

Towards the ATLAS upgrade

LHC on the march

Protvino, November 2011

**A.Zaitsev, Protvino
for ATLAS collaboration**

Scope

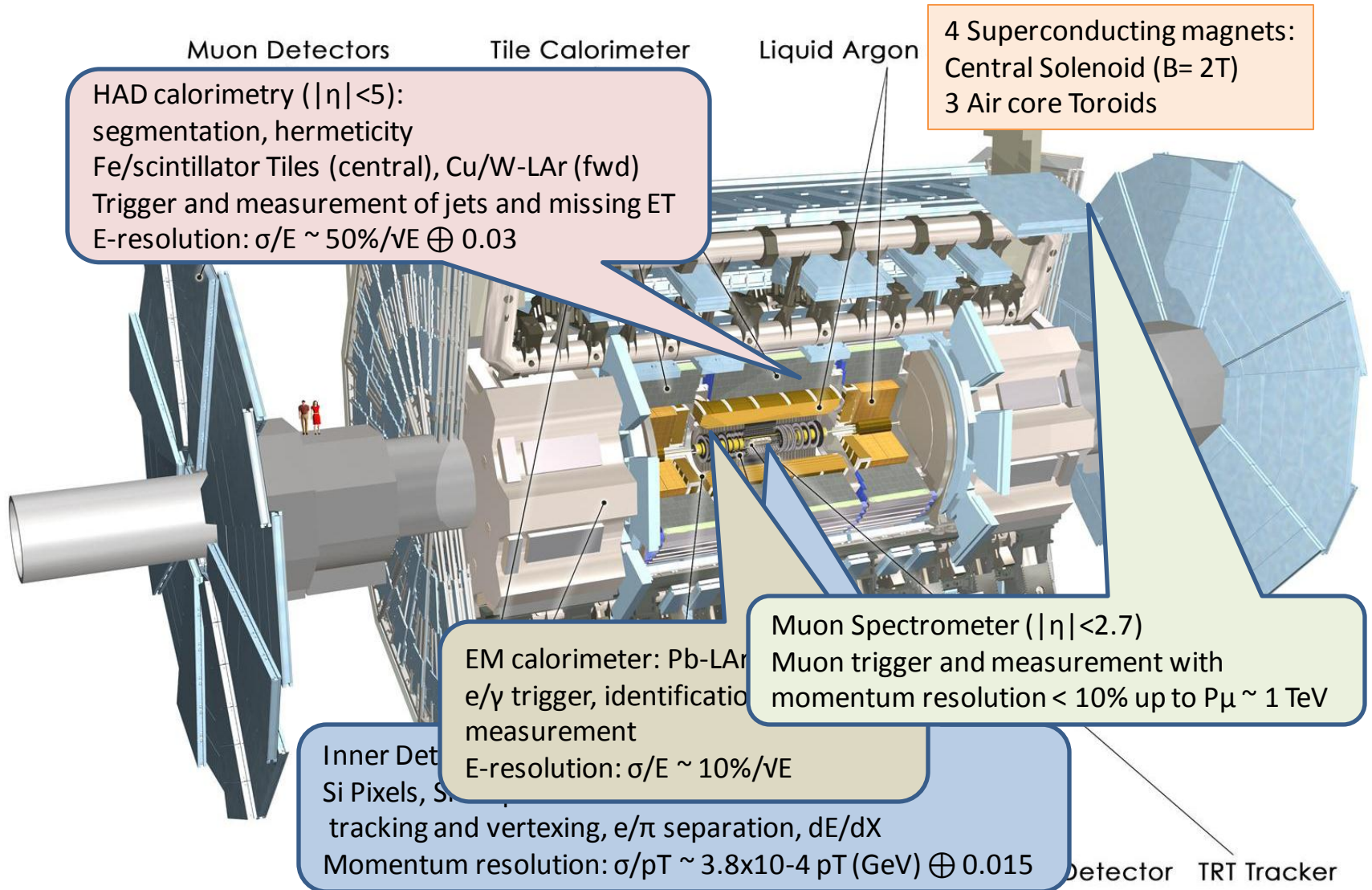
- **The ATLAS detector**
- **Combined performance**
 - Tracking
 - Flavor Tagging
 - Jet/EtMiss
 - Tau
 - e/gamma
 - Muon combined
- **LHC**
 - Milestones
- **The ATLAS upgrade**
 - Phase 0
 - Phase 1
 - Phase 2

38 Countries
174 Institutions
3000 Scientists
1000 Students



Albany, Alberta, NIKHEF Amsterdam, Ankara, LAPP Ancey, Argonne NL, Arizona, UT Arlington, Athens, NTU Athens, Baku, IFAE Barcelona, Belgrade, Bergen, Berkeley LBL and UC, HU Berlin, Bern, Birmingham, UAN Bogota, Bologna, Bonn, Boston, Brandeis, Brasil Cluster, Bratislava/SAS Kosice, Brookhaven NL, Buenos Aires, Bucharest, Cambridge, Carleton, CERN, Chinese Cluster, Chicago, Chile, Clermont-Ferrand, Columbia, NBI Copenhagen, Cosenza, AGH UST Cracow, IFJ PAN Cracow, SMU Dallas, UT Dallas, DESY, Dortmund, TU Dresden, JINR Dubna, Duke, Edinburgh, Frascati, Freiburg, Geneva, Genoa, Giessen, Glasgow, Göttingen, LPSC Grenoble, Technion Haifa, Hampton, Harvard, Heidelberg, Hiroshima IT, Indiana, Innsbruck, Iowa SU, Iowa, UC Irvine, Istanbul Bogazici, KEK, Kobe, Kyoto, Kyoto UE, Lancaster, UN La Plata, Lecce, Lisbon LIP, Liverpool, Ljubljana, QMW London, RHBNC London, UC London, Lund, UA Madrid, Mainz, Manchester, CPPM Marseille, Massachusetts, MIT, Melbourne, Michigan, Michigan SU, Milano, Minsk NAS, Minsk NCPHEP, Montreal, McGill Montreal, RUPHE Morocco, FIAN Moscow, ITEP Moscow, MEPhI Moscow, MSU Moscow, LMU Munich, MPI Munich, Nagasaki IAS, Nagoya, Naples, New Mexico, New York, Nijmegen, Northern Illinois, BINP Novosibirsk, Ohio SU, Okayama, Oklahoma, Oklahoma SU, Olomouc, Oregon, LAL Orsay, Osaka, Oslo, Oxford, Paris VI and VII, Pavia, Pennsylvania, NPI Petersburg, Pisa, Pittsburgh, CAS Prague, CU Prague, TU Prague, IHEP Protvino, Regina, Rome I, Rome II, Rome III, Rutherford Appleton Laboratory, DAPNIA Saclay, Santa Cruz UC, Sheffield, Shinshu, Siegen, Simon Fraser Burnaby, SLAC, South Africa, Stockholm, KTH Stockholm, Stony Brook, Sydney, Sussex, AS Taipei, Tbilisi, Tel Aviv, Thessaloniki, Tokyo ICEPP, Tokyo MU, Tokyo Tech, Toronto, TRIUMF, Tsukuba, Tufts, Udine/ICTP, Uppsala, UI Urbana, Valencia, UBC Vancouver, Victoria, Waseda, Washington, Weizmann Rehovot, FH Wiener Neustadt, Wisconsin, Wuppertal, Würzburg, Yale, Yerevan

The ATLAS detector



Forward detectors

on both side

ATLAS



ALFA at 240 m

$10.6 < |\eta| < 13.5$

ZDC at 140 m

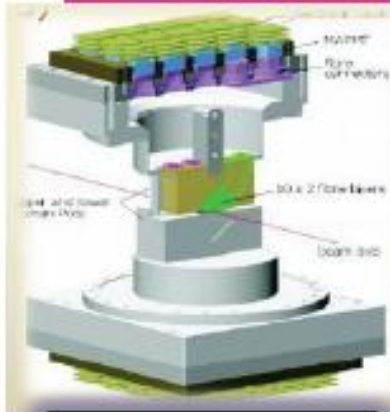
$|\eta| > 8.3$

LUCID at 17 m

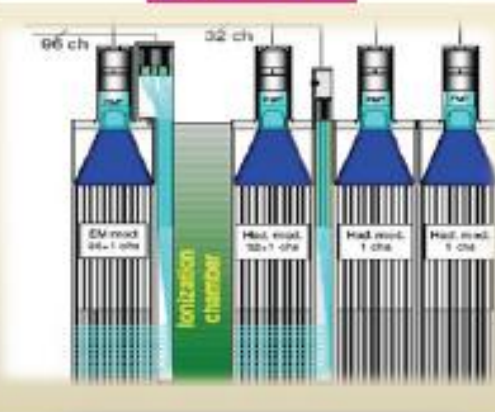
$5.6 < |\eta| < 5.9$

MBTS at 3.6 m

$2.1 < |\eta| < 3.8$



SciFi tracker



W+quartz rods calorimetry



Cerenkov rad.



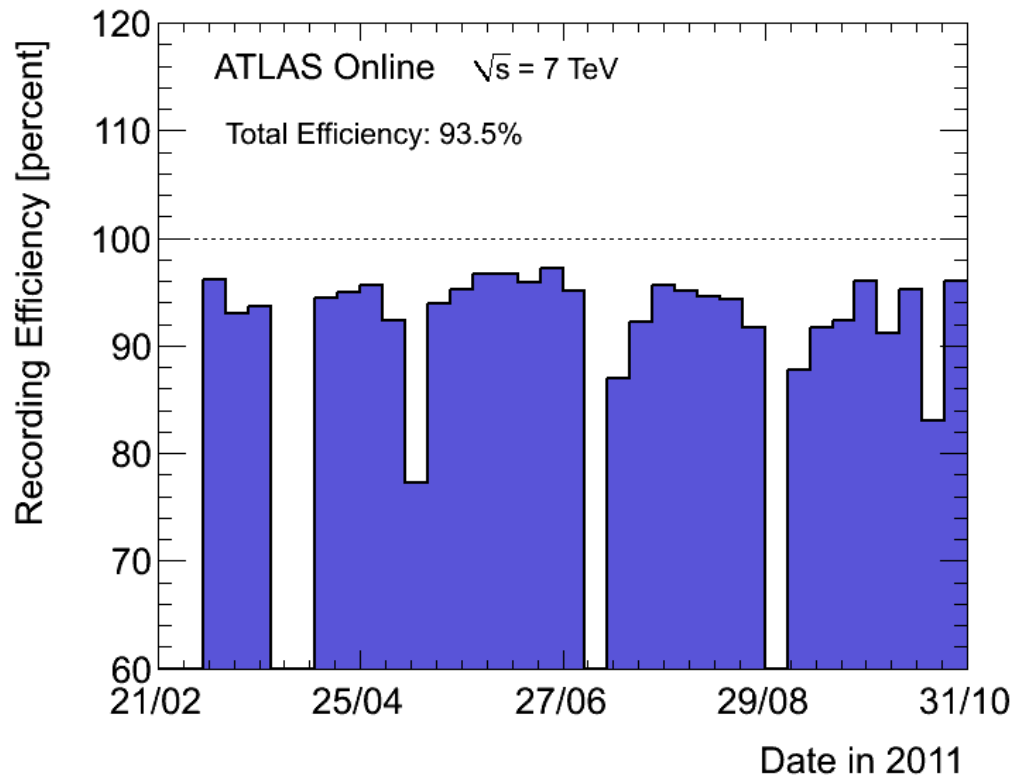
Scintillators

B. Di Girolamo

Data Taking Efficiency

the luminosity delivered (between the declaration of stable beams and the LHC request to turn the sensitive detectors off)

$$\text{Efficiency} = \frac{\text{the luminosity delivered (between the declaration of stable beams and the LHC request to turn the sensitive detectors off)}}{\text{the luminosity recorded by ATLAS}}$$

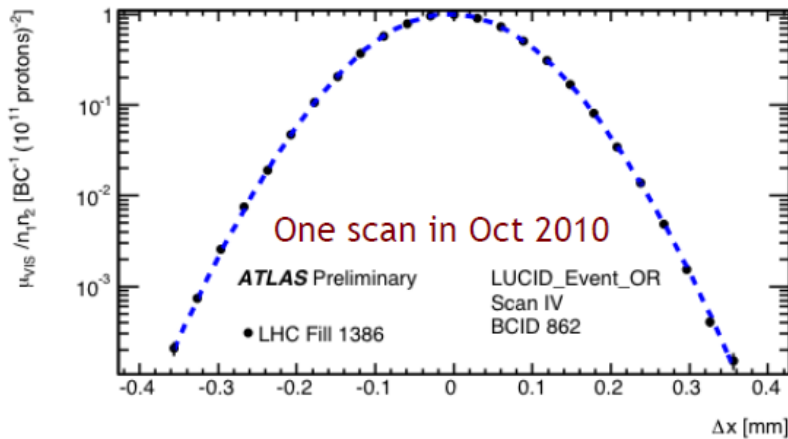


Operational fraction

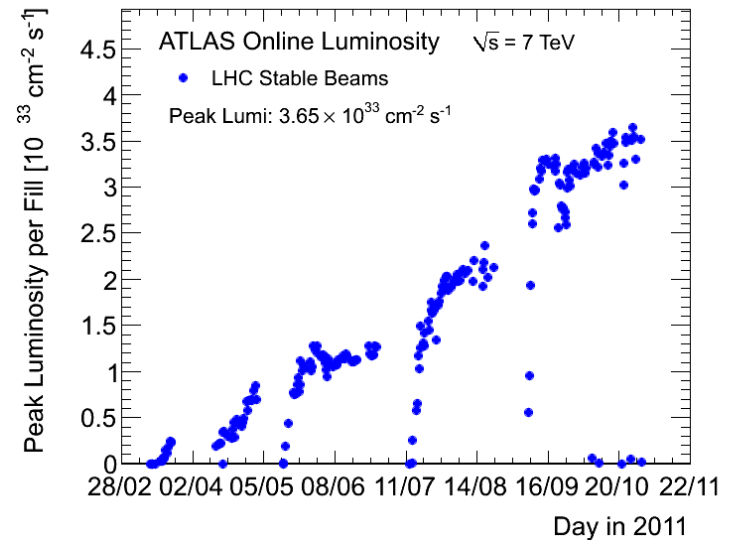
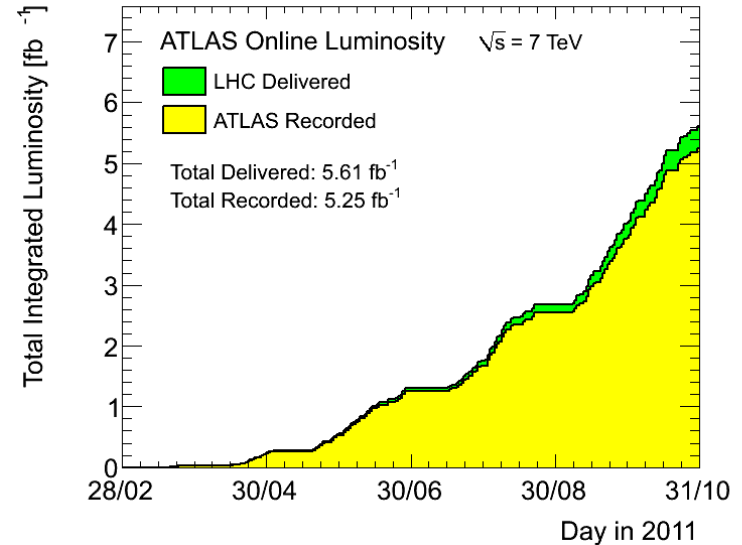
Subdetector	N Channels	Operational Fraction
Pixels	80 M	96.9%
SCT Silicon Strips	6.3 M	99.1%
TRT Transition Radiation Tracker	350 k	97.5%
LAr EM Calorimeter	170 k	99.5%
Tile Calorimeter	9.8 k	97.9%
Hadronic Endcap LAr Calorimeter	5.6 k	99,6%
Forward LAr Calorimeter	3.5 k	99.8%
MDT Muon Drift Tubes	350 k	99.8%
CSC Cathode Strip Chambers	31 k	98.5%
RPC Barrel Muon Chambers	370 k	97.0%
TGC Encap Muon Chambers	320 k	98.4%
LVL1 Calo Trigger	7160	99.9%
LVL1 Muon RPC Trigger	370 k	99.5%
LVL1 Muon TGC Trigger	320 k	100%

