Matter effects in D-mixing

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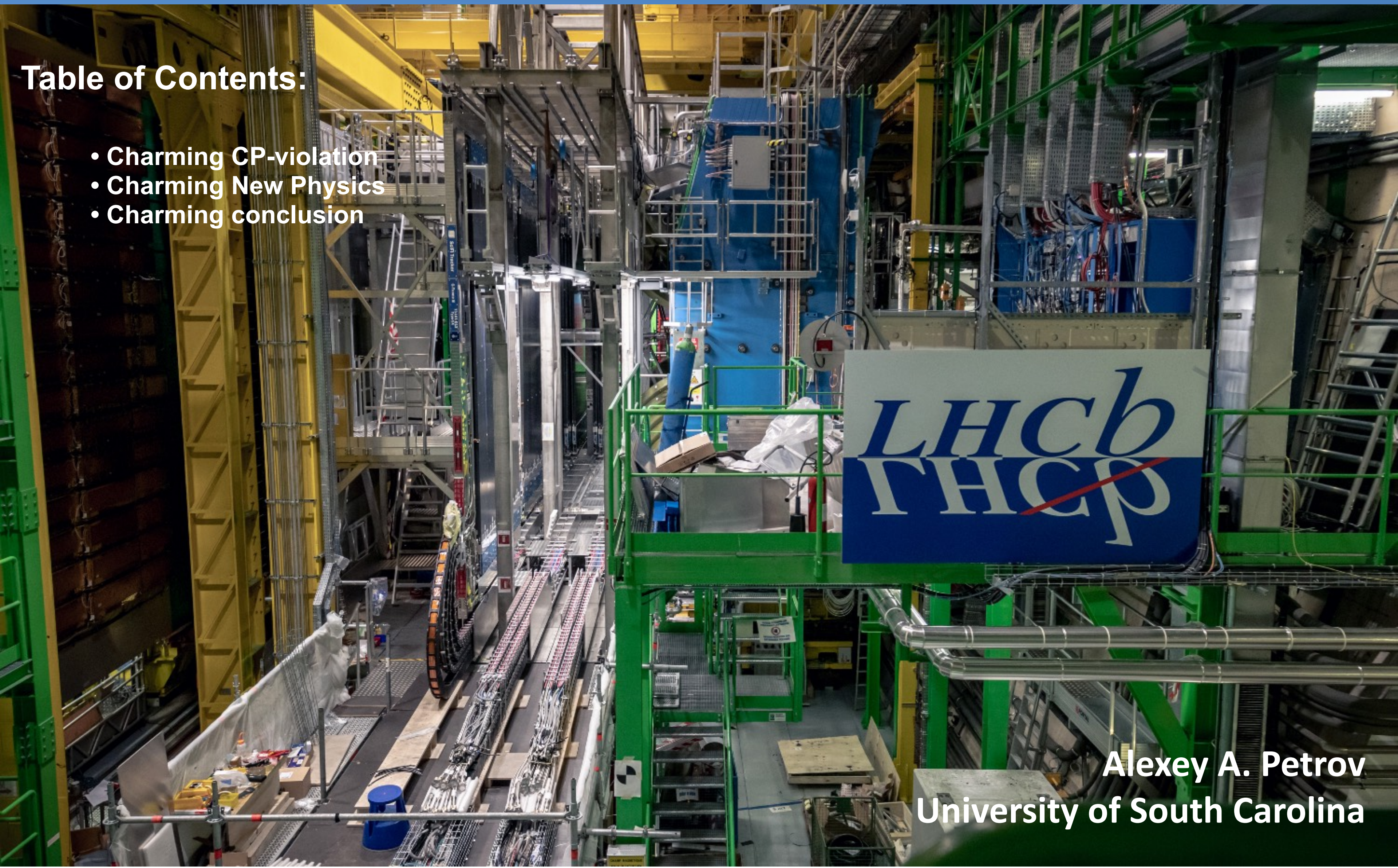

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Charming history

- Fifty years ago, Burt Richter and Sam Ting discovered J/ψ state in November of 1974: “November revolution”

The Arrival of Charm[1]

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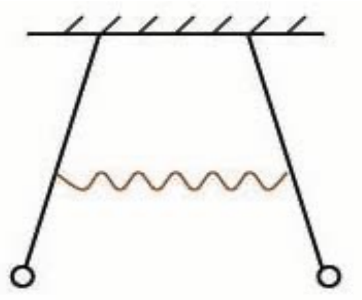
Abstract. Some of the theoretical motivations and experimental developments leading to the discovery of charm are recalled.

