \nu_1/\nu_2$)
- So for α and simple case of scalar field ϕ and linear coupling, ratio oscillates with frequency $f = m_\phi c^2/h$

$$\frac{dR}{R} = [K_{\alpha,1} - K_{\alpha,2}]\frac{d\alpha}{\alpha}$$
$$= [K_{\alpha,1} - K_{\alpha,2}]\frac{\phi_0 \cos(m_\phi t)}{\Lambda}$$

Sensitivity estimates for $\delta\alpha/\alpha$ for different clock transitions… similar for and $\delta\mu/\mu$

Data Analysis Challenge

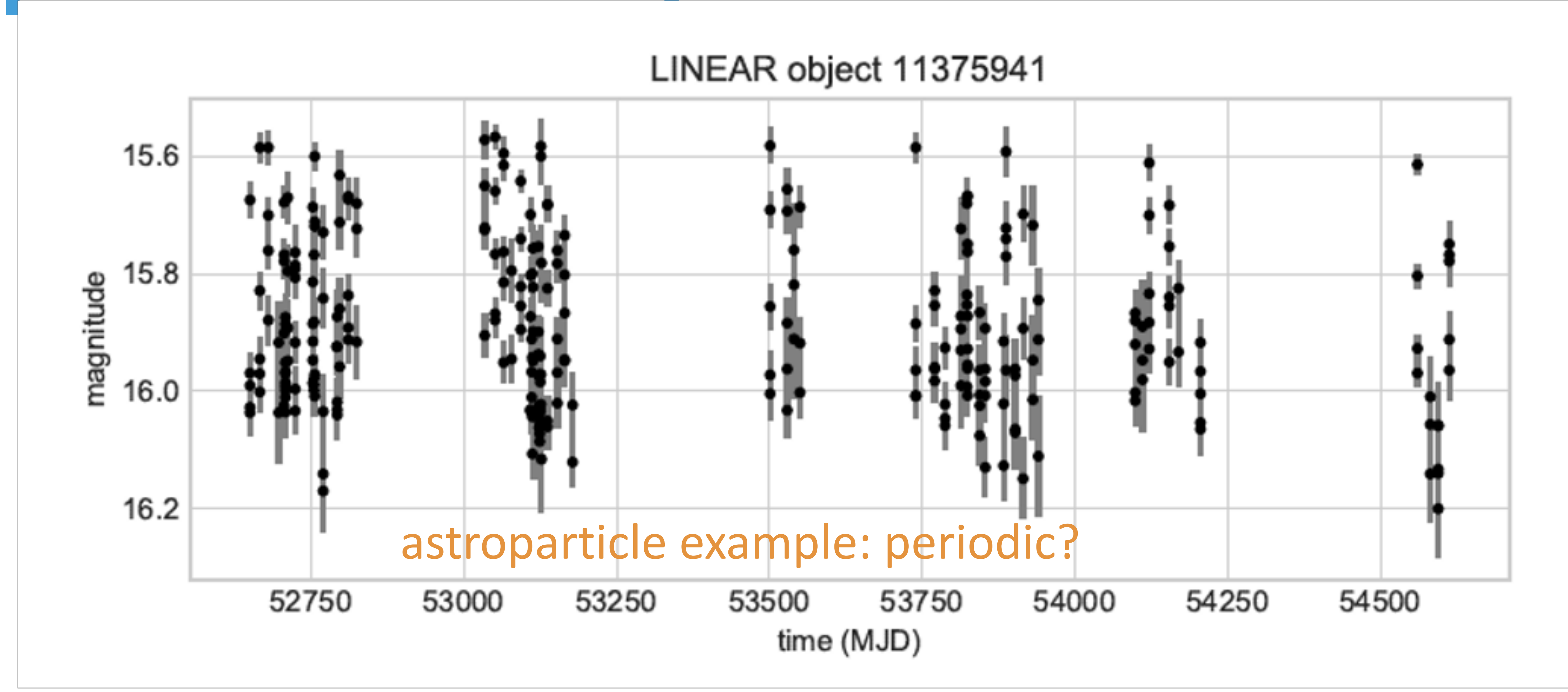
- One frequency ratio R measurement per s for $\mathcal{O}$(1 year), i.e. 3×10^7 data points $\rightarrow$ time series analysis
- We can estimate the error on dR/R by adding in quadrature the fractional frequency error for the two clocks (each with e.g. probe time 1 s)
- Ultimately, we would like to measure/limit the signal frequency (thus m_ϕ) and also Λ

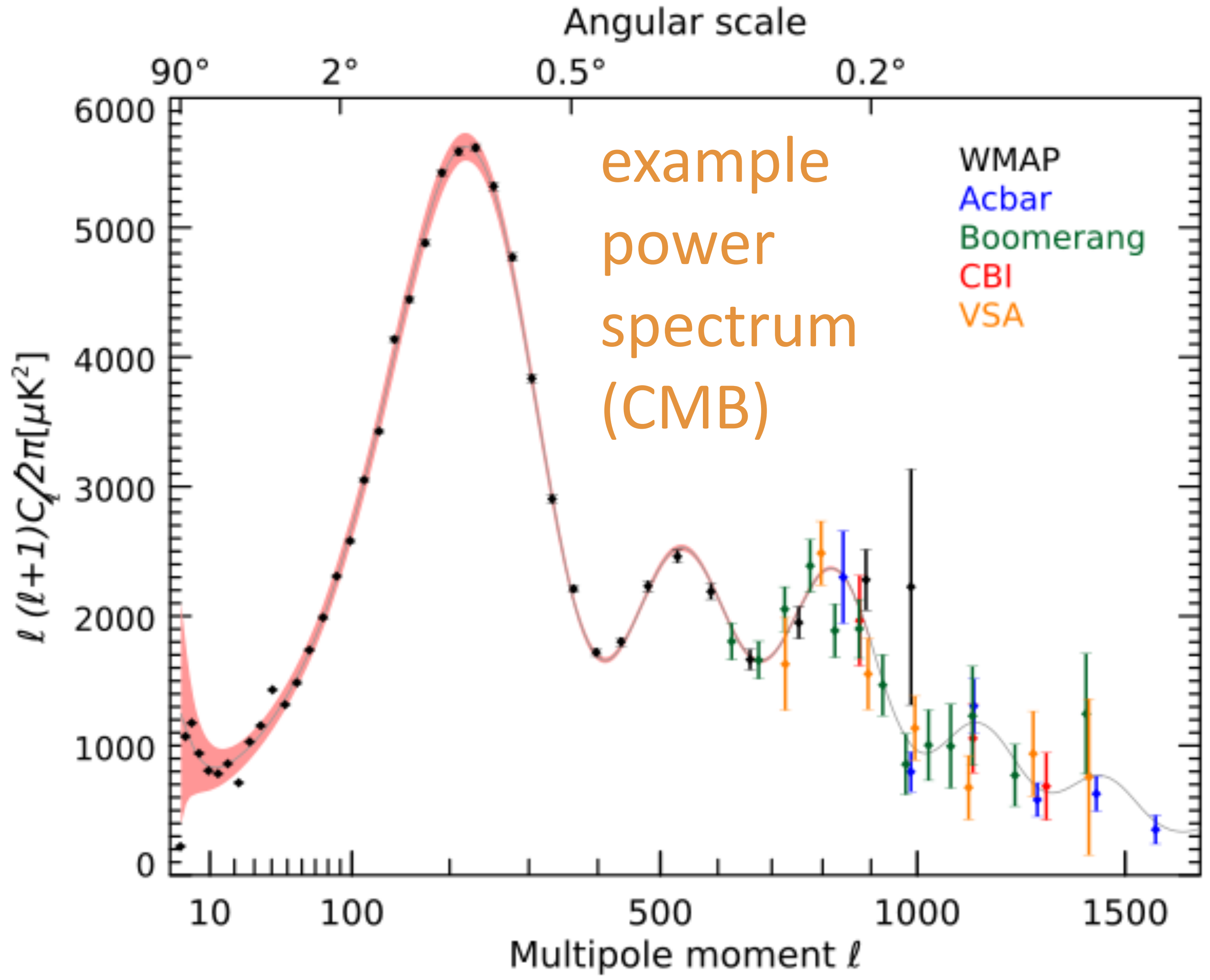
$$\frac{dR}{R} = [K_{\alpha,1} - K_{\alpha,2}]\frac{\phi_0 \cos(m_\phi t)}{\Lambda}$$

- *The challenge:*
- Limited time steps: discreet (~1s) data points
- Limited resolution: can't say anything about periods much smaller than 1s
- Irregular gaps in data-taking periods
- Assumptions/model dependence/systematic errors in $K_{\alpha,1}$, $K_{\alpha,2}$, ϕ_0, relation to Λ

Fourier Transform and Power Spectral Density

- There are many techniques to identify periodicity
- Signal is often folded/convolved with a windowing function resulting from experimental constraints
- Suitable analysis method depends on many factors (frequency known or not, phase known or not, cadence of observations)
- Our case: frequency unknown, but relatively high uptime and regularly spaced measurements





$$\frac{dR}{R} \propto A\cos(\omega t + \phi) = g(t)$$

$$F(\omega) = \int_{-\infty}^{+\infty} g(t)e^{-i\omega t}dt$$ Fourier transform

$$P(\omega) = \lim_{T\to\infty} \frac{1}{T}|F(\omega)|^2$$ power spectral density

[Astro-ph arXiv: 1703.09824; Wikipedia for CMB]

Periodogram and Lomb-Scargle

- Periodogram: serves as an estimate of the power spectral density for the case of discreet data
- A periodogram is useful for finding the dominant periods (or frequencies) of a fixed-interval time series
- Lomb-Scargle Periodogram developed for cases where the sampling is irregular

$$P(\omega) = \lim_{T\to\infty} \frac{1}{T} |F(\omega)|^2 = \lim_{T\to\infty} \frac{1}{T}\left(\int g \sin \omega t dt\right)^2 + \left(\int g \cos \omega t dt\right)^2$$

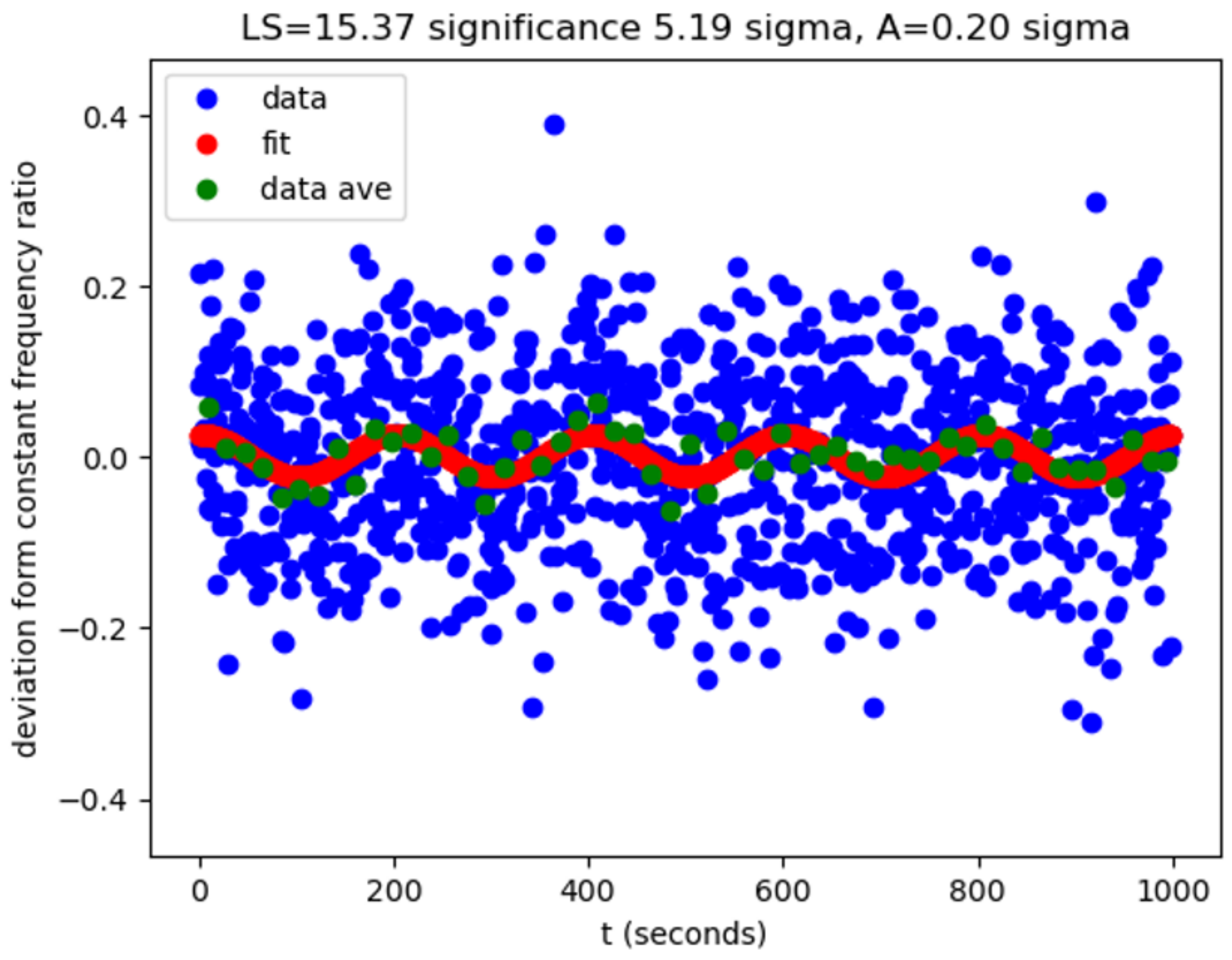
power spectral density

$$\frac{1}{N}\left\{\left(\sum g_n \cos(\omega t_n)\right)^2 + \left(\sum g_n \sin(\omega t_n)\right)^2\right\}$$

classic periodogram

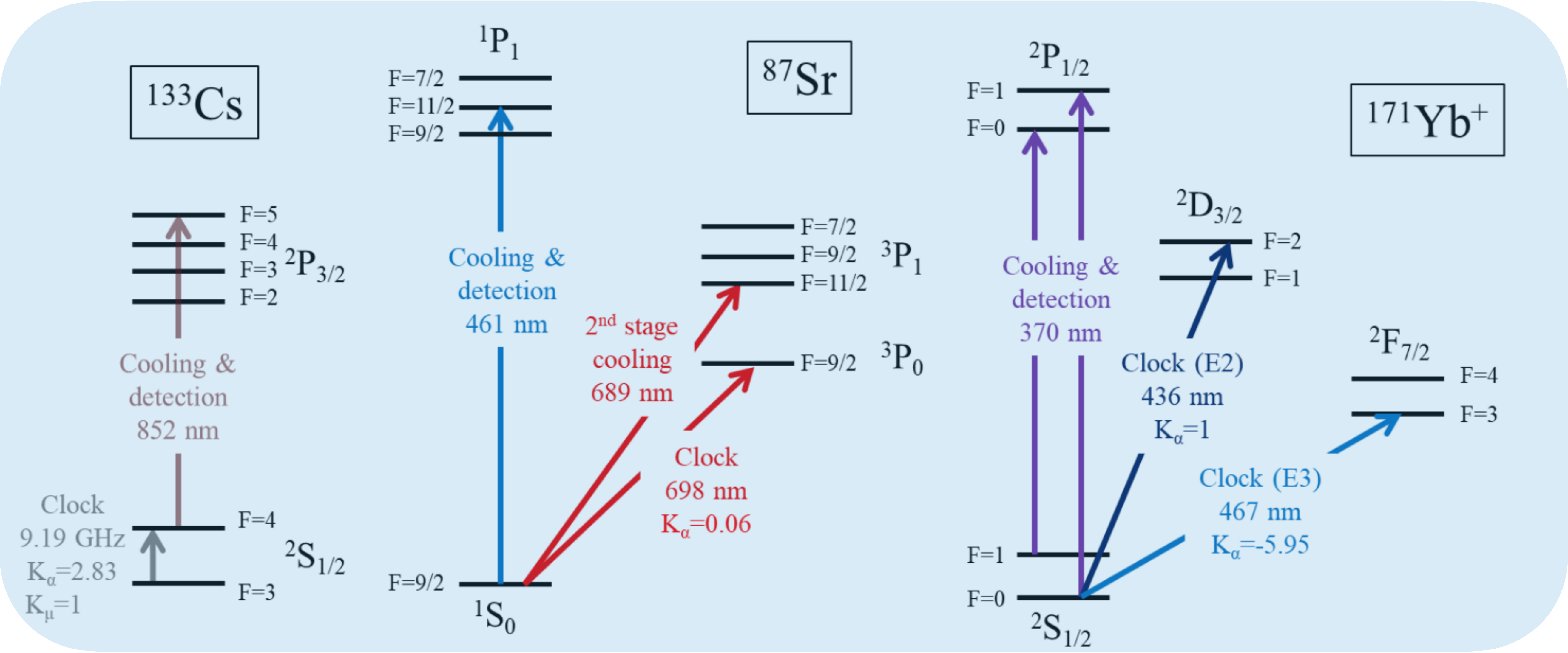
$$\frac{B_1^2}{2}\left(\sum g_n \cos(\omega(t_n - \tau))\right)^2 + \frac{B_2^2}{2}\left(\sum g_n \sin(\omega(t_n - \tau))\right)^2$$

