Özel Görelilik

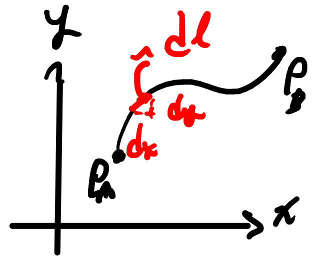
D-boyutlu uzay:

$$\vec{r} = \sum_{i=1}^{D} x^i \hat{e}_i \quad , \quad \hat{e}_i \cdot \hat{e}_j = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Herhangi bir vektör

$$\vec{v} = \sum_{i=1}^{D} v^i \hat{e}_i \rightarrow v^i = \vec{v} \cdot \hat{e}_i$$

Sonsuz küçük çizgi elemanı



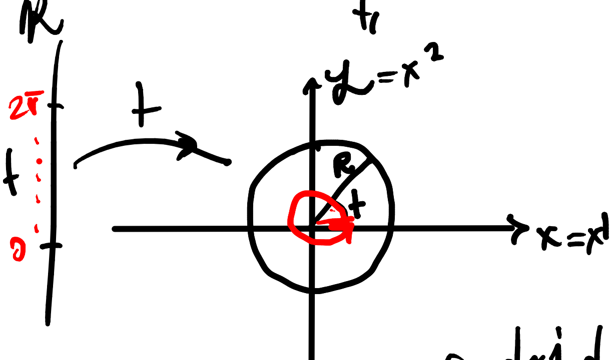
$$dl^2 = \delta_{ij}\, dx^i dx^j$$
$$= (dx^1)^2 + (dx^2)^2 + \dots (dx^D)^2$$

D-boyutlu öklidyen uzayda eğrinin uzunluğu

$$\Delta L = \int_{P_A}^{P_B} dl \quad , \quad P_A = x(t_1) \quad , \quad P_B = x(t_2)$$

$$dl = \sqrt{\delta_{ij}\, dx^i dx^j} = \sqrt{\delta_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}}\, dt \quad , \quad \frac{dx^i}{dt} = \dot{x}^i$$

$$\Delta L = \int_{t_1}^{t_2} \sqrt{\delta_{ij}\, \dot{x}^i \dot{x}^j}$$



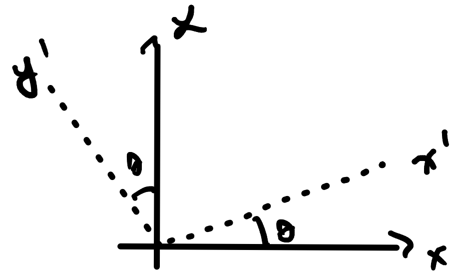
$$C : \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$$

$$x^1 = R \cos t$$
$$x^2 = R \sin t$$

$$\delta_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} = \left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 = (-R \sin t)^2 + (R \cos t)^2$$
$$= R^2$$

$$\Delta L = \int_{t_1}^{t_2} \sqrt{\delta_{ij}\,\dot{x}^i\,\dot{x}^j} = \int_0^{2\pi} R\,dt = 2\pi R$$

* Dönme dönüşümü Öklidyen metriği değişmez bırakır.



$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}$$

$R \in SO(2)$ $\qquad\qquad$ $SO(n)$

special $\quad$ orthogonal

det=1 $\qquad$ $R^T I R = I$

* Sonsuz küçük dönme dönüşümü:

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$\theta \sim 0 \quad \rightarrow \quad \cos\theta \cong 1$$
$$\sin\theta \cong \theta$$

$$\begin{aligned} x' &= x + y\theta \\ y' &= -x\theta + y \end{aligned} \quad \Rightarrow \quad R(\theta) = \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}$$

$$R(\theta) \simeq I + \underbrace{\theta}_{\text{parametre}} \overset{\text{üreteci}}{\Gamma} + \cdots, \quad \Gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$\psi(x_1, x_2 \ldots) = -\psi(x_2, x_1 \ldots)$

$$e^{\alpha x} = 1 + \alpha x + \cdots$$

$$R(\theta) \to e^{\theta r} \simeq I + \theta r + \cdots$$

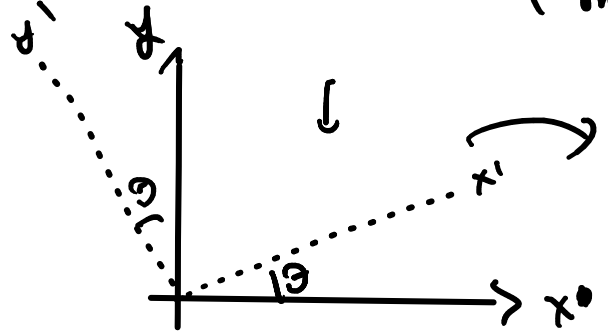
$R^T = -R$ ⚠ anti-simetrik matrisler her zaman dönme dönüşümüne götürür.

