



Equivariant neural networks for CP violation

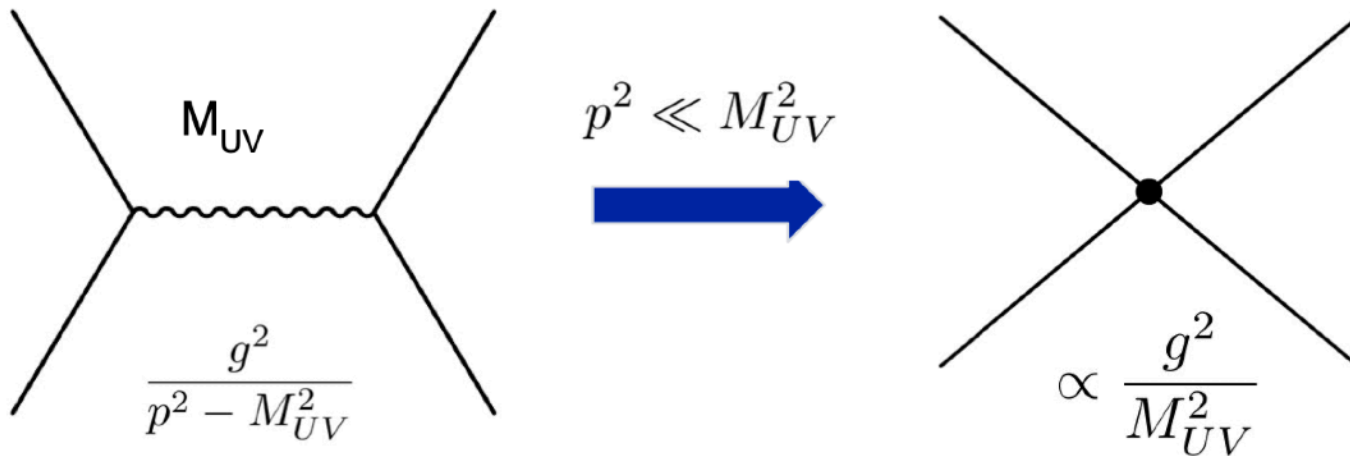
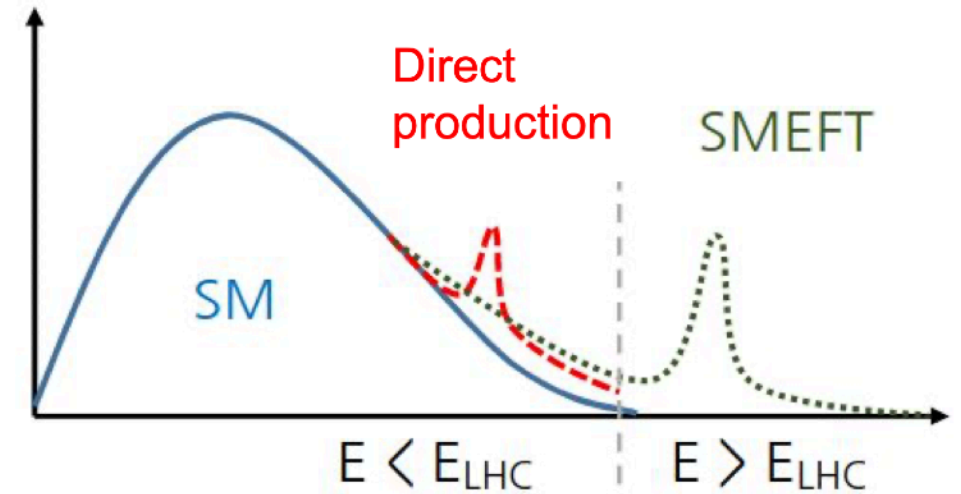
Sergio Sánchez Cruz in collaboration with Marina Kolosova, Giovanni Petrucciani, Clara Ramón, and Pietro Vischia

[arXiv:2405.13524](https://arxiv.org/abs/2405.13524)

10.06.2024

Introduction: SMEFT and CP-violation

- SMEFT is an extension of the SM, adding contributions from high-mass BSM particles
- 1350 CP-even operators, 1149 CP-odd operators
- Plenty of CP-violation sources to study!

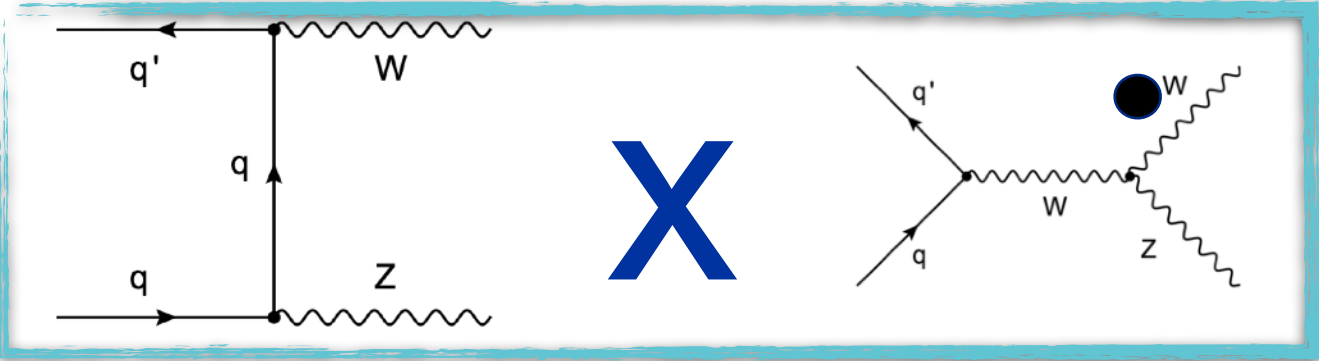
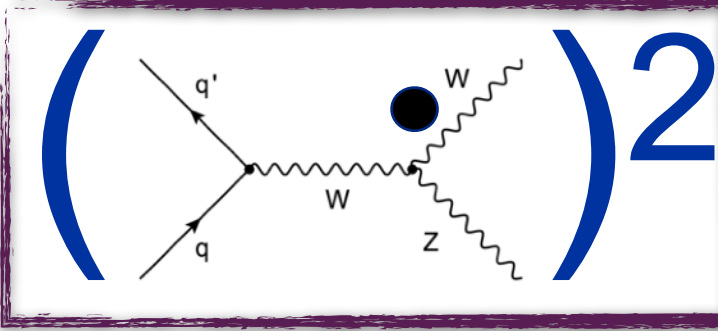
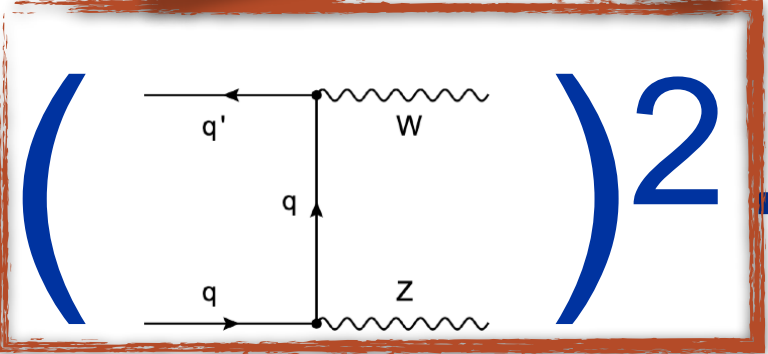