Luminosity

- The maximum instantaneous luminosity: $3.65 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
- Delivered Luminosity: 5.61 fb^{-1}
- ATLAS Ready Recorded: 5.25 fb^{-1}

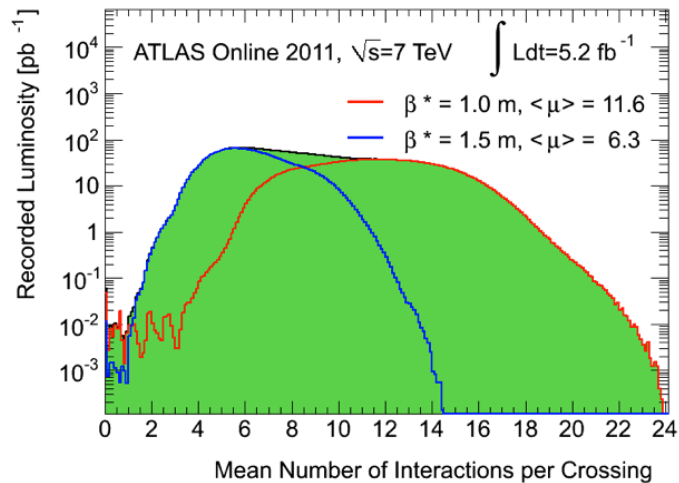
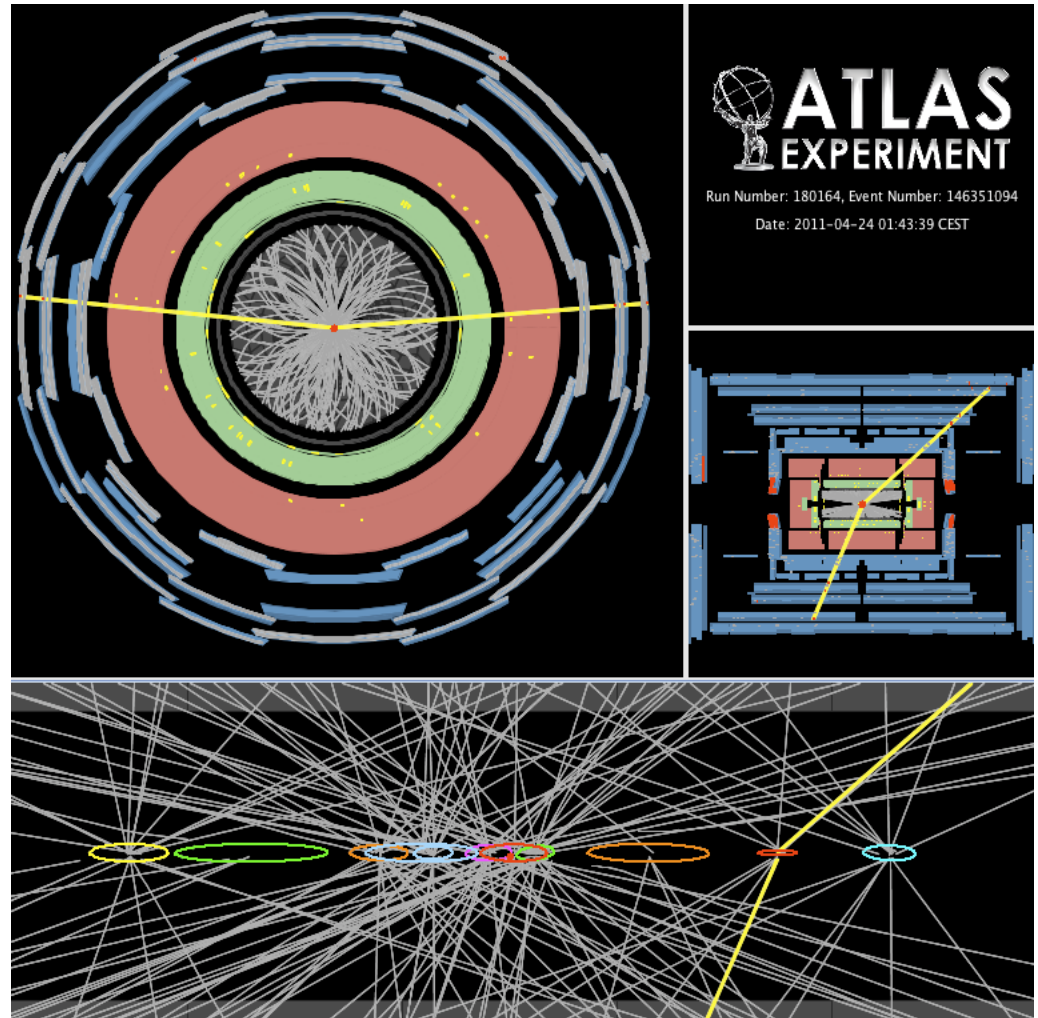


Absolute luminosity calibration by van der Meer scans
 $\Delta L/L = \pm 3.4\%$ (2010, prel)
 $\Delta L/L = \pm 3.7\%$ (2011, prel)



Pile-up

- 50 ns bunch trains for ~all 2011 data
- Substantial in- and out-of-time pileup
- Much progress understanding impact on performance, with data & simulation



$Z \rightarrow \mu\mu$ event with 11 primary vertices

Trigger

Level-1:

Implemented in hardware
Muon + Calo based coarse granularity
 e/γ , μ , π , τ , jet candidate selection
Define regions of interest (ROIs)

Typical rates

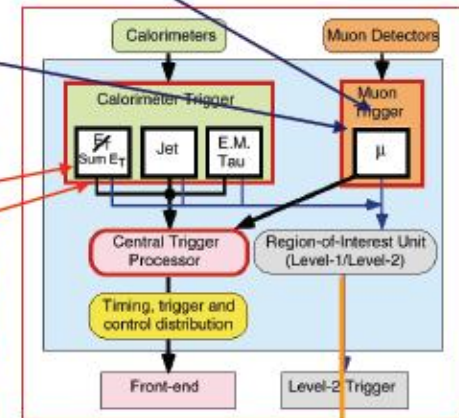
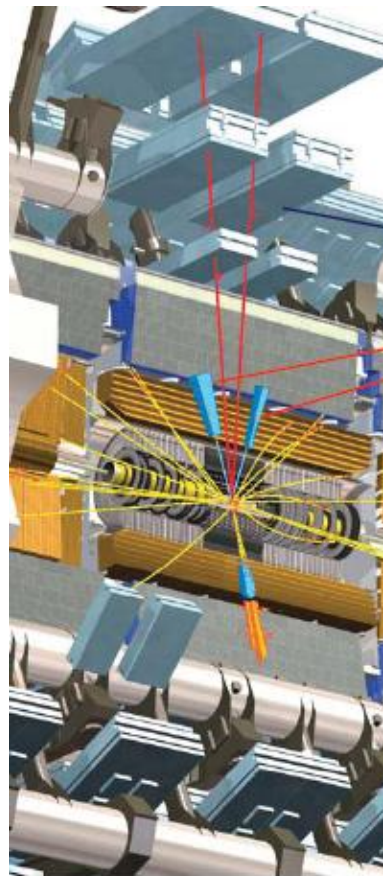
L1: 60 kHz
L2: 5 kHz
EF: 200-400 Hz

Level-2:

Implemented in software
Seeded by level-1 ROIs, full granularity
Inner Detector – Calo track matching

Event Filter:

Implemented in software
Offline-like algorithms for physics signatures
Refine LV2 decision
Full event building

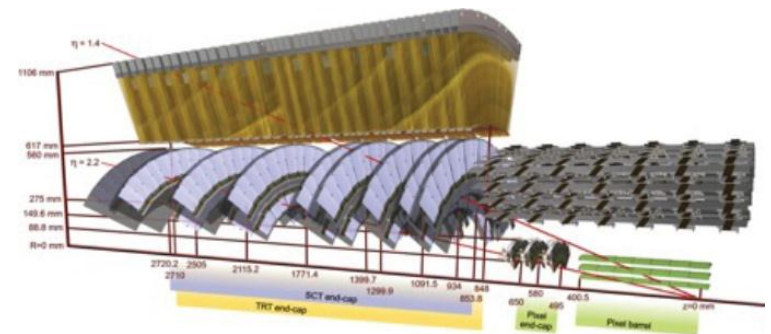
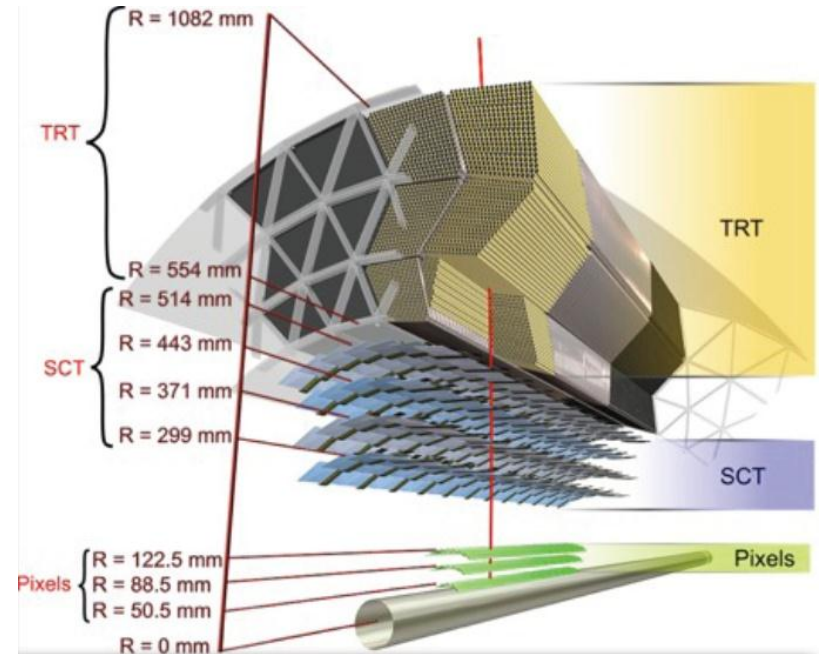


RoI
 η - ϕ addresses

L.Mapelli

Inner detector

- Tasks:
 - Precision tracking covering over 5 units in eta
 - Primary vertex reconstruction
 - b-tagging
 - reconstruction electrons and converted photons,
 - electron identification with TRT
 - tracking of muons combined with toroid Muon Spectrometer
 - V0, b- and c-hadron reconstruction, ...
 - dE/dx from T.o.T. in Pixels and TRT
 - fast tracking for high level trigger



ID Performance

- Reconstruction, p_T resolution**

The inner tracking system measures charged particle tracks at all over $\eta < 2.5$ using the pixel, SCT and TRT detectors.

A pattern recognition “inside-out (silicon \rightarrow TRT) tracking procedure selects track candidates with $p_t > 100$ MeV .

One further step (TRT \rightarrow silicon) selects tracks from secondary interactions with $p_t > 300$ MeV.

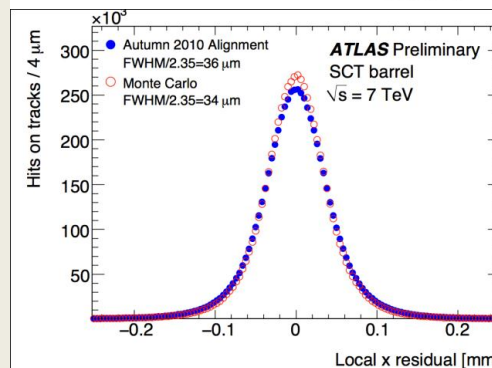
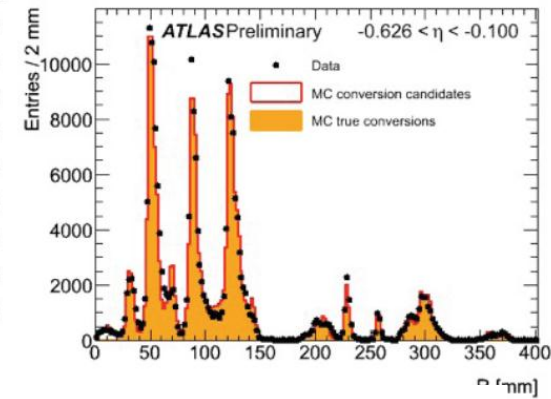
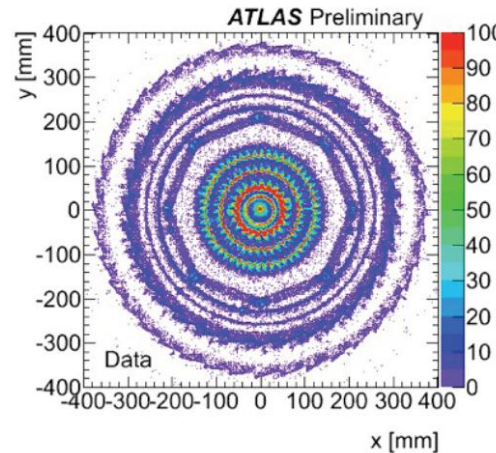
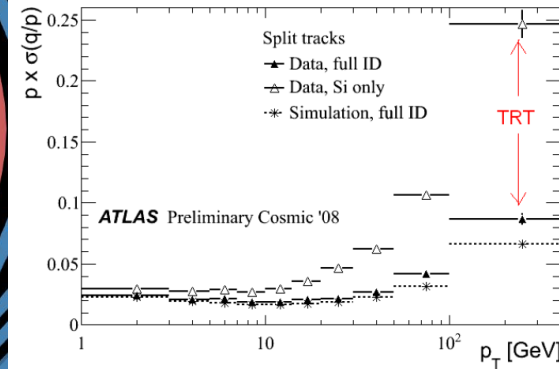
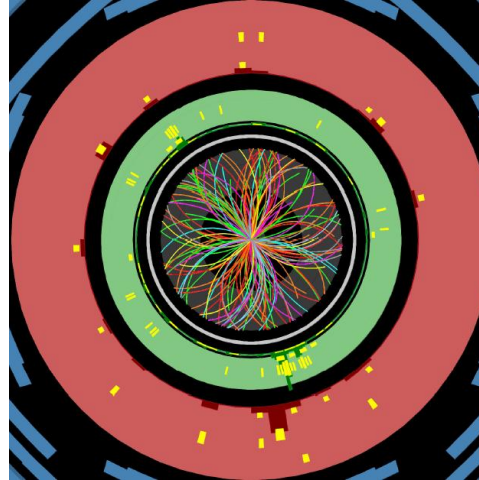
- Material studies**