Simetrik üretici, $L^T = L$

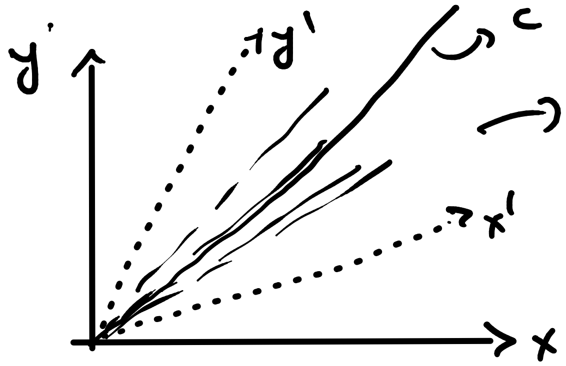
$\xi \to$ parametre, l : üretici

$$L(\xi) \simeq I + \xi l + \cdots, \quad l = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$L(\xi) = e^{\xi l} = \begin{pmatrix} \cosh\xi & -\sinh\xi \\ -\sinh\xi & \cosh\xi \end{pmatrix}$$



$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



Lorentz boost $\quad L = \begin{pmatrix} \gamma & -\beta\gamma \\ -\gamma\beta & \gamma \end{pmatrix}$

$$\beta = \frac{v}{c}$$

Lorentz dönüşümü

Dönme + boost

$$L^{-1}(\xi) = L(-\xi) = \begin{pmatrix} \cosh\xi & \sinh\xi \\ \sinh\xi & \cosh\xi \end{pmatrix}$$

$R^T I R = I$, $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$L^T \eta L = \eta \qquad , \quad \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Minkowski metriği

$$L \in SO(1,3)$$

$$dl^2 = dx^2 + dy^2 + dz^2$$

$$\vec{r} = \vec{r}(t) = x^i(t)\,\hat{e}_i \qquad \dot{x}(t) = v_0 + at$$

1-boyutta serbest parçacık $\rightarrow \dfrac{d^2x(t)}{dt^2} = 0$

$$\Rightarrow \begin{matrix} t' = t \\ x' = x - vt \end{matrix} \quad \Rightarrow \quad \frac{d^2x'(t')}{dt'^2} =$$

$$dt' = dt$$

$$\frac{dx'}{dt'} = \frac{dx'}{dt} = -v \ , \quad \frac{d^2x'}{dt'^2} = 0$$

$\tau = ct \qquad , \quad [\tau] = $ uzunluk.

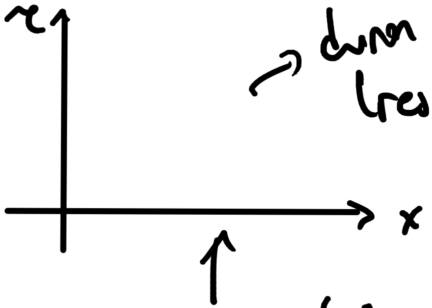
Cizgi elemanı:

$$ds^2 = -d\tau^2 + dr^2$$

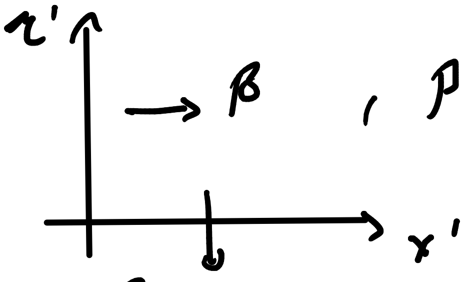
$L(3)$ ds^2'yi invaryant bırakır.

