Quantum Sensing autumn school

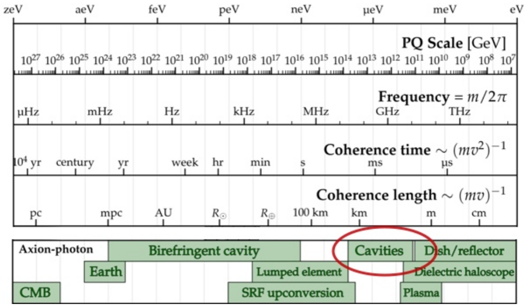
4 Nov 2024, 09:00 → 8 Nov 2024, 14:00 Europe/Zurich

CERN

Superconducting circuits in particle physics

Caterina Braggio
University of Padova and INFN

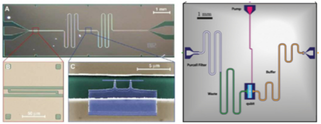
which particle physics: motivation for SC circuits/quantum sensors



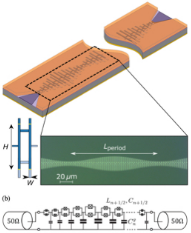
- below 1 eV $\Longleftrightarrow$ wave-like DM
- resonant cavities (μeV - .1 meV): this is most sensitive method, QCD axions can be probed
- broadband haloscopes ($\gtrsim$ .1 meV)

which SC circuits are of interest in this search

- → parametric amplifiers
- → transmons
- → photon counters



Phys. Rev. X **10**, 021038 (2020)



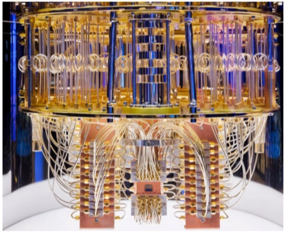
PRX 10, 021021 (2020)

building block: the Josephson Junction

knowledge, skills

- ⊙ circuit QED (starting from cavity QED)
- ⊙ related hamiltonians
- X circuit design
- X nanofabrication
- ⊙⊙ testing and use in particle physics experiments
- ⊙ dilution refrigerators

which SC circuits are of interest in this search



same as QC, but with a smaller number of RF lines...

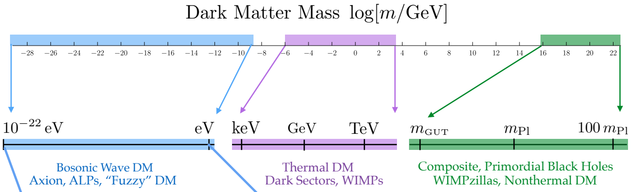
knowledge, skills

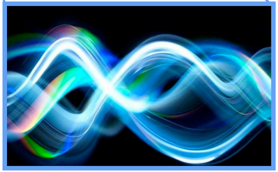
- $\odot$ circuit QED (starting from cavity QED)
- $\odot$ related hamiltonians
- X circuit design
- X nanofabrication
- $\odot\odot$ testing and use in particle physics experiments
- $\odot$ dilution refrigerators

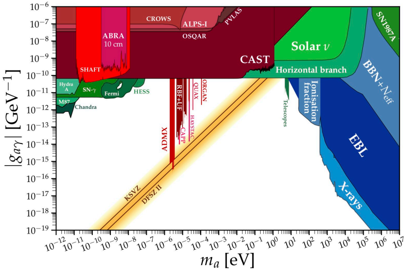
$$^{4}\text{He cryostats} \rightarrow \sim 4.2 - 1.3\,\text{K}$$
$$^{3}\text{He cryostats} \rightarrow \sim 1.3 - 0.4\,\text{K}$$
$$^{3}\text{He-}^{4}\text{He mixture} \rightarrow \sim 0.4\,\text{K to mK}$$

F. Pobell "*Matter and methods at low temperature*"

“wave-like” DM



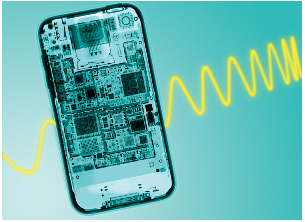


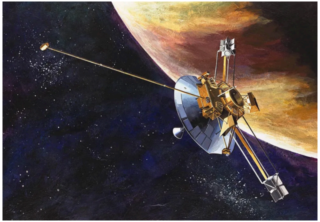


$m \lesssim 10\,\mathrm{eV}$

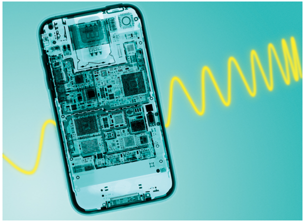
classical field oscillating at the Compton frequency 10^{-6} coherence

to what extent is this detectable with current technology?

