

The electroweak radiation picture of the future multi-TeV muon colliders

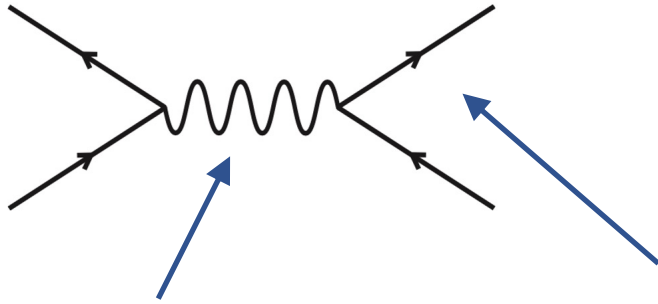
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Early Career Researchers & Muon Colliders (August 28, 2024)



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Lepton colliders are s-channel machines



$$\sigma \sim \frac{1}{s} = \frac{1}{E^2}$$

Resonance peaks:

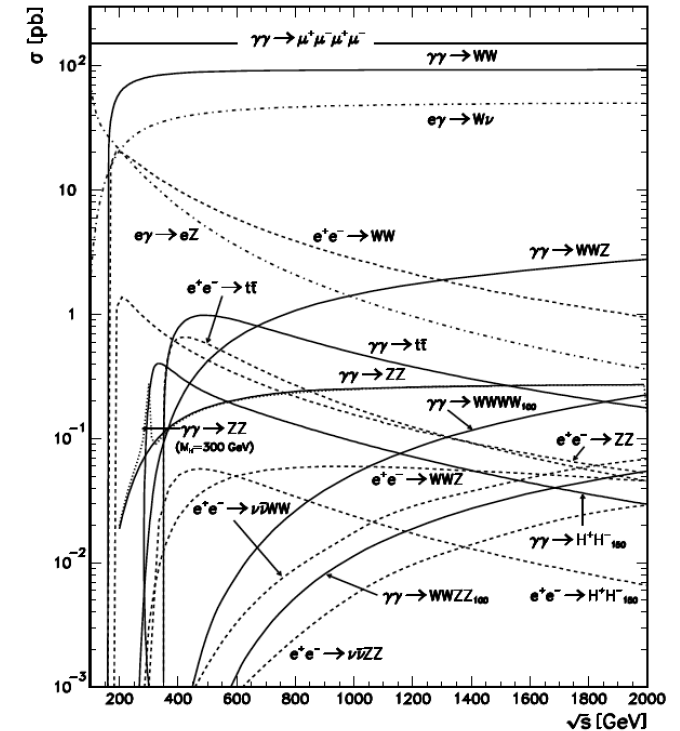
- B Factories: $\Upsilon(4S)$ peak (10.6 GeV) e.g. Babar, Belle II
- Z Factories, Z peak (91.188 GeV) e.g. LEP
- HE machines: BSM resonance peaks?

Pair production:

e.g. e+ e- Higgs Factory at 250 GeV
Look for a Z at 110 GeV, whatever on the other side, it is from Higgs decay!

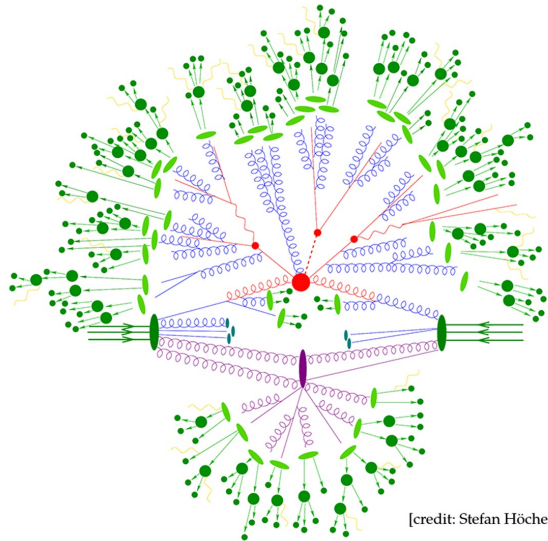
Lepton colliders are clean

Photon-Photon machine



S. Brodsky, SLAC-PUB-11581

Q: Is this still the case for multi-TeV lepton colliders?