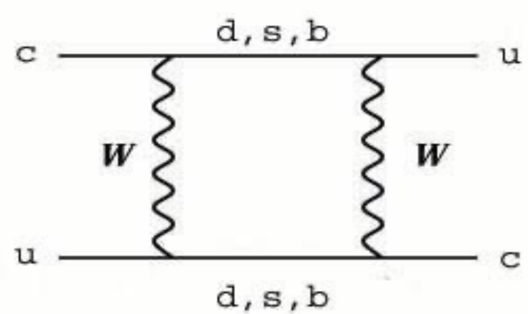
hep-ph/9811359

- At 50, charm quark continue to churn out surprises!

★ In the SM, neutral D-mesons can mix via weak interaction diagrams



Coupled oscillators



D-D mixing

* Not the dominant effect!

- only at one loop in the Standard Model, so can be sensitive to possible quantum effects due to new physics particles
- $\Delta c = 2$ interactions couple dynamics of D^0 and $\bar{D}^0$
- We need to study simultaneous time evolution,

$$|D(t)\rangle = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} = a(t)|D^0\rangle + b(t)|\overline{D}^0(t)\rangle$$

- This is very similar to the case of coupled pendula in classical mechanics

Time dependent decay amplitudes in vacuum

- Time dependence: coupled Schrodinger equations
  - note that CPT-invariance requires that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

$$i\frac{d}{dt}|D(t)\rangle = \left[M - i\frac{\Gamma}{2}\right]|D(t)\rangle \equiv \left[\begin{array}{cc} A & p^2 \\ q^2 & A \end{array}\right]|D(t)\rangle$$

Q: this Hamiltonian is clearly non-hermitian! What is goin on?

- Non-diagonal Hamiltonian: need to diagonalize the mass matrix

$$|D_L\rangle = p|D^0\rangle + q|\overline{D}^0\rangle$$
$$|D_H\rangle = p|D^0\rangle - q|\overline{D}^0\rangle$$

("switch from flavor to mass eigenstates")

- In the mass basis the mass matrix is diagonal, i.e.

$$Q^{-1}\left[M - i\frac{\Gamma}{2}\right]Q = \left(\begin{array}{cc} M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2 \end{array}\right)$$

- … with mass and lifetime differences: $\Delta M = M_H - M_L$ & $\Delta\Gamma = \Gamma_L - \Gamma_H$

Note that $m = \frac{M_H + M_L}{2} = M_{11} = M_{22}$ & $\Gamma = \frac{\Gamma_L + \Gamma_H}{2} = \Gamma_{11} = \Gamma_{22}$

Time dependent decay amplitudes in vacuum

- The transformation matrices that diagonalize the Hamiltonian are

$$Q = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad Q^{-1} = \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- To find the time development of the flavor eigenstates one needs to transform the evolution equation back to the flavor basis

$$\begin{bmatrix} |D^0(t)\rangle \\ |\overline{D}^0(t)\rangle \end{bmatrix} = Q \begin{pmatrix} e^{-iM_L - \Gamma_L/2} & 0 \\ 0 & e^{-iM_H - \Gamma_H/2} \end{pmatrix} Q^{-1} \begin{bmatrix} |D^0\rangle \\ |\overline{D}^0\rangle \end{bmatrix}$$

- … which gives for the time evolution matrix in the flavor basis

$$Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} = \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix}$$

Nierste

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \left[\cosh \frac{\Delta\Gamma\, t}{4} \cos \frac{\Delta M\, t}{2} - i \sinh \frac{\Delta\Gamma\, t}{4} \sin \frac{\Delta M\, t}{2} \right],$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2} \left[- \sinh \frac{\Delta\Gamma\, t}{4} \cos \frac{\Delta M\, t}{2} + i \cosh \frac{\Delta\Gamma\, t}{4} \sin \frac{\Delta M\, t}{2} \right].$$