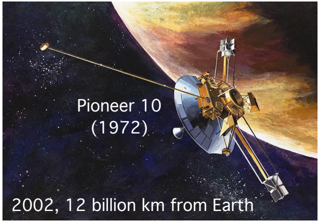




kW

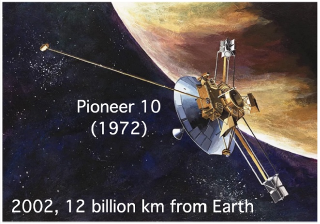


(0.1-2) W



2.5×10^{-21} W

quantum microwaves in DARK MATTER search

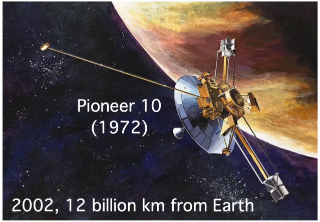


2.5×10^{-21} W

$< 10^{-23}$ W

Unknown frequency (particle mass)

quantum microwaves in DARK MATTER search



2.5×10^{-21} W

$< 10^{-23}$ W

Unknown frequency (particle mass)

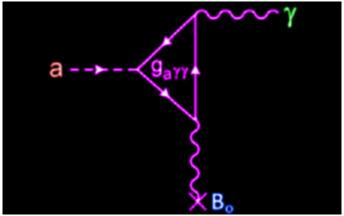
< few photons/s ⇒ QUANTUM 2.0

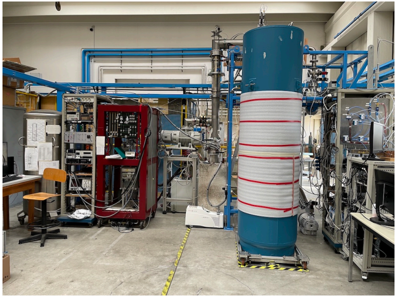
1. **3D** microwave **resonator** for resonant amplification
   -think of an HO driven by an external force-

2. with **tunable frequency** to match the axion mass

3. the **resonator** is within the bore of a **SC magnet** → $\mathbf{B}_0$
   multi-tesla field

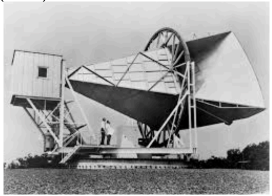
4. it is readout with a **low noise receiver**
   delfridge operation at mK temperatures



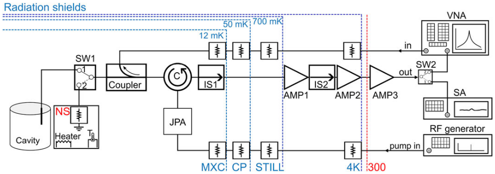


Microwave receivers

(1965) Penzias and Wilson



Green Bank telescope



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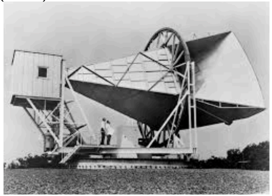

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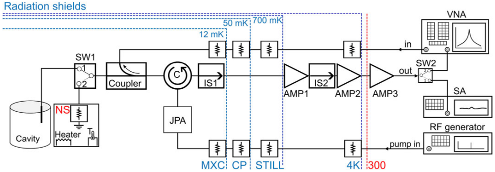
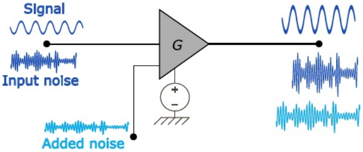


Amplifiers introduce noise

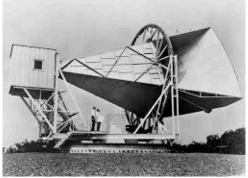


- → Johnson noise $N = kTB$
- → quantum noise (fundamental limit)

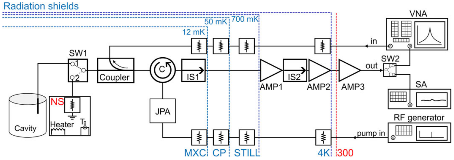
measured in Watt or number of photons

Microwave receivers

(1965) Penzias and Wilson

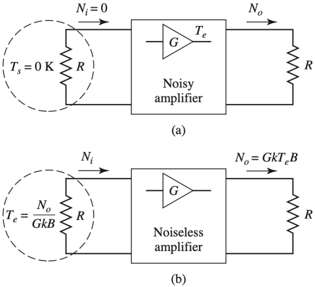


Green Bank telescope



see ch10, Pozar “Microwave engineering”

the **noise introduced by an amplifier** is quantified by its noise temperature T_e

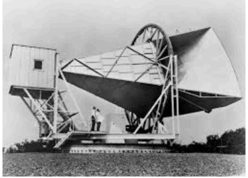


$$T_e = \frac{N_o}{GkB}$$

the same load noise power is obtained by driving an **ideal noiseless amplifier** with a resistor at the temperature T_e

Microwave receivers

(1965) Penzias and Wilson



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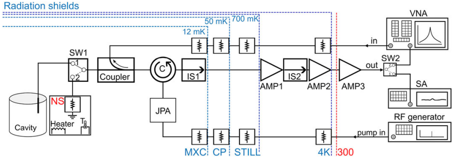
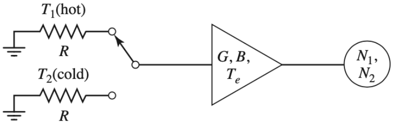


assessing receiver's noise



$$N_1 = GkT_1B + GkT_eB$$

$$N_2 = GkT_2B + GkT_eB$$

$$Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1 \qquad T_e = \frac{T_1 - YT_2}{Y - 1}$$

