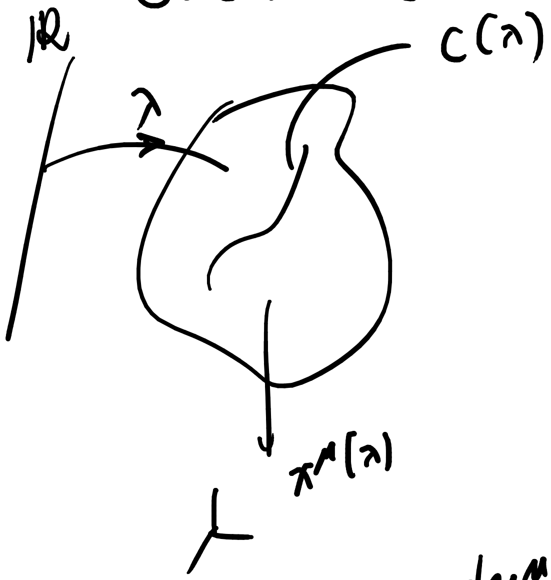
Böl-3| Uzay zamanda ivmeli hareket

$$\vec{F} = m\vec{a} = m\frac{d^2x}{dt^2} \rightarrow \text{Hareket denklemi}$$

Göreli ivme:



$$u^\mu = \frac{dx^\mu}{d\lambda}$$

$$a^\mu = \frac{du^\mu}{d\lambda}$$

$$\partial^\alpha = \eta^{\alpha\beta}\partial_\beta$$
$$\downarrow$$

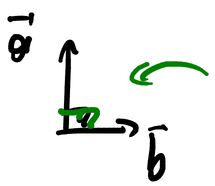
$$a^\mu = \frac{du^\mu}{d\lambda} = \frac{\partial u^\mu}{\partial x^\alpha}\underbrace{\frac{dx^\alpha}{d\lambda}}_{u^\alpha} = u^\alpha \partial_\alpha u^\mu \quad , \quad \partial_\alpha = \frac{\partial}{\partial x^\alpha}$$

$$a = u \cdot \partial u \rightarrow \text{bazdan bağımsız gösterim}$$

$$a^{\mu'} = \Lambda^{\mu'}{}_{\nu}\, a^\nu$$

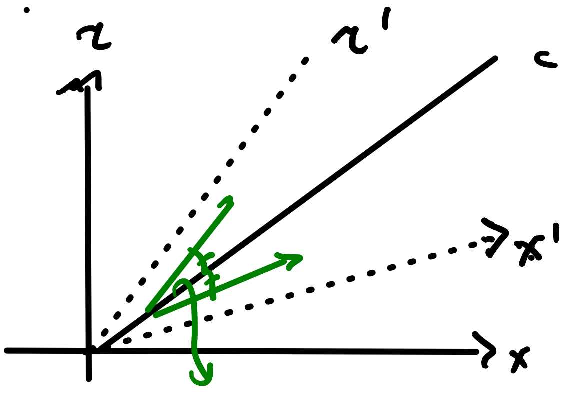
Uzay zaman ivmesi 'a' her zaman uzayzaman hız vektörüne 'u' diktir.

?



$$\vec{a} \cdot \vec{b} = 0$$

$$a \cdot b = \eta_{\mu\nu} a^\mu b^\nu = 0$$
$$= -a^0 b^0 + a^1 b^1 + a^2 b^2 + \ldots$$
$$= 0$$