- γ -conversion
- hadronic interactions

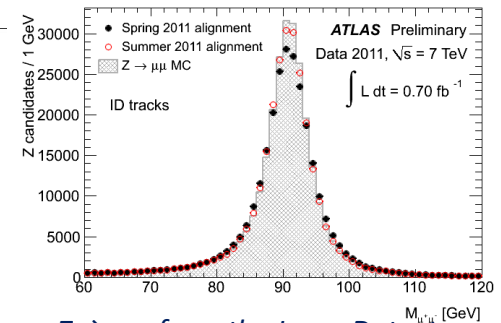
- Very detailed description**

- Detector alignment**

- small residual misalignment
- error scaling to allow for residual misalignments in fit



Residuals of the SCT barrel modules

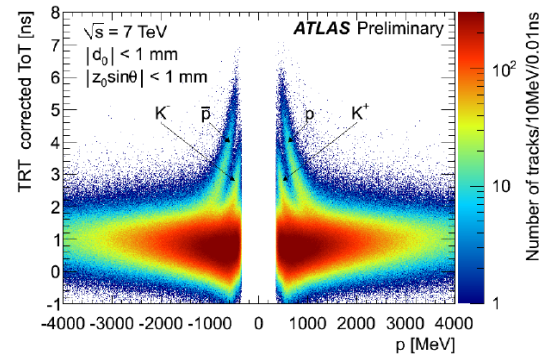
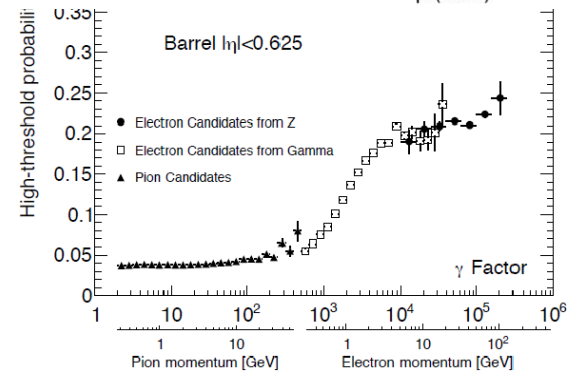
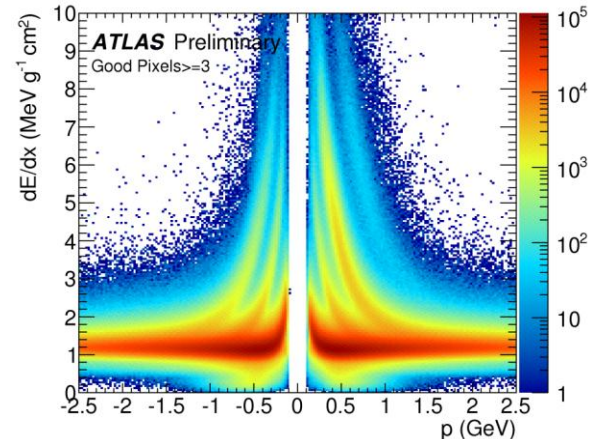


*Z \rightarrow $\mu\mu$ from the Inner Detector
MC: Ideal alignment
Black: spring 2011 alignment
Red: updated alignment*

ID Performance

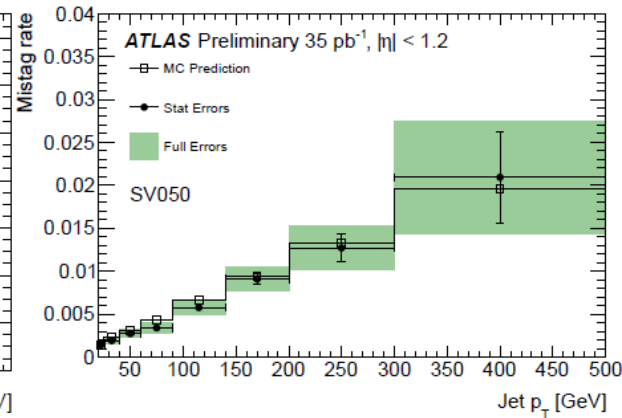
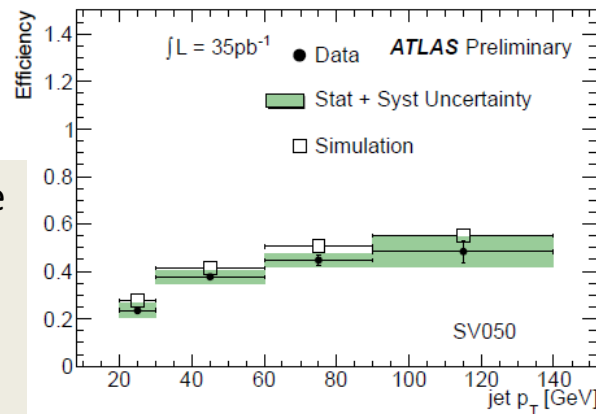
Particle identification

- Energy loss in Pixel
- TRT High Threshold hit fraction provides electron/hadron discrimination over the momentum range between 1 and 150 GeV/c
- Time over Threshold (ToT) is sensitive to dE/dx of charged particle, allowing an independent method of particle identification.



B-tagging

The b-tag efficiency and mistag rate of b-tagging algorithms have been measured with a number of complementary methods using data from the ATLAS detector. Good consistency is observed between the results of all methods.

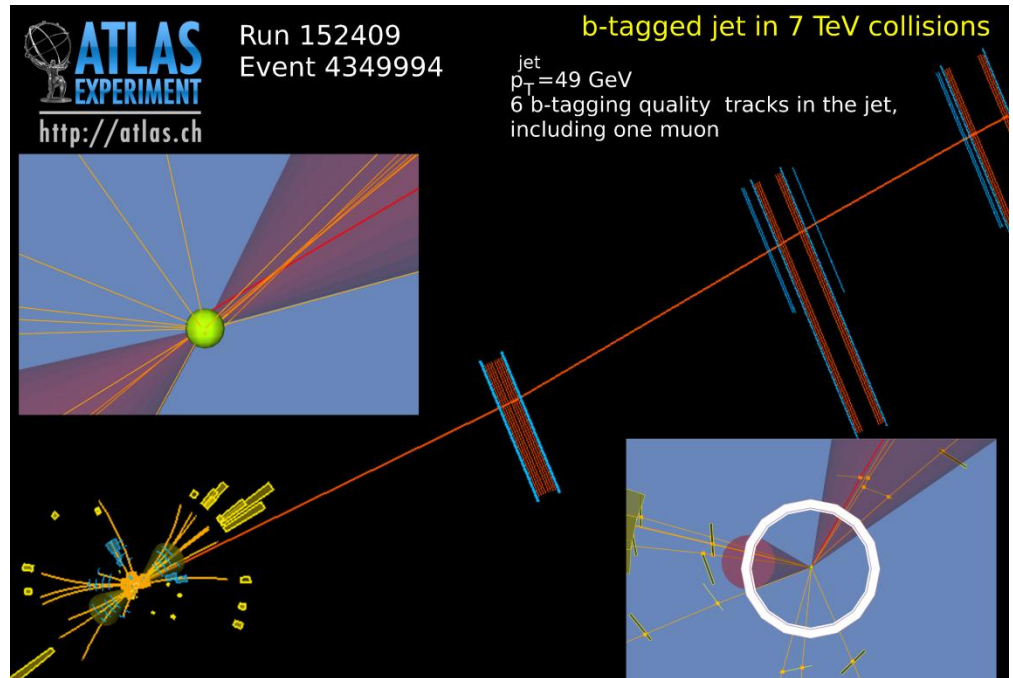


This jet has all the characteristics of a b-jet with a semi-leptonic decay (to a muon).

Muon Tagger:

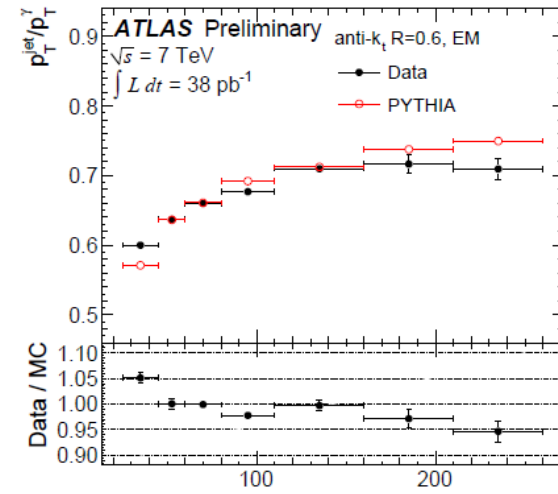
A combined muon with the following properties:

- Muon $p_T = 6 \text{ GeV}$
- Muon $d_0 = 610 \text{ microns}$
- Muon $d_0/\sigma(d_0) = 15$
- The muon is part of the secondary vertex.

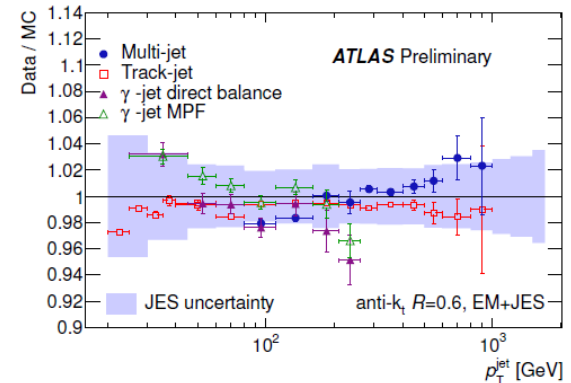


Jet reconstruction

- **Anti-kT algorithm** with resolution parameter $R=0.4 - 1.2$
 - reconstruct jets with simple cone-like geometrical shape from calorimeter clusters or charged particle tracks
 - Infrared and co-linear safe
- Due to the non-compensating nature of the calorimeter, signal losses due to noise thresholds and in dead material the jet energy needs to be calibrated.
- **Jet energy calibration** is validated in-situ by:
 - direct transverse momentum balance between a jet and a photon
 - response to single isolated hadrons
 - multi-jet balance technique
- For jet transverse momenta between 60 and 800 GeV the uncertainty is below 2.5%.



Jet response as determined by the direct p_T balance technique



Jet energy scale uncertainty as a function of p_T^{jet} in $0 \leq |\eta| < 1.2$

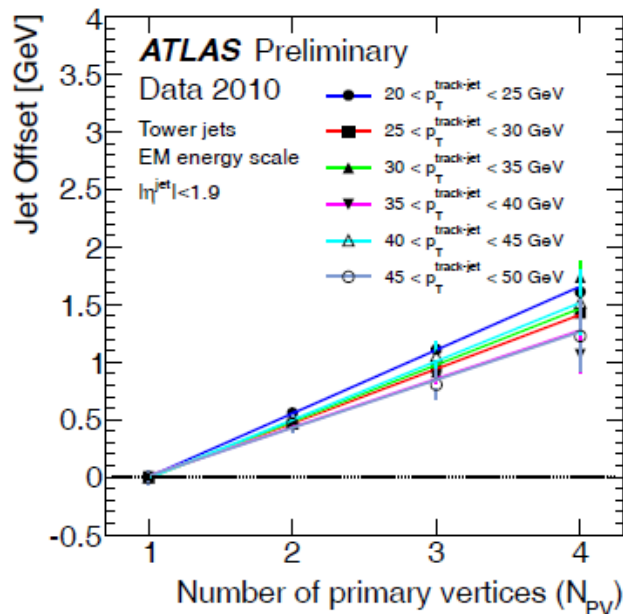
Jet p_T resolution

E_T^{miss} resolution

Jet p_T resolution for QCD jets was measured by Jet p_T balance method.

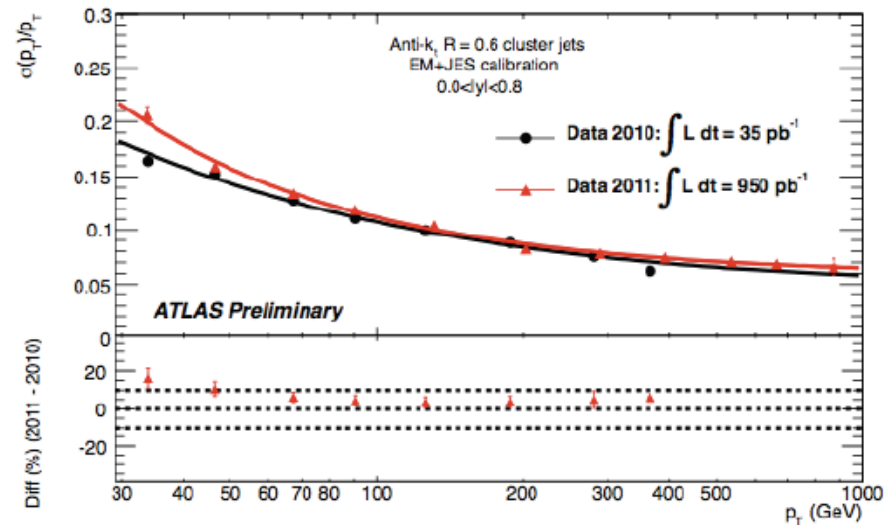
For $p_T \sim 1$ TeV $\sigma(p_T)/p_T = 6\%$

Corrections for jet p_T offset from pile-up is about 0.5 GeV/additional vertex

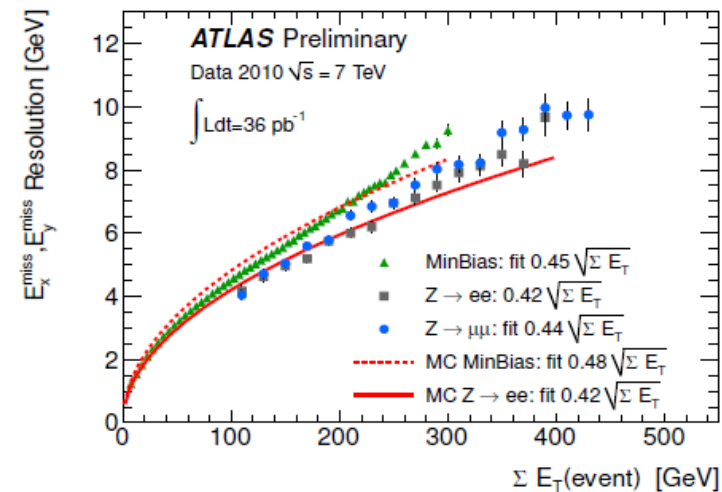


E_x^{miss} and E_y^{miss} resolution as a function of the total transverse energy was measured in minibias events as well as in the events with $W(\text{lv})$ or $Z(\text{ll})$.