$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{C_i}{\Lambda^2} O_i^{(6)}$$

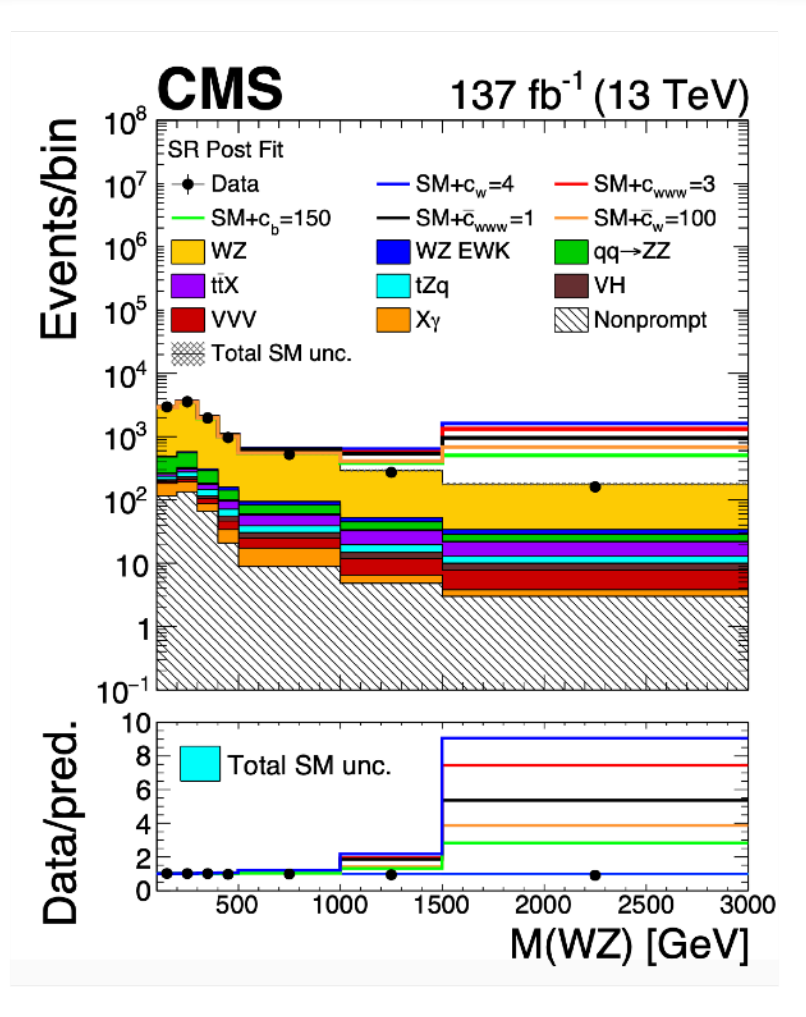
EFT at observable level

SM contribution

Pure BSM contribution



SM-BSM interference



JHEP 07 (2022) 032

CP-violating operators

SM contribution

Pure BSM contribution

SM-BSM interference



- Mostly CP-invariant
- CP-invariant in e.g. the top/Higgs sectors

- Particularly interesting from the phenomenological standpoint
- Odd under CP transformations

- CP-even observables (most of the LHC cross section measurement program) do not provide sensitivity to the interference
- CP-odd observables are robust against signal mismodeling / backgrounds

The algorithm

- The algorithm builds observables that are **equivariant with respect to the CP-symmetry**
 - **CP-invariant observables** are useful to discriminate among backgrounds for searches targeting the pure-BSM part
 - **CP-odd observables** are useful to get sensitivity to the interference term
 - Can be generalized to n_1 CP-invariant and n_2 CP-odd components
- A function $f: D \rightarrow R$ is odd/even under CP transformation if $f(\text{CP}(\text{event})) = \pm f(\text{event})$
 - The function $f(\text{event}) = g(\text{event}) \pm g(\text{CP}(\text{event}))$ trivially satisfies that
- **The space of input features is fully general**
 - Can be the kinematics of a fixed set of particles or a particle set
- We take g to be a fully-connected neural network, could be any function

Training and cost function

- Method inspired in the [SALLY method](#) shown in [2401.10323](#)
 - Equivariant networks can also be used with different cost functions

- Training the algorithm on weighted simulations $w(z) = w_{SM}(z) + cw_{int}(z) + c^2w_{quad}(z)$
 - Function of parton-level kinematics
 - Can be used to compute the (non tractable) likelihood ratio

$$\frac{p(d, z|c_1)}{p(d, z|c = 0)} = \frac{w_{SM} + cw_{lin} + c^2w_{quad}}{w_{SM}}$$

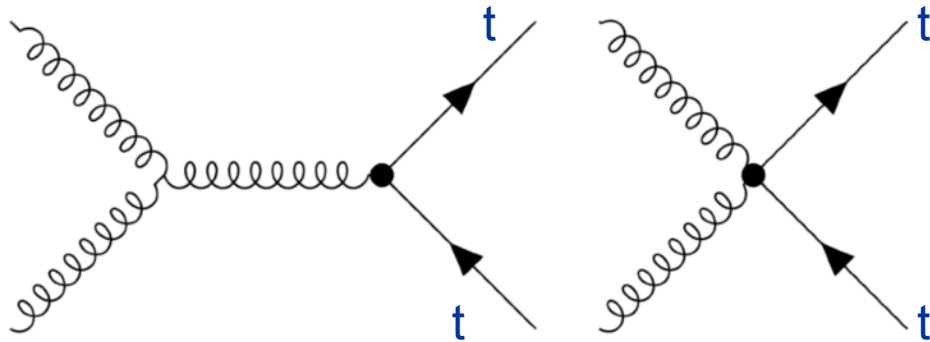
- We are interested in the likelihood score at the SM \rightarrow sufficient statistic for small values of c
 - Small values of c \rightarrow dominated by the interference
- Minimizing the loss function, we obtain a surrogate model of the score