Minkowski diyagramı

<u>İspat:</u> $\rightarrow u.u=-1$ $\quad u^\mu=(\gamma,\gamma\vec{\beta})$

$$u.u=\eta_{\alpha\beta}u^\alpha u^\beta=-(u^0)^2+(u^i)^2$$

$$=-\gamma^2+\gamma^2\beta^2=-\gamma^2(1-\beta^2)$$

$$=-\frac{1}{1-\beta^2}(1-\beta^2)=-1$$

$$u.a=\underline{u}.\,(\underline{u.\partial u})=\eta_{\mu\nu}u^\mu\left(u^\alpha\partial_\alpha u^\nu\right)$$

$$=u^\alpha\left(\eta_{\mu\nu}u^\mu\partial_\alpha u^\nu\right)=u^\alpha\frac{1}{2}\partial_\alpha\left(\eta_{\mu\nu}u^\mu u^\nu\right)$$

$$\frac{1}{2}\frac{d}{dx}(\beta^2)=\frac{1}{2}2\beta\frac{d\beta}{dx}$$

$$\eta_{\mu\nu}\partial_\alpha u^\mu u^\nu+\eta_{\mu\nu}\partial_\alpha u^\nu u^\mu$$

$$u.a=u^\alpha\frac{1}{2}\partial_\alpha\big(\underbrace{\eta_{\mu\nu}u^\mu u^\nu}_{-1}\big)=0$$

Göreli Kuvvet:

serbest parçacık

$$\frac{du}{d\lambda}=u.\partial u=0$$

En basit çözüm $\rightarrow$ $u=(1,0,\dots,0)\leftarrow$

$\rightarrow u=\frac{dx}{d\lambda}\rightarrow$ çözüyor olması lazım

$\rightarrow x^\mu(\lambda)=(\lambda,0,0,\dots,0)\leftarrow$

Öteleme vektör;

$$\rightarrow \Delta x^\mu = (c^0, c^1, \dots, c^D) \quad , c^\mu : \text{sabit}$$

$$x^\mu(\lambda) + \Delta x^\mu = (\lambda + c^0, c^1, c^2, \dots, c^D)$$

Lorentz dönüşümü uygulayalım

(S') çerçevesi (S)'e göre $-\vec{\beta}$ hızıyla hareket ediyor (x^1-yönünde), $x^1 = 0$

$$\tau'(\lambda) = \gamma\tau(\lambda) + \gamma\beta x^1(\lambda) = \gamma\lambda$$

$$x'(\lambda) = \gamma\beta\tau(\lambda) + \gamma x^1(\lambda) = \gamma\beta\lambda$$

(S') çerçevesinde

$$x^\mu(\lambda) = (\gamma\lambda, \gamma\beta\lambda, 0, \dots, 0)$$

Sabit ivmeli hareket eden parçacık:

normalize $\rightarrow$ $-\tau^2 + x^2 = \alpha^2 \leftarrow$ (hiperbolik denk.) , $x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$

$$\tau(\lambda) = \alpha \sinh\left(\frac{\lambda}{\alpha}\right)$$

$$x(\lambda) = \alpha \cosh(\lambda/\alpha)$$

$$x^\mu(\lambda) = (\alpha\sinh(\lambda/\alpha), \alpha\cosh(\lambda/\alpha)) \leftarrow$$

$$u(\lambda) = \left(\frac{d\tau}{d\lambda}, \frac{dx}{d\lambda}\right) = \left(\alpha\frac{1}{\alpha}\cosh(\lambda/\alpha), \alpha\frac{1}{\alpha}\sinh(\lambda/\alpha)\right)$$

$$= (\cosh(\lambda/\alpha), \sinh(\lambda/\alpha))$$

$$a(\lambda) = \left(\frac{d^2 t}{d\lambda^2}, \frac{d^2 x}{d\lambda^2} \right) = \left(\frac{1}{\alpha} \sinh(\lambda/\alpha), \frac{1}{\alpha} \cosh(\lambda/\alpha) \right)$$

$$a \cdot a = \eta_{\mu\nu} a^\mu a^\nu = -(a^0)^2 + (a^1)^2$$

$$= -\frac{1}{\alpha^2} \sinh^2(\lambda/\alpha) + \frac{1}{\alpha^2} \cosh^2(\lambda/\alpha)$$

$$= \frac{1}{\alpha^2} \left(\cosh^2(\lambda/\alpha) - \sinh^2(\lambda/\alpha) \right)$$

$$= \frac{1}{\alpha^2} \quad ,$$

① $dl^2 = dx^2 + dy^2$, $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ | Rosha Men. İU.

