CERN

NexTGen

Next Generation Triggers

Next Generation Triggers 1st Technical Workshop

NOVEMBER 25-27, 2024

NexTGen

Task 1.4: Tensor Networks for Quantum Systems

Enrique Rico Ortega

Monday, 25th November 2024

CERN | NexTGen Next Generation Triggers

Main researchers: Stefano Carrazza, Enrique Rico Ortega

Tensor network algorithms: an overview

A class of tailored variational ansatz states on a lattice many-body quantum system

$$|\Psi_{\text{many-body}}\rangle = \sum_{s_1,\cdots,s_N} \Psi_{s_1,\cdots,s_N} |s_1,\cdots,s_N\rangle \qquad \dim\left(\mathscr{H}\right) = \mathrm{d}^{\mathrm{N}}$$

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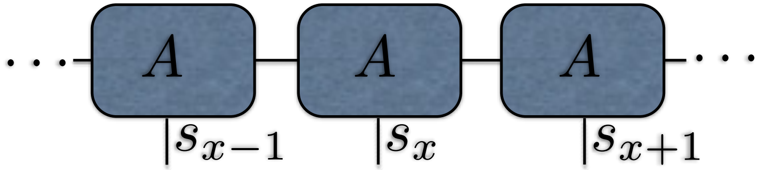
Ψ is obtained contracting smaller tensors over auxiliary indexes

$$|\Psi_{\text{MPS}}\rangle = \sum_{\{s_i\},\{\alpha_i\}} A^{(s_1)}_{\alpha_1} A^{(s_2)}_{\alpha_1,\alpha_2} \cdots A^{(s_N)}_{\alpha_{N-1}} |s_1, s_2, \cdots, s_N\rangle$$



$$\dim(\text{MPS}) = \mathrm{N\,d\,D^2}$$

A simple example

Consider a quantum system in a pure state $|\Psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

$$|\Psi\rangle = \sum_{i,j}^{d} \Psi_{i,j} |i\rangle_A \otimes |j\rangle_B$$

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Schmidt decomposition:

$$|\Psi\rangle = \sum_{j}^{D} c_j |\psi_j\rangle_A \otimes |\psi_j\rangle_B \qquad c_j \geq 0; \qquad \sum_j c_j^2 = 1$$

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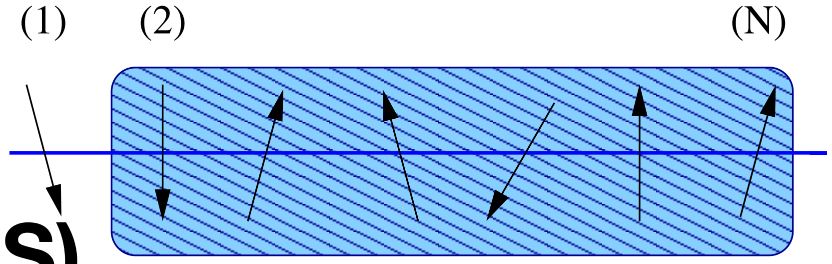
Maximal entangled state: $c_i = c_j = \frac{1}{\sqrt{D}} \rightarrow S_a = \log D$

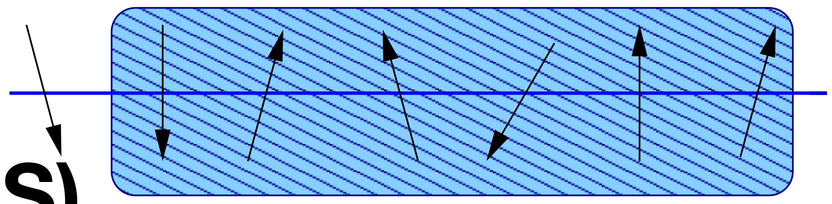
Schmidt picture: Matrix Product State (MPS)



$$|\Psi\rangle = \sum_{a=1}^{D} |\phi_a^{(1)}\rangle \lambda_a^{(1)} |\phi_a^{(2,\ldots,N)}\rangle = \sum_{a=1}^{D} \sum_{s_1=1}^{d} |s_1\rangle\langle s_1|\phi_a^{(1)}\rangle \lambda_a^{(1)} |\phi_a^{(2,\ldots,N)}\rangle$$

$$= \sum_{a=1}^{D} \sum_{s_1=1}^{d} |s_1\rangle A_a[s_1] |\phi_a^{(2,\ldots,N)}\rangle = \sum_{a,b=1}^{D} \sum_{s_1,s_2=1}^{d} |s_1\rangle A_a[s_1] |s_2\rangle A_{ab}[s_2] |\phi_b^{(3,\ldots,N)}\rangle$$

$$= \cdots = \sum_{\{s_j\}=1}^{d} Tr\{A[s_1]\cdots A[s_N]\} |s_1\rangle \cdots |s_N\rangle$$



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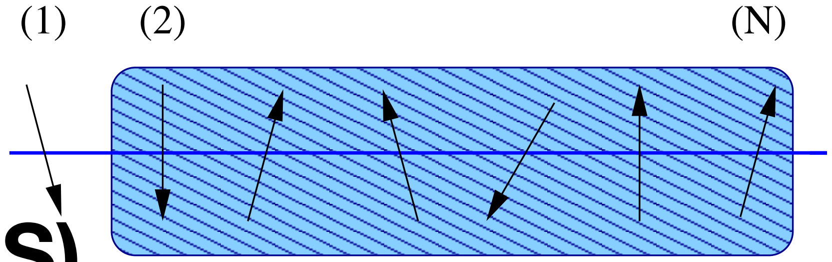
$$D \propto \exp S_L$$



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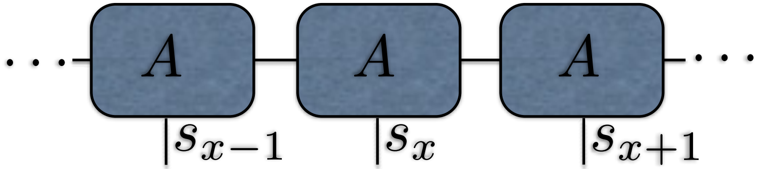
quantum correlations = entanglement =

$$\log(D) \propto \frac{c+\bar{c}}{3} \log(N)$$

C. Holzhey, F. Larsen, F. Wilczek, Nucl. Phys. B (1994)
G. Vidal, J.I. Latorre, E. Rico, A. Kitaev, Phys. Rev. Lett. (2003)
B.-Q. Jin, V.E. Korepin, J. Stat. Phys. (2004)
P. Calabrese, J.J. Cardy, Stat. Mech. (2004)

Tensor network algorithms: an overview



Well-suited to described translational invariant systems

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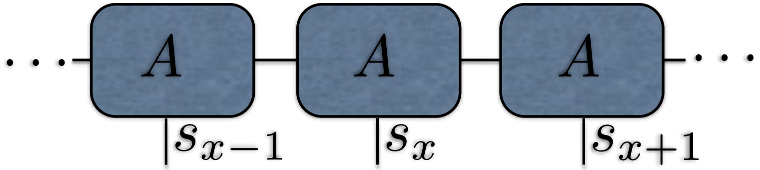