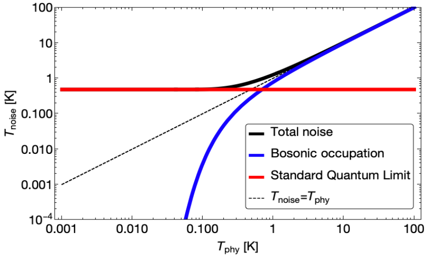
cascaded system

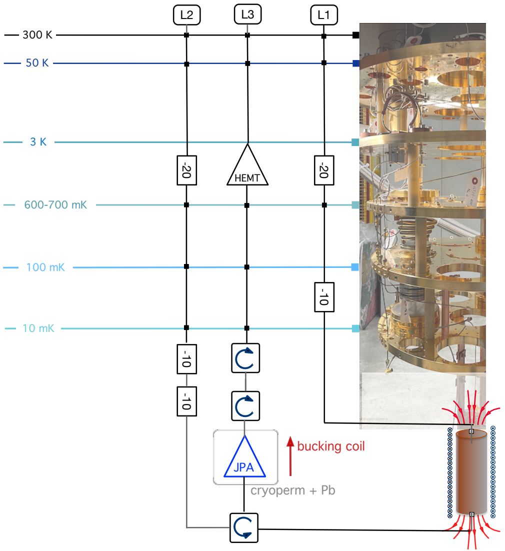
$$T_{\text{cas}} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdots$$

quantum-limited readout

$$k_B T_{sys} = h\nu \left(\frac{1}{e^{h\nu/k_B T} - 1} + \frac{1}{2} + N_a \right), \; N_a \geqslant 0.5$$

$T_{sys} = T_c + T_a$
T_c cavity physical temperature
T_a effective noise temperature of the amplifier



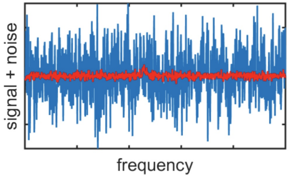


a poor S/N ratio

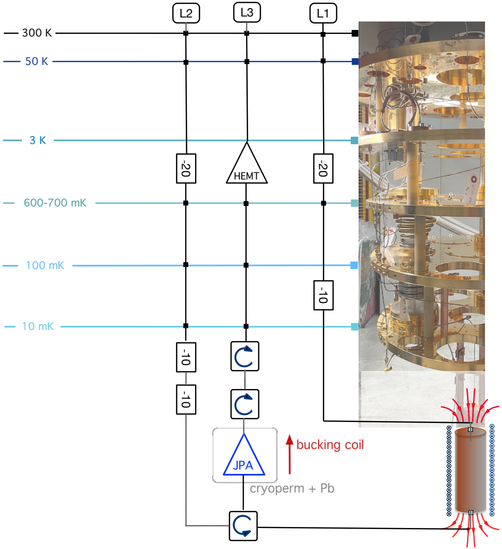
In these searches, **the signal is much smaller than noise**

$$P_n = k_B T \Delta\nu \gg P_s \;\; \propto \; B^2\, V_{\mathrm{eff}}\, Q_L \sim 10^{-23}\,\mathrm{W}$$

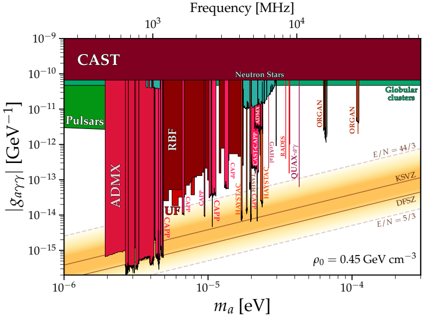
To increase sensitivity we rely on **averaging several spectra** recorded at the same cavity frequency **over a certain integration time**.



$$P_n = k_B T \sqrt{\frac{b}{t_m}}$$



Heavier (axions) & Harder (life)



- heavier axions are well motivated,
  BUT
  the *scan rate df/dt scales unfavourably with f*

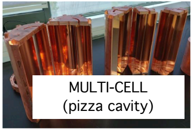
$$\frac{df}{dt} \propto \frac{g_{a\gamma\gamma}^4 B^4 V_{\rm eff}^2 Q_L}{T_{sys}^2} \propto f^{-4}$$

(asm. quantum noise, SC cavities, relax r/L)

- $(df/dt)_{DFSZ} \sim$ **50** $(df/dt)_{KSVZ}$

DIELECTRIC CAVITY
(higher order modes)

THIN SHELL



→ new cavities with larger $V_{\rm eff}$ compared to a pill-box cavity

→ QIS technologies and methods to **reduce the noise** (parametric amplifiers, photon counters)

photon counting vs parametric amplification at standard quantum limit (**SQL**)