$$x' = x\cos\theta + y\sin\theta \quad \rightarrow \quad dx' = dx\cos\theta + dy\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta \quad \quad dy' = -dx\sin\theta + dy\cos\theta$$

$$dl'^2 = dx'^2 + dy'^2$$

$$= (dx\cos\theta + dy\sin\theta)^2 + (-dx\sin\theta + dy\cos\theta)^2$$

$$= dx^2 + dy^2 \quad \checkmark .$$

② $\vec{V} = \underline{\underline{V}}^x e_x + \underline{V}^y e_y = \tilde{V}^x \hat{e}_{\tilde{x}} + \tilde{V}^y \hat{e}_{\tilde{y}}$ | Rukiye Kartal İMU

$$= \tilde{V}^x (e_x\cos\theta + e_y\sin\theta) + \tilde{V}^y(-e_x\sin\theta + e_y\cos\theta)$$

$$= \underline{\underline{(\tilde{V}^x\cos\theta - \tilde{V}^y\sin\theta)}}e_x + \underline{(\tilde{V}^x\sin\theta + \tilde{V}^y\cos\theta)}e_y$$

$$V^x = \tilde{V}^x\cos\theta - \tilde{V}^y\sin\theta \quad / \cos\theta \quad / \sin\theta$$

$$V^y = \tilde{V}^x\sin\theta + \tilde{V}^y\cos\theta \quad / \sin\theta \quad / +\cos\theta$$

③ Mustafa Ateş ODTÜ

$$e^{x}=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\cdots$$

$$x=\theta A$$

$$A=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\quad A^2=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=-I$$

$$A^3=-IA^3=-A,\quad A^4=I,\quad A^5=A,\quad A^6=-I$$

$$e^{\theta A}=\sum_{n=0}^{\infty}\frac{(\theta A)^n}{n!}=I+\theta A+\frac{1}{2!}\theta^2A^2+\frac{1}{3!}\theta^3A^3+\cdots$$

$$=I\Big(\underbrace{\sum_{n=0}^{\infty}(-1)^n\frac{\theta^{2n}}{(2n)!}}_{\cos\theta}\Big)+\Big(\underbrace{\sum_{n=0}^{\infty}(-1)^n\frac{\theta^{2n+1}}{(2n+1)!}}_{\sin\theta}\Big)A$$

$$e^{\theta A}=I\cos\theta+A\sin\theta=\begin{pmatrix}\cos\theta&0\\0&\cos\theta\end{pmatrix}+\begin{pmatrix}0&\sin\theta\\-\sin\theta&0\end{pmatrix}$$

$$=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$$

④ $S=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$, $e^{\xi S}=?$

$$S^2=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}0&-1\\-1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$$

$$S^3=S,\quad S^4=I,\quad S^5=S,\quad S^6=I$$

$$e^{\xi S}=I+\xi S+\frac{1}{2!}\xi^2I+\frac{1}{3!}\xi^3S+\cdots$$

$$=\Big(\underbrace{\sum_{n=0}^{\infty}\frac{\xi^{2n}}{(2n)!}}_{\cosh\xi}\Big)I+\Big(\underbrace{\sum_{n=0}^{\infty}\frac{\xi^{2n+1}}{(2n+1)!}}_{\sinh\xi}\Big)S$$

$$= \begin{pmatrix} \cosh\xi & 0 \\ 0 & \cosh\xi \end{pmatrix} + \begin{pmatrix} 0 & -\sinh\xi \\ -\sinh\xi & 0 \end{pmatrix} = \begin{pmatrix} \cosh\xi & -\sinh\xi \\ -\sinh\xi & \cosh\xi \end{pmatrix}$$