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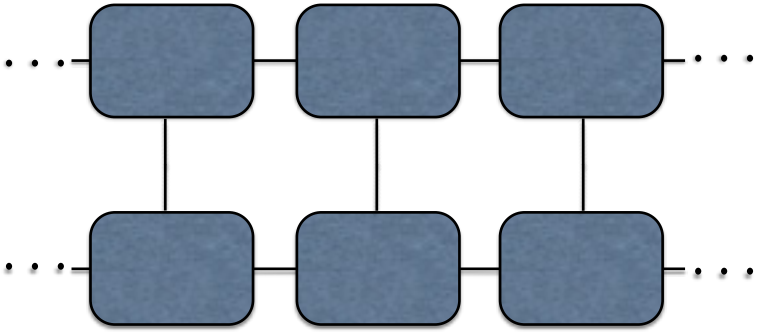
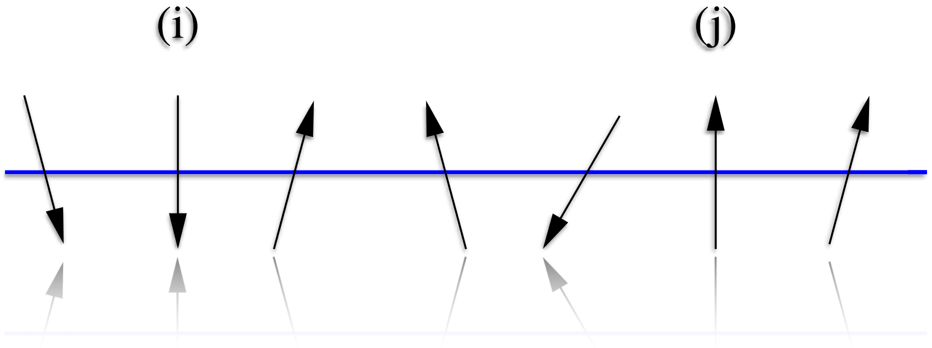
Simple way to obtain any expectation value (Transfer matrix)





$$E = \sum_s A^*[s] \otimes A[s]$$

$$\langle O_i O_j \rangle = \mathrm{Tr}\left\{ E^{N-j+i-2} \tilde{O}_i E^{j-i} \tilde{O}_j \right\}$$

$$\tilde{O} = \sum_{s,s'} A^*[s] \otimes A[s'] \langle s | O | s' \rangle$$

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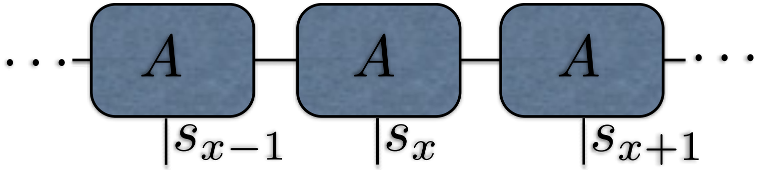

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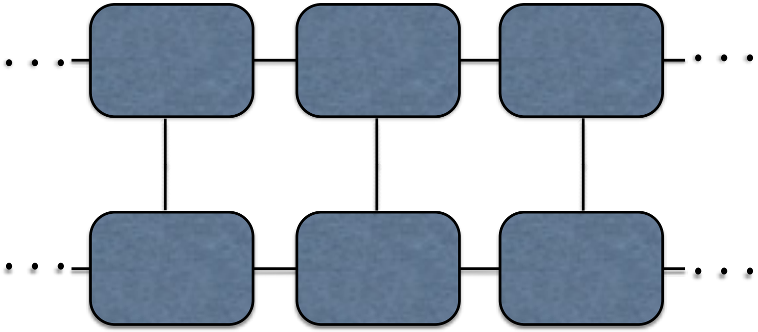

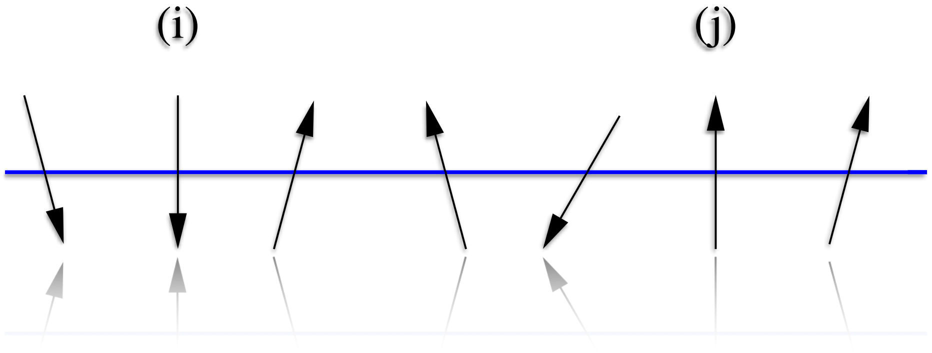

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Optimal to minimize the energy (DMRG)

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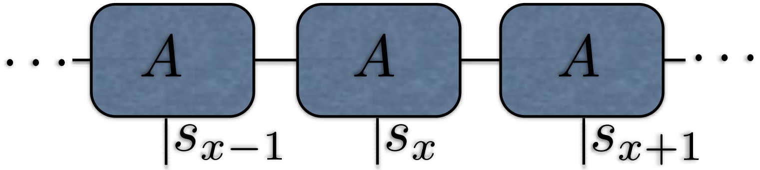

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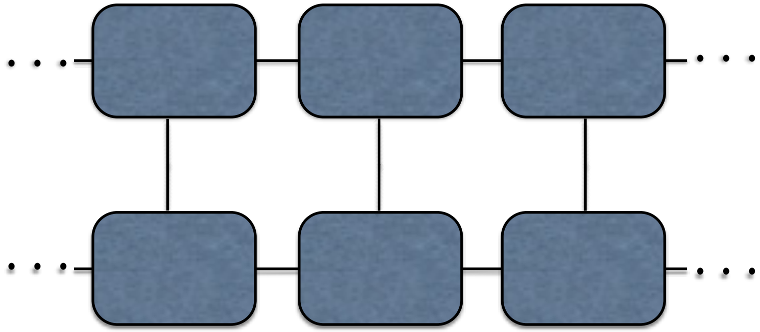

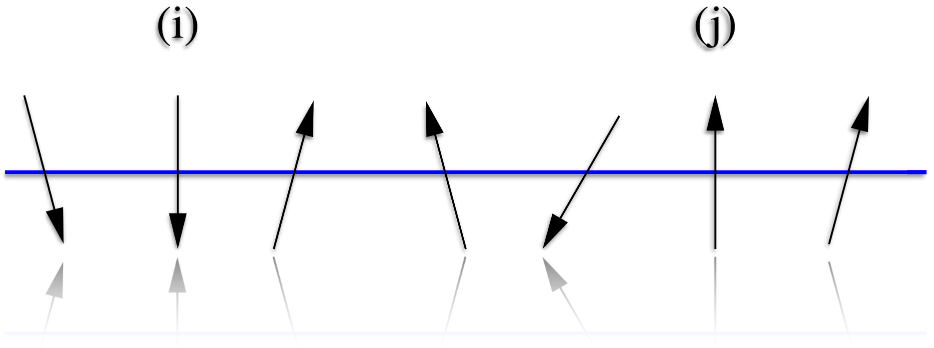

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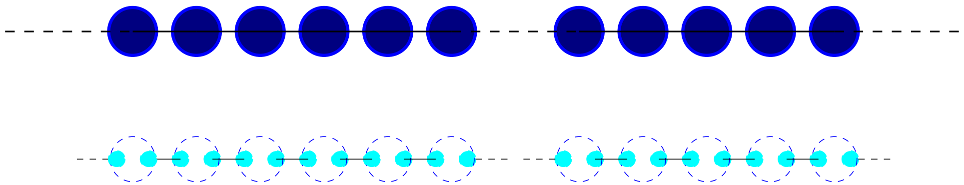
Optimal to minimize the energy (DMRG)