$$L = w_{SM} \left(f(d) - \frac{w_{int}(z)}{w_{SM}(d)} \right)^2$$

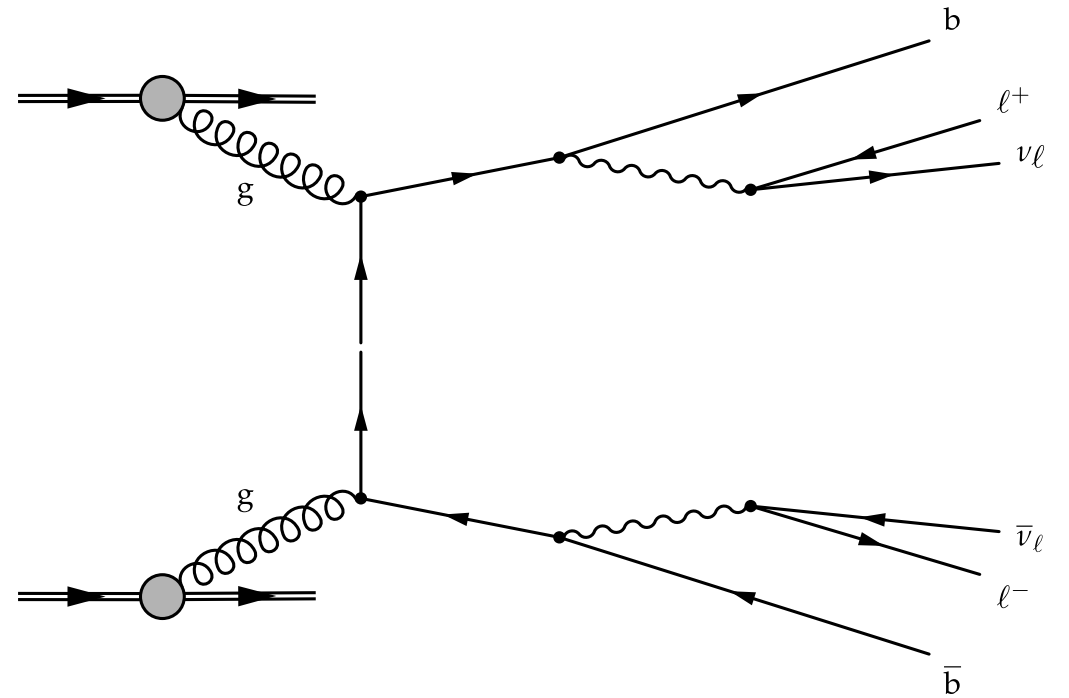
Use case: ttbar production

- Targeting ttbar production in the dileptonic final state
- Potentially affected by the chromoelectric dipole moment operator (CP violating)

$$g_s \frac{\nu}{\sqrt{2}} (\bar{t} \sigma^{\mu\nu} \gamma_5 T^A t) G_{\mu\nu}^A$$

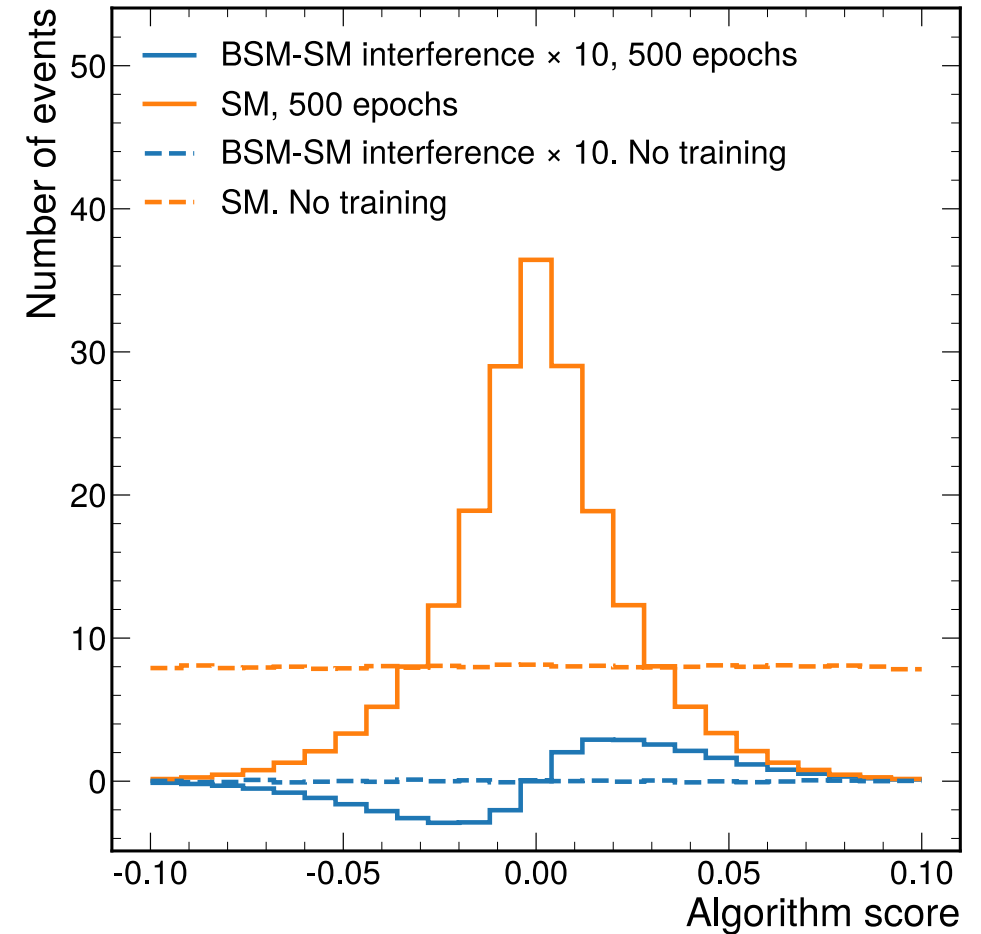


$$\vec{p}_\ell^+, \vec{p}_\ell^-, \vec{p}_{j_1}, \vec{p}_{j_2}, \vec{p}_T^{\text{miss}} \xrightarrow{\text{CP}} -\vec{p}_\ell^-, -\vec{p}_\ell^+, -\vec{p}_{j_2}, -\vec{p}_{j_1}, -\vec{p}_T^{\text{miss}}$$