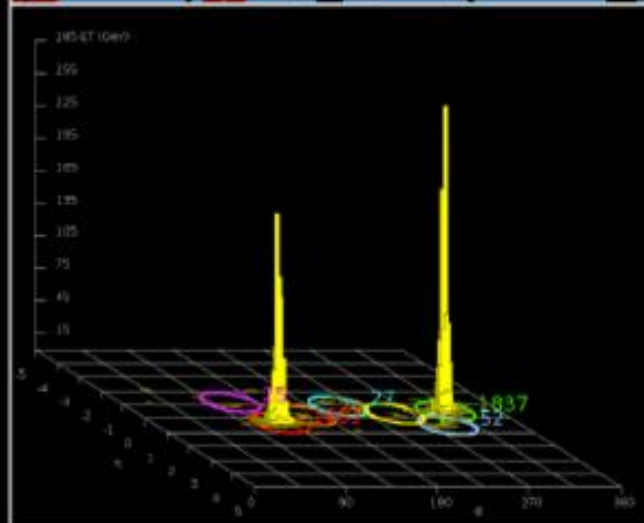
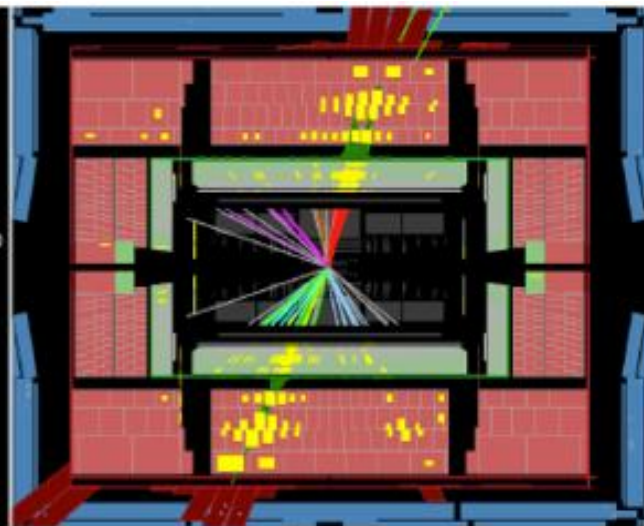
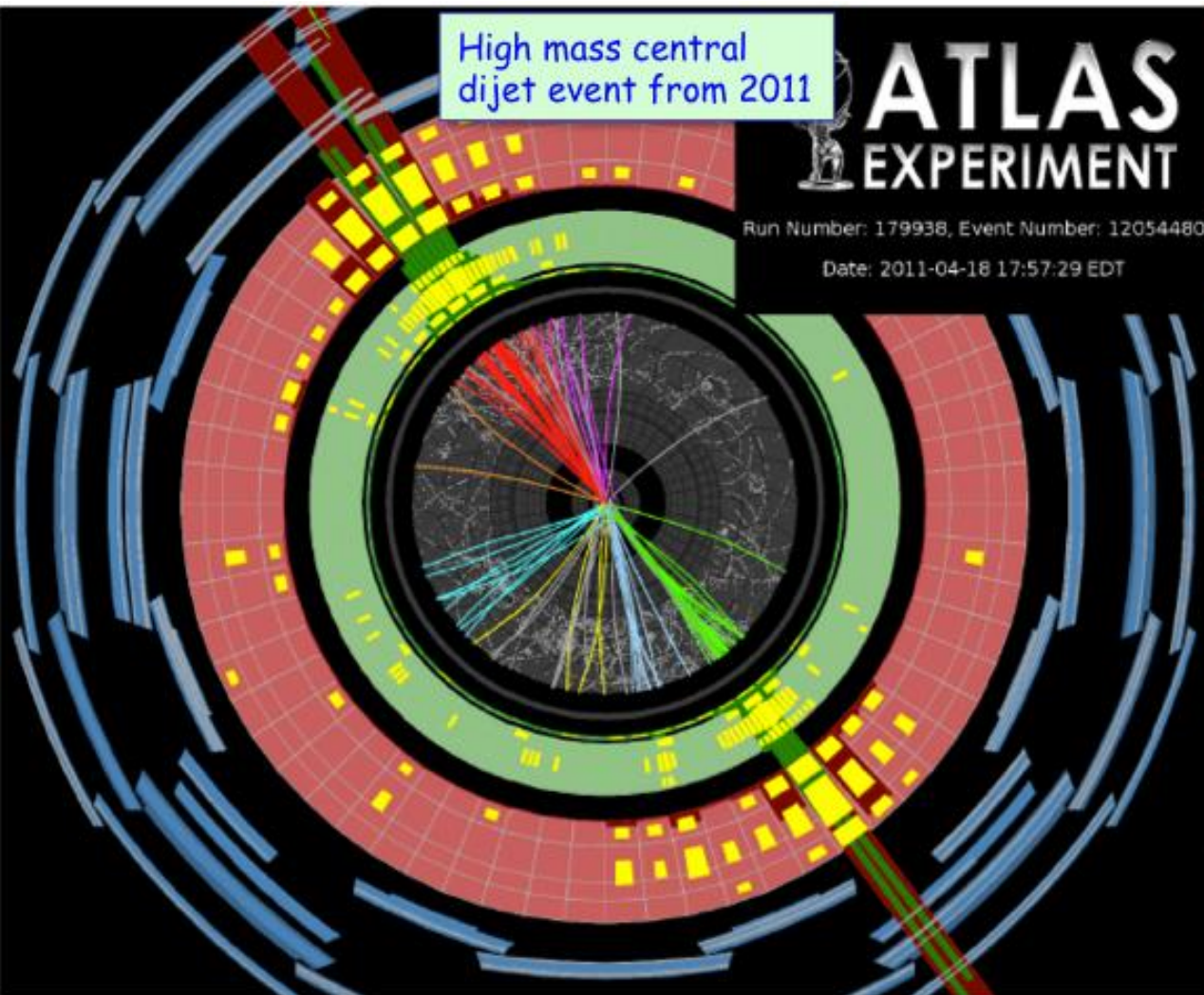
$$\sigma(E_T^{\text{miss}}) = 0.48 \sqrt{\Sigma E_T^{\text{miss}}}$$



Fractional jet energy resolution as a function of the average jet transverse momenta for events with two jet in the same rapidity bin for EM+JES calibration with 2011 (red) and 2010 (black) data.

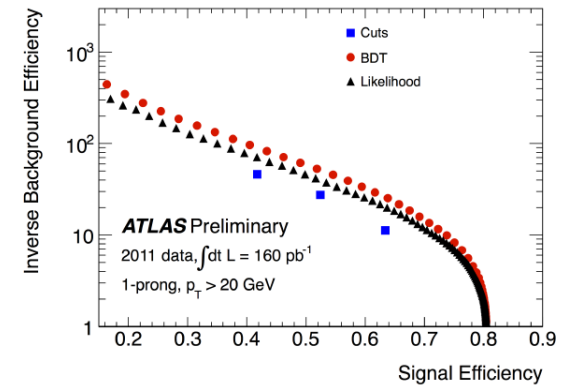


Highest-mass dijet event observed so far

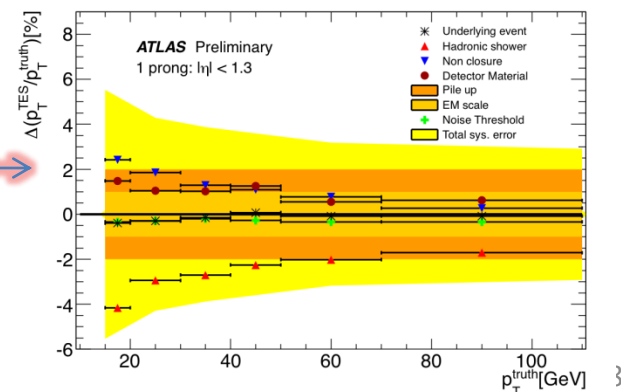
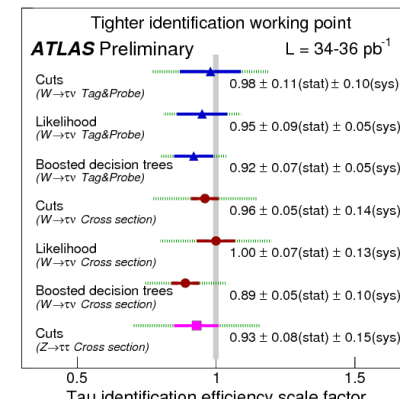


Tau-lepton

- **Tau leptons are recognized** in the ATLAS detector via their hadronic decays.
- Tau leptons versus QCD jets:
 - number of final state particles
 - width of the energy depositions in the calorimeter
 - displacement of the secondary vertex
 - small invariant mass
- The tau identification efficiency has been measured in data using $W \rightarrow \tau\nu$ and $Z \rightarrow \tau\tau$ events
- **The tau energy scale** is obtained from the measured transverse momentum by scaling it to its expected value from Monte Carlo simulation of tau decays.
- Systematic uncertainty on the tau energy scale $\sim 2\%$



Inverse background efficiency (in dijet data) versus signal efficiency (in $W \rightarrow \tau\nu$ and $Z \rightarrow \tau\tau$ Monte Carlo samples)

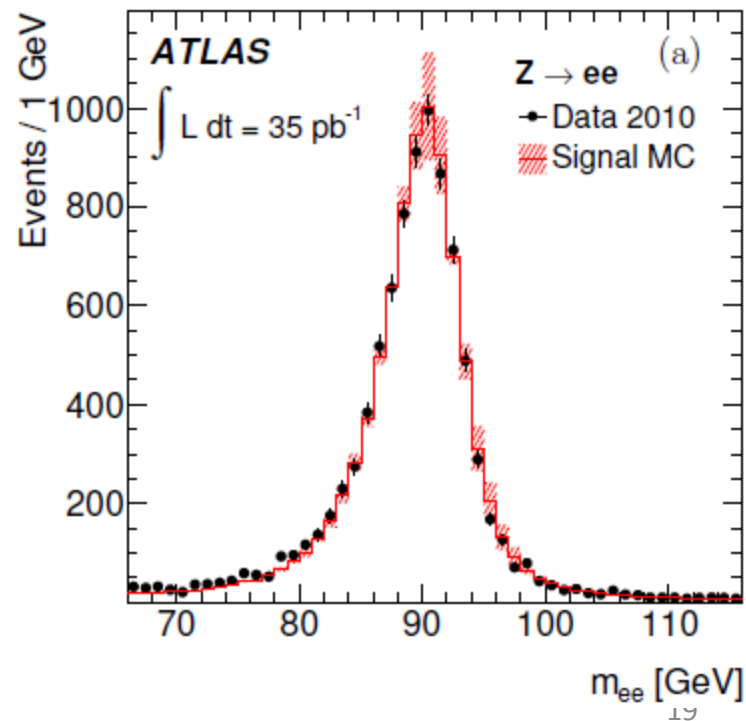
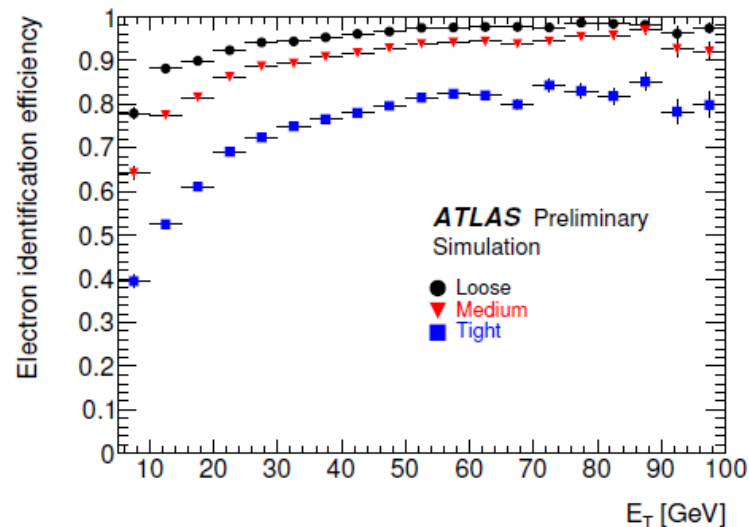


e/γ

Electron identification

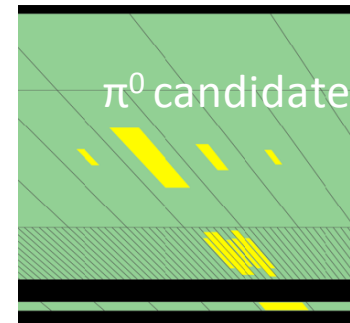
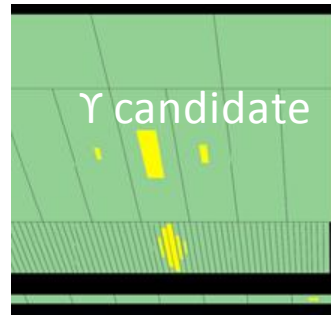
- Three reference sets of requirements provide progressively stronger jet rejection :
- Loose: shower shape in the second layer of the EM calorimeter and energy leakage into the hadronic calorimeters.
- Medium: the energy deposit patterns in the first layer , track quality variables, distance to the primary vertex and a cluster-track matching
- Tight: the ratio of cluster energy to track momentum, the number of hits in the TRT, the ratio of high-threshold hits to the total number of hits in the TRT, at least one hit in the first layer.

MC efficiency and resolution agreed with data



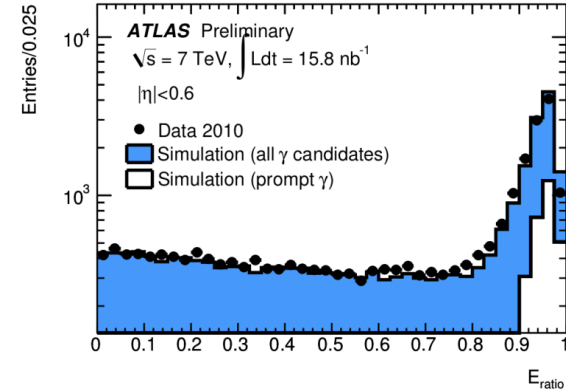
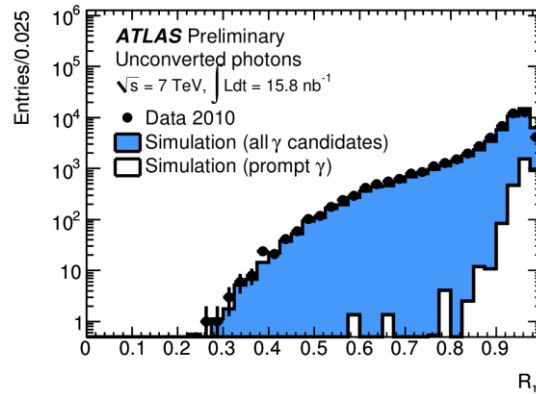
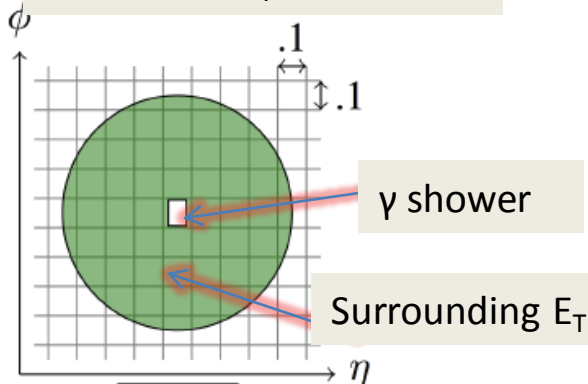
Photon identification

Energy and $m_{\gamma\gamma}$ resolution



loose and tight selections
optimized separately for
unconverted and converted γ

Isolation: $E_T^{\text{iso}} < 3 \text{ GeV}$

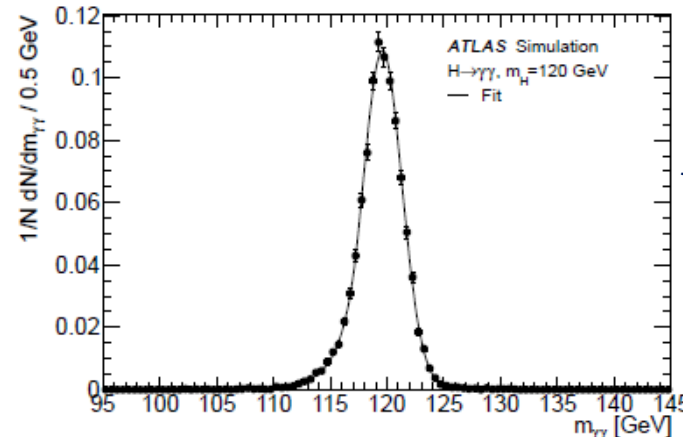


Examples of discriminating shower shape variables

- transverse shape variable
- energy deposit in the first longitudinal compartment of the electromagnetic calorimeter

E_γ and $M_{\gamma\gamma}$ resolution is estimated with Monte-Carlo.