Encoded the entropic boundary law (VBS picture)

$$A_{D\times D}[s] |s\rangle_{s=1}^{d}$$



$$\mathbb{C}^D \otimes \mathbb{C}^D \underset{\rightarrow}{A} \mathbb{C}^d$$

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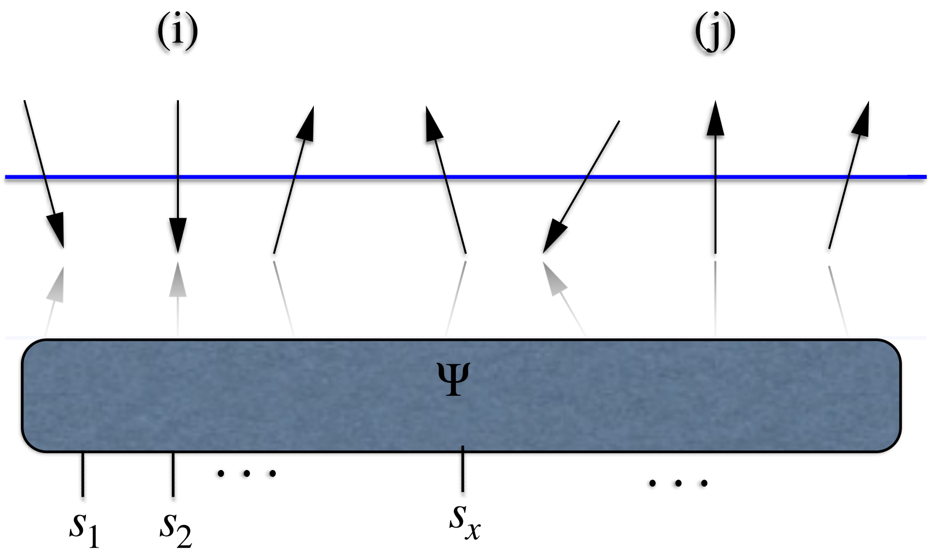
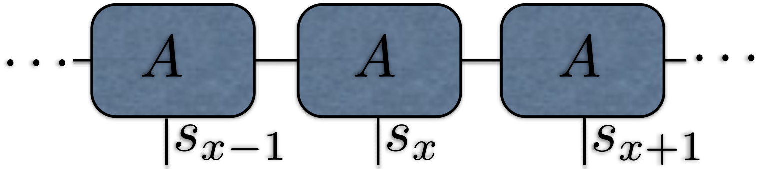


Matrix Product State (MPS)
sequencial generation



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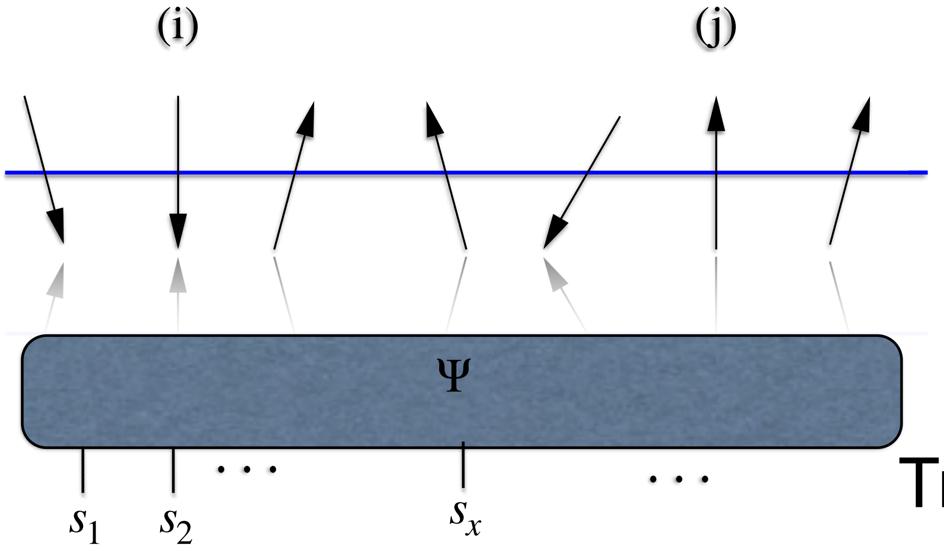
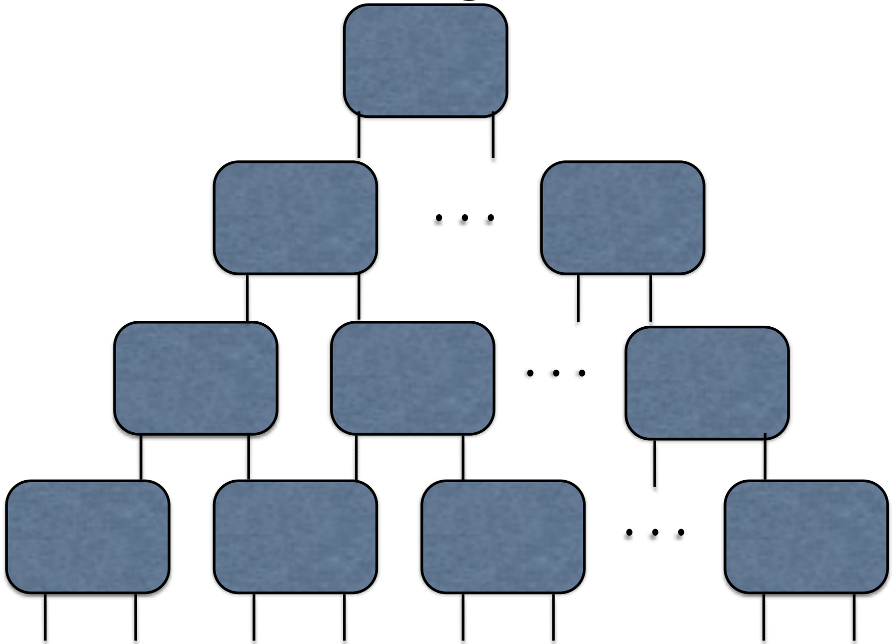


Tree tensor network (TTN)
multiscale generation



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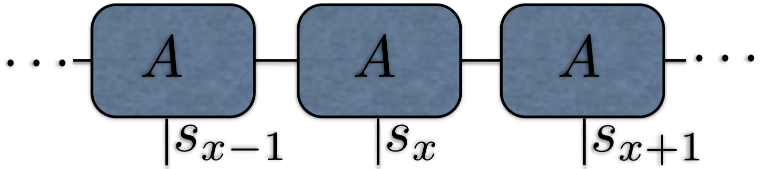