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu \qquad , \quad \mu,\nu : 0,1,2,3 \ , \quad \eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$



duran gözlemci (rest frame)



β , $\beta = \frac{v}{c}$

metric : $ds^2 = -d\tau^2 + dx^2$

$\downarrow$

$dx = 0$

$ds^2 = -d\tau^2$

Bu çerçevedeki gözlemci için

$$\frac{dx'}{d\tau'} = -\beta$$

$$\downarrow$$

$$ds^2 = -(d\tau')^2 + (dx')^2$$
$$= -(d\tau')^2 + \beta^2 (d\tau')^2$$

İki metric birbirine eşit olmalı

$$-d\tau^2 = -(d\tau')^2 + \beta^2 (d\tau')^2$$
$$= -(d\tau')^2 (1-\beta^2)$$

$\beta < 1$

$$d\tau = d\tau' \sqrt{1-\beta^2} \Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$d\tau = d\tau' \frac{1}{\gamma} \Rightarrow \boxed{d\tau' = \gamma\, d\tau} \quad , \gamma > 1$$

$$d\tau' > d\tau$$

$$\begin{pmatrix} \tau' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\zeta & -\sinh\zeta \\ -\sinh\zeta & \cosh\zeta \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$\frac{d\tau'}{d\tau} = \gamma = \cosh\zeta$$

$$\Rightarrow \quad \tau' = \tau\cosh\zeta - x\sinh\zeta$$
$$x' = -\tau\sinh\zeta + x\cosh\zeta$$

$d\tau' = d\tau\cosh\zeta - dx\sinh\zeta$, $dx = 0 \Rightarrow d\tau' = d\tau\cosh\zeta$

$$\cosh\xi = \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

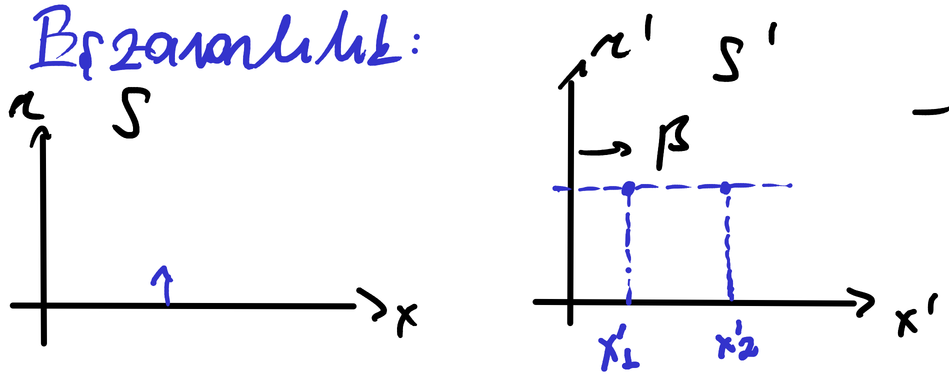
$$dx' = dx\cosh\xi - d\tau\sinh\xi \quad , \; dx \to$$

$$dx' = -d\tau\sinh\xi \Rightarrow dx' = -\frac{d\tau'}{\gamma}\sinh\xi$$

$$\Rightarrow \gamma \underbrace{\frac{dx'}{d\tau'}}_{\beta} = -\sinh\xi \Rightarrow \sinh\xi = -\gamma\beta$$

$$L = \begin{pmatrix} \cosh\xi & -\sinh\xi \\ -\sinh\xi & \cosh\xi \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

– Eşzamanlılık:



$\to \Delta\tau' = 0$

$\Delta x' = x_2' - x_1' \neq 0$

$$\begin{pmatrix} \tau \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \tau' \\ x' \end{pmatrix} \Rightarrow \begin{aligned} \tau &= \gamma\tau' + x'\gamma\beta \\ x &= \tau'\gamma\beta + x'\gamma \end{aligned}$$