Lomb-Scargle periodogram

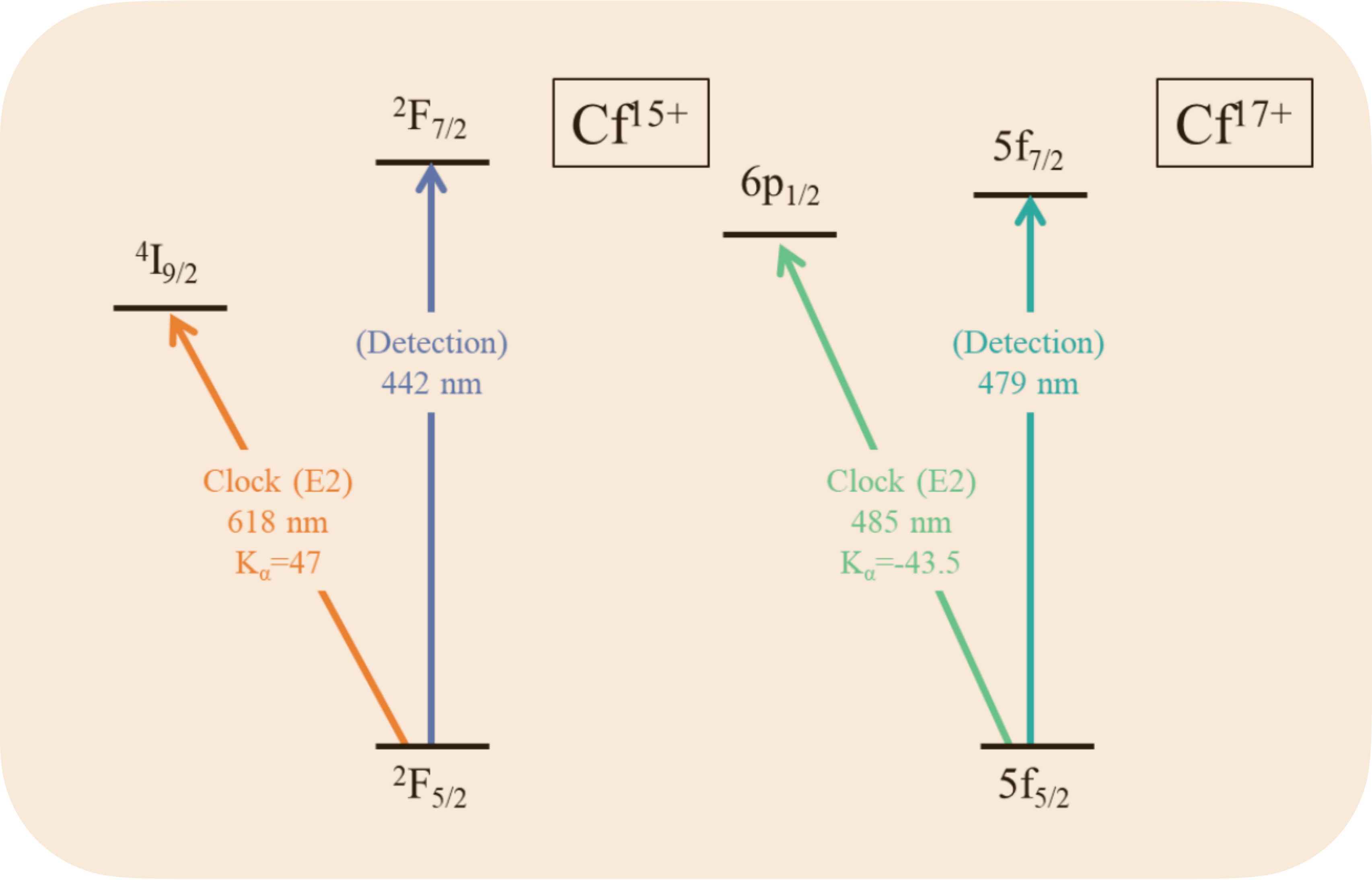


Simulation w/ Lomb-Scargle fit (red)
1 Hz sampling, σ=0.1, N=10^3, T=200 s

Example Clocks

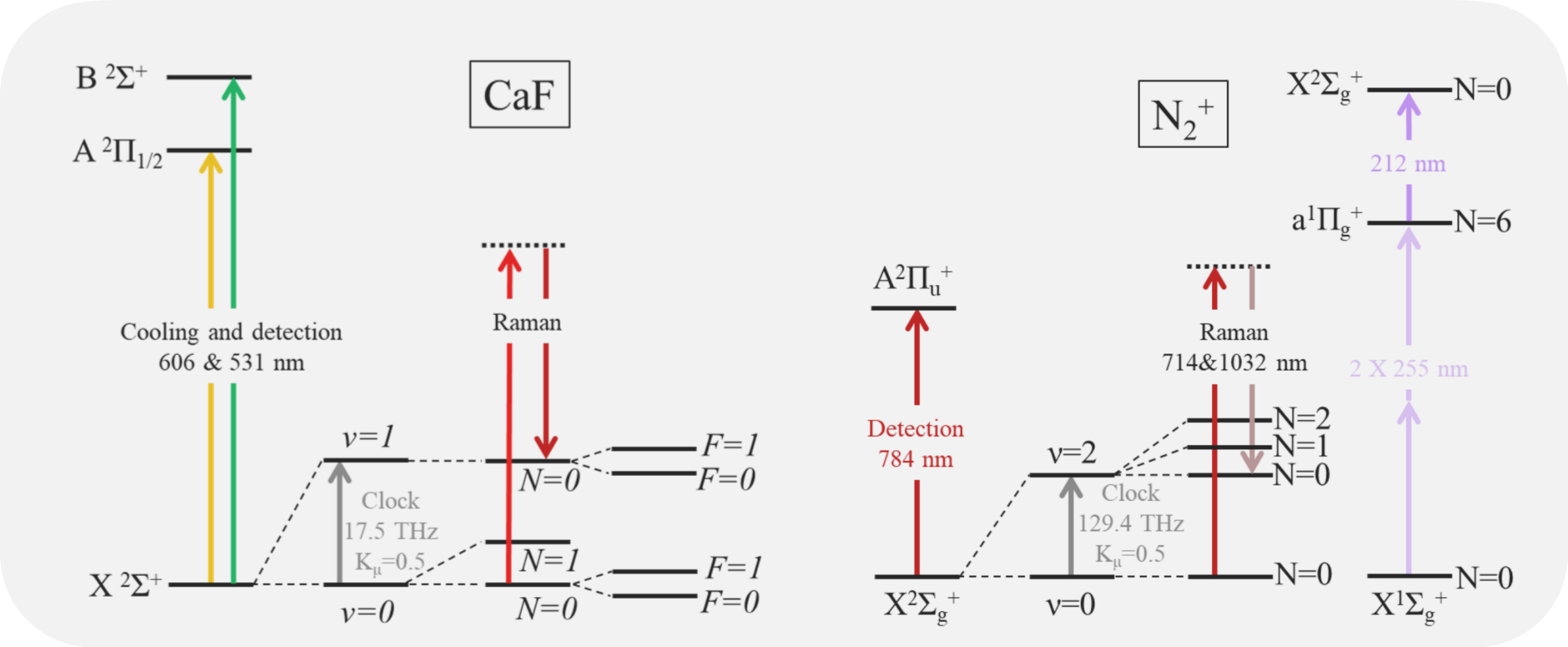


Atomic Clocks



Highly Charged Ion Clock

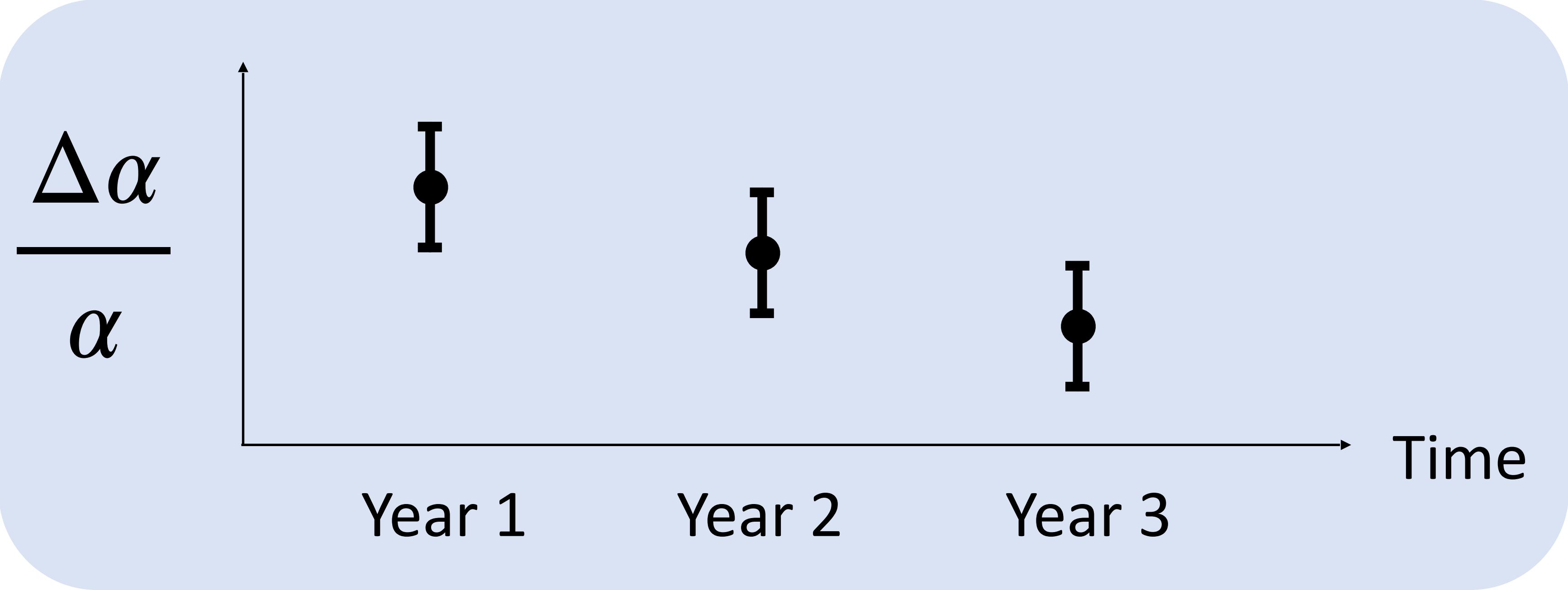
Molecular Clocks



[EPJ Quant.Technol. 9 (2022) 1,12]

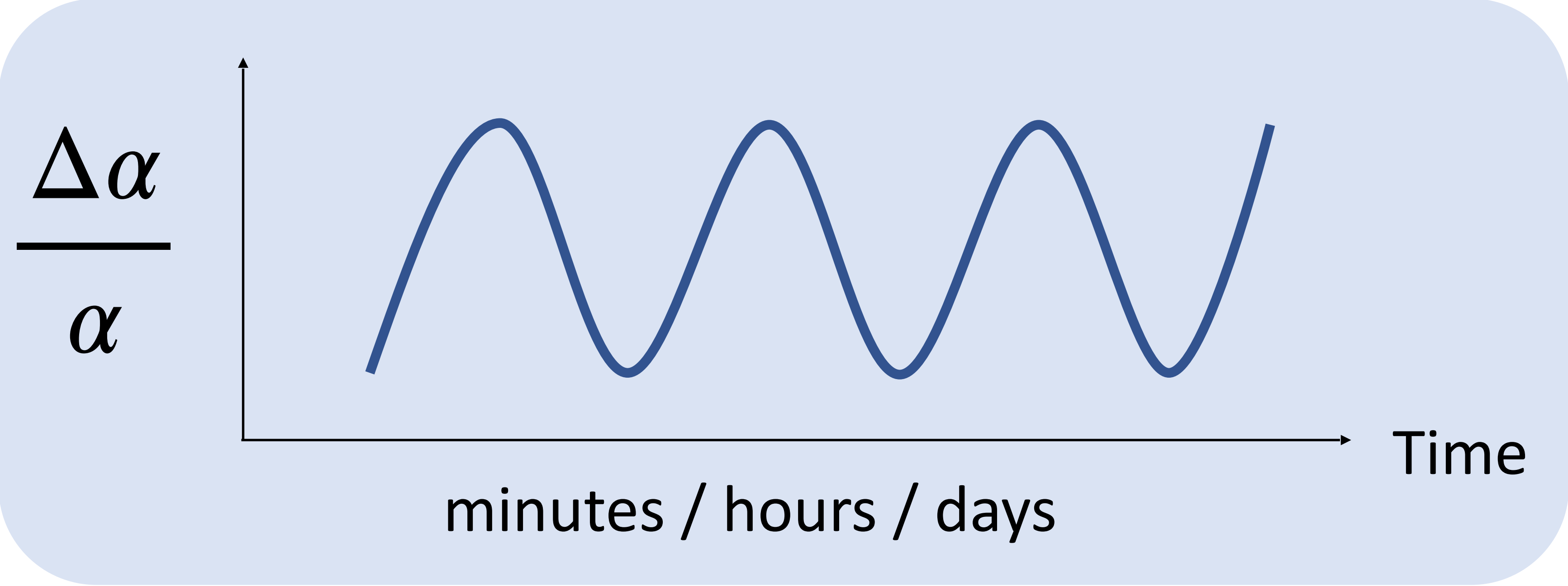
Clock-based Search for α Variations at Different Timescales

