Nötrino Fiziği III

Nötrino salınım deneyleri/sonuçları

Ödev I

- $\sigma_{\nu p} \sim 10^{-38}$ cm^2 E/GeV
- Atmosferik nötrinoların
  - $\Phi^{atm} = 1$ nötrino/cm^2.s and <E> = 1 GeV, $N_A = 6.022 \times 10^{23}$

  Vücudumuzda kaç kere etkileşim yaparlar ?

  - $N_{int} = \Phi_{\nu} \times \sigma_{\nu p} \times N^{hum} \times T^{hum}$
  - $N^{hum} = \frac{M_{hum}}{g} \times N_A = 80 \times 10^3 \times N_A \sim 5 \times 10^{28}$ proton .
  - $T^{hum} = 80$ years $= 2 \times 10^9$ sec
  - $N_{int} = 5 \times 10^{28}\ 2 \times 10^9 \times 10^{-38} \sim 1$ etkileşim

Ödev II

$$P_{\vartheta e \to \vartheta\mu} = sin^2(2\theta) sin^2\left(\frac{\Delta m^2 c^3 l}{4E\hbar}\right)$$

$$hc = 197.327 MeV fm = 1,$$
$$MeV.m = 5.0677 10^{12}$$

$$\frac{\Delta m^2}{4E} L = 1.2627 \left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

$$\frac{\Delta m^2}{4E} L = \frac{1}{4}\left(\frac{\Delta m^2 eV^2}{eV^2}\right)\left(\frac{L\, m}{m}\right)\left(\frac{MeV}{E}\frac{1}{MeV}\right)$$

$$\frac{\Delta m^2}{4E} L = \frac{1}{4}\left(\frac{1eV^2 1m}{1MeV}\right)\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right)$$

$$\frac{1}{4}\left(\frac{1eV^2 1m}{1MeV}\right) = \left(\frac{1}{4}\right) 10^{-12} \frac{MeV^2}{1MeV} 1m = \frac{10^{-12}}{4}(MeV.m) = 0.25 10^{-12} x 5.0677 10^{12} = 1.267$$

Ödev III

t=0 anında $|\nu_\alpha>$, saf nötrino özdurumunda, 3 kütle özdurumunun karışımı

$$|\psi(x=0)>= U_{\alpha 1}|\nu_1> + U_{\alpha 2}|\nu_2> + U_{\alpha 3}|\nu_3>$$

Dalga fonksiyonu zaman içerisindeki gelişimi

$$|\psi(t)>= U_{\alpha 1}|\nu_1> e^{-ip_1\cdot x} + U_{\alpha 2}|\nu_2> e^{-ip_2\cdot x} + U_{\alpha 3}|\nu_3> e^{-ip_3\cdot x}$$

4-momentum

$$p_i \cdot x = E_i t - \mathbf{p_i} \cdot \mathbf{x}$$

L kadar mesafe sonra dalga fonksiyonu

$$|\psi(L)>= U_{\alpha 1}|\nu_1> e^{-i\phi_1} + U_{\alpha 2}|\nu_2> e^{-i\phi_2} + U_{\alpha 3}|\nu_3> e^{-i\phi_3}$$

ÖdevIII

Relativistik nötrino kabülü

$$\phi_i = p_i \cdot x = E_i t - |p_i| L \approx (E_i - |p_i|) L.$$

$$E_i \approx p_i + \frac{m_i^2}{2E_i}$$

$$\phi_i = (E_i - |p_i|) L \approx \frac{m_i^2}{2E_i} L$$

$$\begin{aligned} |\psi(L)> = & U_{\alpha 1} e^{-i\phi_1} (U_{e1}^* |\nu_e> + U_{\mu 1}^* |\nu_\mu> + U_{\tau 1}^* |\nu_\tau>) \\ & + U_{\alpha 2} e^{-i\phi_2} (U_{e2}^* |\nu_e> + U_{\mu 2}^* |\nu_\mu> + U_{\tau 2}^* |\nu_\tau>) \\ & + U_{\alpha 3} e^{-i\phi_3} (U_{e3}^* |\nu_e> + U_{\mu 3}^* |\nu_\mu> + U_{\tau 3}^* |\nu_\tau>) \end{aligned}$$

ÖdevIII

$$|\psi(L)>=(U_{\alpha 1}U_{e1}^{*}e^{-i\phi_1}+U_{\alpha 2}U_{e2}^{*}e^{-i\phi_2}+U_{\alpha 3}U_{e3}^{*}e^{-i\phi_3})|\nu_e>$$
$$+(U_{\alpha 1}U_{\mu 1}^{*}e^{-i\phi_1}+U_{\alpha 2}U_{\mu 2}^{*}e^{-i\phi_2}+U_{\alpha 3}U_{\mu 3}^{*}e^{-i\phi_3})|\nu_\mu>$$
$$+(U_{\alpha 1}U_{\tau 1}^{*}e^{-i\phi_1}+U_{\alpha 2}U_{\tau 2}^{*}e^{-i\phi_2}+U_{\alpha 3}U_{\tau 3}^{*}e^{-i\phi_3})|\nu_\tau>$$