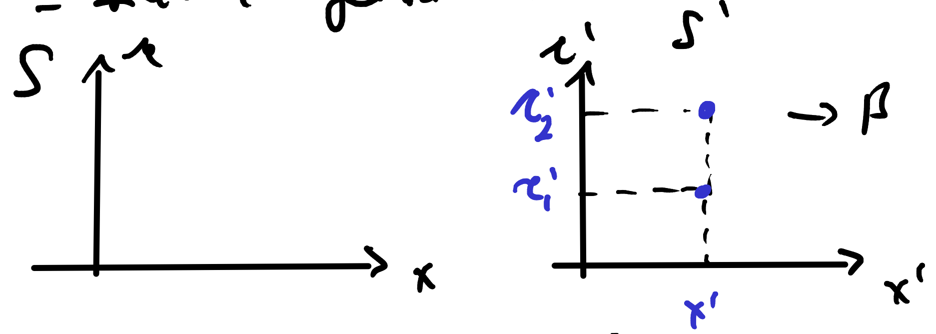
$$\begin{aligned} \tau_1 &= \gamma\tau_1' + \gamma\beta x_1' \\ \tau_2 &= \gamma\tau_2' + \gamma\beta x_2' \end{aligned} \Rightarrow \underbrace{\tau_2 - \tau_1}_{} = \gamma(\overbrace{\tau_2' - \tau_1'}^{0}) + \gamma\beta(x_2' - x_1')$$

$$\Delta\tau = \tau_2 - \tau_1 = \gamma\beta\Delta x' \checkmark$$

$ds^2 < 0 \to$ timelike..

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + dz^2$$

- Zaman genişlemesi:



$\Delta x' = 0$

$\Delta \tau' = \tau_2' - \tau_1' \neq 0$

$$\begin{pmatrix} \tau \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \tau' \\ x' \end{pmatrix}$$

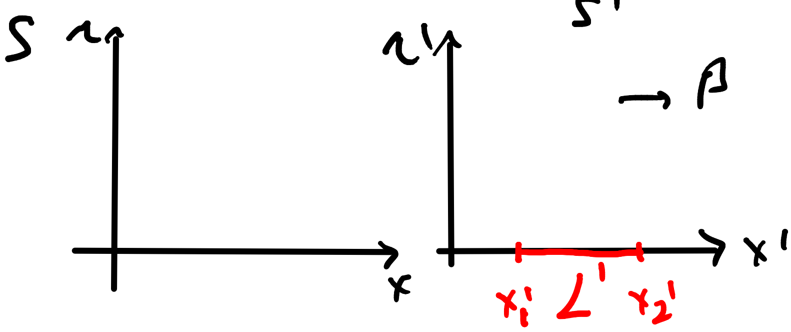
$$\tau = \gamma \tau' + \gamma\beta x' \quad \Rightarrow \quad \begin{aligned} \tau_2 &= \gamma \tau_2' + \gamma\beta x_2' \\ \tau_1 &= \gamma \tau_1' + \gamma\beta x_1' \end{aligned}$$

$$\Delta \tau = \tau_2 - \tau_1 = \gamma \underbrace{(\tau_2' - \tau_1')}_{\Delta \tau'} + \gamma\beta \overbrace{(x_2' - x_1')}^{0}$$

$$\Rightarrow \quad \Delta \tau = \gamma \Delta \tau' \quad , \gamma > 1$$

$$\Delta \tau > \Delta \tau'$$

- Uzunluk daralması:



$\Delta \tau' = 0$

$$\begin{pmatrix} \tau' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix} \Rightarrow \begin{aligned} \tau' &= \gamma \tau - \gamma\beta x \\ x' &= -\gamma\beta \tau + \gamma x \end{aligned}$$

$$\begin{aligned} x_2' &= -\gamma\beta \tau_2 + \gamma x_2 \\ x_1' &= -\gamma\beta \tau_1 + \gamma x_1 \end{aligned} \Rightarrow (x_2' - x_1') = -\gamma\beta(\tau_2 - \tau_1) + \gamma(x_2 - x_1) \ldots (1)$$

$$\begin{aligned} \tau_2' &= \gamma \tau_2 - \gamma\beta x_2 \\ \tau_1' &= \gamma \tau_1 - \gamma\beta x_1 \end{aligned} \Rightarrow \tau_2' - \tau_1' = 0 = \gamma(\tau_2 - \tau_1) - \gamma\beta(x_2 - x_1)$$

$$\Rightarrow \gamma(\tau_2-\tau_1)=\gamma\beta(x_2-x_1) \quad \cdots (2)$$

(2)'yi (1)'de yerine koyalım:

$$\underbrace{x_2'-x_1'}_{L'} = -\beta\left(\gamma\beta(\underbrace{x_2-x_1}_{L})\right)+\gamma(\underbrace{x_2-x_1}_{L})$$