Time dependent decay amplitudes in vacuum

- This procedure provides a picture of how B-states evolve due to flavor oscillations, $|D^0(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p} g_-(t)|\overline{D}^0\rangle$

$$|\overline{D}^0(t)\rangle = \frac{p}{q} g_-(t)|D^0\rangle + g_+(t)|\overline{D}^0\rangle$$

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \left[\cosh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right],$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2} \left[-\sinh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right].$$

- The only thing left is to relate q/p, ΔM and $\Delta\Gamma$ to original parameters of H

secular equation: $$(\Delta M + i\frac{\Delta\Gamma}{2})^2 = 4\left(M_{12} - i\frac{\Gamma_{12}}{2}\right)\left(M_{12}^* - i\frac{\Gamma_{12}^*}{2}\right)$$

Re

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

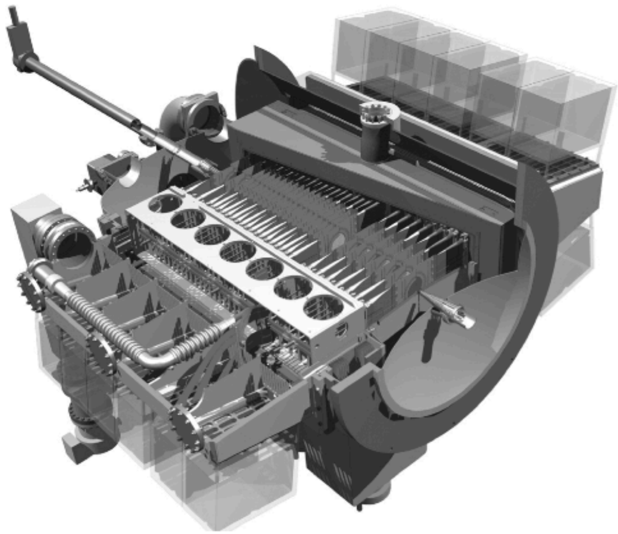
Im

$$\Delta M\, \Delta\Gamma = -4\,\mathrm{Re}\,(M_{12}\Gamma_{12}^*)$$

For $\Gamma_{ik} = 0$: $\Delta M = 2|M_{12}| \equiv 2\epsilon$

- Finally, the ratio $$\frac{q}{p} = -\frac{\Delta M + i\,\Delta\Gamma/2}{2M_{12} - i\,\Gamma_{12}} = -\frac{2M_{12}^* - i\,\Gamma_{12}^*}{\Delta M + i\,\Delta\Gamma/2}$$

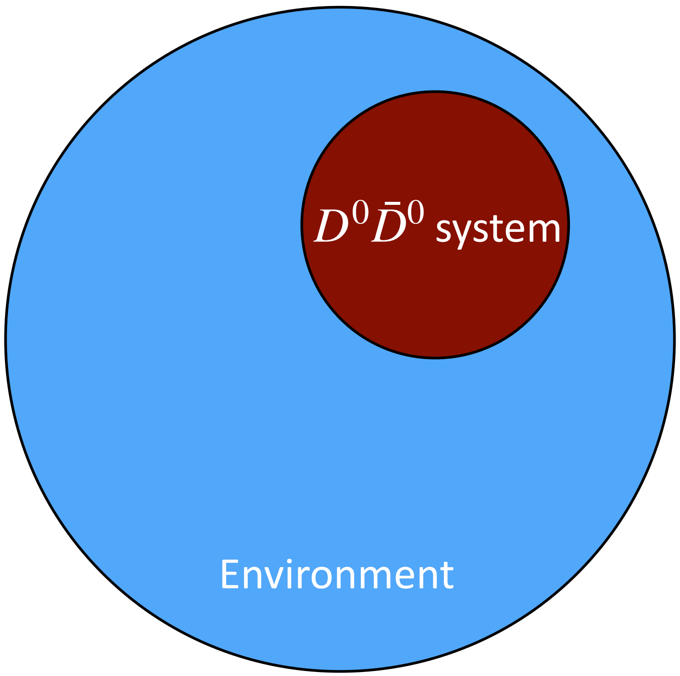
Charm in matter at the LHCb



arXiv:1209.4845

- Explore the proximity of the VELO detector's RF-folis to the interaction
  - some D's traverse the foils
  - can be used as a material target
- Perform experimental study of D-mesons propagating in a material
  - new effects associated with material interactions
  - general formalism