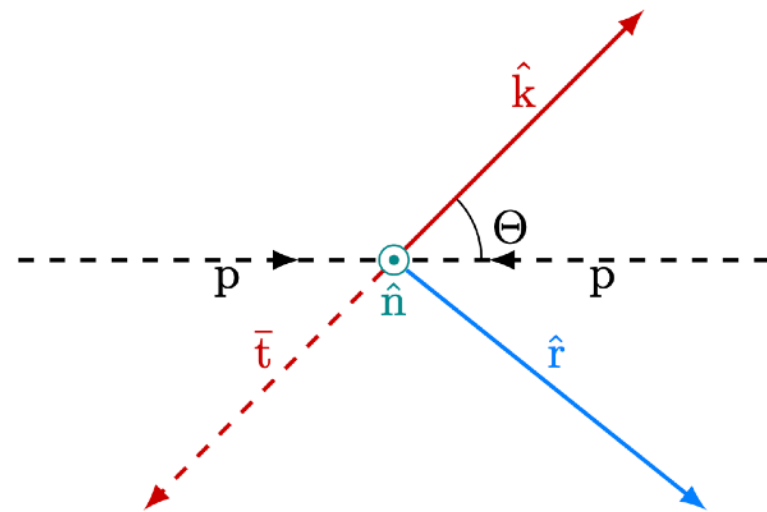
Use case: $t\bar{t}$ production

- Score after the training is a CP-odd observable
 - Symmetric for the SM contribution
 - **Any SM-like mismodeling / background will be symmetric by construction**
 - Interference contributes constructively for positive values and negatively for negative values
- Equivariance respected even during (or before) training
- **Observable is robust even if the training has not converged**



Use case: $t\bar{t}$ production

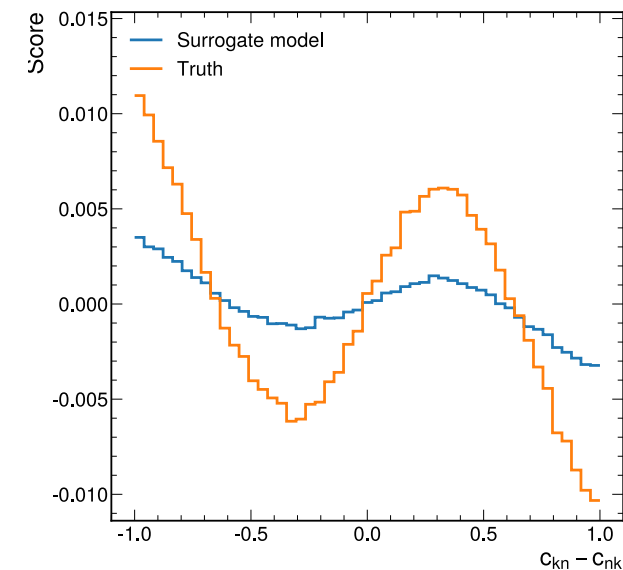
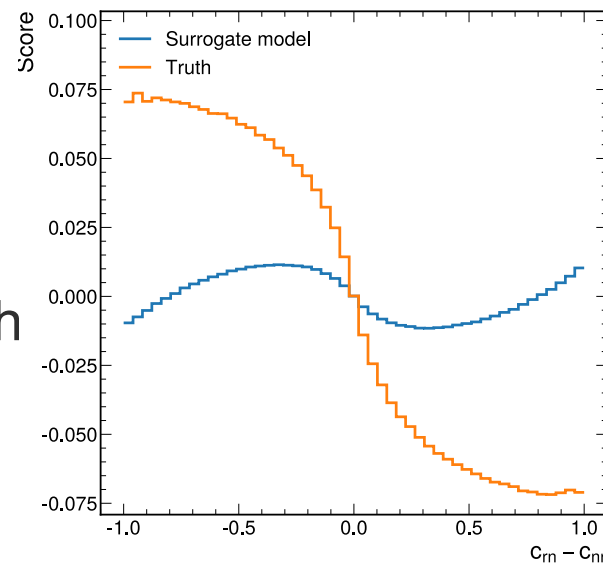
- The algorithm provides a surrogate model of the score
 - Comparing against the true model (from parton level quantities)
- [1508.05271](#) proposes two observables, relying on the reconstruction of the $t\bar{t}$ system, based on angles between leptons and axes



$$C_{rn} - C_{rn} = \cos(l_r^+) \cos(l_n^-) - \cos(l_n^+) \cos(l_r^-)$$

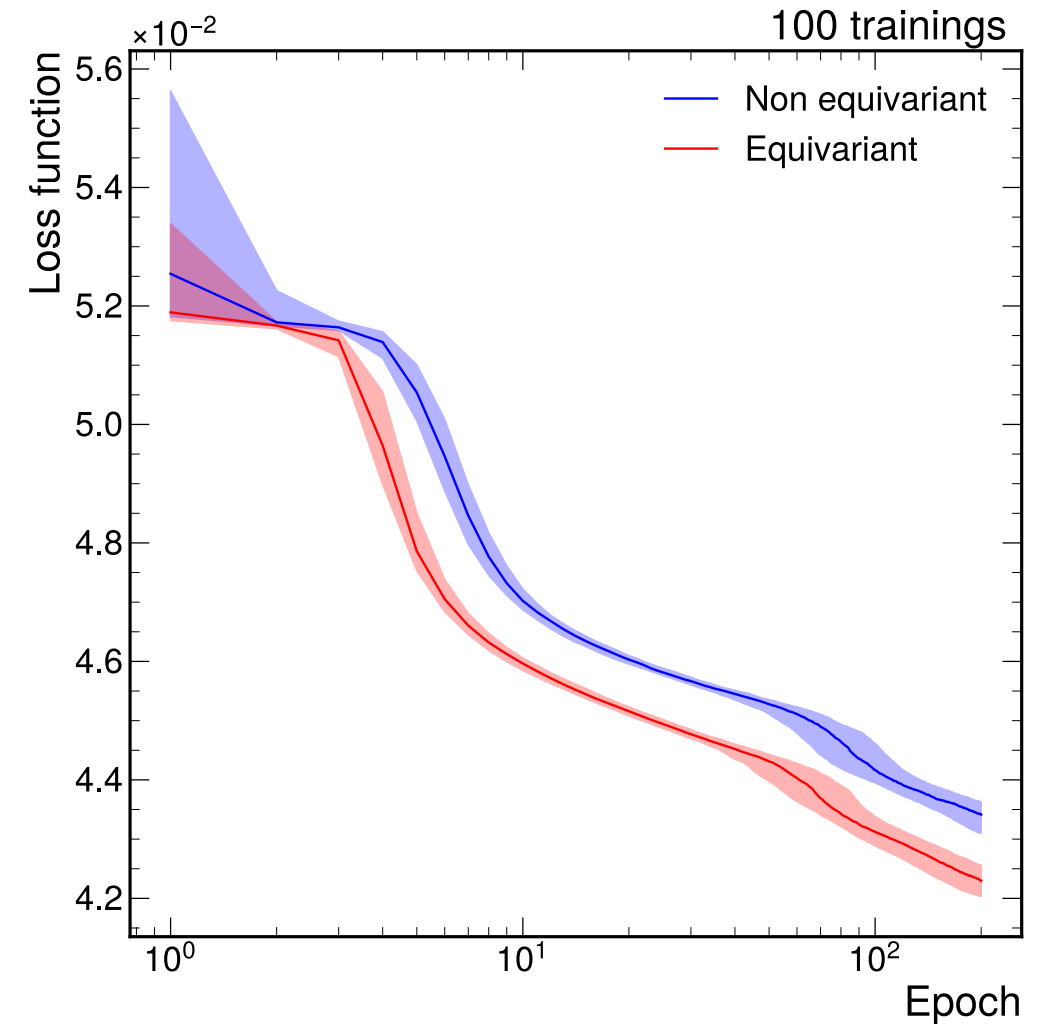
$$C_{kn} - C_{nk} = \cos(l_k^+) \cos(l_n^-) - \cos(l_n^+) \cos(l_k^-)$$