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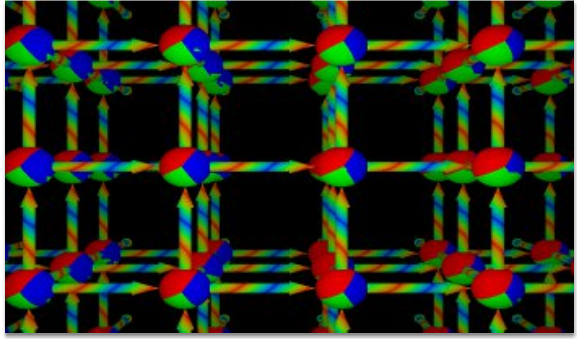
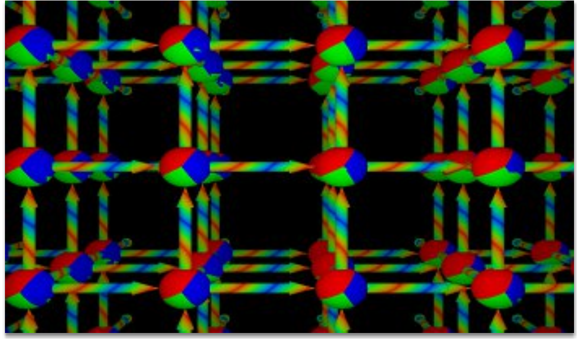


Variational (non-perturbative) method for Hamiltonian systems

Extremely useful in 1D systems (MPS)

Proposals and extensions in higher dimensions (TNS)

Tensor network algorithms: an overview



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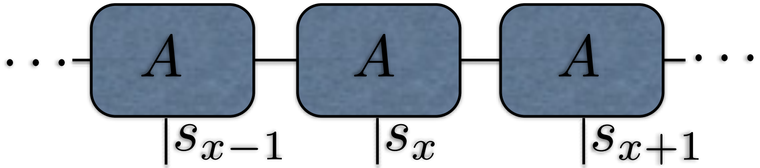


Ground states

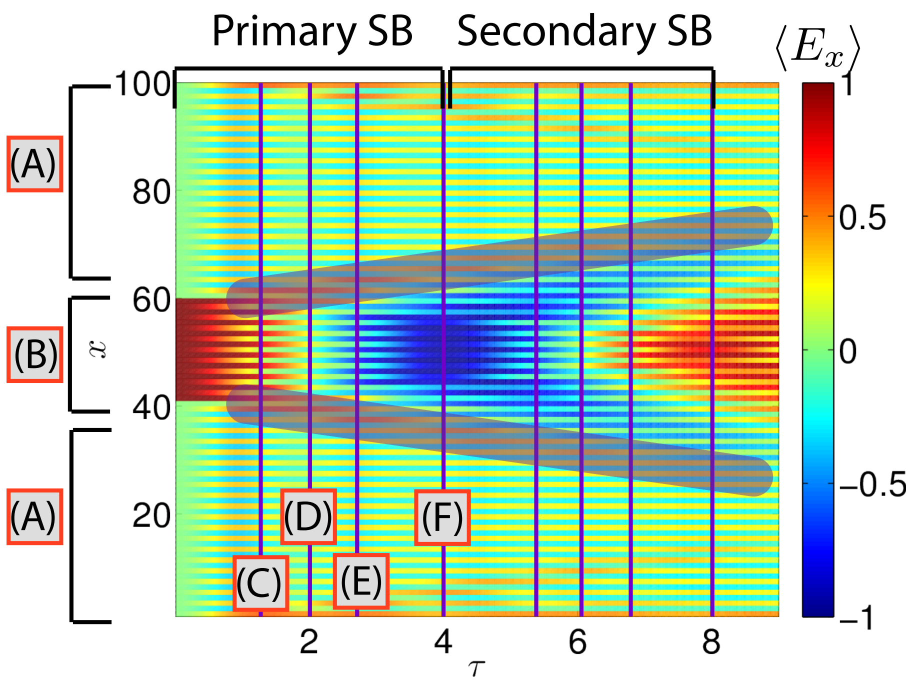
Low-energy excitations

Thermal states

Time evolution

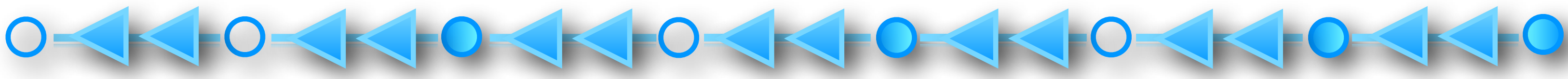
Proposal for fermionic systems

Confinement and string breaking: QED in (1+1)-d (Schwinger model)



(A) Vacuum

(B) String



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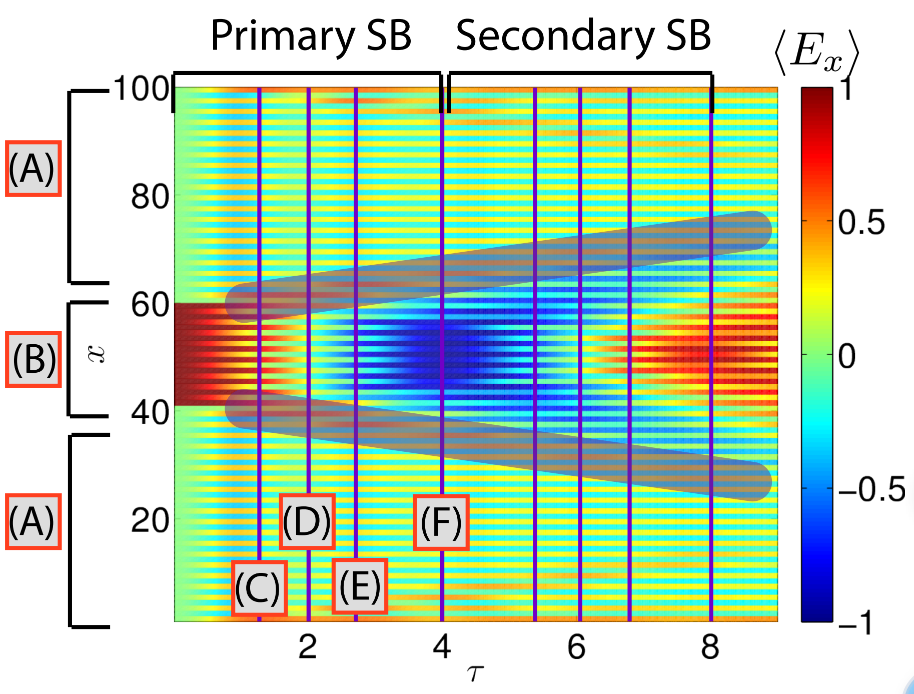