- Matter effects: $D^0\bar{D}^0$ system interacts with the environment (atoms, etc.)
    - Proper quantum-mechanical fomalism: density matrix



$$|D(t)\rangle = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} = a(t)|D^0\rangle + b(t)|\overline{D}^0(t)\rangle$$

with the corresponding density matrix

$$\rho_{\text{pure}} = |D\rangle\langle D| = aa^*|D^0\rangle\langle D^0| + ab^*|D^0\rangle\langle\bar{D}^0| \\ ba^*|\bar{D}^0\rangle\langle D^0| + bb^*|\bar{D}^0\rangle\langle\bar{D}^0|$$

or, in the matrix form,

$$\rho_{\text{pure}}(t) = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} aa^* & ab^* \\ ba^* & bb^* \end{pmatrix}$$

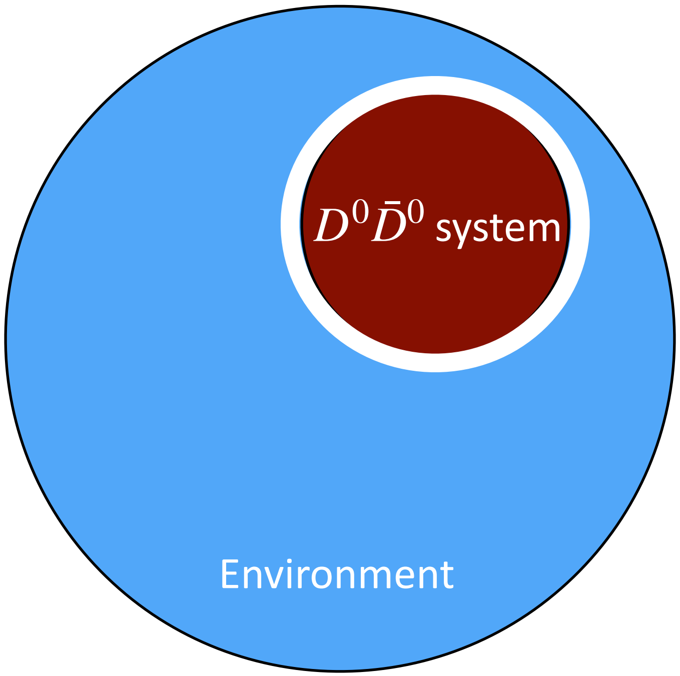
- Quantum-mechanical evolution: $i\dfrac{\partial\hat{\rho}}{\partial t} = \left[\hat{H}, \hat{\rho}\right]$ Neumann-Liouville equation (unitary evolution, Hermitian Hamiltonian)

Density matrix: evolution of pure states

- Evolution equation for unitary evolution (neglect effects of the environment)

i.e., set $\Gamma_D = 0$ (stable D-mesons) and make sure that $H = H^\dagger$

$$i\frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}, \hat{\rho}\right] \quad \text{with} \quad H = \begin{pmatrix} M & \epsilon \\ \epsilon & M \end{pmatrix}$$



- Recast the evolution equation in the Bloch form

rewrite the density matrix as

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2}\left(1 + \vec{R}\cdot\vec{\sigma}\right)$$

Kerbikov

$$\text{with} \quad \vec{R} = \begin{pmatrix} \rho_{12} + \rho_{21} \\ i\left(\rho_{12} - \rho_{21}\right) \\ \rho_{11} - \rho_{22} \end{pmatrix}$$

- Bloch evolution equation: $\frac{\partial \vec{R}}{\partial t} = \vec{X} \times \vec{R}$ with $\vec{X} = \begin{pmatrix} 2\epsilon \\ 0 \\ 0 \end{pmatrix}$

precession of R around X

Density matrix: evolution of pure states

- The Bloch equation for the vacuum evolution

$$\frac{\partial \vec{R}}{\partial t} = \vec{X} \times \vec{R} \quad \Longrightarrow \quad \begin{cases} \dot{R}_1 = 0 \\ \dot{R}_2 = -2\epsilon R_3 \\ \dot{R}_3 = 2\epsilon R_2 \end{cases}$$