IDEAL PHOTON DETECTOR

$$\frac{R_{\text{counter}}}{R_{\text{SQL}}} \approx \frac{Q_L}{Q_a} e^{\frac{h\nu}{k_B T}}$$

Ex. at 7 GHz, 40 mK $\rightarrow$ gain by 10^3

S. K. Lamoreaux *et al.*, Phys Rev D **88** 035020 (2013)

REAL DETECTOR WITH DARK COUNTS Γ_{dc}

$$\frac{R_{\text{counter}}}{R_{\text{SQL}}} \approx \eta^2 \frac{\Delta\nu_a}{\Gamma_{dc}} \qquad \Gamma_{dc} \text{ dark counts}$$

η photon counter efficiency
$\Delta\nu_a$ axion linewidth

$\rightarrow$ ($\times$100s) gain [$\Gamma_{dc} \sim$ 10s count/s, $\eta^2 \sim 70\%$]

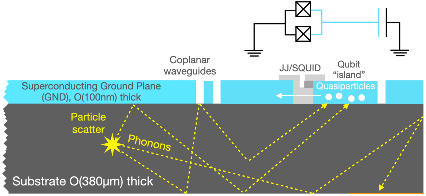
- can probe in a day the same range a linear amplifier at SQL would take more than 3 months-

https://arxiv.org/abs/2403.02321

A transmon-based SMPD needs a JPA

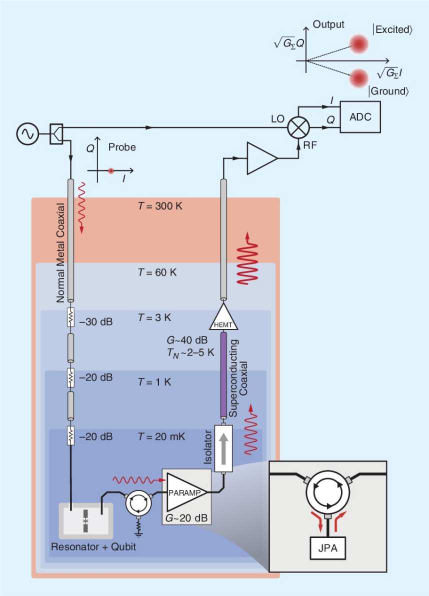
qubits as sensors

very recent proposals to detect DM using phonons excitations, phonons → qubit readout



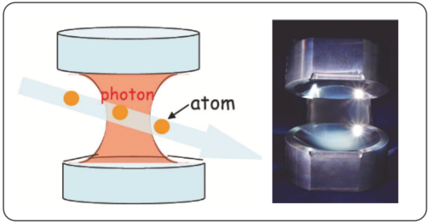
We learn about the **qubit state** by sending **low power microwave pulses** and **ultra low noise microwave amplification** in the output line is accomplished using Josephson Parametric Amplifiers

$\Longrightarrow$ **dispersive readout**

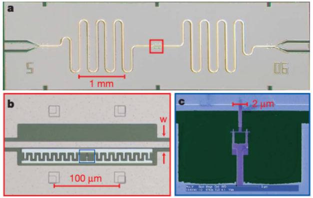


from Cavity-QED to circuit QED

studies of the interaction of a single atom with (few) **microwave photons** has been first studied **atomic cavity quantum electrodynamics experiments** and later on transposed to **circuit QED experiments**



Nature **400**, 239–242 (1999)



Nature **445**, 515–518 (2007)

In both cases **two-level atoms** interact directly with a **microwave field mode** in the cavity

from Cavity-QED to circuit QED

Can the field of a single photon have a large effect on the artificial atom?

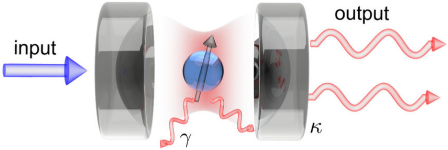
Interaction: $H = -\vec{d} \cdot \vec{E}$, $E(t) = E_0 \cos \omega_q t$

It's a matter of increasing the **coupling strength** g between the atom and the field $g = \vec{E} \cdot \vec{d}$:

$\rightarrow$ work with **large atoms**

$\rightarrow$ **confine the field** in a cavity

$$\vec{E} \propto \frac{1}{\sqrt{V}}, \; V \text{ volume}$$



κ rate of cavity photon decay
γ rate at which the qubit loses its excitation
to modes $\neq$ from the mode of interest

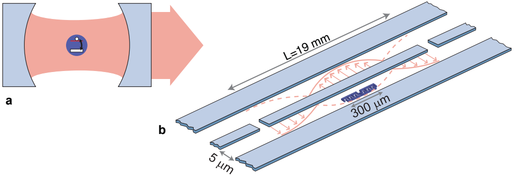
$g \gg \kappa, \gamma \iff$ regime of strong coupling
coherent exchange of a field quantum between the atom (matter) and the cavity (field)

from cavity-QED to circuit-QED

g is significantly increased compared to Rydberg atoms:

$\rightarrow$ artificial atoms are large ($\sim 300\,\mu$m)
$\Longrightarrow$ large dipole moment

$\rightarrow$ $\vec{E}$ can be tightly confined
$\vec{E} \propto \sqrt{1/\lambda^3}$
$\omega^2\lambda \approx 10^{-6}\,\text{cm}^3$ (1D) versus $\lambda^3 \approx 1\,\text{cm}^3$ (3D)
$\Longrightarrow 10^6$ larger energy density



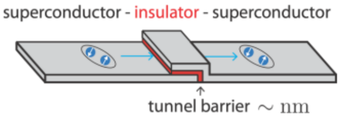
(a) $(g/2\pi)_{\text{cavity}} \sim 50\,\text{kHz}$

(b) $(g/2\pi)_{\text{circuit}} \sim 100\,\text{MHz}$ (typical)

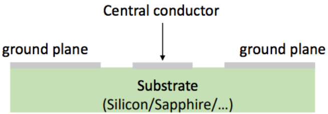
10^4 larger coupling than in atomic systems

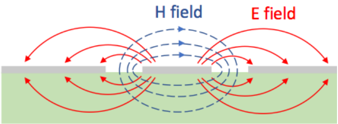
the Josephson Junction

the only circuit element that is both **dissipationless** and **nonlinear** (fundamental properties to make quantum hardware)



It's integrated in superconducting (SC) circuits, solid state electrical circuits fabricated using techniques borrowed from **conventional integrated circuits**.





Exercise: deriving the Josephson current-phase relation

Aperto: lunedì, 1 aprile 2024, 00:00
Data limite: venerdì, 12 aprile 2024, 00:00

A superconducting (SC) condensate can be described by a many-electron wave function Ψ, where $|\Psi|^2$ is the density of Cooper pairs n, and ψ is characterised by a macroscopic
θ:

$$\Psi = \sqrt{n}e^{i\theta} .$$

In a SC tunnel junction, the evanescent overlap of the two wave functions inside the insulating region induces tunnelling between the two islands.

In this exercise, you will derive about the semiclassical Josephson effect starting from the Schroedinger equation.

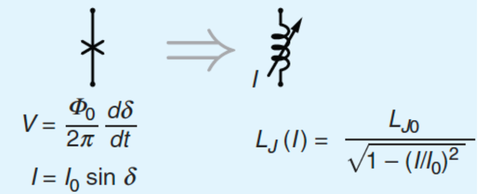
As the two SC regions are small and isolated, the total number of electron pairs in each region (1, 2) can be taken as n_1, n_2.
The two SC wave functions are thus described by:

$$\Psi = \sqrt{n_1}e^{i\theta_1}$$

$$\Psi = \sqrt{n_2}e^{i\theta_2}$$

1. Derive the following two coupled differential equations from the Schroedinger equation:
   $\hbar \frac{d\Psi_1}{dt} = \frac{E}{2}\Psi_2$
   $\hbar \frac{d\Psi_2}{dt} = \frac{E}{2}\Psi_1$
2. Show that the independent differential equations for the number and phase variables are:
   $\frac{dn_1}{dt} = \frac{E}{\hbar}\sqrt{n_1 n_2}\sin\delta$
   $\frac{d\theta_1}{dt} = -\frac{E}{2\hbar}\sqrt{\frac{n_1}{n_2}}\cos\delta$
   $\frac{dn_2}{dt} = -\frac{E}{\hbar}\sqrt{n_1 n_2}\sin\delta$
   $\frac{d\theta_2}{dt} = -\frac{E}{2\hbar}\sqrt{\frac{n_1}{n_2}}\cos\delta$
   \\ [Hint: rewrite the coupled differential equations in terms of $\delta = \theta_2 - \theta_1$ and equate the real and imaginary parts to isolate the desired derivative terms.]
3. Can you make the assumption $n_1 \approx n_2 \equiv n_0$. If it holds, use it to show that: $d\delta/dt = 0$ and $dn_1/dt = -dn_2/dt$
4. Use the relations you have obtained to derive the Josephson current-phase relation: $I = I_0 \sin\delta$. How much is I_0?

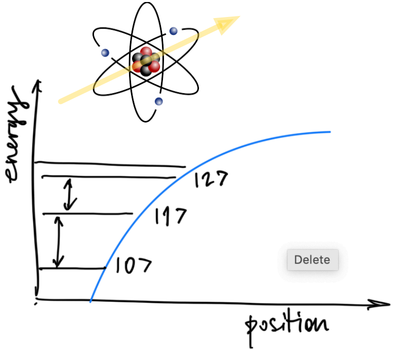
Josephson Junction



- Josephson equations relate the voltage and current in this element to a phase δ across the junction
- I_0 depends on the barrier thickness and superconducting gap energy
- current-dependent inductance $L_J(I)$ that diverges as $I \to I_0$ $\Longrightarrow$ basic **nonlinearity** at the core of many devices