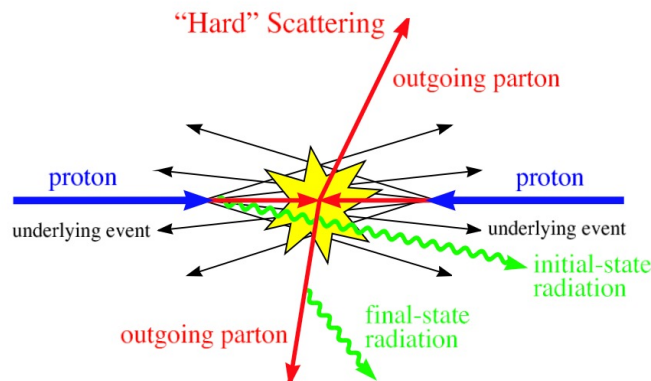
What happens at Tevatron/LHC?

- **Hard interaction:** fixed-order calculation (in QCD $\equiv 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \dots$)
- **All-order radiative corrections:** parton shower emissions
- **Hadronisation** from confinement property (quarks and gluons cannot live individually for long)
- **Multiple parton interactions (MPI)**
- Hadron decays

The QCD factorization formalism

- Hadrons are composite and contains “partons” inside them
- The parton distribution functions (PDF) describes the probability to find a parton with a particular momentum from the beam particle.

$$\sigma(AB \rightarrow X) = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \rightarrow X)$$



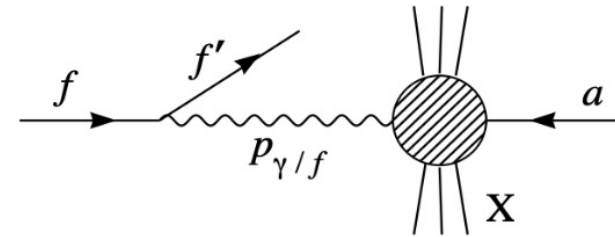
Q: how could an elementary particle contain a “parton” inside itself?

Equivalent photon approximation (EPA)

Treat radiation generated photon as the “parton”

$$\sigma(\ell^- + a \rightarrow \ell^- + X) = \int dx f_{\gamma/\ell} \hat{\sigma}(\gamma a \rightarrow X)$$

$$f_{\gamma/\ell, \text{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi} \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q^2}{m_\ell^2}$$



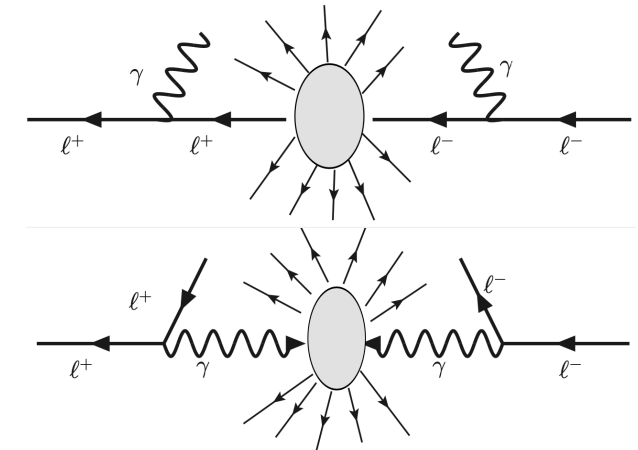
Applications on colliders

- Initial state radiation (ISR)
- Photon-photon collisions

Polarizations

$$\sigma(\ell^+ \ell^- \rightarrow F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(ij \rightarrow F), \tau = \hat{s}/s$$

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$



For unpolarized beam, it is okay to average over parton’s polarization

	e_L^-	e_R^-	$\langle e^- \rangle$	e_L^+	e_R^+	$\langle e^+ \rangle$
γ_-	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$
γ_+	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$
$\Sigma \gamma_\lambda$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$