- Ex., solution for the $R_3 = \rho_{11} - \rho_{22}$, starting with $\overrightarrow{R} = (0,0,1)$

$$R_2 = R_2(0)\cos(\Lambda t) - \frac{2\epsilon}{\Lambda}\sin(\Lambda t)$$

which implies for the R_3

$$R_3(t) = 1 - \frac{8\epsilon^2}{\Lambda^2}\sin^2\left(\frac{\Lambda t}{2}\right)$$

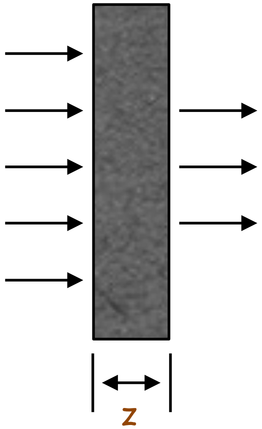
with $\Lambda^2 = 4\epsilon^2$

- Since the state is pure, $Tr(\hat{\rho}) = \rho_{11} + \rho_{22} = 1$

$$\rho_{22} = |b\rangle\langle b| = \frac{1}{2}\left(1 - R_3(t)\right) = \sin^2(\epsilon t)$$

recall that $\epsilon = \Delta M/2$

- Coherent scattering of D-mesons in the forward direction: index of refraction



$$n = 1 + \frac{2\pi N}{k^2} f(0)$$

with N is the number of scatterers per unit volume and k is the wave number

the forward scattering amplitude $f(0)$ is related to the total cross section

$$\sigma_{\rm tot} = \frac{4\pi}{k} Im f(0)$$

(optical theorem)

- A particle traversing a slab of material picks up a phase

$$\phi = k\,(n-1)\,z$$

since D^0 and $\bar{D}^0$ scatter differently in matter: effect!

- Scattering in matter also leads to the loss of quantum coherence

this happens due to entanglement with the medium

- Probability depletion due to decays of D^0 and $\bar{D}^0$ mesons

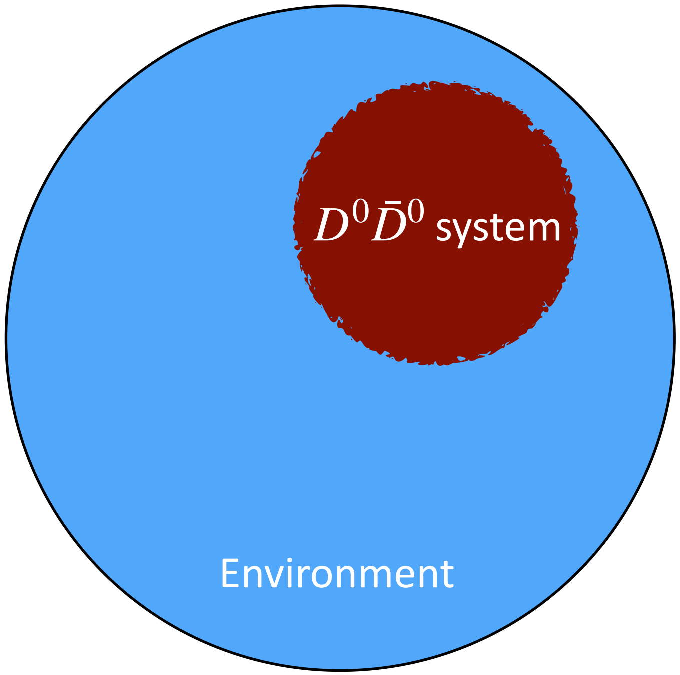
density matrix evolution is no longer described by the Neumann-Liouville equation