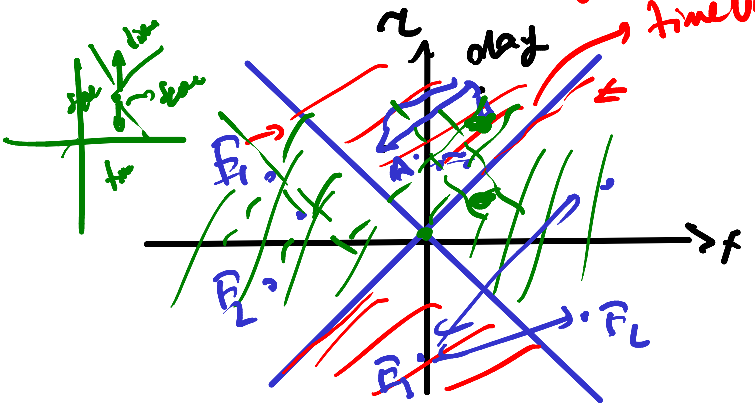
$$L' = -\gamma\beta^2 L+\gamma L = \gamma(1-\beta^2)L$$

$$= \frac{1}{\sqrt{1-\beta^2}}(1-\beta^2)L = \sqrt{1-\beta^2}\,L = \frac{1}{\gamma}L$$

$$\Rightarrow \boxed{L'=\frac{L}{\gamma}} \quad , \gamma>1 \qquad L>L'$$

$$\boxed{L=\gamma L'}$$

Bölüm-21 Uzay zaman geometrisi



$$ds^2=-d\tau^2+dx^2$$

$ds^2<0 \rightarrow$ timelike

$ds^2=0 \rightarrow$ null

$ds^2>0 \rightarrow$ spacelike

$$\Delta S_{AB}^2=-(\tau_B-\tau_A)^2+(x_B-x_A)^2<0$$

Nedensellik: (Causal structure)

i-) $\Delta S_{AB}^2>0$: Spacelike , E_A ve $E_B \rightarrow$

ii-) $\Delta S_{AB}^2=0$: null ,

iii-) $\Delta S_{AB}^2<0$: timelike , E_A ve E_B

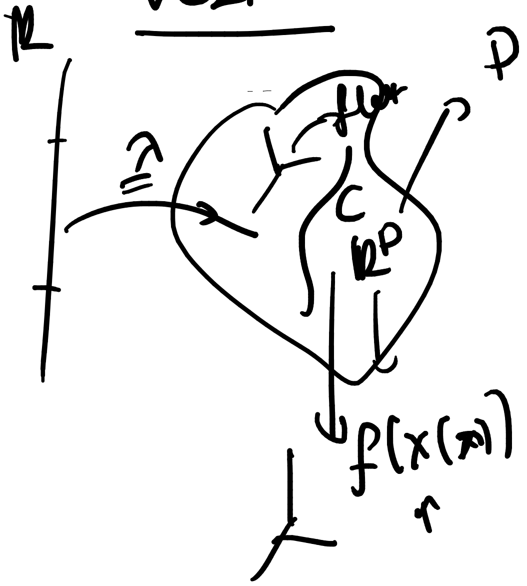
space-like { E_A ve E_B olayı aynı anda meydana gelip ama farklı pozisyonlarda

null { E_A ve E_B olayı

Postula-1: Işık hızı tüm eylemsiz referans çerçevelerinde aynıdır. $c = 3\times10^8$ m/s

time-like { E_A ve E_B aynı pozisyonda farklı zamanlarda olabilir.

Vektör:



$C : \mathbb{R} \to M$

$f : \mathbb{R}^D \to \mathbb{R}$

$$\frac{df}{d\lambda} = \frac{\partial f}{\partial x^i}\underbrace{\frac{dx^i}{d\lambda}} = X[f] \leftarrow$$

vector bileşen

baz vektörler: $\frac{\partial}{\partial x^i}$

$$X = X^i \hat{e}_i = \frac{dx^i}{d\lambda}\frac{\partial}{\partial x^i}$$

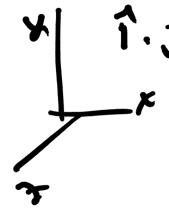
$$X[f] = \frac{dx^i}{d\lambda}\frac{\partial f}{\partial x^i}$$