- Slow drifts



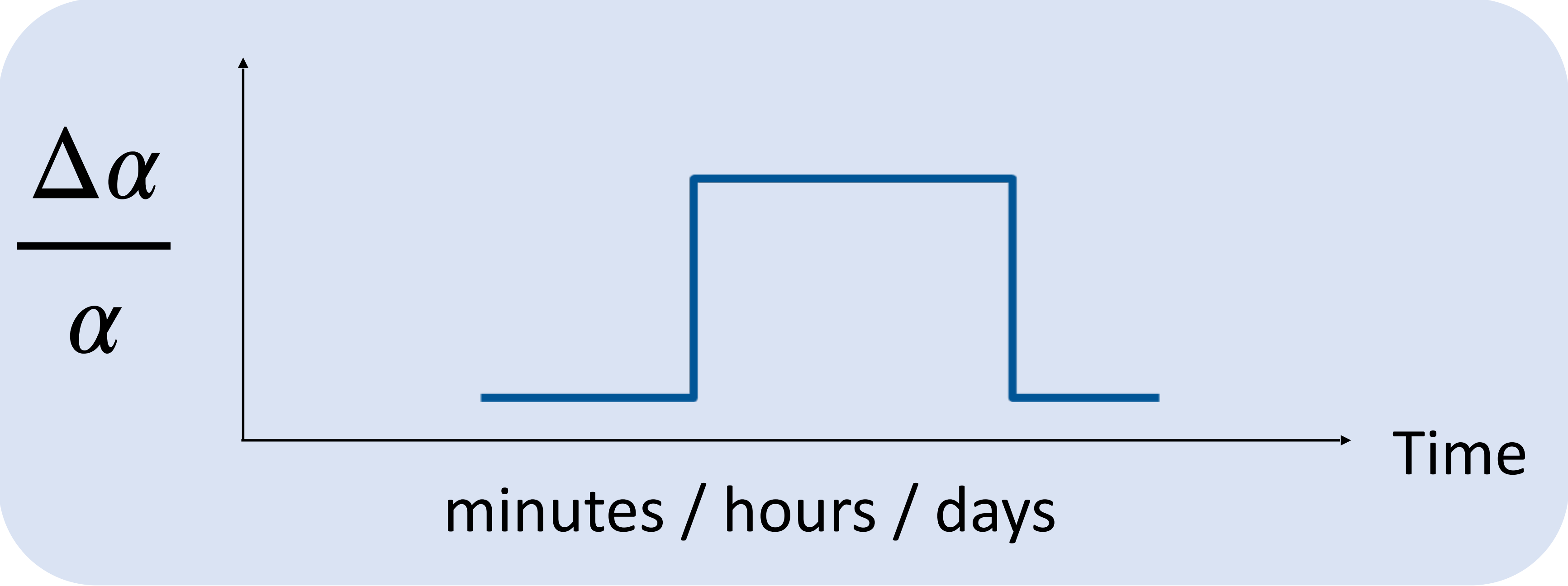
→ New physics

- Oscillations



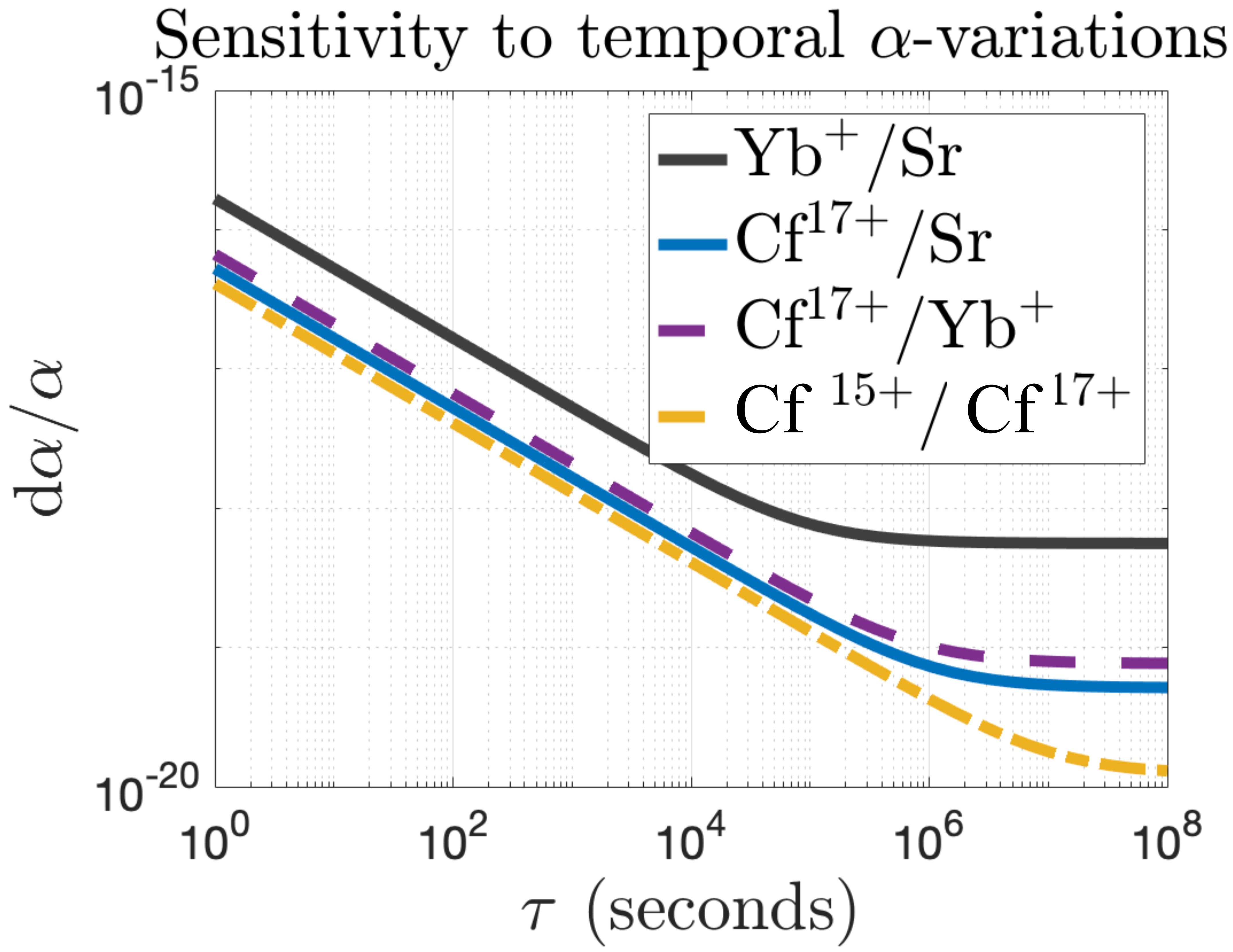
→ Very light dark matter

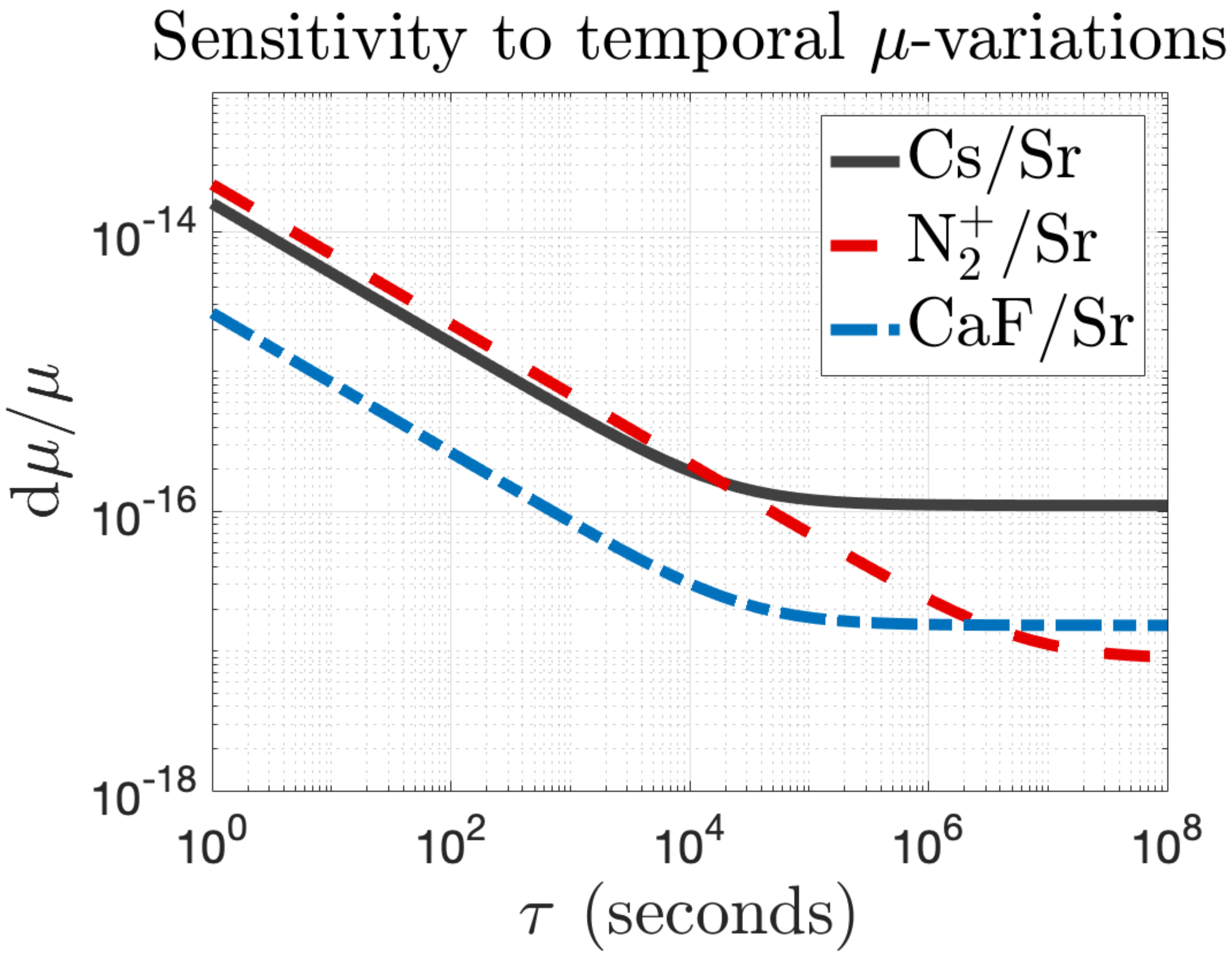
- Fast transients



→ Dark matter, topological defects

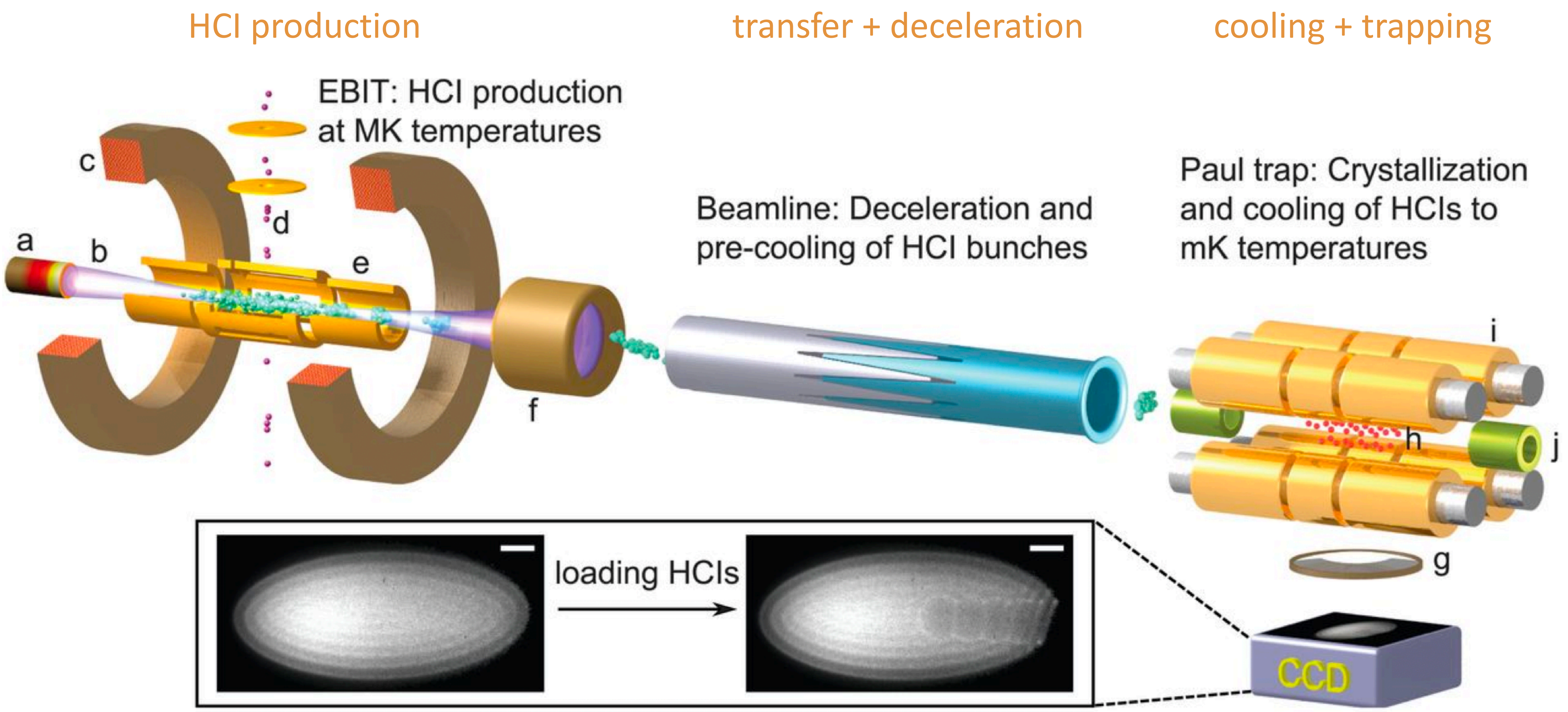
Sensitivities to Temporal Variations of α, μ





[EPJ Quant.Technol. 9 (2022) 1,12]

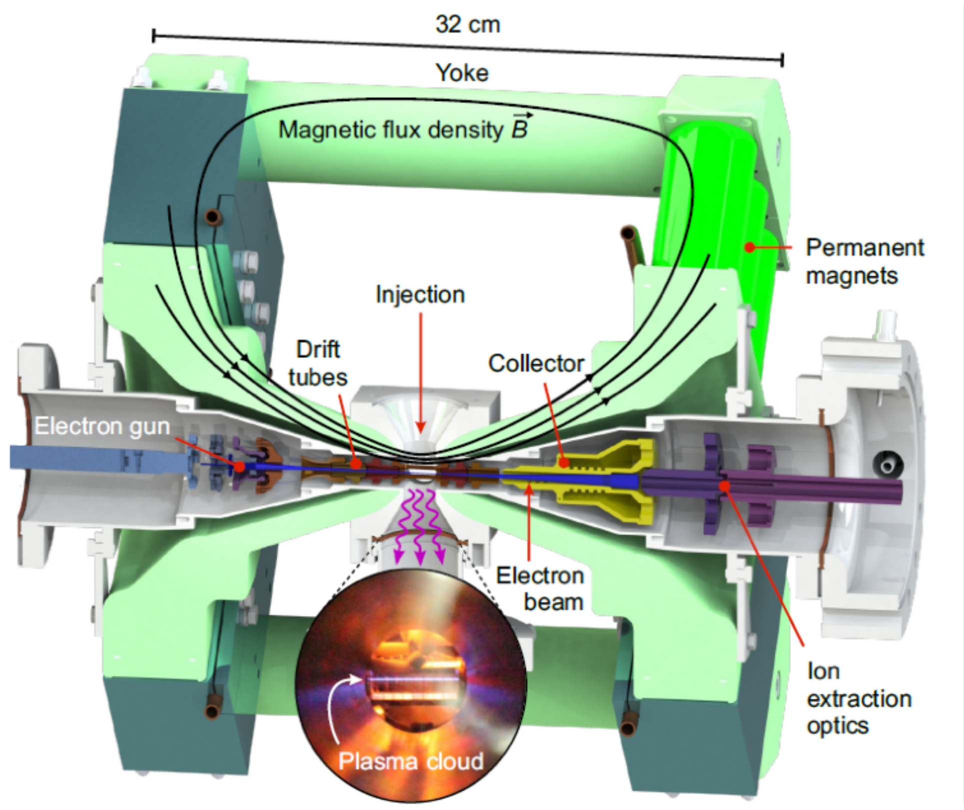
Highly Charged Ions: Production, Cooling and Trapping



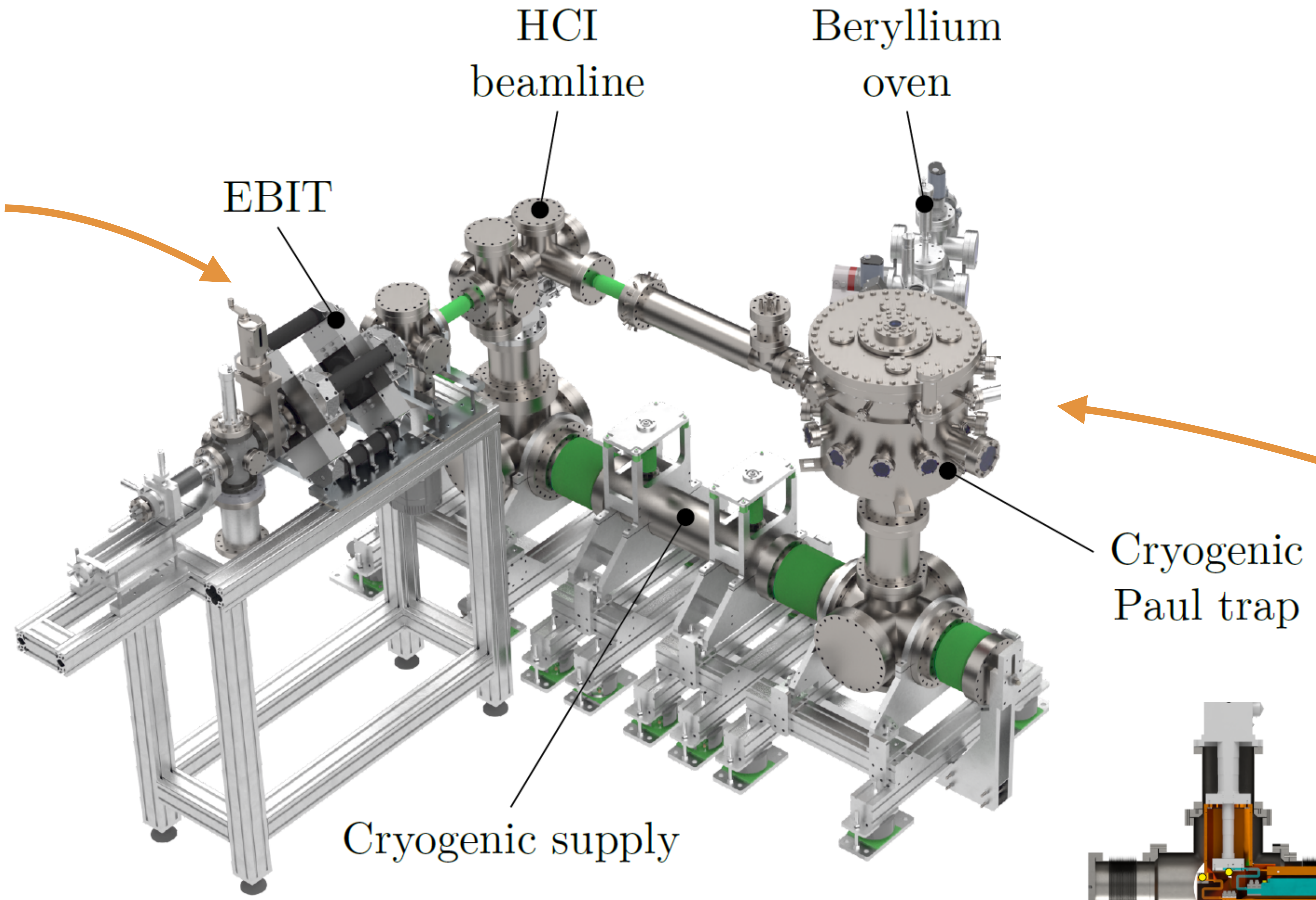
- EBIT: Electron Beam Ion Trap (used as a source of HCI)
- HCI bunches are loaded into a linear Paul Trap on demand (HCI lifetimes can be hours)
- Laser and Sympathetic Cooling e.g. with Be+, Ca+ (lots of lasers for cooling/preparation/readout)

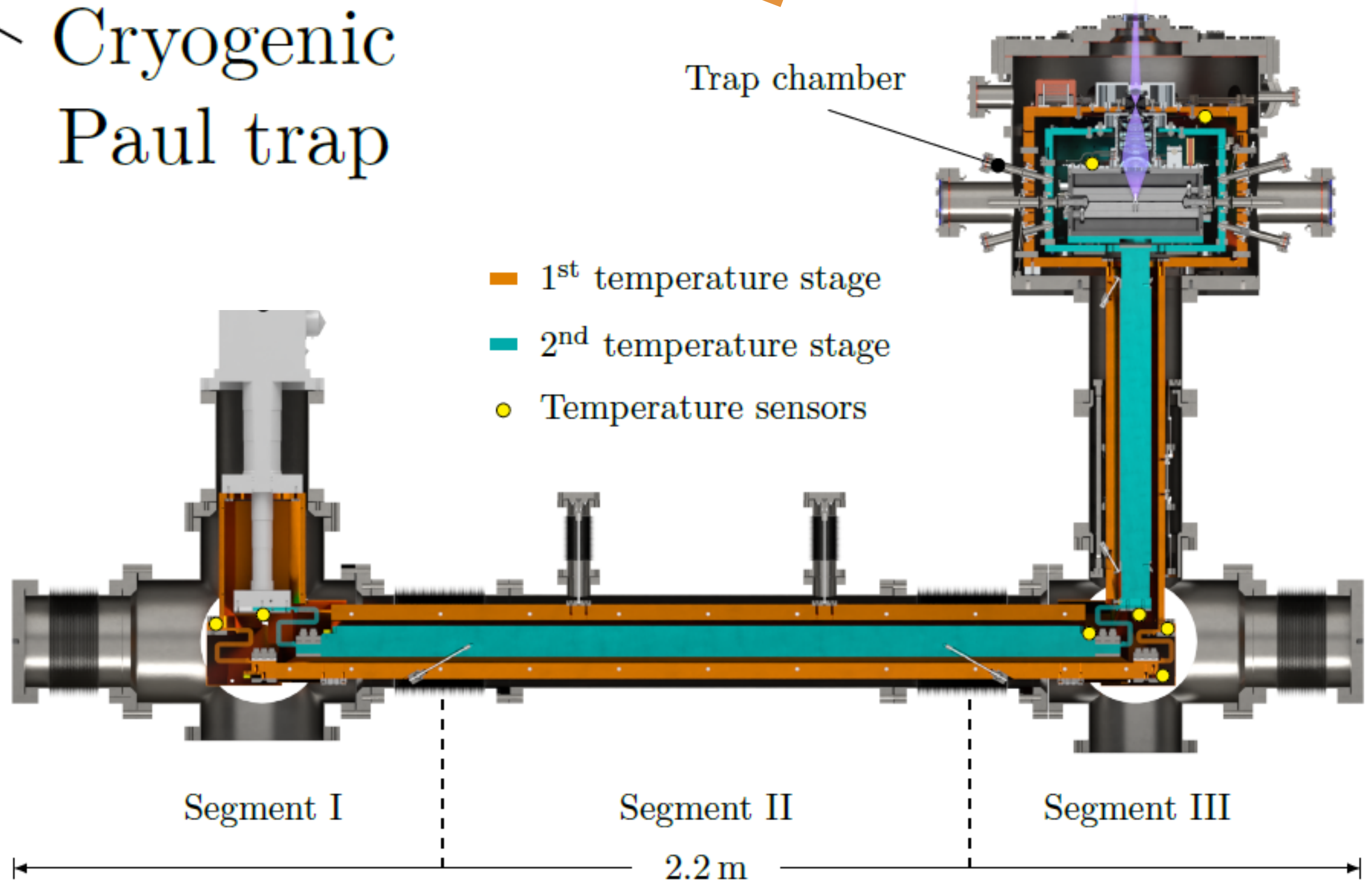
[L. Schmöger et al., Science 347, 6227 (2015); P. Micke et al., Nature 578, 2020]