Density matrix: general evolution

- Evolution equation for “general” evolution: Lindblad equation

assuming markovian process and enthropy increase…

Benatti, Floreanini



$$\frac{\partial \rho}{\partial t} = -i\left[\hat{H},\hat{\rho}\right] + \sum_n \left(\hat{L}_n \hat{\rho} \hat{L}_n^\dagger - \frac{1}{2}\left\{\hat{L}_n^\dagger \hat{L}_n, \hat{\rho}\right\}\right)$$

where $\hat{L}_n$ are the quantum jump operators

- Move the decay effects into the Hamiltonian

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left(\hat{H}\hat{\rho} - \hat{\rho}\hat{H}^\dagger\right) + \hat{L}\hat{\rho}\hat{L}^\dagger - \frac{1}{2}\left\{\hat{L}^\dagger \hat{L}, \hat{\rho}\right\}$$

where $L = \sqrt{Nv}\begin{pmatrix} f(\theta) & 0 \\ 0 & \bar{f}(\theta) \end{pmatrix}$ Feinberg, Weinberg

and $H = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} - \frac{2\pi N}{k}\frac{v}{\sqrt{1-v^2}} Re f(0) & \epsilon \\ \epsilon & M_{22} - \frac{i}{2}\Gamma_{22} - \frac{2\pi N}{k}\frac{v}{\sqrt{1-v^2}} Re \bar{f}(0) \end{pmatrix}$

where the terms with Re f(0) corrsponds to the energy shift due to forward scattering

Density matrix: general evolution

- Evolution equation for "general" evolution: Lindblad equation with non-hermitean Hamiltonian

$$H = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} - \frac{2\pi N}{k}\frac{v}{\sqrt{1-v^2}} Re f(0) & \epsilon \\ \epsilon & M_{22} - \frac{i}{2}\Gamma_{22} - \frac{2\pi N}{k}\frac{v}{\sqrt{1-v^2}} Re \bar{f}(0) \end{pmatrix}$$

Expect two main effects: (1) oscillation suppression due to the level splitting
(2) decoherence: off-diagonal elements of the density matrix are damped

- The Lindblad equation for the components of the density matrix — Kerbikov

$$\dot{\rho}_{11} = i\epsilon(\rho_{12} - \rho_{21}) - \Gamma\rho_{11}$$
$$\dot{\rho}_{12} = i\epsilon(\rho_{11} - \rho_{22}) - (M + iK)\,\rho_{12}$$
$$\dot{\rho}_{21} = -i\epsilon(\rho_{11} - \rho_{22}) - (M - iK)\,\rho_{21}$$
$$\dot{\rho}_{22} = -i\epsilon(\rho_{12} - \rho_{21}) - \Gamma\rho_{22}$$

where

$$K = -\frac{2\pi N v}{k}\left[Re\left(f(0) - \bar{f}(0)\right) - 2k\ Im\left(f\bar{f}^*\right)\right]$$
$$M = \frac{2\pi N v}{k}\left[Im\left(f(0) + \bar{f}(0)\right) - 2k\ Re\left(f\bar{f}^*\right)\right]$$

Density matrix: general evolution

- The Bloch evolution equation $\frac{\partial \vec{R}}{\partial t} = \vec{X} \times \vec{R}$ must now be modified

Harris, Stodolsky

$$\frac{\partial \vec{R}}{\partial t} = \vec{X} \times \vec{R} - D_T \vec{R}_T - \Gamma \vec{R}$$

"precession" "damping" "decay"

with $\vec{X} = \begin{pmatrix} 2\epsilon \\ 0 \\ K \end{pmatrix}$, $D_T = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}$, and $\vec{R}_T = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$

the rate of decoherence

- General solution is hard: approximations? Unaccounted effects?

(1) include lifetime differences

(2) include modifications of ϵ in matter

Stay Tuned!