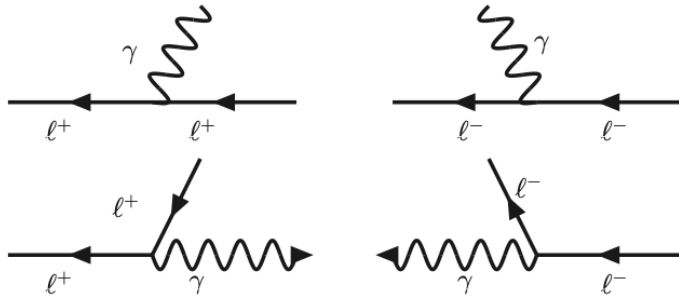
The full EW partonic picture

People have been doing:

- ▶ $l^+ l^-$ annihilation



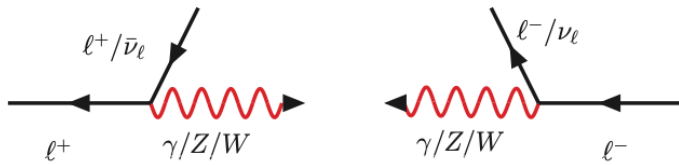
- ▶ EPA and ISR



- ▶ “Effective W Approx.” (EWA)

[G. Kane, W. Repko, and W. Rolnick, PLB148 (1984) 367]

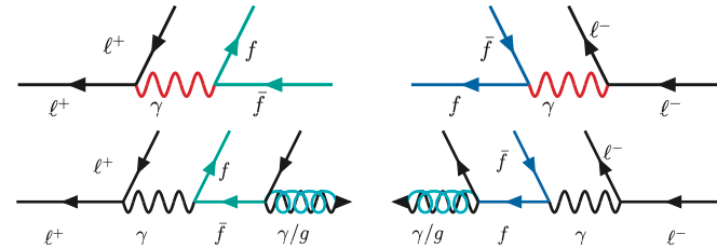
[S. Dawson, NPB 249 (1985) 42]



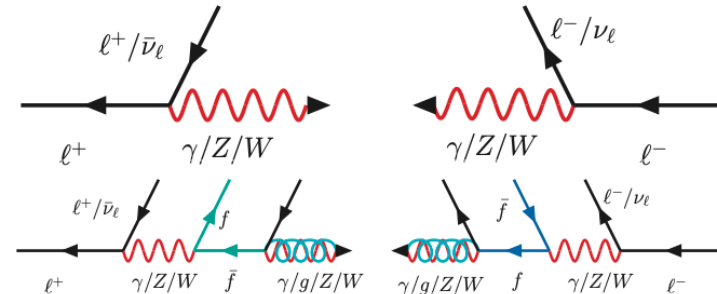
We will add [T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

[F. Garosi, D. Marzocca, S. Trifinopoulos 2303.16964]

- ▶ Above μ_{QCD} : $\text{QED} \otimes \text{QCD}$
 q/g emerge



- ▶ Above $\mu_{\text{EW}} = M_Z$: $\text{EW} \otimes \text{QCD}$
EW partons / corrections to the above



In the end, everything is parton, i.e. need the full SM PDFs.

The Eletroweak PDFs

Below the EW scale

- ▶ **Muon PDFs:** $f_{\mu_{\text{val}}}$, f_{γ} , $f_{\ell_{\text{sea}}}$, f_q , f_g
- ▶ Scale uncertainty: 20% for f_g/μ