HOW TO BUILD AN ARTIFICIAL ATOM

The same underlying physics is at work in cavity and circuit QED:
two-level spin-like systems are interacting with **quantum harmonic oscillators**



spins = two-state particles

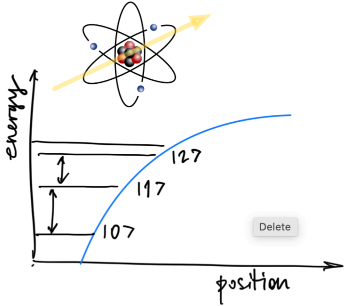
coupled to:

springs = quantum field oscillators

$E_{01} = E_1 - E_0 = \hbar\omega_{01} \neq E_{02} = E_2 - E_1 = \hbar\omega_{21}$
$\rightarrow$ good **two-level atom** approximation

control internal state by shining laser tuned at the transition frequency:
$H = -\vec{d} \cdot \vec{E}(t)$, with $E(t) = E_0 \cos \omega_{01} t$

qubits from “**artificial atoms**”: LC circuit



$E_{01} = E_1 - E_0 = \hbar\omega_{01} \quad \neq \quad E_{02} = E_2 - E_1 = \hbar\omega_{21}$
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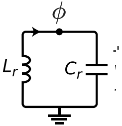
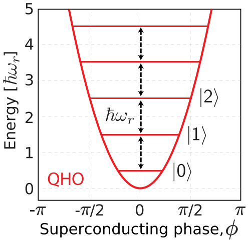
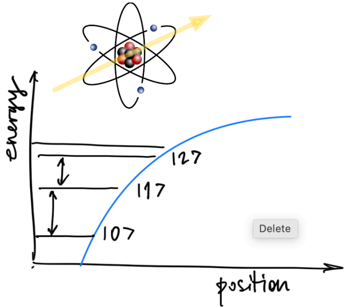
control internal state by shining laser tuned at the transition frequency:
$H = -\vec{d}\cdot\vec{E}(t)$, with $E(t) = E_0 \cos\omega_{01} t$





toolkit: capacitor, inductor, wire (all SC)
$\omega_{01} = 1/\sqrt{LC} \sim 10\,\text{GHz} \sim 0.5\,\text{K}$

$\rightarrow$ simple LC circuit is not a good **two-level atom** approximation qubits from "**artificial atoms**": LC circuit **with NL inductance** of the Josephson Junction



$E_{01} = E_1 - E_0 = \hbar\omega_{01} \neq E_{02} = E_2 - E_1 = \hbar\omega_{21}$
$\rightarrow$ good **two-level atom** approximation

control internal state by shining laser tuned at the transition frequency:
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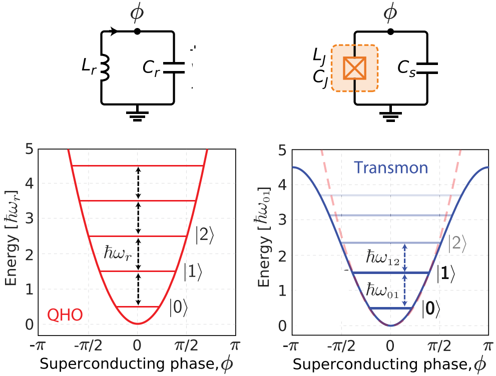


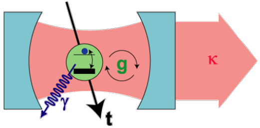
toolkit: capacitor, inductor, wire (all SC) + JJ

JJ is a **nonlinear** and dissipationless element

$$L_J = \frac{\phi_0}{2\pi} \frac{1}{I_c \cos\phi}$$

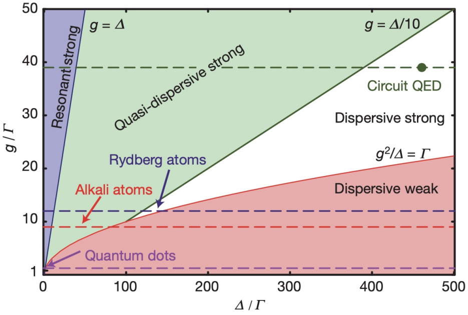
Jaynes-Cummings model

Interaction of a **two state system** with **quantized radiation in a cavity**



$$\mathcal{H}_{\rm JC} = \frac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)$$

Parameter space diagram for cavity-QED



$$\Delta = |\omega_r - \omega_q|$$

$$\Gamma = \min\{\gamma,\ \kappa,\ 1/T\}$$

- $\omega_r \sim \omega_q$ *resonance* case
- $\Delta = |\omega_r - \omega_q| \gg g$ *dispersive limit* case

Dispersive regime $\quad g/\Delta \ll 1$

$$\begin{aligned}\hat{H}_{\mathrm{JC}}^{\mathrm{eff}} &= \hbar\omega_r \hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z + \hbar\chi\hat{a}^\dagger\hat{a}\hat{\sigma}_z \\ &= (\hbar\omega_r + \hbar\chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z \\ &= \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}(\omega_q' + \cancel{\omega_r}\,\hat{a}^\dagger\hat{a})\hat{\sigma}_z\end{aligned}$$