The core component of the mass resolution ranges from 1.4 GeV in the “Unconverted central” category to 2.1 GeV in the “Converted transition” category.

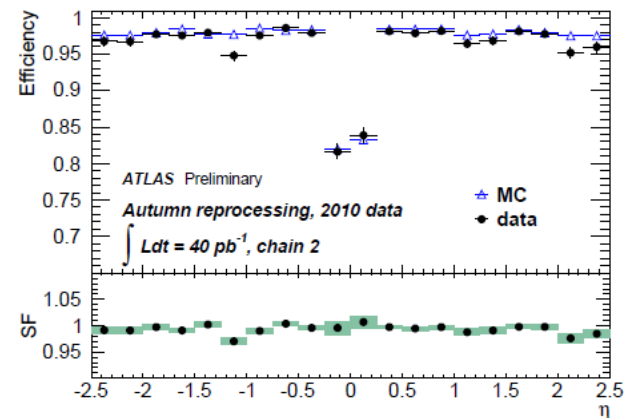
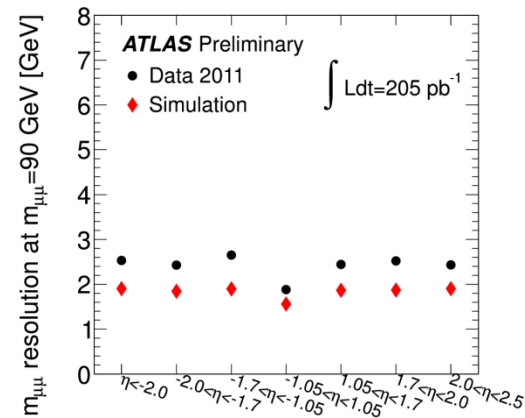
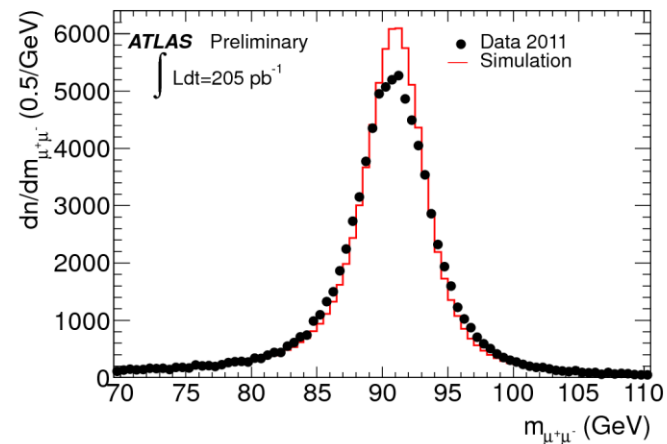


Monte-Carlo
for $H \rightarrow \gamma\gamma$
 $M_H = 120 \text{ GeV}$

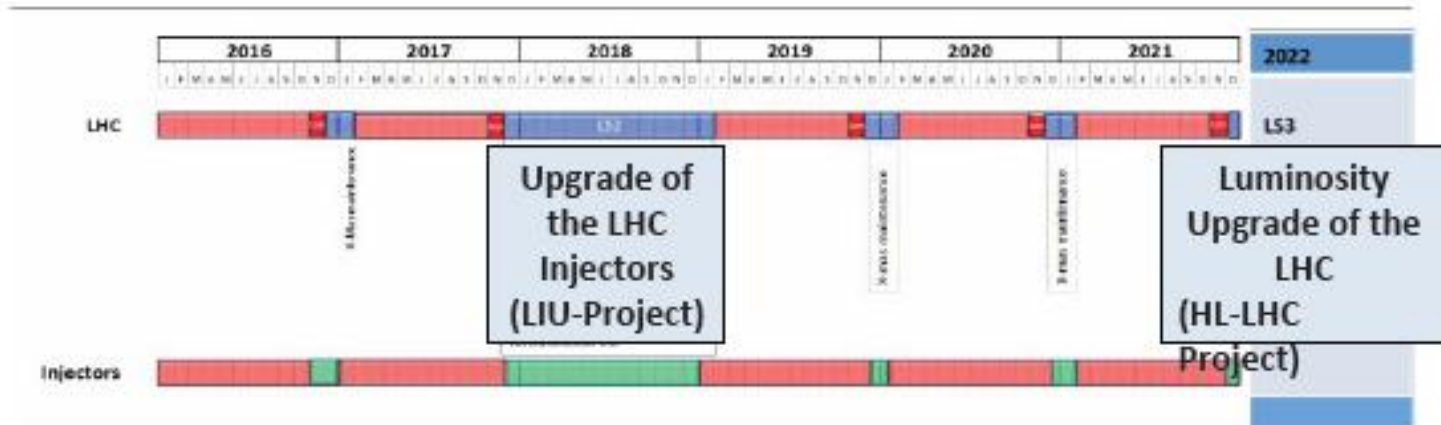
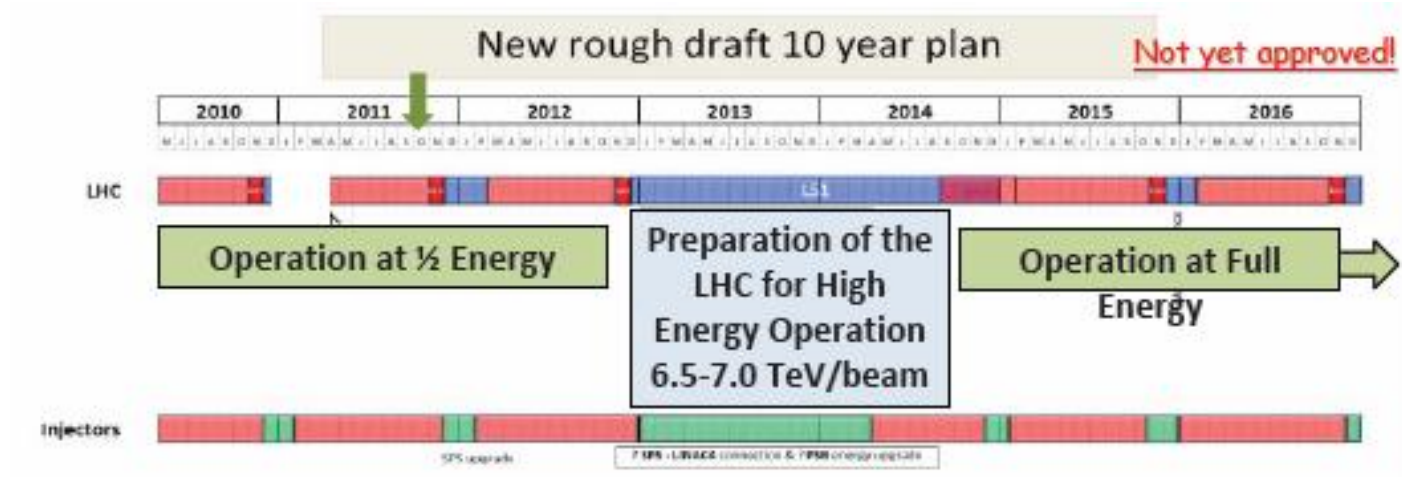
Muon combined

The momentum resolution is extracted from the width of the di-muon mass distribution in $Z \rightarrow \mu\mu$ decays and the comparison of the independent measurements of muons from $Z \rightarrow \mu\mu$ and $W \rightarrow \mu\nu$ decays provided by the two ATLAS tracking systems, the Inner Detector and Muon Spectrometer.

The muon reconstruction efficiencies are measured with $Z \rightarrow \mu+\mu-$ decays in which one of the decay muons is reconstructed in both systems and the other is identified by just one of the systems in order to probe the efficiency of the other.



LHC plans



Aim to produce $\sim 3000 \text{ fb}^{-1}$ delivered to the experiments over 10 years.

luminosity around 5×10^{34} and leveling

ATLAS

Why upgrade?

- LHC luminosity goes beyond the nominal luminosity
- New technologies → better detectors
- Experience with ATLAS → better understanding of needs and opportunities
- Aging

Tentative time schedule

- The multi-phase detector upgrade: Phase 0, I, II
- **2010-2012 Run 1: $\sqrt{s} = 7$ TeV (2012: 8-9 TeV ?), $L = 3 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$**
- **2013-2014 Shutdown. Phase 0:**
 - Detector consolidation
 - New pixel layer
- **2014-2017 Run 2: $\sqrt{s} = 14$ TeV, $L = 1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$**
- **2018 Shutdown. Phase 1:**
 - Small muon wheels
 - Trigger upgrade
- **2019-2022 Run 3: $\sqrt{s} = 14$ TeV, $L = 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$**
- **2022 Shutdown. Phase 2**
 - New inner tracker
 - Warm forward calorimeter
 - New muon chambers
 - Trigger upgrade
- **2023-2030? Run 4: $\sqrt{s} = 14$ TeV, $L = 5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$**

Phase 0

Consolidation:

- New beam pipe (steel → Al, Be → Be):
reduction of background in cavern
- Complete installation of EE muon chambers
- New LV power for LiArgon
- New LV power for TileCal
- Repairing

**Installation of new pixel layer
ATLAS TDR 19, CERN/LHCC 2010-013**

Phase 0

Insertable B-Layer

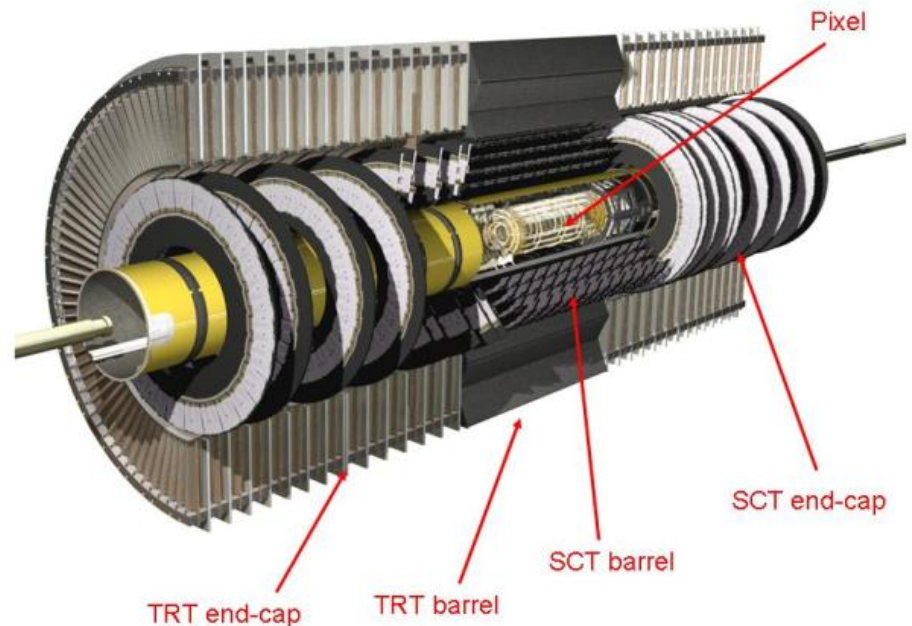
- Pixel (n+-on-n sensor) :
3 barrels + 2x3 discs ($5 < R < 15\text{cm}$)
- Strip (SCT) (p+-on-n sensor) :
4 layers + 2x9 discs ($30 < R < 51\text{cm}$)
- TRT (straw drift tubes) :
Barrel + Wheel ($55 < R < 105\text{cm}$)

Designed for fluences of :

- Pixel B-layer :
 1×10^{15} 1MeV neq/cm²
- SCT layer 1 :
 2×10^{14} 1MeV neq /cm²
- TRT outer rad :
 3×10^{13} 1MeV neq/cm²

Plan:

- Exchange beampipe with thinner one
- Use additional space for a 4th pixel layer:
Insertable B-Layer (IBL)



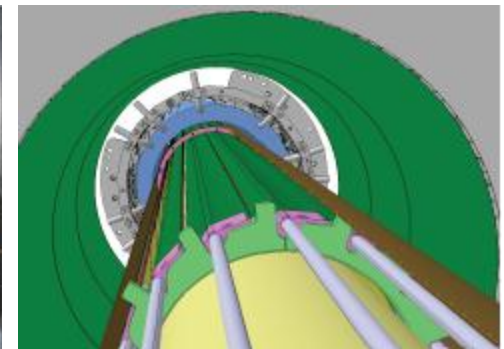
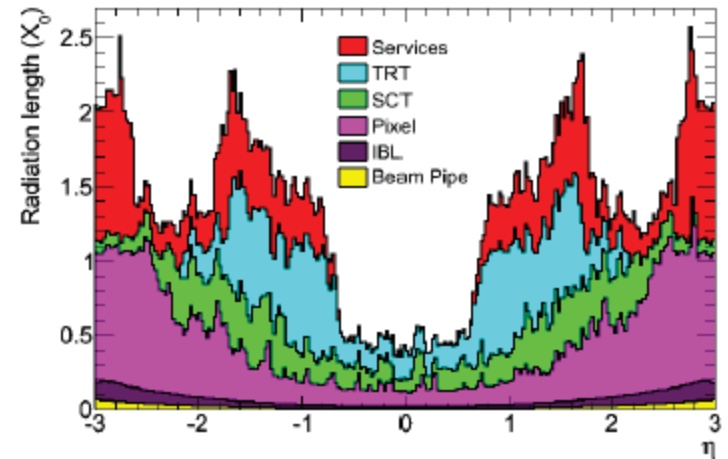
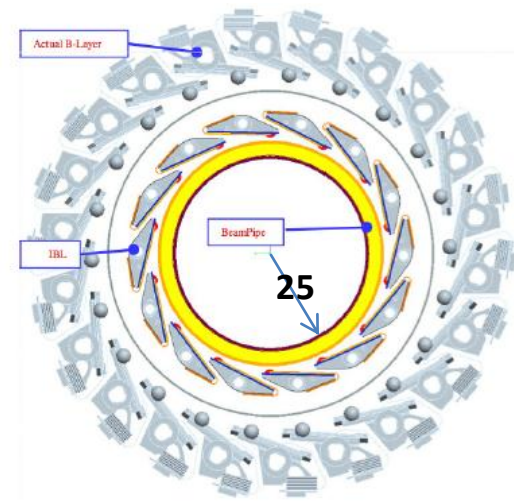
IBL

The IBL fulfils several functions:

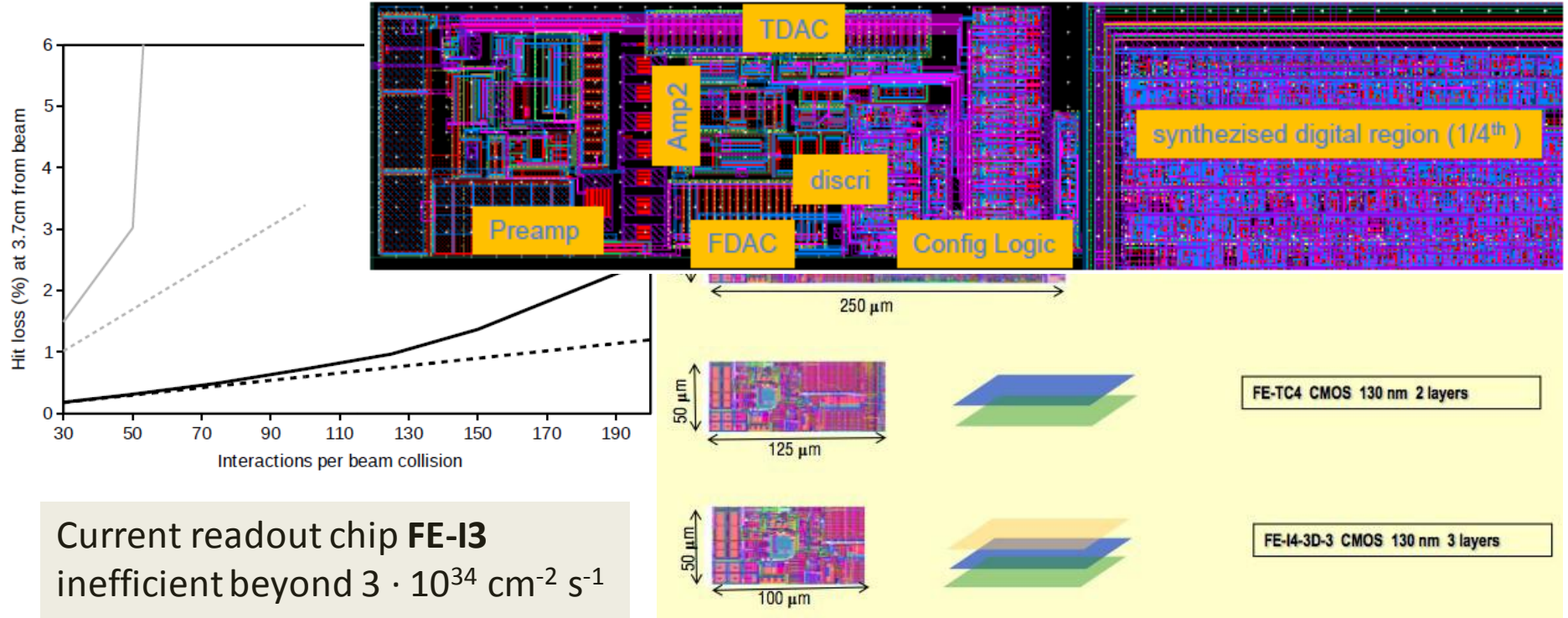
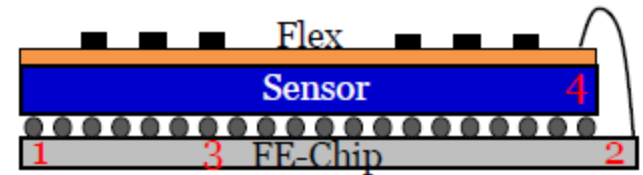
- improved determination of secondary vertices → better b-tagging
- 'hot spare' for existing b-layer
- 4th pixel hit → improved tracking

Challenges:

- New sensors
- New readout
- Material budget
- Space budget



IBL readout



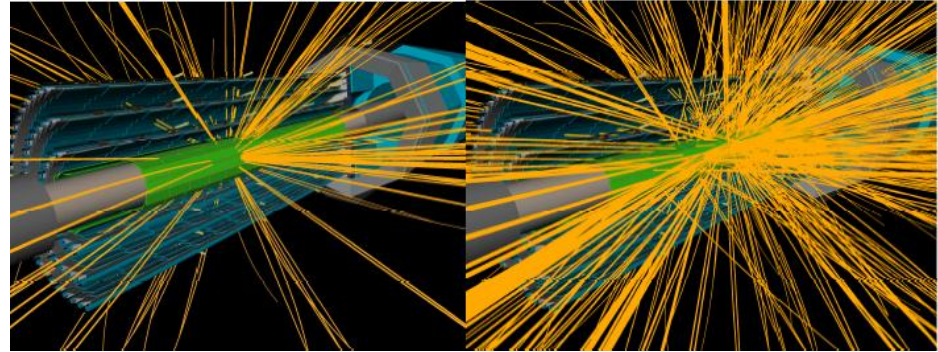
Current readout chip **FE-I3**
inefficient beyond $3 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

New chip **FE-I4** assets:

- local memory cells
- larger active fraction
- higher data rate
- more radiation hard (130 nm)

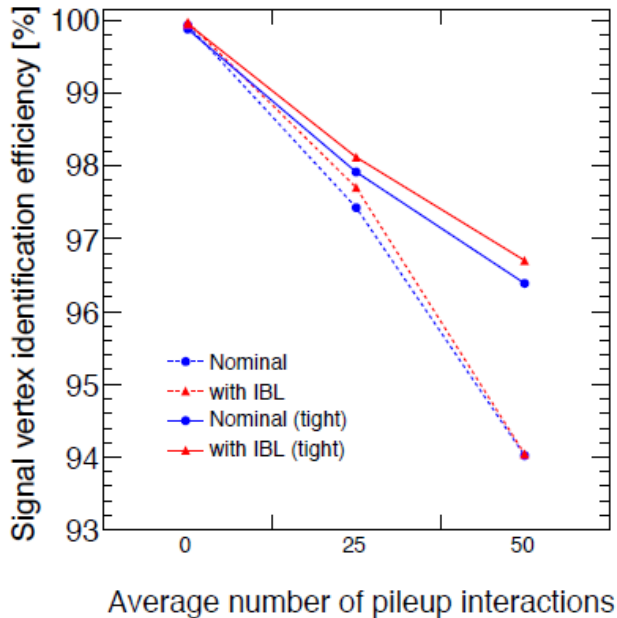
	FE-I3	FE-I4
Pixel Size [μm^2]	50×400	50×250
Pixel Array	18×160	80×336
Chip Size [mm^2]	7.6×10.8	20.2×19.
Active Fraction	74 %	89 %

IBL performance

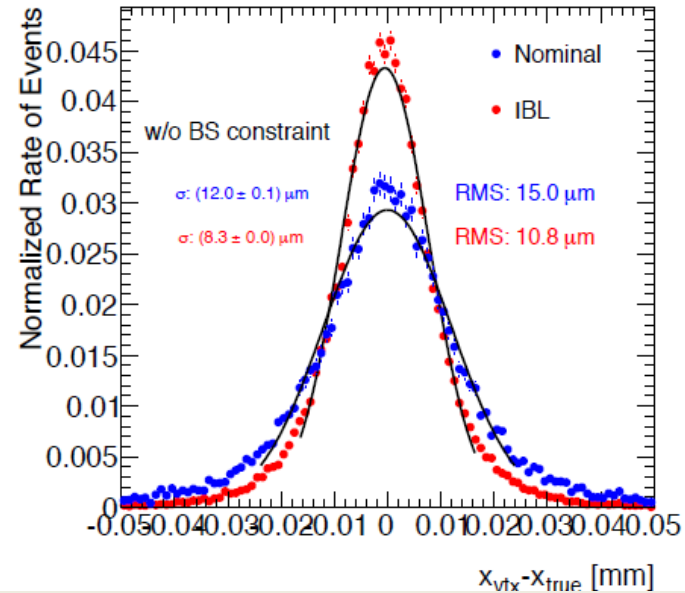


Event with 2 jets of 500 GeV
tracks $p_T > 0.5$ GeV and
cluster > 1 Pixel+IBL

same event adding pileup for
 $L=2 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$
same track selection applied



Efficiency for reconstructing the primary vertex in $t\bar{t}$ events with and without the IBL

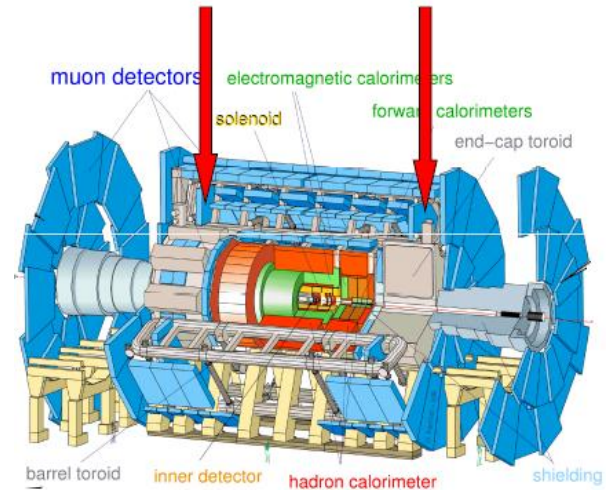


Resolution in x of the reconstructed primary vertex without beam spot constraint for $t\bar{t}$ events with and without the IBL

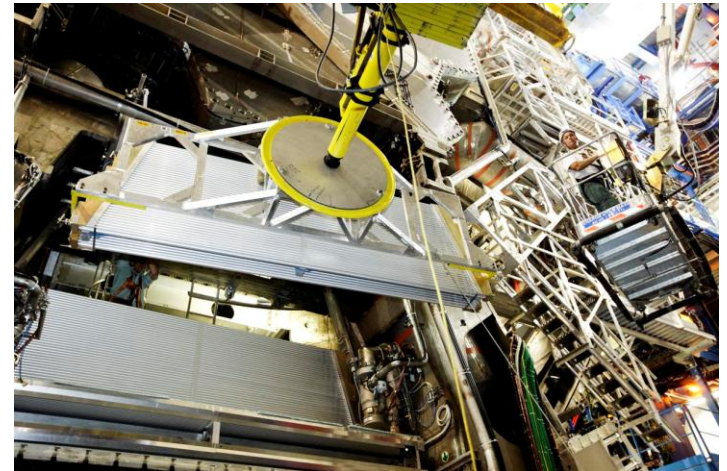
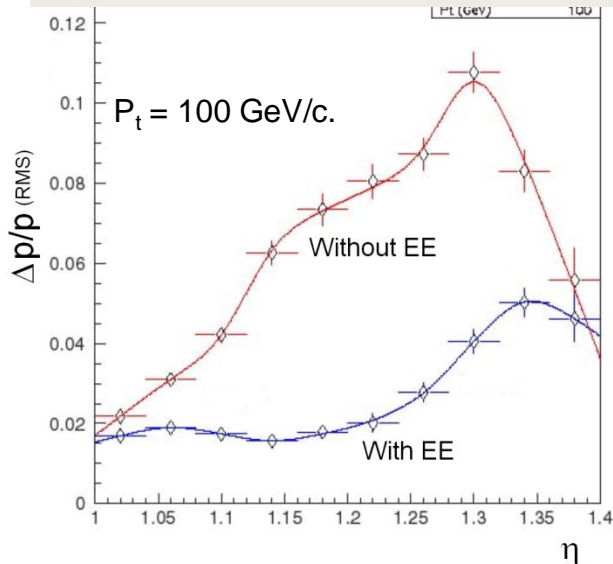
Muon chambers (EE)

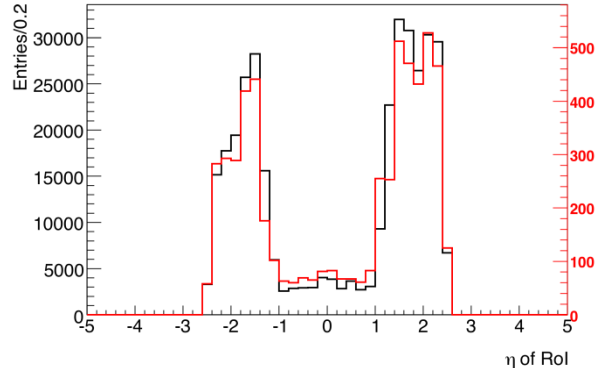
Complete installation of EE muon chambers

EE chambers



These chambers improve momentum resolution at $1 < \eta < 1.4$





L1 mu rate dominated by endcap
 L1MU20 rate
 extrapolation to 1×10^{34}

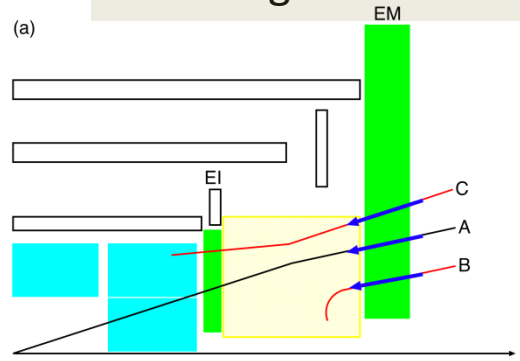
- 10 kHz at 7 TeV
- 20 kHz (?) at 14 TeV
- 60 kHz at 3×10^{34}

Phase 1 Small wheels

- removing background
- improving p_T resolution

New **small wheels** for phase-1:

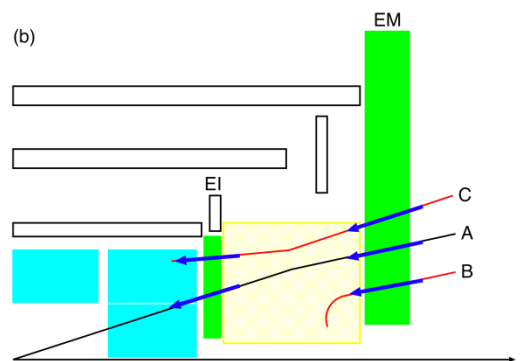
- precision tracker with high rate capability
- fast segment reconstruction for L1 upgrade



Present muon L1

Based on EM (Big wheel) segments:

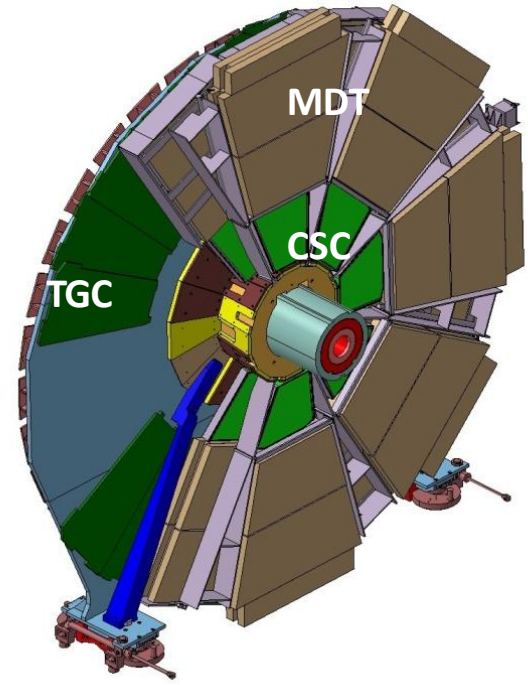
- many background tracks
- fake high p_T muons



Upgrade

Integrate EI (Small wheel) segments in trigger:
 requiring an IP pointing segment on EI matched to the EM segment

T. Kawamoto



The small wheels options

sMDT + sTGC

sMDT : 15 mm tube instead of 30 mm (x7 increase of rate capability)