Above the EW scale

- ▶ **All SM particles are partons**

[T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

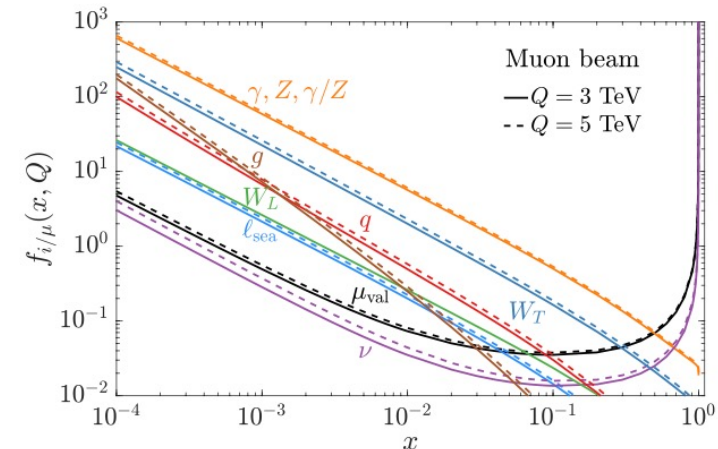
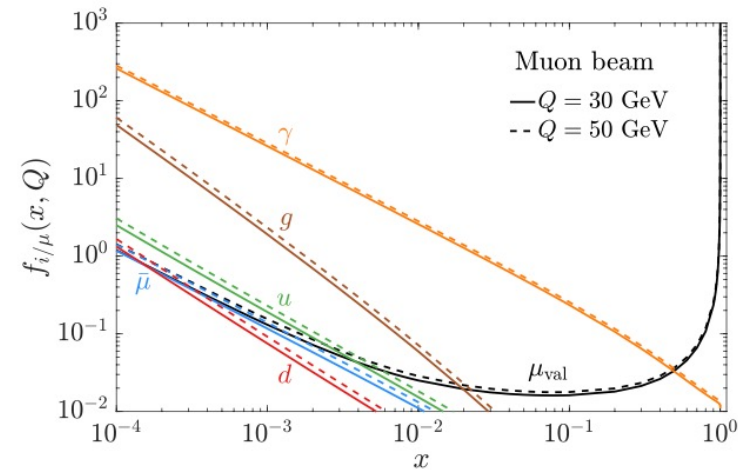
- ▶ The sea leptonic and quark PDFs show up

$$\nu = \sum_i (\nu_i + \bar{\nu}_i),$$

$$\ell_{\text{sea}} = \bar{\mu} + \sum_{i \neq \mu} (\ell_i + \bar{\ell}_i),$$

$$q = \sum_{i=d}^t (q_i + \bar{q}_i)$$

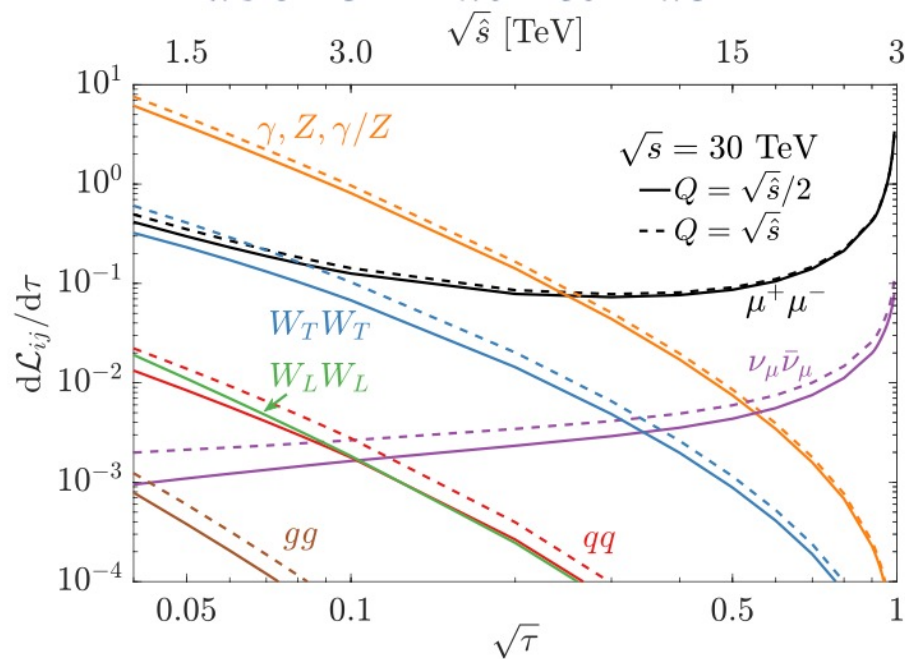
There is even neutrino due to the EW sector