- Partially learning them
 - Limited by the possibility of reconstructing the system



Use case: $t\bar{t}$ production (II)

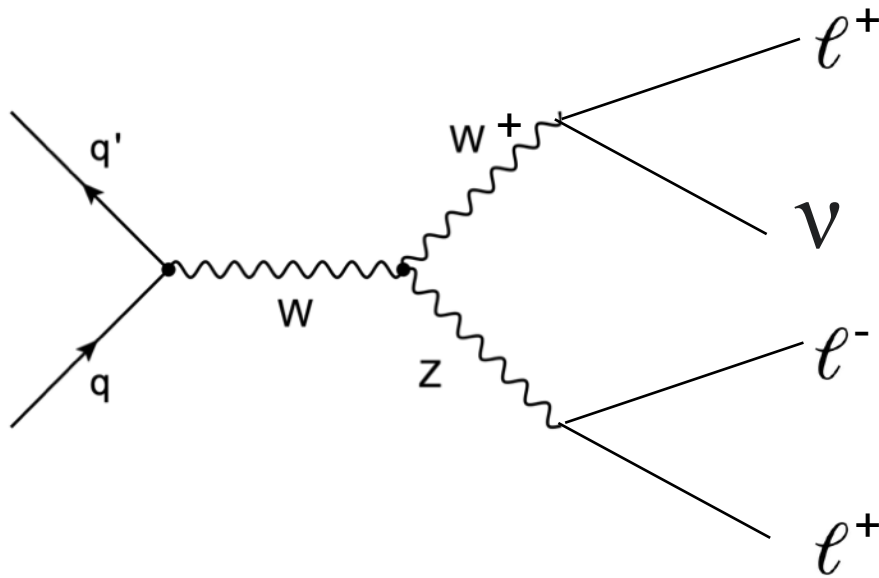
- Imposing equivariance as inductive bias improves the convergence of the model
- Training 100 instances of equivariant and non-equivariant model
- Smaller variance in the first steps of the training
- Overall, between 40 and 300% less iterations needed to achieve the same loss function



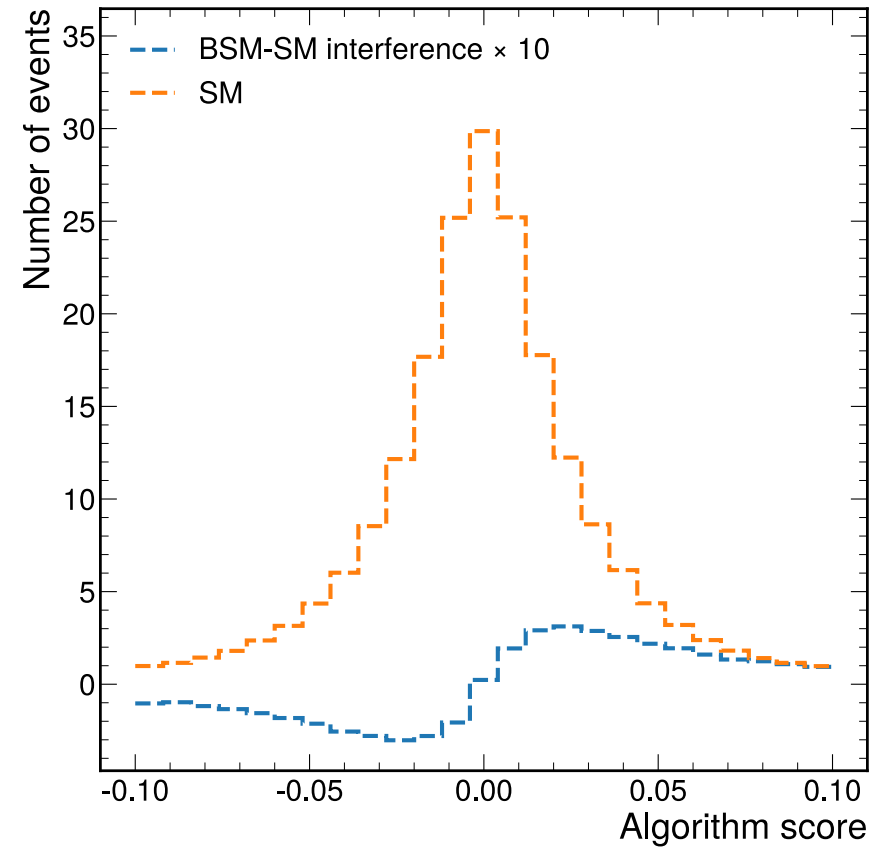
Use case: WZ production

- Targeting WZ production in the three lepton final state
- Potentially affected by the cWtilde operator