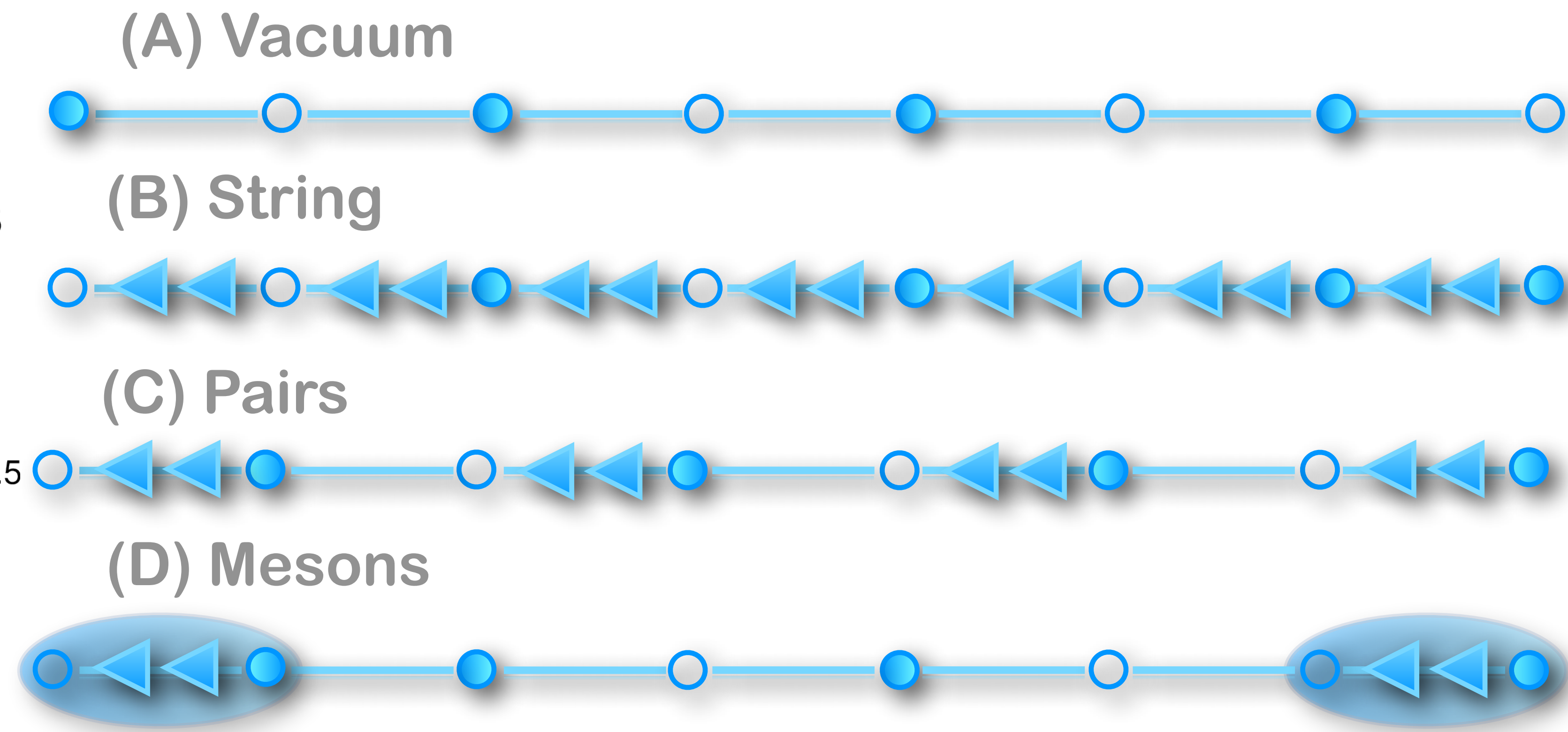

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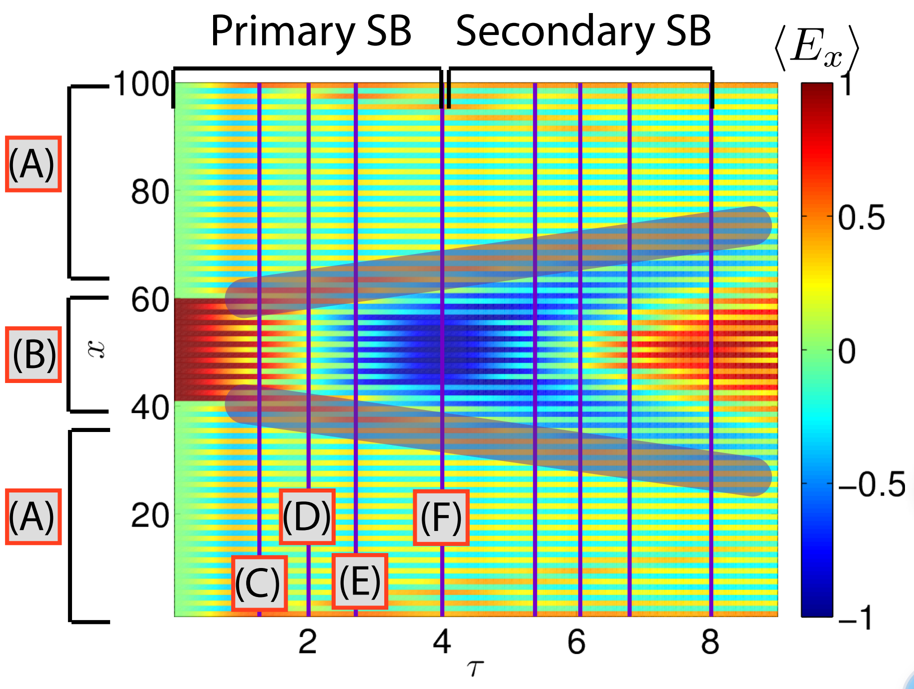