Tao Han, YM, Keping Xie, arXiv:2007.14300

Partonic luminosity of a 30 TeV MuC

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j) \right]$$

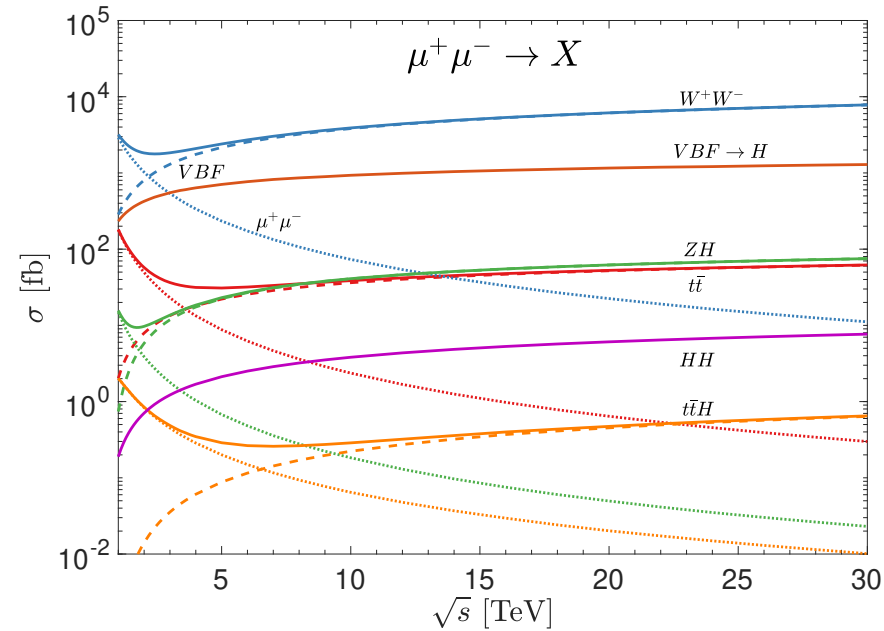


The polarization is counted

Semi-inclusive processes

Just like in hadronic collisions:

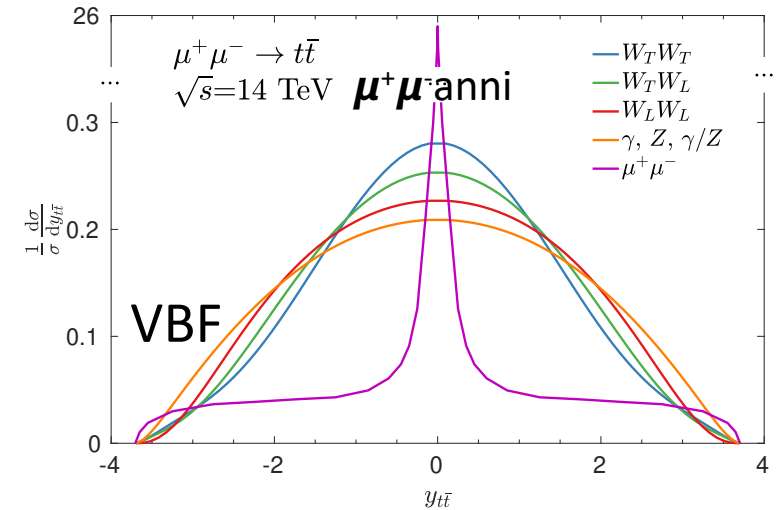
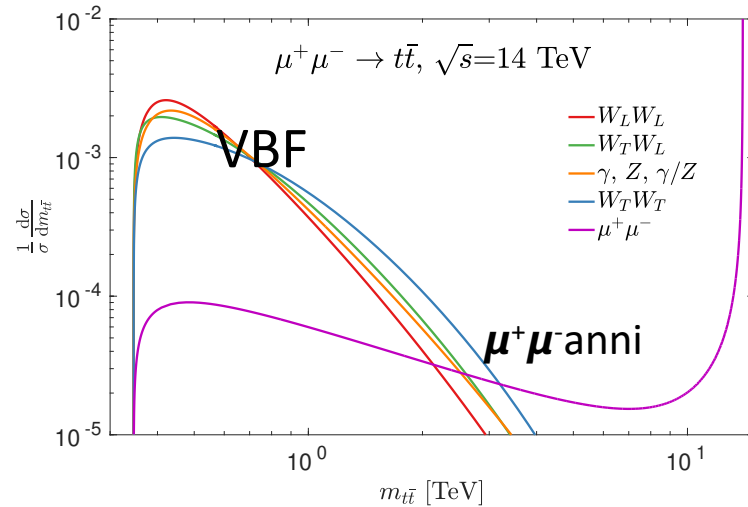
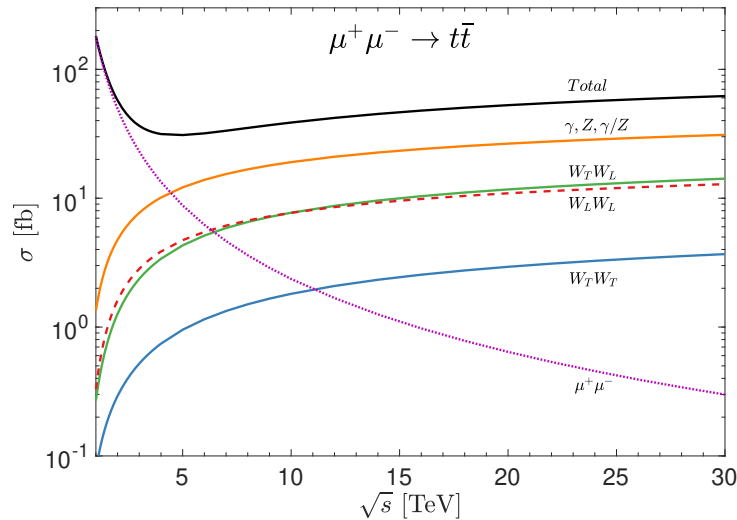
$\mu^+ \mu^- \rightarrow$ exclusive particles + remnants