Things to take home

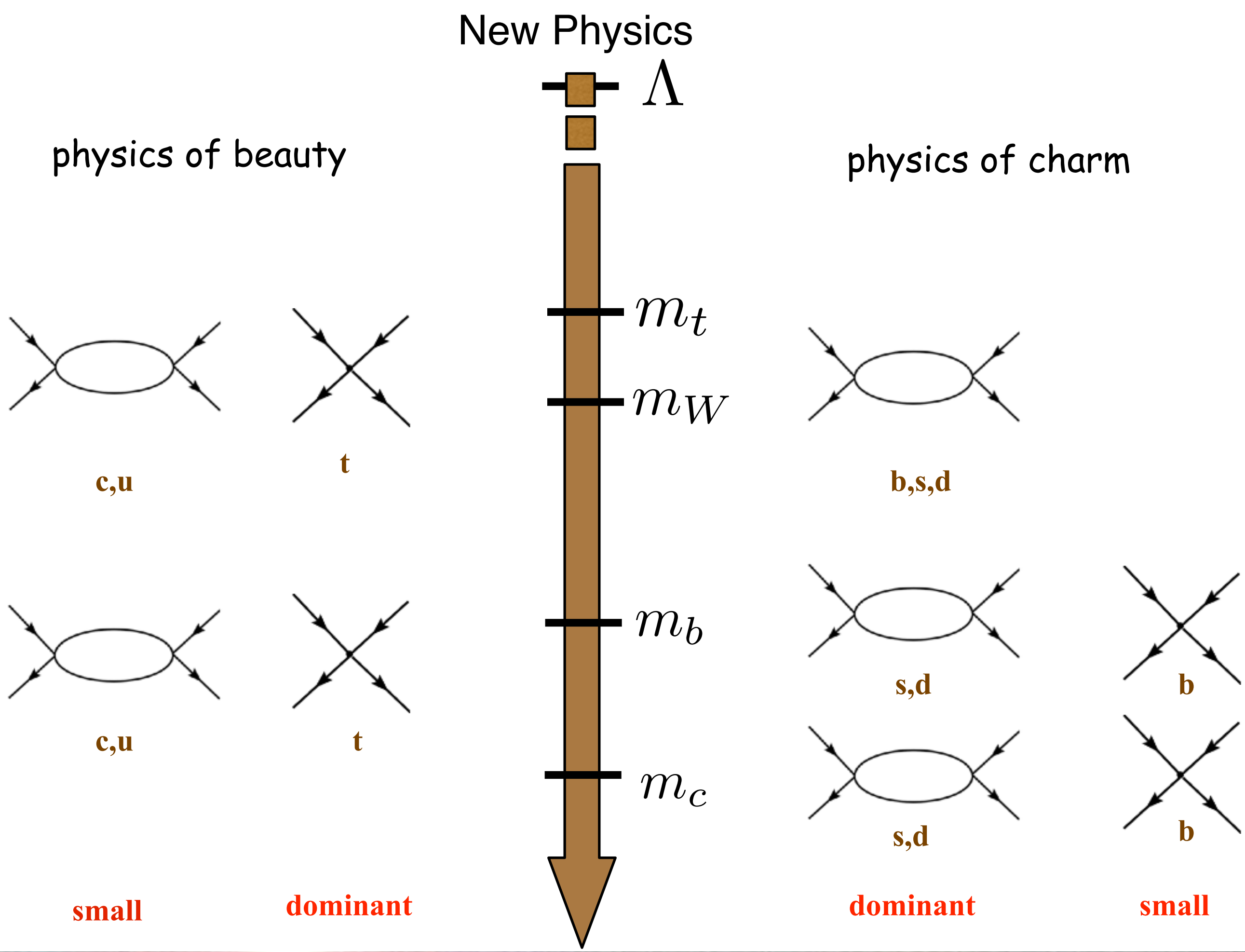
- Features of the VELO detector allow studies of matter effects on D-mesons
  - possibility to study matter effects outside of the kaon system
  - probe different physics scenarios/quantum mechanical effects
- Generic computation of matter effects is a difficult task
  - some approximations are required: how good?
  - better inputs on matter properties
  - with better input numerical solution is also possible
  - general parameterizations?
- Many parameters still need to be fixed experimentally or computed

"Charm physics"
Eur. Phys. J. ST 233 (2024) 2, 439-456


EXTRA!

★ Main goal of the exercise: understand physics at the most fundamental scale

★ It is important to understand relevant energy scales for the problem at hand



★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start, mass and lifetime differences of mass eigenstates...

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \; y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

★ ...can be calculated as real and imaginary parts of a correlation function

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} {\rm Im}\, \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\Big\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\Big\}|D^0\rangle$$

bi-local time-ordered product

$$x_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} {\rm Re}\, \left[2\langle \overline{D^0}|H^{|\Delta C|=2}\,|D^0\rangle + \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\Big\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\Big\}|D^0\rangle\right]$$

local operator (b-quark, NP): small?

bi-local time-ordered product

★ ... or can be written in terms of hadronic degrees of freedom...

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$