Maxwell denklemleri:

$$\vec{\nabla}\cdot\vec{E} = 0 \quad , \quad \vec{\nabla}\cdot\vec{B} = 0$$

$$\rightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad , \quad \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad , \quad \mu_0\varepsilon_0 = \frac{1}{c^2}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0}) - \vec{\nabla}^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow -\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \nabla^2\vec{E} = 0 \rightarrow \text{Dalga denklemi}$$

$$\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} - \nabla^2\vec{B} = 0 \rightarrow \text{Dalga denk.}$$

(5) Merve Dedeoğlu (Ege Uni)

(6) Bilge Tin Ören (İTÜ)

$$\frac{\partial\psi}{\partial t'} = \frac{\partial\psi}{\partial t} + v\frac{\partial\psi}{\partial x} \quad , \quad \frac{\partial\psi}{\partial x'} = \frac{\partial\psi}{\partial x} \quad , \quad \frac{\partial\psi}{\partial y'} = \frac{\partial\psi}{\partial y}$$

$$\frac{\partial\psi}{\partial t'} = \underline{\chi} \quad , \quad \frac{\partial^2\psi}{\partial t'^2} = \frac{\partial\chi}{\partial t'}$$

$$\frac{\partial\chi}{\partial t'} = \frac{\partial\chi}{\partial t} + v\frac{\partial\chi}{\partial x} = \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t'}\right) + v\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial t'}\right)$$

$$= \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t} + v\frac{\partial\psi}{\partial x}\right) + v\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial t} + v\frac{\partial\psi}{\partial x}\right)$$

$$= \frac{\partial^2\psi}{\partial t^2} + 2v\frac{\partial^2\psi}{\partial t\partial x} + v^2\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial t'^2}$$

$$\frac{\partial^2\psi}{\partial x'^2} = \frac{\partial^2\psi}{\partial x^2}, \quad \frac{\partial^2\psi}{\partial y'^2} = \frac{\partial^2\psi}{\partial y^2}$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x'^2} + \frac{\partial^2\psi}{\partial y'^2} - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t'^2} = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - \frac{1}{c^2}\left(\frac{\partial^2\psi}{\partial t^2} + 2v\frac{\partial^2\psi}{\partial t\partial x} + v^2\frac{\partial^2\psi}{\partial x^2}\right)$$

$$= \left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - \frac{2v}{c^2}\frac{\partial^2\psi}{\partial t\partial x} - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

(7) Mustafa Ates (ODTÜ)

$$\tau = \tau'\cosh\zeta + x'\sinh\zeta$$

$$x = \tau'\sinh\zeta + x'\cosh\zeta$$

$$y = y'$$

$$\frac{\partial\psi}{\partial\tau'} = \frac{\partial\psi}{\partial\tau}\frac{\partial\tau}{\partial\tau'} + \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial\tau'} + \frac{\partial\psi}{\partial y}\cancelto{0}{\frac{\partial y}{\partial\tau'}} = \cosh\zeta\frac{\partial\psi}{\partial\tau} + \sinh\zeta\frac{\partial\psi}{\partial x}$$

$$\frac{\partial\psi}{\partial x'} = \frac{\partial\psi}{\partial\tau}\frac{\partial\tau}{\partial x'} + \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial\psi}{\partial y}\cancelto{0}{\frac{\partial y}{\partial x'}} = \sinh\zeta\frac{\partial\psi}{\partial\tau} + \cosh\zeta\frac{\partial\psi}{\partial x}$$

$$\frac{\partial\psi}{\partial y'} = \frac{\partial\psi}{\partial\tau}\cancelto{0}{\frac{\partial\tau}{\partial y'}} + \frac{\partial\psi}{\partial x}\cancelto{0}{\frac{\partial x}{\partial y'}} + \frac{\partial\psi}{\partial y}\frac{\partial y}{\partial y'} = \frac{\partial\psi}{\partial y}$$