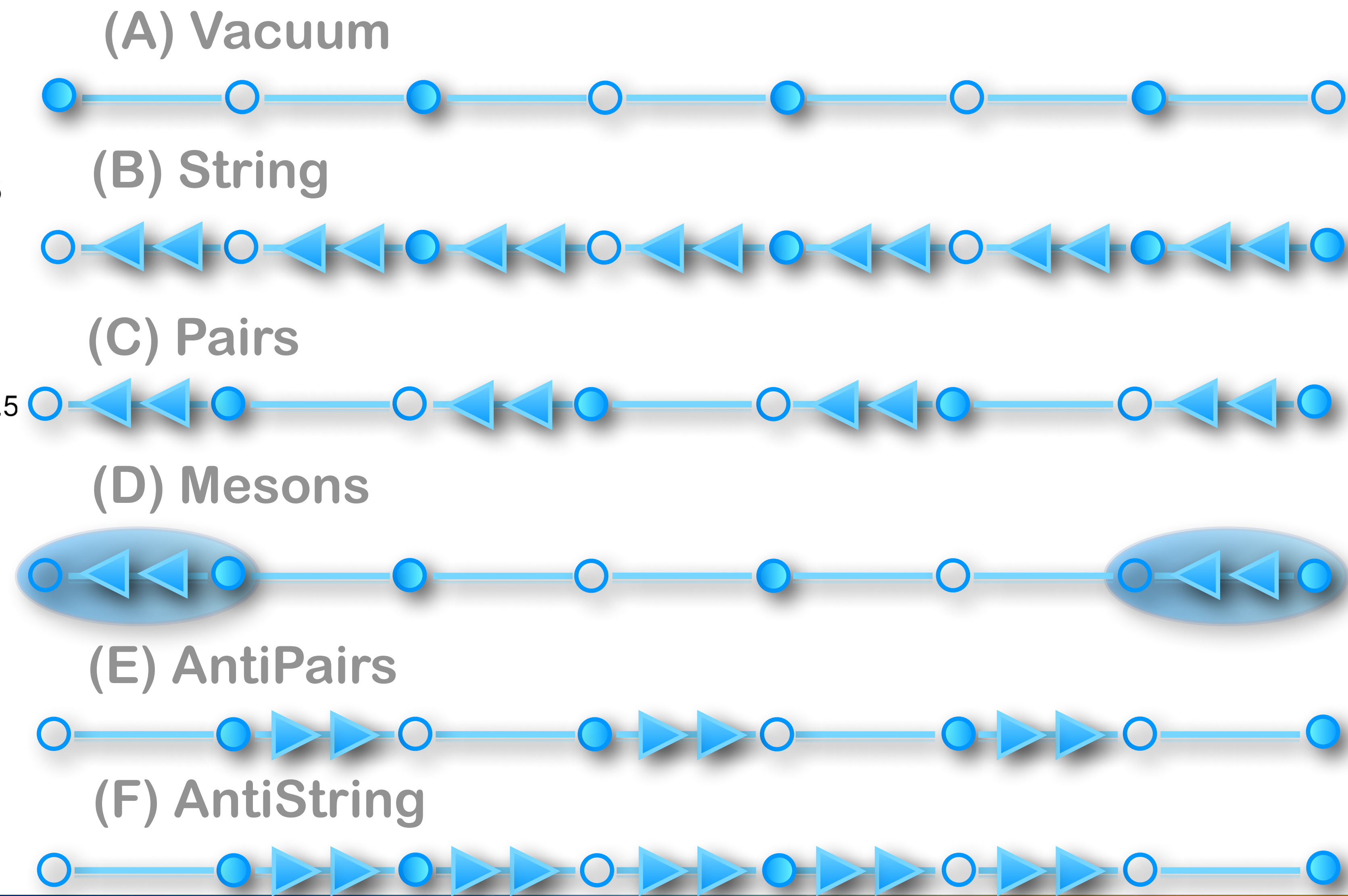

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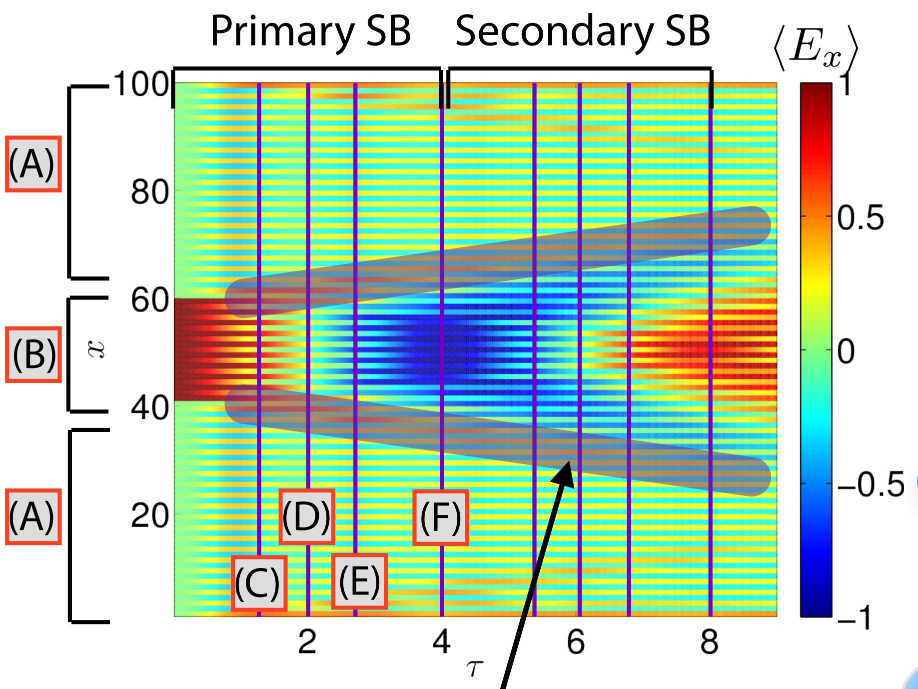
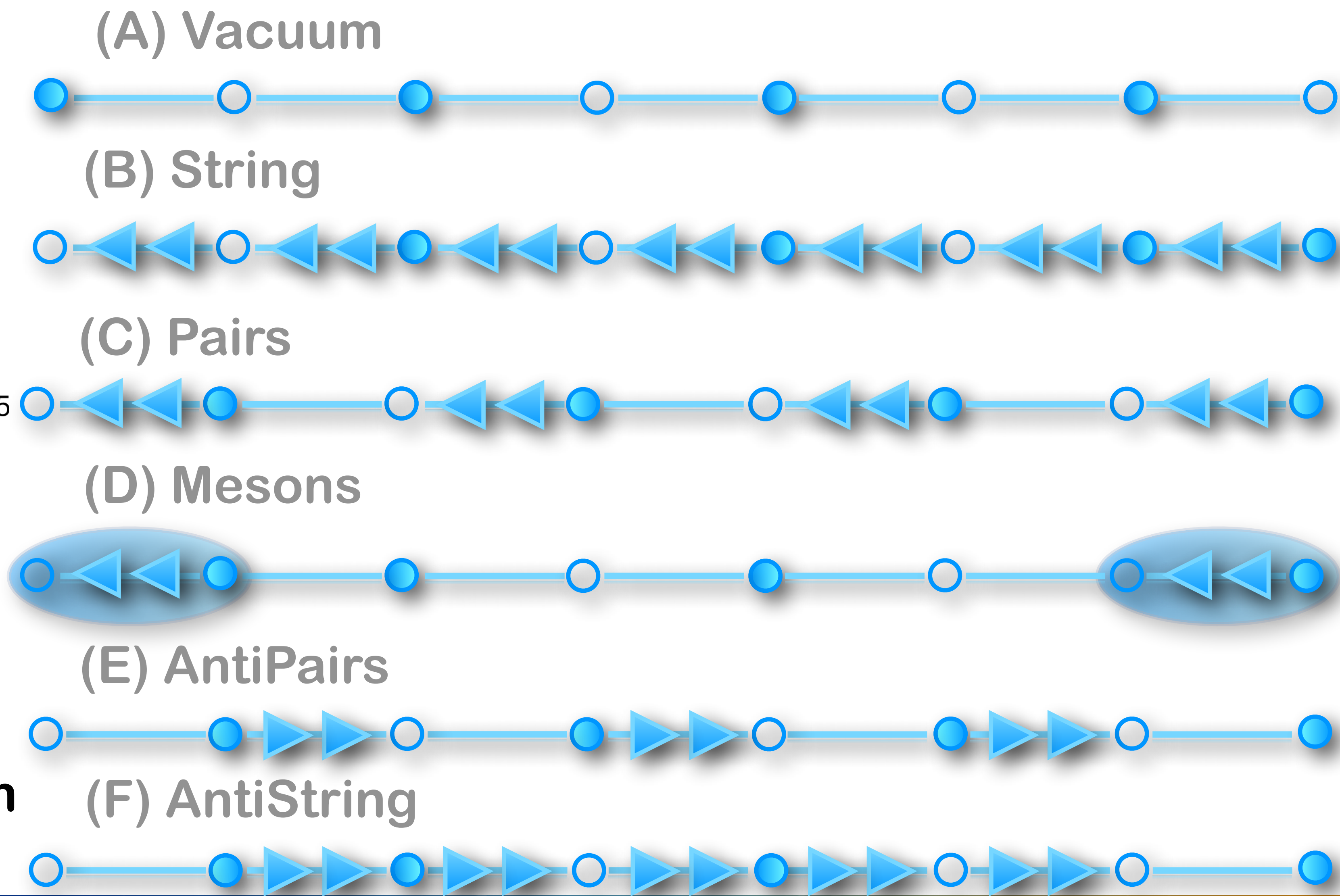
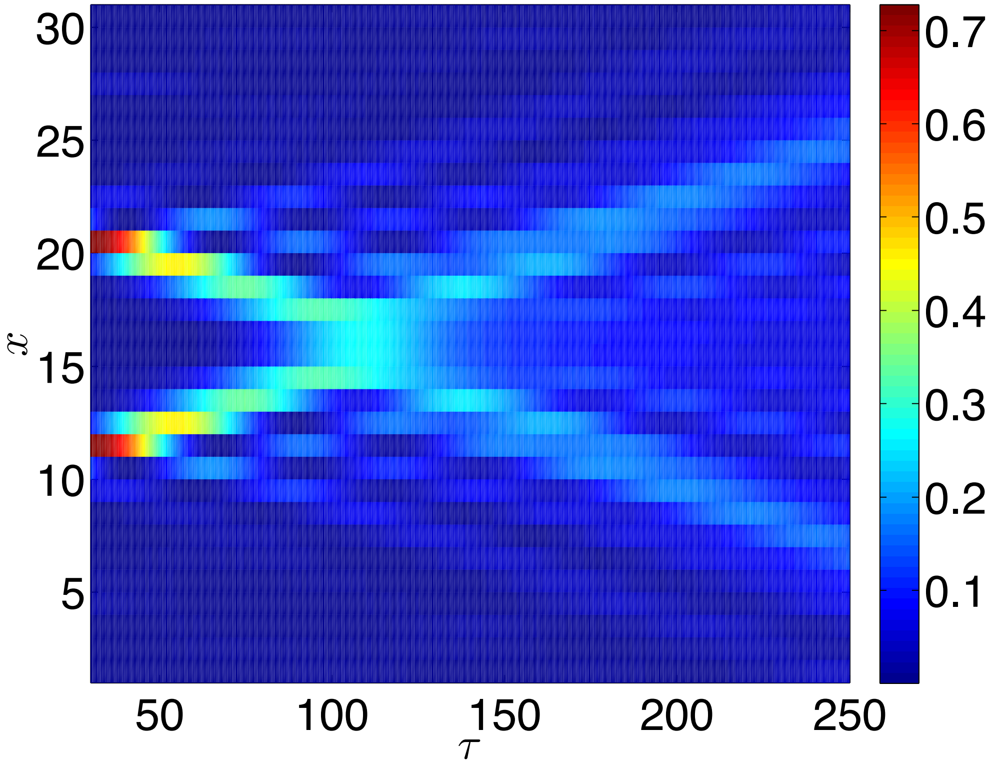