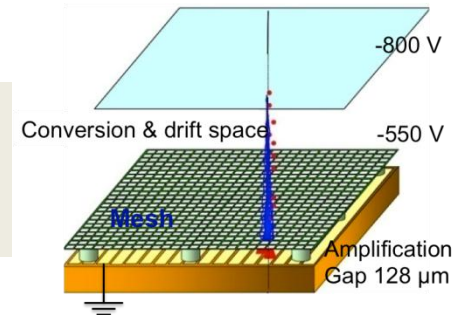
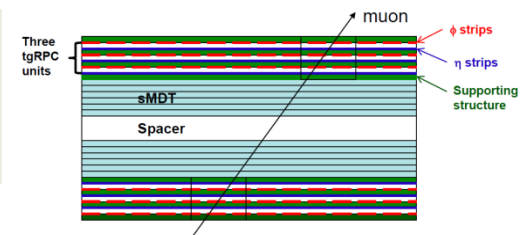
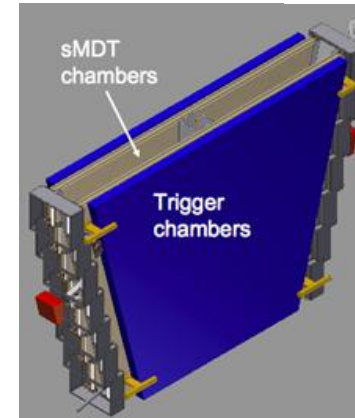
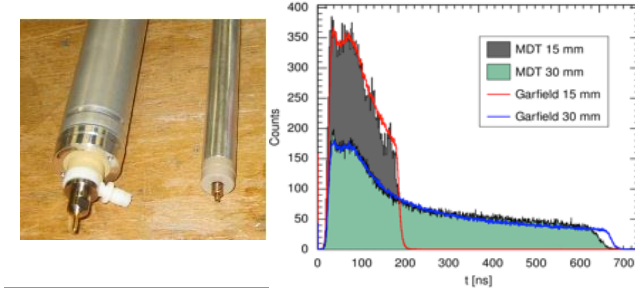
sTGC : strip readout for precision coordinates ($\sim 100 \mu$) for 1 mrad resolution

sMDT + mRPC

mRPC : 2x1 mm gap instead of a single 2 mm gap combined with low noise amplifier
big reduction of charge \rightarrow high rate, long life

Micromegas

Based on a single technology for both tracking and trigger
Very high rate capability, good position resolution
New technology for the use in large scale (~ 1 m size)



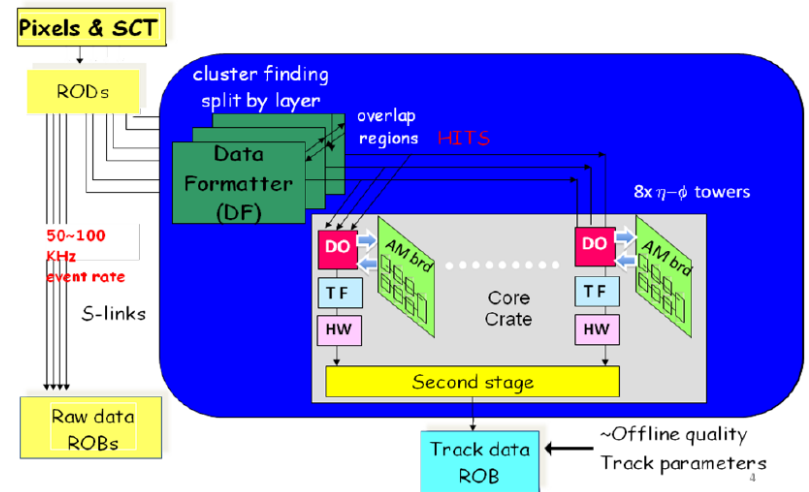
T. Kawamoto

The Fast Tracker (FTK)

FTK is a hardware track finder up to 3 orders of magnitude faster than L2 processor farm

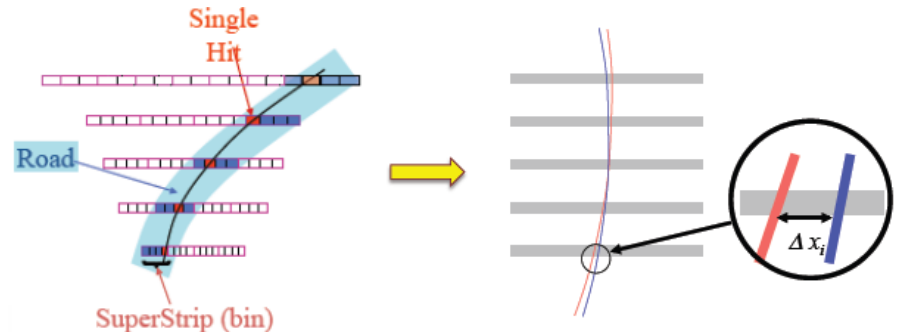
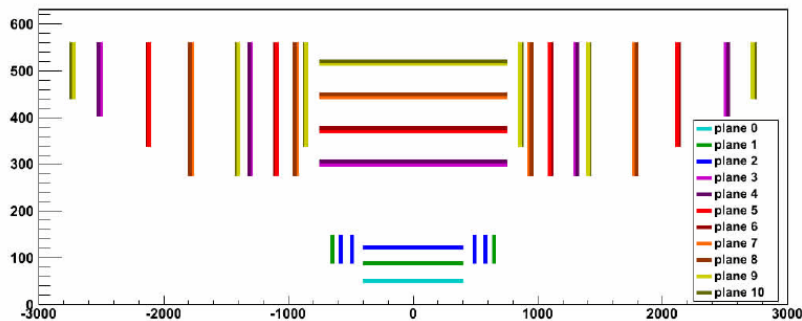
Technical Proposal April 13, 2010

- frees L2 farm time
- could scale to $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- track parameters comparable to offline tracking
- off-detector → upgrade without long access
- will not reduce L1 trigger rate but help at L2
- template technology interesting for SLHC



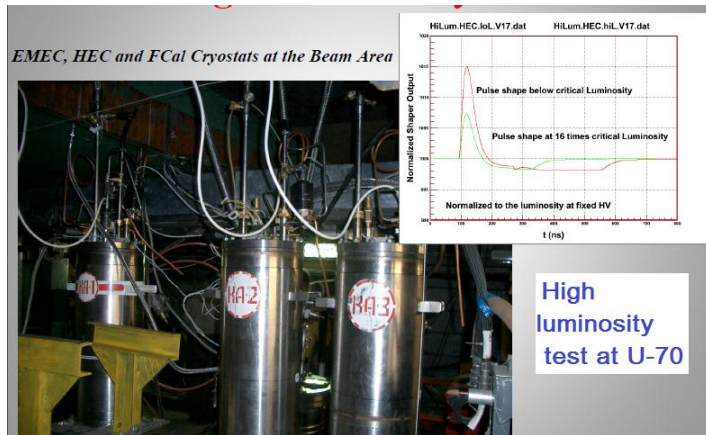
global tracking in two steps:
pattern recognition and track fit

FTK Logical Layer assignment (R-Z View)

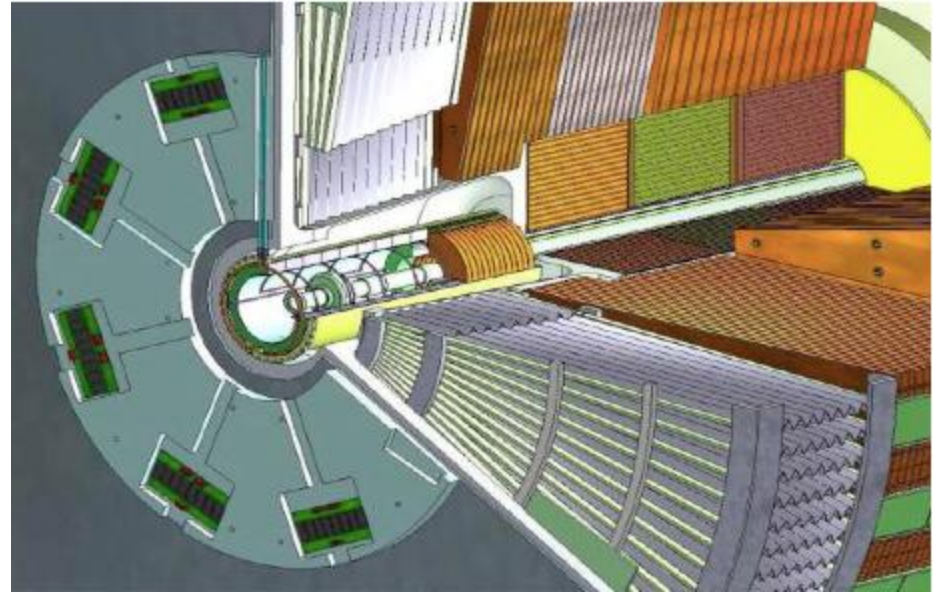


Challenges:

- Charge build-up in Liquid Argon gap
- Higher current draw → HV drop
- High ionization load → Lar boiling



Forward calorimeter



Baseline: small warm calorimeter in front of the LAr FCAL in order to protect it from heating, ion build-up

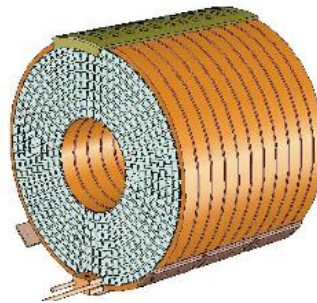
Copper absorbers +1 cm² diamond sensors on ceramic highly segmented readout

Options: Warm High-pressure Xe
LAr mini-Fcal

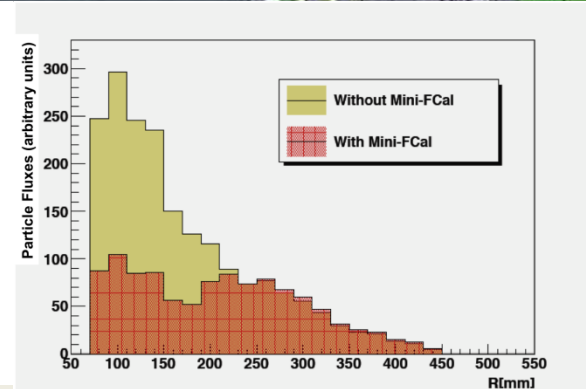
Or:

Replace FCal with super FCal (sFCal)

- Smaller gaps
- New HV protection resistors
- Additional cooling



Phase 1 – Phase 2 ?



Simulation results on the energy deposited on FCal1.

Will cold electronics inside end-cap survive 3000 fb-1?

The ATLAS Forward Physics Project (AFP)

~210 m from ATLAS IP consisting of 3 main parts

- Movable Beam Pipe
- Si Detectors (momentum)
- Timing (backgrounds)

Si Detectors:

Baseline:

- IBL 3D Si sensors; dead edge $\sim 225 \mu\text{m}$
Very promising R&D for edgeless/active-edge
3D Si; dead edge $\leq 50 \mu\text{m}$.
- FE-I4b Readout chips,

Timing detectors

Crucial to reduce backgrounds from
 ≥ 2 single-diffraction events in same crossing

Baseline:

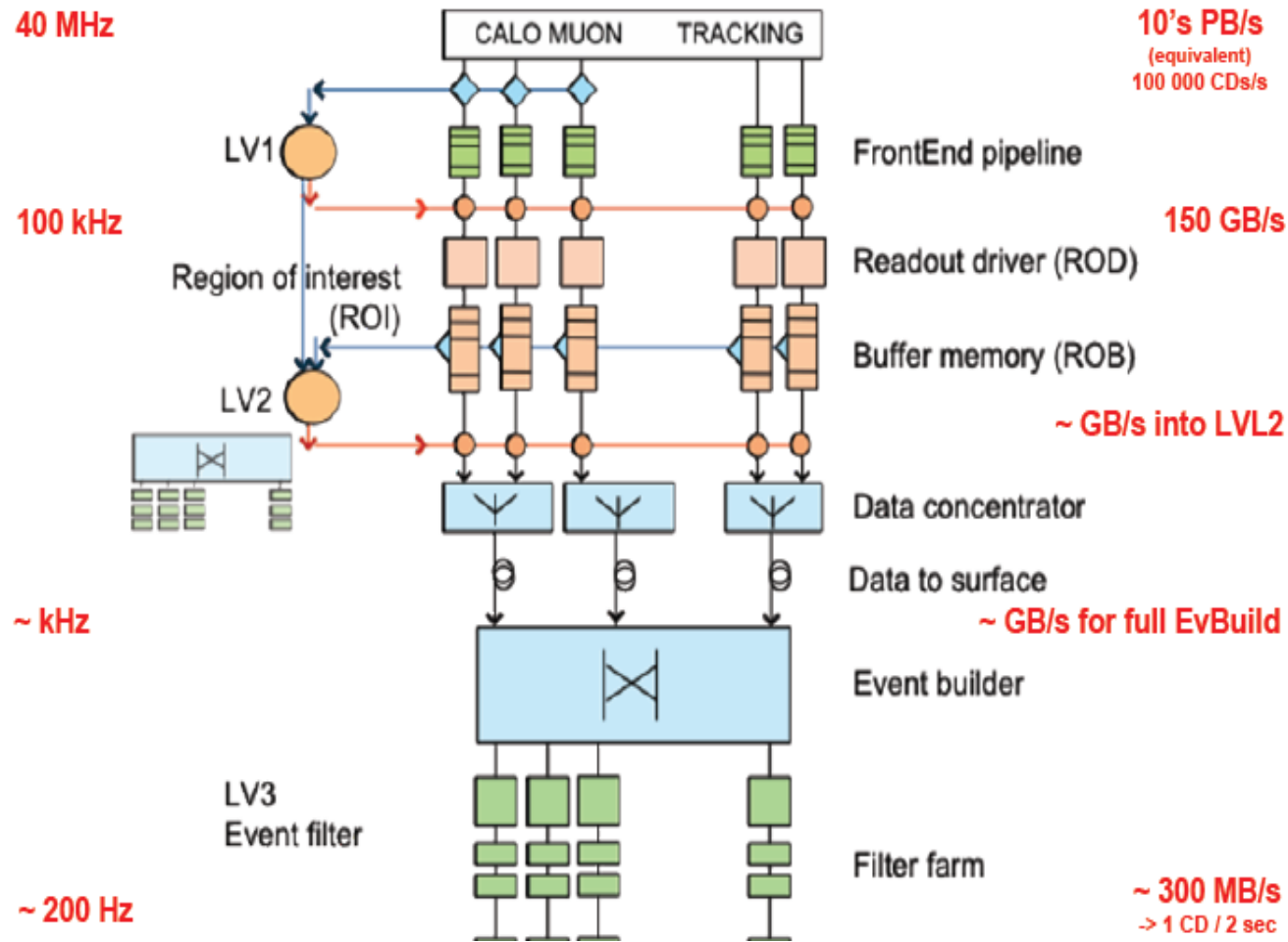
- quartz bars
- MCP-PMT

To be approved

Phase 2 Trigger

- More processing at L1 :
- sharpening threshold for leptons and calorimeter energy
 - full readout of digitized energy from all cells over high speed optical link
 - bringing higher granularity for L1 calorimeter trigger
 - L1Track?