2χ

$$\chi = \frac{g^2}{\Delta}$$

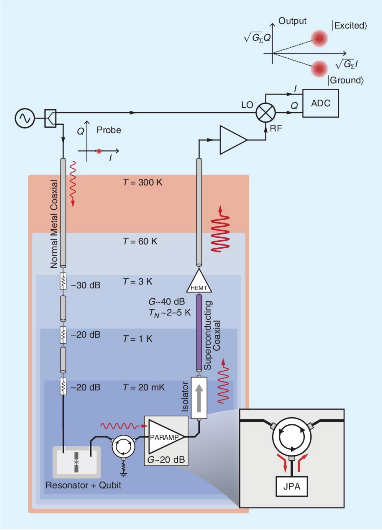
$\rightarrow \hbar\chi\hat{\sigma}_z$ dispersive qubit state readout

$\rightarrow 2\chi a^\dagger a$ number splitting

→ **qubit frequency** is a function of the **cavity photon number**

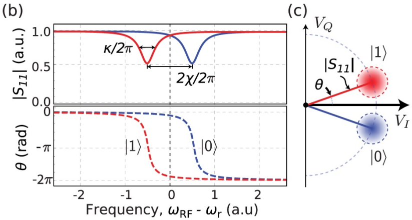
→ measuring the **qubit frequency** is equivalent to measuring the **number of photons** in the cavity

dispersive qubit readout of qubits



$$H/\hbar = (\omega_r + \chi\sigma_z)\left(aa^\dagger + \frac{1}{2}\right) + \frac{\omega_q'}{2}\sigma_z$$

$\chi = \frac{g^2}{\Delta}$, qubit-state dependent frequency shift

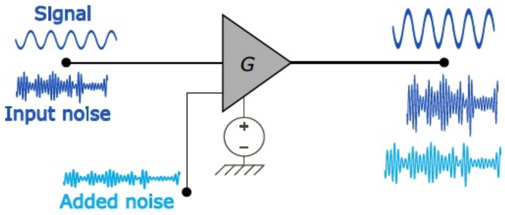


$\Longrightarrow$ **Amplitude readout**: the frequency of the microwave probe pulse is at either at $\omega_r + \chi$ or $\omega_r - \chi$. Depending on T($|S_{12}|$)/R$|S_{11}|$ power you know what the qubit state is

$\Longrightarrow$ **Phase readout**: the probe is at 0 (reflected power same for $|1\rangle$ and $|1\rangle$). All info is in the phase $\theta = \pm\arctan(\chi/\kappa)$

Q: How well can we discriminate qubit states?

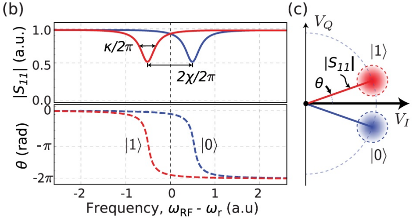
A: Depends on the **noise added by the amplifier**

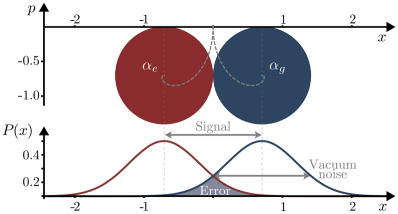


Q: How well can we discriminate qubit states?

A: Depends on the **noise added by the amplifier**

Any amplifier is going to add some noise → added noise will affect the standard deviation of these two distributions (gaussian) and in turn **fidelity**.

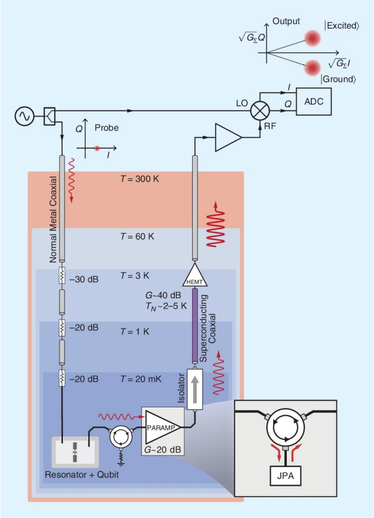


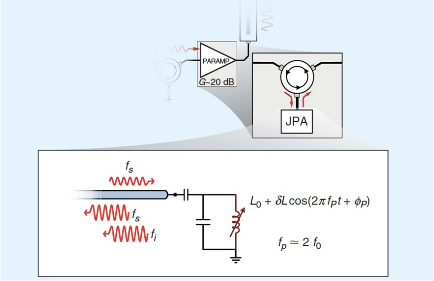


Motivation: using amplifiers that add **minimum amount of noise**.