String wave-front characterised by:
-Electric field spreading
-Entanglement propagation
O(100) sites simulation

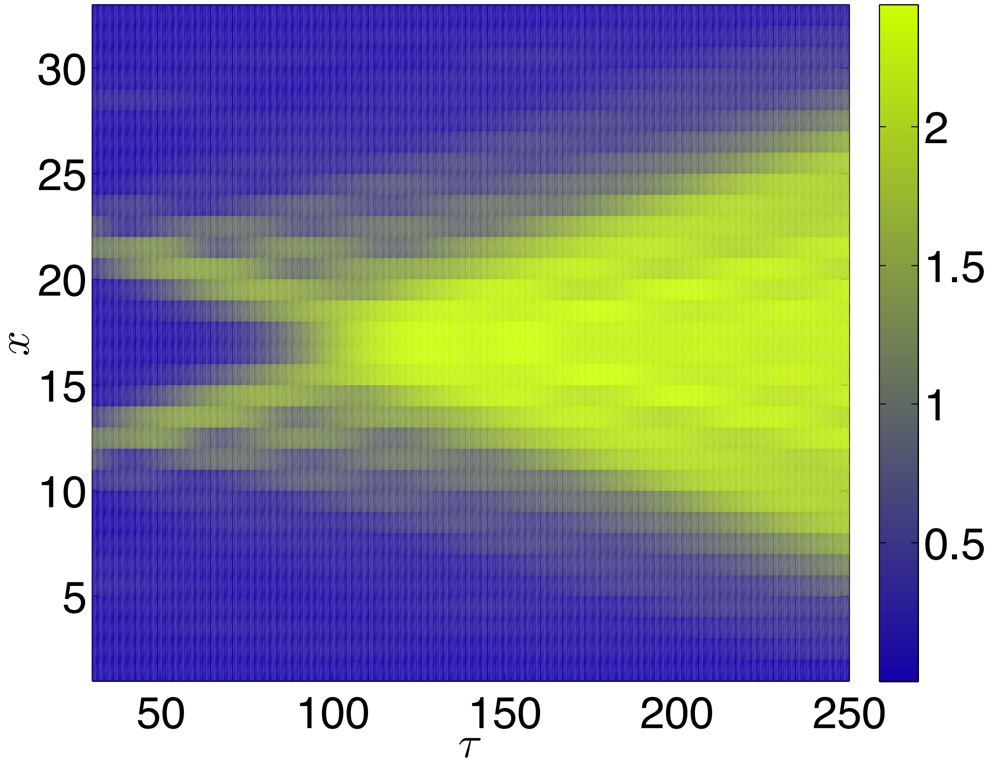


Confinement and string breaking: QED in (1+1)-d (Schwinger model)

We prepare two mesons in a dynamical state giving them momentum towards the center



Electric field of two mesons during the scattering evolution

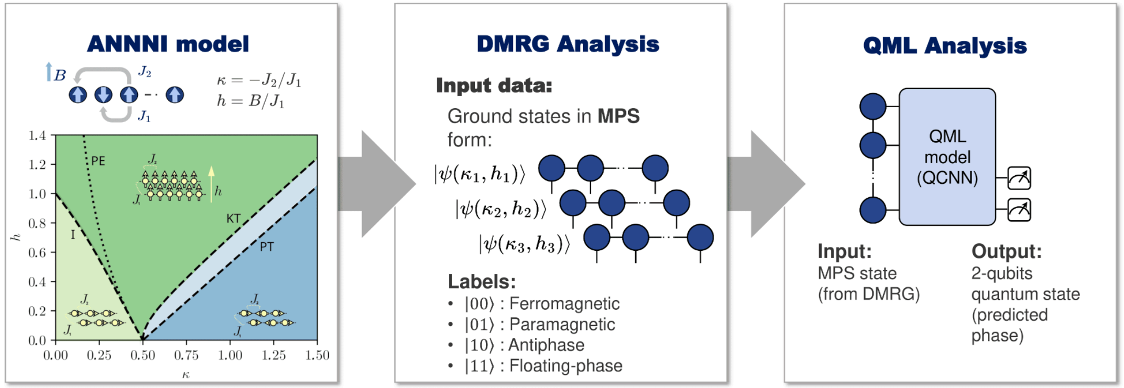


Entanglement entropy during the scattering

Tensor network algorithms and machine learning

Exploring the Phase Diagram of the quantum one-dimensional ANNNI model

M. Cea,[1,2,*] M. Grossi,[3,†] S. Monaco,[4,5,‡] E. Rico,[6,7,8,9,§] L. Tagliacozzo,[10,¶] and S. Vallecorsa[3,**]



arXiv:2402.11022 (2024)