Salınım olasılığı:

$$P(\nu_\alpha \to \nu_\beta) = |<\nu_\beta|\psi(L)>|^2$$
$$= (U_{\alpha 1}U_{\beta 1}^{*}e^{-i\phi_1}+U_{\alpha 2}U_{\beta 2}^{*}e^{-i\phi_2}+U_{\alpha 3}U_{\beta 3}^{*}e^{-i\phi_3})^2$$

$$|z_1+z_2+z_3|^2 = |z_1|^2+|z_2|^2+|z_3|^2+2\Re(z_1z_2^*+z_1z_3^*+z_2z_3^*)$$

ÖdevIII

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$+ 2\sum_{i>i} Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) sin(\Delta m^2_{ij}\frac{L}{2E})$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

ÖdevIII

CP ihlalinin olmadığı durumda δ_{CP} =0,

$$P(\nu_\alpha \to \nu_\beta) \;=\; \delta_{\alpha\beta} - 4\sum_{i>j}(U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j})sin^2(\Delta m_{ij}^2\frac{L}{4E})$$

üç kütle özdurumuna karşılık iki bağımsız kütle karesi farkı var.

$$\Delta m_{23}^2 \text{ and } \Delta m_{12}^2.$$

$$\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$$

ÖdevIII

Deneysel ölçümlerden $\Delta m^2_{31} = 2.45 \times 10^{-3}$ eV^2 ve $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV^2

Görünür kanalda çeşni salınımı

$$P(\nu_\alpha \to \nu_\beta) = -4\sum_{i>j}(U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j})sin^2(1.27\Delta m^2_{ij}\frac{L}{E})$$

$$= -4[(U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\alpha 2})sin^2(1.27\Delta m^2_{12}\frac{L}{E})$$

$$+ (U_{\alpha 1}U_{\beta 1}U_{\alpha 3}U_{\alpha 3})sin^2(1.27\Delta m^2_{13}\frac{L}{E})$$

$$+ (U_{\alpha 2}U_{\beta 2}U_{\alpha 3}U_{\alpha 3})sin^2(1.27\Delta m^2_{23}\frac{L}{E})]$$

ÖdevIII

Şayet L/E küçükse (~0), $\Delta m^2_{12} = 8 \times 10^{-5}\mathrm{eV}^2$

$$sin^2(1.27\Delta m^2_{12}\frac{L}{E}) \to 0$$

$$\Delta m^2_{13} \approx \Delta m^2_{23}$$

$$P(\nu_\alpha \to \nu_\beta) = -4[U_{\alpha 1}U_{\beta 1}U_{\alpha 3}U_{\beta 3} + U_{\alpha 2}U_{\beta 2}U_{\alpha 3}U_{\beta 3}]sin^2(1.27\Delta m^2_{23}\frac{L}{E})$$

Karışım matriks değerlerini yerine yazınca

$$P(\nu_\mu \to \nu_\tau) = \cos^2(\theta_{13})\sin^2(2\theta_{23})\sin^2(1.27\Delta m^2_{23}\frac{L}{E})$$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(1.27\Delta m^2_{23}\frac{L}{E})$$

$$P(\nu_e \to \nu_\tau) = \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2(1.27\Delta m^2_{23}\frac{L}{E})$$

ÖdevIII

Şayet L/E büyükse, Δm^2_{23} ve Δm^2_{13} değişkenlerine sahip terimler hızlı salınım yapıp 0.5 ortalama değere geliyorlar.

$$sin^2(1.27\Delta m^2_{23}\frac{L}{E}) \rightarrow < sin^2(1.27\Delta m^2_{23}\frac{L}{E}) >= \frac{1}{2}$$

$$sin^2(1.27\Delta m^2_{13}\frac{L}{E}) \rightarrow < sin^2(1.27\Delta m^2_{13}\frac{L}{E}) >= \frac{1}{2}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4[(U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\beta 2})sin^2(1.27\Delta m^2_{12}\frac{L}{E}) + \frac{1}{2}(U_{\alpha 1}U_{\beta 1}U_{\alpha 3}U_{\beta 3} + U_{\alpha 2}U_{\beta 2}U_{\alpha 3}U_{\beta 3})]$$

$$P(\nu_e \rightarrow \nu_{\mu,\tau}) = \cos^2(\theta_{13})\sin^2(2\theta_{12})\ \sin^2(1.27\Delta m^2_{12}\frac{L}{E}) + \frac{1}{2}\sin^2(2\theta_{13})$$

ÖdevIII

Şayet $\theta_{13} \approx 0$

Küçük L/E durumu

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{23})\sin^2(1.27\Delta m^2_{23}\frac{L}{E})$$

$$P(\nu_e \rightarrow \nu_\mu) = 0$$

$$P(\nu_e \rightarrow \nu_\tau) = 0$$

Büyük L/E durumu

$$P(\nu_e \rightarrow \nu_{\mu,\tau}) = \sin^2(2\theta_{12})\,\sin^2(1.27\Delta m^2_{12}\frac{L}{E})$$

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