Longer L1 latency :
 $3\mu\text{s} \rightarrow 6\mu\text{s}$ or $9\mu\text{s}$ or even longer



Inner tracker

Rate :

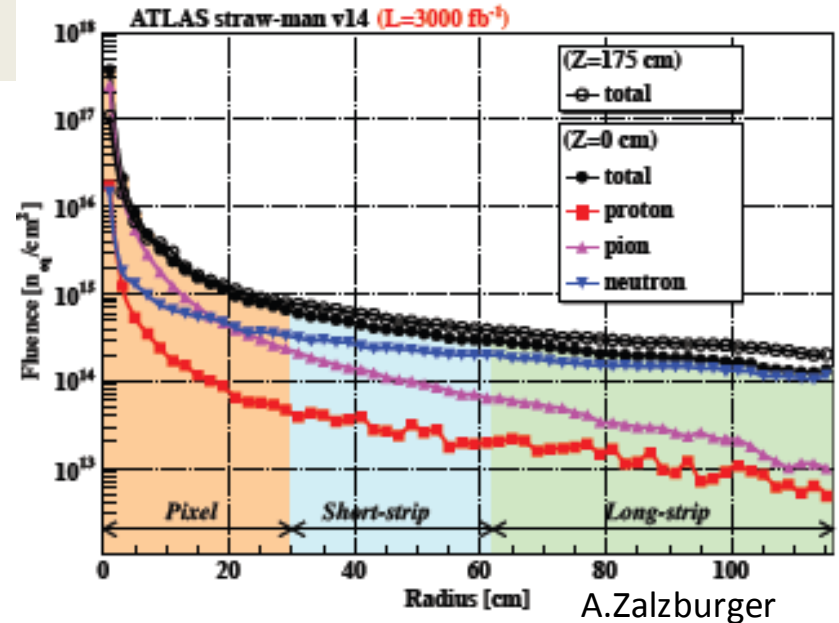
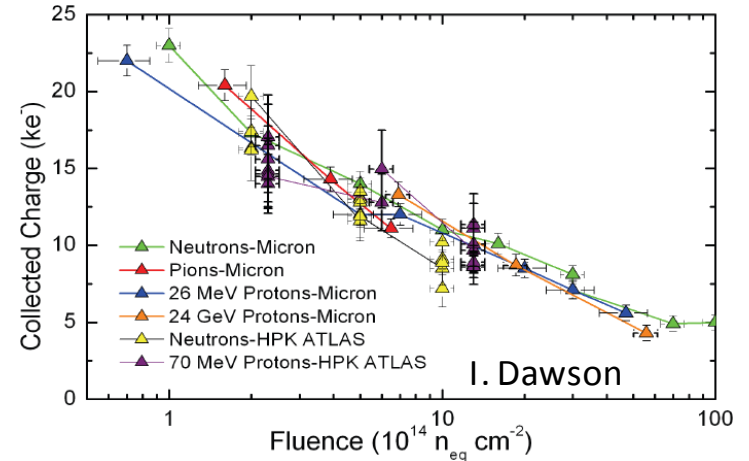
- Pixel B-layers will become inefficient at 2×10^{34}
- SCT (strip), bandwidth limitation
- TRT occupancy will become very high

Radiation damage:

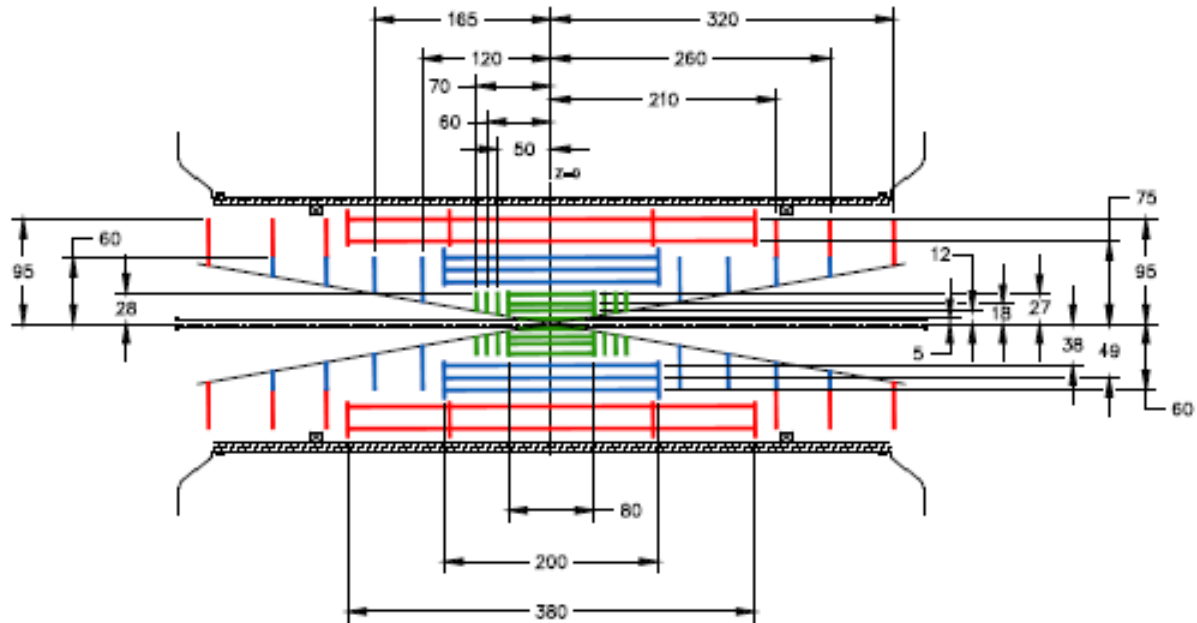
- SCT designed for 700 fb⁻¹
- Much shorter life for B-layer

All new inner tracker for Phase2

- higher granularity to keep occupancy low
- improved radiation hardness
- improved material budget
- baseline : all silicon strip + pixel



Inner tracker



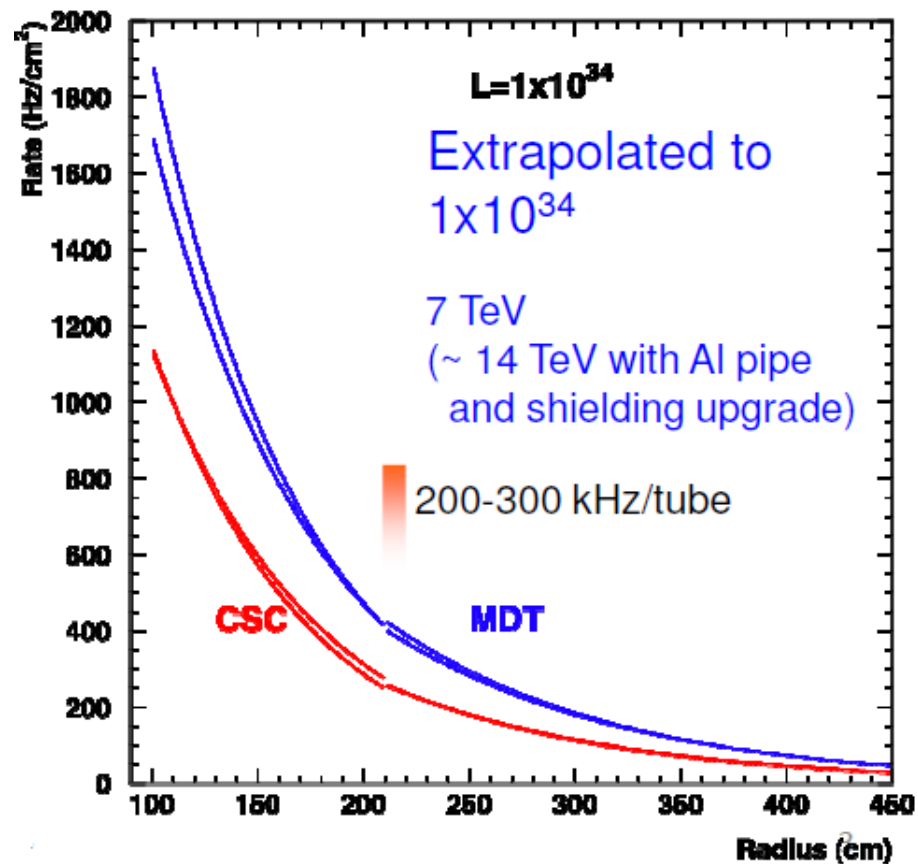
All silicon: pixel (4 layers), short strips (3.5 cm, 3 layers), long strips (9 cm, 2 layers)
40M channels in Si strips (6M in the current detector)
160M channels in Si pixel (80M in the current detector)

n-in-p sensors in full size available
Breakdown voltage > 1000 V
Radiation hardness verified up to 10^{15} neq cm^{-2}

Muon detectors

Limitation of the present MDT :
200-300 kHz/tube

- More shielding
- New/additional muon chambers

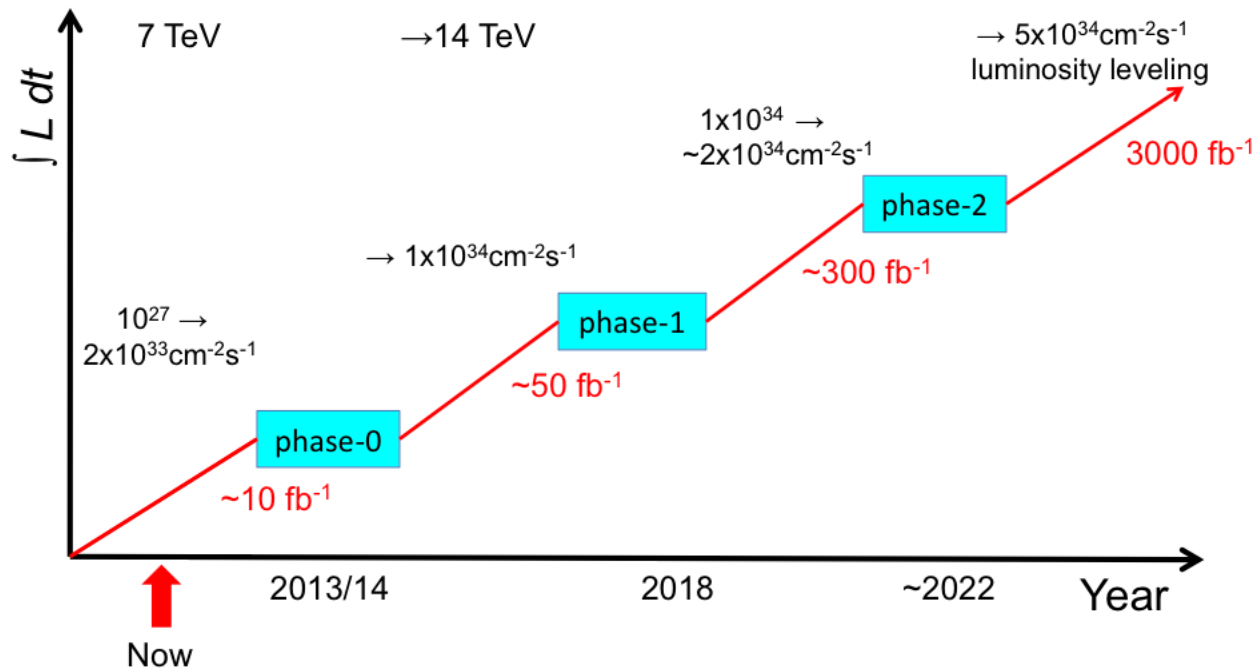


Conclusion

1 fb⁻¹ → 3000 fb⁻¹

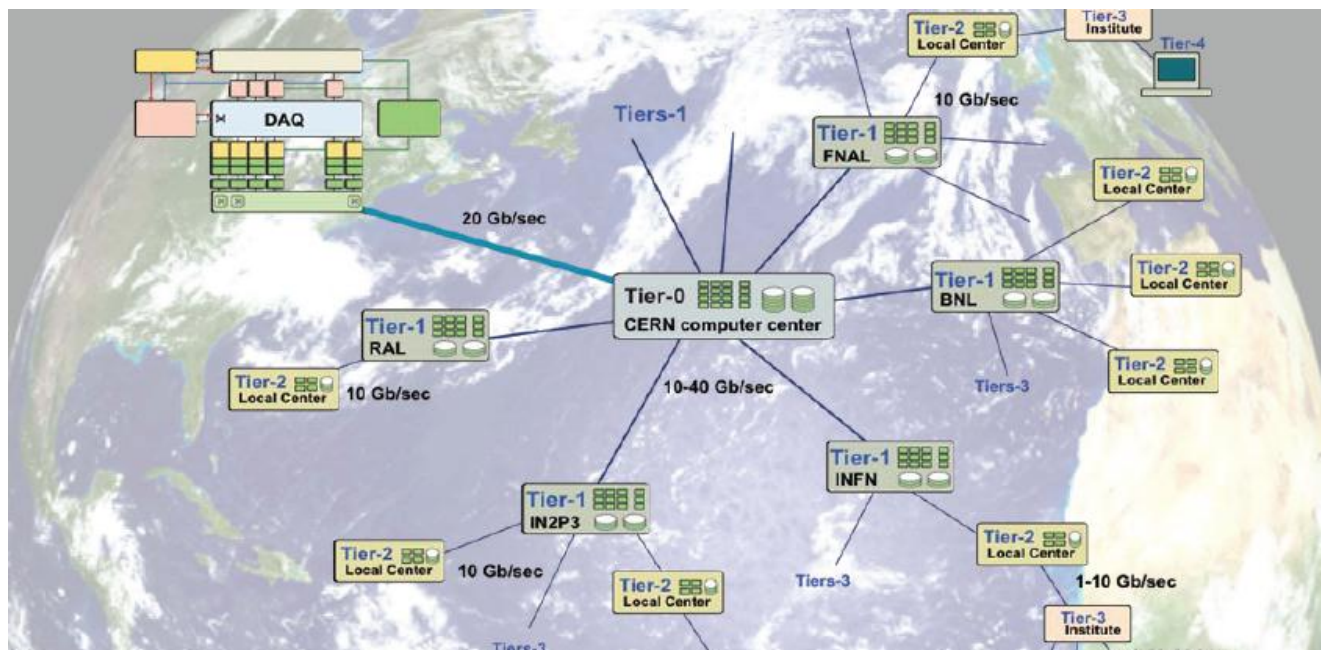
0.03% → 100%

Possible upgrade timeline



Spare

Data analysis



Grid: A distributed computing infrastructure, uniting resources of HEP institutes around the world to provide institutes seamless access to CPU and storage for the LHC experiments

- Tier-0 (CERN) : recording –reconstruction –distribution
- Tier-1 (~10 centres) : storage -reprocessing –analysis
- Tier-2 (~140 centres) : simulation –end-user analysis

- **Aplanarity:**

the smallest eigenvalue of the momentum tensor

- $H_{T,3p}$: the transverse momentum of all but the two leading jets, normalized to the sum of absolute values of all longitudinal momenta in the event

$$M_{ij} = \frac{\sum_{k=1}^{N'_{\text{objects}}} P_{ik} P_{jk}}{\sum_{k=1}^{N'_{\text{objects}}} P_k^2}$$

$$H_{T,3p} = \frac{\sum_{i=3}^{N_{\text{jets}}} |p_{T,i}|}{\sum_{j=1}^{N_{\text{objects}}} |p_{z,j}|}$$

- **The anti-kt algorithm** constructs, for each input object (either energy cluster or particle) i , the quantities d_{ij} and d_{iB} as follows:

where

k_{ti} is the transverse momentum of object i with respect to the beam direction.

A list containing all the d_{ij} and d_{iB} values is compiled. If the smallest entry is a d_{ij} , objects i and j are combined (their four-vectors are added) and the list is updated. If the smallest entry is a d_{iB} , this object is considered a complete “jet” and is removed from the list.

$$d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2}) \frac{(\Delta R)_{ij}^2}{R^2}$$

$$d_{iB} = k_{ti}^{-2}$$

$$(\Delta R)_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$