$$\epsilon^{ijk} \widetilde{W}_{\mu}^{i\nu} W_{\nu}^{j\rho} W_{\rho}^{k\mu}$$

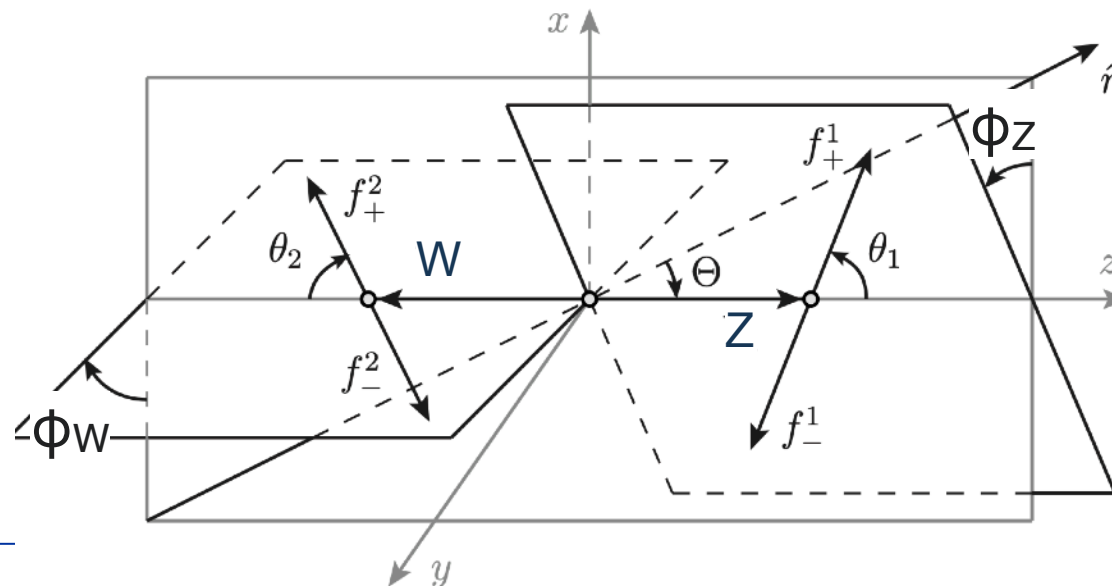


$$\vec{p}_{\ell^+}^Z, \vec{p}_{\ell^-}^Z, \vec{p}_{\ell}^W, Q^W, p_T^{\text{miss}} \xrightarrow{\text{CP}} -\vec{p}_{\ell^-}^Z, -\vec{p}_{\ell^+}^Z, -\vec{p}_{\ell}^W, -Q^W, -p_T^{\text{miss}}$$

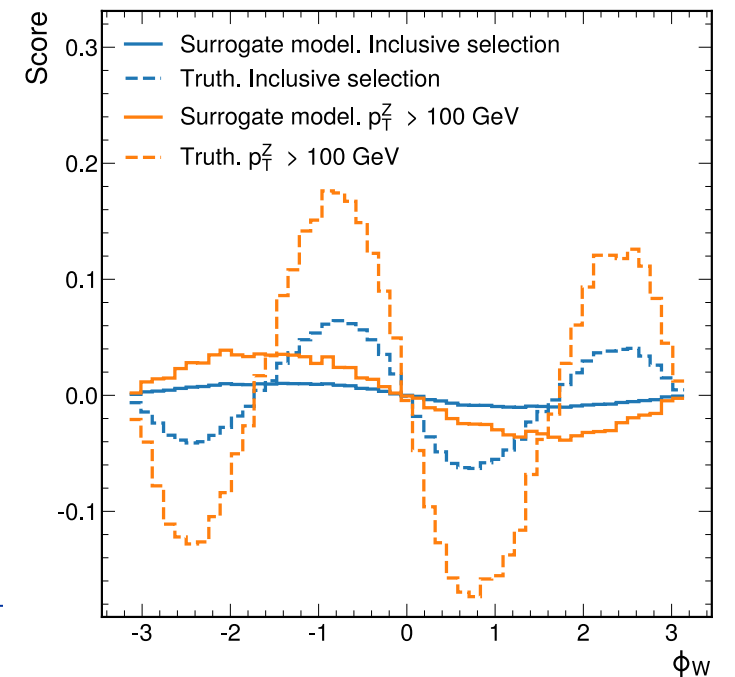
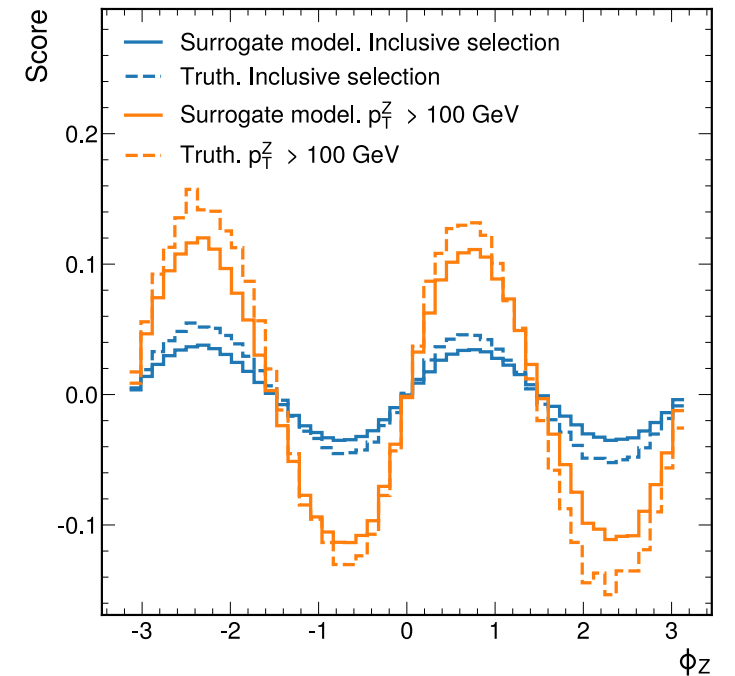


Use case: WZ production (II)

- Checking if the model has learned dedicated observables based on spin correlation
- Modulation introduced in ϕ_z is well captured
- Mostly insensitive to ϕ_w due to ambiguity in the W decay reconstruction
- **More sensitive than dedicated observables, can capture energy growth**

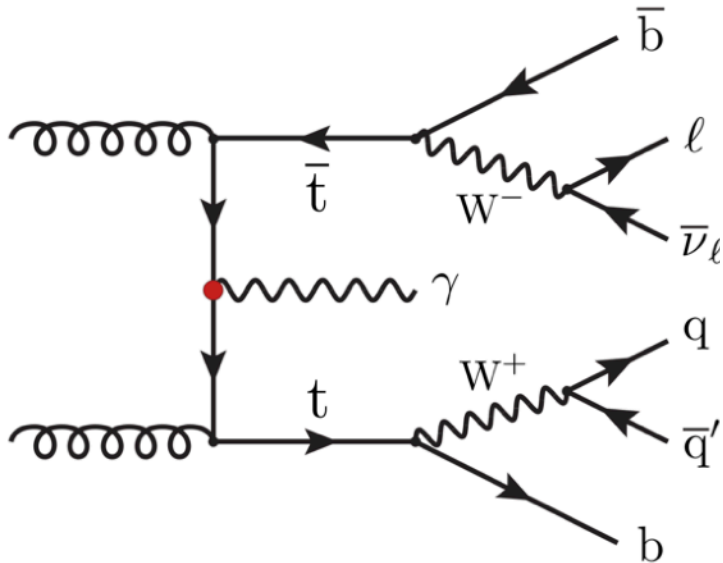


[arXiv:1708.07823](https://arxiv.org/abs/1708.07823)