$pp \rightarrow tt = gg \rightarrow tt + qq \rightarrow tt$

$\mu^+ \mu^- \rightarrow tt =$ annihilation + VBFs

Partonic contributions



Tao Han, YM, Keping Xie, arXiv:2007.14300

**$\mu^+\mu^-$ Collider -- “Buy one, get one free”:
Annihilation + VBF**

Thank you



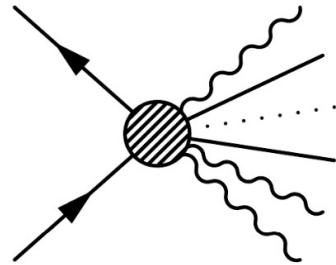
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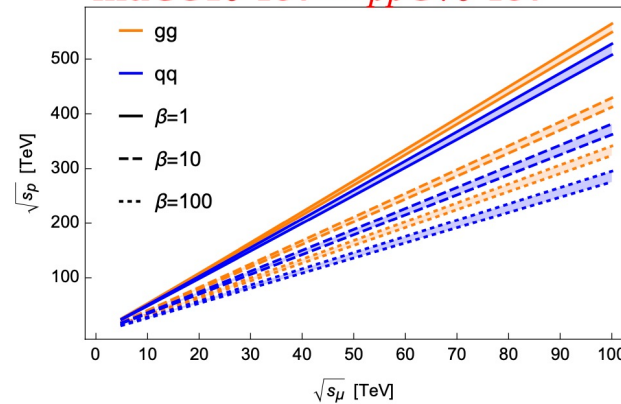
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The general physics picture of muC

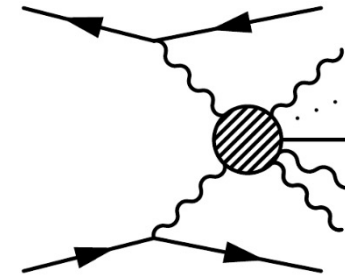
Annihilation



muC@10 TeV ~ pp@70 TeV



VBF



Take luminosity 10 ab^{-1}
 10M H, 500K HH @ 10 TeV

- ▶ l^+l^- annihilation **probes TeV scale directly**
- ▶ VBF **scans physics in the full spectrum of energy**
 From the threshold to up to 2 orders of magnitude above EW scale.
- ▶ It produces a lot of H , top quarks, W/Z , ... as a **“factory” for SM precision test**
- ▶ **An “EW jet factory”**
 In addition to QCD jets, there are W/Z jet, H jet, t jet, neutrino jet, ...
 Even neutrino collision is not impossible!

Challenges:

Be careful about the radiation!

EW NLO shall be necessary, just like the NLO QCD at LHC.

More Examples