$N^{\text{HEMT}} \sim 20$ photons
$N^{\text{JPA}} \sim 1$ photon $\Longrightarrow$ order of magnitude smaller added noise $\Longrightarrow$ 99% fidelity in $\sim 100\,\text{ns}$

SUPERCONDUCTING PARAMETRIC AMPLIFIERS: a basic example





→ a lossless parallel LC resonator connected to a transmission line (adds delay to signal at frequency f_s tuned within a linewidth)

PARAMETRIC AMPLIFICATION

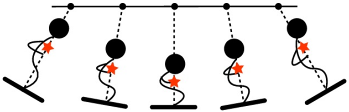
systems that can periodically convert energy between **conjugate field variables**

$$\mathrm{I} \Longleftrightarrow \mathrm{V}$$

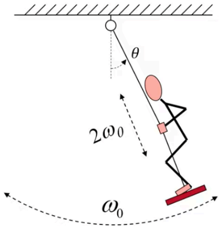
$$\mathbf{E} \Longleftrightarrow \mathbf{B}$$

$$x \Longleftrightarrow p$$

can exhibit parametric behavior when the corresponding mediating elements can be modulated.



MECHANICAL SYSTEM



PARAMETRIC AMPLIFICATION

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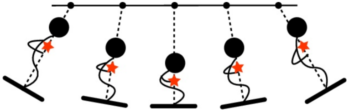
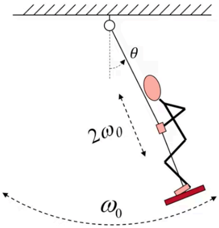


MECHANICAL SYSTEM



- $\rightarrow$ change the moment of inertia (parameter) at $2f_0$
- $\rightarrow$ pump (energy source): work done against F_c
- $\rightarrow$ nonlinear resonator: the restoring force $\propto \sin\theta$ and not just θ

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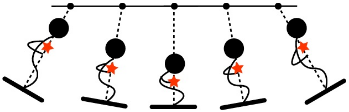
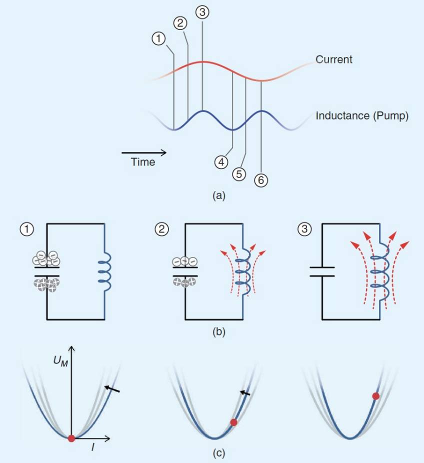
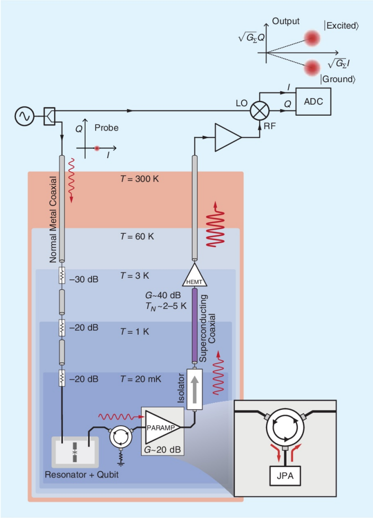
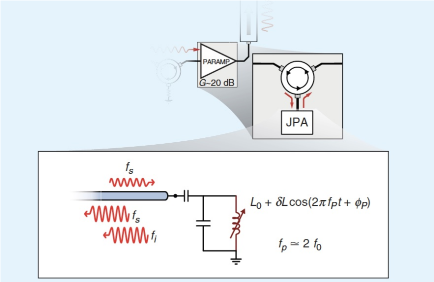




SUPERCONDUCTING PARAMETRIC AMPLIFIERS: a basic example



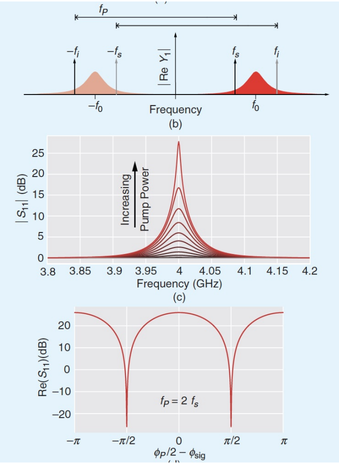


→ a lossless parallel LC resonator connected to a transmission line (adds delay to signal at frequency f_s tuned within a linewidth)

→ inductance (or capacitance) modulation (pump) at f_p generates mixing with sidebands $f_i = f_s \pm f_p$

→ power is drawn from the pump source to produce gain at f_i and f_s

PARAMETRIC AMPLIFICATION



$f_p \simeq 2f_0$

degenerate parametric amplification ($f_s = f_i$)

$\rightarrow$ gain response:

$$BW(G) \simeq \frac{BW_0}{\sqrt{G}}$$

e.g. $BW_0 = 100\,\text{MHz}$, reduced to $10\,\text{MHz}$ for $G = 20\,\text{dB}$

$\rightarrow$ sensitivity to the pump phase

More complex circuit designs exploit **different types of nonlinearities**, to accomplish:

- large gain ($\simeq 20\,\text{dB}$ in practice)
- dynamic and tuning bandwidth
- near-quantum-limited added noise
- other things