$$\frac{\partial\psi}{\partial\tau'} = \chi, \quad \frac{\partial^2\psi}{\partial\tau'^2} = \frac{\partial\chi}{\partial\tau'}$$

$$\Rightarrow \frac{\partial^2\psi}{\partial\tau'^2} = \frac{\partial\chi}{\partial\tau'} = \cosh\zeta\frac{\partial\chi}{\partial\tau} + \sinh\zeta\frac{\partial\chi}{\partial x}$$

$$= \cosh\zeta\frac{\partial}{\partial\tau}\left(\frac{\partial\psi}{\partial\tau'}\right) + \sinh\zeta\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial\tau'}\right)$$

$$= \cosh\zeta\frac{\partial}{\partial\tau}\left(\cosh\zeta\frac{\partial\psi}{\partial\tau} + \sinh\zeta\frac{\partial\psi}{\partial x}\right)$$

$$+\sinh\xi \frac{\partial}{\partial x}\left(\cosh\xi \frac{\partial \phi}{\partial \tau} + \sinh\xi \frac{\partial \phi}{\partial x}\right)$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial \tau'^2} = \cosh^2\xi \frac{\partial^2 \phi}{\partial \tau^2} + 2\cosh\xi \sinh\xi \frac{\partial^2 \phi}{\partial \tau \partial x} + \sinh^2\xi \frac{\partial^2 \phi}{\partial x^2}$$

$$\frac{\partial \phi}{\partial x'} = \frac{\partial \phi}{\partial \tau}\frac{\partial \tau}{\partial x'} + \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial y'} = \sinh\xi \frac{\partial \phi}{\partial \tau} + \cosh\xi \frac{\partial \phi}{\partial x}$$

$$\to \frac{\partial^2 \phi}{\partial x'^2} = \sinh^2\xi \frac{\partial^2 \phi}{\partial \tau^2} + \cosh^2\xi \frac{\partial^2 \phi}{\partial x^2} + 2\sinh\xi \cosh\xi \frac{\partial^2 \phi}{\partial \tau \partial x}$$

$$\frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial y'^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial \tau'^2} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial \tau^2} \quad ✓.$$

(8) Arda Ers (ODTÜ)

$$\tau = \frac{1}{2}(u+v), \quad x = \frac{1}{2}(u-v)$$

$$-d\tau^2 + dx^2 + dy^2 = -\frac{1}{4}(du+dv)^2 + \frac{1}{4}(du-dv)^2 + dy^2$$

$$= -\frac{1}{4}(\cancel{du^2} + \cancel{dv^2} + 2\,du\,dv)$$

$$+ \frac{1}{4}(\cancel{du^2} + \cancel{dv^2} - 2\,du\,dv) + dy^2$$

$$= -du\,dv + dy^2 \quad ✓.$$

$$ds^2 = -du\,dv + dy^2$$

(9) Uveys Turhan (Boğaziçi Üni)

$$L = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

$$S \to S'$$

$$\begin{pmatrix} \tau' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma_v & -\gamma_v \beta_v \\ -\gamma_v \beta_v & \gamma_v \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$S' \to S''$$

$$\to \begin{pmatrix} \tau'' \\ x'' \end{pmatrix} = \begin{pmatrix} \gamma_u & -\gamma_u \beta_u \\ -\gamma_u \beta_u & \gamma_u \end{pmatrix} \begin{pmatrix} \tau' \\ x' \end{pmatrix}$$