Highly Charged Ions: Production, Cooling and Trapping



Compact EBIT @ MPIK

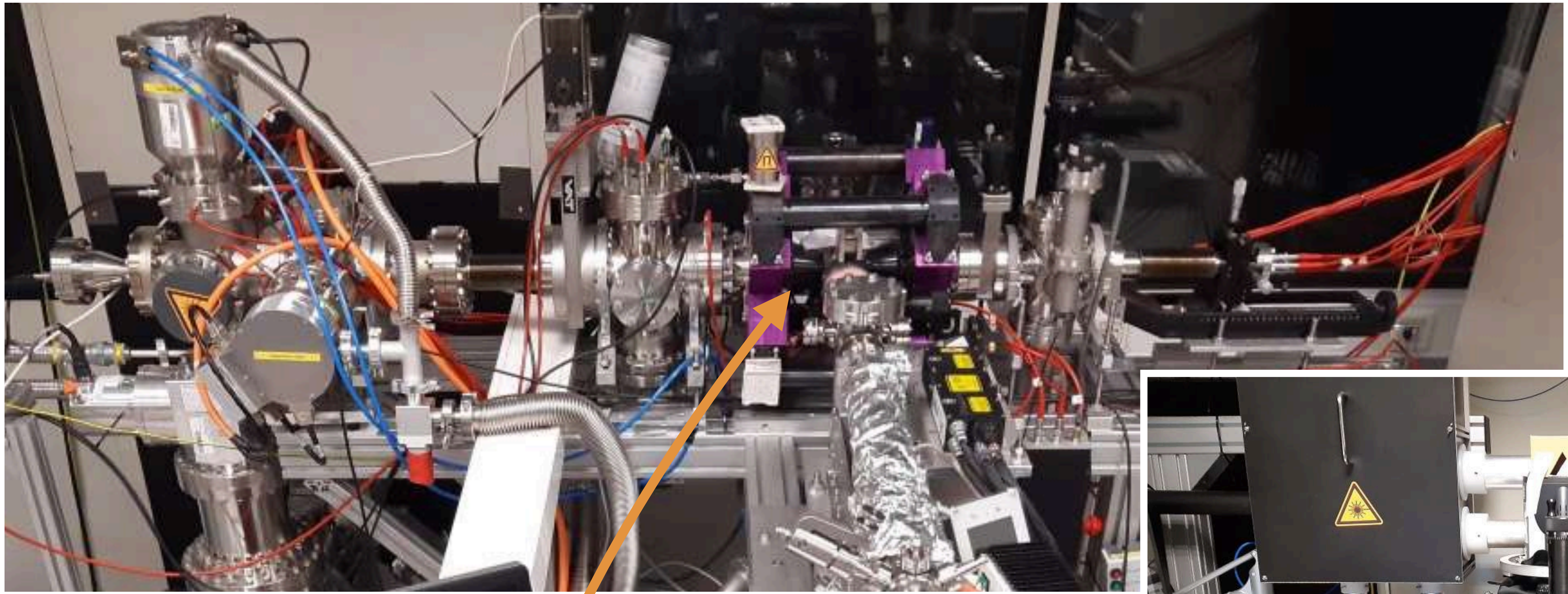




MK → μK

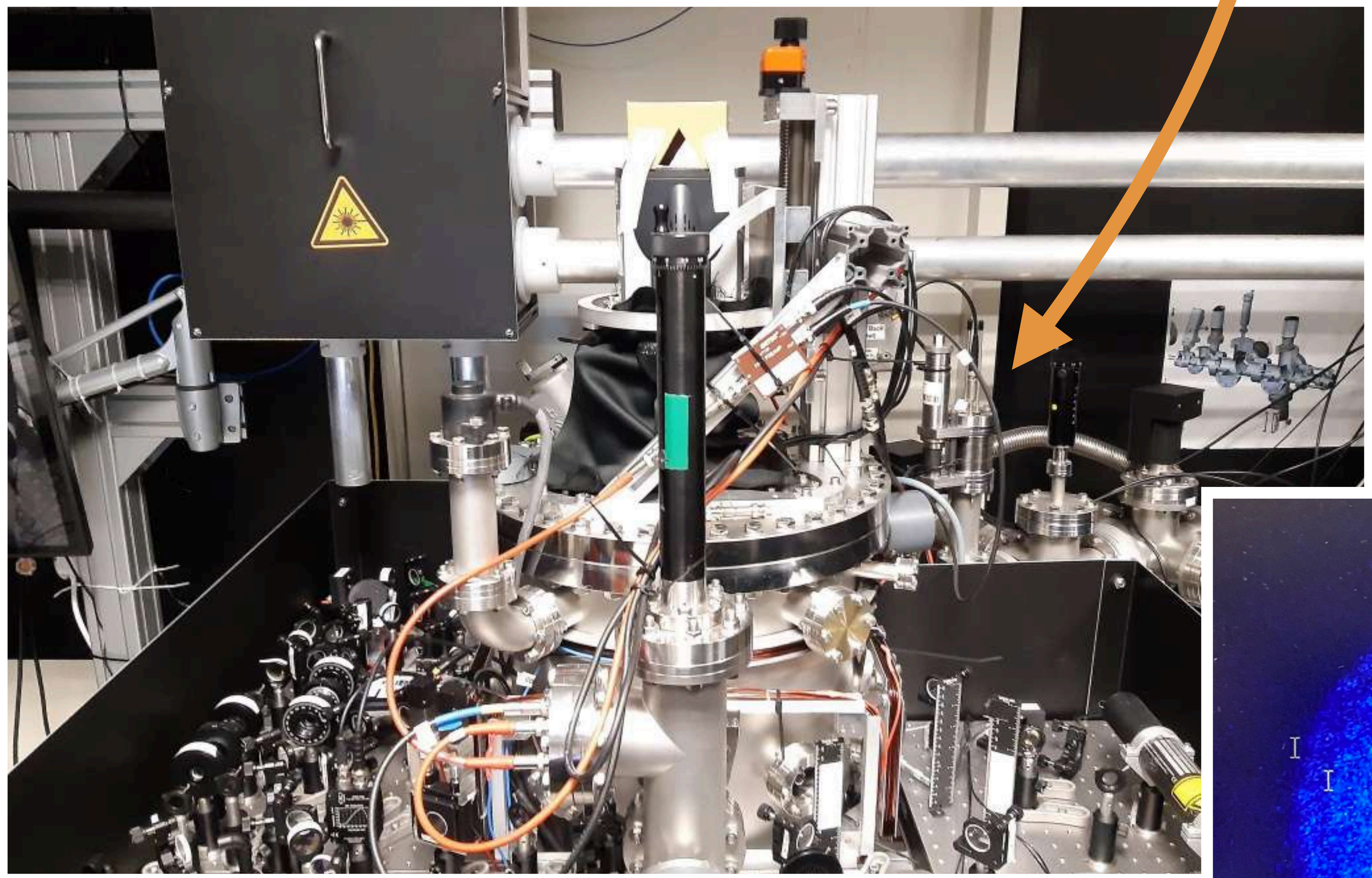
Ultra-low vibration cryogenic vacuum

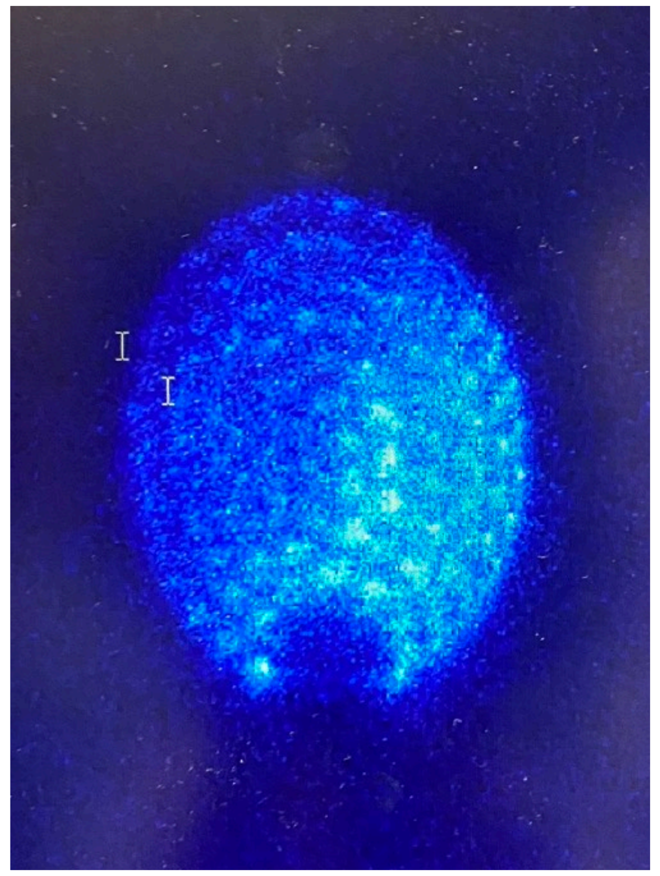
Highly Charged Ions: Production, Cooling and Trapping



EBIT test stand @MPIK

Paul trap w/ optics





Be crystal with highly charged ion (Ar)

[MPIK: José R. Crespo López-Urrutia]

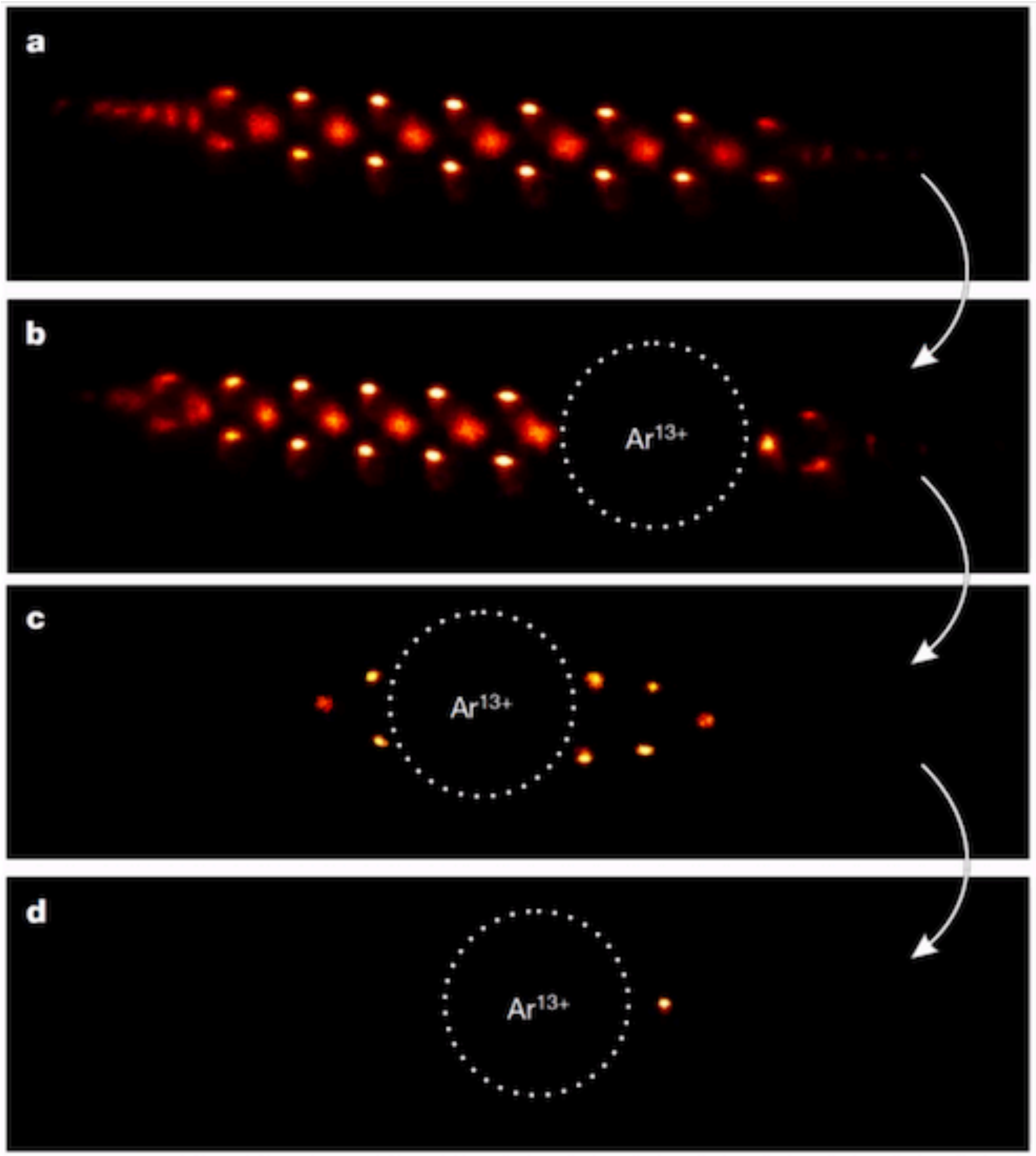
Highly Charged Ions: Production, cooling and trapping

HCI ions implanted in a Coulomb crystal

- Sympathetic cooling with the crystal ions (eg $^{9}Be^{+}$)
- Clock-based experiments with highly charged ions
- Also possible: Quantum Logic Spectroscopy (QLS) using the co-trapped ions

Sympathetic Resolved-Sideband Cooling

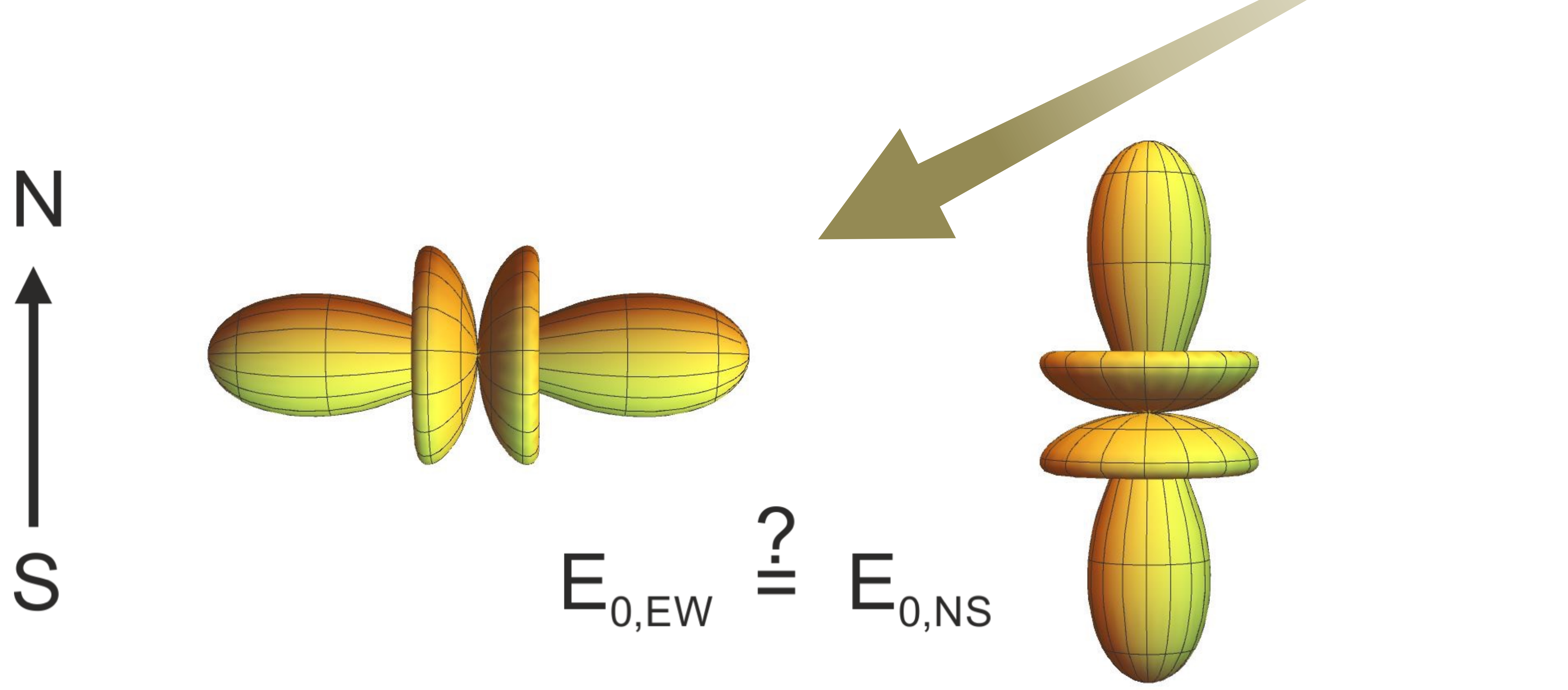
a. Laser-cooled Coulomb crystal of fluorescing $^{9}Be^{+}$ ions in the Paul trap
b. Single Ar^{+13} ion is injected, sympathetically cooled, and co-crystallized with $^{9}Be^{+}$
c. Excess $^{9}Be^{+}$ ions removed by modulating the Paul trap radio-frequency potential
d. Ar^{+13} – $^{9}Be^{+}$ two-ion crystal prepared



[Science 347 (6227), 1233-1236 (2015), Nature 578 (7793), 60-65 (2020)]