$\{\hat{e}_i\} \in T_p(M) \to$ baz vektörleri

$\{\bar{e}^i\} \in T_p^*(M) \to$ dual bazis vektörler

$\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$



İç çarpım: $\langle \hat{e}_j, \bar{e}^i \rangle = \delta^i_j$

$\langle \frac{\partial}{\partial x^i}, dx^j \rangle = \delta^i_j$, dx^i: one-form

$X \in T_p(M) \to X = \alpha^i \frac{\partial}{\partial x^i}$ co-variant

$\bar{\omega} \in T_p^*(M) \to \bar{\omega} = \omega_i dx^i$ contra-variant

$\partial_i = \frac{\partial}{\partial x^i}$

$$a^i = L^i{}_j \alpha^j$$

$$\omega_i = L_i{}^j \omega_j$$

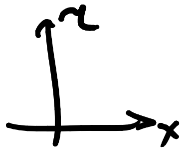
$\Rightarrow L^i{}_k L^k{}_j = \delta^i_j$

$$\langle \vec{v}, \bar{\omega} \rangle = \langle v^i \partial_i , \omega_j dx^j \rangle = v^i \omega_j \underbrace{\langle \partial_i , \omega^j \rangle}_{\delta^i_j} = v^i \omega_i$$

Metric tensor: $g : T_p(M) \otimes T_p(M)$

$$g = g_{ij} dx^i dx^j$$

$g_{ij} = g(\partial_i , \partial_j)$ ve $g(\vec{u}, \vec{v}) = g_{ij} v^i u^j$



Uzay zamanda vektörler:

$\vec{V} = V^\mu \frac{\partial}{\partial x^\mu} = V^t \frac{\partial}{\partial t} + V^i \frac{\partial}{\partial x^i}$, $L = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$

$L^{0'}{}_0 = \gamma$

$L^{0'}{}_i = -\gamma\beta^i$, $L^{i'}{}_0 = -\gamma\beta^i$ $\Rightarrow$ En genel Lorentz Boost

$L^{i'}{}_j = (\gamma - 1) \frac{\beta^i \beta^j}{\beta^2} + \delta^{ij}$, $i,j = 1,2,3,\dots$

$(\gamma - 1)\frac{\cancel{\beta^2}}{\cancel{\beta^2}} + 1 = \gamma$

$$L = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

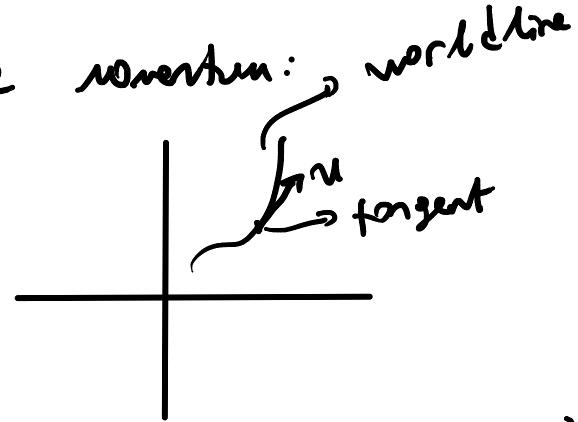
$$V^{\mu'} = L^{\mu'}{}_{\nu} V^{\nu} \quad , \mu, \nu = 0, 1, 2, 3$$

Öklid uzayı → $\vec{u} \cdot \vec{v} = v^1 v^1 + v^2 v^2 + v^3 v^3 = \underline{\delta_{ij}} v^i v^j$ $\quad \eta_{\mu\nu} = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

$$\vec{u} \cdot \vec{v} = \eta_{\mu\nu} u^\mu u^\nu = \eta_{00} u^0 v^0 + \eta_{11} v^1 u^1 + \cdots$$
$$= -u^0 v^0 + v^1 u^1 + v^2 u^2 + \cdots$$
$$= -u^0 v^0 + \sum_{i=1}^{D} u^i v^i$$

$$v^2 = \vec{v} \cdot \vec{v} = \eta_{\mu\nu} v^\mu v^\nu = -(v^0)^2 + \sum_{i=1}^{D} (v^i)^2 = \begin{cases} < 0, & \text{timelike} \\ = 0, & \text{null} \\ > 0, & \text{spacelike} \end{cases}$$