$$\begin{pmatrix} \tau'' \\ x'' \end{pmatrix} = \begin{pmatrix} \gamma_u & -\gamma_u \beta_u \\ -\gamma_u \beta_u & \gamma_u \end{pmatrix} \begin{pmatrix} \gamma_v & -\gamma_v \beta_v \\ -\gamma_v \beta_v & \gamma_v \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$= \gamma_u \gamma_v \begin{pmatrix} 1 + \beta_u \beta_v & -(\beta_u + \beta_v) \\ -(\beta_u + \beta_v) & 1 + \beta_u \beta_v \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$= \gamma_u \gamma_v (1 + \beta_u \beta_v) \begin{pmatrix} 1 & -\frac{\beta_u + \beta_v}{1 + \beta_u \beta_v} \\ -\frac{\beta_u + \beta_v}{1 + \beta_u \beta_v} & 1 \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$\Rightarrow \gamma_u \gamma_v (1 + \beta_u \beta_v) = \frac{\sqrt{(1 + \beta_u \beta_v)^2}}{\sqrt{1 - \beta_u^2}\,\sqrt{1 - \beta_v^2}}$$

$$= \frac{\sqrt{(1 + \beta_u \beta_v)^2}}{\sqrt{1 - \beta_u^2 - \beta_v^2 + \beta_u^2 \beta_v^2 + 2\beta_u \beta_v - 2\beta_u \beta_v}}$$

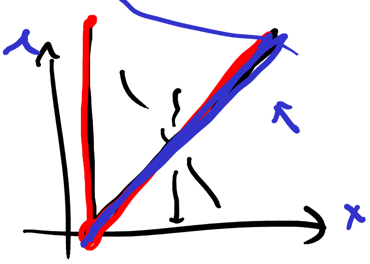
$$= \frac{\sqrt{(1 + \beta_u \beta_v)^2}}{\sqrt{(1 + \beta_u \beta_v)^2 - (\beta_u + \beta_v)^2}} = \sqrt{\frac{\cancel{(1 + \beta_u \beta_v)^2}}{\cancel{(1 + \beta_u \beta_v)^2} \left(1 - \left(\frac{\beta_u + \beta_v}{1 + \beta_u \beta_v} \right)^2 \right)}}$$

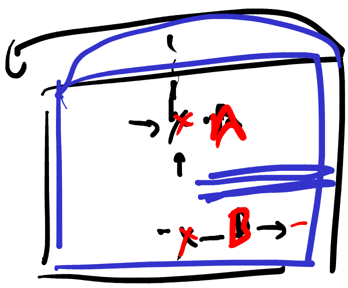
$$= \frac{1}{\sqrt{1-\left(\frac{\beta_u+\beta_v}{1+\beta_u\beta_v}\right)^2}}$$

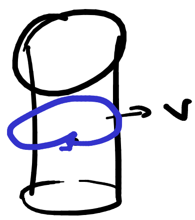
$$\begin{pmatrix} \tau'' \\ x'' \end{pmatrix} = \frac{1}{\sqrt{1-\left(\frac{\beta_u+\beta_v}{1+\beta_u\beta_v}\right)^2}} \begin{pmatrix} 1 & -\frac{\beta_u+\beta_v}{1+\beta_u\beta_v} \\ -\frac{\beta_u+\beta_v}{1+\beta_u\beta_v} & 1 \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$\begin{pmatrix} \tau'' \\ x'' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}$$

$$\gamma_w = \left(1-\left(\frac{\beta_u+\beta_v}{1+\beta_u\beta_v}\right)\right)^{-1/2}, \quad \beta_w = \frac{\beta_u+\beta_v}{1+\beta_u\beta_v}$$

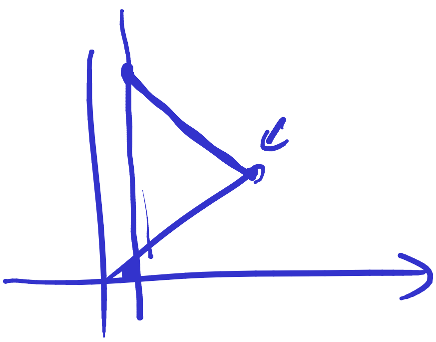






ds^2 invariant $\qquad ds^2 = -dt^2 + dx^2 + y \cdot$

g



$ds^2 < 0$

$R =$