Test of Lorentz symmetry

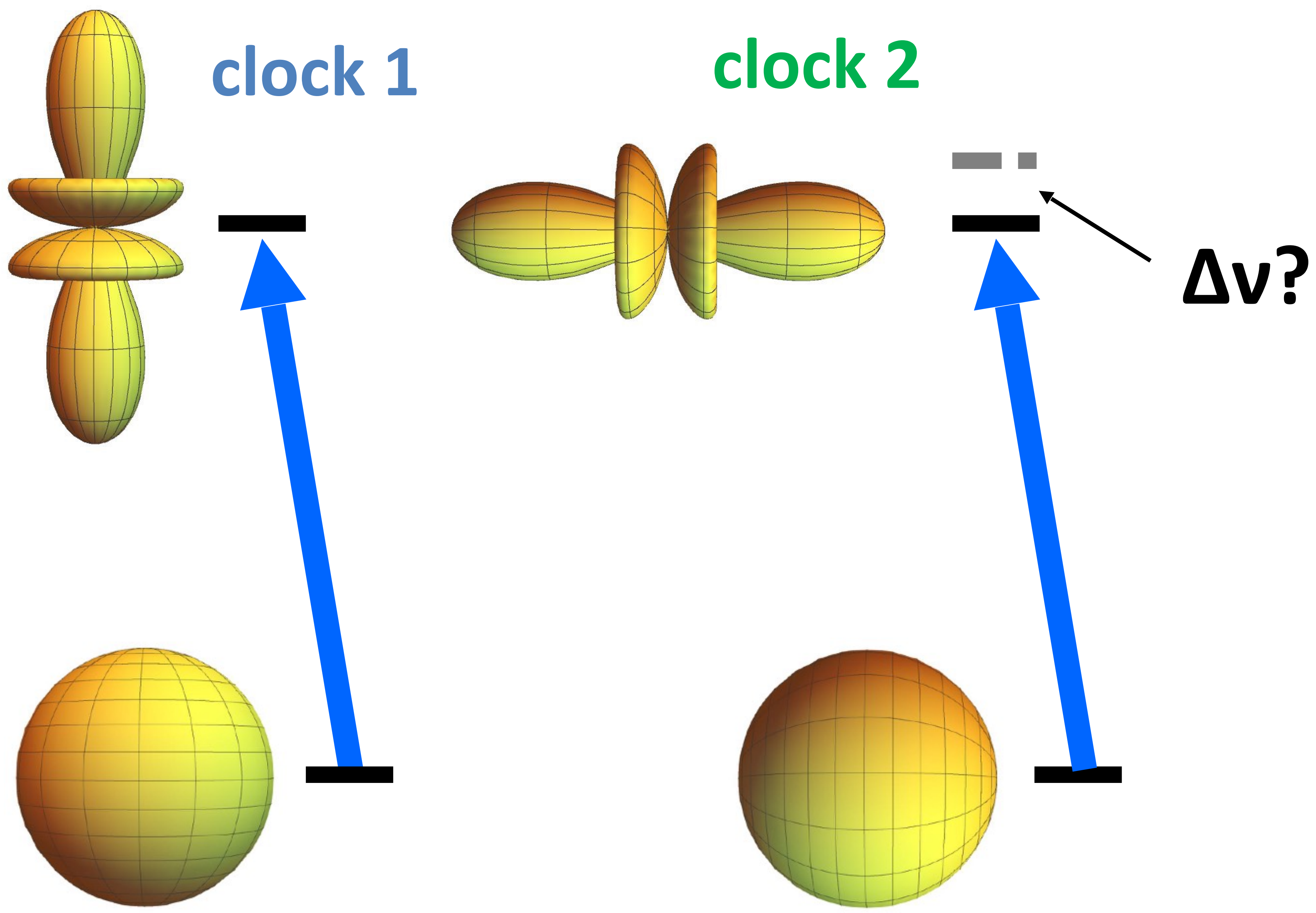
Is the electron's dispersion relation isotropic in space?



There is a large variety of electron containers...

$$^{171}\mathrm{Yb}^+, 4f^{13}6\mathrm{s}^2, {}^2\mathrm{F}_{7/2}, F=3, m_F=0$$

Measure the energy relative to an isotropic state.



$$^{171}\mathrm{Yb}^+, 4f^{14}6\mathrm{s}^1, {}^2\mathrm{S}_{1/2}, F=3, m_F=0$$

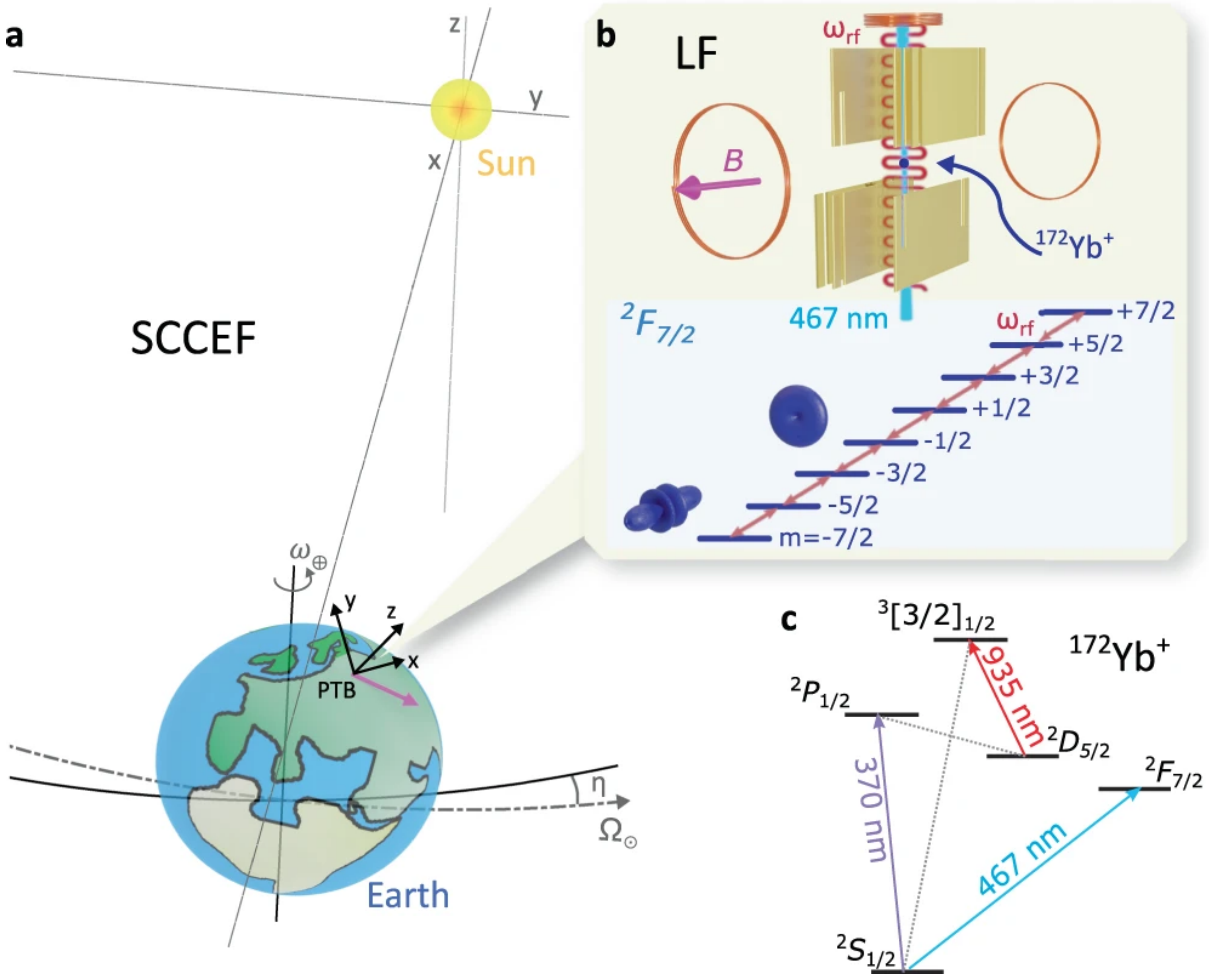
Lorentz violation → frequency offset

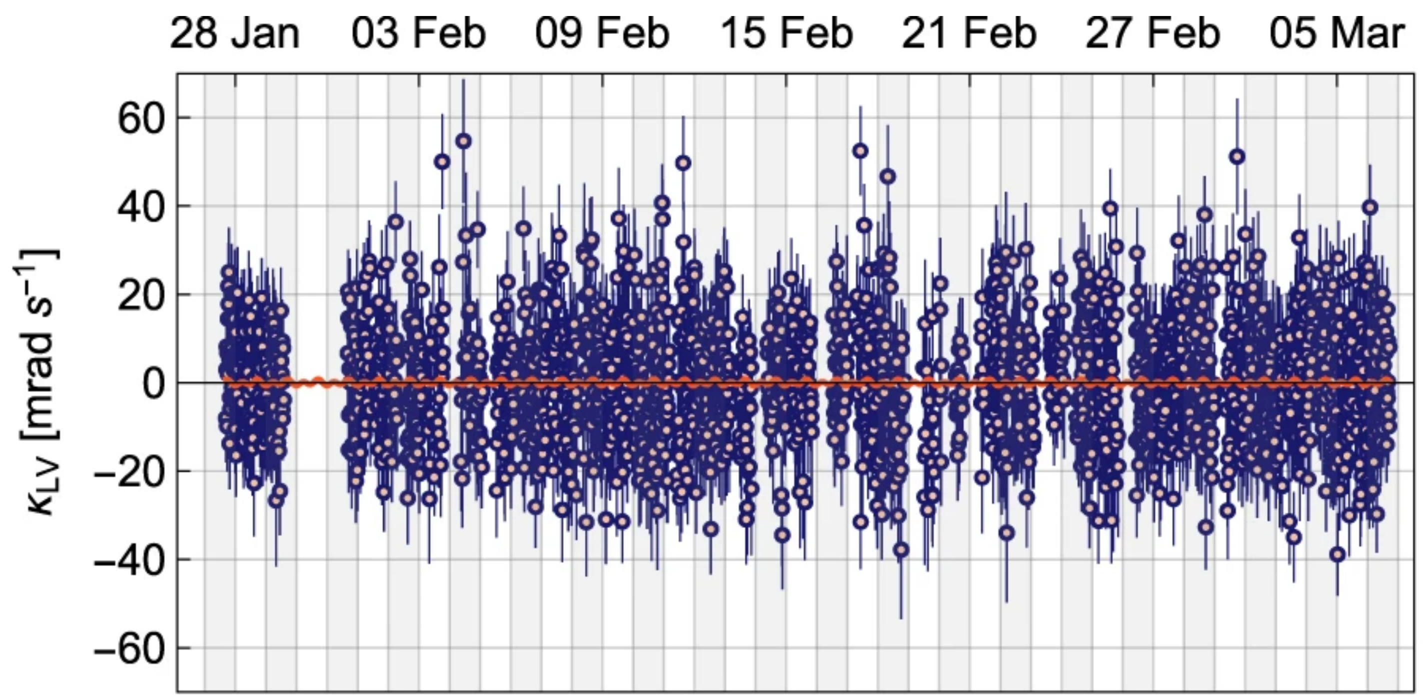
Ch. Sanner *et al*, Nature **567**, 204 (2019)

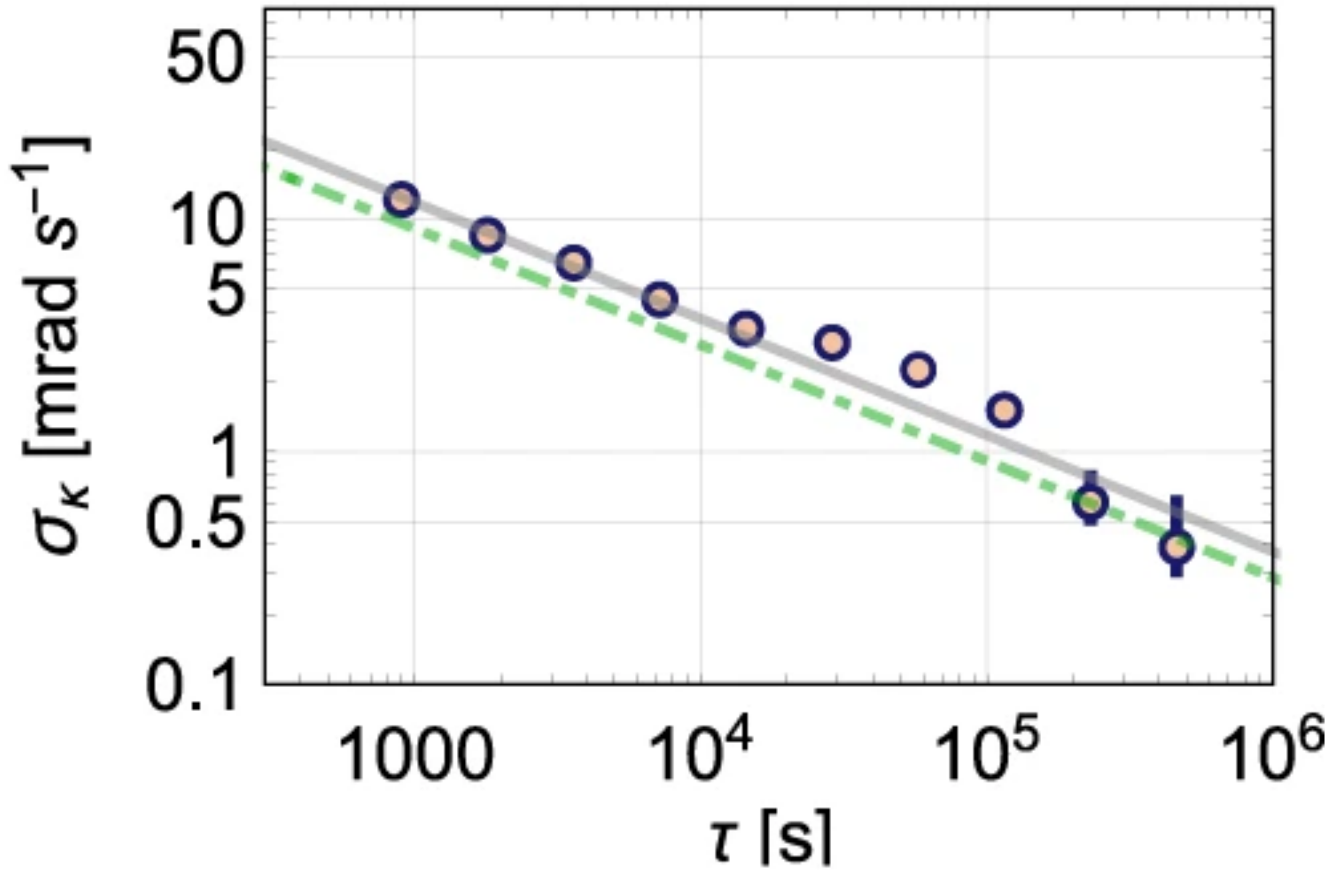
[courtesy: N. Huntemann]

Lorentz Invariance: Recent Results

- Lorentz Invariance: outcome of experiment does not depend on the velocity or orientation of inertial frame
- Can move your clock around, but it is sensitive to *everything*
- Typical experiments pick a clock transition that is sensitive, fix a magnetic field, and look for variations as the earth rotates around the sun