Use case: $t\bar{t}\gamma$ production

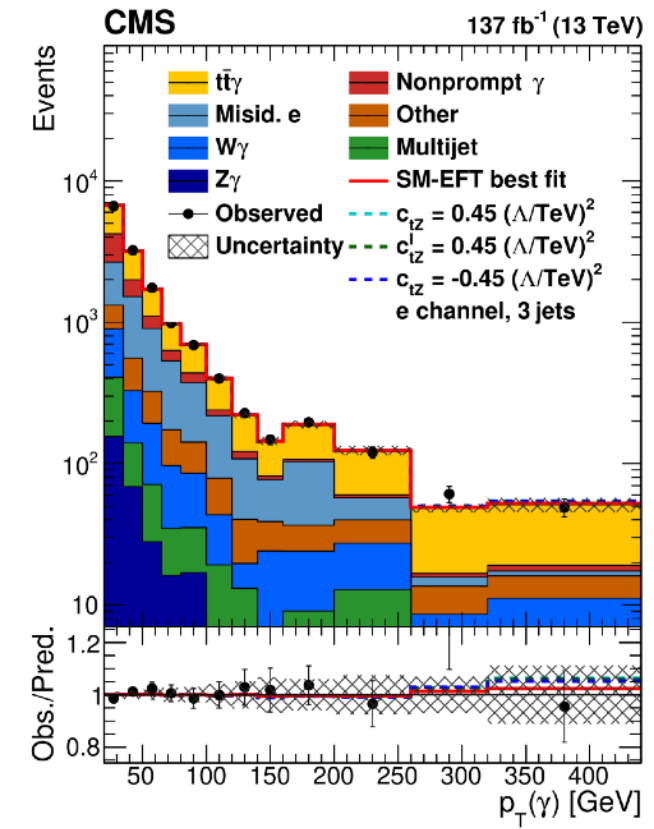
- Checking $t\bar{t}\gamma$ in the single lepton channel, affecting c_{tZ}^I
- Operator related to $(\bar{Q}\sigma^{\mu\nu}t)\tilde{H}B_{\mu\nu}$
- Often looked for using the p_T of the photon
 - Not a CP-violating observable



$$\vec{p}_\gamma, \vec{p}_\ell, Q_\ell, \vec{p}_{b_1}, \vec{p}_{b_2}, \vec{p}_{j_1}, \vec{p}_{j_2}$$

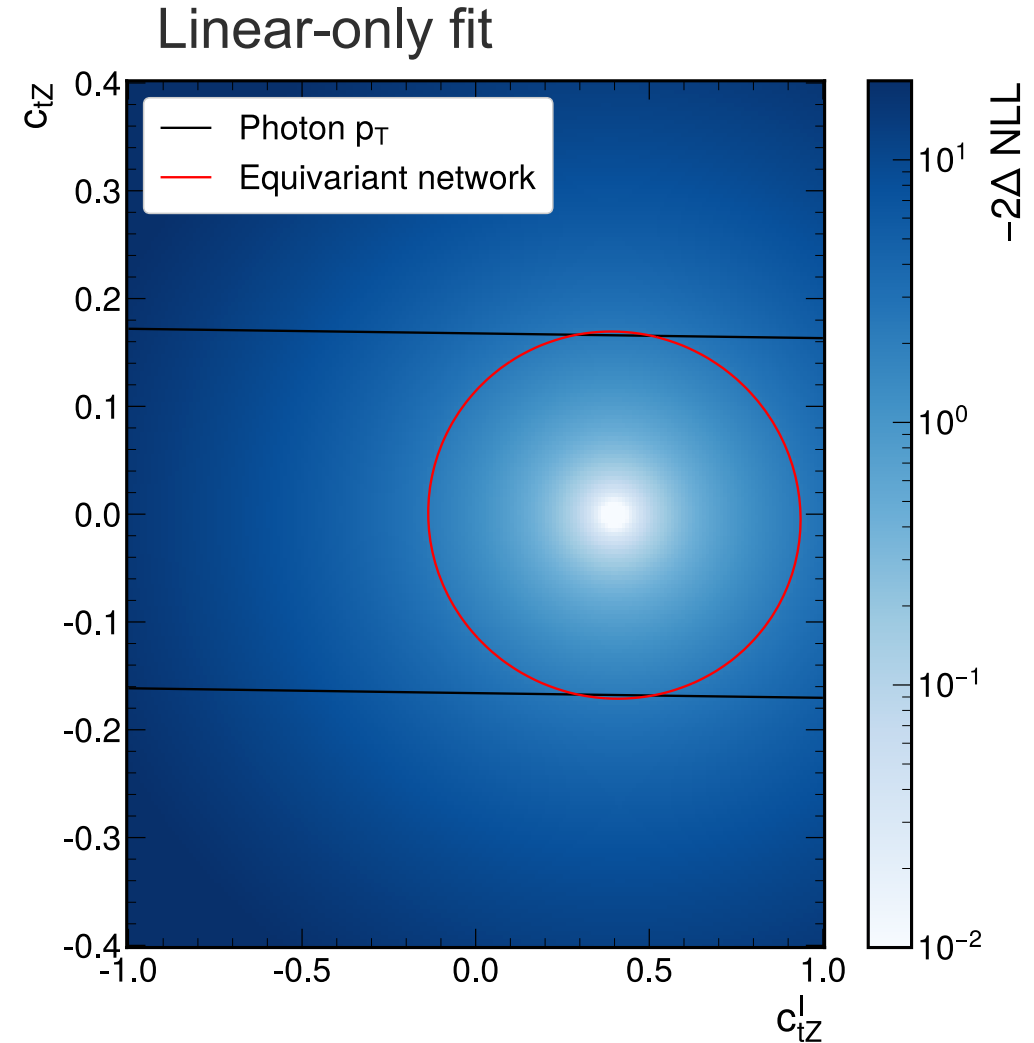


$$-\vec{p}_\gamma, -\vec{p}_\ell, -Q_\ell, -\vec{p}_{b_2}, -\vec{p}_{b_1}, -\vec{p}_{j_2}, -\vec{p}_{j_1}$$