O(400) sites simulation

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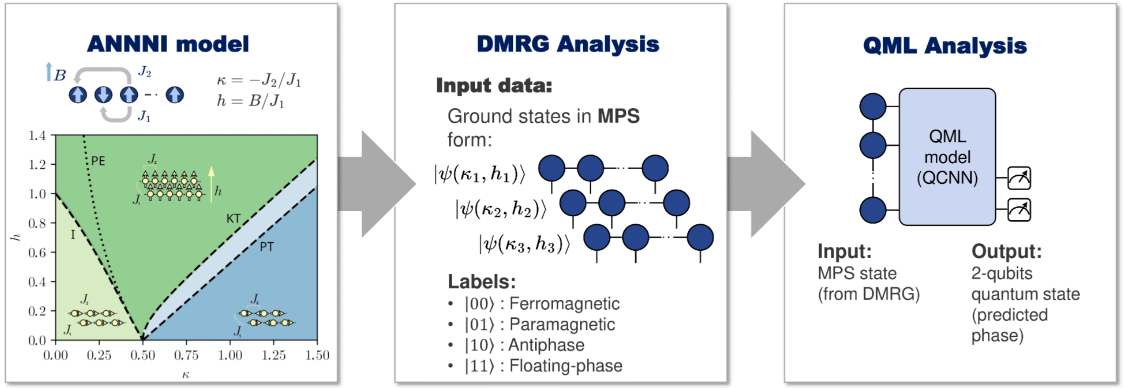


arXiv:2402.11022 (2024)

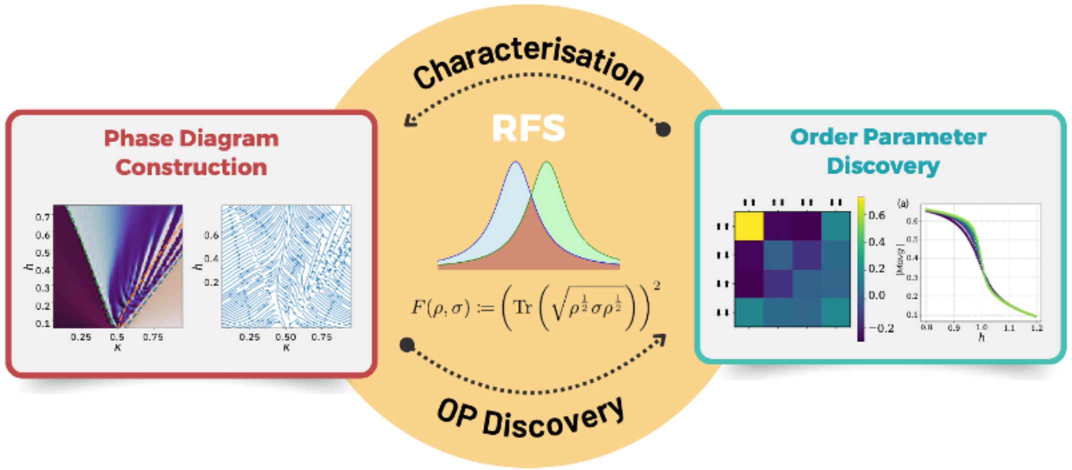
O(400) sites simulation

Description of Quantum Phases of Matter with Quant Info

Order Parameter Discovery for Quantum Many-Body Systems

Nicola Mariella+,[1,*] Tara Murphy+,[1,2,†] Francesco Di Marcantonio,[3,‡] Khadijeh Najafi,[4,§] Sofia Vallecorsa,[5,¶] Sergiy Zhuk,[1,**] and Enrique Rico[3,6,7,††]

arXiv:2408.01400



Conclusions and outlook

-First results (preprints) on Tensor Networks and Quantum Machine Learning analysis with O(400) sites.

-Ongoing projects on real-time dynamics of High-Energy Physics and classical simulation within GPUs architectures

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-First results (preprints) on Tensor Networks and Quantum Machine Learning analysis with O(400) sites.

-Ongoing projects on real-time dynamics of High-Energy Physics and classical simulation within GPUs architectures

- Benchmark TNML approach with other existing ML models, to investigate the advantage of the method.
- Applications (mainly HEP-oriented):

    use TN for the L1-CMS trigger (continuing the present efforts using FPGAs): how this approach compares to standard ML in terms of performance, accuracy, trainability and explainability

    explore TN usage for tracking applications, for rare events (anomaly detection)

    use TN for quantum circuit simulation and hybrid quantum hardware applications: quantum error correction

    explore potential TNML generative applications (or even reproducing other ML applications with TNML, e.g. auto-encoders)

Workshop Outcomes and Opportunities for Collaboration

Legeza showed the application of tensor networks (TNs) to nuclear and atomic physics, with a particular emphasis on optimising hardware architectures.

His work aligns with Task 1.4 (hardware optimisation) and potentially Task 1.7.

We will meet with Legeza to determine the value of closer collaboration, focusing on hardware-related insights for Task 1.4 and 1.7. (*meeting this week to be determined*)

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Presentations by Montangero's and Marco Zanetti's groups showcased recent advances, including work like arXiv:2409.16075, which reviewed FPGA-implemented machine learning (ML).

This overlaps with Task 1.4 (ML on FPGAs) and possibly Task 1.2, suggesting a bridging opportunity between the two tasks.

Discuss hosting short visits (1 week) from Zanetti's collaborators to foster integration with Tasks 1.2 and 1.4, potentially evolving into longer stays. (*I have already asked Montangero, no answer yet*)

Work Package 1: Infrastructure, Algorithms and Theory

Task 1.4: Tensor Networks for Quantum Systems.

This task will develop and apply quantum-inspired methodology, in particular Tensor Network algorithms, to simulate quantum many-body problems unreachable by classic approaches and benchmark future applications of quantum hardware on low-entangled systems to O(100) qubits, progressing towards the development of a software stack for quantum machine learning model design, simulation, and deployment.

Task 1.7: Common software developments for heterogeneous architectures

To make efficient use of accelerator (GPU and FPGA) devices in the software designed for the High-Luminosity LHC, various common developments and improvements are needed in the frameworks and code bases of the experiments and Monte Carlo generators. Frameworks need to make efficient use of all available computing resources of single compute nodes, and even possibly multiple nodes at the same time. Existing implementations should be harmonised between the experiments, and optimisation efforts should be shared.

Task 1.2: Development framework towards fast inference of complex network architectures on LHC online systems

In this task, we will work with existing expertise in the experiment collaboration on ongoing work on tools such as hls4ml, and on expertise from selected academic and industrial partners to develop ML->FPGA model synthesis tools, addressing the needs of WP2 and WP3. The work will also focus on integrating modern ML tooling while maintaining the strict latency requirements set forth by LHC experiments' online selection system. All task items are supposed to be co-developed by CERN researchers and external partners with qualified expertise on the topic.