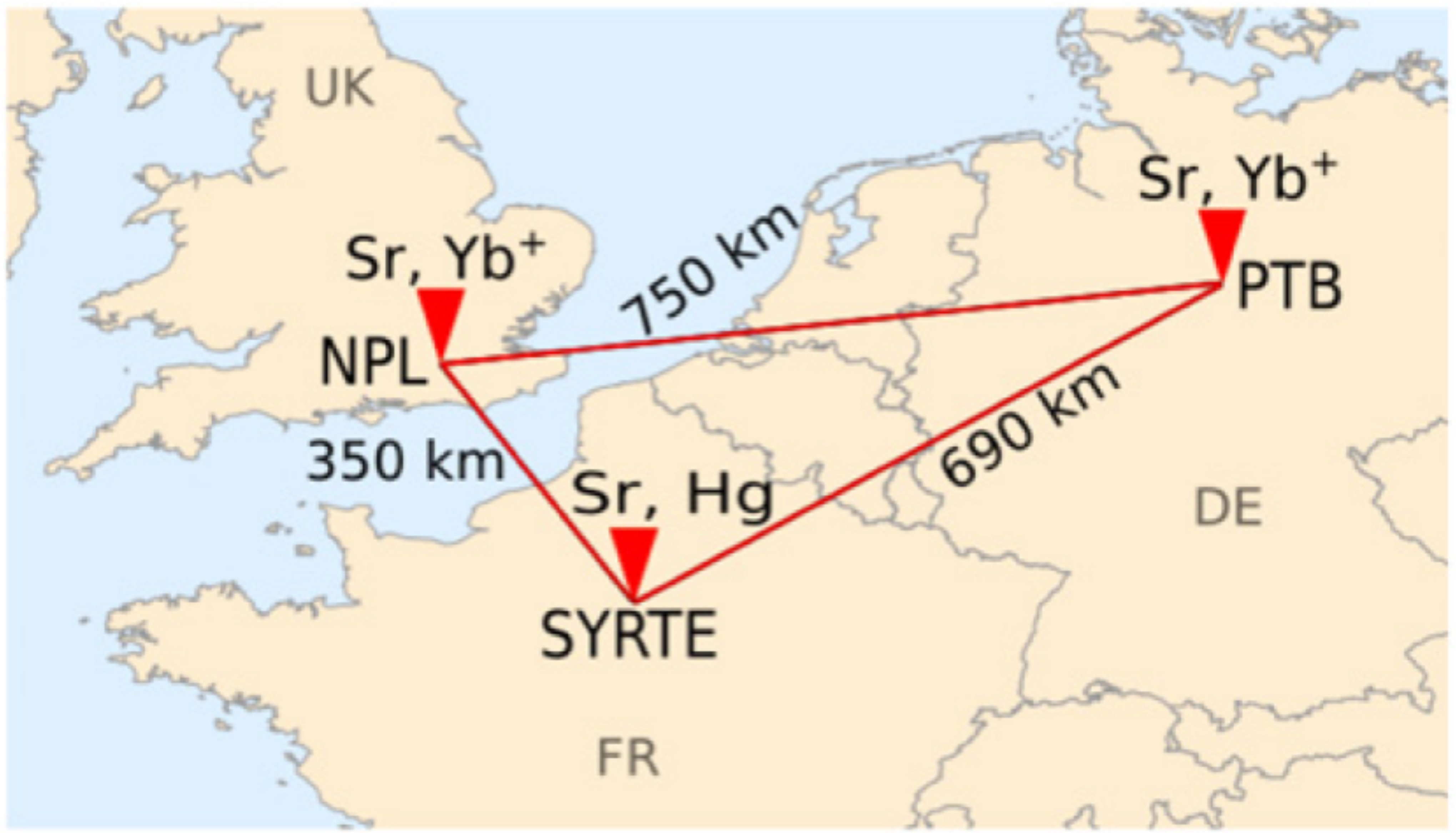


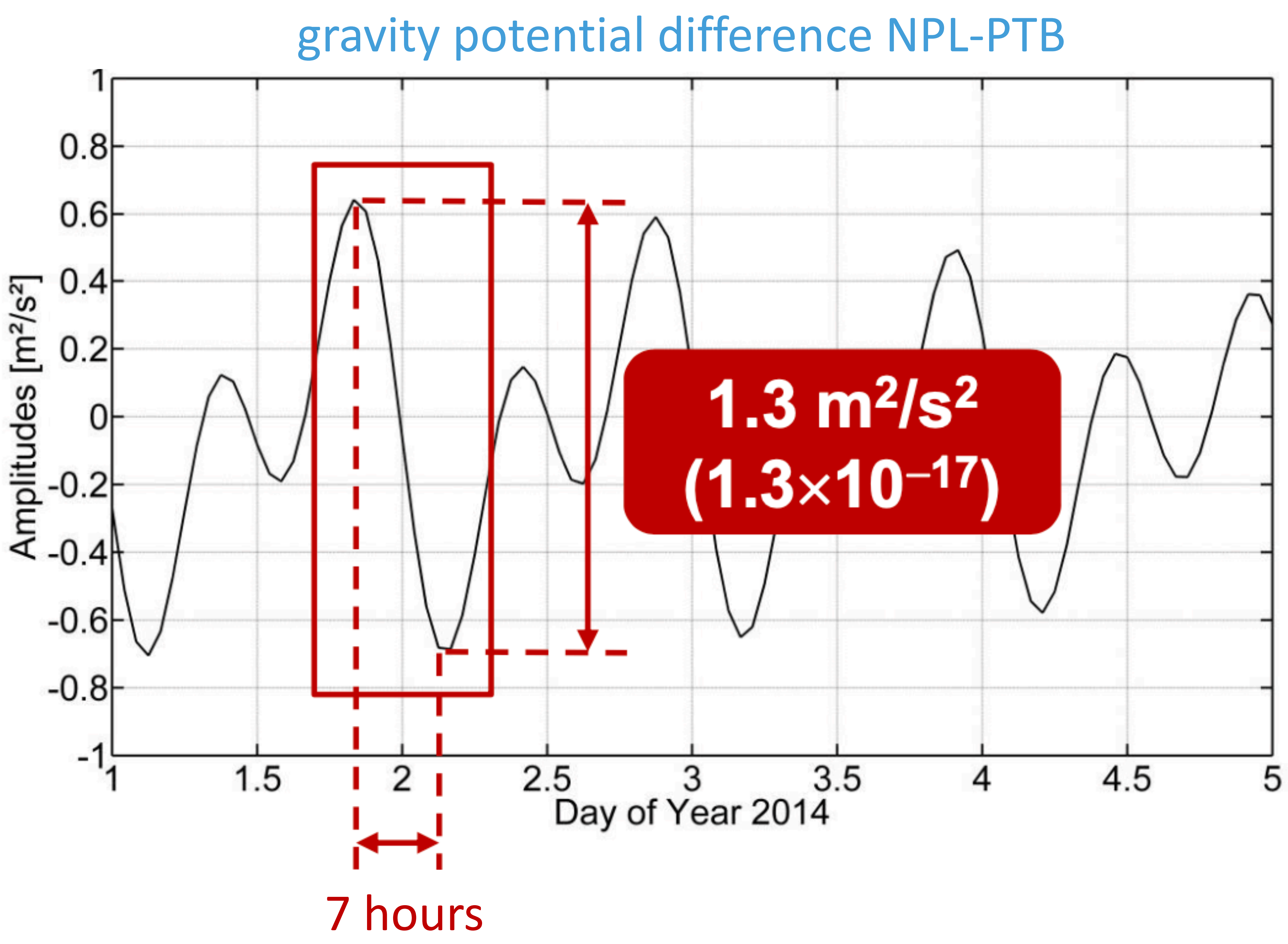


[Dreissen et al., Nature Communications 13, 7314 (2022)]

Clock Networks and Fast Transients

- NPL-SYRTE-PTB demonstrated an optical clock network with dark fibres
- Comparing clocks with different sensitivities to variations of α
- Clock-clock comparisons over optical fibres features excellent long-term stability
- Previously unconstrained parameter space for quadratic coupling

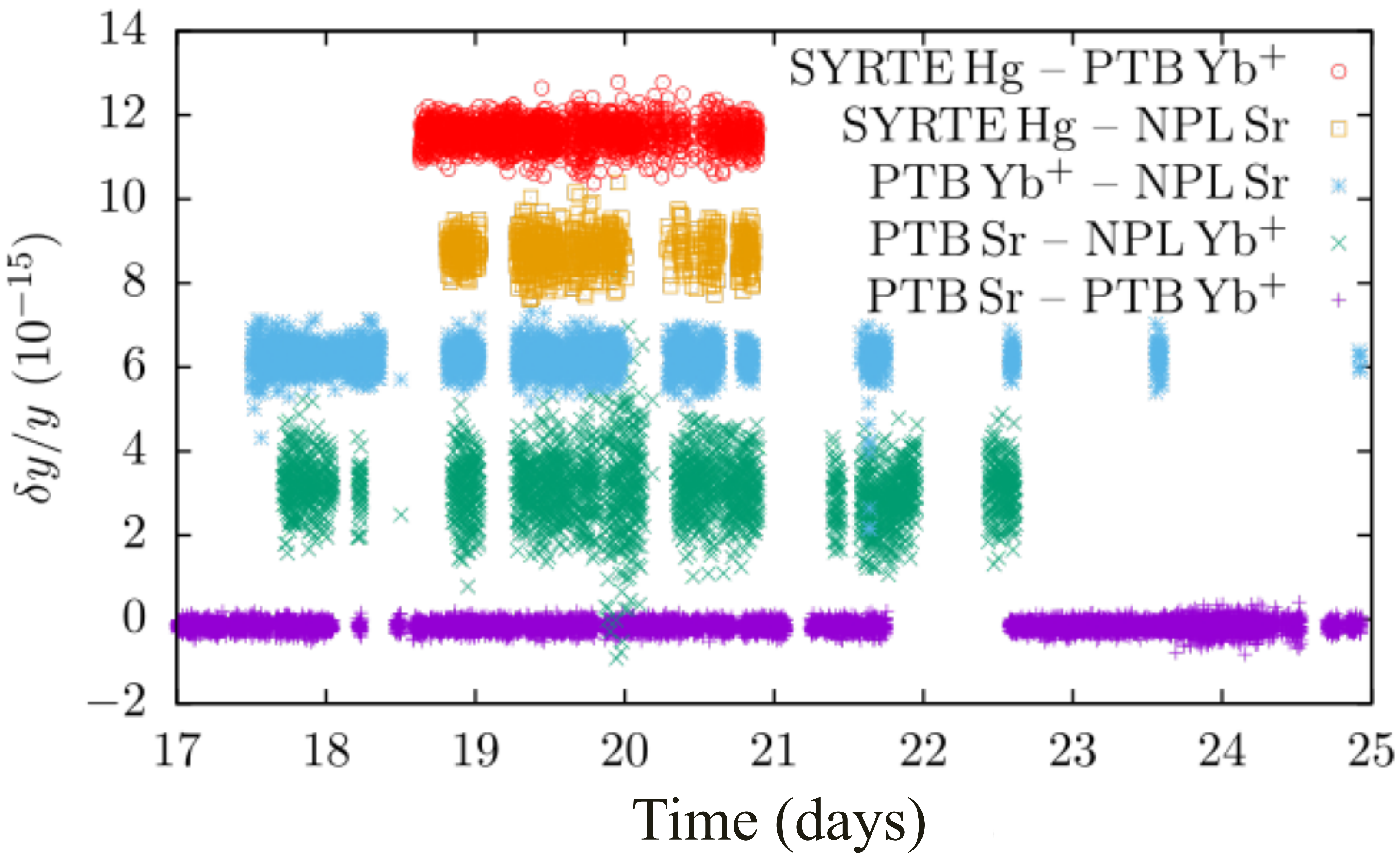


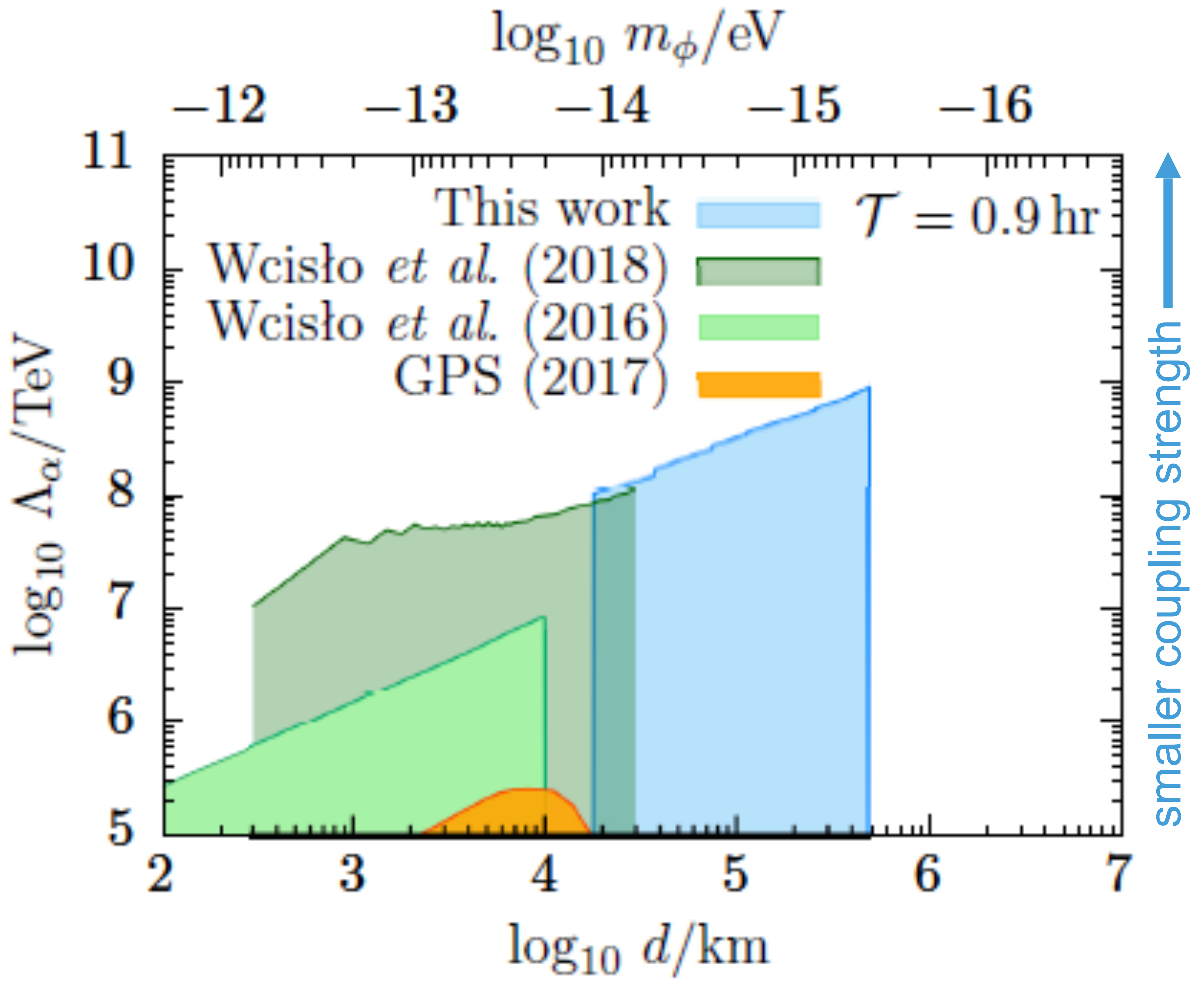


[Roberts et al, New J. Phys. 22, 093010 (2020); P. Delva et al., Phys. Rev. Lett., 118 221102 (2017)]

International Clock Comparison Data

- Constraints on energy scale, Λ_α of dark matter interactions
- Results for T = 0.9, 12, 45 hours
- Collaboration between PTB, SYRTE and NPL

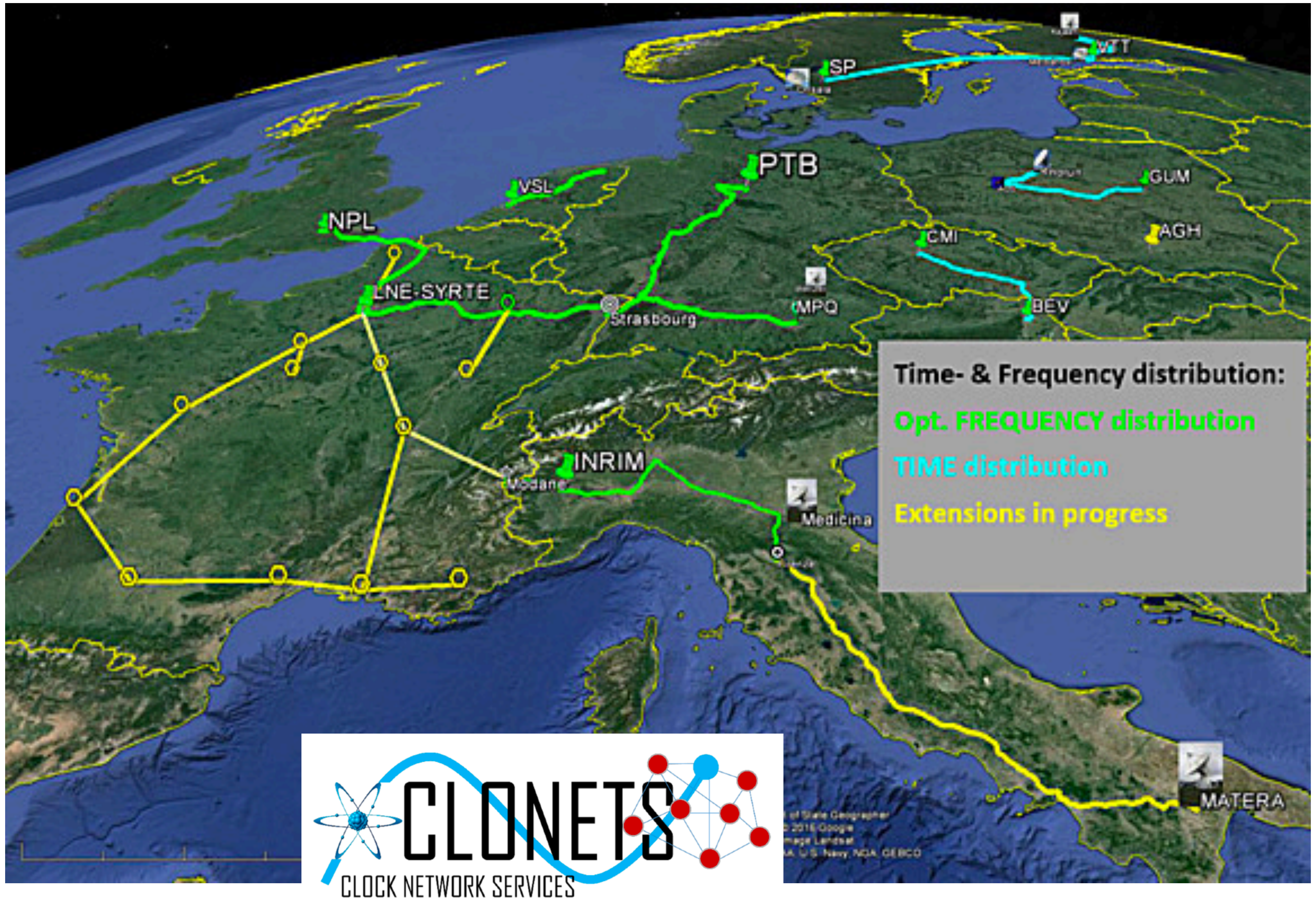




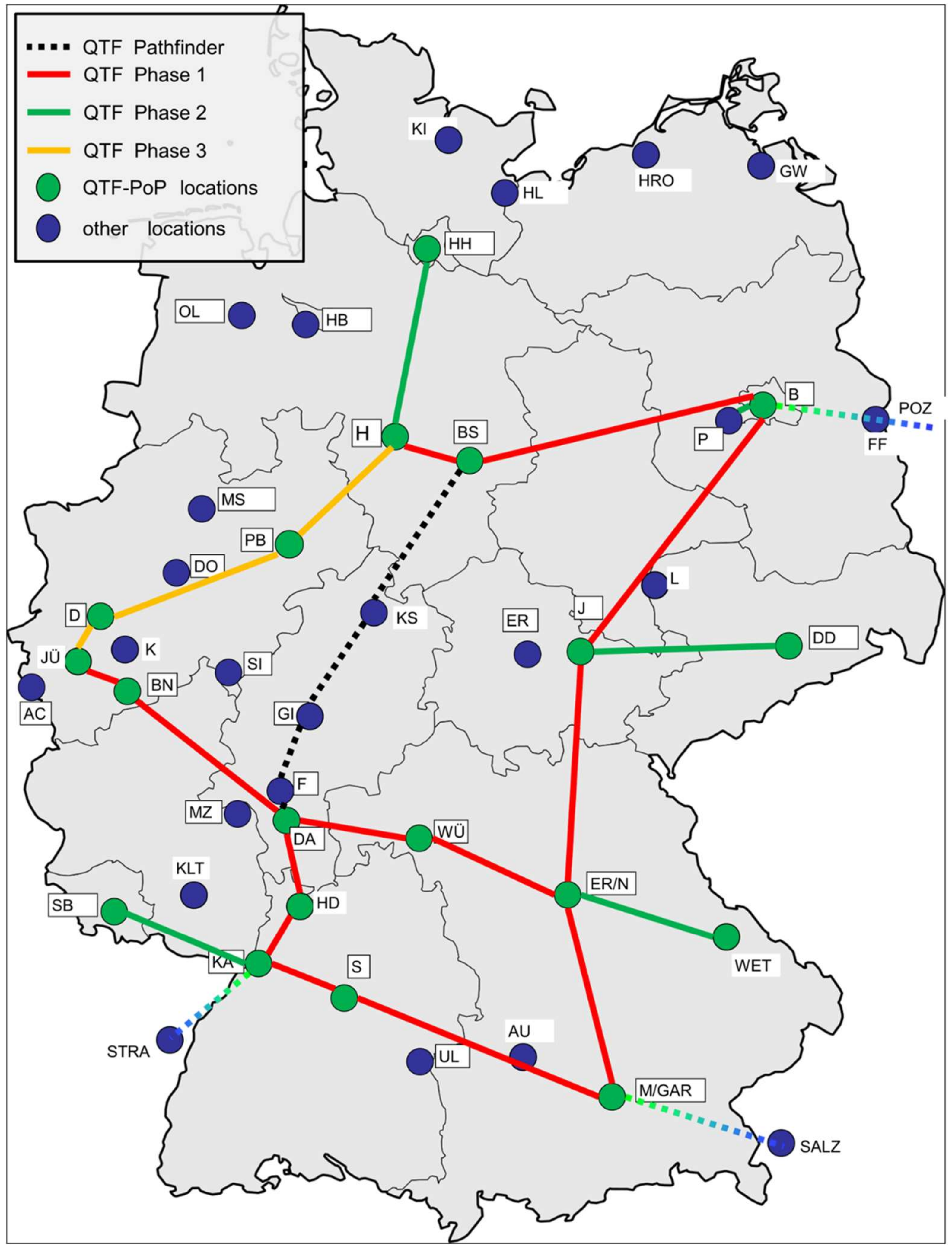
[B.M. Roberts *et al.* arXiv:1907.02661]