Use case: $t\bar{t}$ production

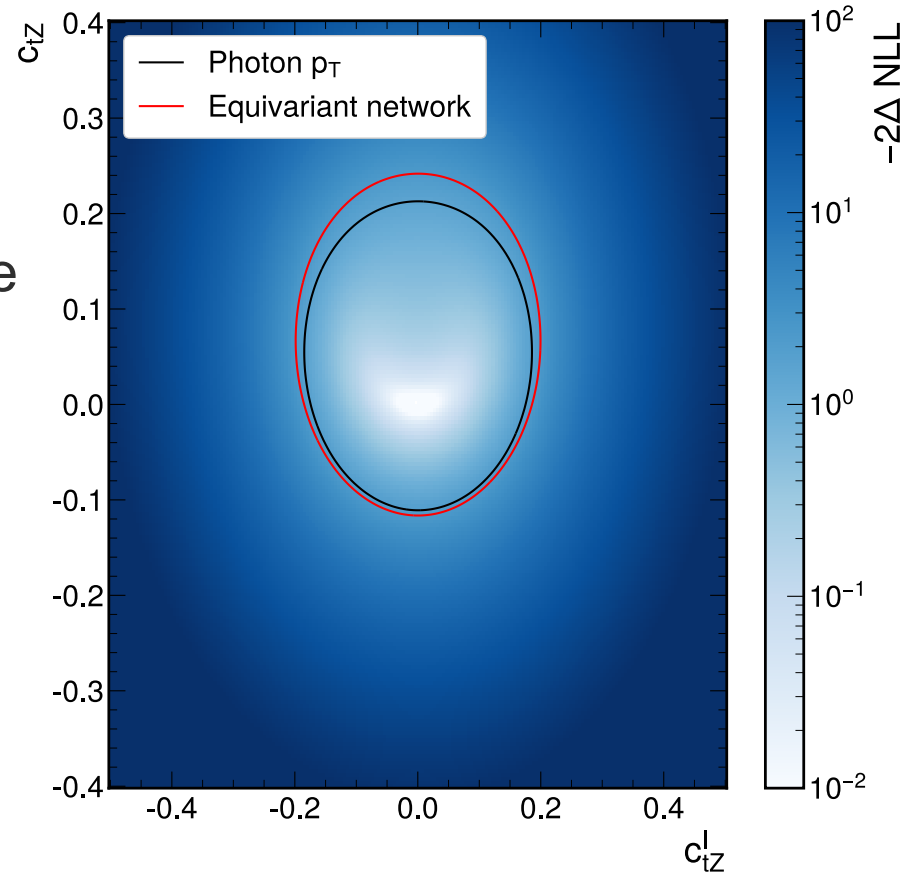
- Comparing counting analyses binning on the score of the equivariant network and photon p_T
- Setting expected limits on c_{tZ}^I and c_{tZ}
- Using Poissonian likelihood
 - **When considering CP-odd observables systematic uncertainties are suppressed**
- When considering linear contributions only, the **equivariant network provides sensitivity to c_{tZ}^I**
 - **Photon p_T is completely blind to this operator**
 - Both observables give similar sensitivity to c_{tZ}



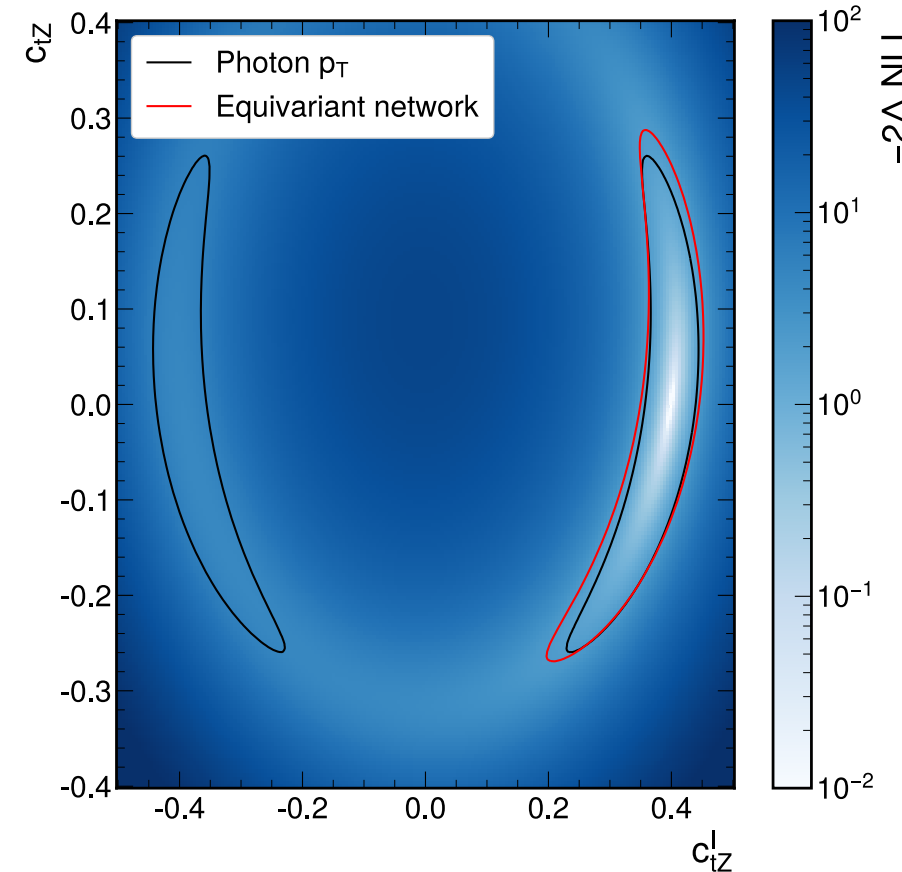
Use case: $t\bar{t}$ production

- Assuming the SM, both observables give similar sensitivity
 - Equivariant network targets the interference
- In BSM cases, the equivariant network can disentangle positive and negative c_{tZ}^I values

Linear+quadratic, true value (0,0)



Linear+quadratic, true value (0.4,0)



- Overall, the equivariant network is more powerful, even if it has not been trained to capture quadratic effects

Conclusions

- Showcased the properties of equivariant neural networks to search for CP-violation
- Using equivariance as an inductive bias, we obtain robust CP-odd observables
 - Robust observables, regardless of the convergence of the training
 - Better numerical convergence properties than non-equivariant algorithms
- Produced optimal CP-odd observables for $t\bar{t}$, WZ and $t\bar{t}\gamma$ production
 - Observables developed in WZ and $t\bar{t}\gamma$ improve the existing state-of-the-art observables
 - Highly relevant for analysis in the top, Higgs and electroweak sectors targeting CP violation, potential for improving any such analysis
- Possible trivial extensions
 - Many of the physics we look at are CP-even \rightarrow improving convergence by considering CP-invariant networks?