- Uzay zamanda hız ve momentum:



u : timelike

$g(u,u) < 0$ →

Eğri → $C(\lambda) = (\tau(\lambda), x^1(\lambda), x^2(\lambda), \ldots, x^D(\lambda))$

λ: proper time → , $\tau = ct$

zaman

$$-d\lambda^2 = -d\tau^2 + \sum (dx^i)^2 \quad / \quad d\lambda^2\text{'ye bölelim}$$

$$-1 = -\left(\frac{d\tau}{d\lambda}\right)^2 + \sum \left(\frac{dx^i}{d\lambda}\right)^2$$

$$u^\mu = \frac{dx^\mu}{d\lambda} \rightarrow u^0 = \frac{dx^0}{d\lambda} = \frac{d\tau}{d\lambda} \quad , \quad u^i = \frac{dx^i}{d\lambda}$$

$$u = u^\mu \frac{\partial}{\partial x^\mu}$$ ⇝ uzayzamanda parçacığın worldline'ına tanjant olan vektör.

$$u^2 = \eta_{\mu\nu} u^\mu u^\nu = -1$$

$d=4$ için $\quad u$: four-velocity

$$\frac{dx^i}{d\lambda} = \frac{dx^i}{d\tau}\underbrace{\frac{d\tau}{d\lambda}}_{\gamma} \quad , \quad d\tau = \gamma\, d\lambda$$

time dilation

$$= \gamma \frac{dx^i}{d\tau}$$

$$u = u^\mu \partial_\mu = u^0 \partial_0 + u^i \partial_i \quad , \quad u^\mu = \frac{dx^\mu}{d\lambda}$$

$$= \frac{dx^0}{d\lambda}\partial_0 + \frac{dx^i}{d\lambda}\partial_i$$

$$x^0 = \tau = ct$$

$$= \frac{dx^0}{d\tau}\underbrace{\frac{d\tau}{d\lambda}}_{\gamma}\partial_0 + \underbrace{\frac{dx^i}{d\tau}}\underbrace{\frac{d\tau}{d\lambda}}_{\gamma}\partial_i \quad , \quad \frac{dx^i}{d\tau} = \beta^i = \frac{1}{c}\frac{dx^i}{dt}$$

$$= \gamma \frac{\partial}{\partial \tau} + \gamma \beta^i \frac{\partial}{\partial x^i} \quad , \quad u^\mu = (\gamma, \gamma\beta^i)$$

$$p = mu \quad , \quad u^2 = \eta_{\mu\nu} u^\mu u^\nu = -1 \quad , \quad u^\mu = (\gamma, \gamma\beta^i)$$

$$p^2 = \eta_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + |\vec{p}|^2 = -m^2$$

$$p^0 = \gamma m$$

$$p^i = \gamma m \vec{\beta}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$, çok küçük β için

$$\gamma = (1-\beta^2)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \dots$$

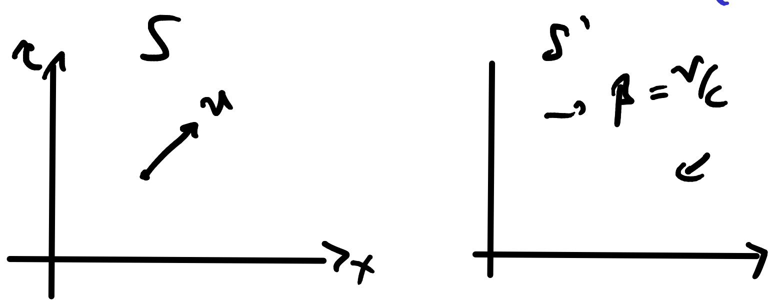
$$p^0 \simeq m\left(1+\frac{1}{2}\beta^2+\dots\right) = m\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) = m + \frac{1}{2} m \frac{v^2}{c^2}$$

$$= \frac{1}{c^2}\left(mc^2 + \frac{1}{2} m v^2 \right)$$

$$\vec{p} \sim m\left(1+\frac{1}{2}\beta^2+\dots\right)\beta = m\frac{\vec{v}}{c} + \dots$$

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) \quad , \quad E = \gamma m c^2, \quad \vec{p} = \gamma m \vec{v}$$