Optical Fibre Networks

- CLONETS: Clock Network Services
- EU Programme from 2017-2019
- http://www.clonets.eu



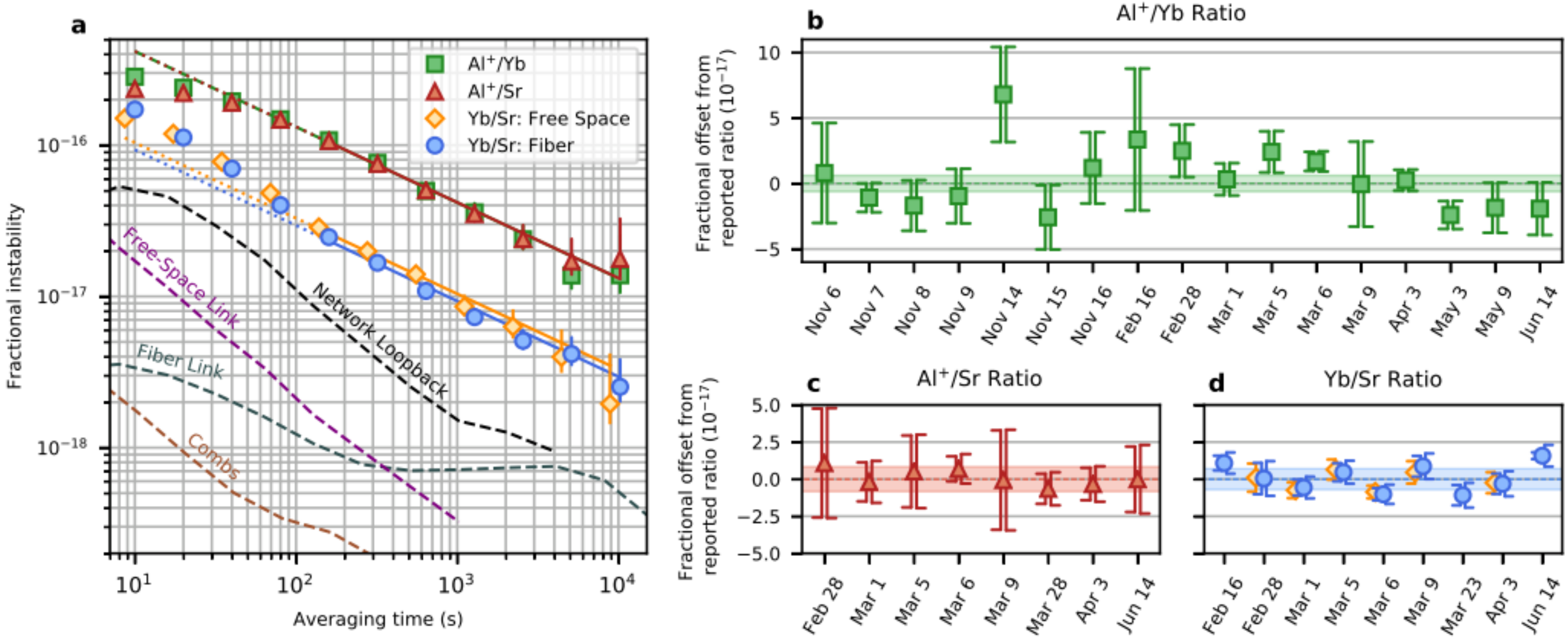
- German Dark Fibre planning:



Boulder Atomic Clock Optical Network

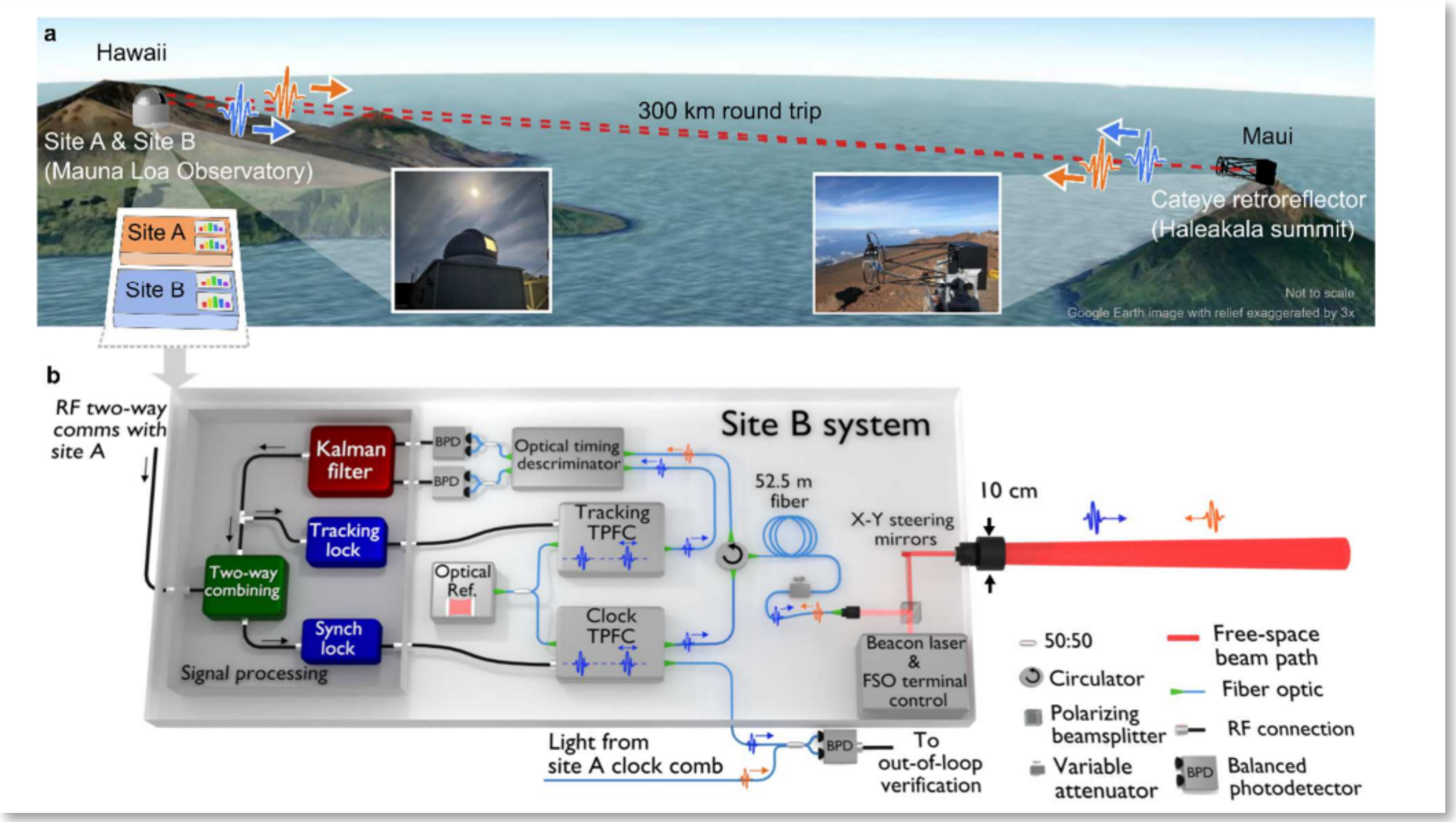
Beloy *et al.*, Nature 591, 564 (2021)

BACON Measurement Campaign

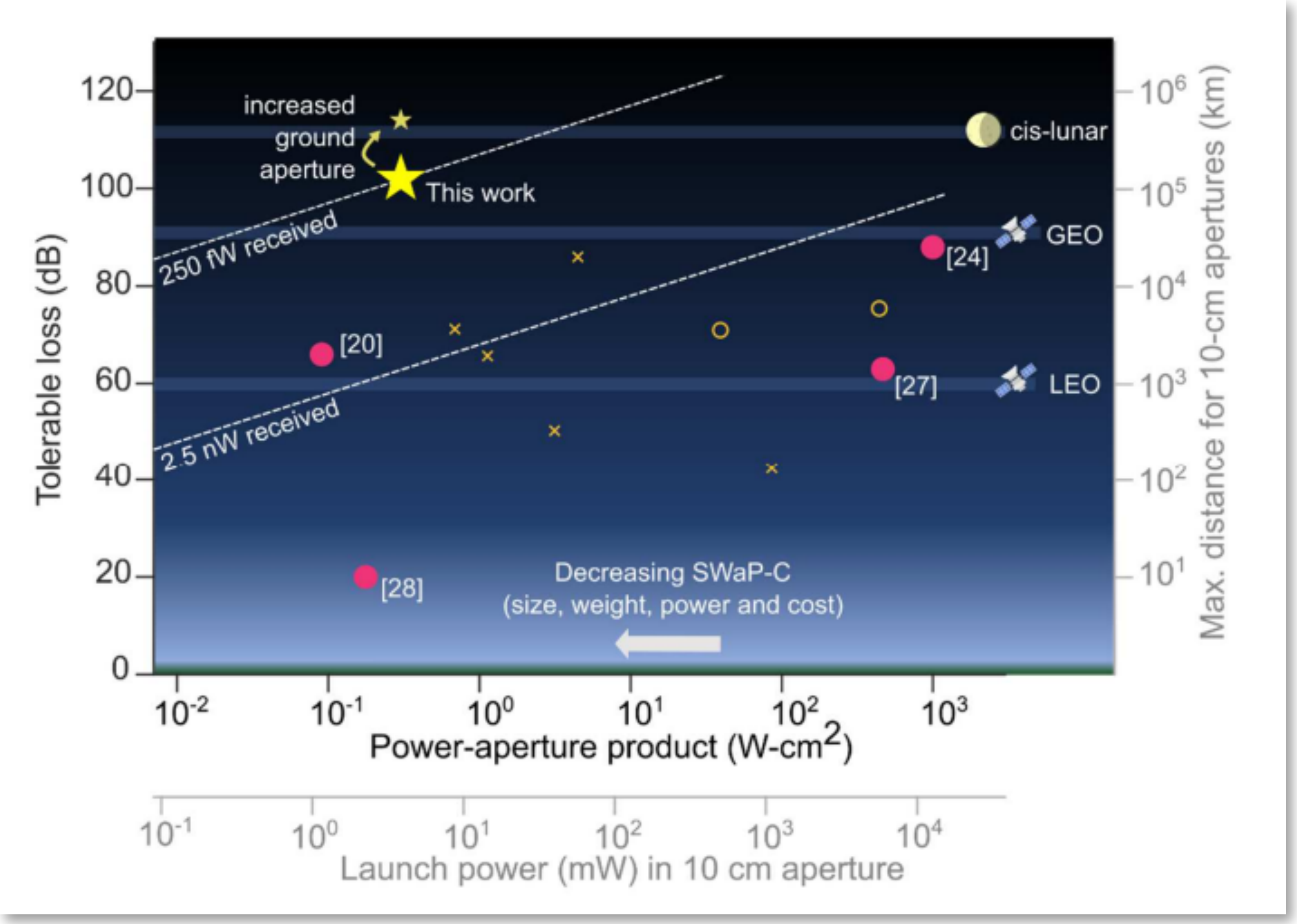


Beloy *et al.*, Nature 591, 564 (2021)

Longer Distance Networks?



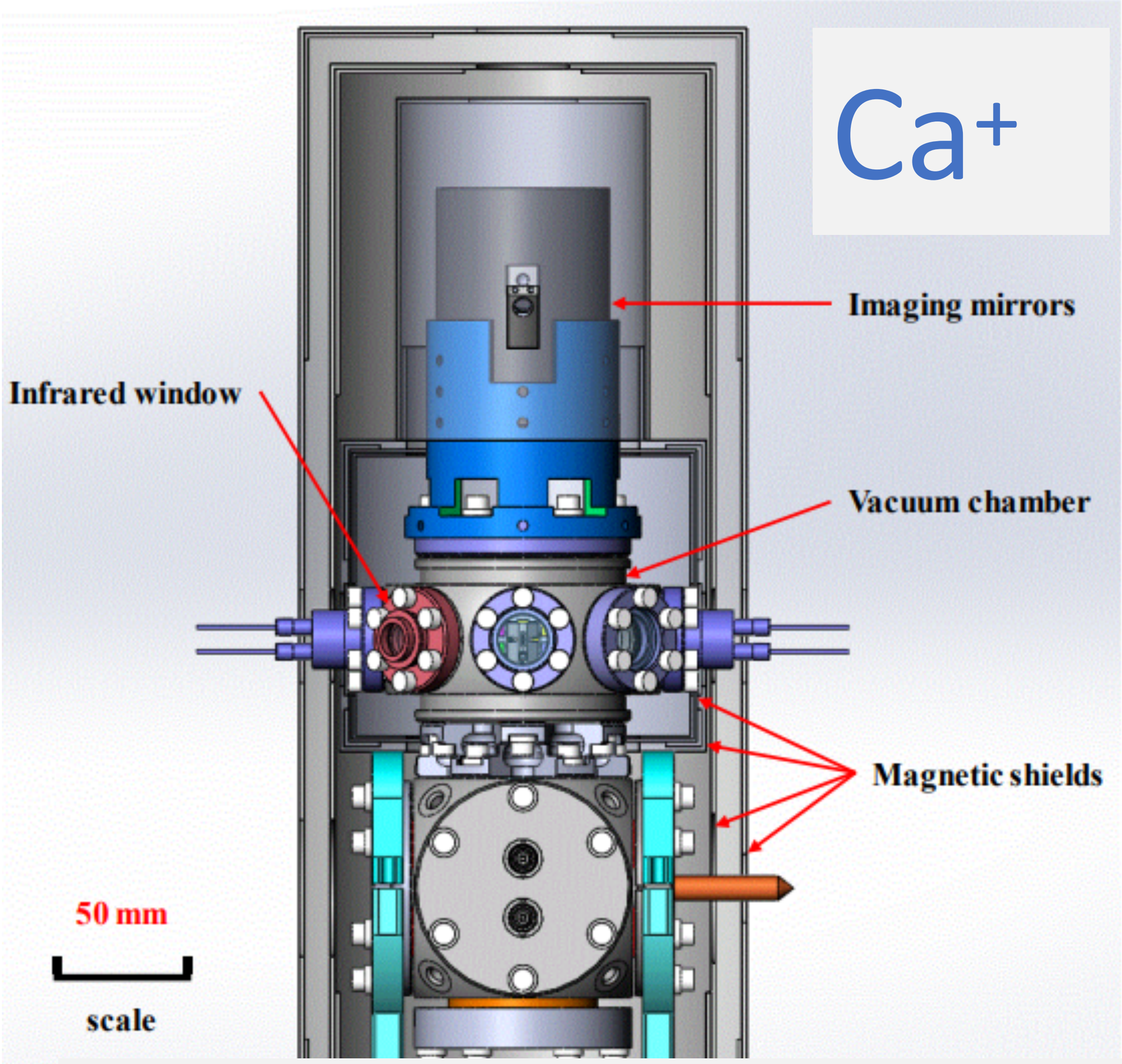
Quantum-limited performance over 300 km



Space is next!

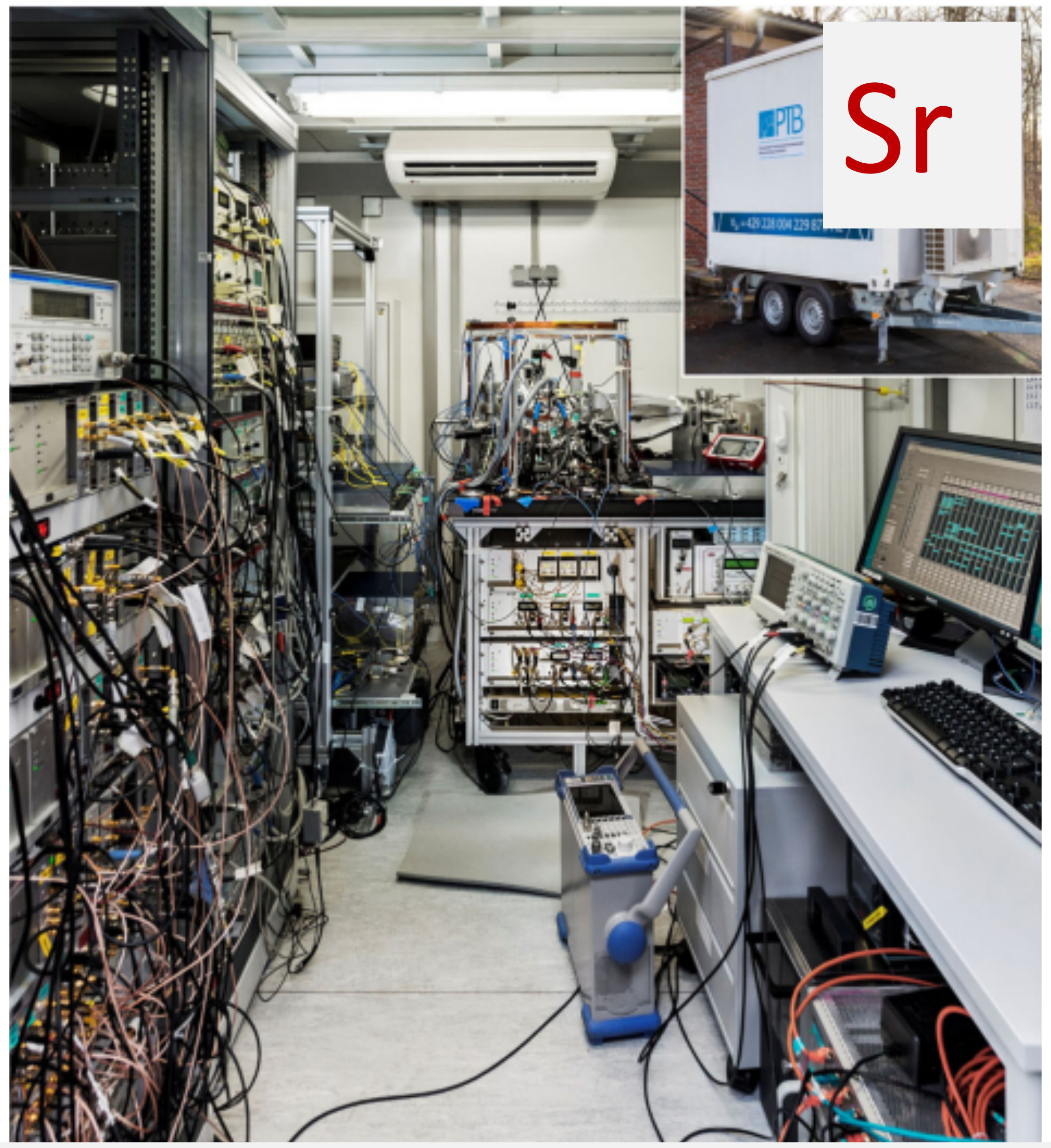
Caldwell et al., arXiv:2212.12541 (2022); Q. Shen et al. Nature 610, 661 (2022)

Transportable Clocks

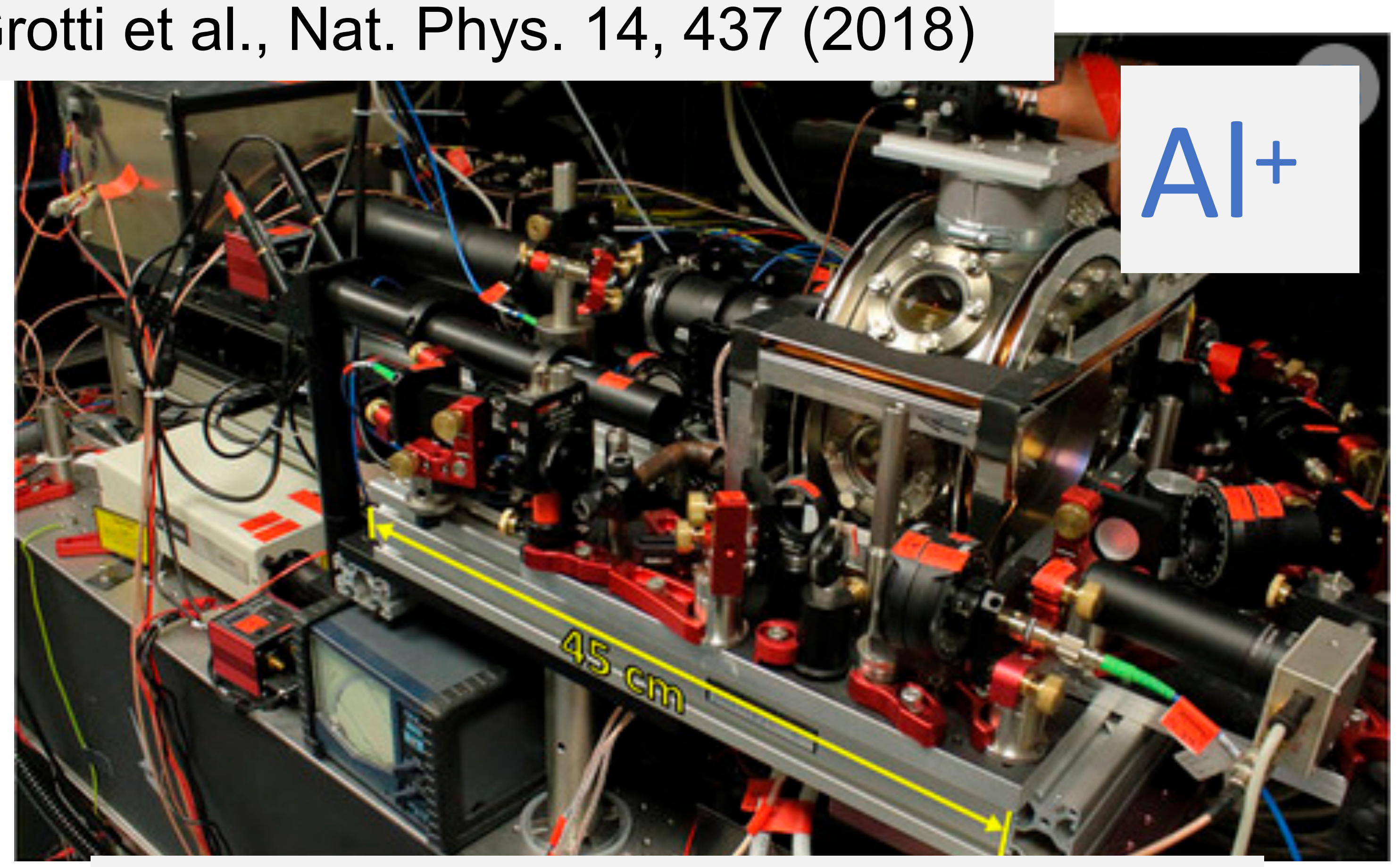


Zeng et al., arXiv:2303.07566 (2023)

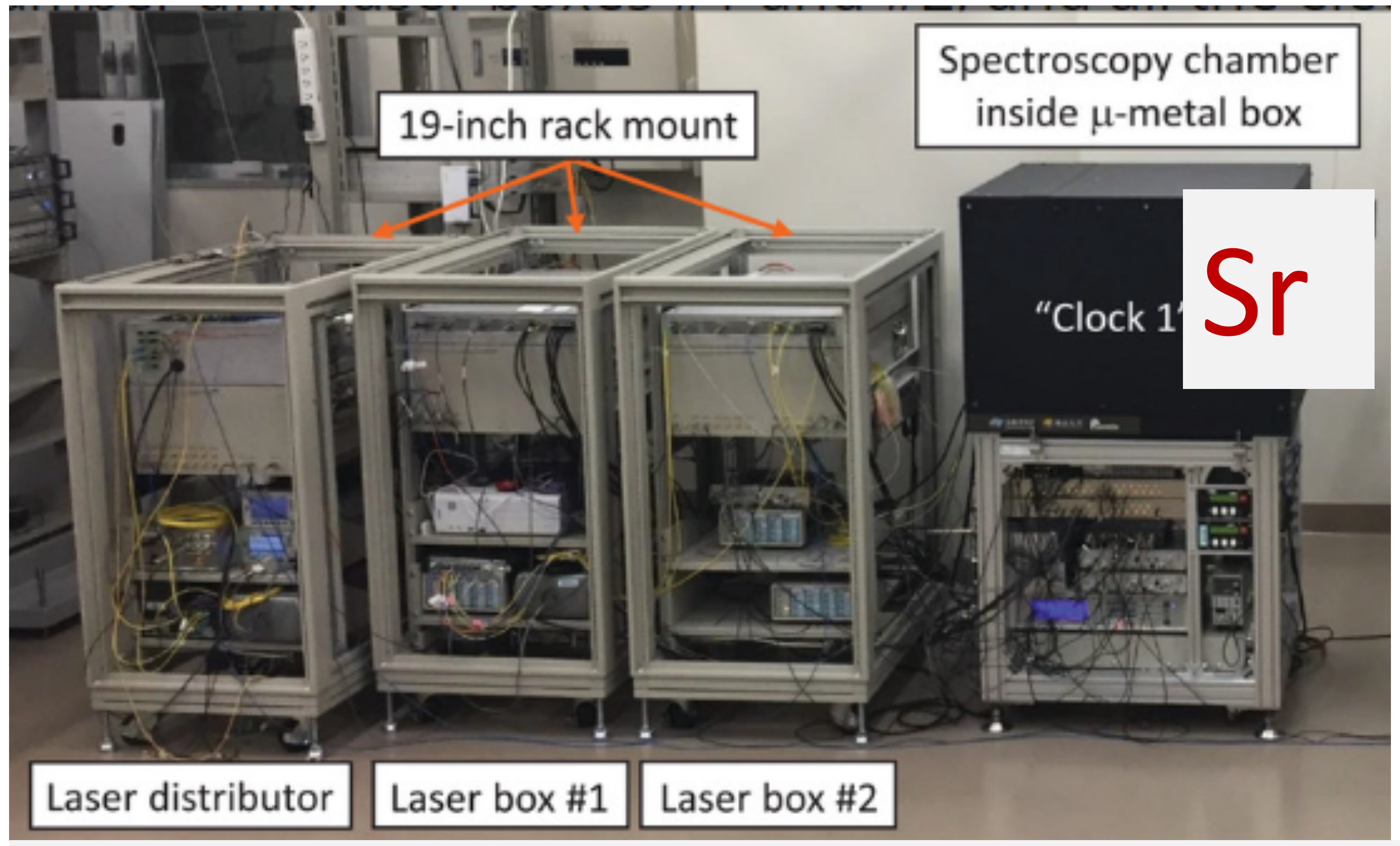
NIST Transportable Yb-lattice clock
Fasano PhD thesis (2021)



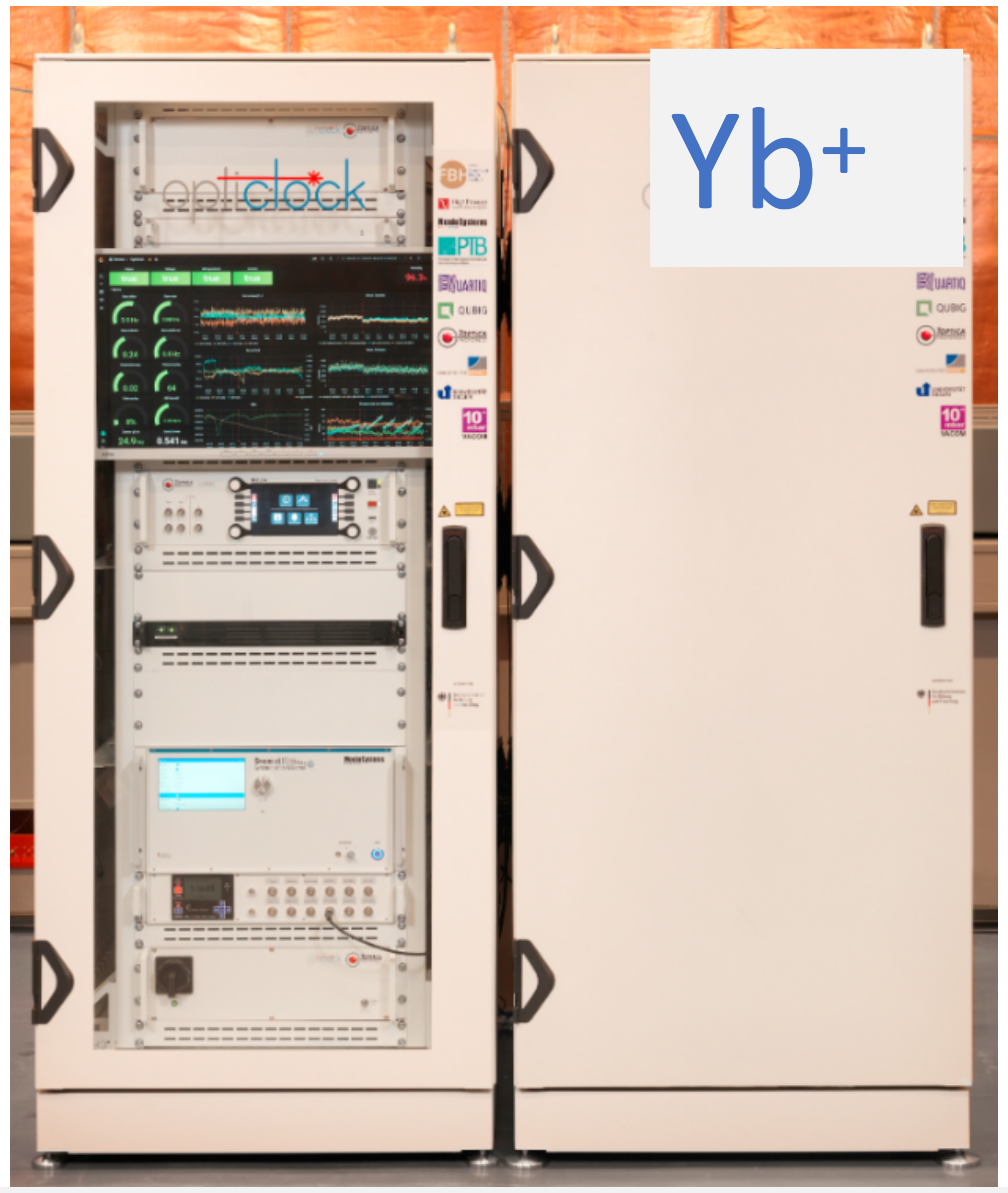
Grotti et al., Nat. Phys. 14, 437 (2018)



Hannig et al., RSI 90, 053204 (2019)



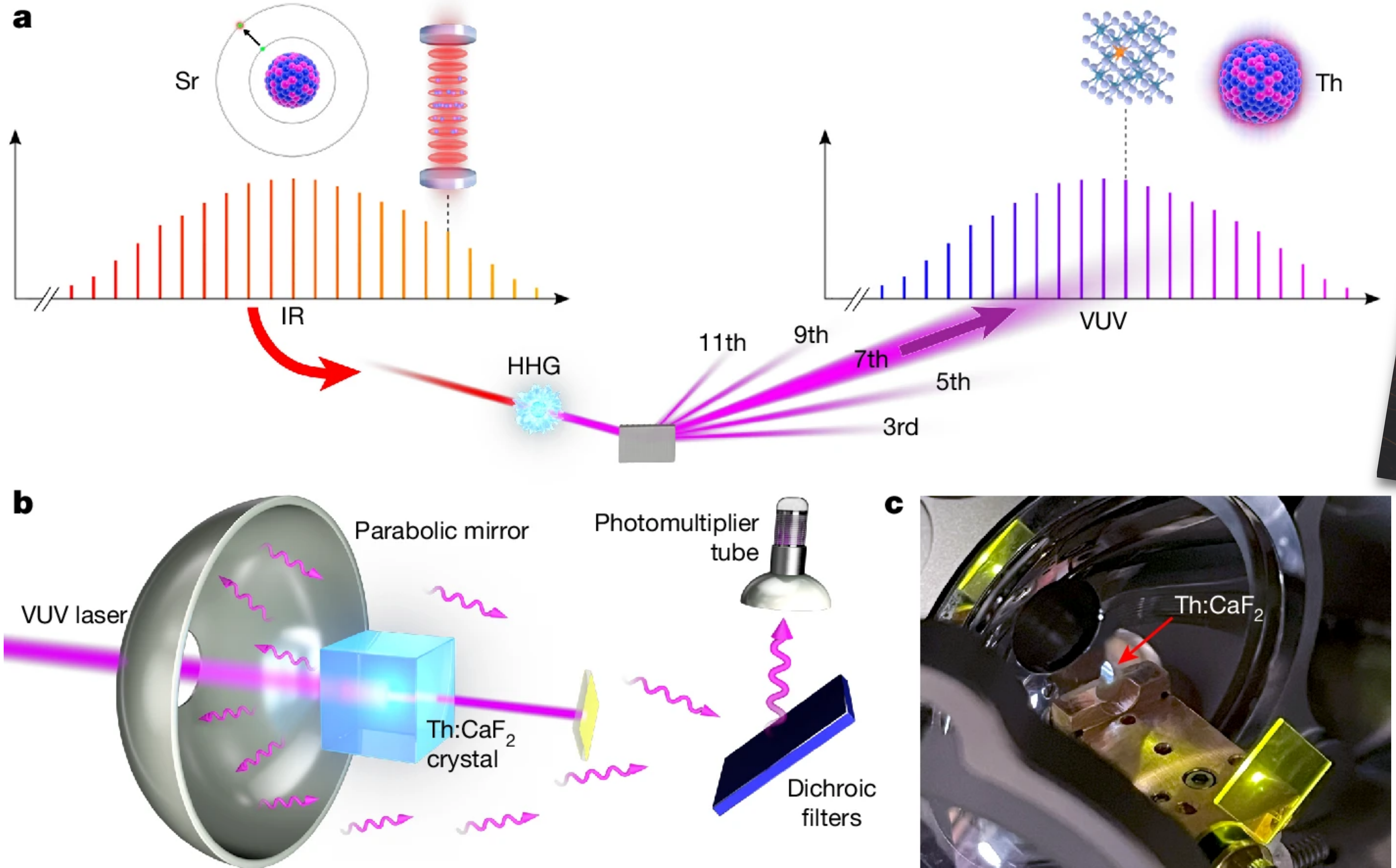
Ohmae et al., Adv. Quant. Tech. 4, 2100015 (2021)



Stuhler et al., Measurement: Sensors 18, 100264 (2021)

[D. Hume]

^{229m}Thorium Clock



Nuclear clocks are here!

C. Zhang et al., Nature 633, 63 (2024)

Conclusions

- Optical Atomic Clocks are some of the most sensitive instruments available
- Fantastic testbed for all sorts of new physics, for example ultra-light dark matter
- Lots of new ideas to follow and new innovations… new clocks, networks, and opportunities!