- Hızların Lorentz Boost'u (dönüşümü)



$$\gamma_u = \frac{1}{\sqrt{1-\beta_u^2}} \quad , \quad \beta_u = \frac{u}{c}$$

u'nun minkowski koordinatları $(\underbrace{\gamma_u}_{u^0}, \underbrace{\gamma_u \vec{\beta}_u}_{u^i})$

$$\rightarrow u^{\mu'} = L^{\mu'}{}_{\nu} u^\nu$$

$$\begin{cases} L^{0'}{}_0 = \gamma \\ L^{0'}{}_i = -\gamma\beta^i \quad , \quad L^{i'}{}_0 = -\gamma\beta^i \\ L^{i'}{}_j = (\gamma-1)\dfrac{\beta^i\beta^j}{\beta^2} + \delta^{ij} \end{cases}$$

$$(\gamma-1)\frac{\cancel{\beta^2}}{\cancel{\beta^2}} + 1 = \gamma$$

$$u^{0'} = L^{0'}{}_\nu u^\nu$$
$$= L^{0'}{}_0 u^0 + L^{0'}{}_i u^i$$
$$= \gamma u^0 - \gamma \beta^i u^i$$
$$= \gamma\gamma_u - \gamma \vec{\beta}^i \gamma_u \beta_u^i \quad ,$$

$$\vec{\beta} = \frac{v}{c} \quad , \quad \vec{\beta}_u = \frac{u}{c}$$

$$= \gamma\gamma_u - \gamma\gamma_u \vec{\beta}\cdot\vec{\beta}_u$$

$$= \gamma\gamma_u (1 - \vec{\beta}\cdot\vec{\beta}_u) = \frac{(1-\vec{\beta}\cdot\vec{\beta}_u)}{\sqrt{1-\beta^2}\sqrt{1-\beta_u^{\,2}}}$$

[illegible] $u^{0'}$ in ← gamma

$$u^{i'} = L^{i'}{}_{\mu} u^{\mu} = L^{i'}{}_{0} u^0 + L^{i'}{}_{j} u^j$$

$$= -\gamma\beta^i u^0 + \left((\gamma-1)\underline{\underline{\frac{\beta^i\beta^j}{\beta^2}}} + \delta^{ij}\right) u^j$$

$$= -\gamma\beta^i u^0 + (\gamma-1)\frac{\beta^i \underline{\beta^j}}{\beta^2}\underline{u^j} + u^i$$

$$u^{i'} = u^i - \gamma\beta^i u^0 + (\gamma-1)\frac{\vec{\beta}\cdot\vec{u}}{\beta^2}\beta^i$$

$$\vec{\beta} = \beta\hat{e}_x \qquad , \vec{\beta}\cdot\vec{u} = \beta u^x$$

$$u^{x'} = u^x - \gamma\beta u^0 + (\gamma-1)\frac{\cancel{\beta} u^x}{\cancel{\beta^2}}\cancel{\beta}$$

$$= \cancel{u^x} - \gamma\beta u^0 + \gamma u^x \cancel{- u^x}$$

$$= \gamma u^x - \gamma\beta u^0 = \gamma(u^x - \beta u^0) \qquad , u = (\gamma_u, \gamma_u\vec{\beta}_u)$$

$$= \gamma\left(\gamma_u(\beta_u)^x - \beta\gamma_u\right) = \gamma\gamma_u\left((\beta_u)^x - \beta\right) , \beta_u^x = \frac{u^x}{c}$$

$$(\beta_u)^{x'} = \frac{u^{x'}}{u^{0'}} = \frac{\cancel{\gamma}\gamma_u\left((\beta_u)^x - \beta\right)}{\cancel{\gamma}\gamma_u(1-\vec{\beta}\cdot\vec{\beta}_u)} = \frac{(\beta_u)^x - \beta}{1-\vec{\beta}\cdot\vec{\beta}_u} \leftarrow \text{boost un}$$

$$(\beta_u)^{y'} = \frac{u^{y'}}{u^{0'}} = \frac{\gamma_u(\beta_u)^y}{\gamma\gamma_u(1-\vec{\beta}\cdot\vec{\beta}_u)} = \frac{(\beta_u)^y}{\gamma(1-\vec{\beta}\cdot\vec{\beta}_u)} \leftarrow \text{boost sol.}$$